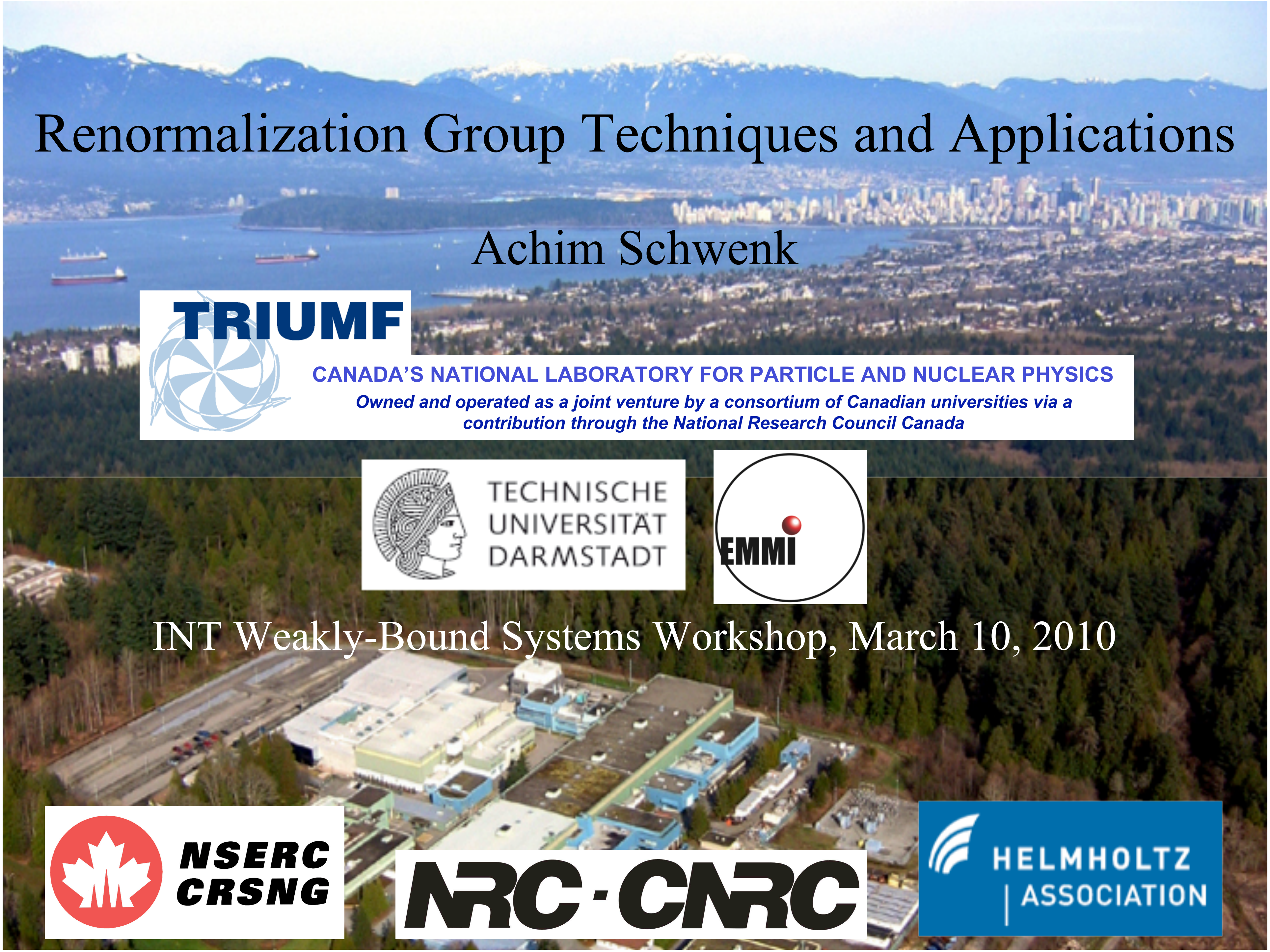

Renormalization Group Techniques and Applications

Achim Schwenk

TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

Owned and operated as a joint venture by a consortium of Canadian universities via a contribution through the National Research Council Canada

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

INT Weakly-Bound Systems Workshop, March 10, 2010

NSERC CRSNG

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HELMHOLTZ | ASSOCIATION

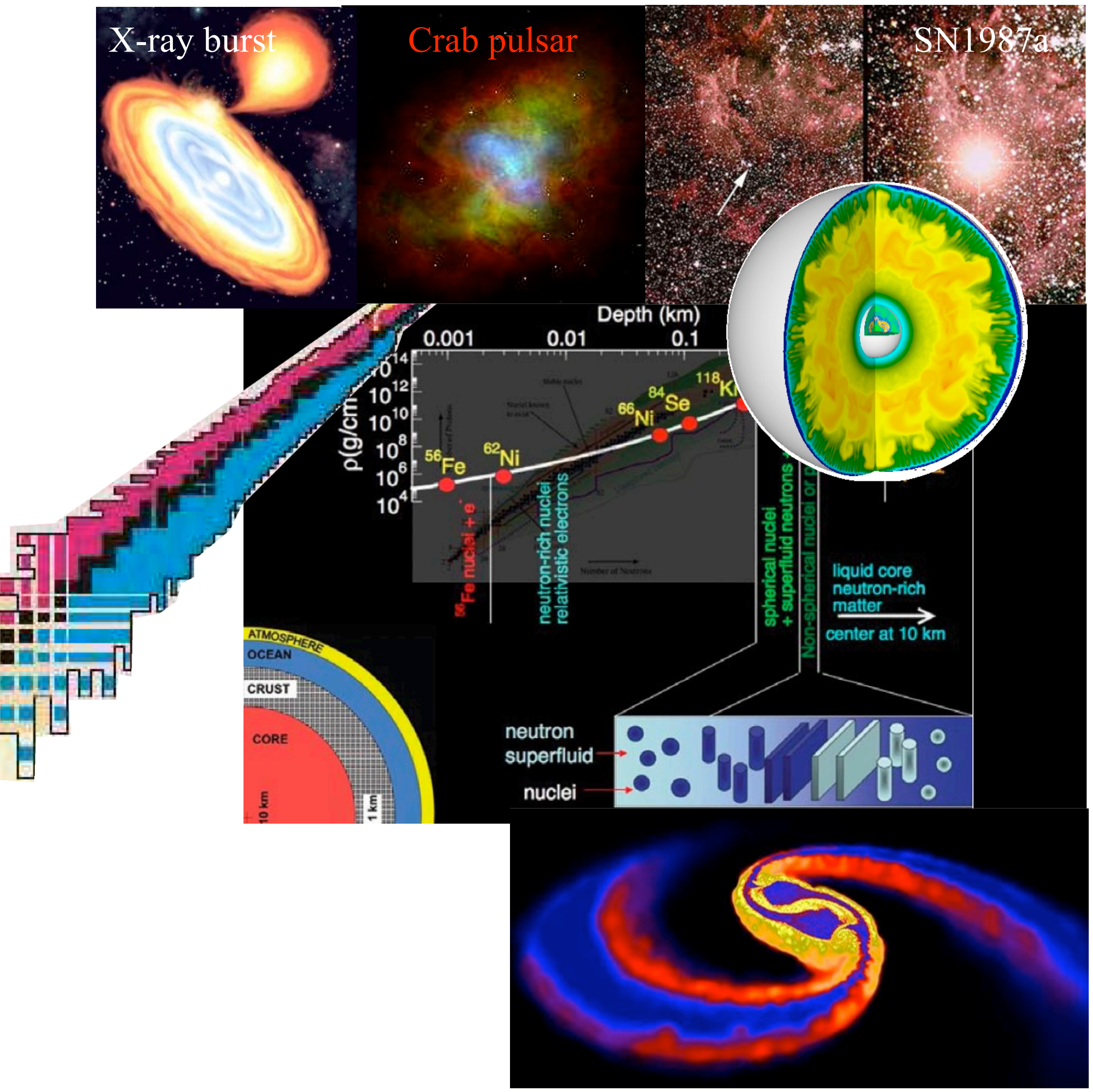
Strong interaction physics in the lab and cosmos

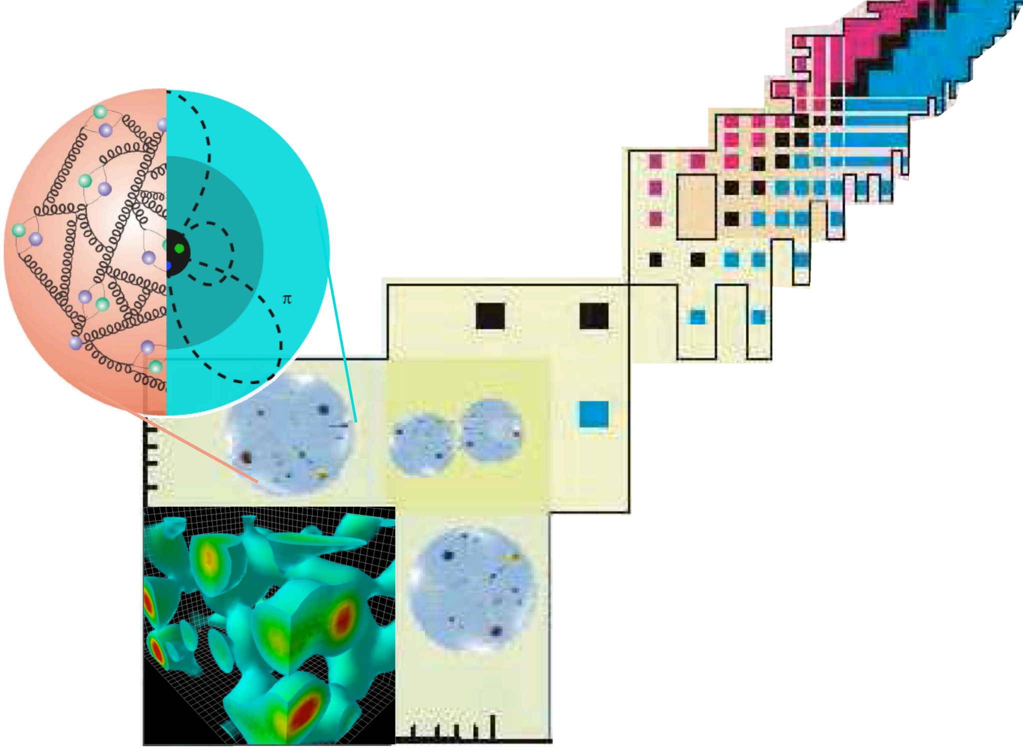
Matter at the extremes:

density ρ ~…10^{15} g/cm^3

proton-rich, neutron-rich,
^{8}He to Z/N ~0.05

temperatures T ~…100 MeV





Outline

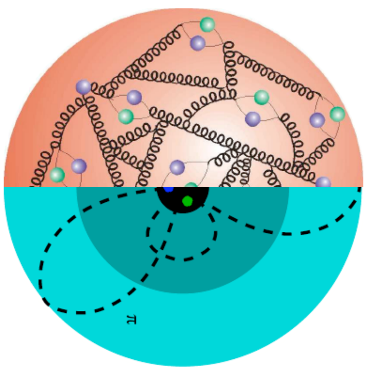
Effective field theory and renormalization group
for nuclear forces

Applications to weakly-bound and neutron-rich nuclei

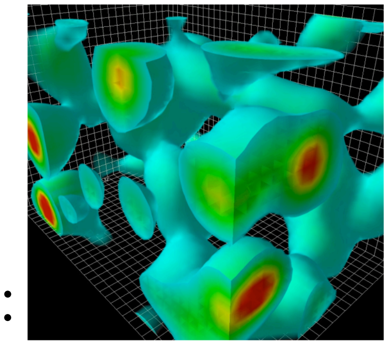
Similarity renormalization group for nuclei

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons



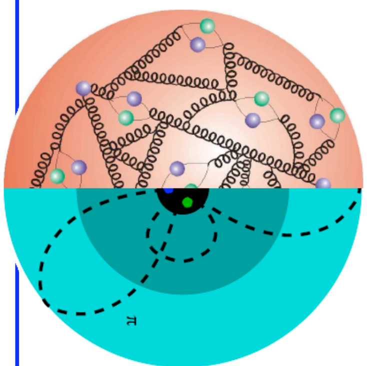
at low energies:
complex QCD vacuum



lowest energy excitations:
pions, nearly massless, m_{π}=140 MeV
"phonons" of QCD vacuum

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons



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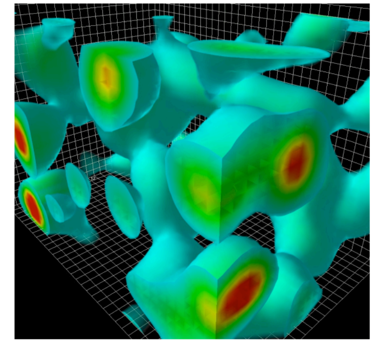

lowest energy excitations:
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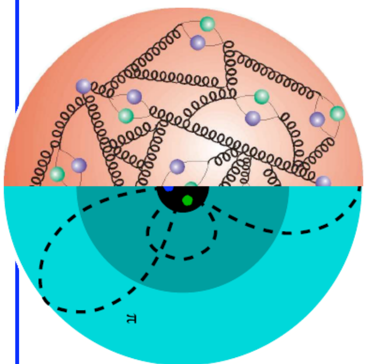
"phonons" of QCD vacuum

Λ_{chiral}
momenta $Q \sim \lambda^{-1} \sim m_\pi$

$\Lambda_{pionless}$
$Q \ll m_\pi$

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons



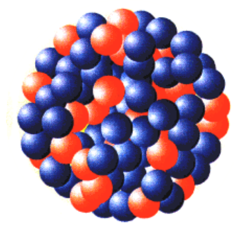
Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/Λ-dependent

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

$\Lambda_{\rm chiral}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory (EFT)

neutrons and protons interacting via pion exchanges
and shorter-range contact interactions



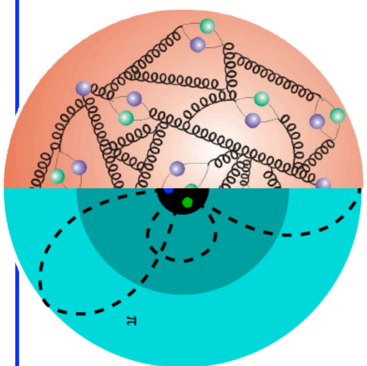
typical momenta in nuclei $\sim m_\pi$

$\Lambda_{\rm pionless}$

$Q << m_\pi$

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons



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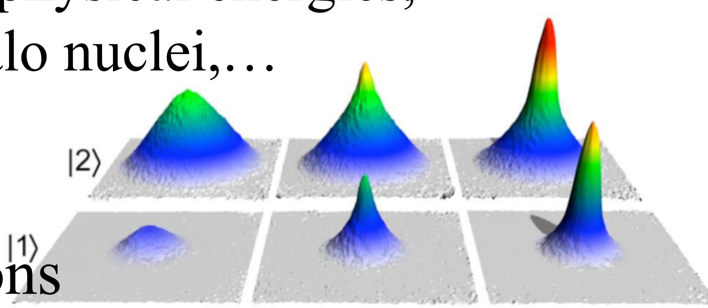
$\Lambda_{\rm chiral}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$

universal properties of neutrons and cold atoms,
reactions at astrophysical energies,
loosely-bound halo nuclei,…



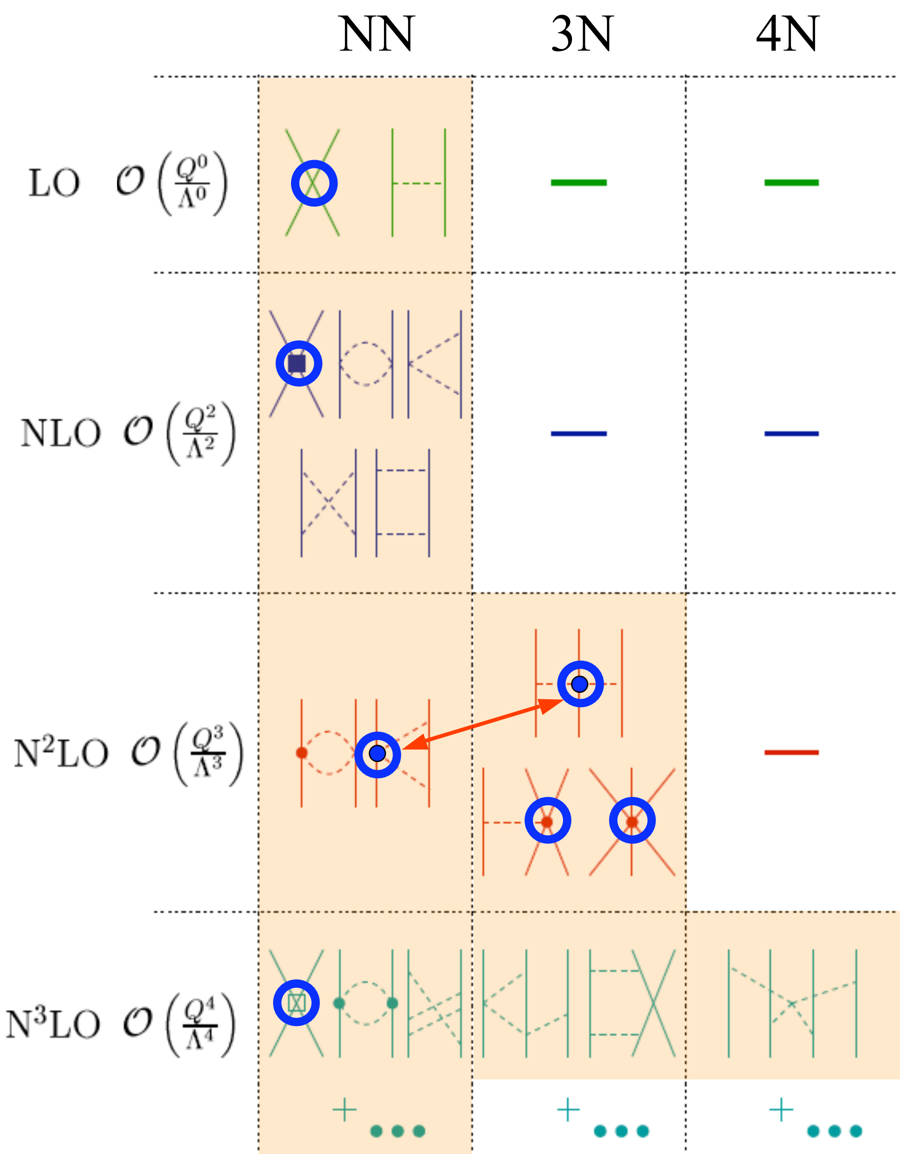
$\Lambda_{\rm pionless}$

$Q \ll m_\pi$: pionless effective field theory

large scattering length physics and corrections

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



limited resolution at low energies, can expand in powers Q/Λ_b

include long-range pion physics

details at short-distance not resolved

capture in few short-range couplings, fit to experiment once, Λ-dependent

systematic: can work to desired accuracy and obtain error estimates from truncation order and Λ variation

several open problems

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner,…

Chiral EFT for nuclear forces

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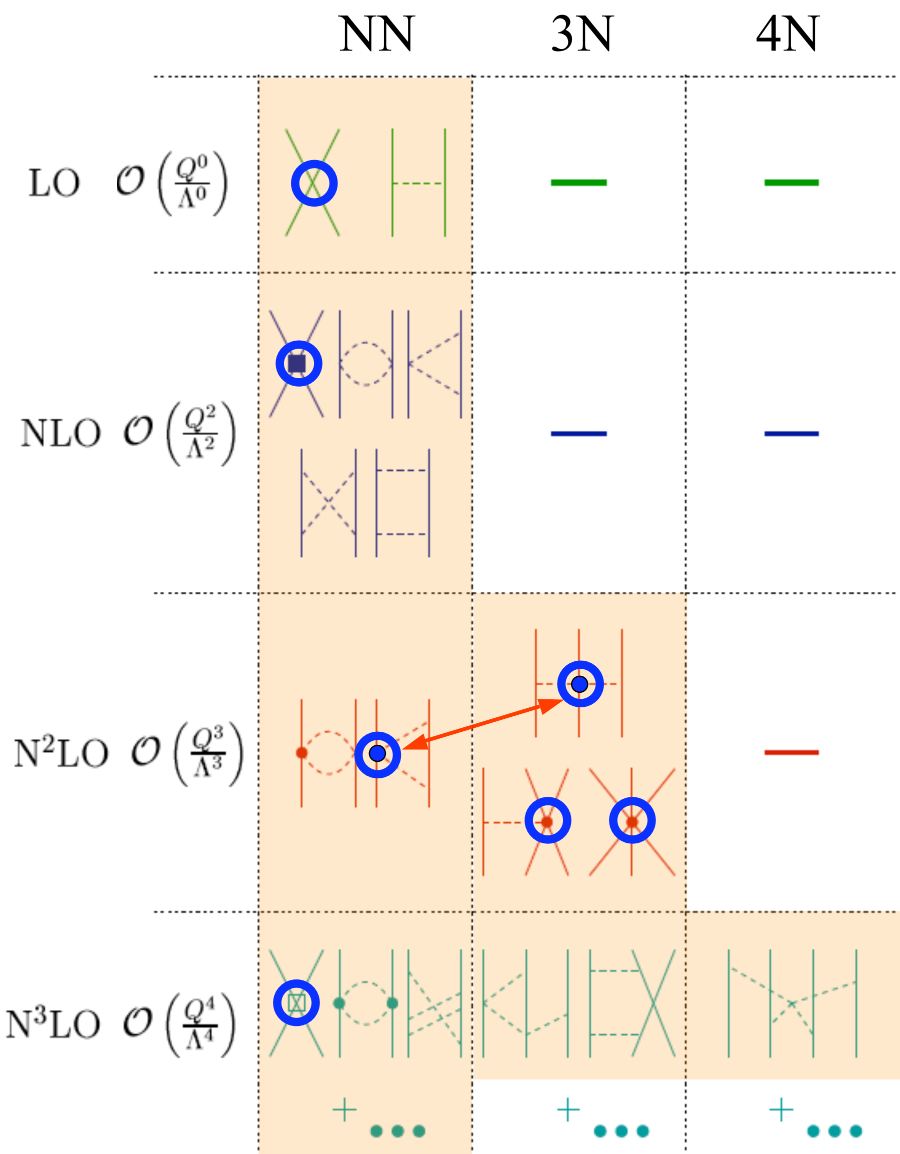
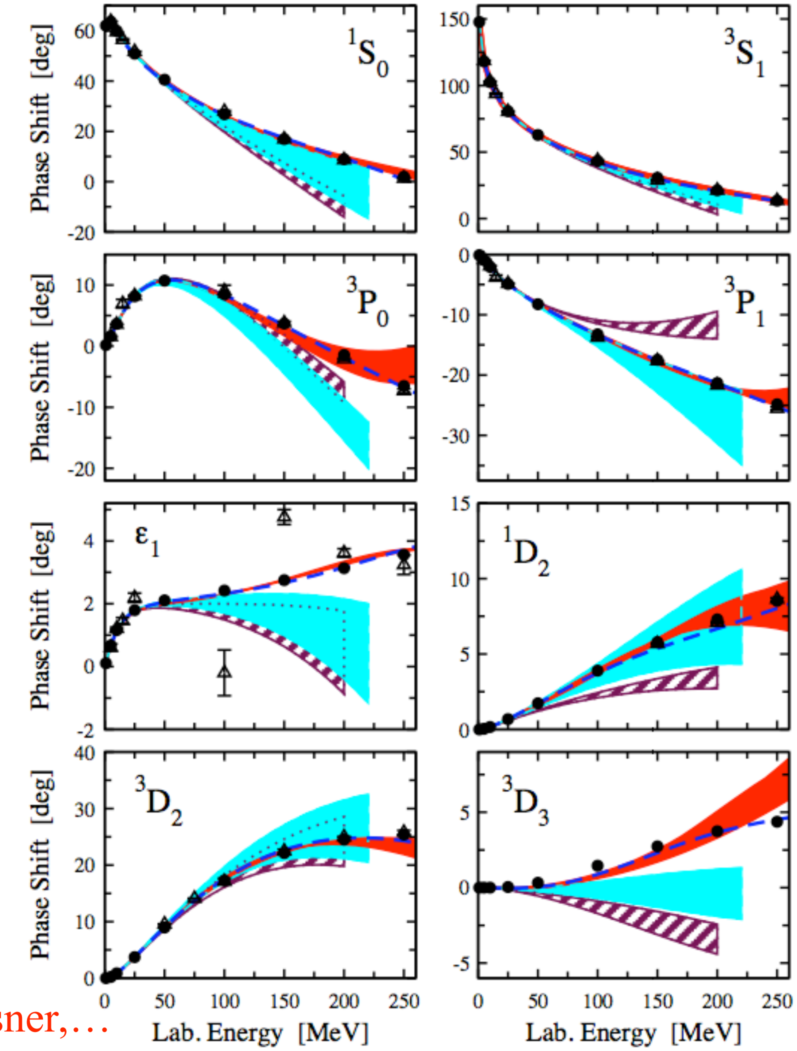
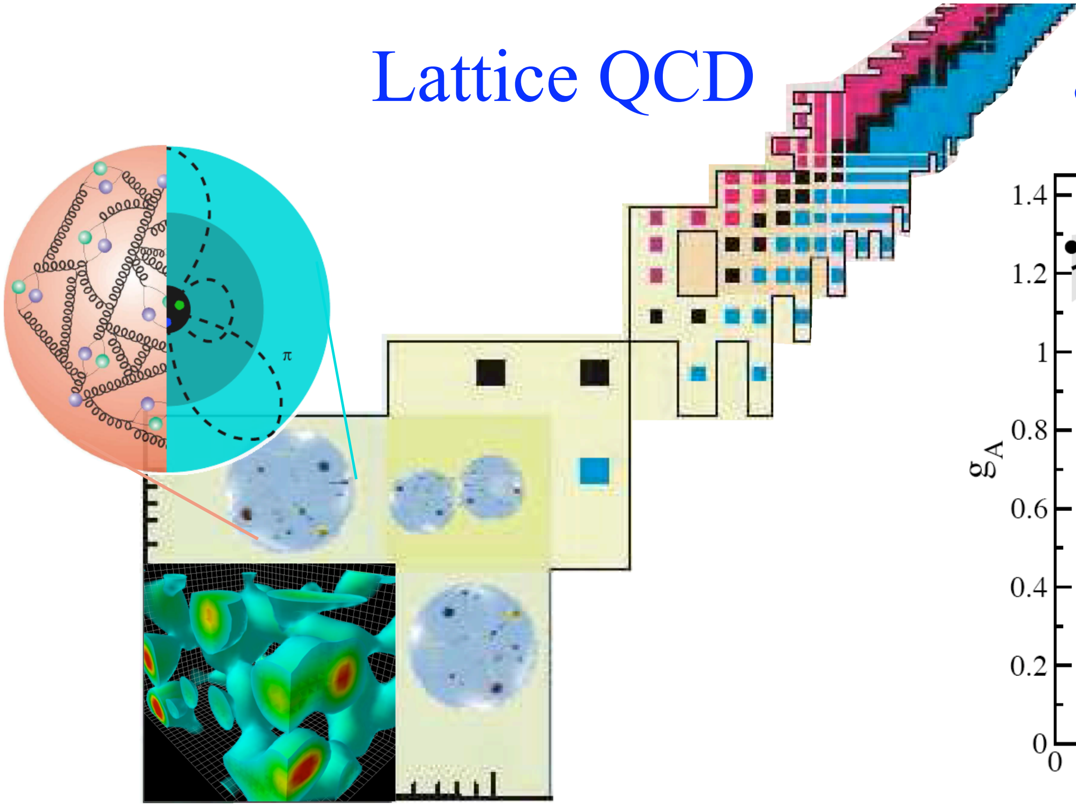


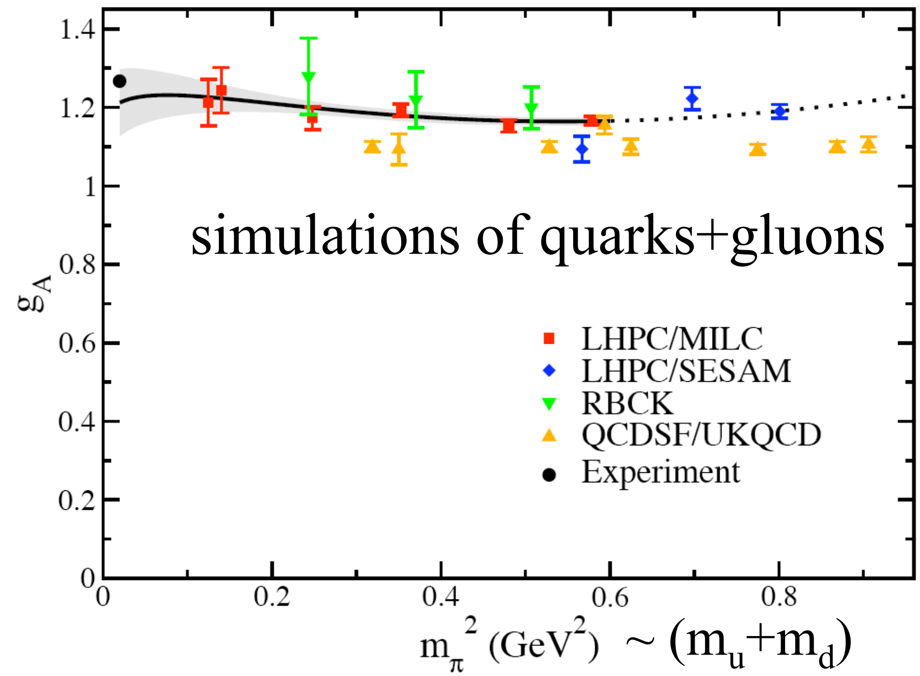
accurate reproduction of
low-energy NN scattering at N^{3}LO



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner,…

Lattice QCD and nuclear forces





First, exciting efforts to connect nuclear forces to underlying QCD

Long-range couplings: pion-nucleon coupling g_a Edwards et al. (2006)

chiral EFT extrapolation to physical pion mass agrees with experiment

variation of nuclear forces with quark masses Beane et al. (2006)

Future possibility: access/constrain 3-neutron forces (3n exp difficult)

Nuclear forces and the Renormalization Group

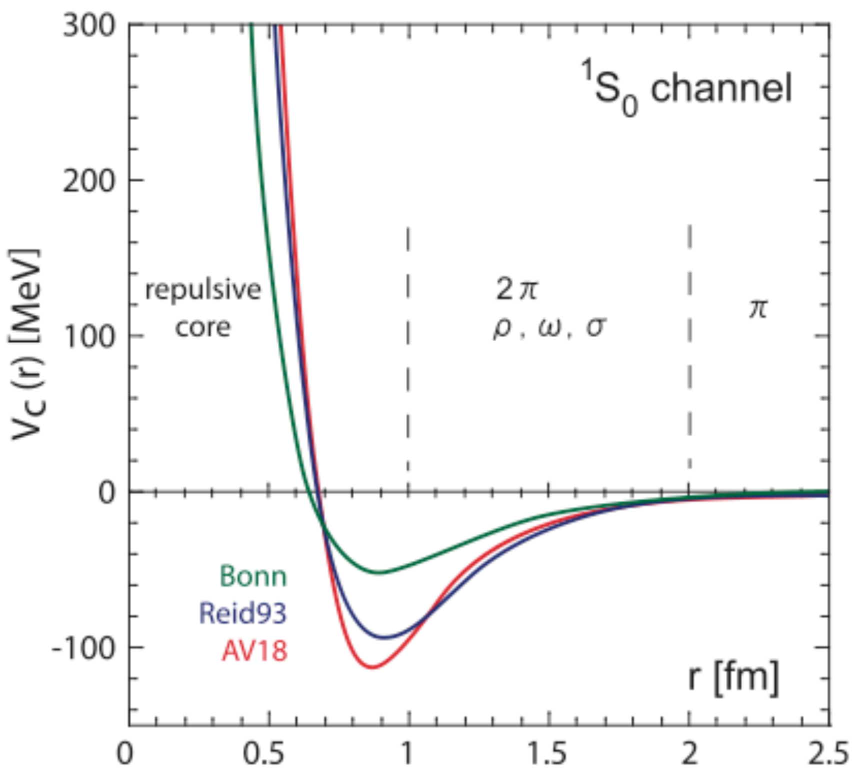
RG evolution to lower resolution/cutoffs

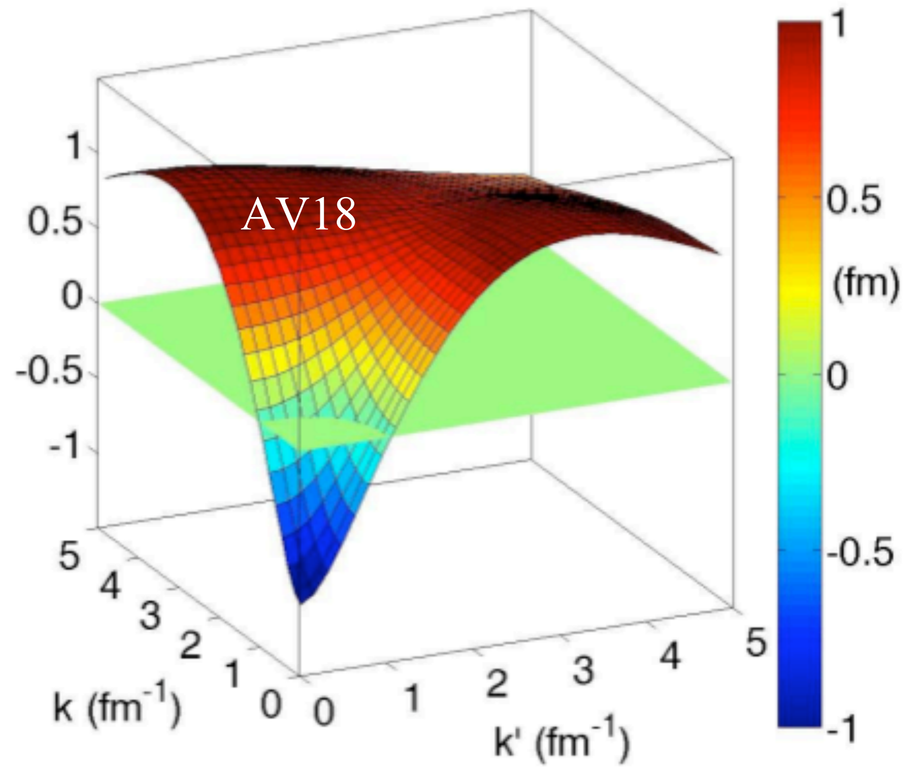
$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

exact RG for NN interactions

Bogner, Kuo, AS, Furnstahl, …

$$\frac{d}{d\Lambda} V_{{\rm low}\,k}^{\Lambda}(k',k) = \frac{2}{\pi} \frac{V_{{\rm low}\,k}^{\Lambda}(k',\Lambda) T^{\Lambda}(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$$





red = short-range repulsion

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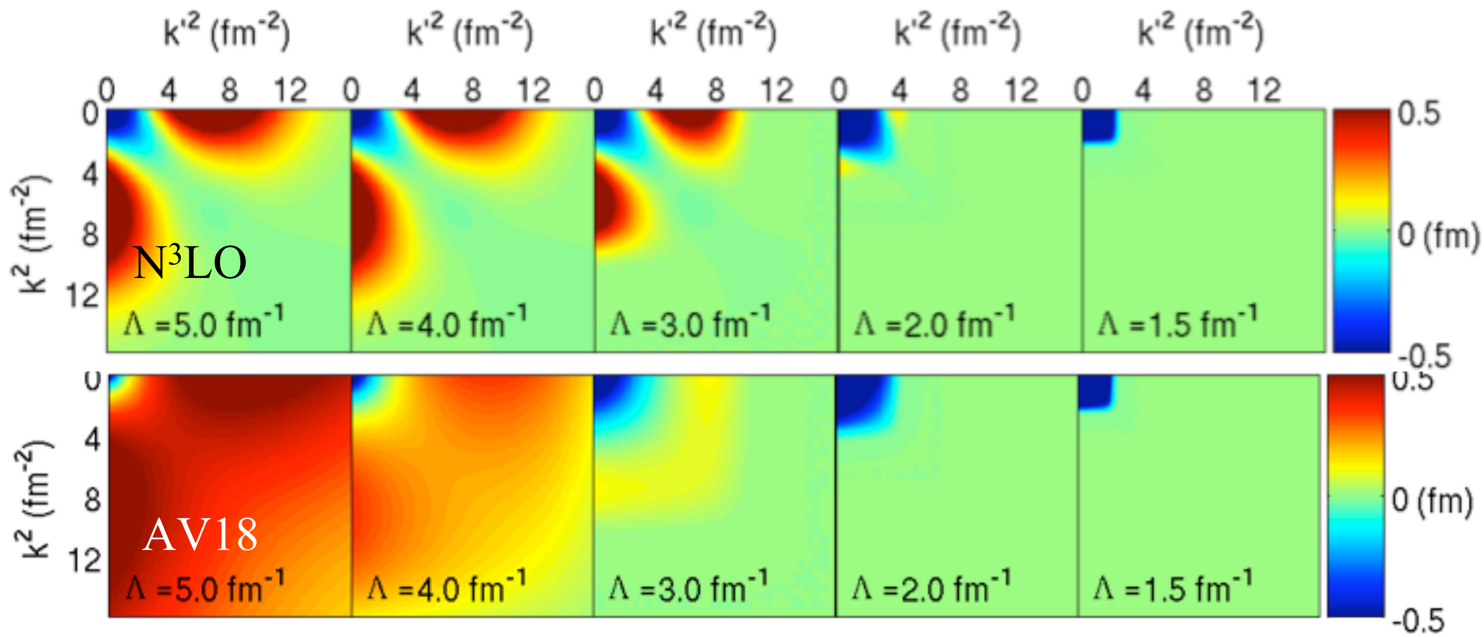
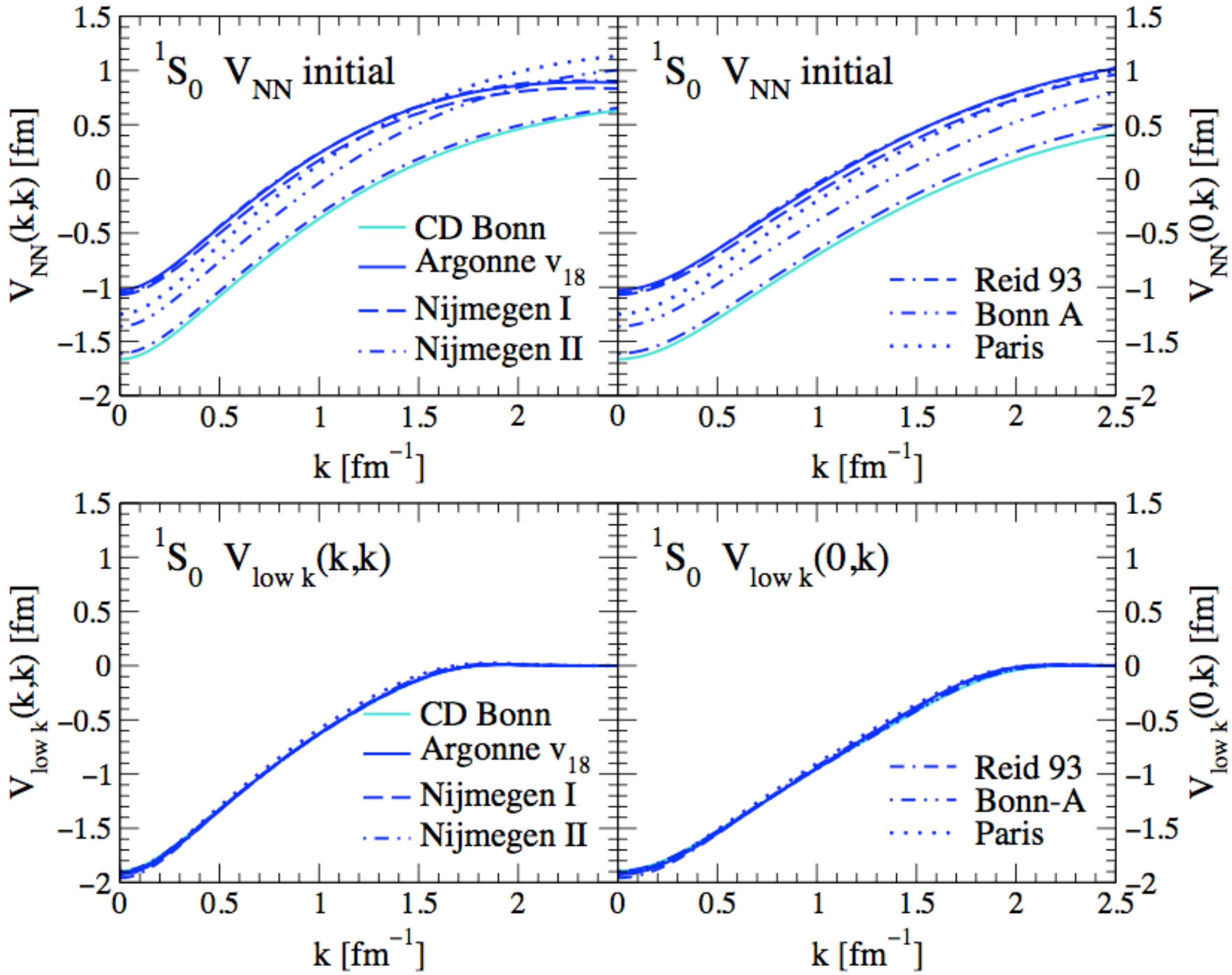


low-momentum interactions $V_{\text{low}\,k}(\Lambda)$ with sharp or smooth regulators

decouples low-momentum physics from high momenta

red = short-range repulsion and short-range tensor parts

Low-momentum universality

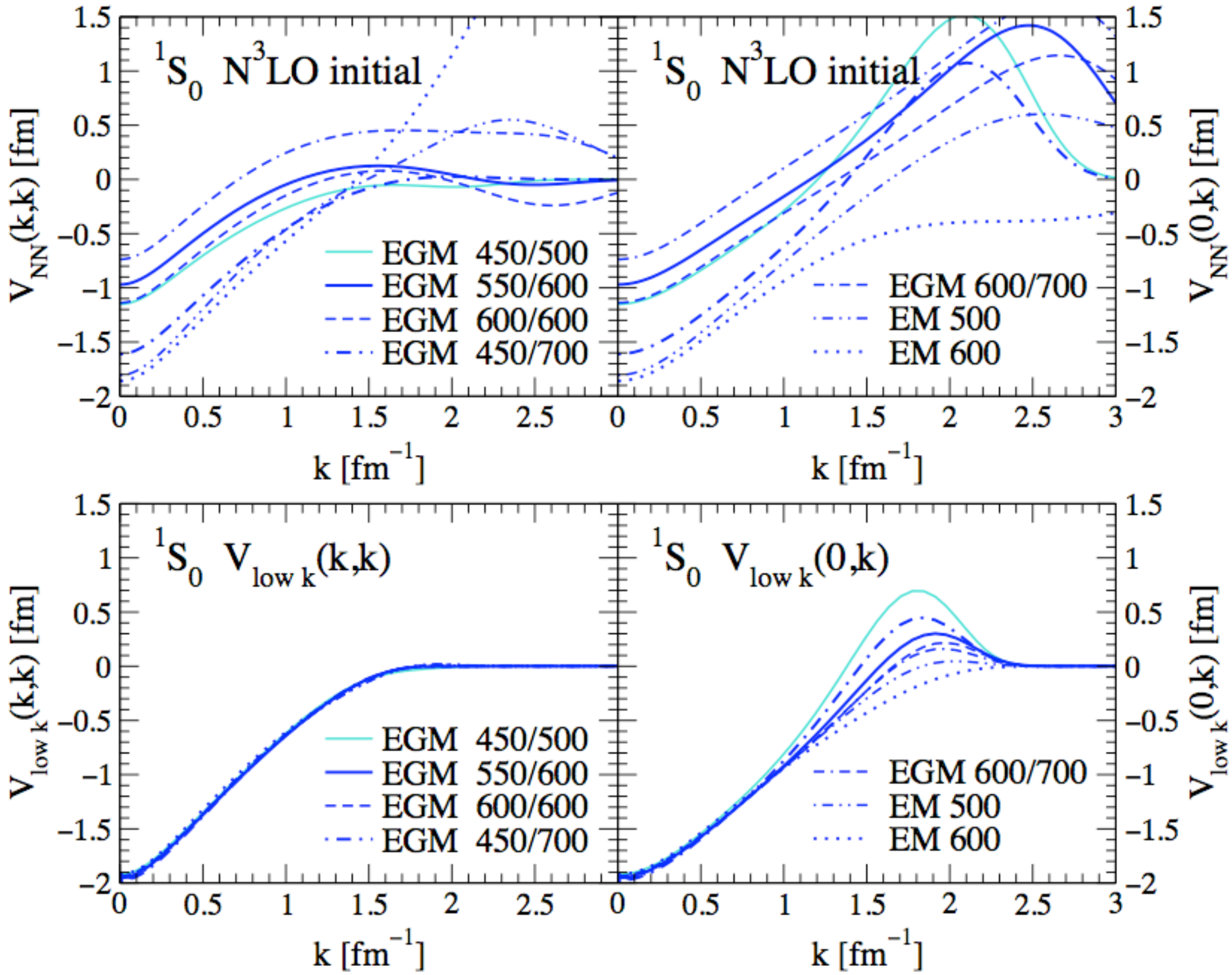


≈ universality from different phenomenological potentials

RG preserves NN observables and long-range parts

decouples low-momentum physics from high momenta

Low-momentum universality



≈ universality from different chiral N^3LO potentials

RG preserves NN observables and long-range parts

decouples low-momentum physics from high momenta

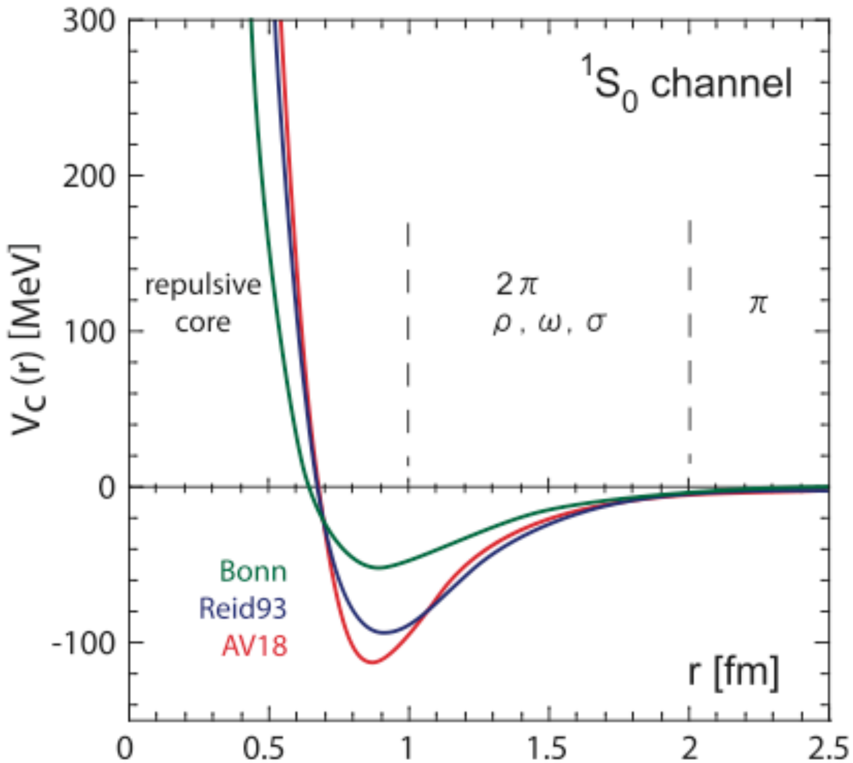
Weinberg eigenvalue diagnostic

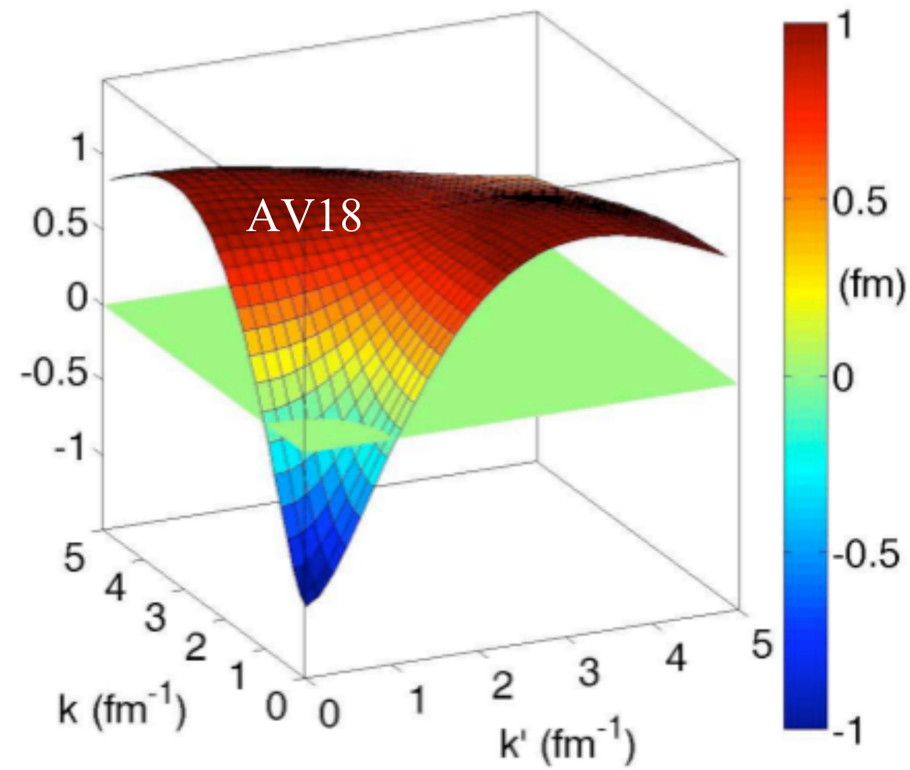
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

can write as Schrödinger equation $\left(H_0+\dfrac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

large cutoffs lead to flipped-potential bound states of -λV for small λ





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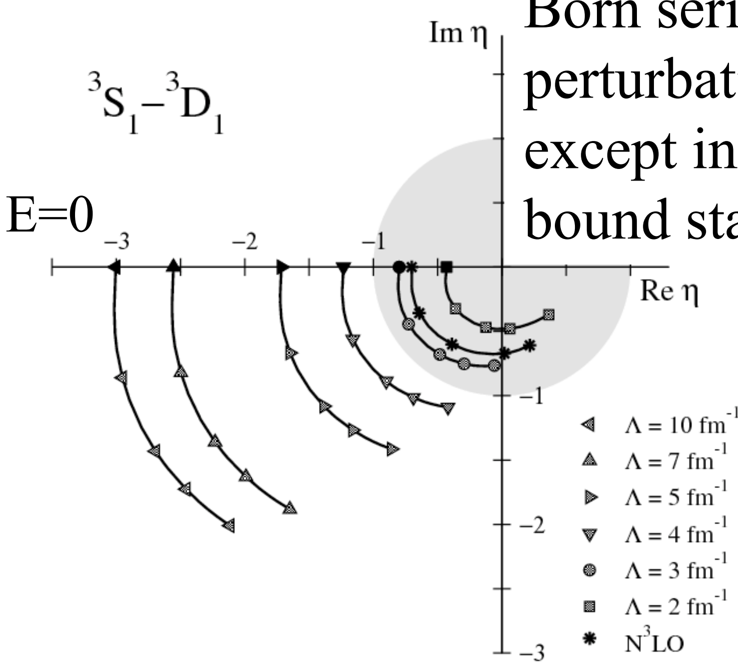
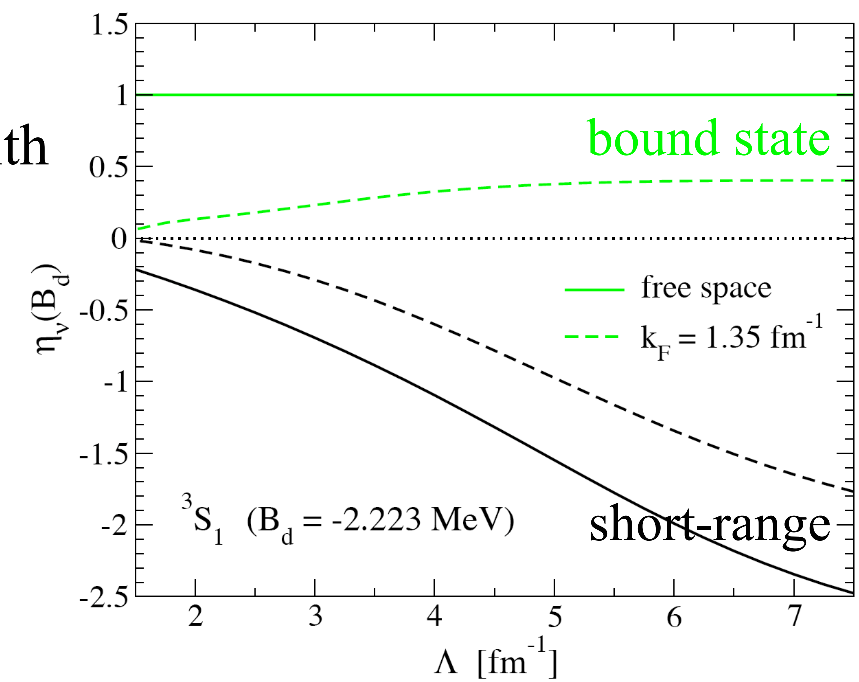
large cutoffs lead to flipped-potential bound states of -λV for small λ
→ large η → strong coupling to high momenta and Born series nonpert.

leads to slow convergence for all nuclei



Born series becomes perturbative after RG except in channels with bound states



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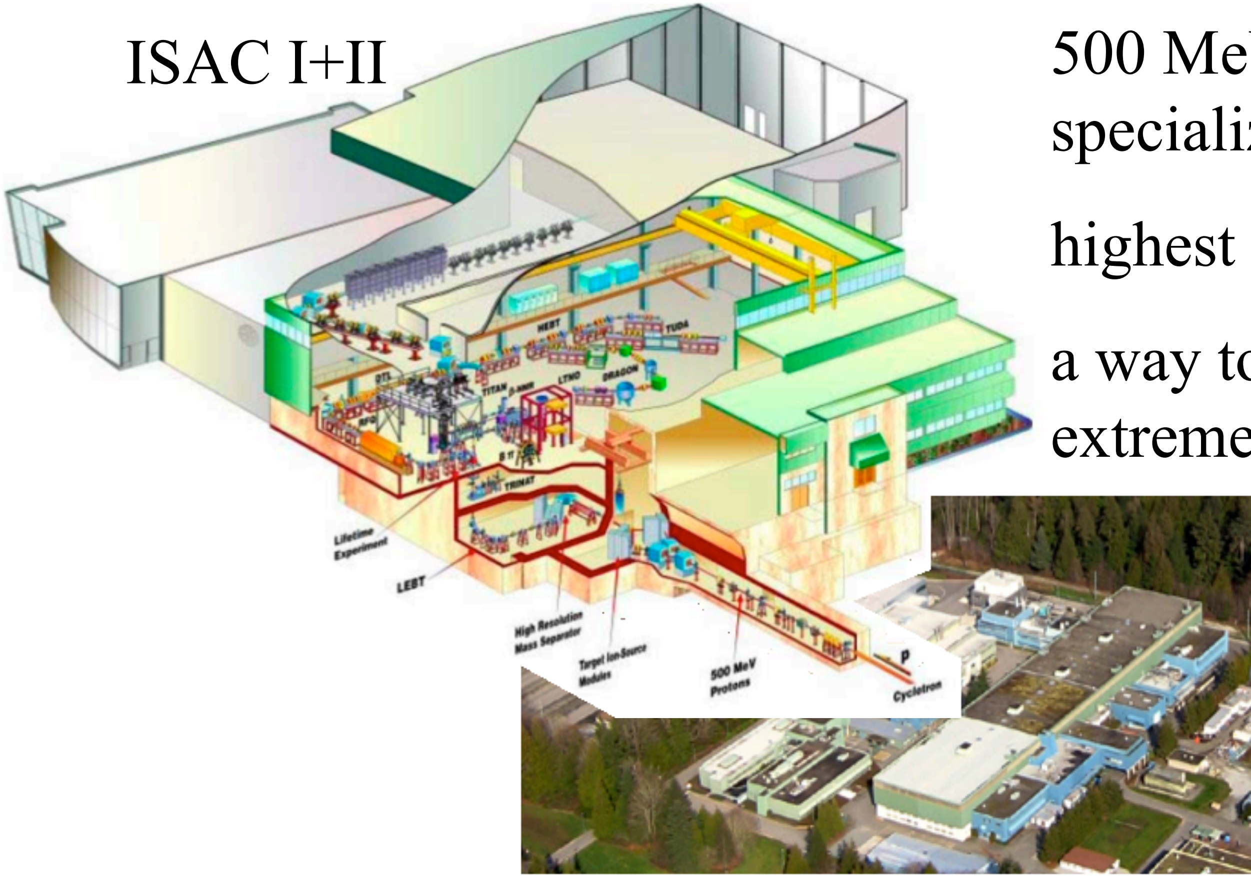
Applications to weakly-bound and neutron-rich nuclei

Similarity renormalization group for nuclei

Neutron-rich nuclei in the laboratory

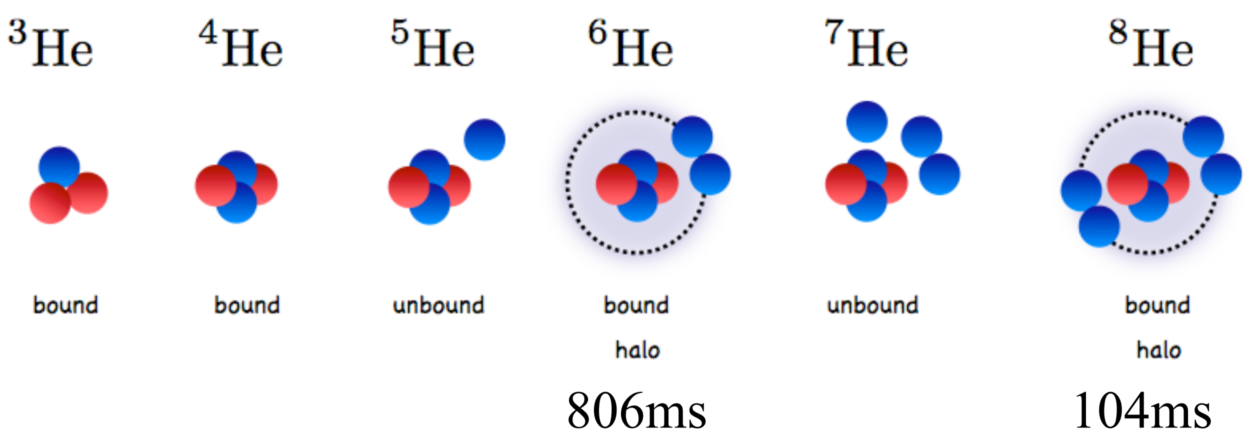
ISAC I+II



500 MeV proton beam on specialized targets

highest power at TRIUMF-ISAC

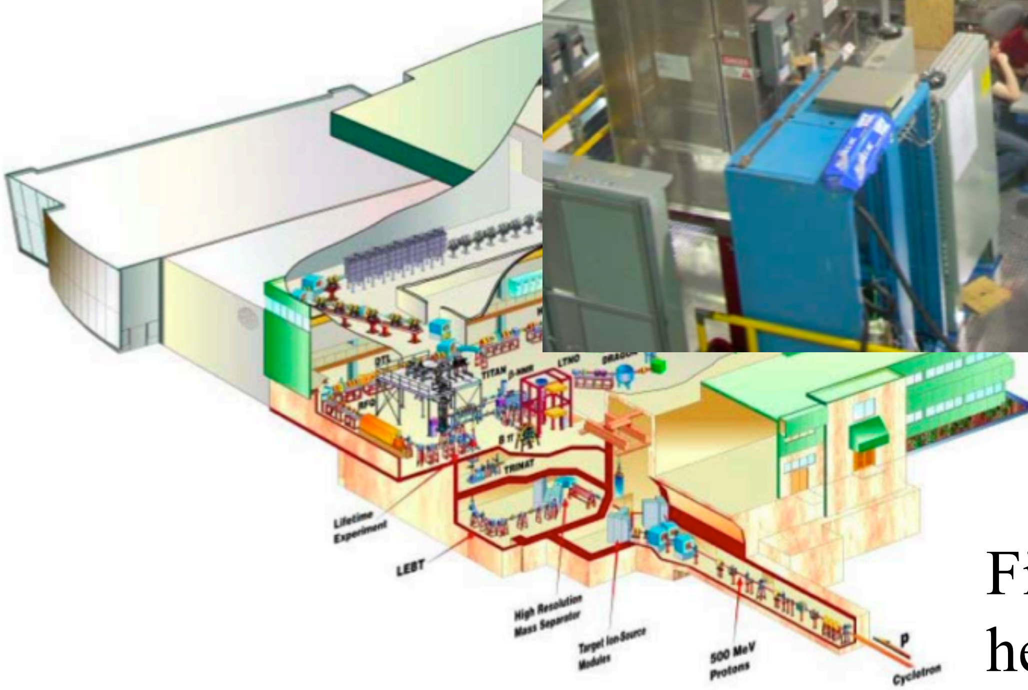
a way to add neutrons to reach extreme neutron-rich nuclei



Novel forms of matter: halo nuclei

^{8}He: most neutron-rich nucleus in the lab

TITAN Penning trap see talk by S. Ettenauer

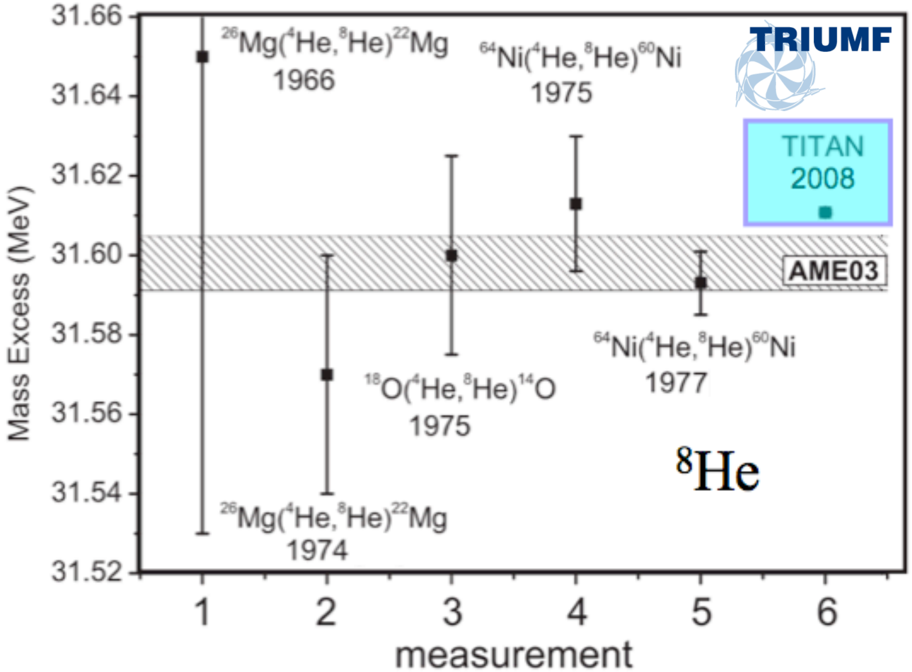


First direct mass measurement of helium and lithium halo nuclei

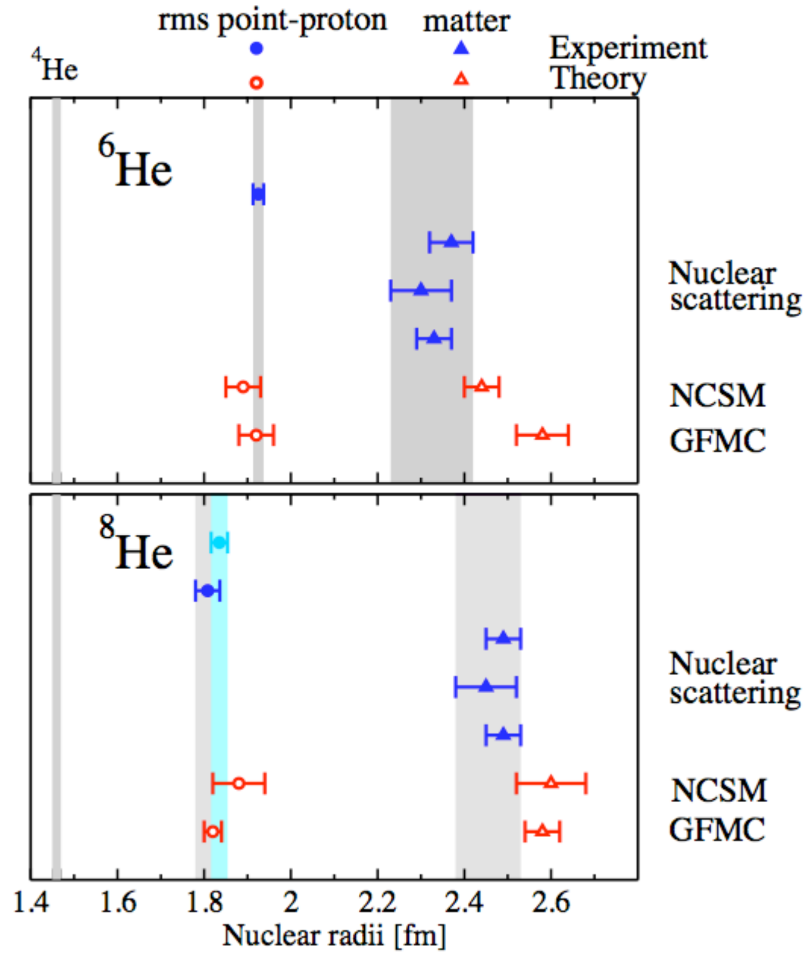
high precision $\delta m/m \sim 10^{-8}$

Neutron halos

New precision era for masses and charge radii (from isotope shifts)



TRIUMF: Ryjkov et al., PRL (2006)



ANL: Wang et al., PRL (2004)

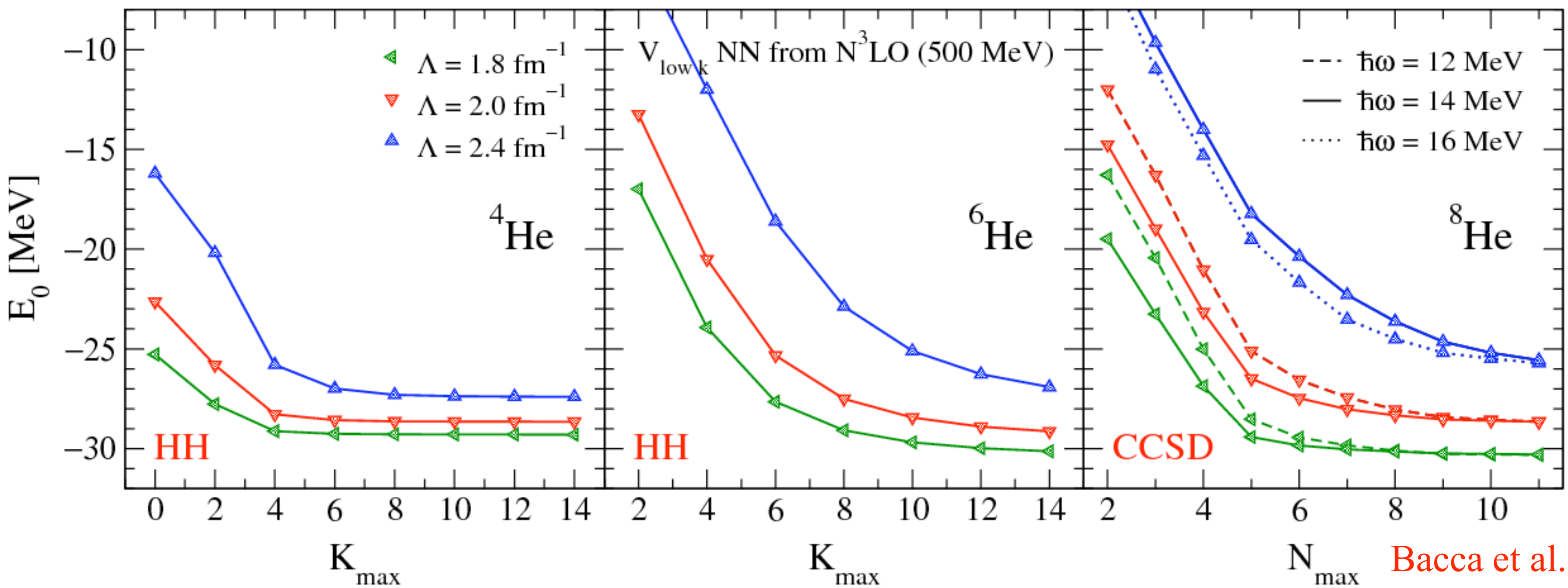
GANIL: Mueller et al., PRL (2007)

poses extraordinary challenges for theory

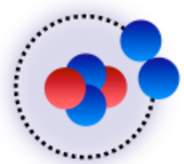
Neutron halos

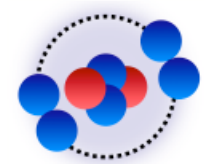
Hyperspherical Harmonics for ^{6}He and Coupled-Cluster theory for ^{8}He describe weakly-bound nuclei with correct asymptotics

but compare convergence to stable ^{4}He!



Bacca et al. (2009)



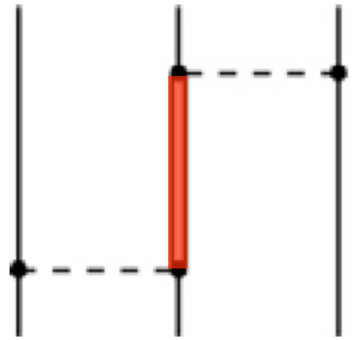


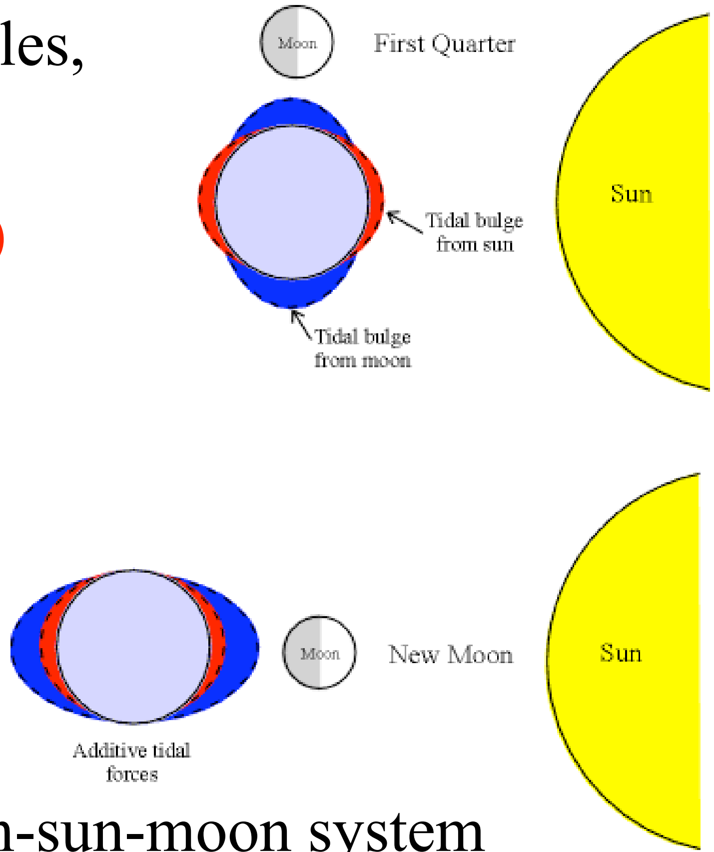
based on N^{3}LO NN potential, RG cutoff variation → need 3N forces

Why are there three-nucleon (3N) forces?

Nucleons are finite-mass composite particles, can be excited to resonances

dominant contribution from Δ(1232 MeV)



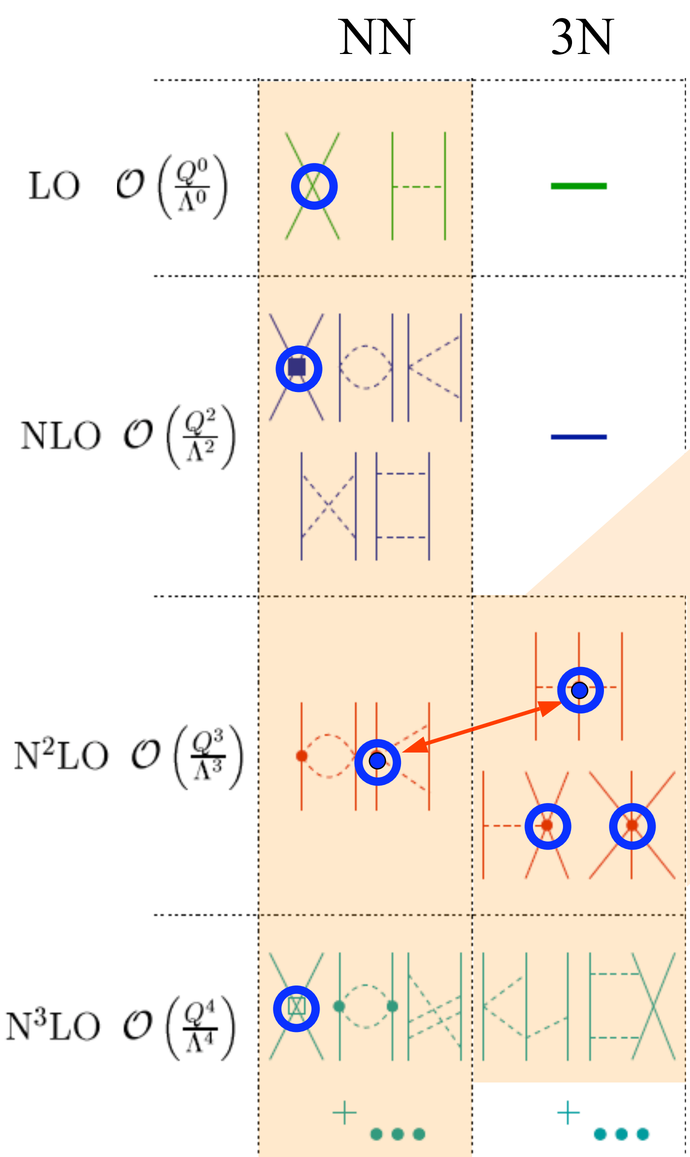


+ shorter-range parts

tidal effects leads to 3-body forces in earth-sun-moon system

Chiral EFT for 3N forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV

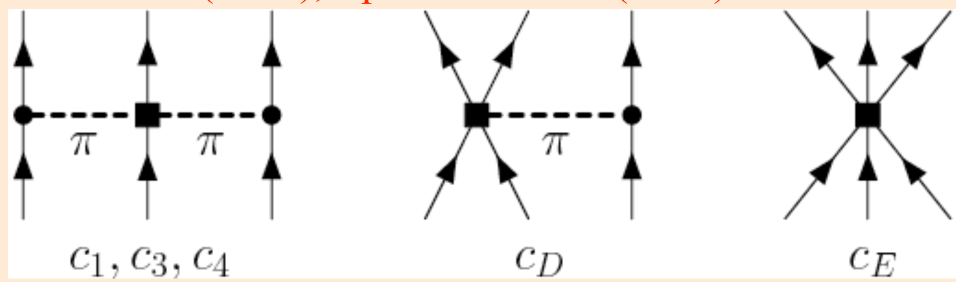


consistent NN-3N interactions

3N,4N: only 2 new couplings to N^3LO

leading 3N: N^2LO

van Kolck (1994), Epelbaum et al. (2002)



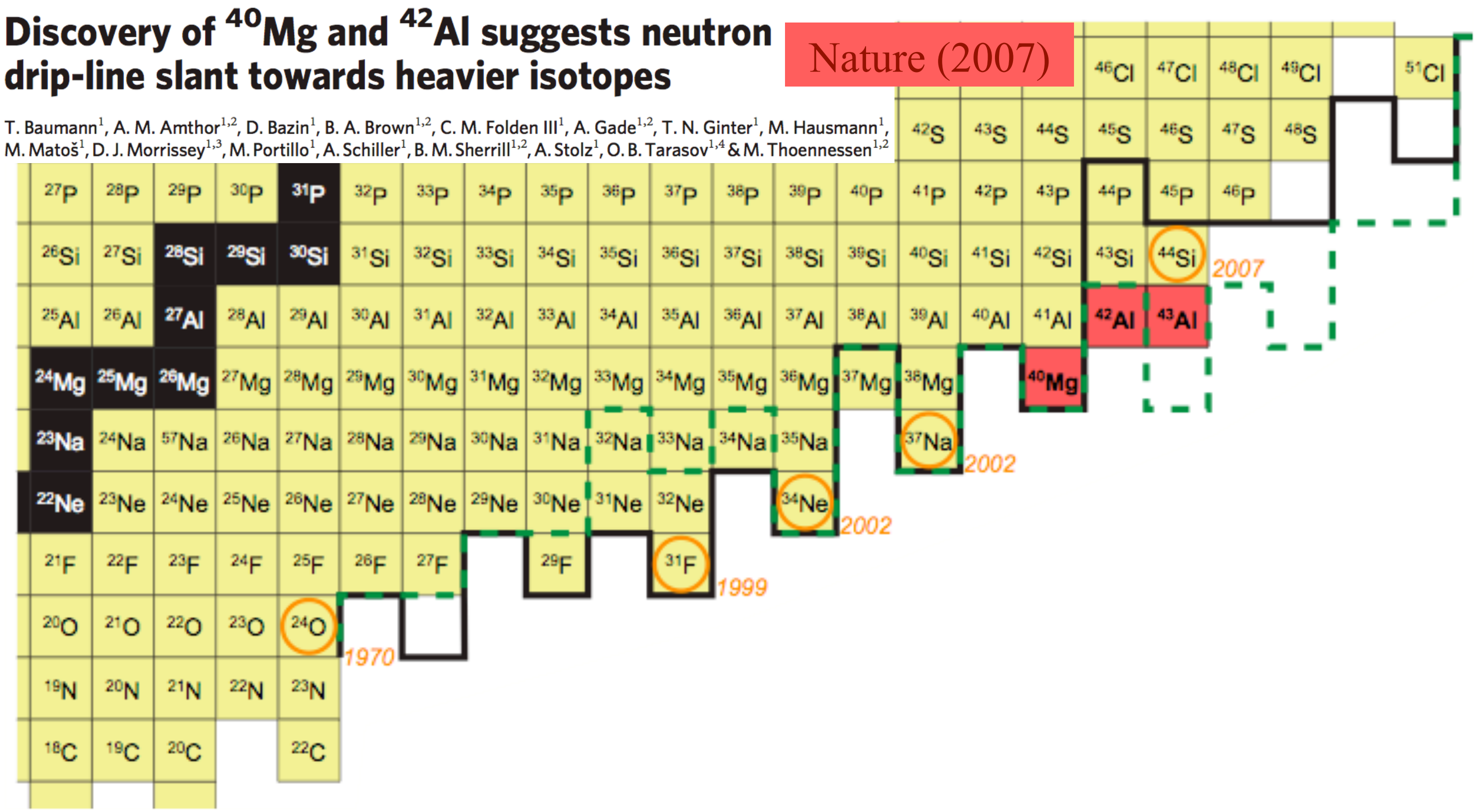
c_i from πN and NN from Meissner (2007)

$$c_1 = -0.9^{+0.2}_{-0.5}\,, \quad c_3 = -4.7^{+1.2}_{-1.0}\,, \quad c_4 = 3.5^{+0.5}_{-0.2}$$

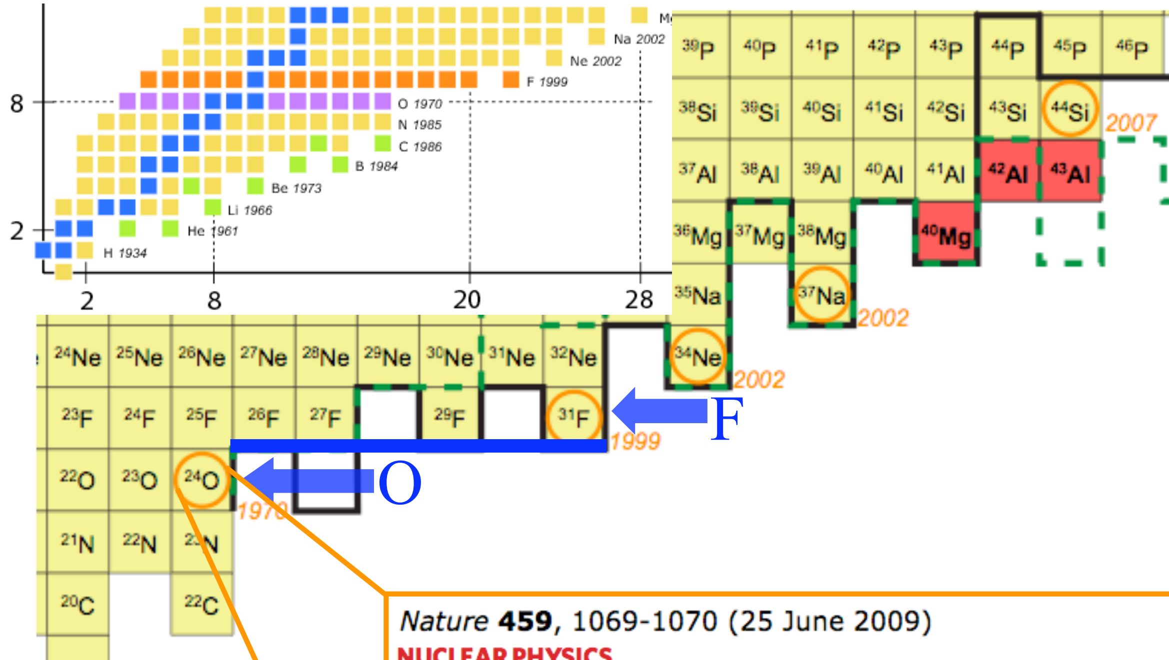
single-Δ excitation = particular c_i

c_D, c_E fit to ^3H binding energy and ^4He radius (or ^3H beta decay half-life)

Towards the limits of existence - the neutron drip-line



The oxygen anomaly



Nature **459**, 1069-1070 (25 June 2009)

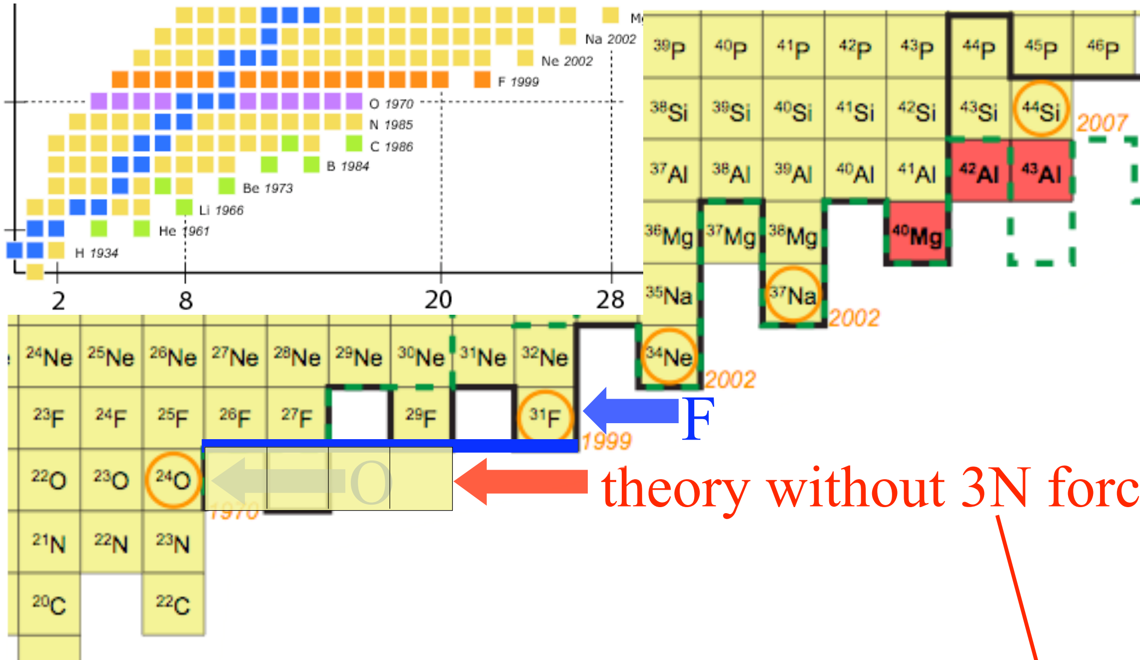
NUCLEAR PHYSICS

Unexpected doubly magic nucleus

Robert V. F. Janssens

Nuclei with a 'magic' number of both protons and neutrons, dubbed doubly magic, are particularly stable. The oxygen isotope ^{24}O has been found to be one such nucleus — yet it lies just at the limit of stability.

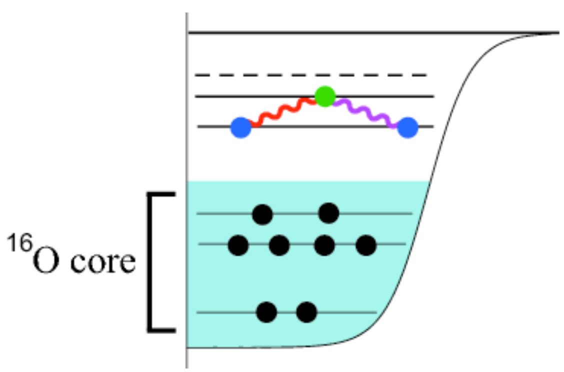
The oxygen anomaly - not reproduced without 3N forces

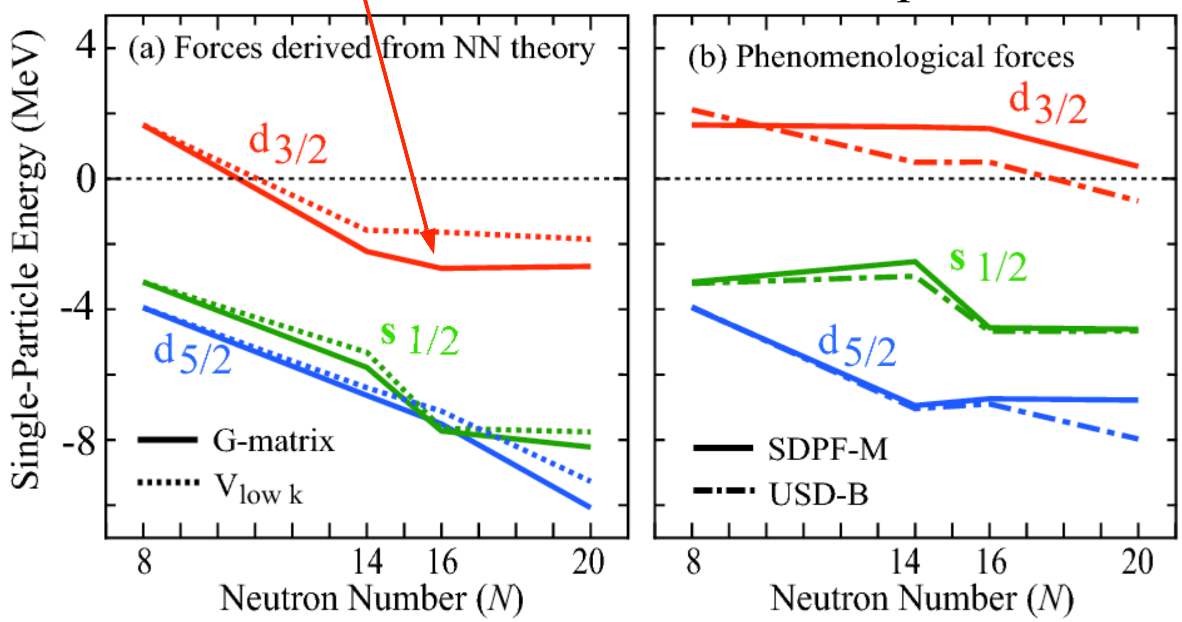


theory without 3N forces

fit to experiment

many-body theory based on two-nucleon forces:
drip-line incorrect at ^{28}O

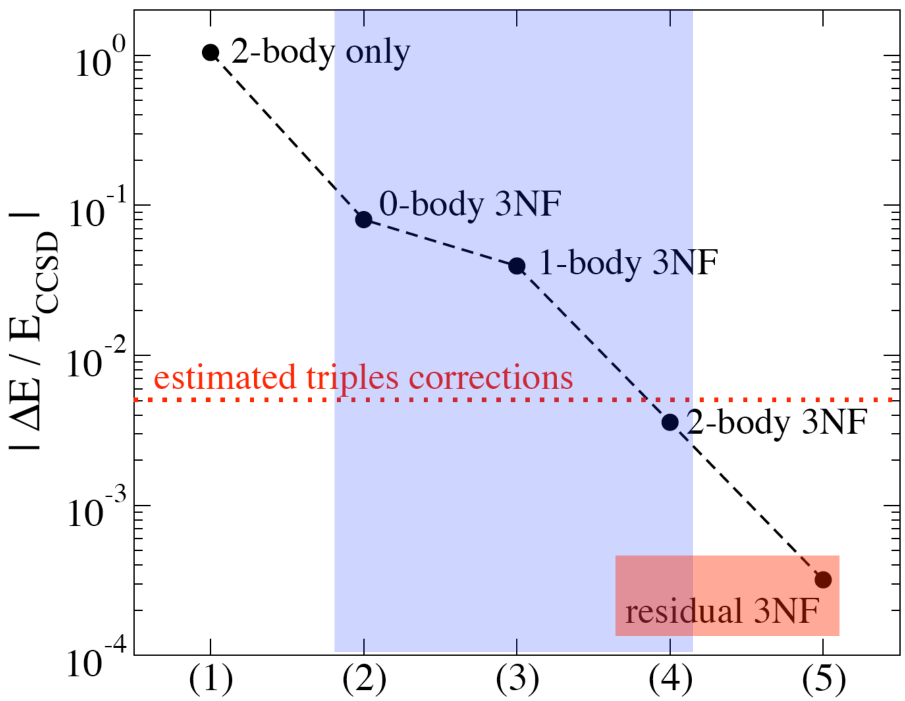




Towards 3N forces in medium-mass nuclei

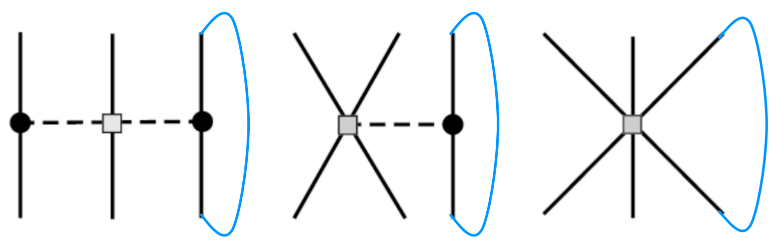
Coupled-Cluster theory with 3N forces Hagen et al. (2007)

first benchmark for ^{4}He, work on ^{16}O in progress



normal-ordered 0-, 1- and 2-body parts of 3N forces dominate

2-body part

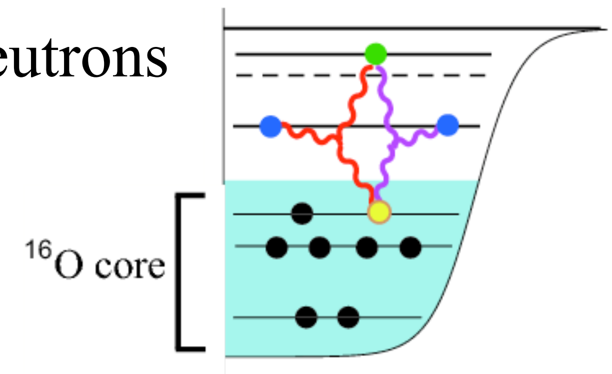


residual 3N interaction can be neglected: very promising

The oxygen anomaly - impact of 3N forces

include normal-ordered 2-body part of 3N forces (enhanced by core A)

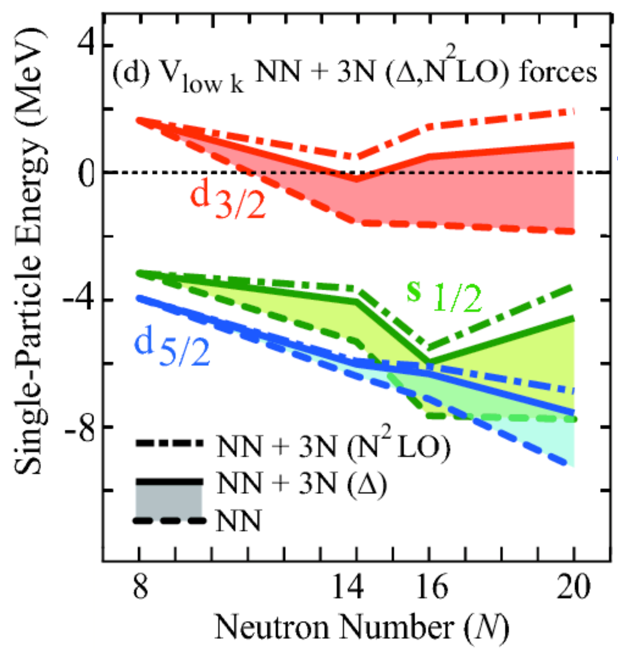
leads to repulsive interactions between valence neutrons
(repulsive based on the Pauli principle)

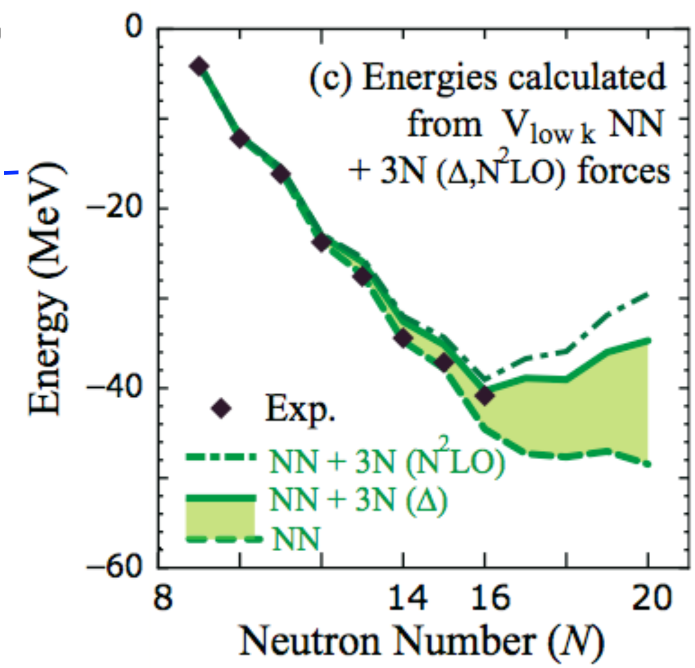


$d_{3/2}$ orbital remains unbound

first microscopic explanation of the oxygen anomaly

Otsuka, Suzuki, Holt, AS, Akaishi (2009)



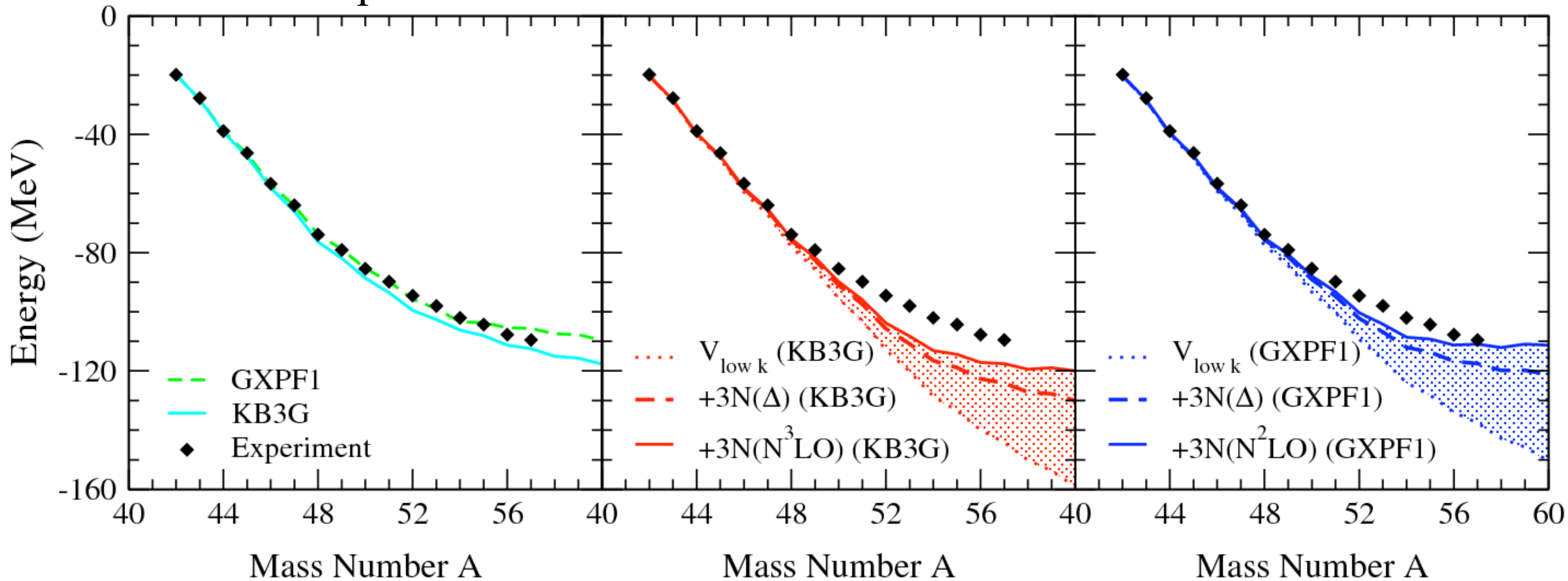


Evolution to neutron-rich calcium isotopes

repulsive 3N contributions also key for calcium ground-state energies

Holt, Otsuka, AS, Suzuki, in prep.

fit to experiment



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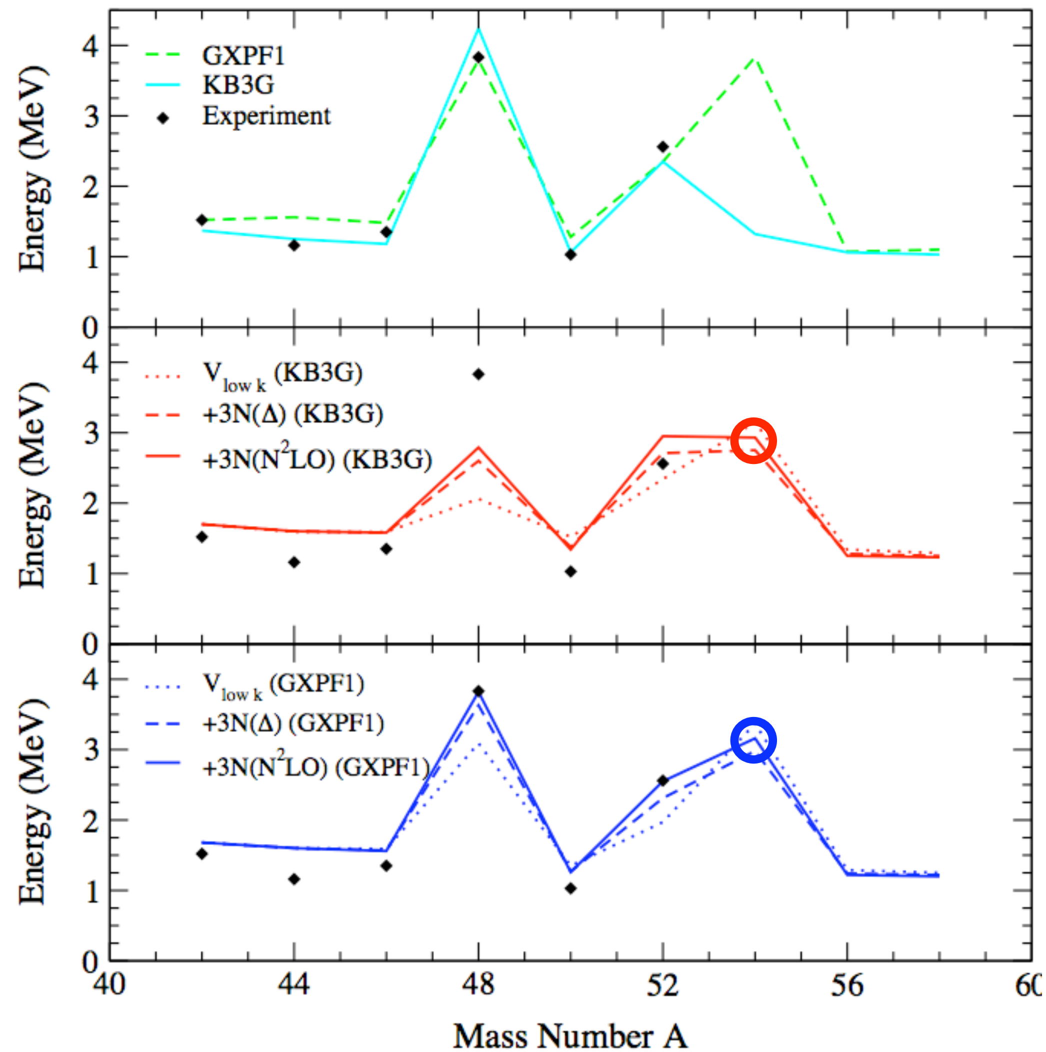
3N mechanism important for shell structure: 2^+ excitation energy in ^{48}Ca

N=28 shell closure due to 3N and single-particle effects (^{41}Ca)

predict 2^+ excitation energy in ^{54}Ca at ~3 MeV



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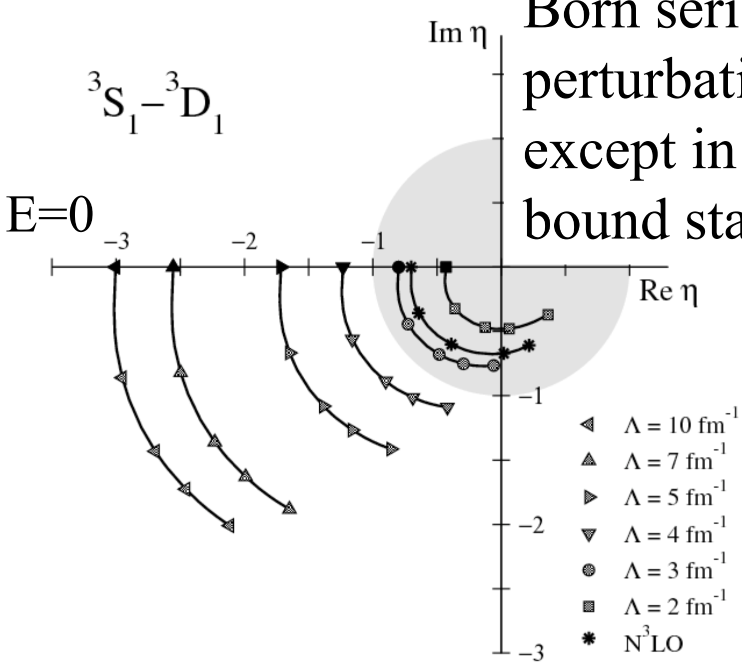
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

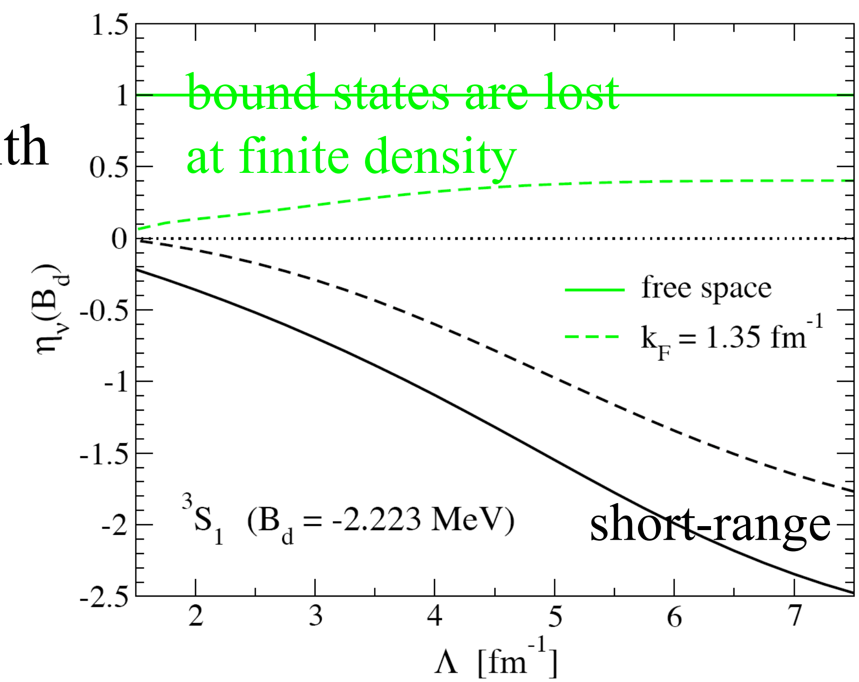
can write as Schrödinger equation $\left(H_0+\frac{1}{\eta_\nu(z)}V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

large cutoffs lead to flipped-potential bound states of -λV for small λ
→ large η → strong coupling to high momenta and Born series nonpert.

leads to slow convergence for all nuclei



Born series becomes perturbative after RG except in channels with bound states

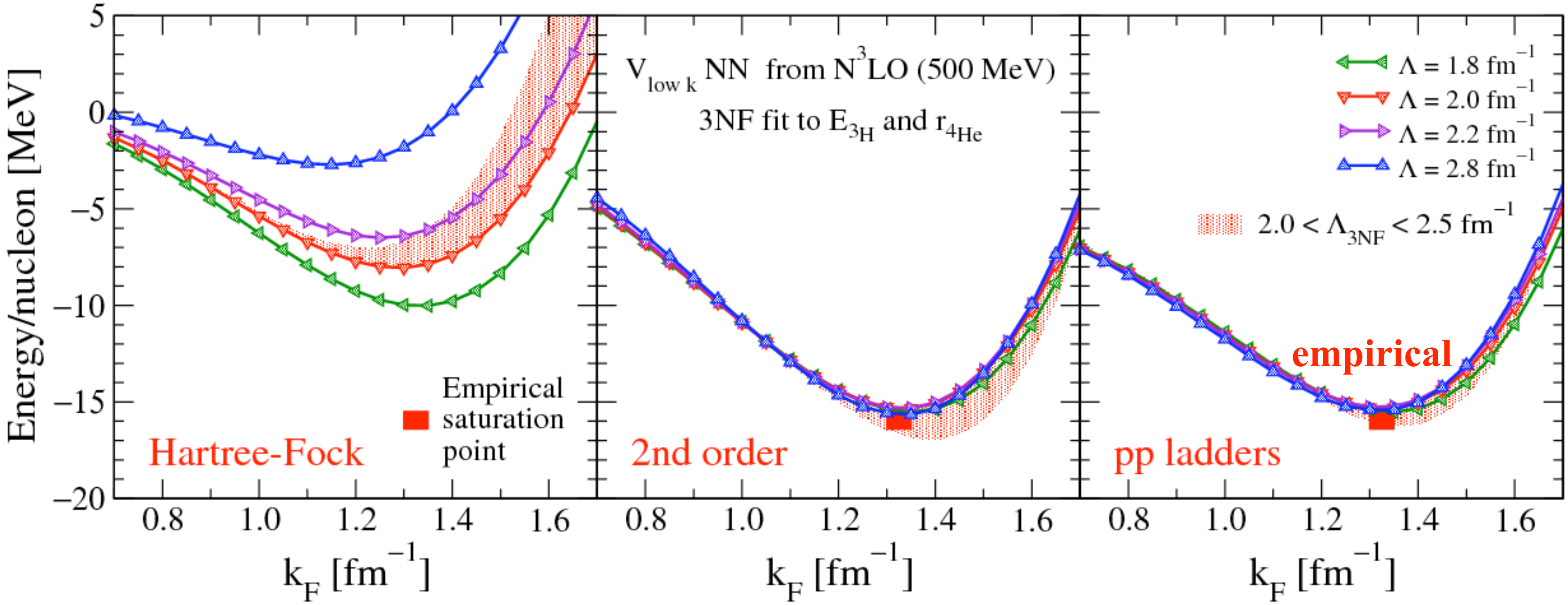


Is nuclear matter perturbative with chiral EFT and RG?

conventional Bethe-Brueckner-Goldstone expansion (sums ladders):
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling

start from chiral EFT and RG evolution:
nuclear matter converged at ≈ 2nd order, 3N drives saturation

weak cutoff dependence, but need to improve 3N treatment



Bogner, AS, Furnstahl, Nogga (2009)

see talk by K. Hebeler for neutron matter and more

Outline

Effective field theory and renormalization group for nuclear forces

Applications to weakly-bound and neutron-rich nuclei

Similarity renormalization group for nuclei

Similarity RG

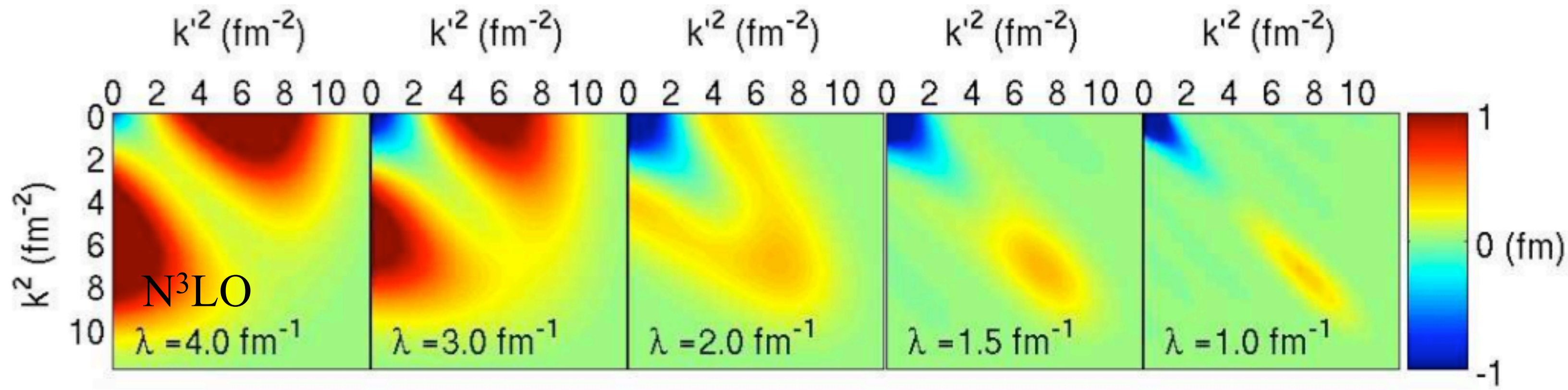
unitary transformations to band-diagonal $V_{srg}(\lambda)$ from flow equations
Glazek, Wilson (1993), Wegner (1994)

$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

evolution driven towards nonzero part of generator G_s

with flow operator $G_s=T_{rel}$ and resolution $\lambda=s^{-1/4}$
Bogner, Furnstahl, Perry,…

$$\frac{dV_s(k,k')}{ds} = -(k^2-k'^2)^2\, V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2+k'^2-2q^2)\, V_s(k,q)\, V_s(q,k')$$

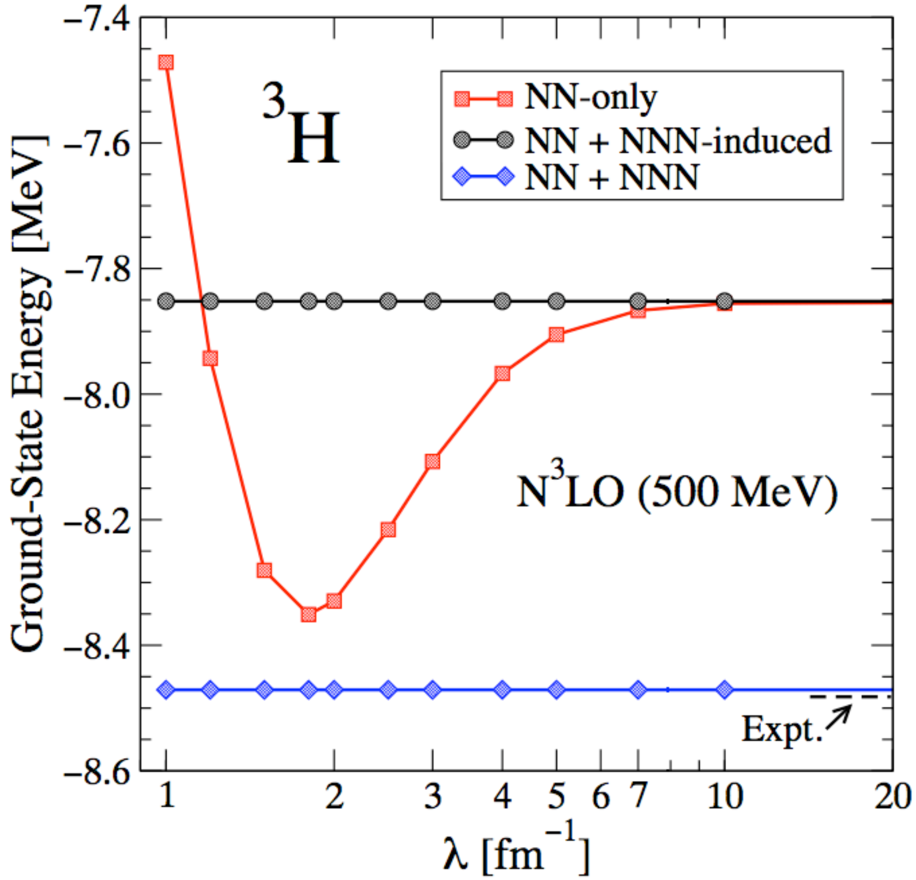


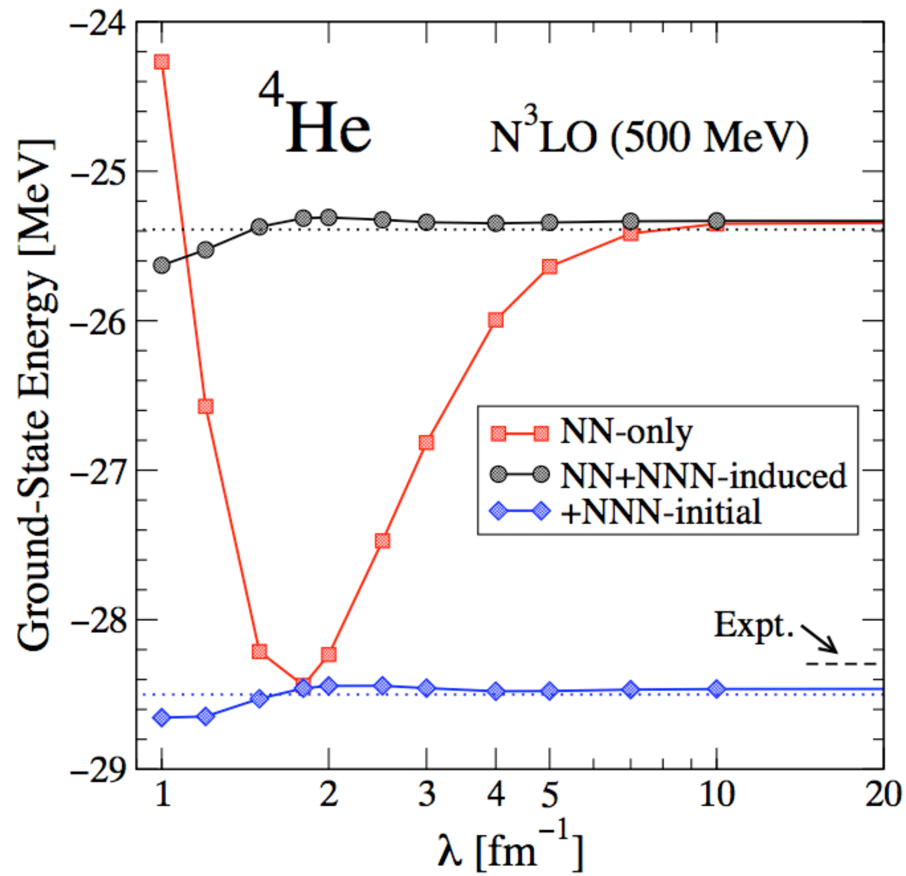
SRG decouples high momenta with similar low-momentum universality

SRG evolution of 3N forces

start from chiral NN and 3N interactions,
SRG evolution in harmonic oscillator basis

Jurgenson, Navratil, Furnstahl (2009)





induced many-body interactions consistent with truncation error in EFT

In-medium SRG for nuclei

$$H = \sum_{12} T_{12} a_1^\dagger a_2 + \frac{1}{(2!)^2} \sum_{1234} \langle 12|V|34\rangle a_1^\dagger a_2^\dagger a_4 a_3 + \frac{1}{(3!)^2} \sum_{123456} \langle 123|V^{(3)}|456\rangle a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4$$

normal-order Hamiltonian with respect to reference state
(e.g., Hartree-Fock ground state)

$$H = E_0 + \sum_{12} f_{12} \{a_1^\dagger a_2\} + \frac{1}{(2!)^2} \sum_{1234} \langle 12|\Gamma|34\rangle \{a_1^\dagger a_2^\dagger a_4 a_3\} + \frac{1}{(3!)^2} \sum_{123456} \langle 123|\Gamma^{(3)}|456\rangle \{a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4\}$$

with 0-, 1- and 2-body normal-ordered parts

$$E_0 = \langle \Phi|H|\Phi\rangle = \sum_1 T_{11} n_1 + \frac{1}{2} \sum_{12} \langle 12|V|12\rangle n_1 n_2 + \frac{1}{3!} \sum_{123} \langle 123|V^{(3)}|123\rangle n_1 n_2 n_3$$

$$f_{12} = T_{12} + \sum_i \langle 1i|V|2i\rangle n_i + \frac{1}{2} \sum_{ij} \langle 1ij|W|2ij\rangle n_i n_j \,,$$

$$\langle 12|\Gamma|34\rangle = \langle 12|V|34\rangle + \sum_i \langle 12i|V^{(3)}|34i\rangle n_i \,,$$

In-medium SRG for nuclei

$$H = \sum_{12} T_{12} a_1^\dagger a_2 + \frac{1}{(2!)^2} \sum_{1234} \langle 12|V|34\rangle a_1^\dagger a_2^\dagger a_4 a_3 + \frac{1}{(3!)^2} \sum_{123456} \langle 123|V^{(3)}|456\rangle a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4$$

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with 0-, 1- and 2-body normal-ordered parts and in-medium SRG eqns
e.g., for nuclear matter with $\eta=[f,\Gamma]$ see Bogner et al., Kehrein (2006)

$$\frac{dE_0}{ds} = \frac{1}{2} \sum_{1234} (f_{12} - f_{34}) |\Gamma_{1234}|^2 \, n_1 n_2 \bar{n}_3 \bar{n}_4 \,,$$

$$\frac{df_1}{ds} = \sum_{234} (f_{41} - f_{23}) |\Gamma_{4123}|^2 (\bar{n}_2 \bar{n}_3 n_4 + n_2 n_3 \bar{n}_4) \,,$$

$$\frac{d\Gamma_{1234}}{ds} = -(f_{12} - f_{34})^2 \, \Gamma_{1234} + \frac{1}{2} \sum_{ab} (f_{12} + f_{34} - 2 f_{ab}) \Gamma_{12ab} \Gamma_{ab34} (1 - n_a - n_b) + \sum_{ab} (n_a - n_b) \times \Big\{ \Gamma_{a1b3} \Gamma_{b2a4} \big[(f_{a1} - f_{b3}) - (f_{b2} - f_{a4}) \big] - \Gamma_{a2b3} \Gamma_{b1a4} \big[(f_{a2} - f_{b3}) - (f_{b1} - f_{a4}) \big] \Big\} \,,$$

approx. includes many-body forces and sums pp, hh, ph diagrams

In-medium SRG for nuclei Tsukiyama, Bogner, AS, in prep.

decouple 1p1h, 2p2ph,… ApAh sectors from reference state

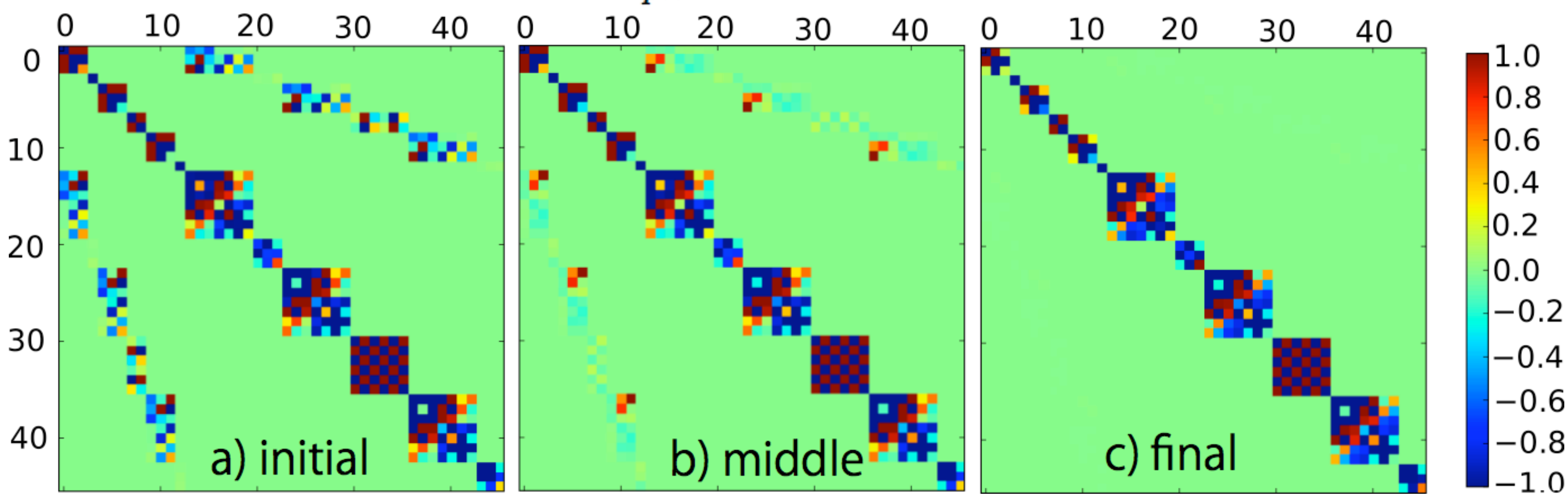
want to suppress pphh and ph couplings,
all other (normal-ordered) couplings annihilate reference state

minimal choice: $\eta(s) = [H^d(s), H(s)] = [H^d(s), H^{od}(s)]$

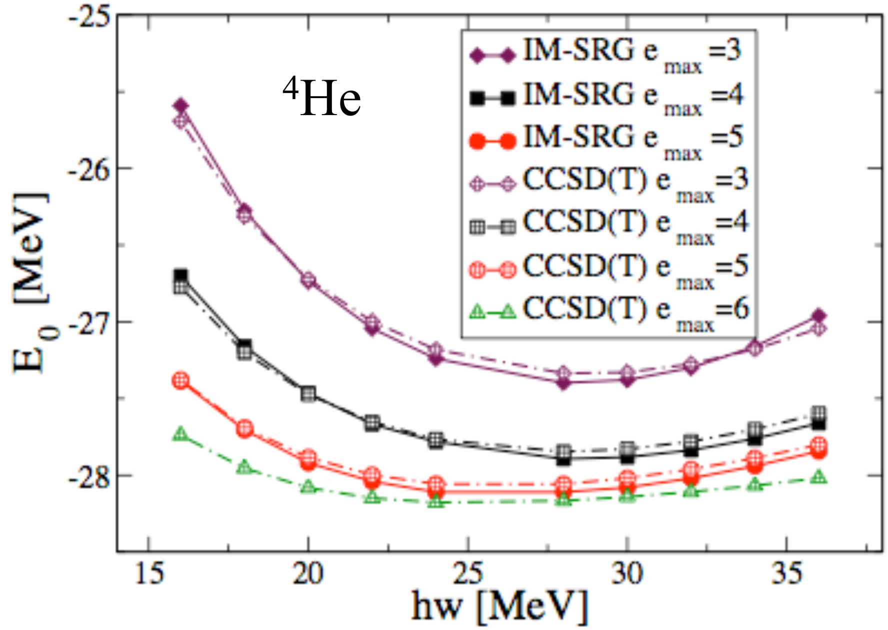
$$H^{od}(s) = g^{od}(s) + \Gamma^{od}(s)$$

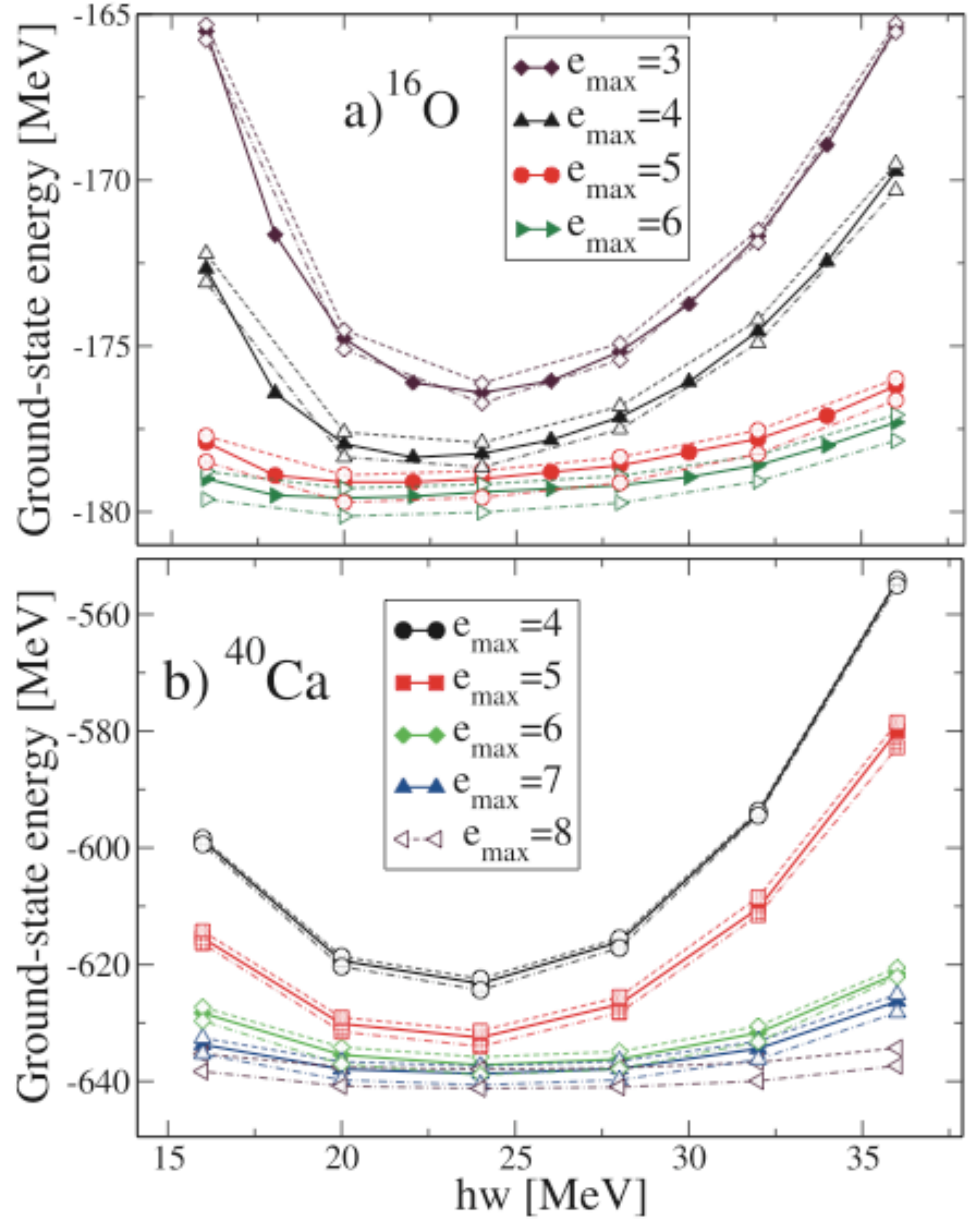
$$\Gamma^{od}(s) = \sum_{pp'hh'} \Gamma_{pp'hh'}(s) a_p^\dagger a_{p'}^\dagger a_h a_{h'} + h.c.$$

$$g^{od}(s) \sum_{ph} g_{ph}(s) a_p^\dagger a_h + h.c.$$



In-medium SRG for nuclei Tsukiyama, Bogner, AS, in prep.





first results for closed-shell nuclei

very promising convergence,
results comparable to CCSD(T)

can be used to derive nonperturbative valence-shell effective interactions

Thanks to collaborators

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日本大学 T. Suzuki

NORDITA Niels Bohr Institutet C.J. Pethick

GSI B. Friman

Summary

development of effective field theory and the renormalization group

with approaches from light to heavy nuclei to matter in astrophysics based on the same interactions

3N forces are a frontier for weakly-bound and neutron-rich nuclei

exciting intersections with problems in many related areas

exciting experiments to study neutron-rich matter in the laboratory at rare isotope beam (RIB) facilities worldwide