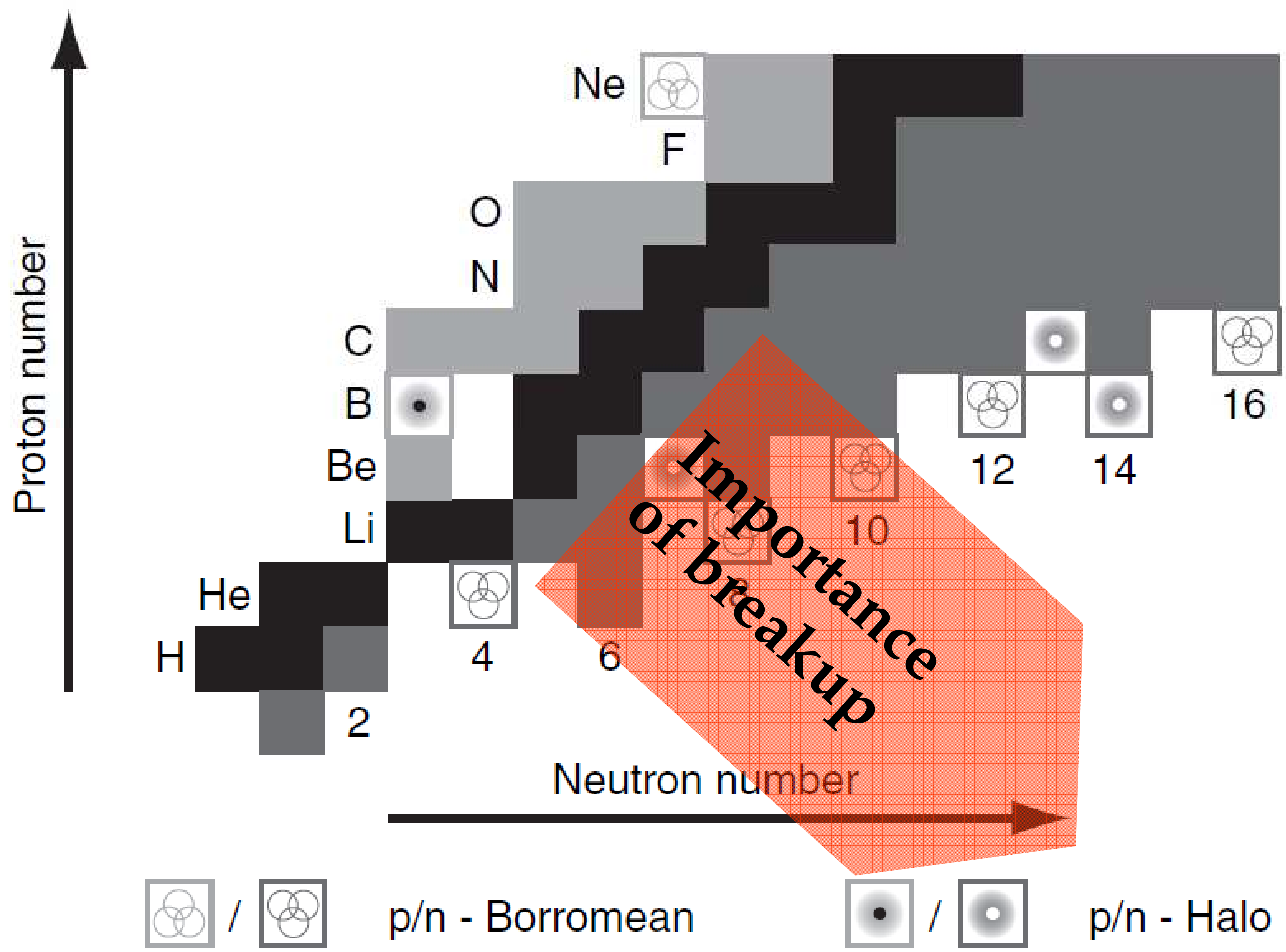
Reactions with Exotic Nuclei

Filomena Nunes

INT, Mar 2010

funded by: NSF and DOE

what are exotic nuclei?



why do reactions? elastic

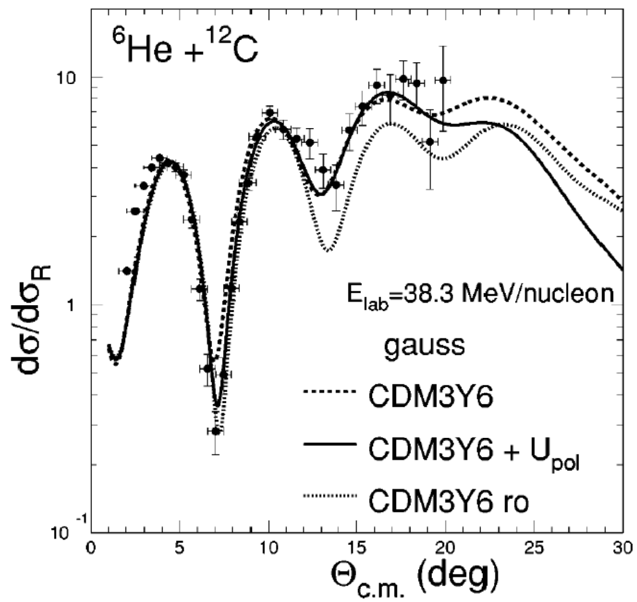
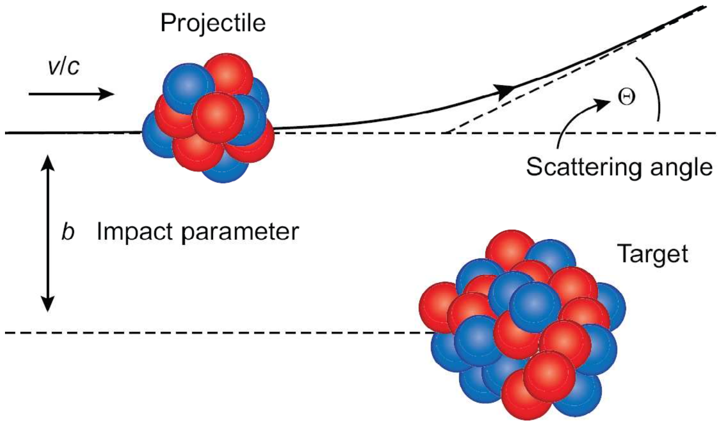


FIG. 10. Elastic scattering for ^{6}He+ ^{12}C at 38.3 MeV/nucleon in comparison with the OM results given by the real folded potential (obtained with the CDM3Y6 interaction and the Gaussian *ga* density for ^{6}He). The dashed curve is obtained with the unrenormalized folded potential only. The solid curve is obtained by adding a complex surface polarization potential to the real folded potential. Its parameters, and those of the imaginary part, are explained in the text. The dotted line is obtained by folding the CDM3Y6 interaction with the compact Gaussian density *ro*.

[Lapoux et al, PRC 66 (02) 034608]

traditionally used to extract optical potentials, rms radii, density distributions.

why do reactions? inelastic



traditionally used to extract electromagnetic transitions or nuclear deformations

example: ^{11}Be B(E1)

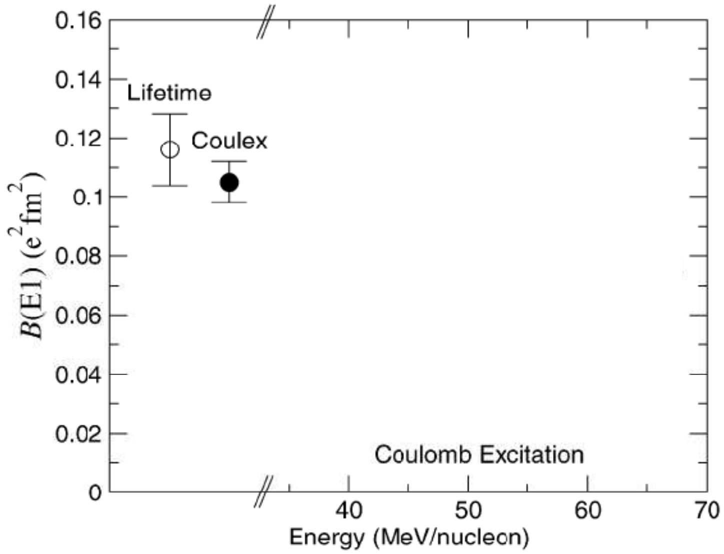
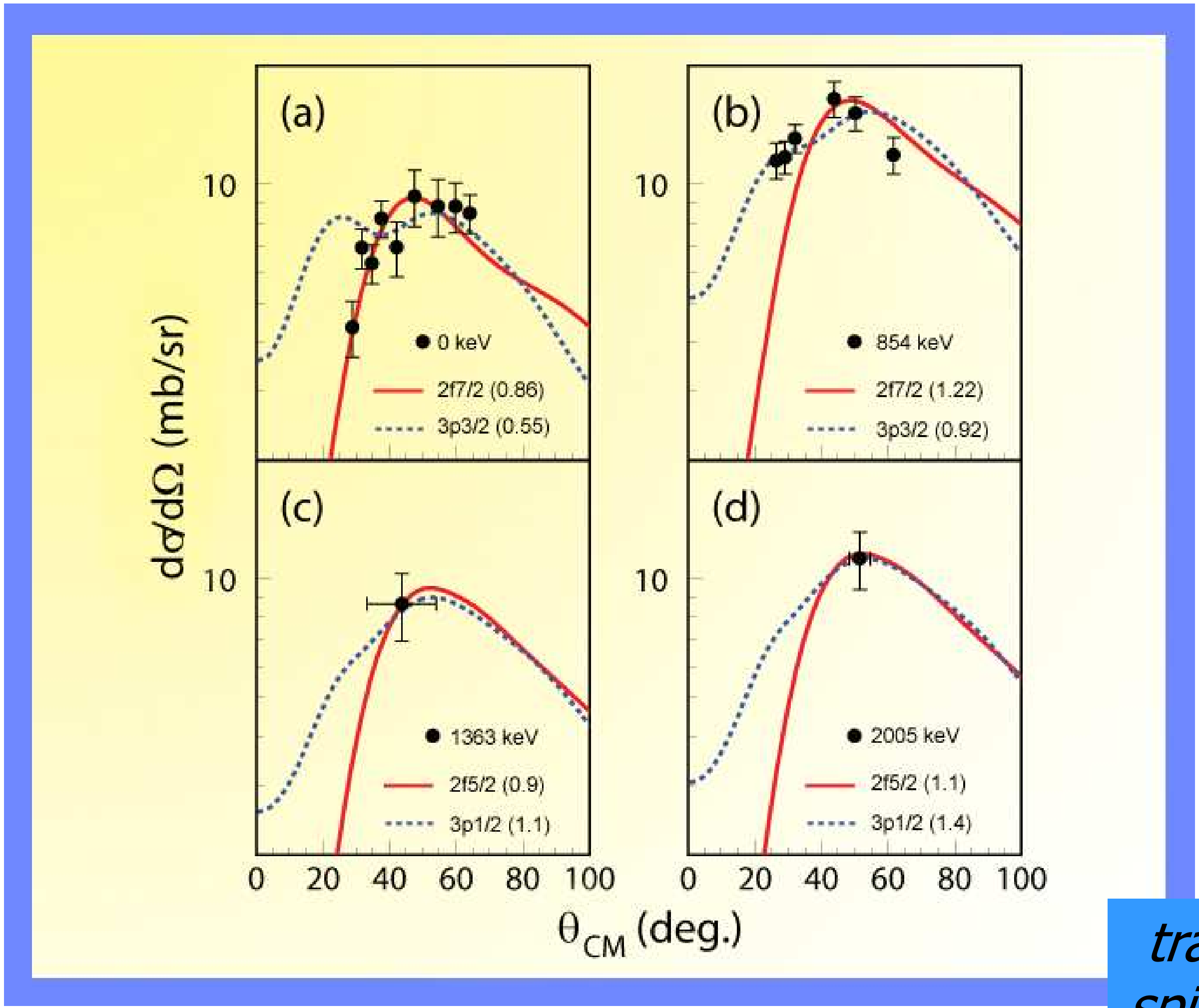


Fig. 2. Comparison of B(E1) values obtained from lifetime and Coulomb excitation measurements. The weighted average of lifetime measurements [3] (open circle) is plotted on the left along with the weighted average (solid circle) of three Coulomb excitation measurements (solid symbols). The individual Coulomb excitation measurements, GANIL (this work, square), MSU (up triangle) [6], RIKEN (down triangle) [7], and a previous GANIL experiment (diamond) [4], are plotted versus the beam energy.

[Summers et al, PLB 650 (2007) 124]

why do reactions? transfer



example:
$^{132}Sn(d,p)^{133}Sn$

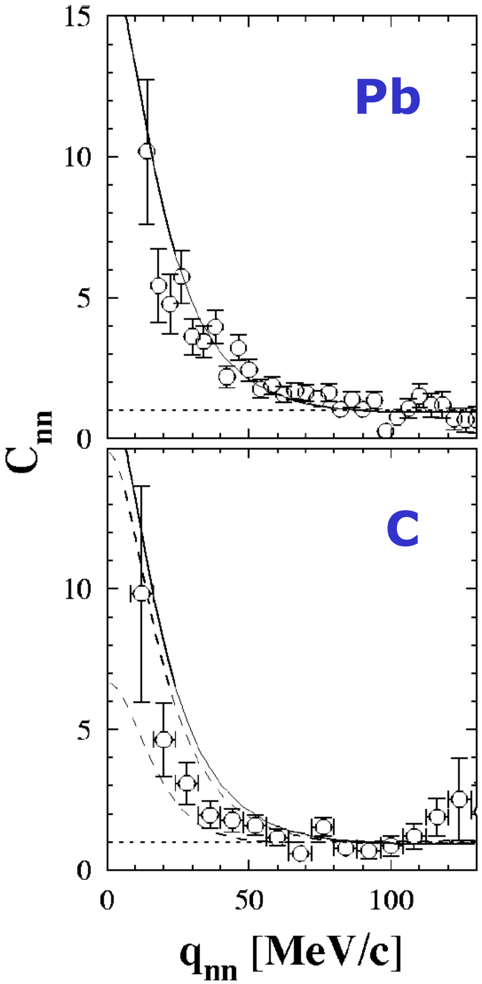
traditionally used to extract spin,parity and spectroscopic factors

[K. Jones et al, to appear in Nature 2010]

why do reactions? breakup

two nucleon correlation function

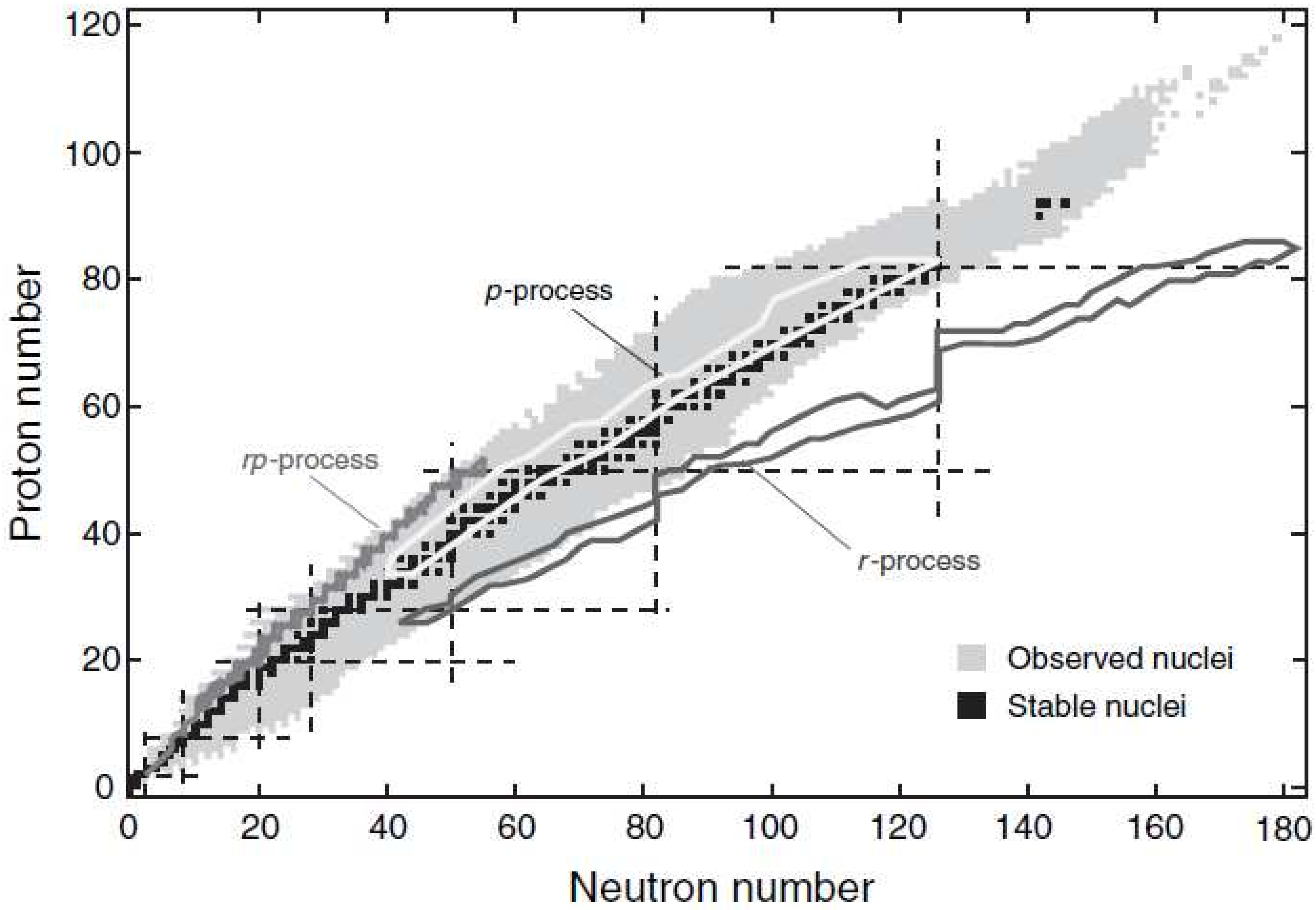
$^{14}Be \rightarrow n+n+^{12}Be$



[Marques et al, PRC 64 (2001) 061301]

can be used to extract properties of ground state, resonant states, correlations, astrophysics

exotic nuclei and astrophysics

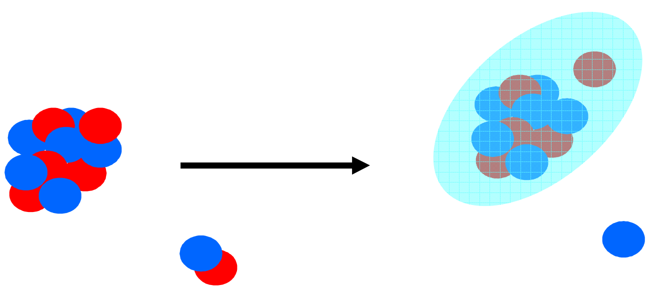


[Nuclear Reactions for Astrophysics, Thompson and Nunes 2009

nuclear reactions and astrophysics

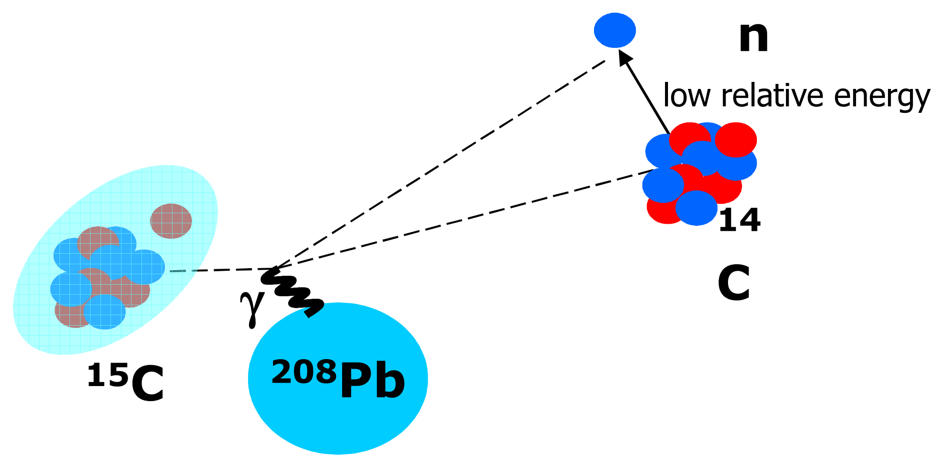
- direct measurement $^{14}C(n,\gamma)^{15}C$

- transfer reaction



$^{14}C(d,p)^{15}C$

- Coulomb dissociation



coulomb dissociation method applied to $^{7}Be(p,\gamma)^{8}B$

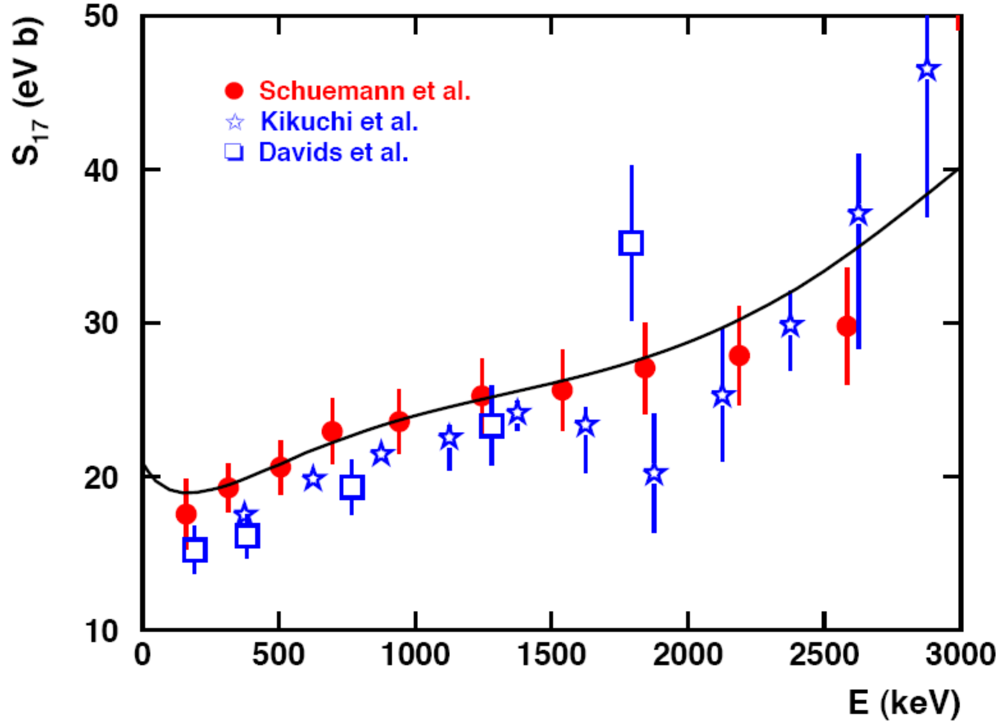


FIG. 8 (Color online) S_{17} values from CD experiments. Full red circles: latest analysis of the GSI CD experiment (Schümann *et al.*, 2006); open blue stars: Kikuchi *et al.* (1998) analyzed in first-order perturbation theory; open blue squares: Davids and Typel (2003). The error bars include statistical and estimated systematic errors. The curve is taken from the cluster-model theory of Descouvemont *et al.* (2004), normalized to $S_{17}(0) = 20.8$ eV b. Note the suppressed zero.

[Haxton et al, RMP (2010)]

outline

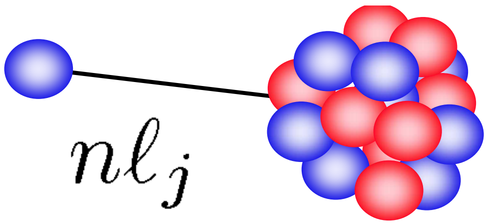
- continuum discretized coupled channels
- a test-bed: ^{8}B breakup
- breakup scaling with ANC
- breakup and (n,γ)
- core excitation and breakup
- breakup of three-body projectile
- summary

meeting point between reactions and structure

overlap function

$$\phi_{I_A:I_B}(\mathbf{r}) = \langle \Phi^A_{I_A}(\xi_A) | \Phi^B_{I_B}(\xi_A, \mathbf{r}) \rangle \rightarrow \boxed{I_{\ell j}(r),} \quad \int_0^\infty [I_{\ell j}(r)]^2 dr = S(J_i, J_f \ell j)$$

spectroscopic factor



$$B = A + v$$

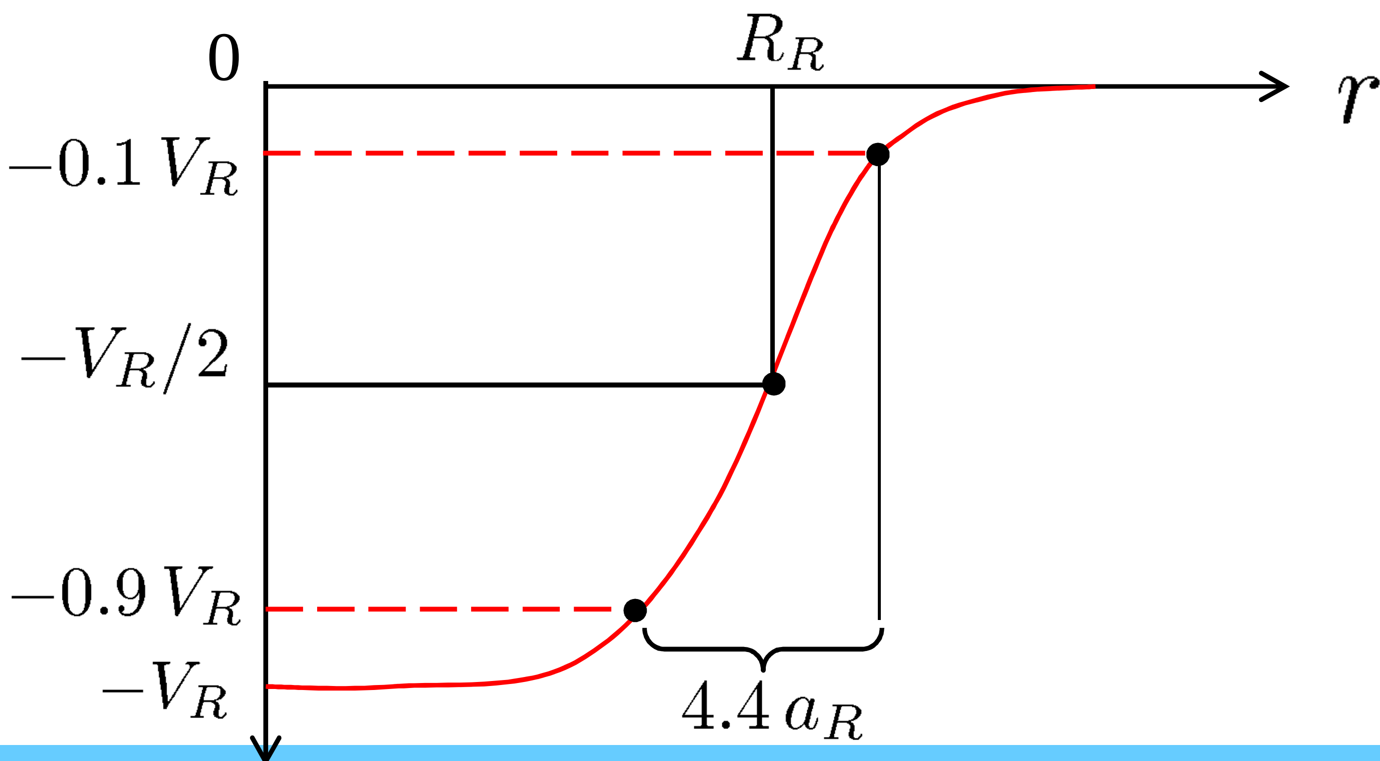
$$I\ (r) =_{r>R_n} C_\ell W_{-\eta,\ell+\frac{1}{2}}(-2k_I r)$$

asymptotic normalization coefficient

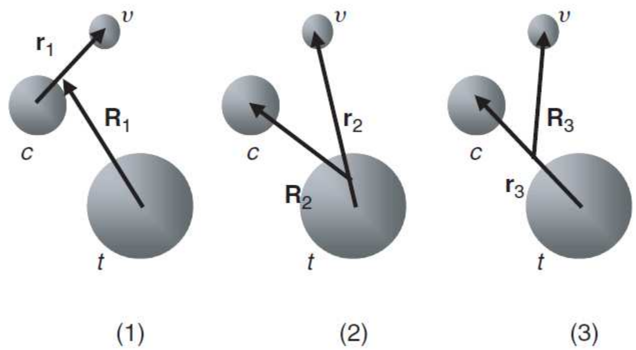
effective binding potential: fit to bound state properties

$$U(r) = V_C(r) + V(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_R = \frac{r - R_R}{a_R}$$



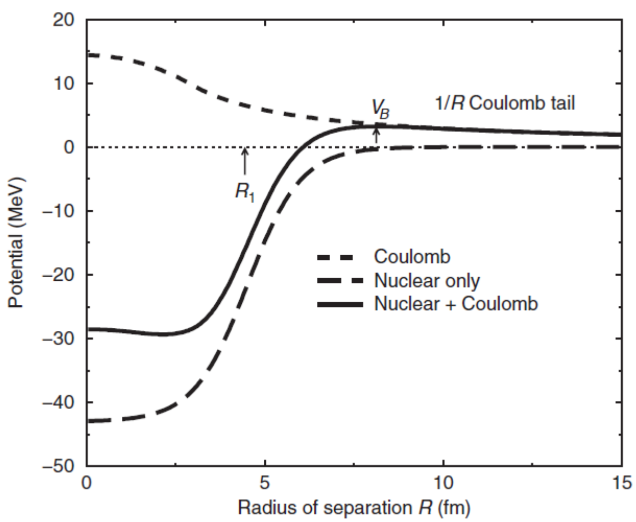
breakup with loosely bound nuclei: 3-body model



3 jacobi coordinate sets

3-body Hamiltonian for the problem:

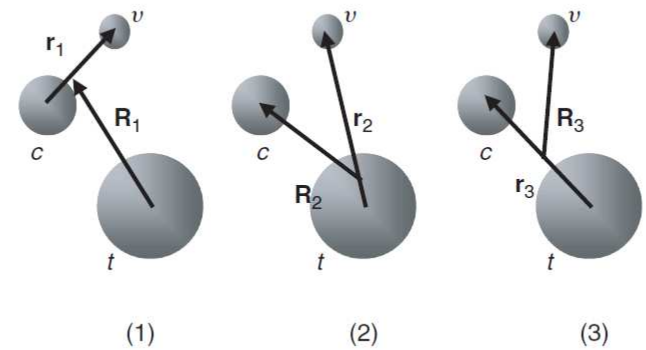
$$H_{3\mathrm{b}} = \hat{T} + V_{vc} + V_{vt} + V_{ct}$$



the interactions: nuclear and Coulomb

breakup within Faddeev

$$\Psi = \sum_{n=1}^{3} \Psi^{(n)}(\mathbf{r}_n, \mathbf{R}_n)$$



3 jacobi coordinate sets

3-body Hamiltonian for the problem:

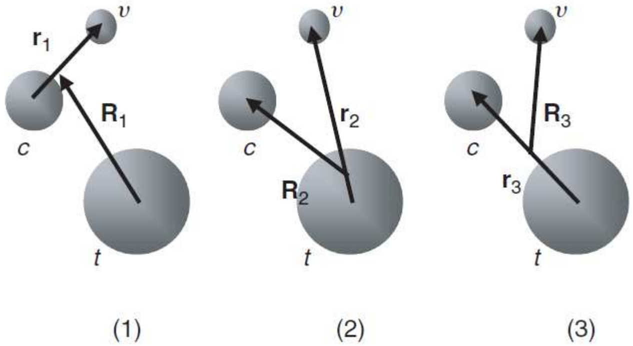
$$H_{3b} = \hat{T} + V_{vc} + V_{vt} + V_{ct}$$

Faddeev Equations:

$$(E - T_1 - V_{vc})\Psi^{(1)} = V_{vc}(\Psi^{(2)} + \Psi^{(3)})$$
$$(E - T_2 - V_{ct})\Psi^{(2)} = V_{ct}(\Psi^{(3)} + \Psi^{(1)})$$
$$(E - T_3 - V_{tv})\Psi^{(3)} = V_{tv}(\Psi^{(1)} + \Psi^{(2)})$$

breakup and transfer channels

$$\Psi = \sum_{n=1}^{3} \Psi^{(n)}(\mathbf{r}_n, \mathbf{R}_n)$$



Transfer components are asymptotically separated:

$$\Psi^{(n)} \rightarrow \sum_{p} \phi_p^{(n)}(\mathbf{r}_n)\psi_p^{(n)}(\mathbf{R}_n) + \text{breakup; when } R_n \rightarrow \infty$$

Usual nuclear physics case:
U_{ct} and U_{vt} are optical potentials; thus no bound states in transfer components.

optical potentials: fit to elastic scattering

$$U(r) = V_C(r) + V(r) + iW(r) + V_{so}(r)\vec{\ell}\cdot\vec{s}$$

$$V(r) = -\frac{V_R}{[1+\exp(X_R)]}\ , \qquad X_i = \frac{r-R_i}{a_i}$$

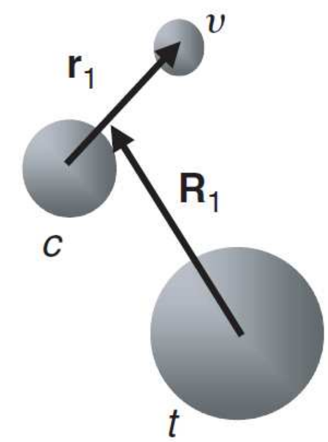
$$V_{so}(r) = -\frac{2V_{so}}{r\,a_{so}}\frac{\exp(X_{so})}{[1+\exp(X_{so})]^2}\ ,$$

$$W(r) = -\frac{W_V}{[1+\exp(X_V)]} - \frac{4W_S\exp(X_S)}{[1+\exp(X_S)]^2}\ ,$$

$$R_i = r_i\,A_2^{1/3} \quad \text{or} \quad R_i = r_i\left[A_1^{1/3}+A_2^{1/3}\right]$$

reduction to one jacobi set

$$[H_{3b} - E]\Psi^{(1)}(\mathbf{r}_1, \mathbf{R}_1) = 0$$



Expand wfn in eigenstates of projectile's internal Hamiltonian:

$$\Psi^{(1)}_{\mathbf{K}_0}(\mathbf{r}_1, \mathbf{R}_1) = \sum_{p=1}^{n_b} \phi_p(\mathbf{r}_1)\psi_p(\mathbf{R}_1) + \int \mathrm{d}\mathbf{k}\ \phi_{\mathbf{k}}(\mathbf{r}_1)\psi^{\mathbf{k}}_{\mathbf{K}}(\mathbf{R}_1)$$

Expand in partial waves:

$$\phi^M_{(p,k)}(\mathbf{r}) = \frac{u_{(p,k)}(r)}{r}\left[\left[Y_\ell(\hat{\mathbf{r}}) \otimes \mathcal{X}_s\right]_j \otimes \mathcal{X}_{I_c}\right]_{I_p M}$$

$$H_{proj}\phi_p = \varepsilon\phi_p$$

Radial wavefunctions for projectile:

$$\left[-\frac{\hbar^2}{2\mu_{vc}}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{\ell(\ell+1)}{r^2}\right) + V_{vc}(r) - \epsilon\right] u_{(p,k)}(r) = 0$$

bound states with $\epsilon_p < 0$

continuum states with energy $\epsilon_k > 0$

$$E_{\mathrm{cm}} + \epsilon_0 = E = \frac{\hbar^2 k^2}{2\mu_{vc}} + \frac{\hbar^2 K^2}{2\mu_{(vc)t}}$$

continuum bins

average method

$$\tilde{u}_p(r) = \sqrt{\frac{2}{\pi N_p}} \int_{k_{p-1}}^{k_p} g_p(k) u_k(r)\, \mathrm{d}k$$

- non overlaping continuum intervals - continuum bins are orthogonal
- square integrable

analytic form if potential is zero and l=0:

$$\tilde{u}_p(r) \propto \sin(k_p r) \frac{\sin((k_p - k_{p-1})r)}{r}$$

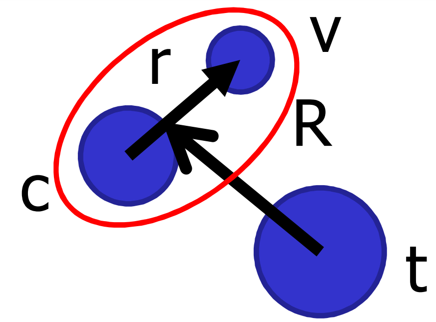
breakup: continuum discretized coupled channels

CDCC 3-body wavefunction:

$$\Psi^{\rm CDCC}(\mathbf{r},\mathbf{R}) = \sum_{p=0}^{N} \tilde{\phi}_p(\mathbf{r})\psi_p(\mathbf{R})$$

$$p = \{lsjI_cI_p; (k_{p-1}, k_p)\}$$

$$(H_{3b} - E)\Psi^{\rm CDCC}(\mathbf{r},\mathbf{R}) = 0$$



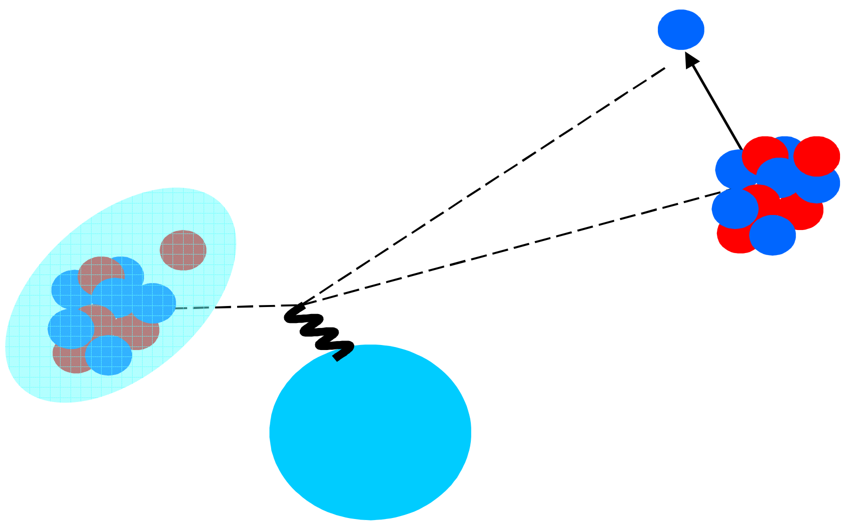
Coupled channel equations:

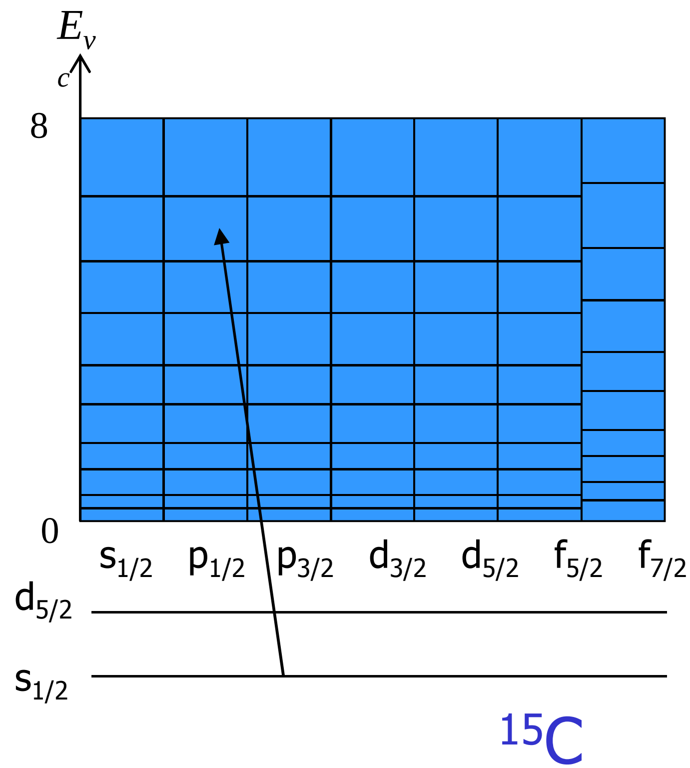
$$[\hat{T}_R + V_{pp}(R) - E_p]\psi_p(\mathbf{R}) + \sum_{p'\neq p} V_{pp'}(R)\psi_{p'}(\mathbf{R}) = 0$$

Coupling potentials: $V_{pp'}(R) = \langle\tilde{\phi}_p(r)|U_{vt}+U_{ct}|\tilde{\phi}_{p'}(r)\rangle$

Energies: $E_p = E - \tilde{\epsilon}_p$ $\quad \tilde{\epsilon}_p = \langle\tilde{\phi}_p(\mathbf{r})|H_{\rm int}|\tilde{\phi}_p(\mathbf{r})\rangle$

standard 3-body CDCC in one slide



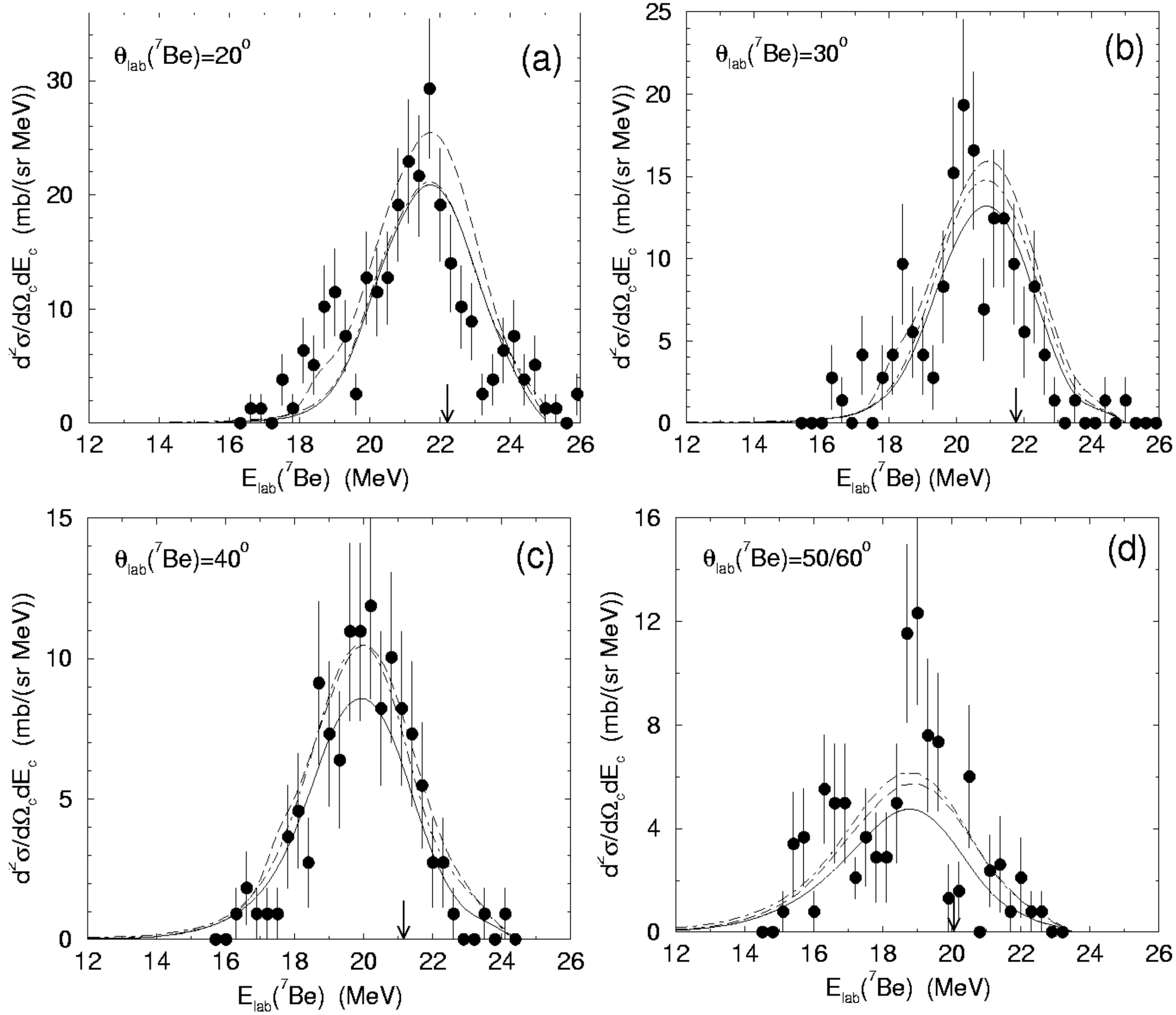


- three-body Hamiltonian for reaction
- three-body wavefunction expanded in term of projectile states
- projectile described in terms of single particle states
- optical potentials from elastic scattering

breakup of ^{8}B : theory versus data

^{8}B breakup on ^{58}Ni
(E_{beam}=26 MeV)

Results of CDCC calculations assuming a single particle structure for ^{8}B=^{7}Be+p



[PRC (2001) 024617]

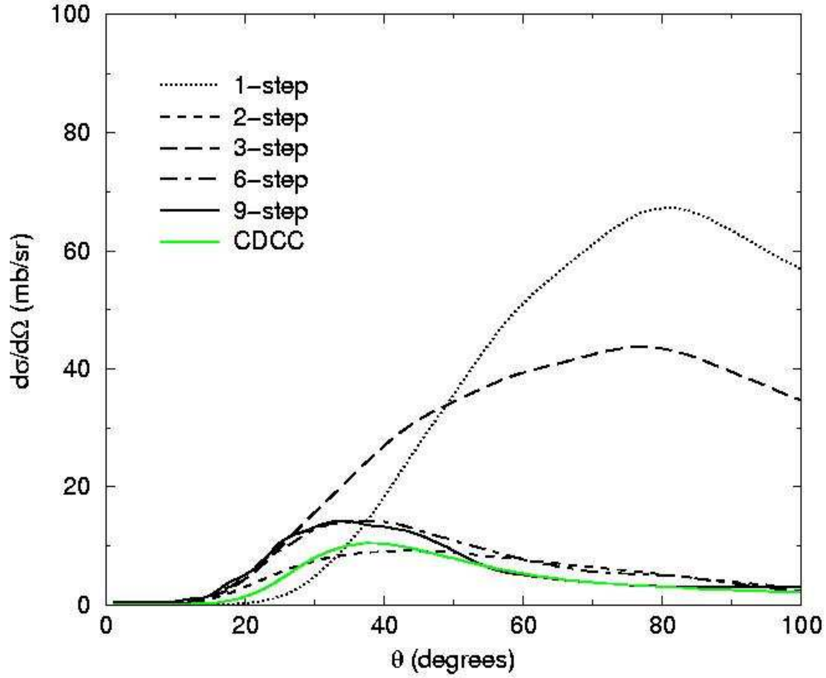
breakup ^{8}B: multistep effects

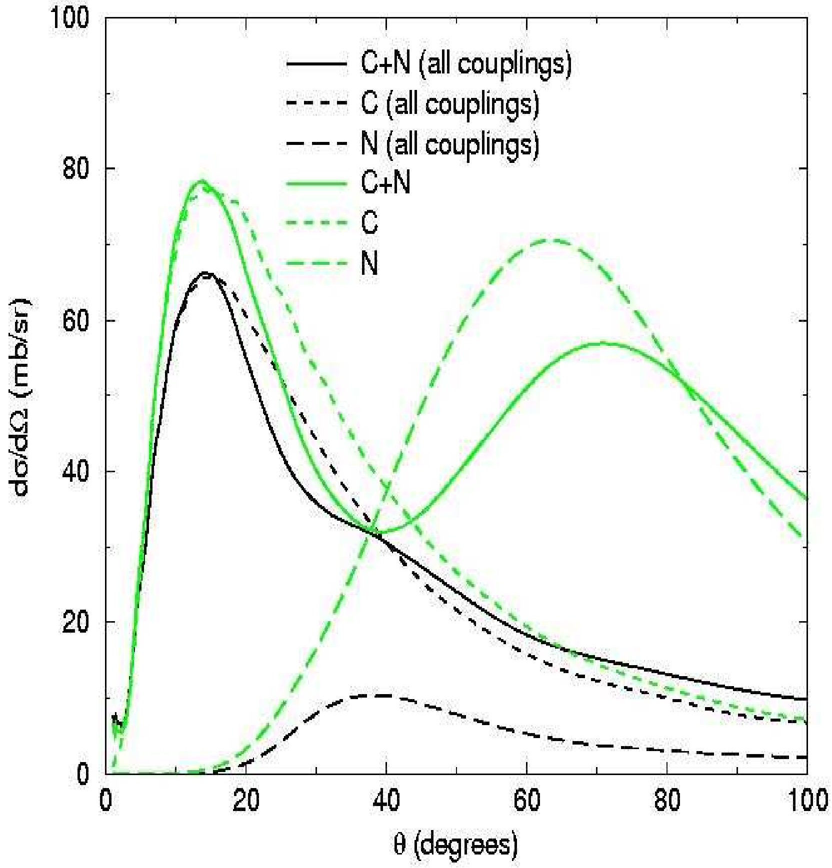
CDCC $^{8}B + ^{58}Ni \rightarrow ^{7}Be+p + ^{58}Ni$ (E_{beam}=26 MeV)

$^{8}B^*$ angular distributions show multi-step effects are crucial
[PRC 59 (1999) 2652]

continuum-continuum couplings are critical!!

strong nuclear destructive interference





breakup reactions with loosely bound nuclei

- breakup reactions are very peripheral
- cross sections depend essentially on the ANC of the bound state
- for low relative energies, small residual dependence on continuum properties

σ/C^2 is constant!

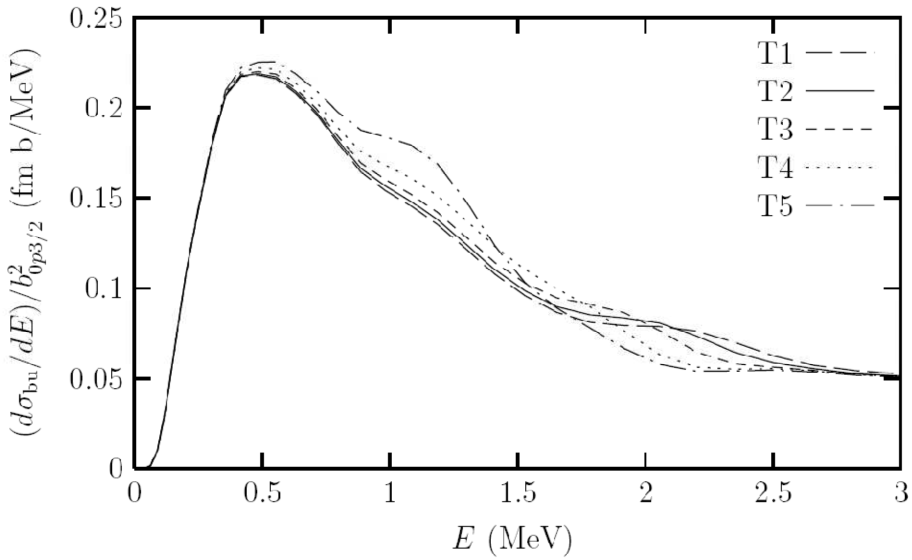


FIG. 1. Cross section for the breakup of ^8B on ^{58}Ni at 25.75 MeV divided by the square of the single-particle ANC. The cross section is given as a function of the ^7Be-p relative energy E after breakup. Calculations are performed with the different ^7Be-p potentials listed in Table I.

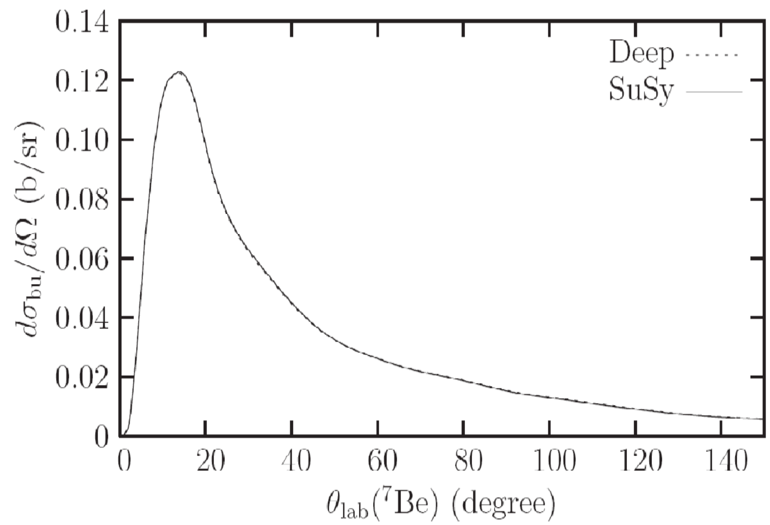


FIG. 3. Breakup cross section of ^8B on Ni at 26 MeV as a function of the ^7Be scattering angle after dissociation. The results are obtained using either the deep potential (dotted line) or its supersymmetric partner (full line). The difference is so small that both curves are superimposed.

[Capel and Nunes, Rev. C **73**, 014615 (2006)]
[Capel and Nunes, Rev. C **75**, 054609 (2007)]

breakup reactions and (n,γ): methodology

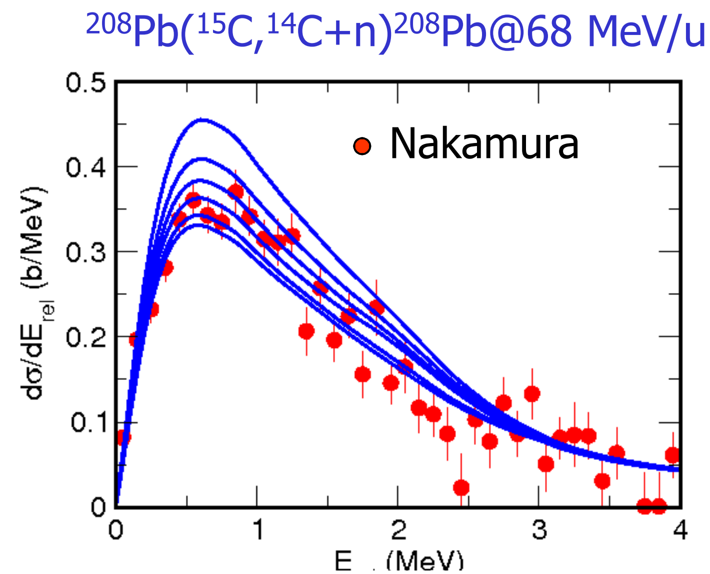
CDCC + set of single particle parameters

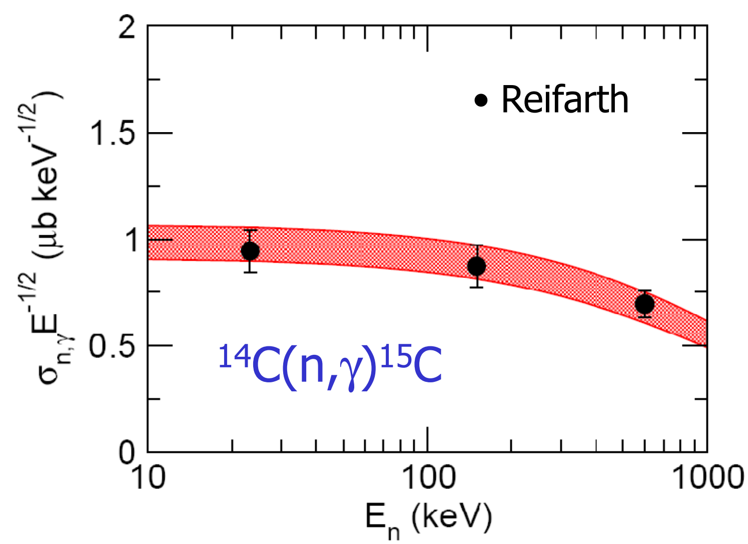
- extract ANC from χ^2 minimum
- error from $\varepsilon=\chi_{min}^2+1$

Yao, JPG33 (2006) 1

$$ANC = 1.32 \pm 0.07 \ \mathrm{fm}^{-1/2}$$

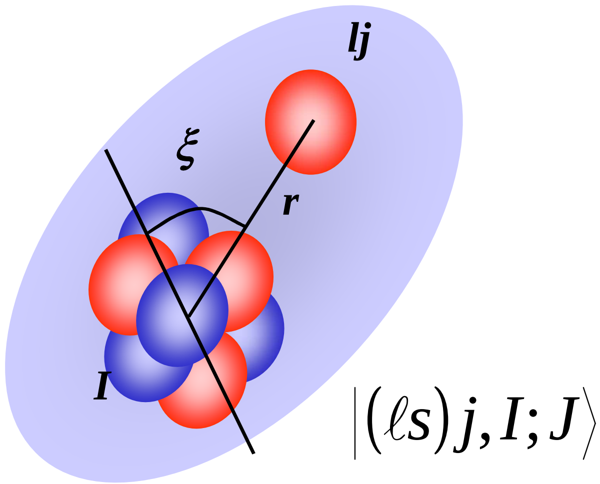
Summers and Nunes, PRC78(2009)069908



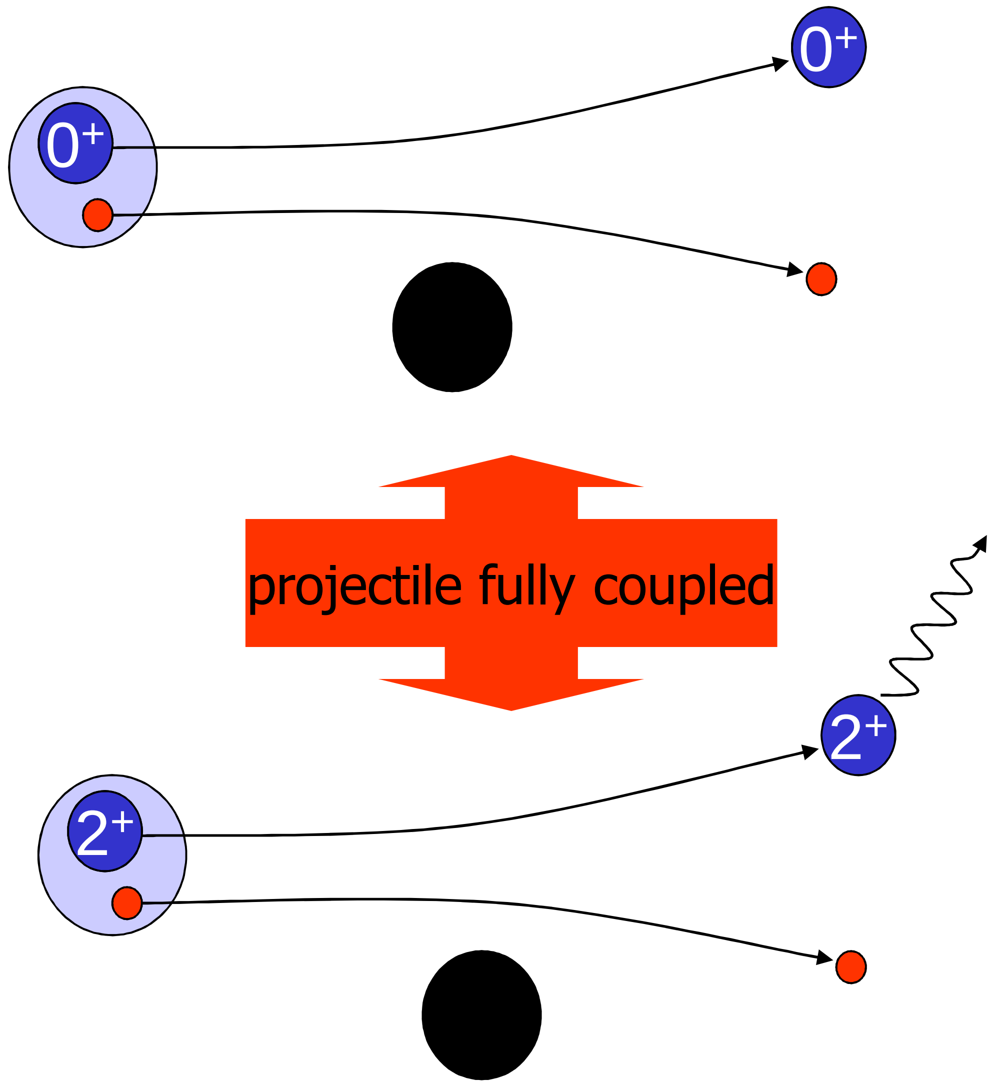


Nakamura et al, NPA722(2003)301c
Reifarth et al, PRC77,015804 (2008)

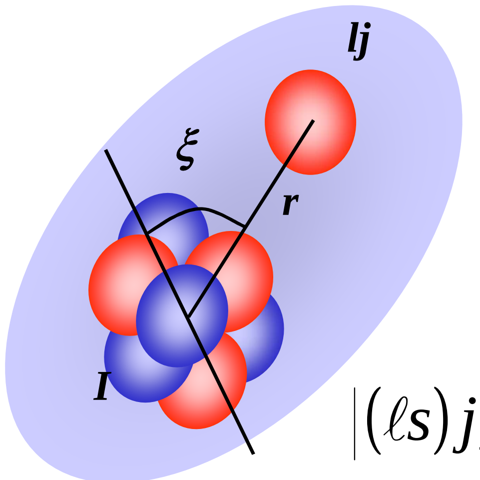
core excitation in breakup



^{11}Be example

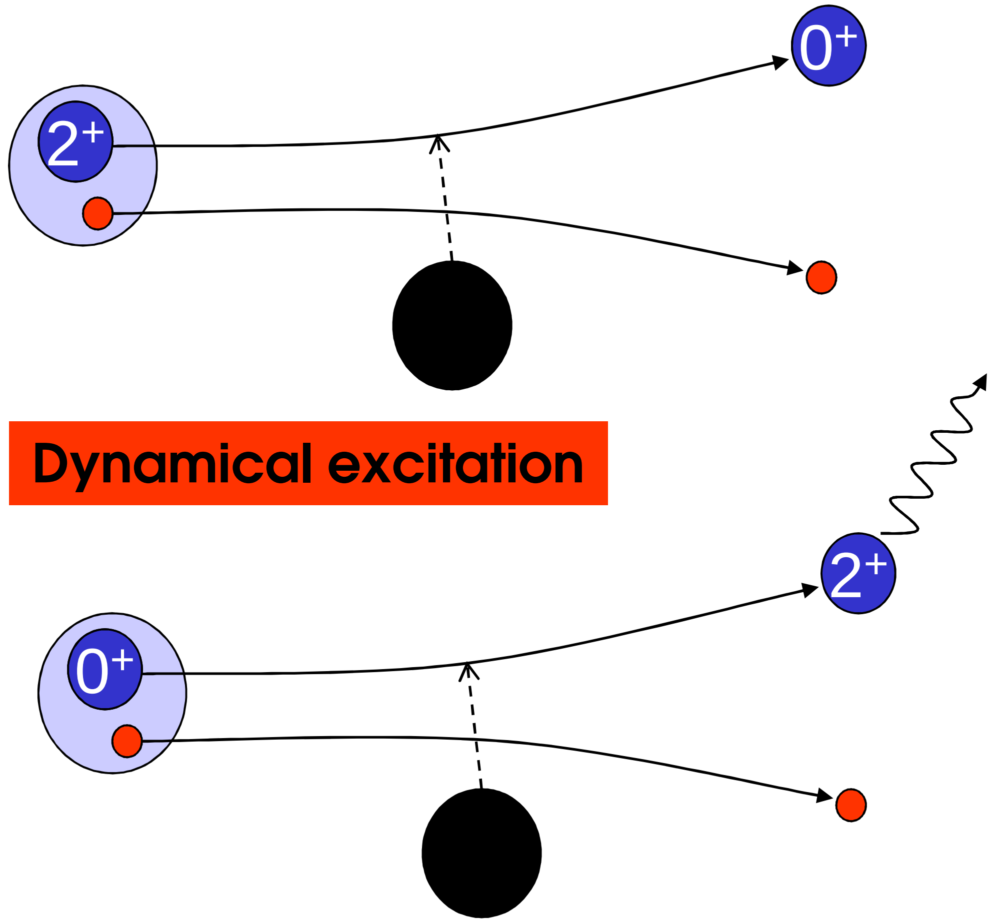


Core excitation in breakup



$$|(\ell s)j, I; J\rangle$$

^{11}Be example



Core excitation in breakup: XCDCC results

^{9}Be(^{11}Be,^{10}Be)X @ E=60 MeV/A

Comparison with other models

Model	σ_{0+}	σ_{2+}	σ
Eikonal	105 mb	3.4 mb	108 mb
CDCC	109 mb	1 mb	110 mb
XCDCC	109 mb	8 mb	117 mb

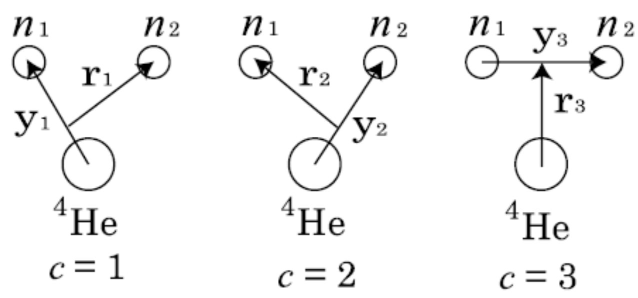
core state	σ_{bu}	σ_{st}	σ_{th}	σ_{exp}
0^+	109 mb	91 mb	200	203(31)
2^+	8 mb	6 mb	14	16(4)

Stripping cross section taken from eikonal calculations (J.A. Tostevin 2005)

Data: Aumann *et al.*, PRL84, 35 (2000)

[Summers, Nunes and Thompson, PRC 73 (2006) 031603R]

4-body CDCC:



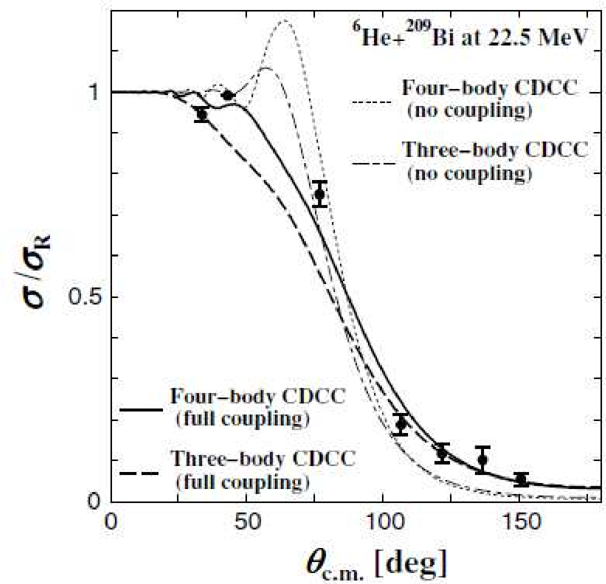


FIG. 3. The same as in Fig. 2 but for ^{6}He + ^{209}Bi scattering at 22.5 MeV. The experimental data are taken from Refs. [1,2]. We take the incident energy of 22.5 MeV shown in the first paper of Aguilera *et al.* [1].

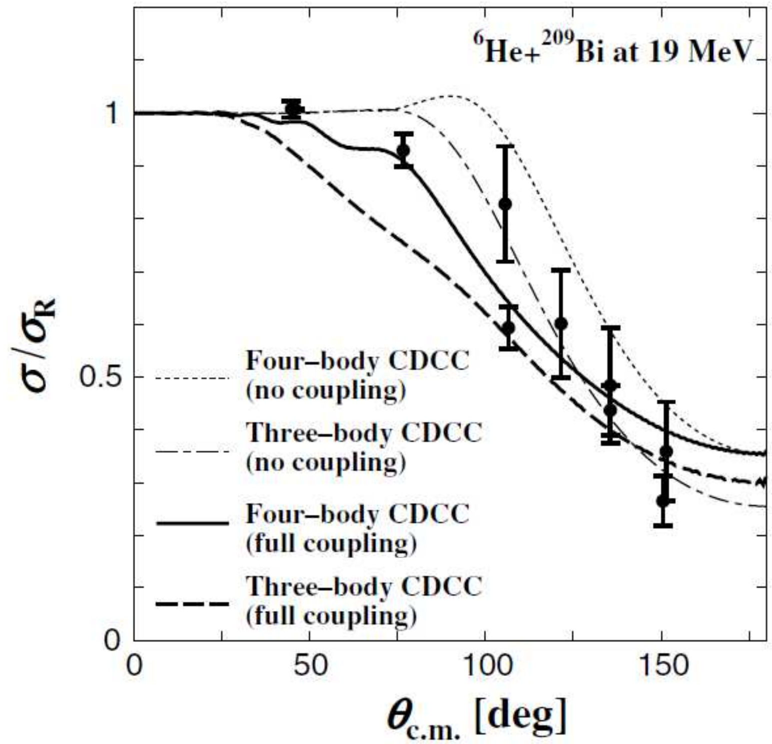


FIG. 2. Angular distribution of the elastic differential cross section as the ratio to the Rutherford cross section for ^{6}He + ^{209}Bi scattering at 19 MeV. The solid (dashed) and dotted (dot-dashed) lines show the results of the four-body CDCC (three-body CDCC) calculation with and without breakup effects, respectively. The experimental data are taken from Refs. [1,2]. The incident energy for the experimental data in the laboratory frame is shown to be 19 and 19.1 MeV in the first [1] and second [2] papers of Aguilera *et al.*, respectively; in the present study we take 19 MeV.

[Matsumoto et al, PRC 73, 051602]

CDCC advantages and limitations

- **advantages**
  - implementation in coordinate space
  - non perturbative
  - treats resonant and non-resonant continuum
  - can handle nuclear and Coulomb on equal footing
  - no restriction in mass (heavy nuclei)
  - can be mapped into underlying many-body theories

- **limitations**
  - not trivial to include non-local interactions
  - only up to four body reactions
  - grows with number of channels cubed
  - inaccurate when transfer channels are important

breakup: summary of recent activities

- **Continuum discretized coupled channels method (CDCC)**
  - nuclear and Coulomb to all orders
  - many applications to weakly bound nuclei: good description of data
  - scaling with square of ANC
- **Coulomb dissociation can be used to extract peripheral (n,γ)**
  - new methodology based on extracting the ANC
  - $^{14}C(n,\gamma)^{15}C$ from Coulomb dissociation consistent with direct capture data
- **Extensions of CDCC**
  - including core excitation (XCDCC)
  - including 3-body projectiles (4-body CDCC)
- **Breakup and transfer on the same footing**
  - CDCC and Faddeev calculations were compared
  - excellent agreement for cases where transfer coupling is weak
  - mismatches for the case of ^{11}Be on protons…

Challenges for reaction theory

- large basis sets – need decoupling approximations
- lack of accurate optical potentials – need more data
- lack of rigorous theories connecting short-range properties and long-range dynamics - need microscopic overlap functions with correct asymptotic behaviour

does this sound familiar?
talk by Roman Krems

thanks to:

few-body group at MSU: Bik Nguyen, Jun Hong, Pierre Capel

collaborators: Ian Thompson and Neil Summers (LLNL)
Akram Mukhamedzhanov (TAMU)
Ivan Brida (ANL)
Charlotte Elster (OU)
Arnas Deltuva and Antonio Fonseca (CFNUL)
Antonio Moro (Seville)
Ron Johnson (Surrey)