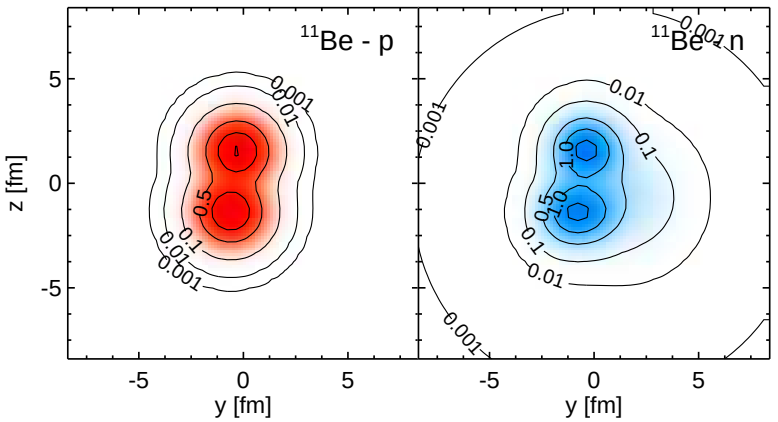
Fermionic Molecular Dynamics for Weakly-Bound Systems



Thomas Neff

INT Workshop
Weakly-Bound Systems in Atomic & Nuclear Physics

Seattle, USA
March 12, 2010

HELMHOLTZ ASSOCIATION

GSI

Overview

Effective Nucleon-Nucleon interaction:

Unitary Correlation Operator Method

- **Short-range Correlations**
- **Correlated Interaction**

Many-Body Method:

Fermionic Molecular Dynamics

- **Model**
- **Beryllium Isotopes**
- **Cluster States in ^{12}C**

Nucleon-Nucleon Interaction

Short-range Correlations

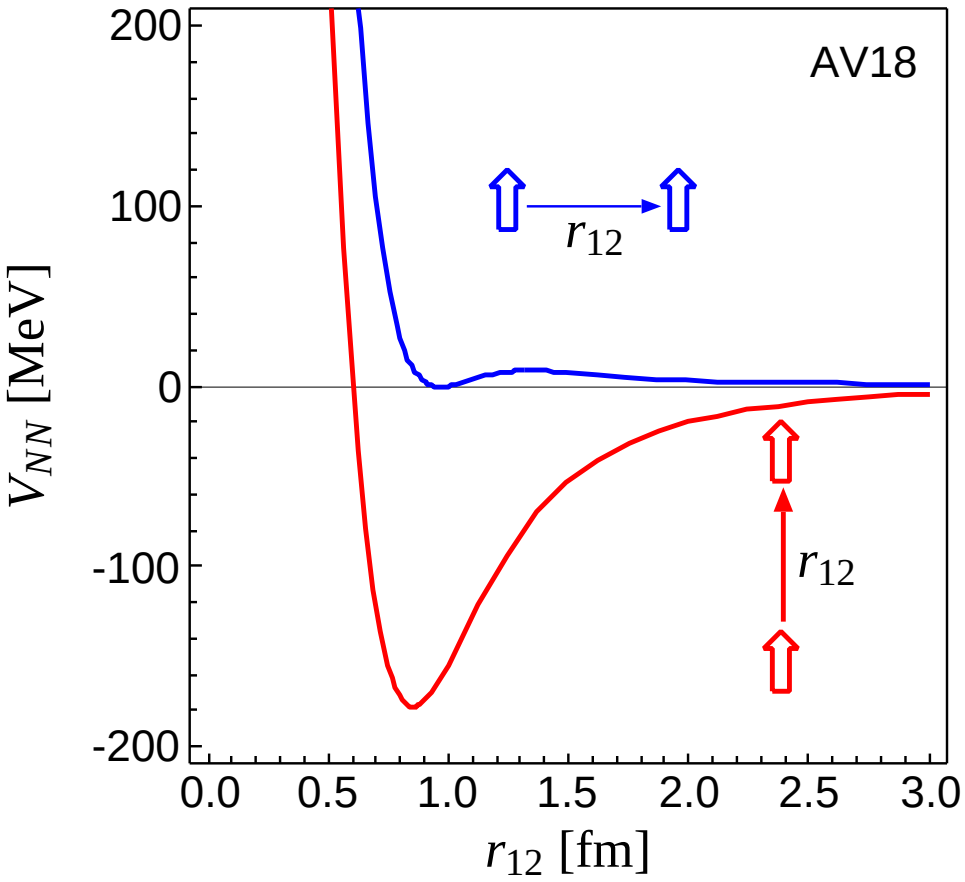
Unitary Correlation Operator Method

- **Correlation Operators**
- **Interaction in Momentum Space**
- ***ab initio* calculations**

Nuclear Force

Argonne V18 (T=0)

spins aligned parallel or perpendicular to the relative distance vector



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➨ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➨ **tensor correlations**

the nuclear force will induce **strong short-range correlations** in the nuclear wave function

UCOM

Unitary Correlation Operator Method

Correlation Operator

- induce short-range (two-body) central and tensor correlations into the many-body state

$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r = \exp\Big[-i \sum_{i<j} \underset{\sim}{g}_{\Omega,ij}\Big] \exp\Big[-i \sum_{i<j} \underset{\sim}{g}_{r,ij}\Big] \quad , \quad \underset{\sim}{C}^\dagger \underset{\sim}{C} = \underset{\sim}{1}$$

- correlation operator should conserve the symmetries of the Hamiltonian and should be of finite-range, correlated interaction **phase shift equivalent** to bare interaction by construction

Correlated Operators

- correlated operators will have contributions in higher cluster orders

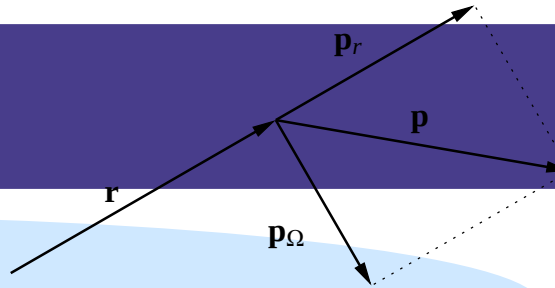
$$\underset{\sim}{C}^\dagger \underset{\sim}{O} \underset{\sim}{C} = \underset{\sim}{\hat{O}}^{[1]} + \underset{\sim}{\hat{O}}^{[2]} + \underset{\sim}{\hat{O}}^{[3]} + \ldots$$

- two-body approximation: correlation range should be small compared to mean particle distance

Correlated Interaction

$$\underset{\sim}{C}^\dagger (\underset{\sim}{T} + \underset{\sim}{V}) \underset{\sim}{C} = \underset{\sim}{T} + \underset{\sim}{V}_{\mathrm{UCOM}} + \underset{\sim}{V}^{[3]}_{\mathrm{UCOM}} + \ldots$$

UCOM
Central and Tensor Correlations



$$\underset{\sim}{C} = \underset{\sim}{C}_\Omega \underset{\sim}{C}_r$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

$$\mathbf{p}_r = \tfrac{1}{2}\left\{\frac{\mathbf{r}}{r}\left(\frac{\mathbf{r}}{r}\mathbf{p}\right) + \left(\mathbf{p}\frac{\mathbf{r}}{r}\right)\frac{\mathbf{r}}{r}\right\}, \qquad \mathbf{p}_\Omega = \frac{1}{2r}\left\{\mathbf{l}\times\frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r}\times\mathbf{l}\right\}$$

Central Correlations

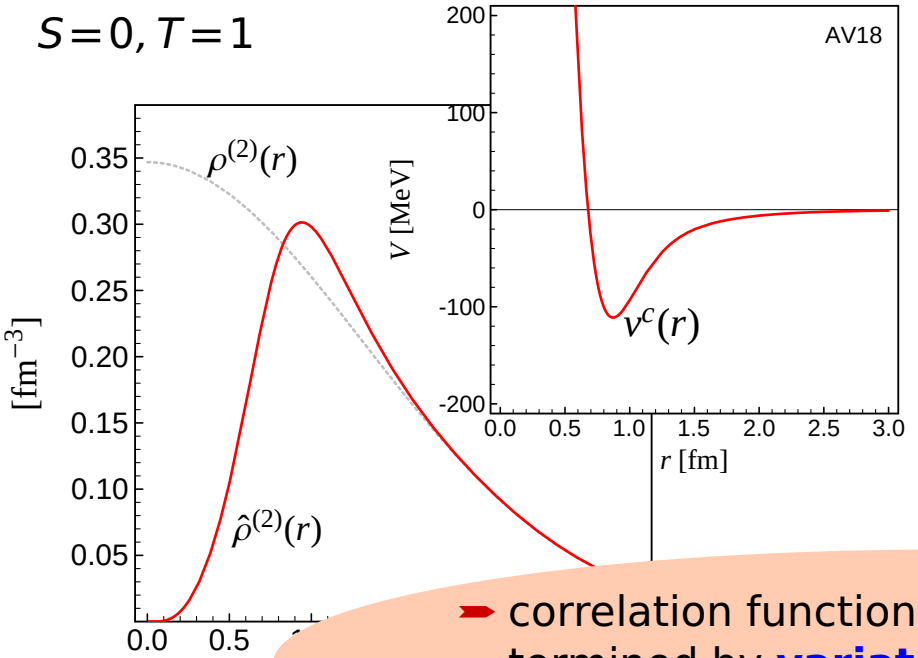
$$\underset{\sim}{c}_r = \exp\left\{-\tfrac{i}{2}\{p_r s(r) + s(r) p_r\}\right\}$$

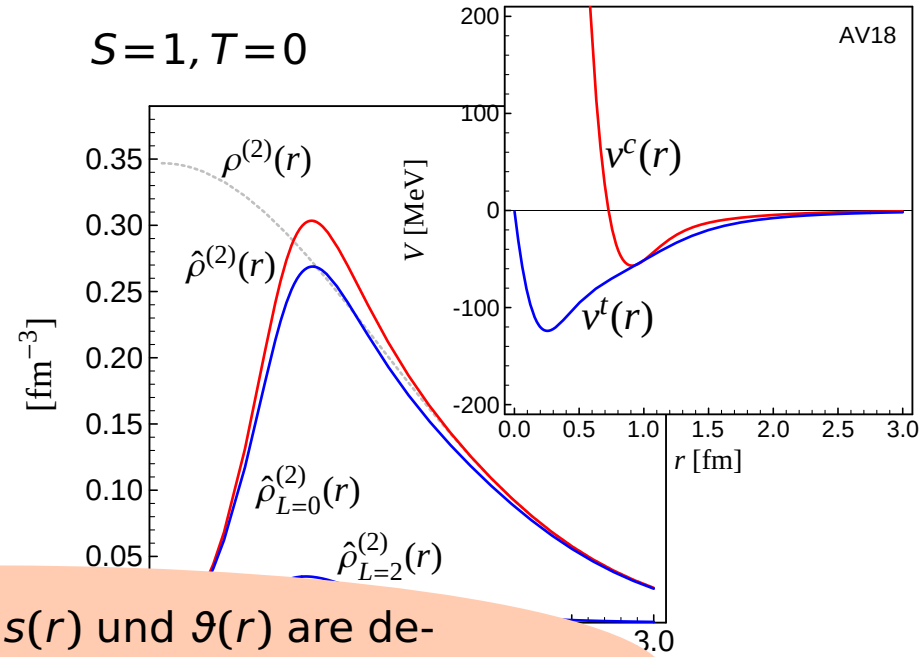
- probability density shifted out of the repulsive core

Tensor Correlations

$$\underset{\sim}{c}_\Omega = \exp\left\{-i\vartheta(r)\left\{\tfrac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{p}_\Omega)(\boldsymbol{\sigma}_2\cdot\mathbf{r}) + \tfrac{3}{2}(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_\Omega)\right\}\right.$$

- tensor force admixes other angular momenta

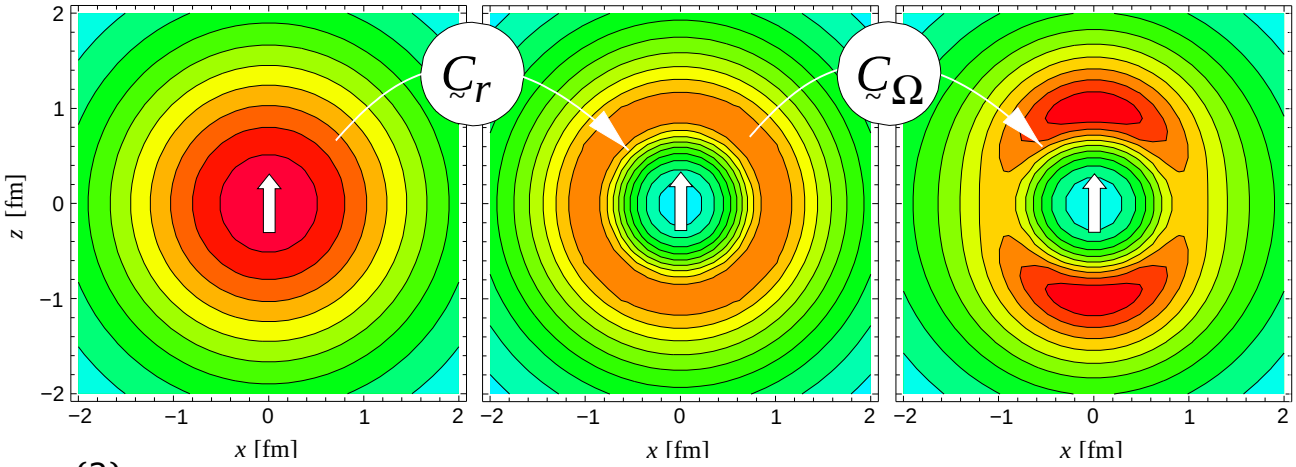




- correlation functions $s(r)$ und $\vartheta(r)$ are determined by **variation** of the energy in the **two-body system** for each S, T channel

Realistic Effective Interaction

two-body densities



$\rho^{(2)}_{S,T}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$

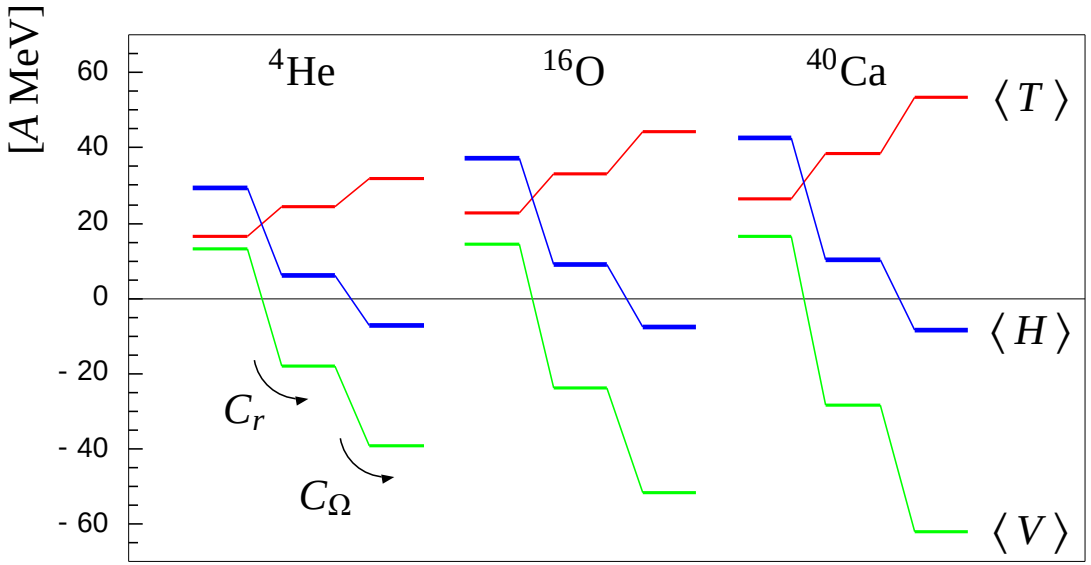
central correlator C_r shifts density out of the repulsive core

tensor correlator C_Ω aligns density with spin orientation

both central and tensor correlations are essential for binding

energies

$0\hbar\omega$ Harmonic Oscillator

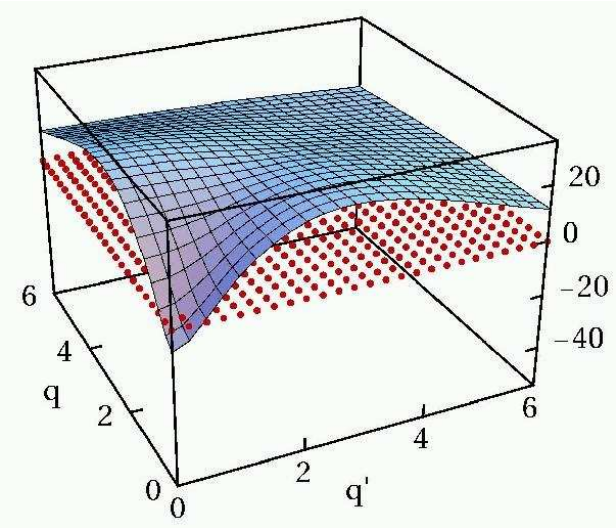


Neff and Feldmeier, Nucl. Phys. **A713** (2003) 311

Unitary Correlation Operator Method

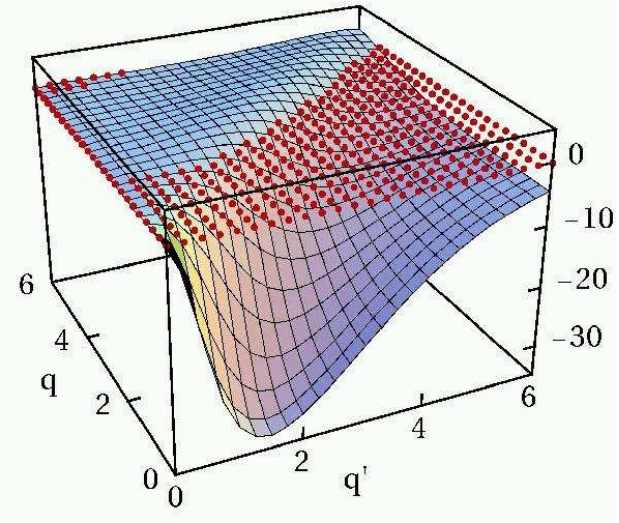
Correlated Interaction in Momentum Space

$^3S_1$ bare

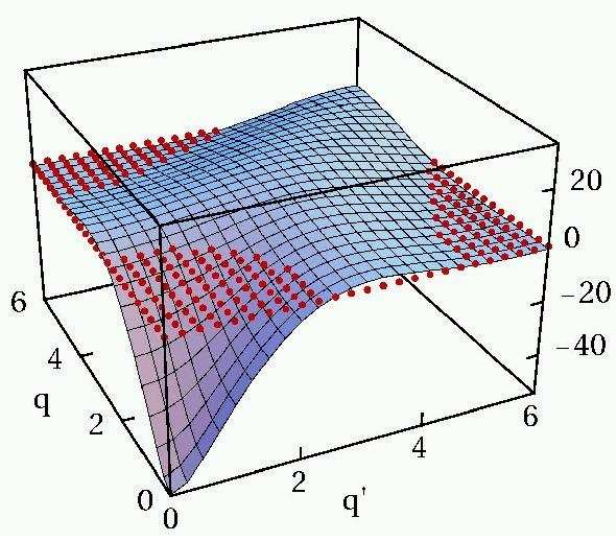


bare interaction has **strong off-diagonal** matrix elements connecting to high momenta

$^3S_1$ - $^3D_1$ bare

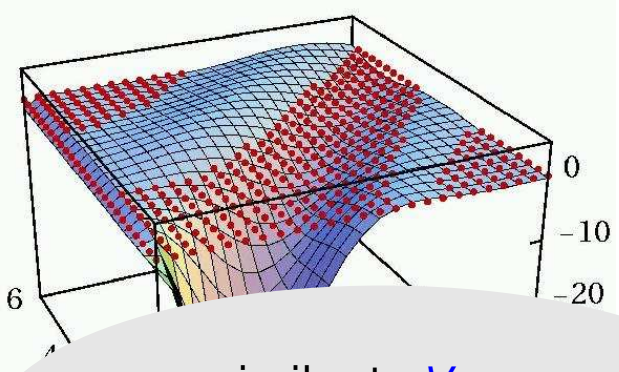


$^3S_1$ correlated



correlated interaction is **more attractive** at low momenta

$^3S_1$ - $^3D_1$ correlated

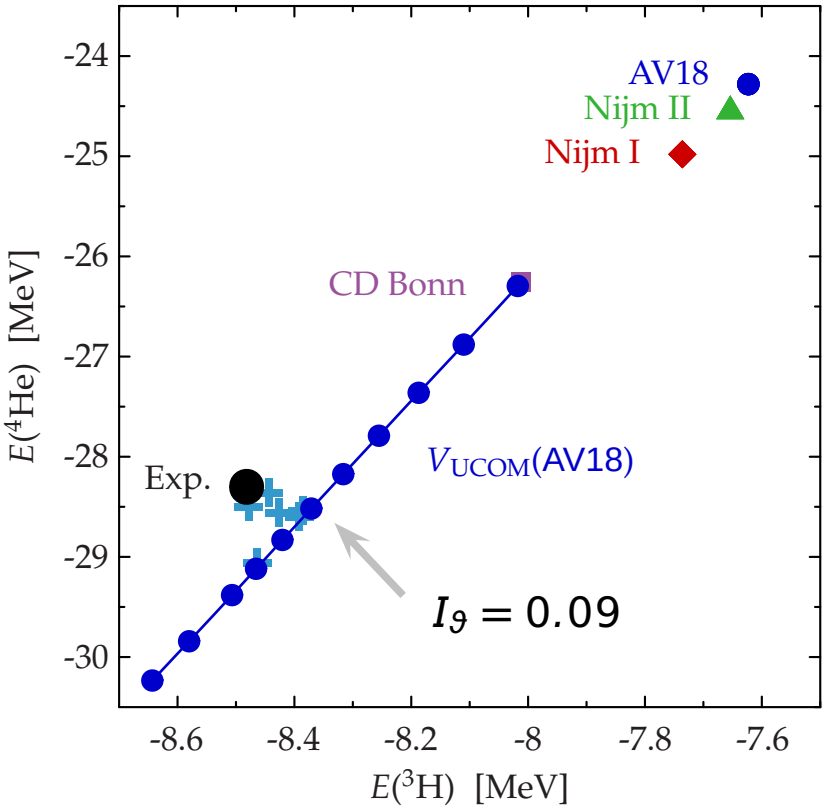


off-diagonal matrix elements connecting low- and high- momentum states are **strongly reduced**

similar to $V_{low\text{-}k}$
Bogner, Kuo, Schwenk, Phys. Rep. **386**, 1 (2003)

Roth, Hergert, Papakonstaninou, Neff, Feldmeier, Phys. Rev. C **72**, 034002 (2005)

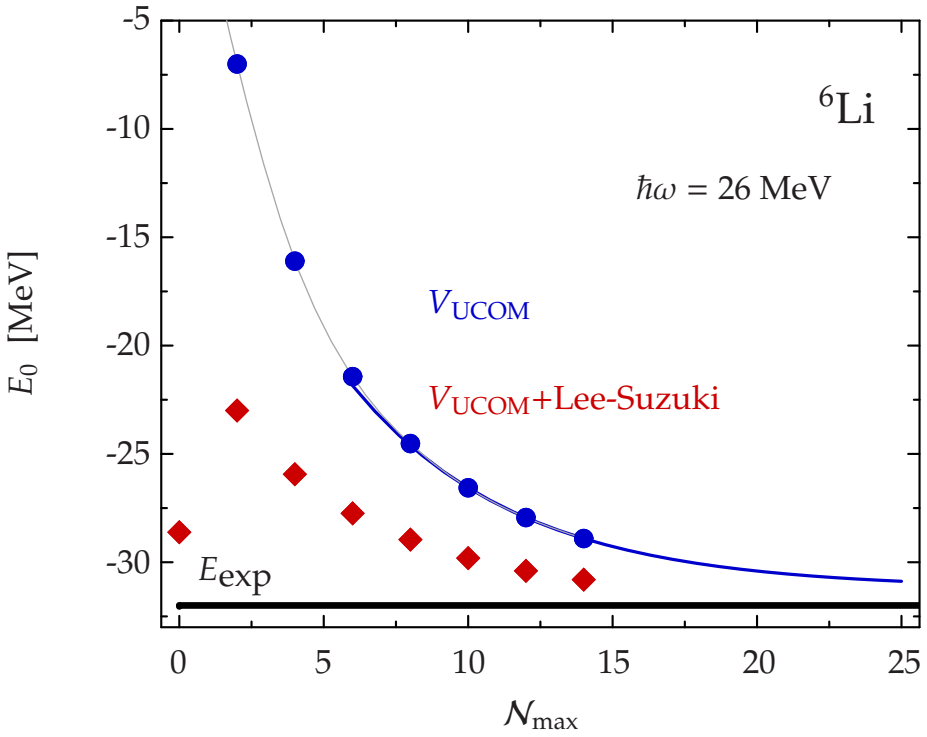
Tjon Line and Three-body Contributions

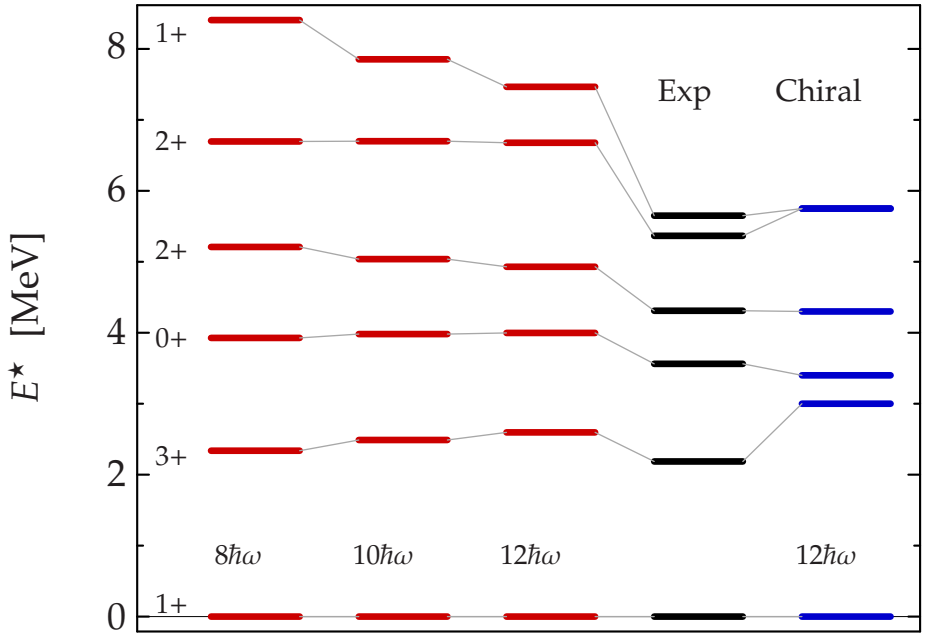


- choose tensor correlation range $I_\vartheta = 0.09$ such that **need for three-body forces is minimized**

➨ **different perspective**: don't try to reproduce the results with the bare interaction but consider **V_{UCOM} as a realistic potential**

UCOM
NCSM ^{6}Li





calculations by Petr Navrátil, LLNL

- NCSM calculations with “bare” V_{UCOM} and Lee-Suzuki effective interaction derived from V_{UCOM} show consistent convergence pattern
- Binding energy close to experiment
- Spectra with V_{UCOM} are of similar quality than with other modern NN forces

Fermionic Molecular Dynamics

Motivation

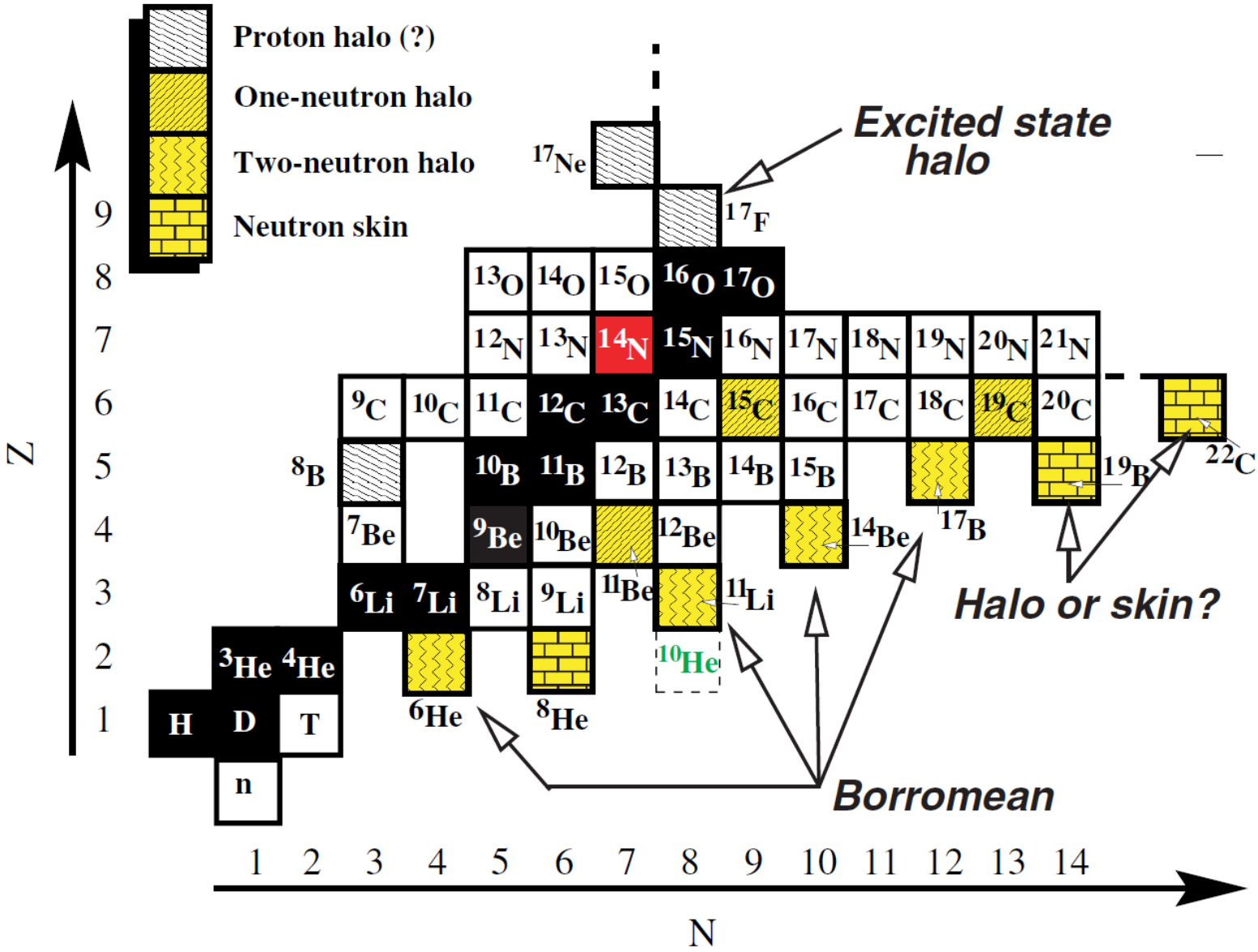
FMD Wave Functions

Nucleon-Nucleon Interaction

Mean-Field Calculations

**Projection After Variation,
Variation After Projection
and Multiconfiguration**

Exotica: Special Challenges



Al-Khalili, Nunes, J. Phys. G **29**, R89 (2003)

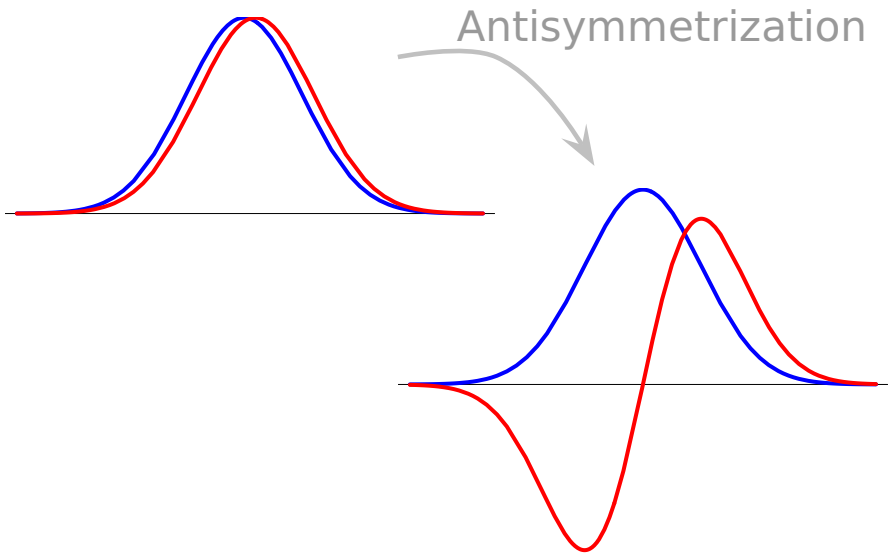
FMD

Fermionic Molecular Dynamics

Fermionic

Slater determinant

$$|Q\rangle = \mathcal{A}\Big(|q_1\rangle \otimes \cdots \otimes |q_A\rangle\Big)$$



- antisymmetrized A-body state

Molecular

single-particle states

$$\langle \mathbf{x}|q\rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i}\right\} \otimes |\chi^{\uparrow}_i, \chi^{\downarrow}_i\rangle \otimes |\xi\rangle$$

- Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed
- width a_i is an independent variational parameter for each wave packet
- use one or two wave packets for each single particle s

see also
Antisymmetrized Molecular Dynamics
H. Horiuchi, Y. Kanada-En'yo

Feldmeier, Schnack, Rev. Mod. Phys. **72** (2000) 655

Neff, Feldmeier, Nucl. Phys. **A738** (2004) 357

Evaluation of Matrix Elements

- non-orthogonal basis, use inverse overlap matrix

One-Body Operators

$$\frac{\langle Q \left| \underset{\sim}{T}^{[1]} \right| Q \rangle}{\langle Q \left| Q \right. \rangle} = \sum_{k,l}^{A} \langle q_k \left| \underset{\sim}{T}^{[1]} \right| q_l \rangle \, o_{lk}$$

Two-Body Operators

$$\frac{\langle Q \left| \underset{\sim}{V}^{[2]} \right| Q \rangle}{\langle Q \left| Q \right. \rangle} = \frac{1}{2} \sum_{k,l,m,n}^{A} \langle q_k, q_l \left| \underset{\sim}{V}^{[2]} \right| q_m, q_n \rangle \, (o_{mk} o_{nl} - o_{ml} o_{nk})$$

$$o = n^{-1} = \left(\langle q_i \left| q_j \right. \rangle \right)^{-1}$$

Interaction Matrix Elements

(One-body) Kinetic Energy

$$\langle q_k \left| \underset{\sim}{T} \right| q_l \rangle = \langle a_k \mathbf{b}_k \left| \underset{\sim}{T} \right| a_l \mathbf{b}_l \rangle \langle \chi_k | \chi_l \rangle \langle \xi_k | \xi_l \rangle$$

$$\langle a_k \mathbf{b}_k \left| \underset{\sim}{T} \right| a_l \mathbf{b}_l \rangle = \frac{1}{2m} \left(\frac{3}{a_k^\star + a_l} - \frac{(\mathbf{b}_k^\star - \mathbf{b}_l)^2}{(a_k^\star + a_l)^2} \right) R_{kl}$$

(Two-body) Potential

- fit radial dependencies by (a sum of) Gaussians

$$G(\mathbf{x}_1 - \mathbf{x}_2) = \exp\left\{ -\frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{2\kappa} \right\}$$

- Gaussian integrals

$$\langle a_k \mathbf{b}_k, a_l \mathbf{b}_l \left| \underset{\sim}{G} \right| a_m \mathbf{b}_m, a_n \mathbf{b}_n \rangle = R_{km} R_{ln} \left(\frac{\kappa}{\alpha_{klmn} + \kappa} \right)^{3/2} \exp\left\{ -\frac{\boldsymbol{\rho}_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\}$$

- analytical formulas for matrix elements

$$\alpha_{klmn} = \frac{a_k{}^\star a_m}{a_k{}^\star + a_m} + \frac{a_l{}^\star a_n}{a_l{}^\star + a_n}$$

$$\boldsymbol{\rho}_{klmn} = \frac{a_m \mathbf{b}_k{}^\star + a_k{}^\star \mathbf{b}_m}{a_k{}^\star + a_m} - \frac{a_n \mathbf{b}_l{}^\star + a_l{}^\star \mathbf{b}}{a_l{}^\star + a_n}$$

$$R_{km} = \langle a_k \mathbf{b}_k \left| a_m \mathbf{b}_m \right. \rangle$$

Effective interaction based on V_{UCOM}

Effective two-body interaction

- FMD model space can't describe correlations induced by residual medium-long ranged tensor forces

➨ use **long ranged tensor correlator – "low cutoff"** to partly account for that

- no three-body forces, missing spin-orbit strength, radii tend to be too small

➨ add phenomenological two-body correction term with a **momentum-dependend** central and (isospin-dependend) **spin-orbit** part (about 15% contribution to potential)

➨ fit correction term to binding energies and radii of "closed-shell" nuclei (^{4}He, ^{16}O, ^{40}Cα), (^{24}O, ^{34}Si, ^{48}Cα)

➨ **Outlook:**
use **three-body** or **density dependent two-body force** instead of two-body correction term

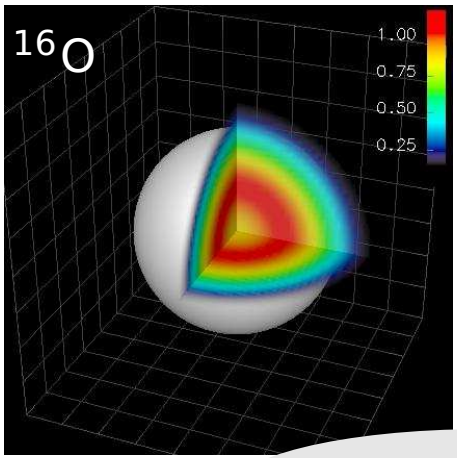
Mean-Field Calculations

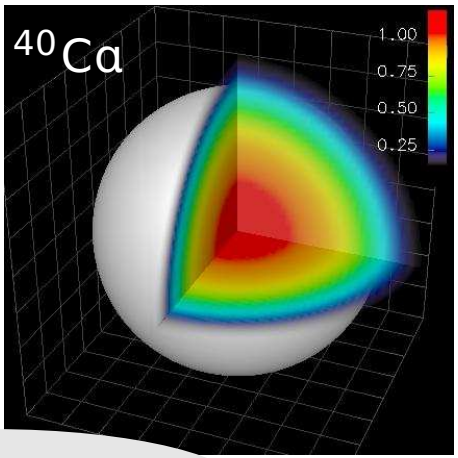
Minimization

- minimize Hamiltonian expectation value with respect to all single-particle parameters q_k

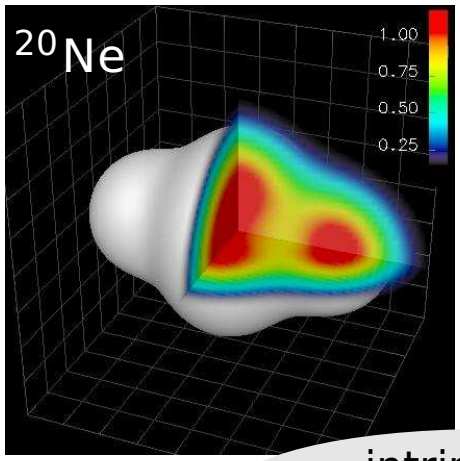
$$\min_{\{q_k\}} \frac{\langle Q \left| \underset{\sim}{H} - \underset{\sim}{T}_{cm} \right| Q \rangle}{\langle Q \left| Q \right\rangle}$$

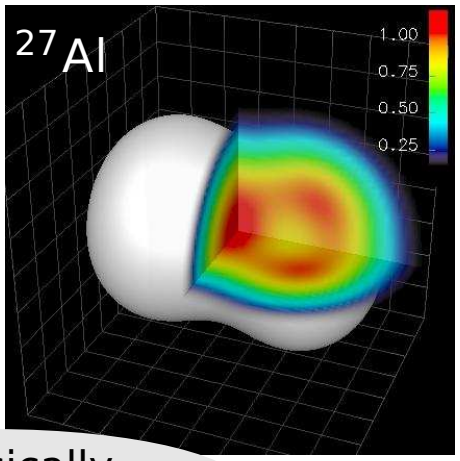
- this is a Hartree-Fock calculation in our particular single-particle basis
- the mean-field may break the symmetries of the Hamiltonian





spherical nuclei





intrinsically deformed nuclei

PAV, VAP and Multiconfiguration

Projection After Variation (PAV)

$$\underset{\sim}{P}^{\pi} = \frac{1}{2}(1 + \pi \underset{\sim}{\Pi})$$

- mean-field may break symmetries of Hamiltonian
- restore inversion, translational and rotational symmetry by projection on parity, linear and angular momentum

$$\underset{\sim}{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int \mathrm{d}^3\Omega \, D^{J}_{MK}{}^{\star}(\Omega) \, \underset{\sim}{R}(\Omega)$$

Variation After Projection (VAP)

$$\underset{\sim}{P}^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int \mathrm{d}^3X \, \exp\{-i(\underset{\sim}{\mathbf{P}} - \mathbf{P})\cdot\mathbf{X}\}$$

- effect of projection can be large
- full Variation after Angular Momentum and Parity Projection (VAP) for light nuclei
- perform VAP in GCM sense by applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimizing the energy in the projected energy surface for heavier nuclei

Multiconfiguration Calculations

- **diagonalize** Hamiltonian in a set of projected intrinsic states

$$\left\{ \left| Q^{(\alpha)} \right\rangle, \quad \alpha = 1, \ldots, N \right\}$$

$$\sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{J^{\pi}}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b} =$$

$$E^{J^{\pi}\alpha} \sum_{K'b} \left\langle Q^{(a)} \right| \underset{\sim}{P}^{J^{\pi}}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{\alpha}_{K'b}$$

Beryllium Isotopes

Results still preliminary !

Questions

- α**-clustering, halos in** ^{11}Be **and** ^{14}Be**,** $N = 8$ **shell closure ?**

Calculation

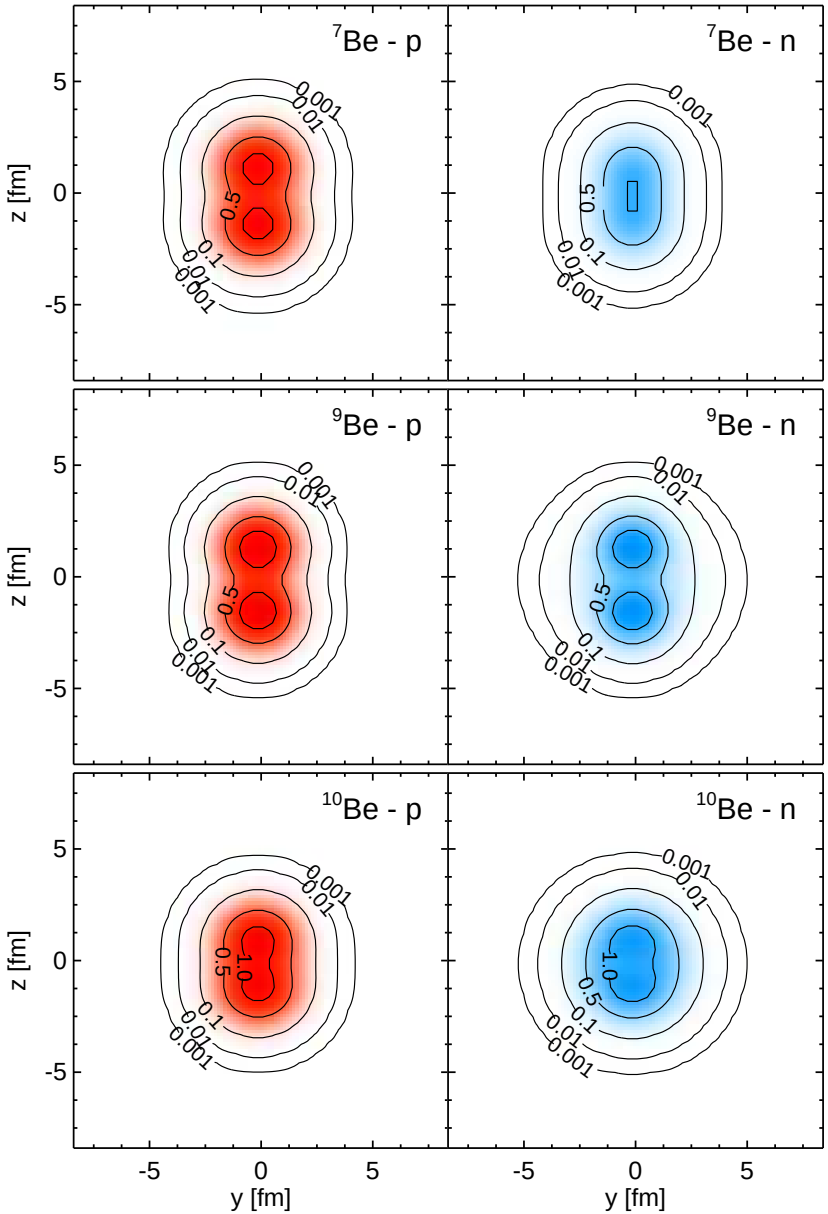
- **FMD wave functions with two Gaussians per sp-state**
- **mean field, variation after projection, variation after multiconfiguration mixing**
- **VAP and multiconfiguration-VAP configurations with mean proton distance as generator coordinate**

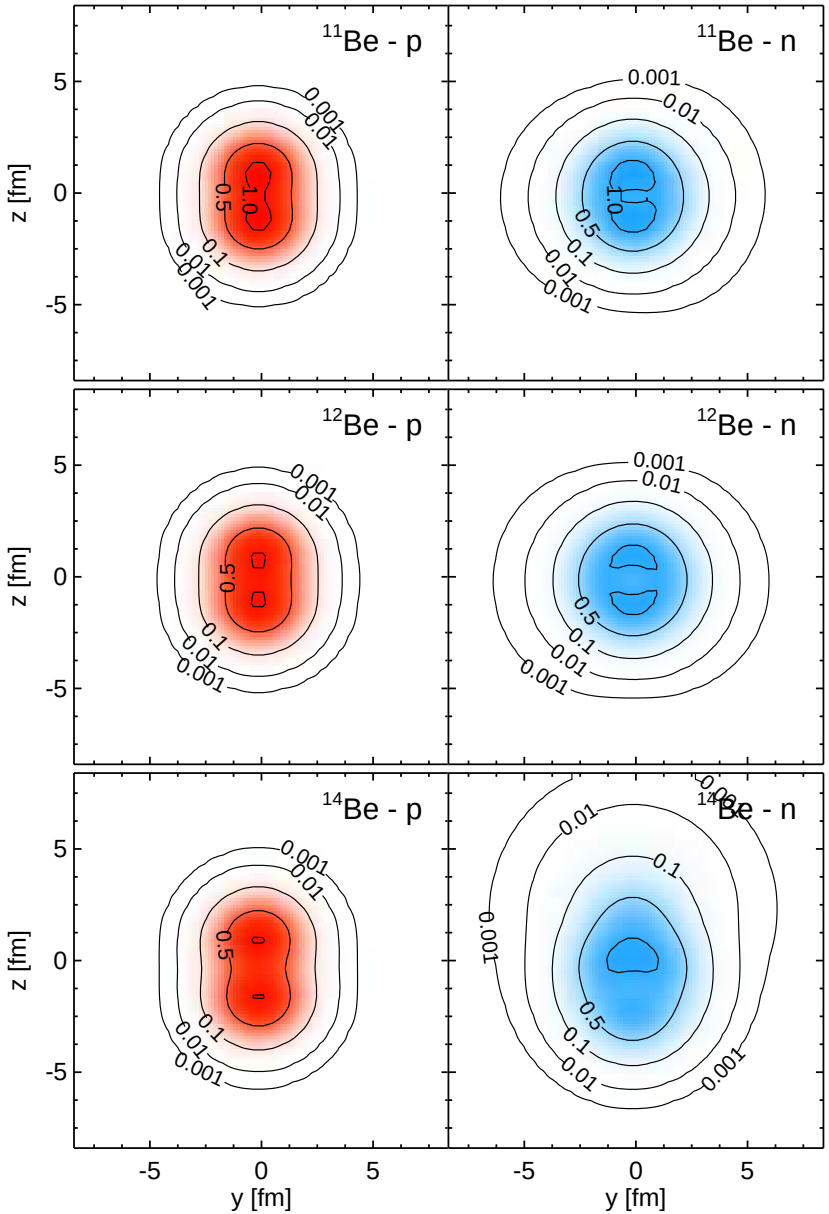
Observables

- **energies**
- **charge and matter radii, electromagnetic transitions**

Beryllium Isotopes

Mean field



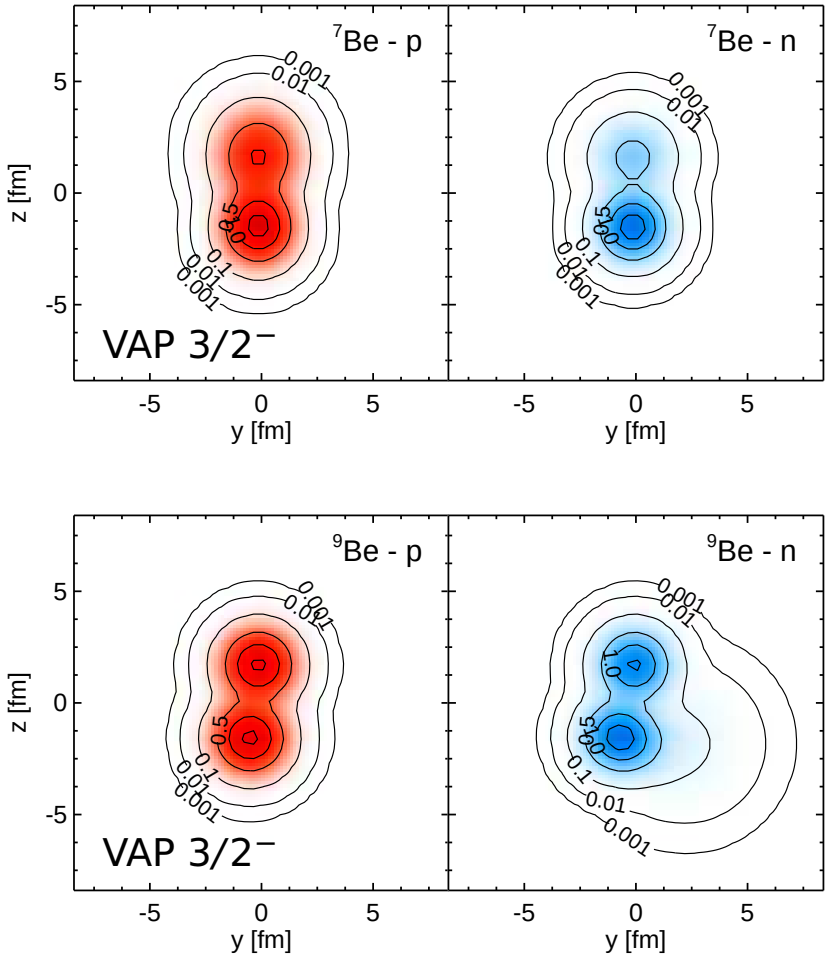


Variation after Projection

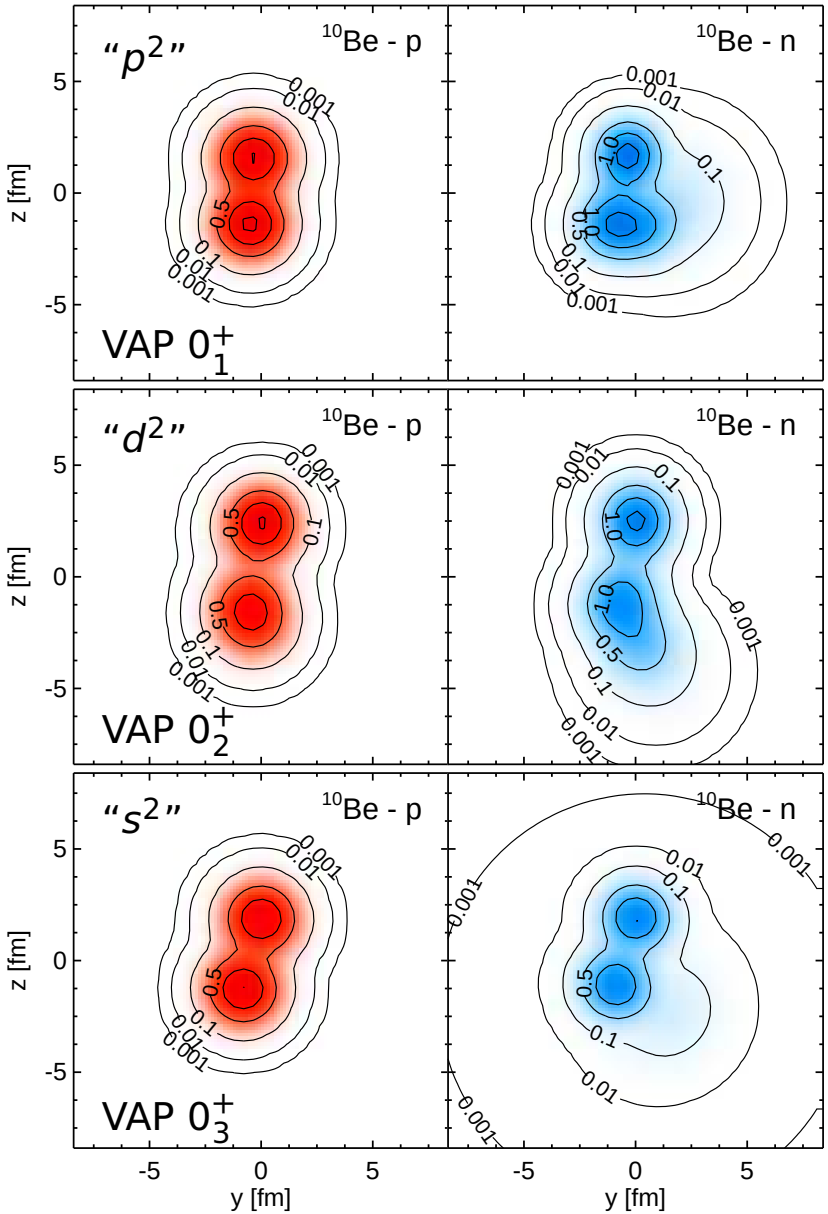
- create configurations by variation after parity and angular momentum projection

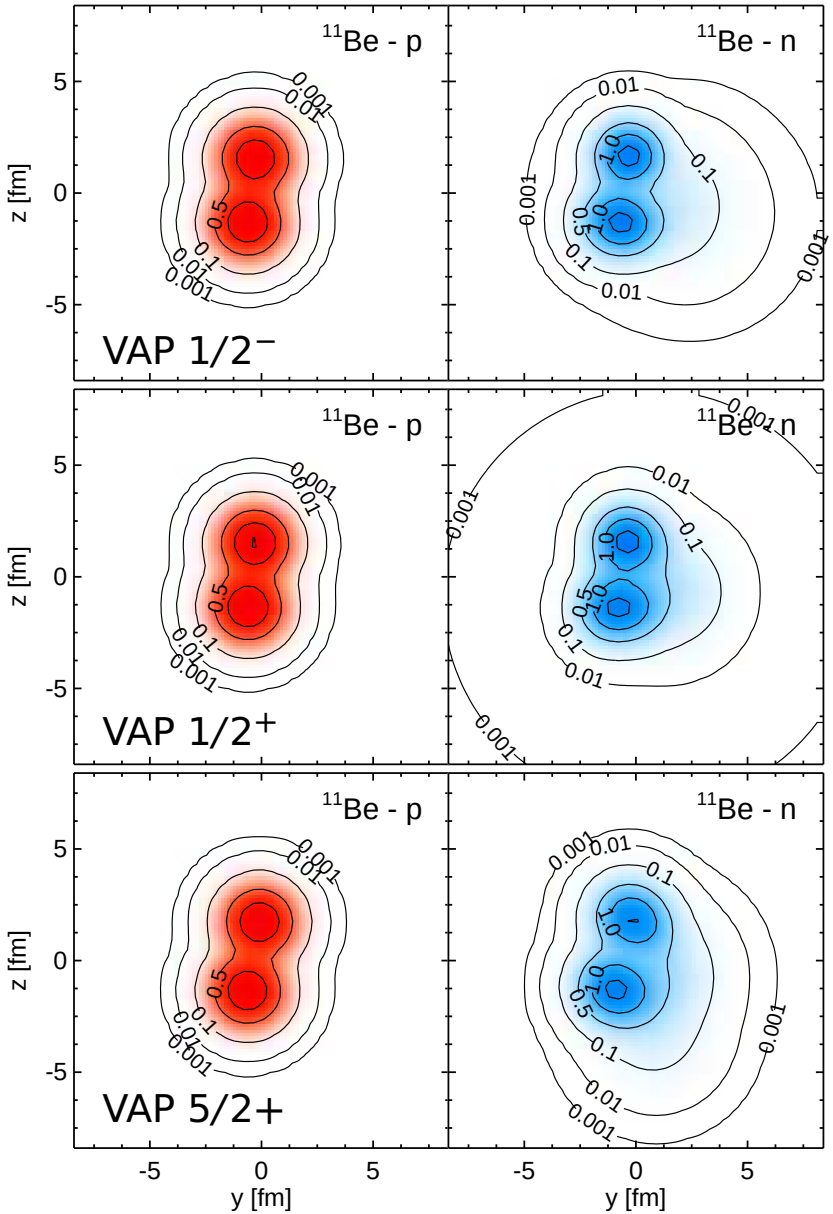
➨ large gain in binding energy compared to mean-field result

➨ intrinsic states show pronounced cluster structure. Parameters of ^{4}He and ^{3}He clusters are close to those of the free clusters

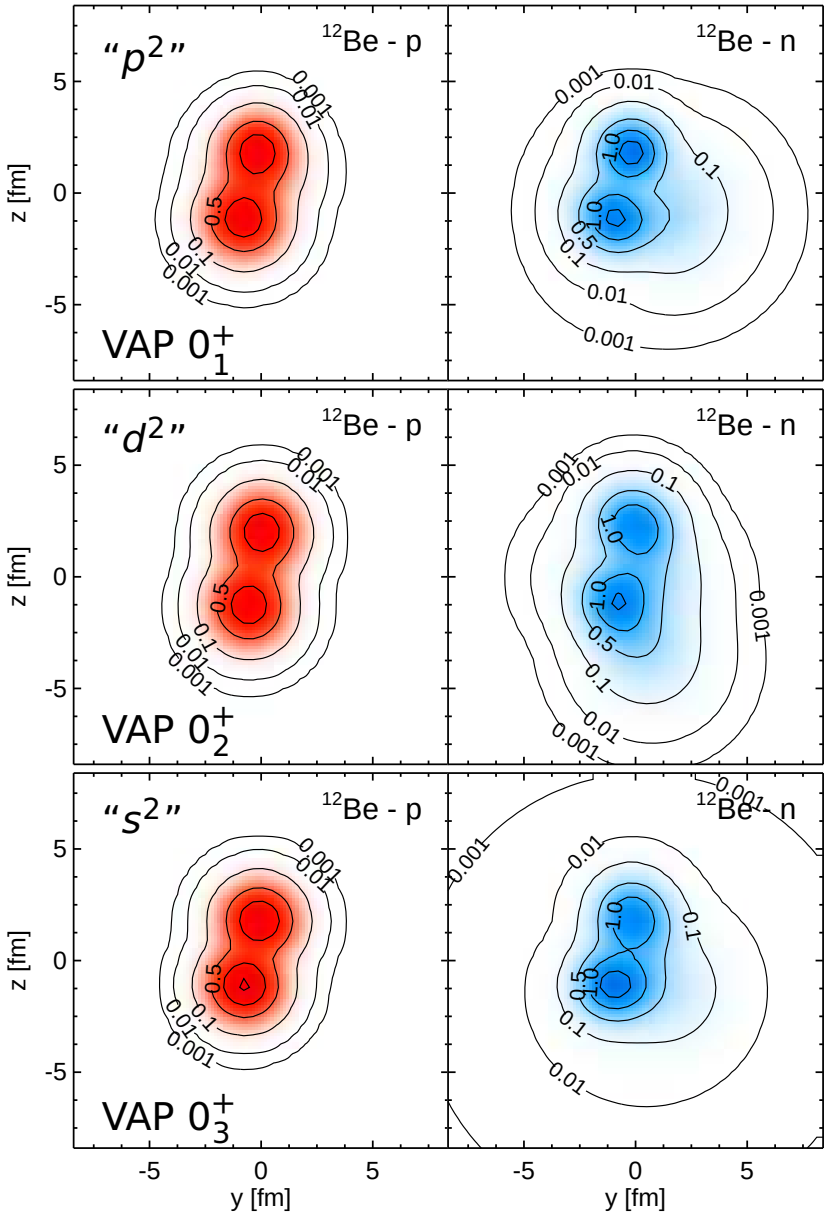


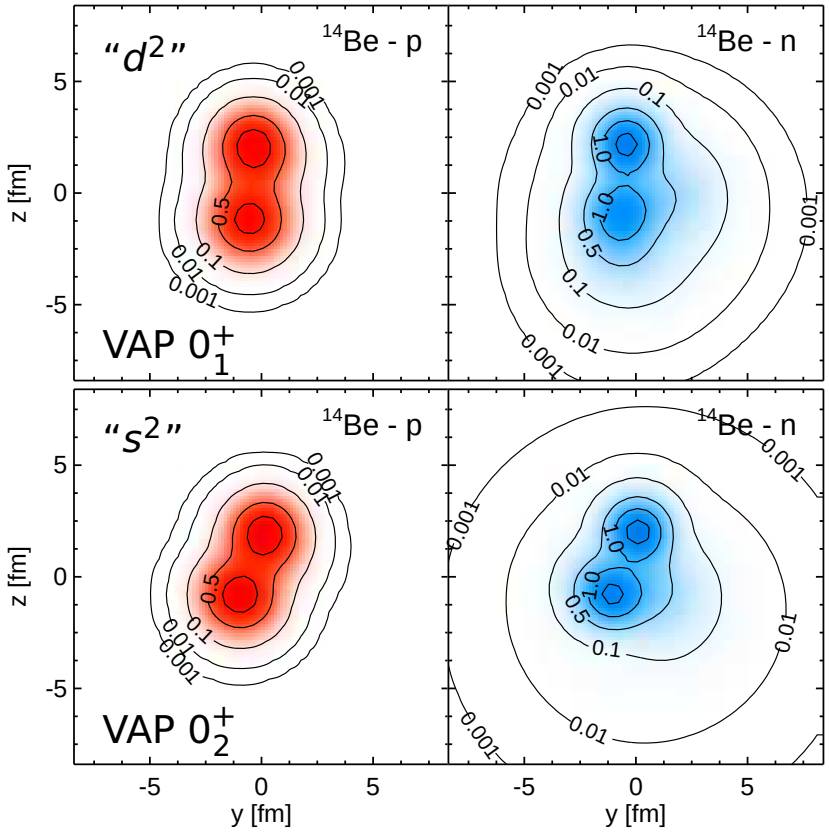
Variation after Projection



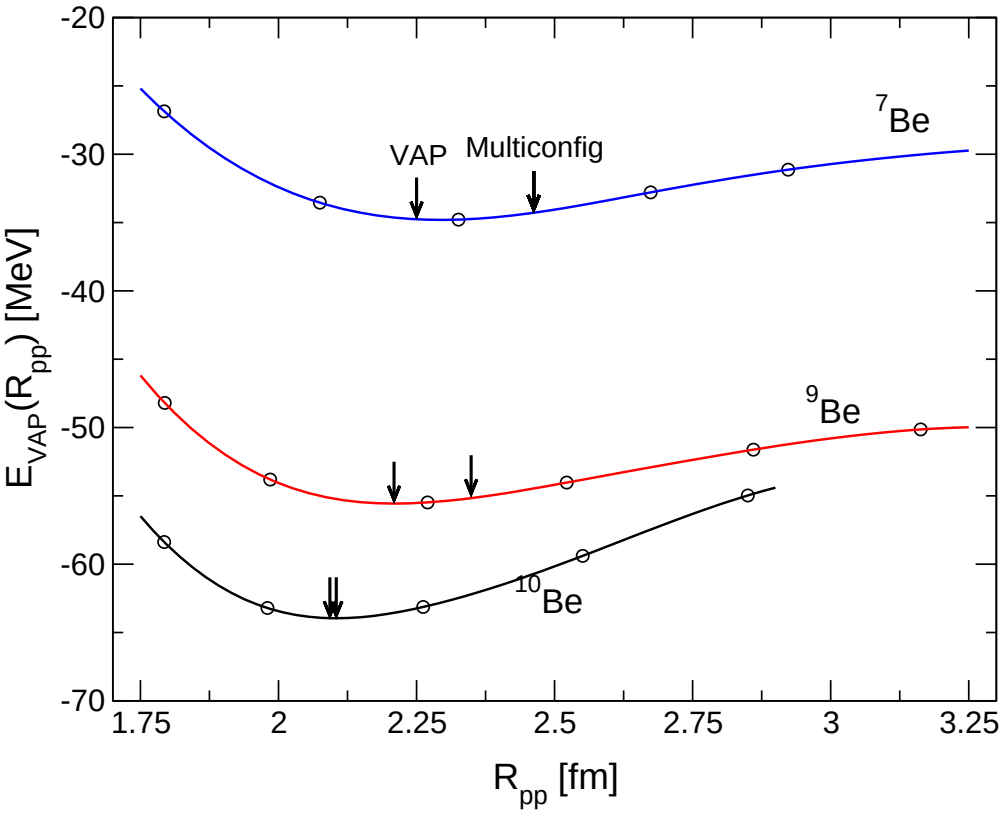


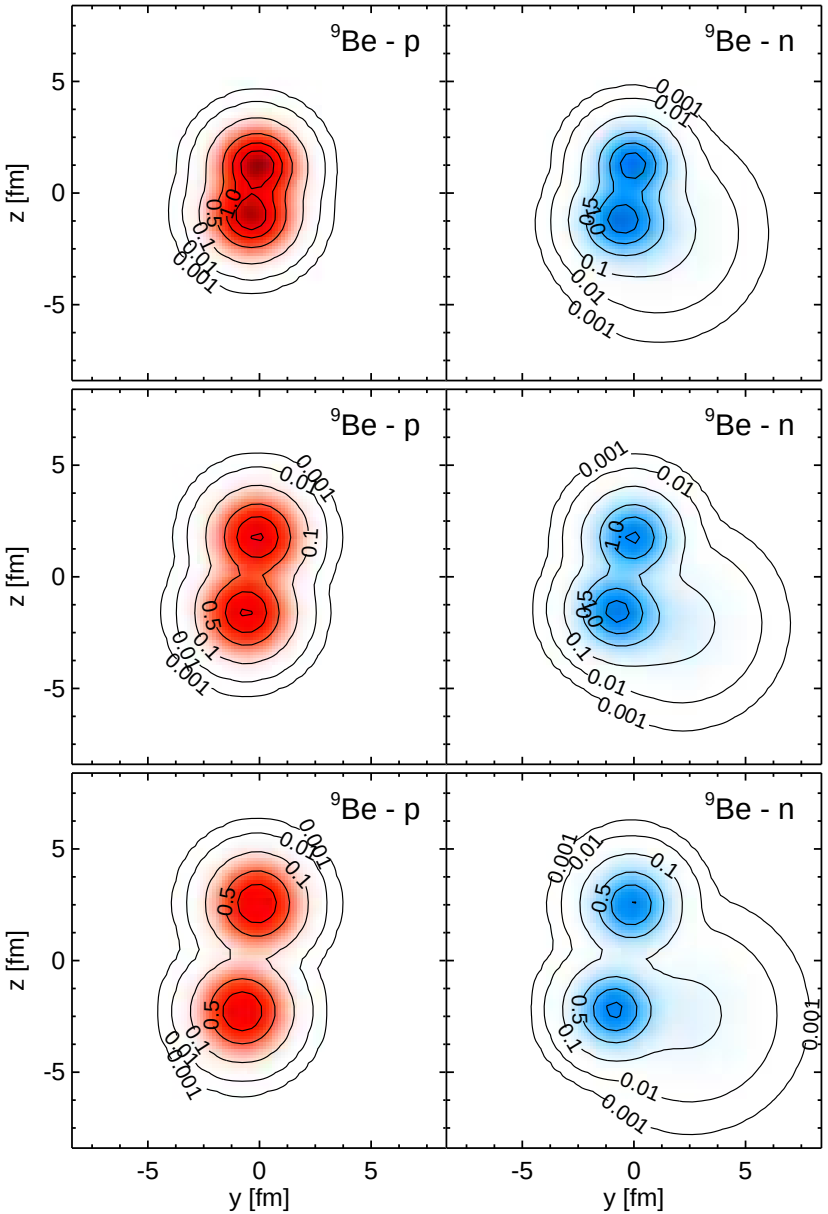
Variation after Projection





Mean proton distance as generator coordinate



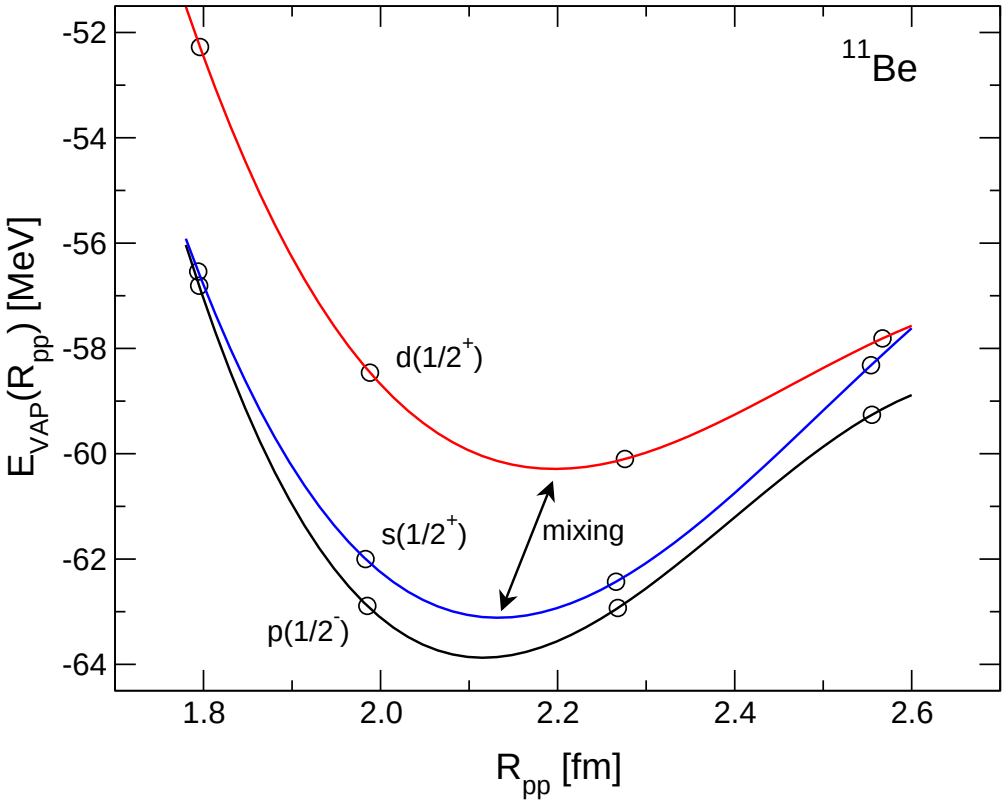


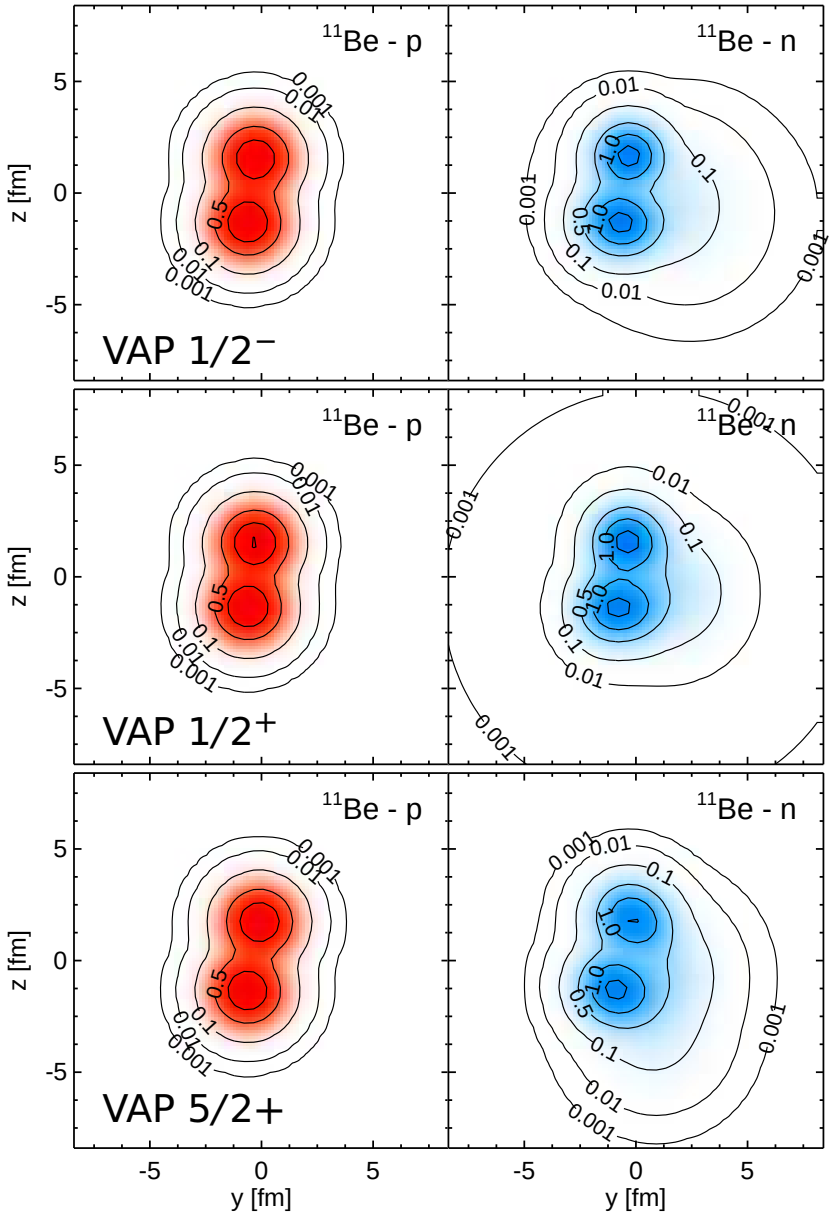
Mean proton distance

$$R_{pp}^2 = \frac{1}{Z^2}\langle \sum_{i<j}^{\text{protons}} (\mathbf{r}_i - \mathbf{r}_j)^2 \rangle$$

R_{pp} as a measure of
α-cluster distance

Mean proton distance as generator coordinate



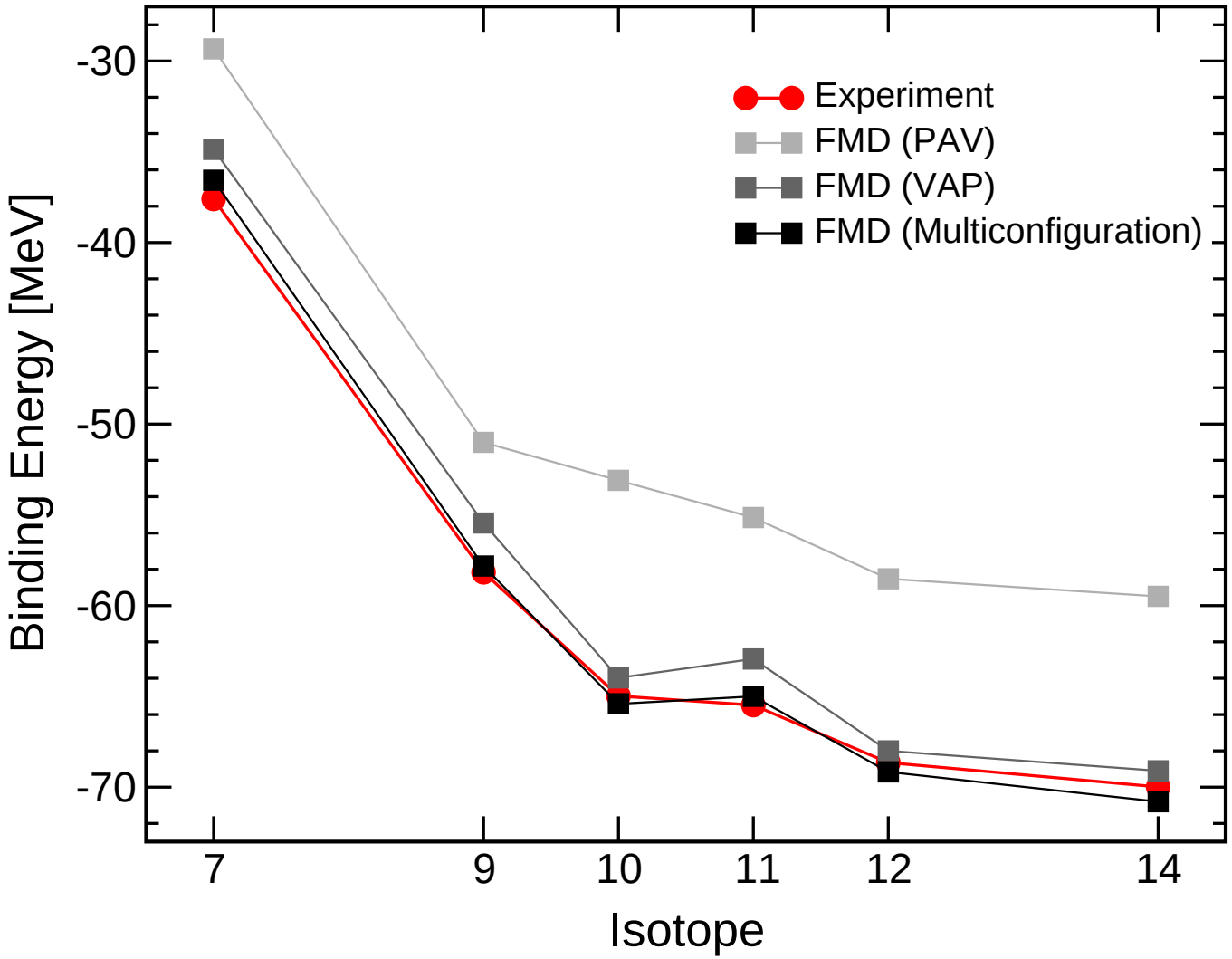


^{11}Be – "*p*", "*s*" and "*d*"-configurations

- "*s*"- and "*d*"-configurations will mix in 1/2$^+$ state
- energy surfaces for "*p*" and "*s*" similar to those in ^{10}Be
- "*d*" surface has minimum at larger cluster distance → *d*-configuration has a polarized ^{10}Be core

Beryllium Isotopes

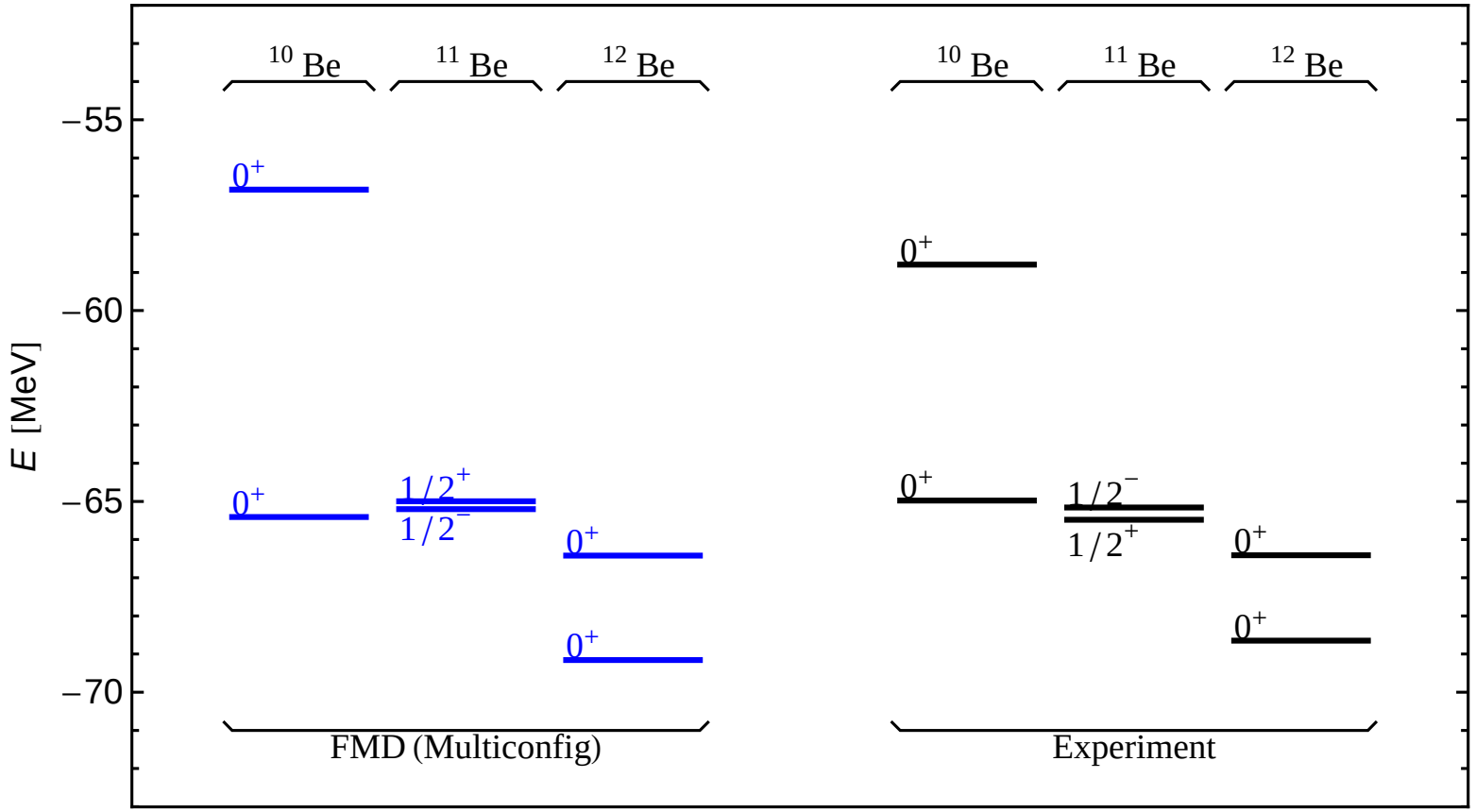
Binding energies



- large correlation energies due to cluster structure
- loosely bound systems gain most by configuration mixing

Beryllium Isotopes

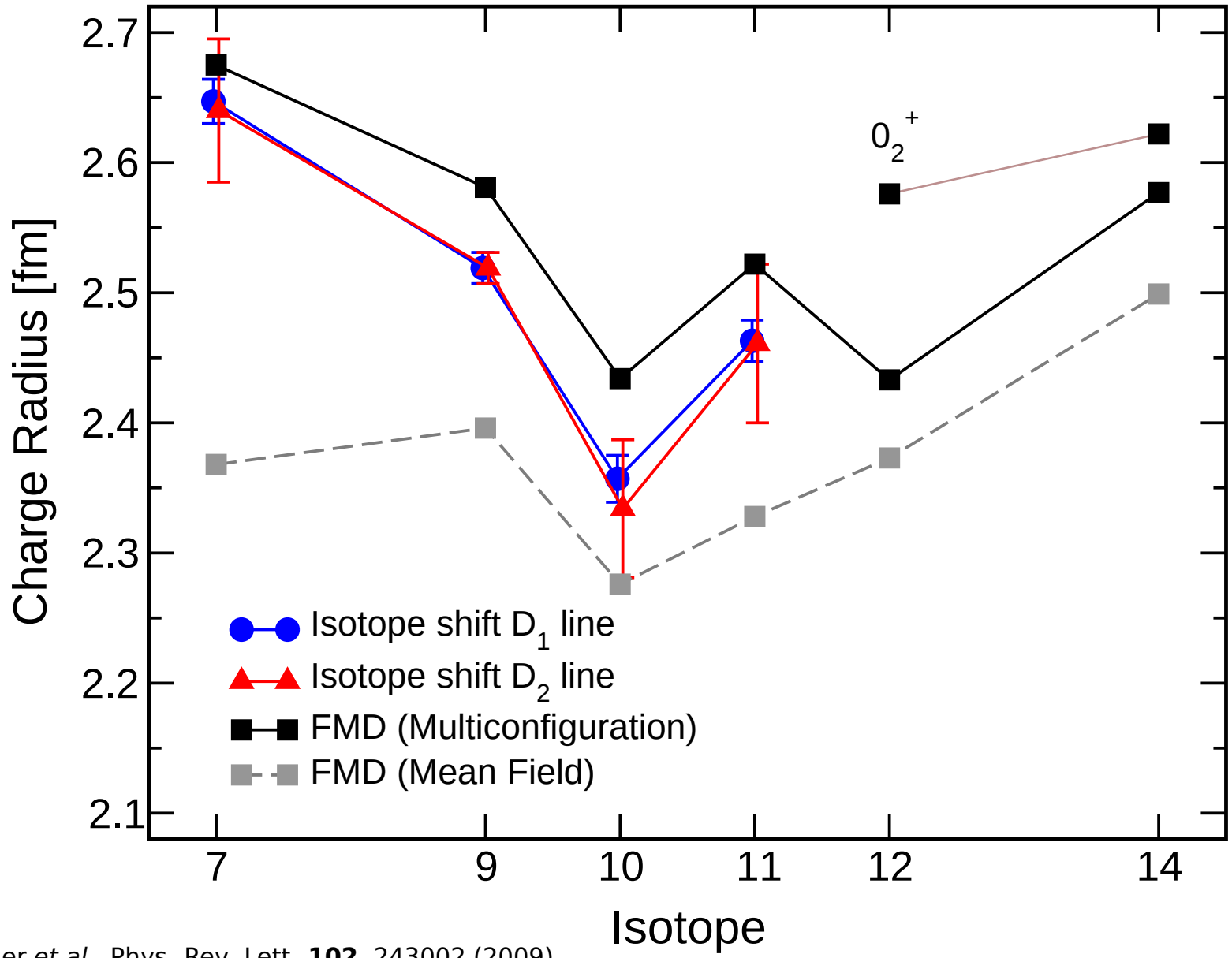
$N = 8$ Shell Closure ?



- "almost correct" level ordering in ^{11}Be
- ^{12}Be ground state dominated by p^2 configuration, sizeable admixture of s^2 and d^2 configurations which stronly mix

Beryllium Isotopes

Charge Radii

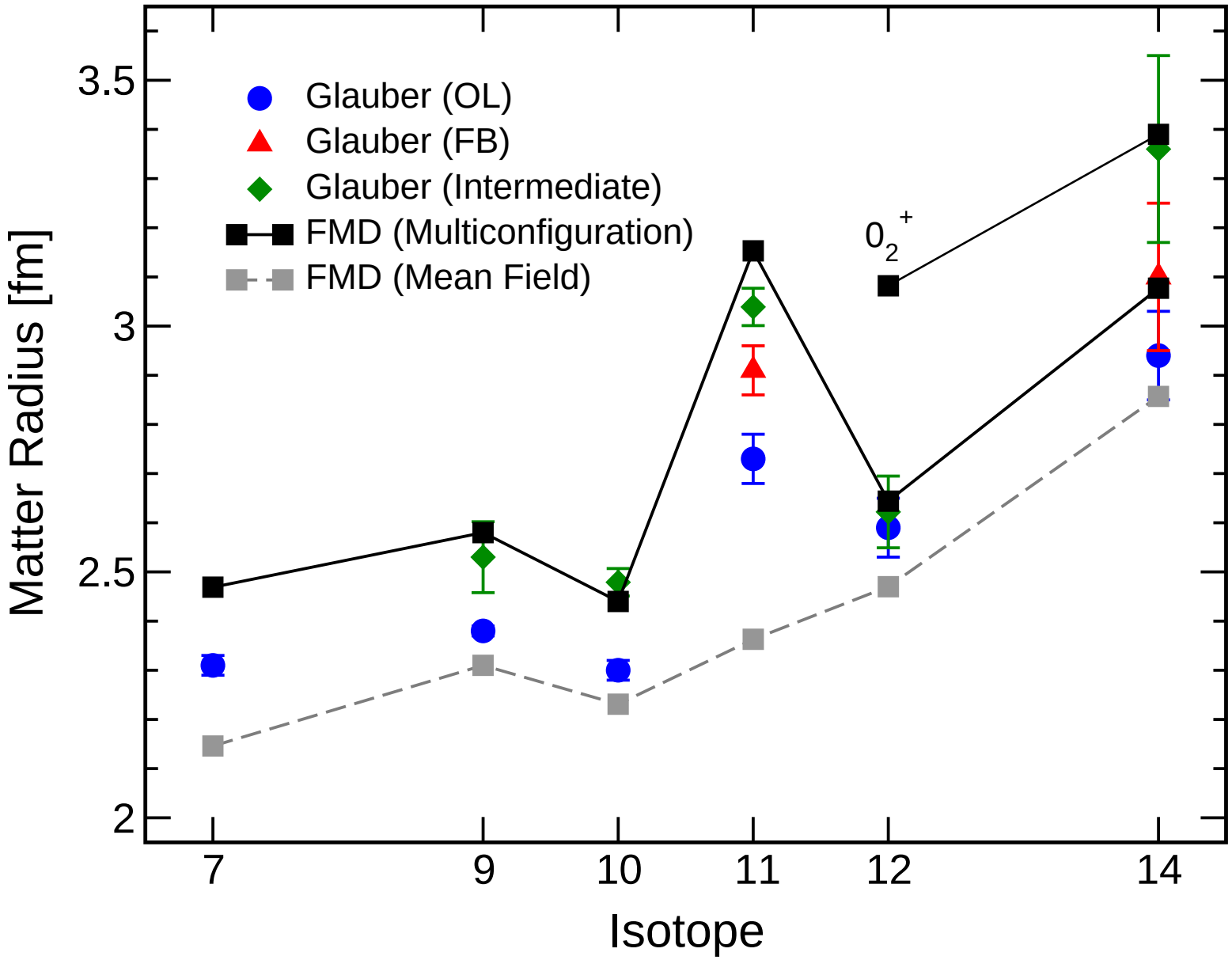


Nörtershäuser *et al.*, Phys. Rev. Lett. **102**, 243002 (2009)

Zakova, Neff, *et al.*, J. Phys. G, accepted for publication

Beryllium Isotopes

Matter Radii



Ozawa *et al.*, Nucl. Phys. **A693**, 32 (2001).

Electromagnetic transitions

^{10}Be

	FMD(Multiconfig)	Experiment
$B(E2; 2_1^+ \to 0_1^+)$	11.27 e^2fm^4	10.2 ± 1.0 e^2fm^4
$B(E2; 0_2^+ \to 2_1^+)$	4.99 e^2fm^4	3.2 ± 1.9 e^2fm^4
$B(E1; 0_2^+ \to 1_1^-)$	0.013 e^2fm^2	0.013 ± 0.004 e^2fm^2

^{11}Be

	FMD(Multiconfig)	Experiment
$B(E1; 1/2_1^+ \to 1/2_1^-)$	0.020 e^2fm^2	0.099 ± 0.010 e^2fm^2

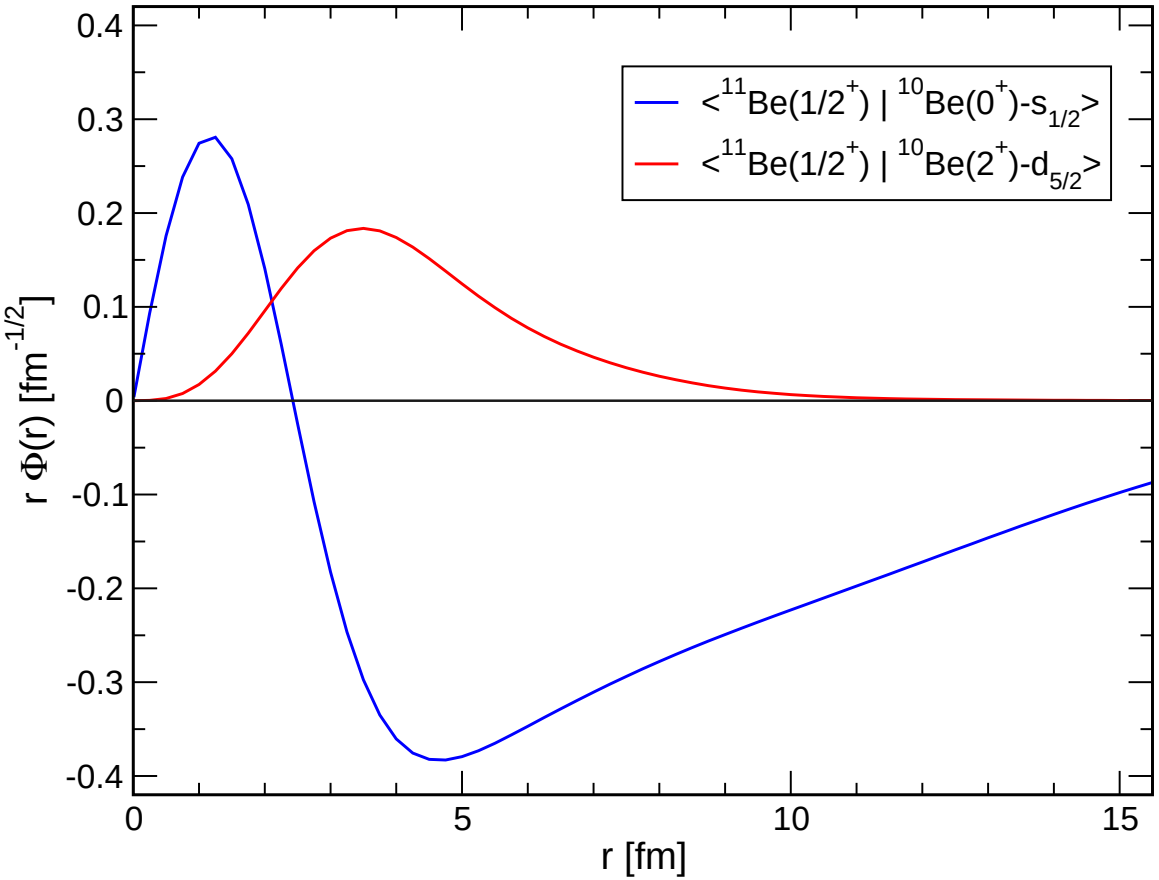
^{12}Be

	FMD(Multiconfig)	Experiment
$B(E2; 2_1^+ \to 0_1^+)$	8.27 e^2fm^4	8.0 ± 3.0 e^2fm^4
$B(E2; 0_2^+ \to 2_1^+)$	6.50 e^2fm^4	7.0 ± 0.6 e^2fm^4
$M(E0; 0_1^+ \to 0_2^+)$	1.05 $e$fm^2	0.87 ± 0.03 $e$fm^2
$B(E1; 0_1^+ \to 1_1^-)$	0.08 e^2fm^2	0.051 ± 0.003 e^2fm^2

Nakamura *et al.*, Phys. Lett. **B394**, 11 (1997).
Shimoura *et al.*, Phys. Lett. **B654**, 87 (2007).

Iwasaki *et al.*, Phys. Lett. **B491**, 8 (2000).

^{11}Be-^{10}Be **Overlaps**



Spectroscopic Factors

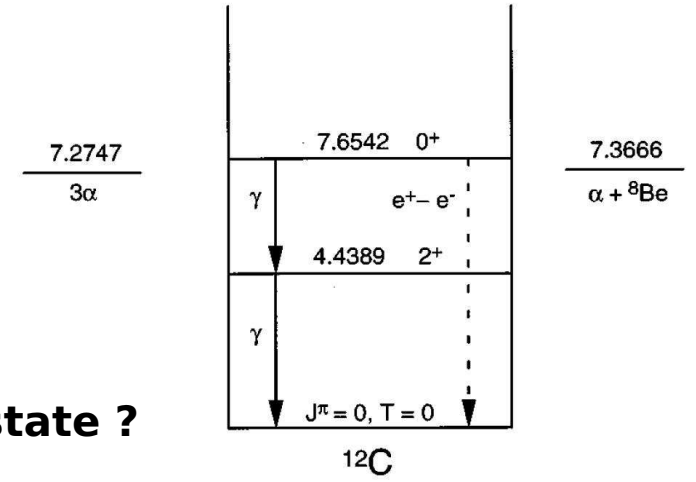
^{11}Be	^{10}Be	l_j	S
$1/2^+$	0^+	$s_{1/2}$	0.937
	2^+	$d_{5/2}$	0.094
	2^+	$d_{3/2}$	0.007
$5/2^+$	0^+	$d_{5/2}$	0.543
	2^+	$s_{1/2}$	0.329
	2^+	$d_{5/2}$	0.243
$1/2^-$	0^+	$p_{1/2}$	0.805
	2^+	$p_{3/2}$	0.779

- extended *s*-wave halo
- $s_{1/2}$ spectroscopic factor larger than results obtained from knockout and transfer reactions

Cluster States in ^{12}C

Astrophysical Motivation

- **Helium burning: triple alpha-reaction**



Structure

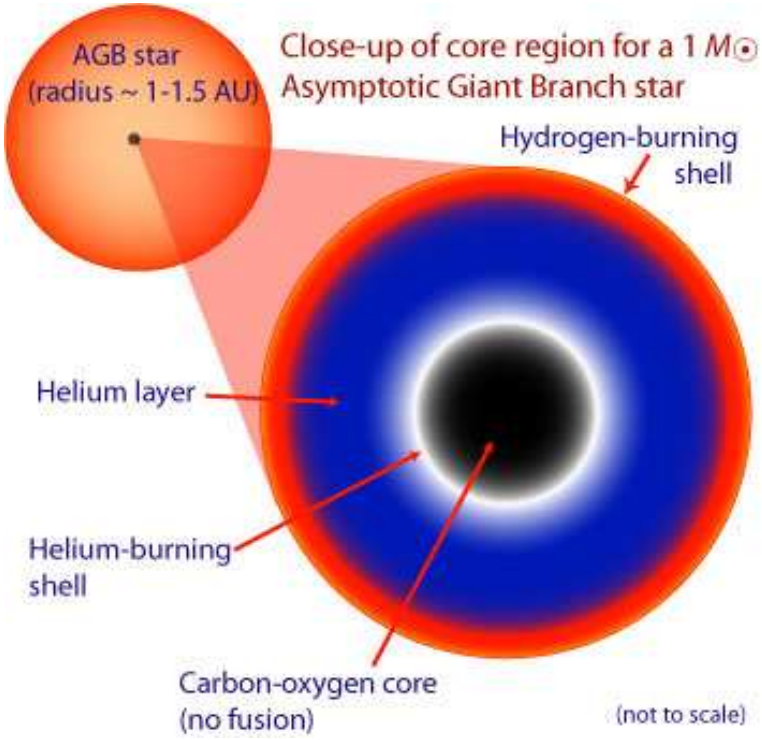
- **Is the Hoyle state a pure α-cluster state ?**
- **Other excited 0^+ and 2^+ states**

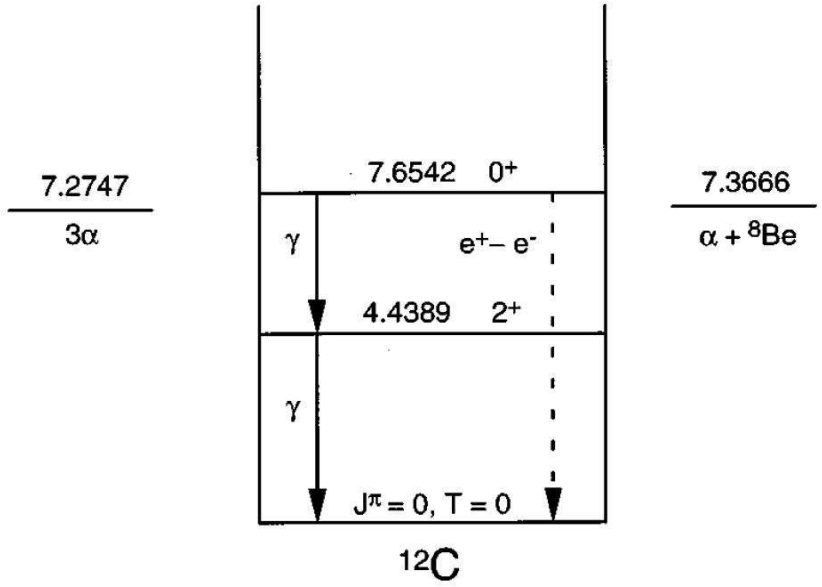
➨ **Compare FMD results to microscopic α-cluster model**

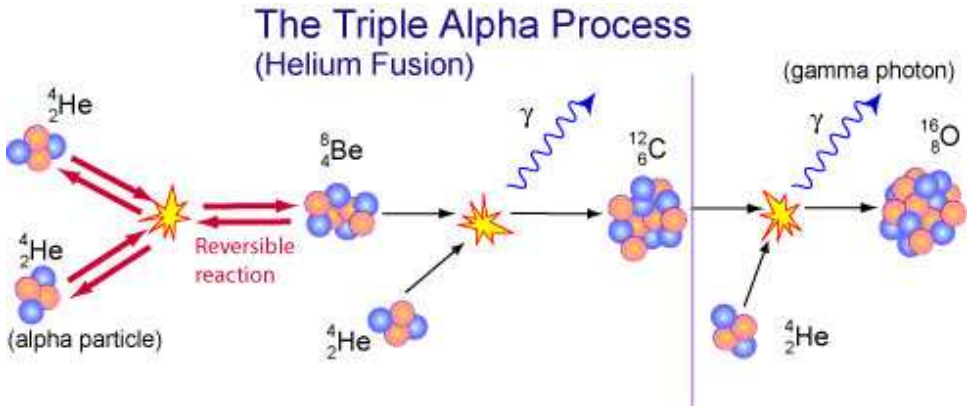
➨ **Analyze wave functions in harmonic oscillator basis**

➨ **No-Core Shell Model Calculations ?**

Cluster States in ^{12}C
Triple α Reaction

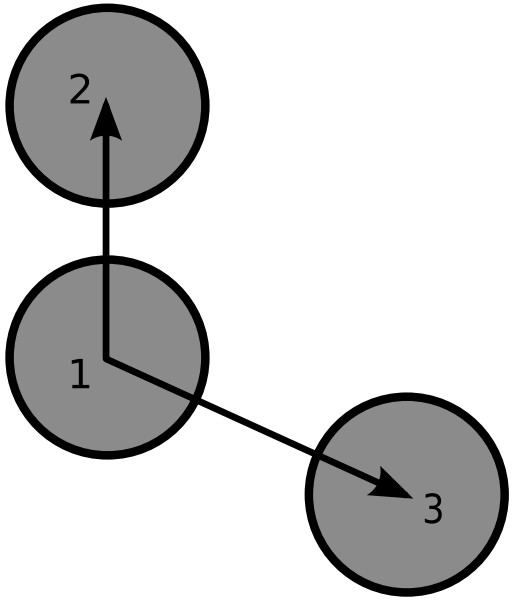






http://outreach.atnf.csiro.au/education/senior/astrophysics/stellarevolution_postmain.html

Cluster States in ^{12}C
Microscopic α-Cluster Model



$$R_{12} = (2, 4, \ldots, 10)\,\text{fm}$$
$$R_{13} = (2, 4, \ldots, 10)\,\text{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \ldots, -1.0)$$

alltogether 165 configurations

Basis States

- describe Hoyle State as a system of 3 ^{4}He nuclei

$$\left|\Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3); JMK\pi\right\rangle = P^J_{MK} P^\pi \mathcal{A}\left\{\left|\psi_\alpha(\mathbf{R}_1)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_2)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_3)\right\rangle\right\}$$

Volkov Interaction

- simple central interaction
- parameters adjusted to reproduce α binding energy and radius, $\alpha - \alpha$ scattering data and C12 ground state energy

✘ only reasonable for ^{4}He, ^{8}Be and ^{12}C nuclei

'BEC' wave functions

- interpretation of the Hoyle state as a Bose-Einstein Condensate of α-particles by Funaki, Tohsaki, Horiuchi, Schuck, Röpke
- same interaction and α-cluster parameters used

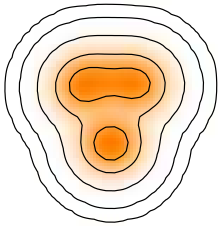
Kamimura, Nuc. Phys. **A351** (1981) 456
Funaki et al., Phys. Rev. C **67** (2003) 051306(R)

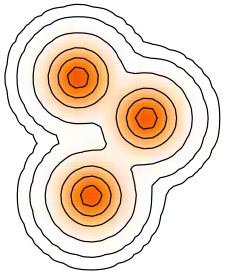
FMD

Basis States

- 20 FMD states obtained in Variation after Projection on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in Variation after Projection on parity with constraints on radius and quadrupole deformation
- 165 α-cluster configurations

➨ projected on angular momentum and linear momentum



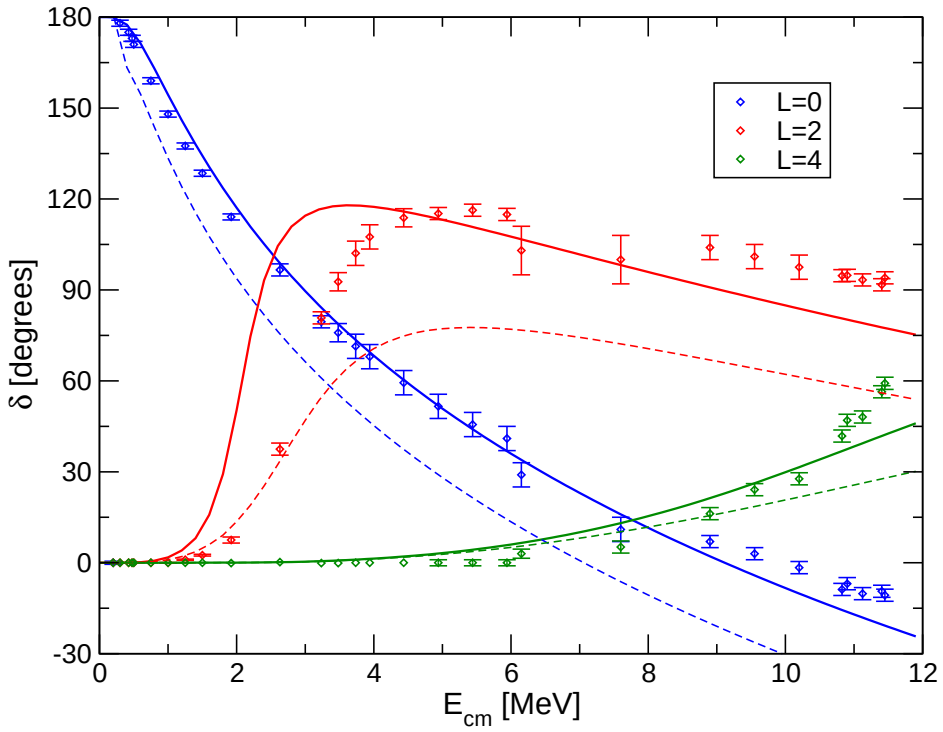


Interaction

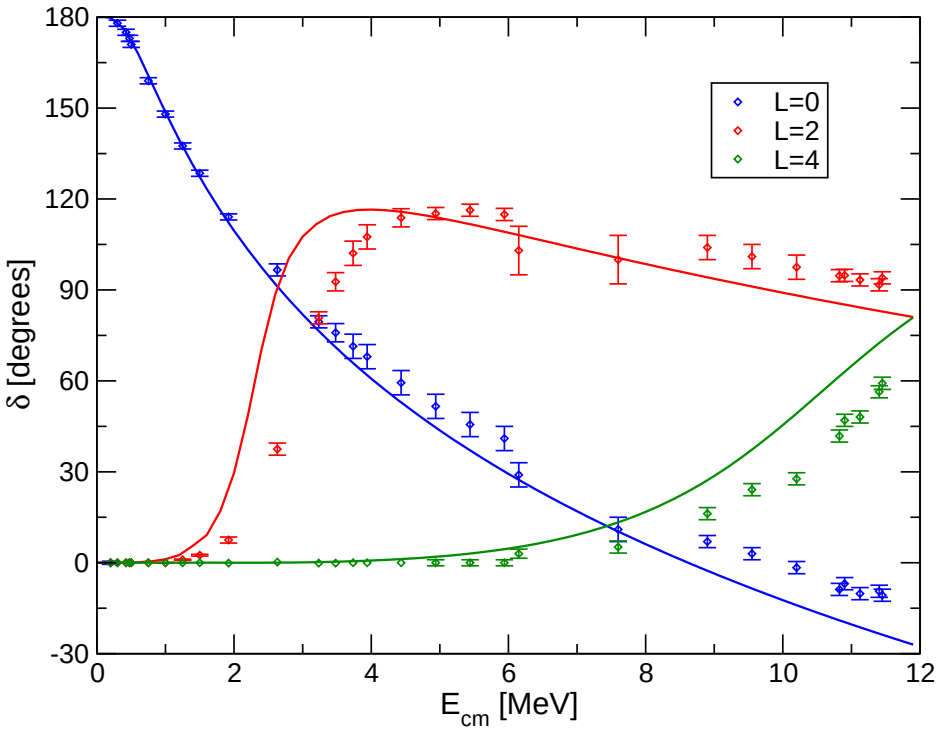
- not tuned for α-α scattering or ^{12}C properties

Cluster States in ^{12}C
α-α Phaseshifts

FMD



Cluster Model

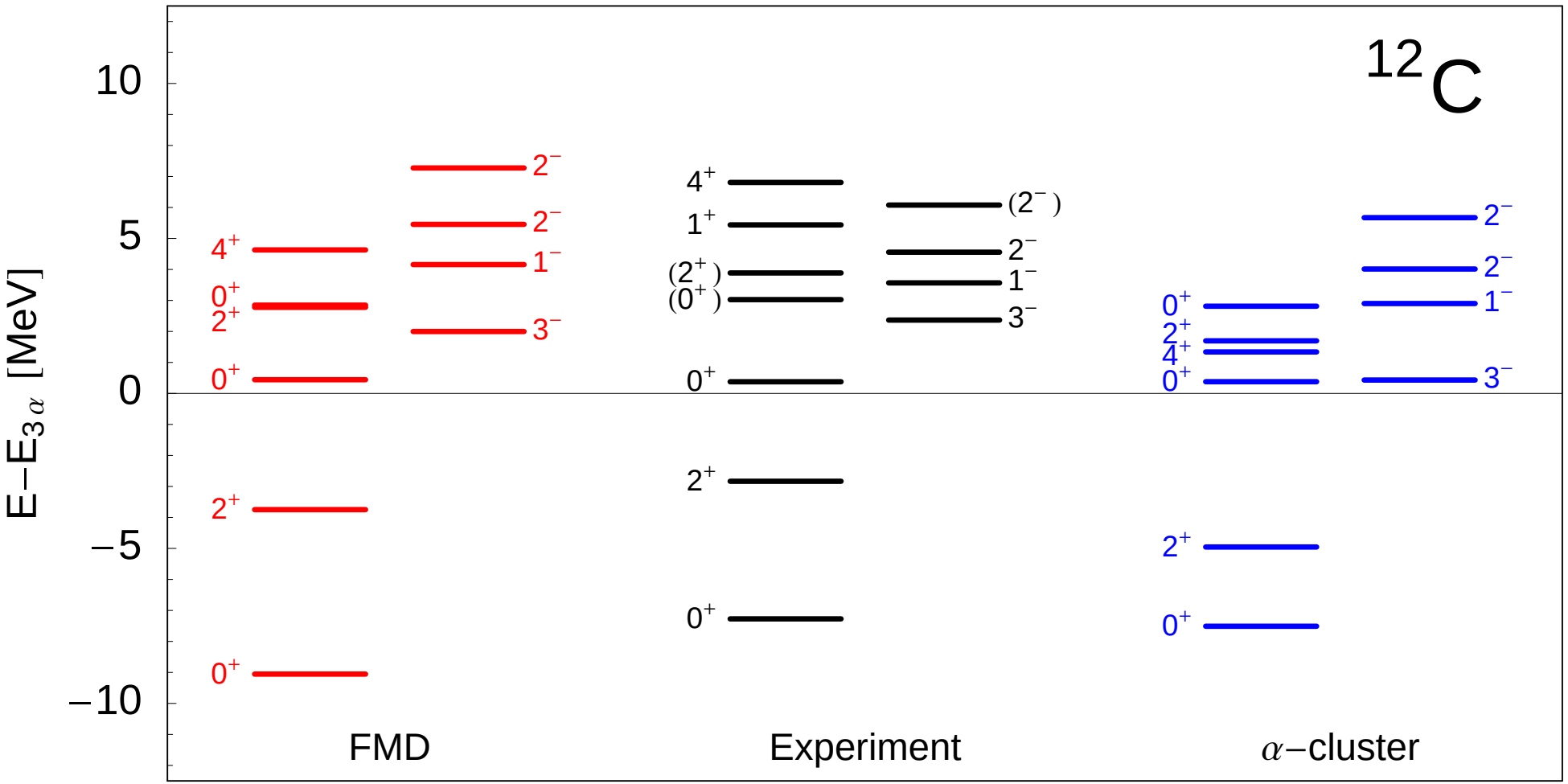


- Phaseshifts calculated with cluster configurations only (dashed lines)
- Phaseshifts calculated with additional FMD VAP configurations in the interaction region (solid lines)

- only cluster configurations included

➨ similar quality for description of α-α-scattering

Cluster States in ^{12}C
Comparison



Cluster States in ^{12}C
Comparison

	Exp[1]	Exp[2]	Exp[3]	FMD	α-cluster	'BEC'[4]
$E(0_1^+)$	-92.16			-92.64	-89.56	-89.52
$E^*(2_1^+)$	4.44			5.31	2.56	2.81
$E(3\alpha)$	-84.89			-83.59	-82.05	-82.05
$E(0_2^+) - E(3\alpha)$	0.38			0.43	0.38	0.26
$E(0_3^+) - E(3\alpha)$	(3.0)	2.7(3)	3.96(5)	2.84	2.81	
$E(2_2^+) - E(3\alpha)$	(3.89)	2.6(3)	6.63(3)	2.77	1.70	
$r_{\text{charge}}(0_1^+)$	2.47(2)			2.53	2.54	
$r(0_1^+)$				2.39	2.40	2.40
$r(0_2^+)$				3.38	3.71	3.83
$r(0_3^+)$				4.62	4.75	
$r(2_1^+)$				2.50	2.37	2.38
$r(2_2^+)$				4.43	4.02	
$M(E0, 0_1^+ \to 0_2^+)$	5.4(2)			6.53	6.52	6.45
$B(E2, 2_1^+ \to 0_1^+)$	7.6(4)			8.69	9.16	
$B(E2, 2_1^+ \to 0_2^+)$	2.6(4)			3.83	0.84	

experimental situation for 0_3^+ and 2_2^+ states still unsettled

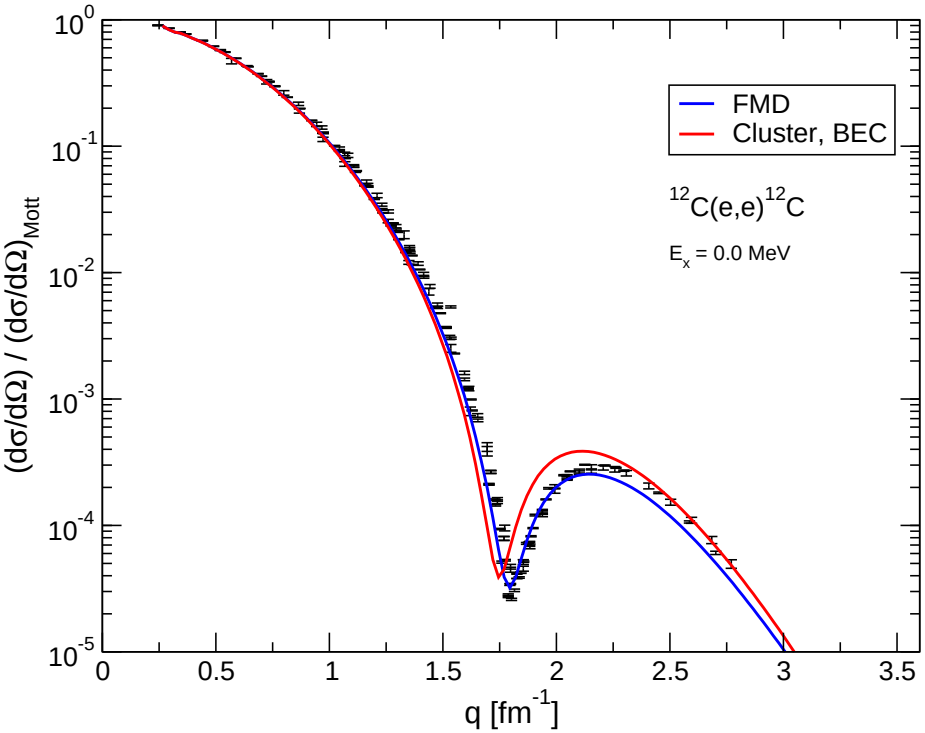
2_2^+ resonance at 1.8 MeV above treshold included in NACRE compilation

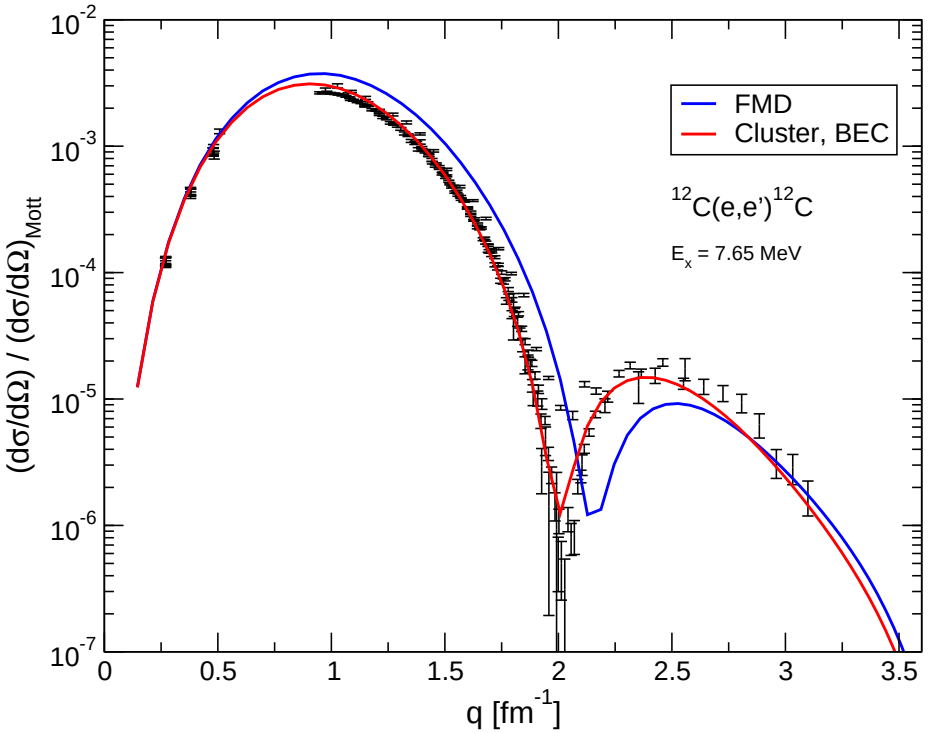
calculated in bound state approximation

➨ include ^{8}Be+^{4}He channels for two-body decay

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Fynbo et al., Nature **433**, 137 (2005). Diget et al., Nuc. Phys. **A738**, 760 (2005)
[4] Funaki et al., Phys. Rev. C **67**, 051306(R) (2003)

Cluster States in ^{12}C
Electron Scattering Data





- compare with precise electron scattering data up to high momenta in Distorted Wave Born Approximation
- use intrinsic density

$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \langle \Psi \left| \delta(\underset{\sim}{\mathbf{x}}_k - \underset{\sim}{\mathbf{X}} - \mathbf{x}) \right| \Psi \rangle$$

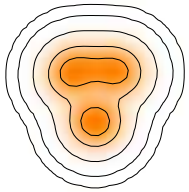
- elastic cross section described very well by FMD
- transition cross section better described by cluster model

M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98** (2007) 032501

Cluster States in ^{12}C

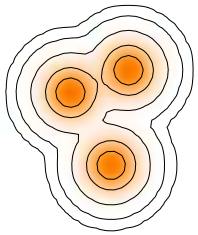
Important Configurations

- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state

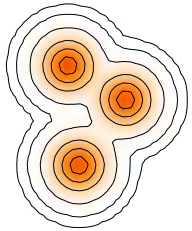


$\left|\left\langle \cdot \mid 0_1^+ \right\rangle\right| = 0.94$

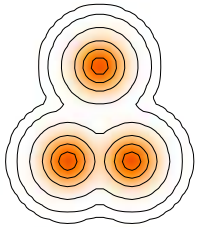
$\left|\left\langle \cdot \mid 2_1^+ \right\rangle\right| = 0.93$



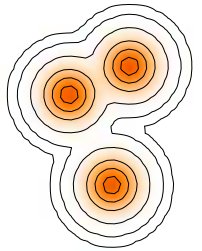
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.72$



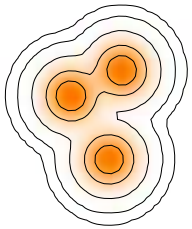
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.71$



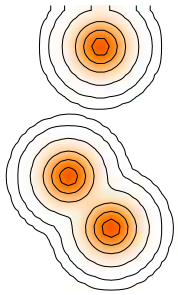
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



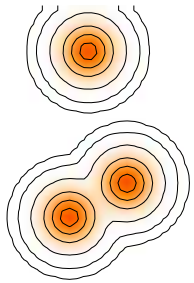
$\left|\left\langle \cdot \mid 0_2^+ \right\rangle\right| = 0.61$



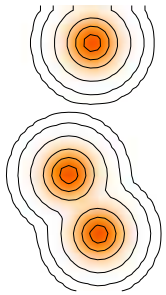
$\left|\left\langle \cdot \mid 3_1^- \right\rangle\right| = 0.83$



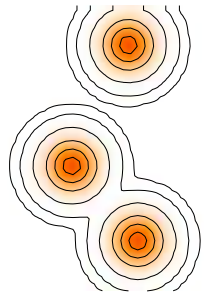
$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.50$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.49$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.44$



$\left|\left\langle \cdot \mid 0_3^+ \right\rangle\right| = 0.41$

FMD basis states are not orthogonal!

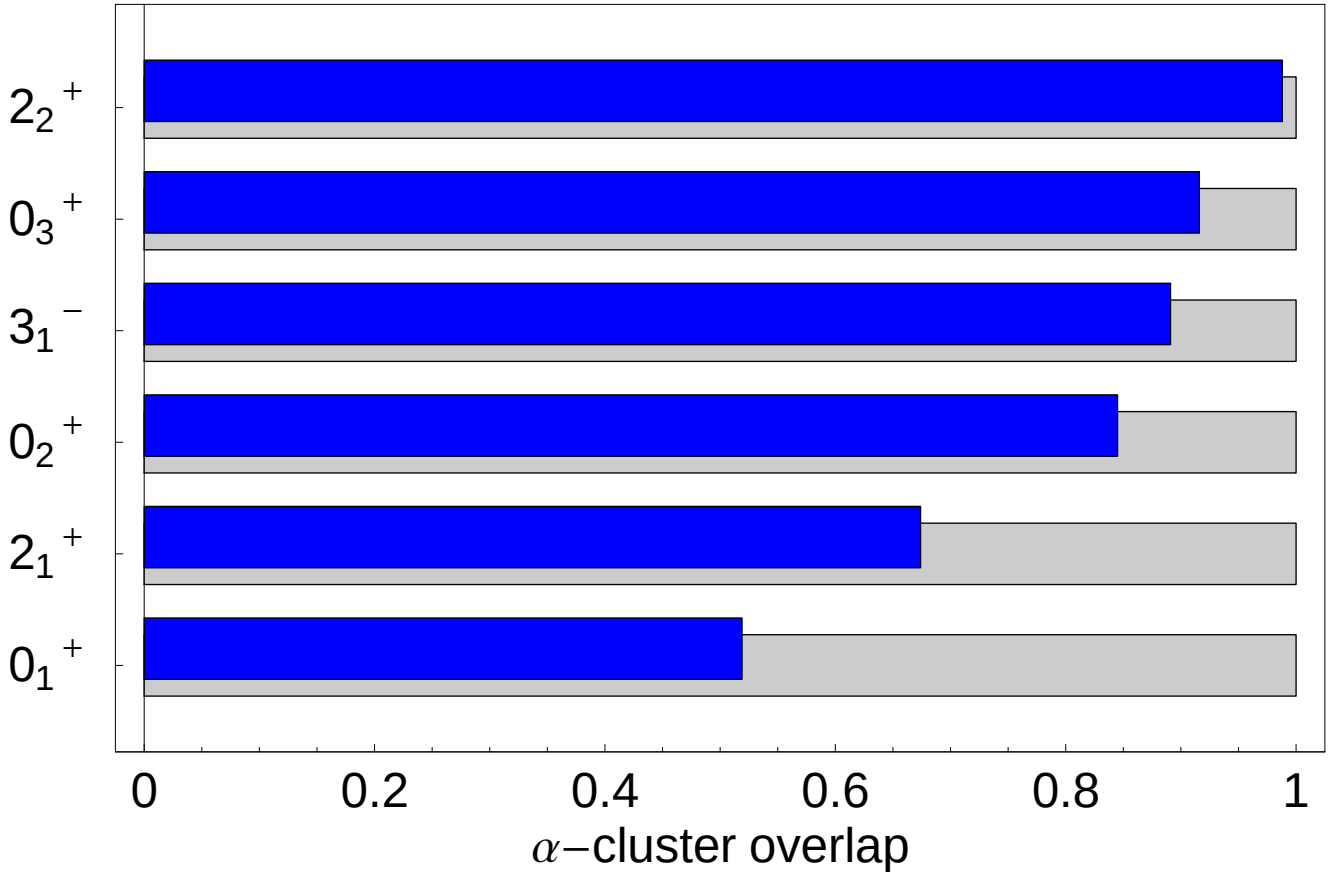
loosely bound, gas-like states

Cluster States in ^{12}C

Overlap with Cluster Model Space

Calculate the overlap of FMD wave functions with pure α-cluster model space

$$N_\alpha = \langle \Psi \left| \underset{\sim}{P}_{3\alpha} \right| \Psi \rangle$$



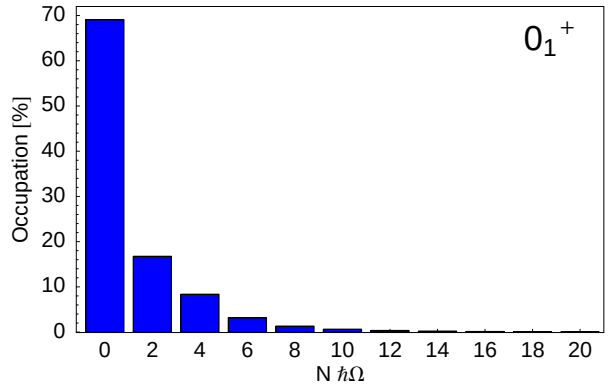
Hoyle state has 15% non-alpha admixtures

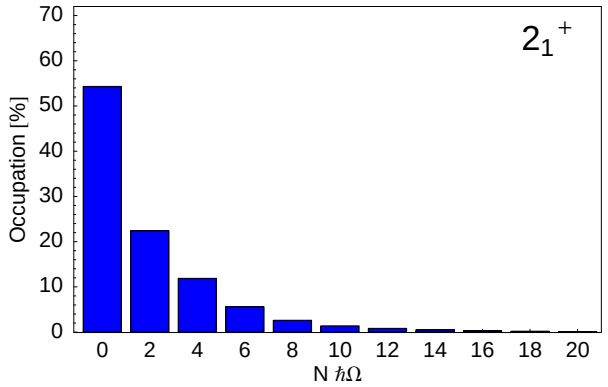
Cluster States in ^{12}C
Harmonic Oscillator $N\hbar\Omega$ Excitations

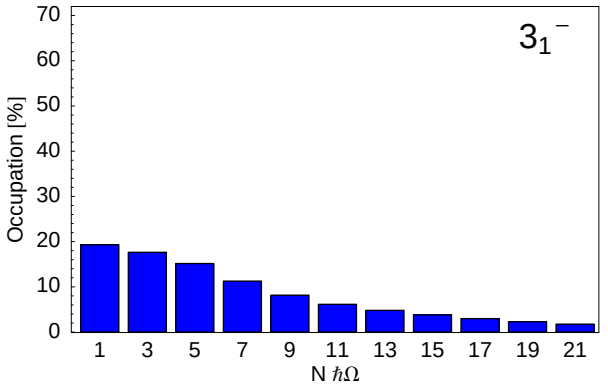
Y. Suzuki et al., Phys. Rev. C **54** (1996) 2073

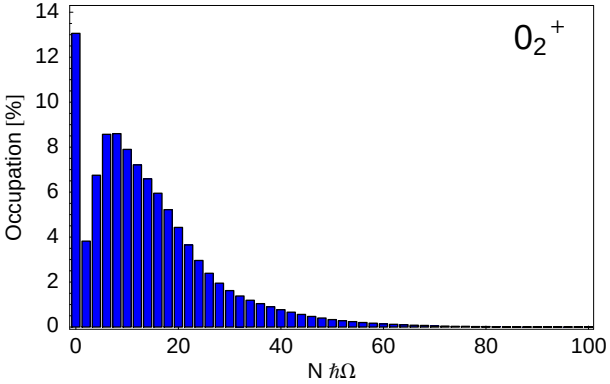
$$\mathrm{Occ}(N) = \langle \Psi \left| \delta \left(\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N \right) \right| \Psi \rangle$$

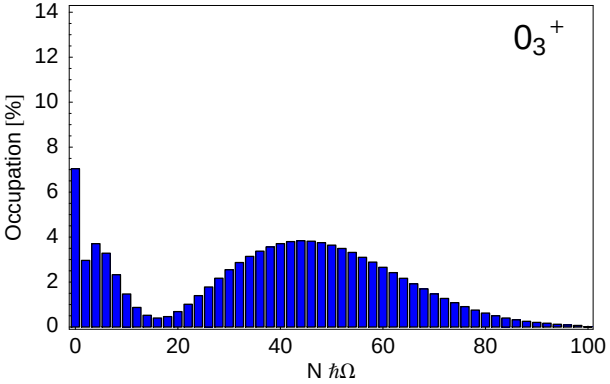
FMD

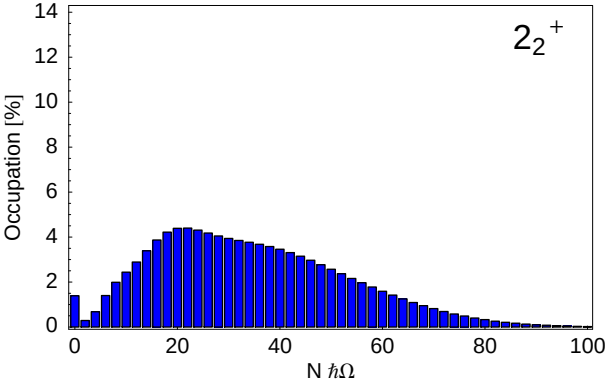










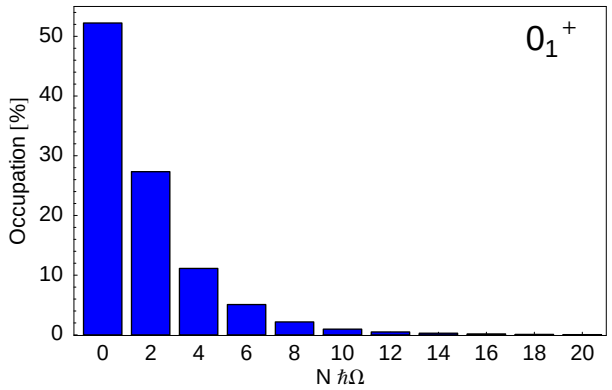


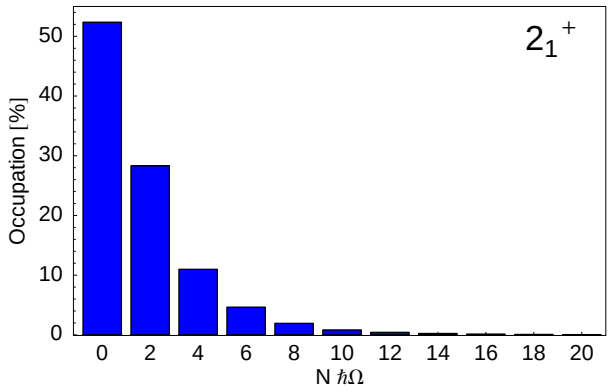
Cluster States in ^{12}C
Harmonic Oscillator $N\hbar\Omega$ Excitations

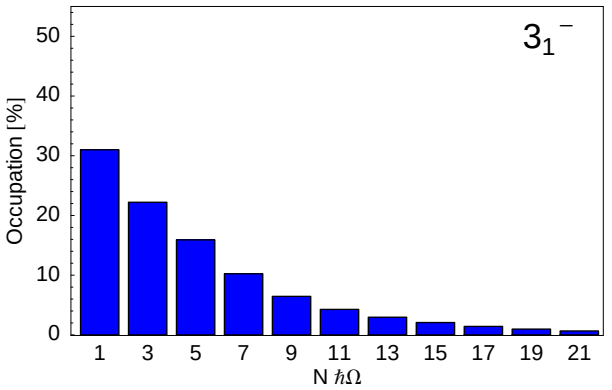
Y. Suzuki *et al*, Phys. Rev. C **54**, 2073 (1996).

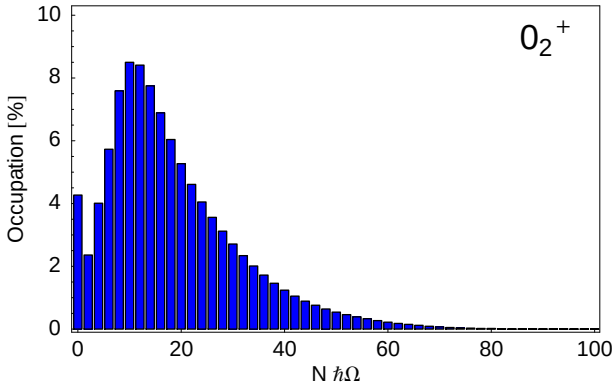
$$\mathrm{Occ}(N) = \langle \Psi \left| \delta \left(\sum_i (\underset{\sim}{H}_i^{HO}/\hbar\Omega - 3/2) - N \right) \right| \Psi \rangle$$

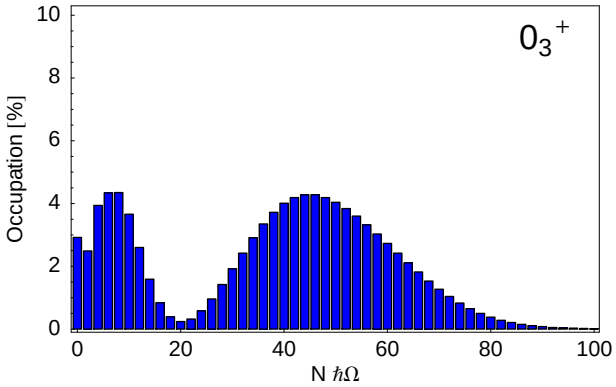
Cluster Model

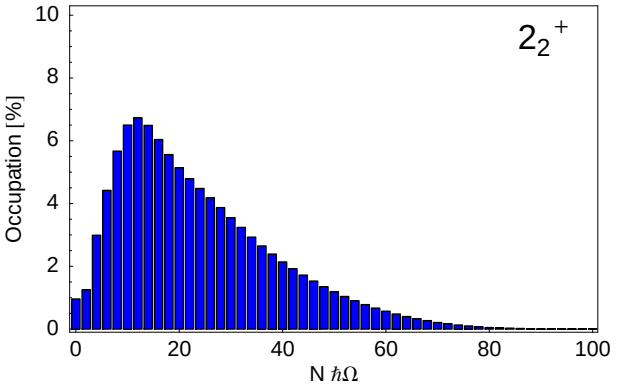












Cluster States in ^{12}C

α-cluster states in the No-Core Shell Model ?

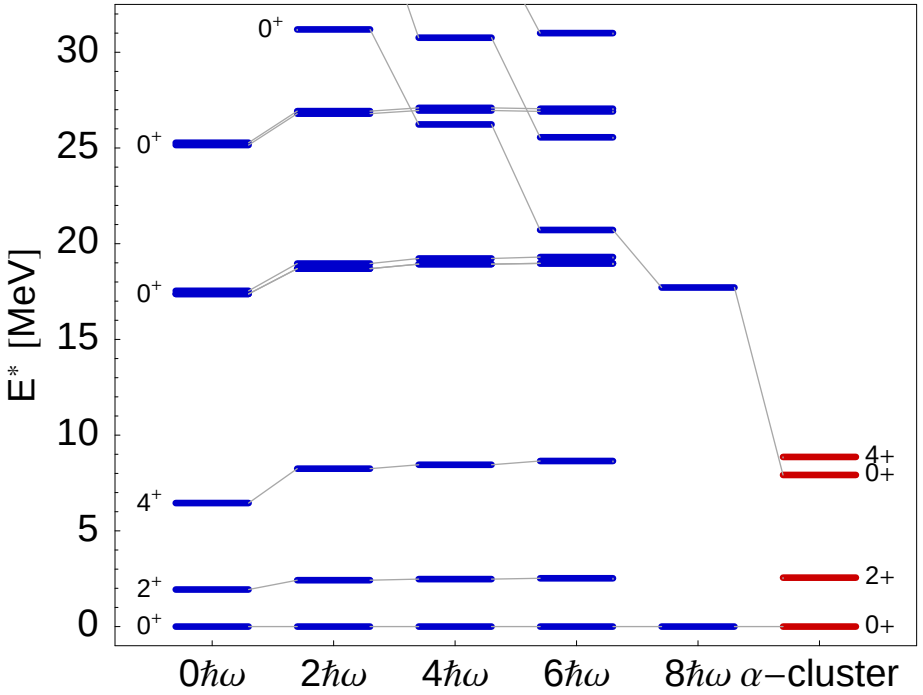
- compare spectra in NCSM and α-cluster model using the Volkov interaction
- bare interaction used in NCSM calculations

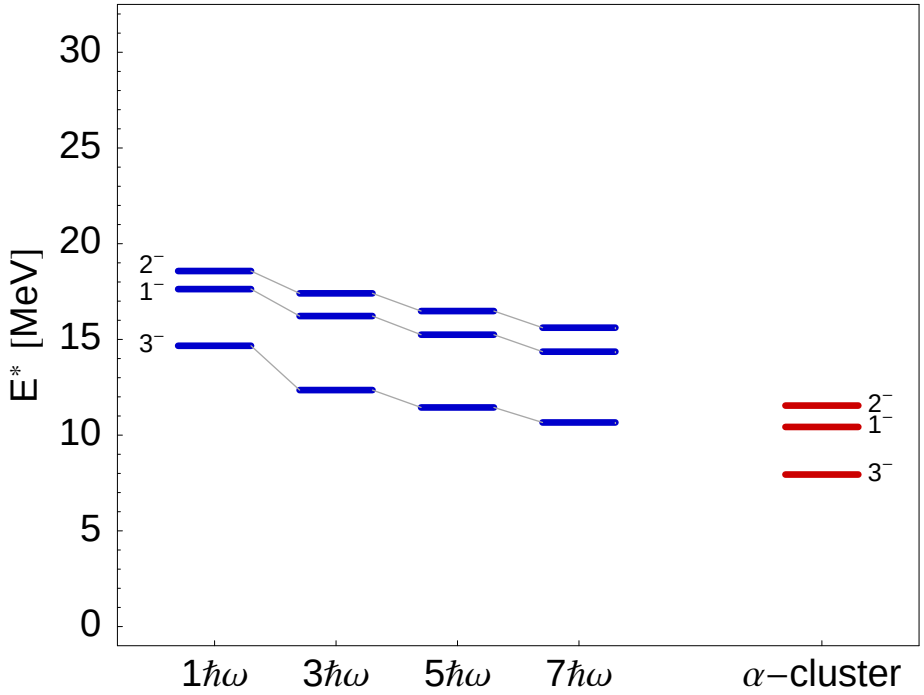
➥ good agreement for ground state band (0^+_1, 2^+_1, 4^+_1)

➥ very slow convergence for cluster states

Binding energies

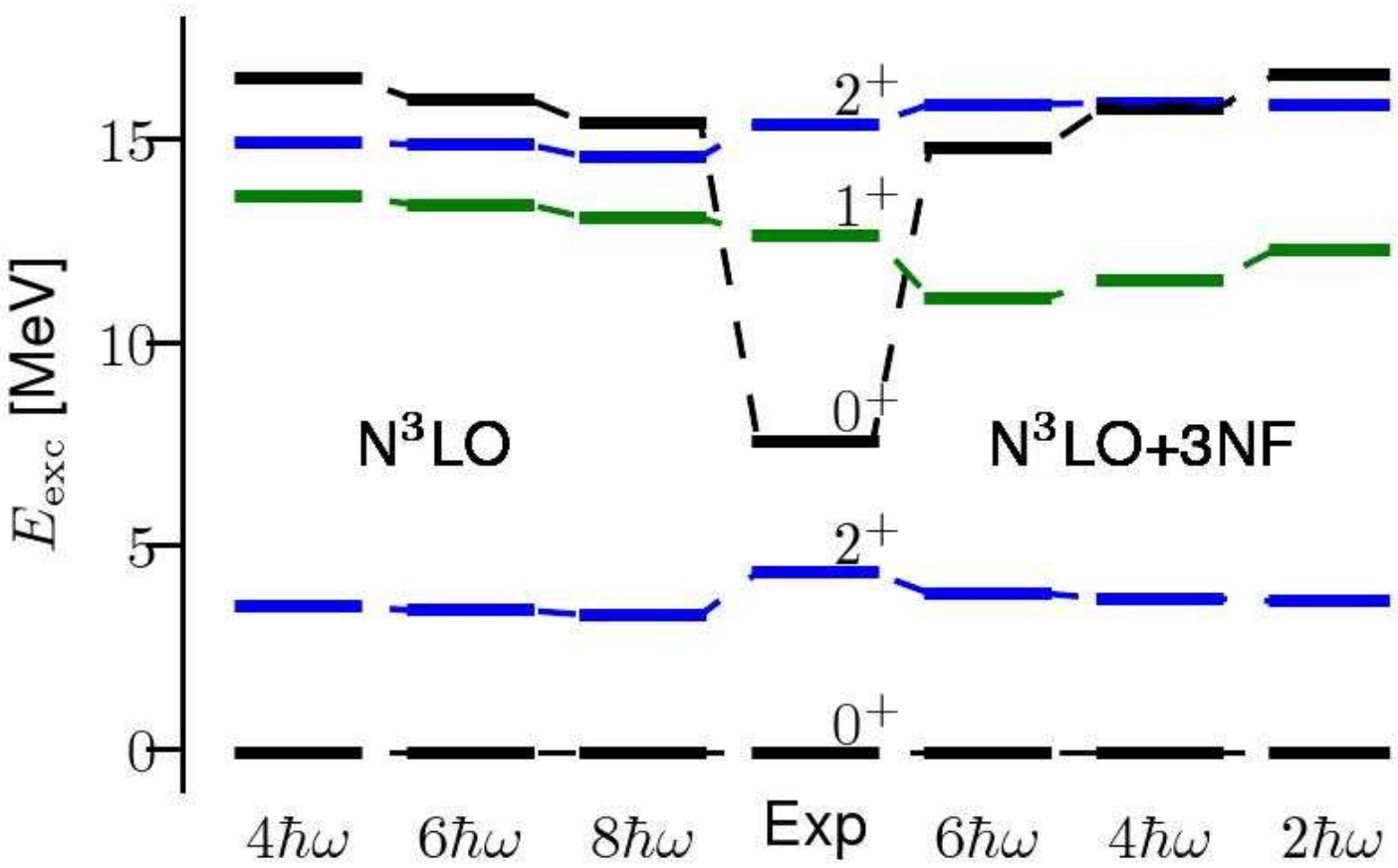
	^{4}He	^{12}C
Cluster	-27.3 MeV	-89.6 MeV
NCSM	-28.3 MeV	-95.4 MeV





Cluster States in ^{12}C

α-cluster states in the No-Core Shell Model ?



➨ three-body forces do not help !

Navrátil, Gueorgiev, Vary, Ormand, Nogga, Phys. Rev. Lett. **99** (2007) 042501

Advantages/Disadvantages of FMD approach

FMD vs *ab initio*

Advantages

- basis very flexible, clusters and halo structure can be described
- can be used for light *p*/*sd*-shell nuclei
- many observables can be calculated
- intrinsic states provide an “intuitive” picture of the nucleus

Disadvantages

- interaction has to be soft and given in operator representation
- does not provide “exact” results for given interaction, not straightforward to check convergence by “increasing model space size”

FMD vs few-body models

Advantages

- microscopic - antisymmetrization
- cluster structure appears naturally, includes polarization effects
- uses nucleon-nucleon interaction, no need for optical potentials

Disadvantages

- numerical effort, “exact” calculations are not possible
- much more difficult to include boundary conditions for resonance or scattering states

Thanks

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