Effective Range Corrections to Few-Body Universality

in an Effective Field Theory Approach

Chen Ji

Department of Physics and Astronomy
Ohio University

Weakly Bounds Systems in Atomic and Nuclear Physics
INT-10-46W, Seattle, March 9, 2010

in collaboration with **Daniel Phillips** and **Lucas Platter**

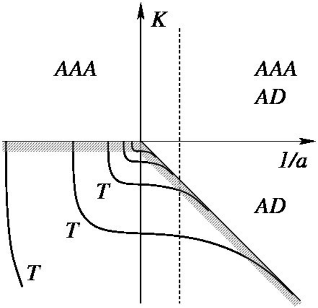
Outline

1. Universality in three-body systems
2. EFT analysis of three-body systems at LO
3. A perturbative analysis of range effects

Universality in 3-body systems

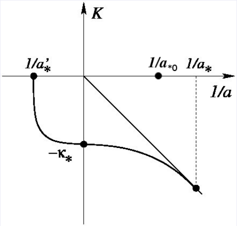
OHIO UNIVERSITY

Universality exists in low-energy 3-body systems with $|a| \gg \ell$



Braaten, Hammer (2006)

On the n_*th branch:



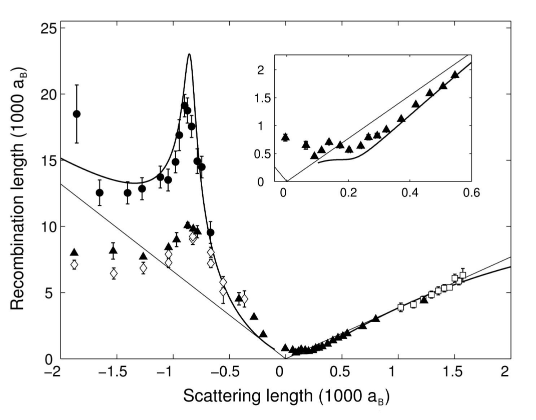
a'_*: 3-atom resonance (AAA↔T)
a_*: atom-dimer resonance (AD↔T)
a_{*0}: recombination minimum (AAA↔AD)

- Universal relations: $\kappa_*^{-1} = 3.12 a_{*0} = -0.641 a'_* = 14.1 a_*$
- Efimov spectrum in the unitary limit $|a| \to \infty$: $E_n = (e^{-2\pi/s_0})^{n-n_*} \hbar^2 \kappa_*^2 / m$

Efimov (1970); Braaten, Hammer (2006)

Three-body effects in cold atomic gases

Observation of three-body recombination rate in ultracold gas of ^{133}Cs atoms



Kraemer *et al.* (2006)

Other recent observations:

- Observation of an Efimov-like trimer resonance in ultracold atom-dimer scattering
  Knoop *et al.*, Nature Physics **5**, 227 - 230 (2009)
- Observation of universality in ultracold 7Li three-body recombination
  Gross *et al.*, Phys. Rev. Lett. 103, 163202 (2009)
- Universality in Three- and Four-Body Bound States of Ultracold Atoms
  Pollack *et al.*, Science 326, 1683 (2009)
- Observation of an Efimov spectrum in an atomic system
  Zaccanti *et al.*, Nature Physics 5, 586 - 591 (2009)

Effective field theory in three-body physics

- Expansion of r_0/a is applied to the study of three-body low-energy phenomena
- LO approach successfully describes three-body systems near the unitary limit $a \to \infty$
- **Effective-range corrections still need to be considered at finite scattering length**

Effective range in different systems

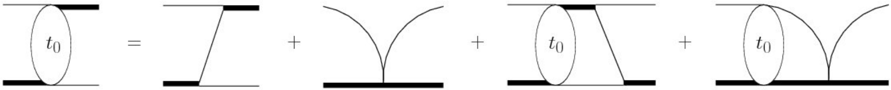
- NN, NNN, ... (nucleons): $r_0/a \sim 1/3$
- ^7Li (cold atoms): a varies, $r_0 \sim 30a_B$ (Van der Waals interaction)

A perturbative analysis to effective-range effects:

- effects from range of potential ℓ and effective range r_0 are separated
- cutoff scale: $\Lambda \sim 1/\ell$
- expansion parameter: r_0
- r_0 is finite even at $\ell \sim 0$
- we apply perturbative analysis at $\ell \sim 0$ ($\Lambda \to \infty$)

EFT analysis of three-boson systems at LO

Solving atom-dimer scattering amplitude at LO:



Solve LO S-wave t_0 from integral equation

$$t_0(q,p;E) = M(q,p;E) + \frac{2}{\pi}\int^{\Lambda} dq' \; \frac{q'^2}{-1/a + \sqrt{3q'^3/4 - mE^+}} M(q',p;E)\, t_0(q,q';E)$$

$$M(q,p;E) = \frac{1}{qp}\log\left(\frac{q^2+p^2+qp-mE}{q^2+p^2-qp-mE}\right) + \frac{2H_0(\Lambda)}{\Lambda^2}$$

Skorniakov, Ter-Martirosian (1957)
Bedaque, Hammer, van Kolck (1999)

LO three-body force and renormalization

The LO 3-body force H_0 represents short-distance physics

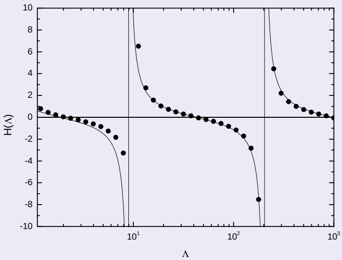
renormalization:

- adjust H_0 at fixed Λ
- reproduce one 3-body observable

$$H_0(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$

where $\Lambda_* = 2.61\kappa_*$.

Bedaque, Hammer, van Kolck (1999)



Asymptotics of the amplitude, where $p \gg k, 1/a$, and $k \sim 1/a$

$$t_0(k,p) \sim \frac{\sin[s_0 \ln(p/\Lambda_*)]}{p} + 8|B_{-1}| \frac{\sin[s_0 \ln(p/\Lambda_*) + \arg(B_{-1})]}{ap^2}$$

Bedaque, Rupak, Griesshammer, Hammer (2003)

Three-body observables at LO

OHIO UNIVERSITY

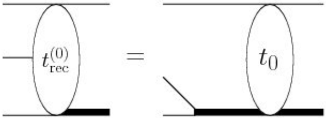
- **Binding energy**

$$t_0(q,p;E) = \frac{\Gamma(q)\Gamma(p)}{E+B_0} + \mathcal{R}(q,p;E)$$

$$\Gamma(q) = \frac{2}{\pi}\int^{\Lambda} dq' \; M(q,q';-B_0)\frac{q'^2}{-1/a+\sqrt{3q'^2/4+mB_0}}\Gamma(q')$$

at $a = a_*,\ B_0 = -1/(ma_*^2)$ $\leftarrow$ atom-dimer resonance $(a > 0)$
at $a = a'_*,\ B_0 = 0$ $\leftarrow$ 3-atom resonance $(a < 0)$

- **Recombination rate**



$$\alpha_0 = \frac{512\pi^2}{\sqrt{3}m}\left|t_0(0,2/(\sqrt{3}a);0)\right|^2, \qquad \frac{dn}{dt} = -3\alpha n^3$$

at $a = a_{*0},\ t_0(0,2/(\sqrt{3}a_{*0});0) = 0$ $\leftarrow$ recombination minimum $(a > 0)$

A perturbative analysis of range effects

Effective-range expansion of physical quantities

$$\begin{aligned} 1/a &= \gamma + \frac{1}{2} r_0 \gamma^2 + \cdots , \\ \Delta(p) &= \Delta^{(0)}(p) + \Delta^{(1)}(p) + \cdots , \\ t(k,p) &= t^{(0)}(k,p) + t^{(1)}(k,p) + \cdots , \\ H(\Lambda,\gamma) &= H^{(0)}(\Lambda) + H^{(1)}(\gamma,\Lambda) + \cdots \end{aligned}$$

NLO dimer propagator:

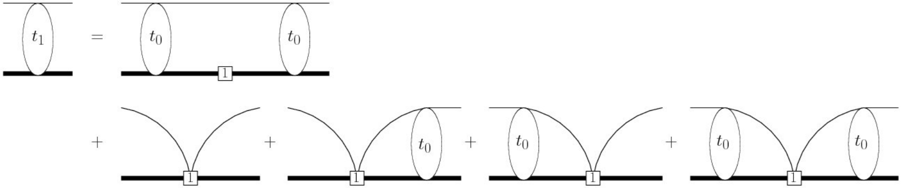
$$\Delta^{(1)}(q) = \frac{r_0}{2} \frac{1/a + \sqrt{-mq_0 + \vec{q}^2/4 - i\epsilon}}{-1/a + \sqrt{-mq_0 + \vec{q}^2/4 - i\epsilon}}$$

c.f. Hammer, Mehen (2001)

A perturbative analysis of range effects

OHIO UNIVERSITY

The NLO correction to dimer propagator is inserted between LO wave functions



$$
\begin{aligned}
t_1(k,k;E) \quad = \quad & \frac{r_0}{\pi}\int^{\Lambda} dq'\, q'^2 \frac{1/a+\sqrt{3q'^2/4-mE}}{-1/a+\sqrt{3q'^2/4-mE^+}}\, t_0^2(k,q';E) \\
& + \frac{2H_1(\Lambda)}{\Lambda^2}\left(1+\frac{2}{\pi}\int^{\Lambda} dq' \frac{q'^2 t_0(k,q';E)}{-1/a+\sqrt{3q'^2/4-mE^+}}\right)^2
\end{aligned}
$$

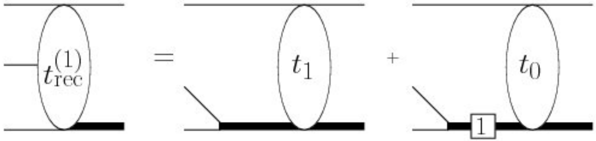
c.f. Hammer, Mehen (2001)

NLO range corrections to 3-body observables

- **NLO binding energy shift**

$$\Delta B = -\frac{r_0}{\pi}\int^{\Lambda} dq\; q^2 \frac{1/a+\sqrt{3q^2/4+mB_0}}{-1/a+\sqrt{3q^2/4+mB_0}}\Gamma^2(q) - \frac{8H_1(\Lambda)}{(\pi\Lambda)^2}\left[\int^{\Lambda} dq\; \frac{q^2}{-1/a+\sqrt{3q^2/4+mB_0}}\Gamma(q)\right]^2$$

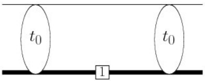
- **NLO recombination rate**



$$\alpha^{NLO} = \frac{512\pi^2}{\sqrt{3}m}\left|(1+r_0/a)t_0(0,\frac{2}{\sqrt{3}a};0) + t_1(0,\frac{2}{\sqrt{3}a};0)\right|^2$$

3-body force and renormalization at NLO

Using the asymptotic behavior of t_0



$$\sim r_0\Lambda + \frac{r_0}{a}\ln(\Lambda/\Lambda_*) + \cdots$$

The NLO 3-body force H_1 absorbs cutoff dependencies:

$$H_1(a,\Lambda) = r_0\Lambda h_{10}(\Lambda) + \frac{r_0}{a}\ln(\Lambda/\Lambda_*)h_{11}(\Lambda)$$

where $h_{10}(\Lambda)$ and $h_{11}(\Lambda) \sim \mathcal{O}(\Lambda^0)$

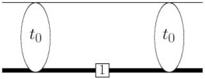
NLO renormalization scheme

$\mathbf{h_{10}}$ fits the observable chosen at LO

$\mathbf{h_{11}}$ fits one extra observable

3-body force and renormalization at NLO

Using the asymptotic behavior of t_0



$$\sim r_0\Lambda + \frac{r_0}{a}\ln(\Lambda/\Lambda_*) + \cdots$$

The NLO 3-body force H_1 absorbs cutoff dependencies:

$$H_1(a,\Lambda) = r_0\Lambda h_{10}(\Lambda) + \frac{r_0}{a}\ln(\Lambda/\Lambda_*)h_{11}(\Lambda)$$

where $h_{10}(\Lambda)$ and $h_{11}(\Lambda) \sim \mathcal{O}(\Lambda^0)$

NLO renormalization scheme

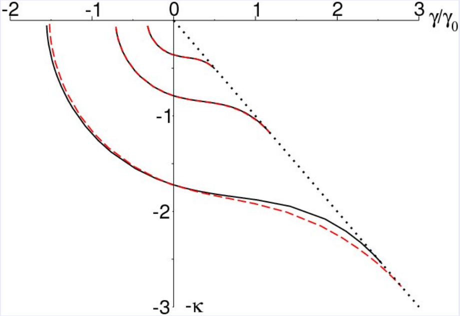
h_{10} fits the observable chosen at LO

h_{11} fits one extra observable

Two observables at different scattering lengths are needed at NLO

NLO shift to 3-body spectrum

Fixing κ_* in the unitary limit at NLO:



where $\gamma \sim 1/a$, $\ell \sim 0$

In the unitary limit:

- $\Delta B^{(n)} = 0$ for all n, as $\Delta B^{(n_*)} = 0$

Platter, Ji, Phillips (2009)

NLO shift to positions of 3-body observables

- atom-dimer resonance at $a > 0$:

$$\Delta a_* = -\frac{\Delta B(a_*)}{2/a_*^3 + B_0'(a_*)}$$

- three-atom recombination resonance at $a < 0$:

$$\Delta a_*' = -\frac{\Delta B(a_*')}{B_0'(a_*')}$$

- three-atom recombination minimum at $a > 0$:

$$\Delta a_{*0} = -\frac{t_1(a_{*0})}{t_0'(a_{*0})}$$

NLO corrections to universal relations

Semi-universal relations in ultra-cold ^7Li atoms at fixed r_o:

$^7Li \quad |F=1, m_F=1>$

- $a_* = 608(11)a_B$
- $a_{*0} = 2676(67)a_B$

Pollack *et al.* (2009)

Renormalization scheme

LO: fit to exp $a_* \quad \Rightarrow \kappa_*$
NLO: keep κ_*, a_*; solve a_{*0}^{NLO}

$$\mathbf{a_{*0}^{NLO} = 4.53a_* + 2.48r_0}$$

theo: $(\text{take } r_0 \sim -7a_B)$

- $a_{*0}^{LO} = 2754a_B$
- $a_{*0}^{NLO} = 2737a_B$

$^7Li \quad |F=1, m_F=0>$

- $a_{*0} \simeq 1160a_B$
- $a'_* = -264(11)a_B$

Gross *et al.* (2009)

Renormalization scheme

LO: fit to exp $a_{*0} \quad \Rightarrow \kappa_*$
NLO: keep κ_*, a_{*0}; solve $a'_{*\ NLO}$

$$\mathbf{a'_{*\ NLO} = -0.215a_{*0} + 1.36r_0}$$

theo: $(\text{take } r_0 \sim -20a_B)$

- $a'_{*\ LO} = -249a_B$
- $a'_{*\ NLO} = -276a_B$

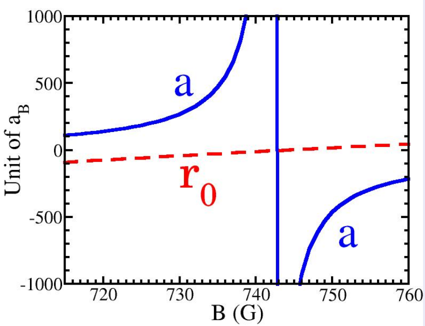
Variable effective range

a and r_0 are both functions of the magnetic field

Gross *et al.* (2009)

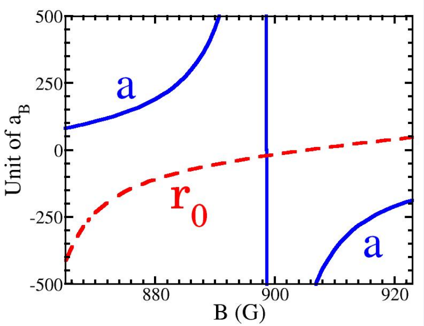
$^7Li \quad |F = 1, m_F = 1>$

Pollack *et al.* (2009)



$^7Li \quad |F = 1, m_F = 0>$

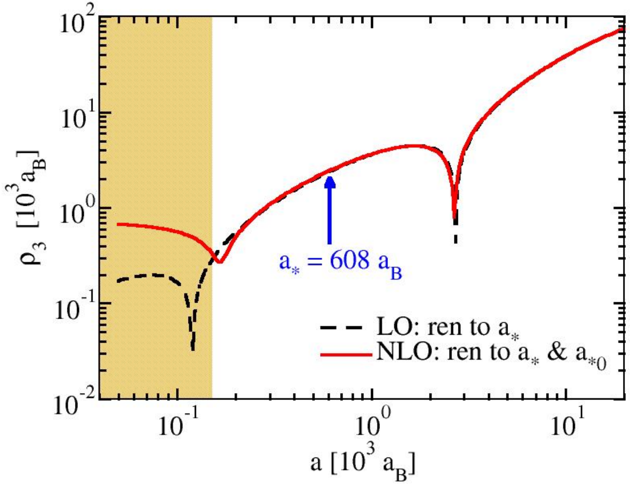
Gross *et al.* (2009)



3-body recombination in ^{7}Li ($m_F = 1$)

experimental data: $a_* = 608 a_B$ $a_{*0} = 2676 a_B$ $a_{(n_*-1)0} = 119 a_B$

Pollack *et al.* (2009)



Includes $r_0 = r_0(a)$:
$r_0/a > 0.5$ at $a < 150 a_B$

- a_{*0} agrees
- $a_{(n_*-1)0}$ agrees with LO, but not NLO

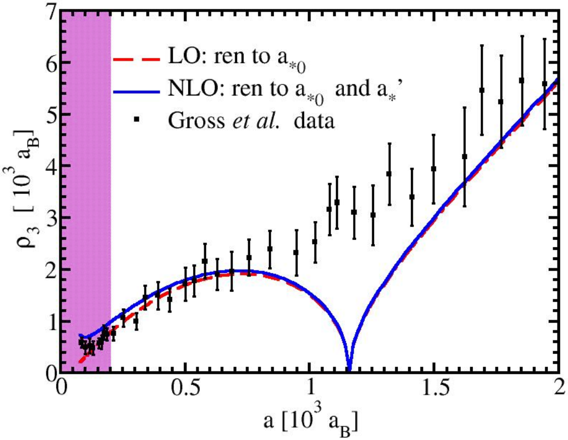
3-body recombination in ^{7}Li ($m_F = 0$)

experimental data: $a_{*0} = 1160 a_B \quad a'_* = -264 a_B$

Gross *et al.* (2009)

Deep dimer and finite temperature effects are not included

c.f. Braaten, Hammer, Kang, Platter (2008)



Includes $r_0 = r_0(a)$:
$r_0/a > 0.5$ at $a < 200 a_B$

- recombination rate at $a < 0$ needs deep dimer
- position of a_{*0} and a'_* can be determined without deep dimer

Summary

- Universality connects various 3-body systems

  A consistent study of range effects distinguish different 3-body systems (essential in nuclear physics)

- 3-body force in EFT encodes short-distance physics

  NLO 3BF depends on a, needs to fit to extra data
  $H_1(a,\Lambda) = r_0\Lambda h_{10}(\Lambda) + (r_0/a)\ln(\Lambda/\Lambda_*)h_{11}(\Lambda)$

- Effective-range corrections connect thee-body parameters beyond universality

  The perturbative analysis relates r_0/a expansion to LO deviation from experiments

Discussion

- The perturbative NLO analysis doesn't work well when r_0/a is too large
- The impact of the variable effective range model needs to be further studied
- A complete study of 3-body recombination needs to include deep dimer and finite temperature effects
- More experimental data would be useful to test EFT above LO

c.f. Bedaque, Hammer, van Kolck (1999)