Three-body forces in nucleonic matter

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TRIUMF

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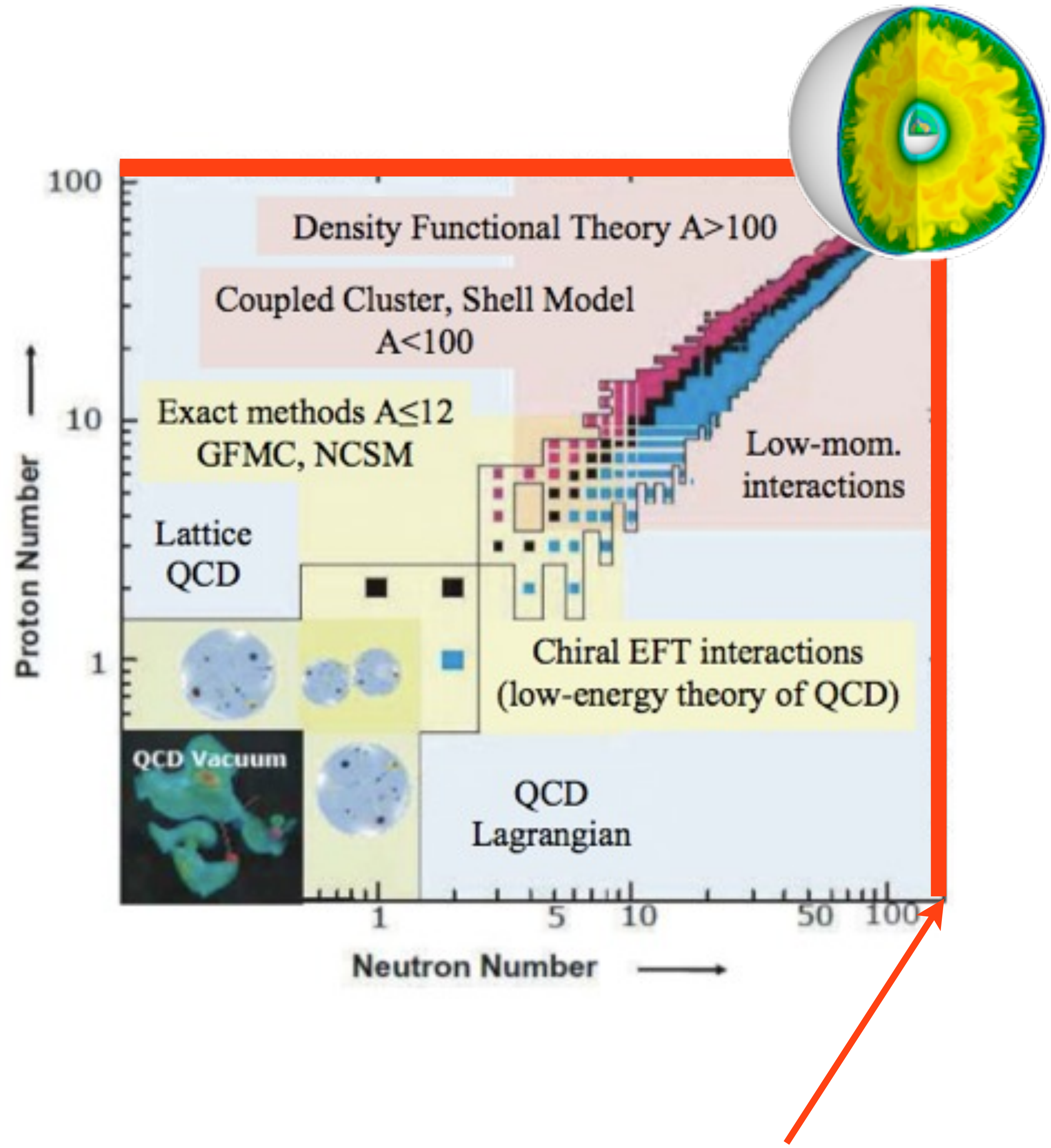
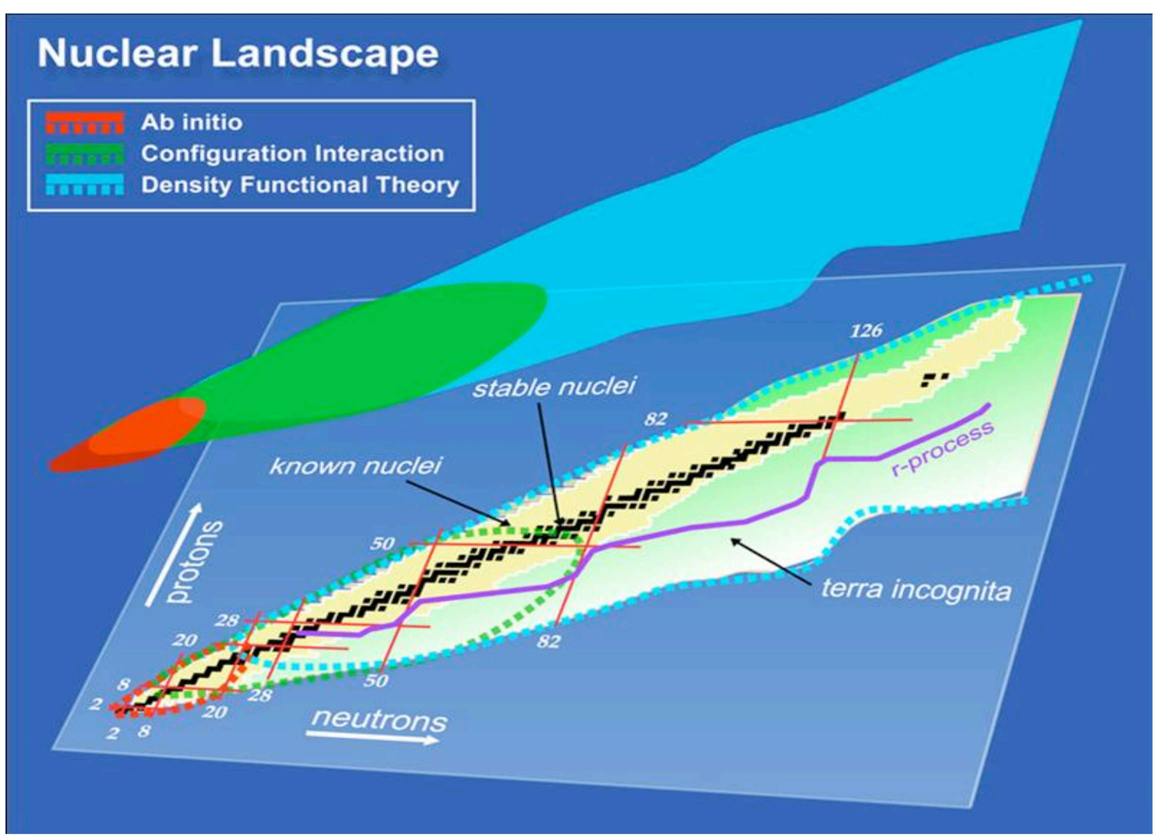
Weakly-Bound Systems
in Atomic and Nuclear Physics

The nuclear landscape

- Nuclear systems are complex many-body systems with rich properties
- No "one size fits all" method
- All theoretical approaches need to be linked





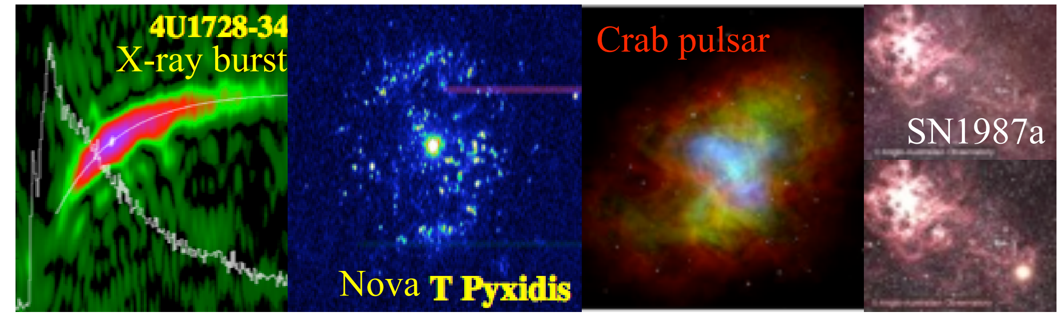
Nucleonic matter:
Infinite system of interacting neutrons and protons in the thermodynamic limit.

Significance of nuclear and neutron matter results

• for the extremes of astrophysics:

neutron stars, supernovae,

neutrino interactions with nuclear matter



• microscopic constraints of nuclear energy-density functionals,

next-generation Skyrme functionals

UNEDF

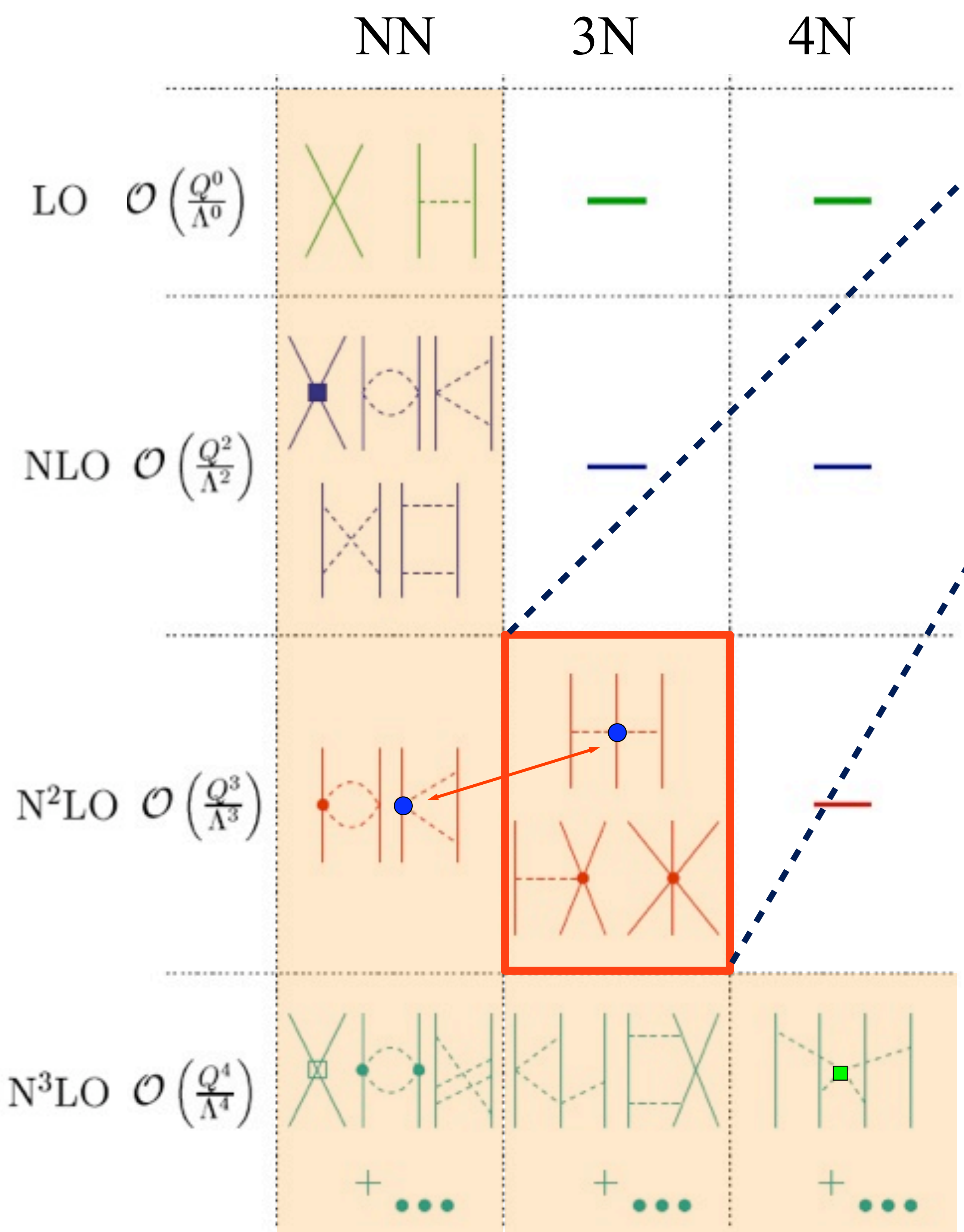
$$\mathcal{E} = \frac{\tau}{2M} + A[\rho] + B[\rho]\tau + C[\rho]|\nabla\rho|^2 + \cdots$$

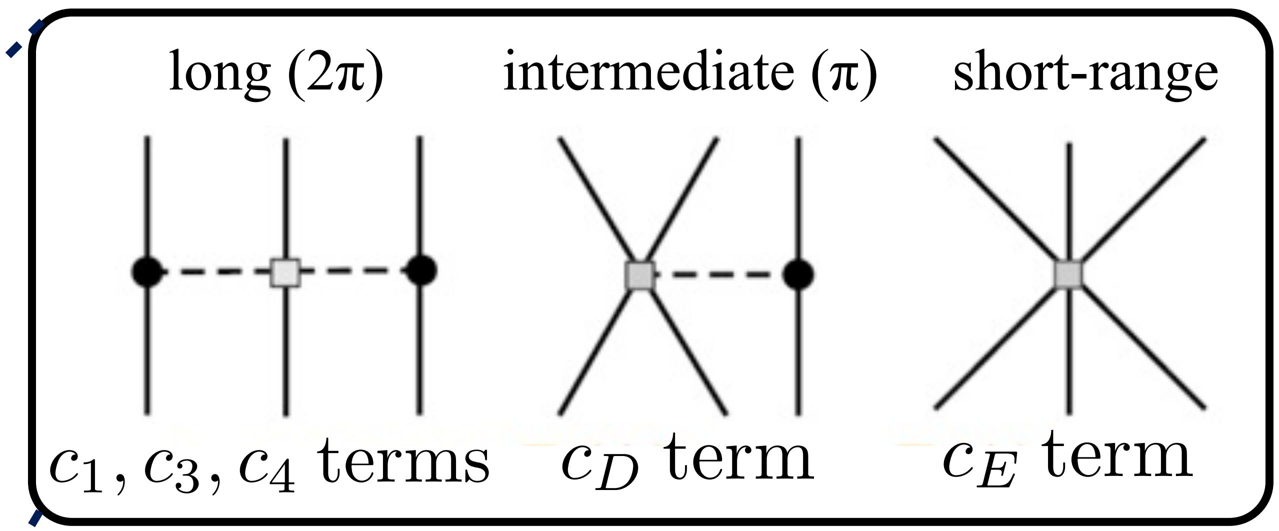
• universal properties at low densities ⟶ ultracold Fermi gases

• my focus: development of efficient methods to include 3N forces in microscopic many-body calculations of neutron and nuclear matter

⟶ applications to finite systems

Reminder: Chiral EFT for nuclear forces





large uncertainties in coupling constants at present:

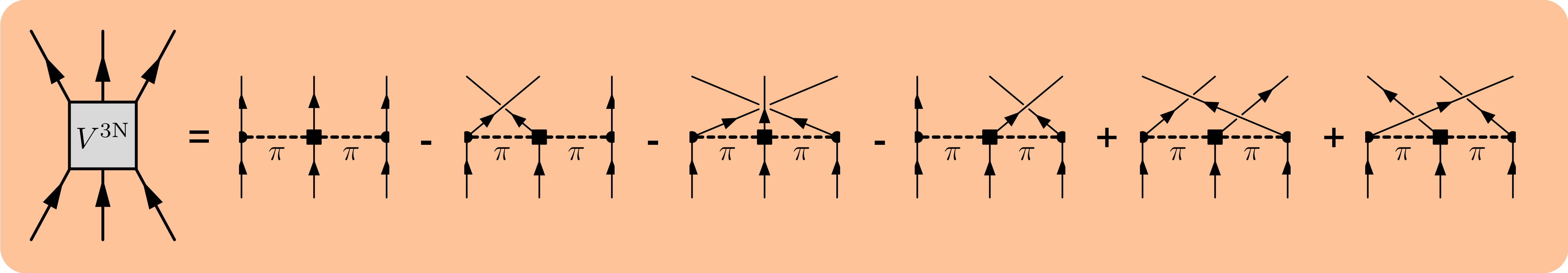
$$c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.5}_{-1.0}\,,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

Meissner et al. , Machleidt

Chiral 3N interaction as density-dependent two-body interaction

antisymmetrized 3N interaction (at N²LO) in neutron matter:

$$V^{3\mathrm{N}} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \sum_{i\neq j\neq k} A_{ijk} \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{q}_i)(\boldsymbol{\sigma}_j \cdot \mathbf{q}_j)}{(\mathbf{q}_i^2 + m_\pi^2)(\mathbf{q}_j^2 + m_\pi^2)} \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q}_i \cdot \mathbf{q}_j\right]$$



c_4 , c_D and c_E terms vanish in neutron matter

in nuclear matter all terms contribute

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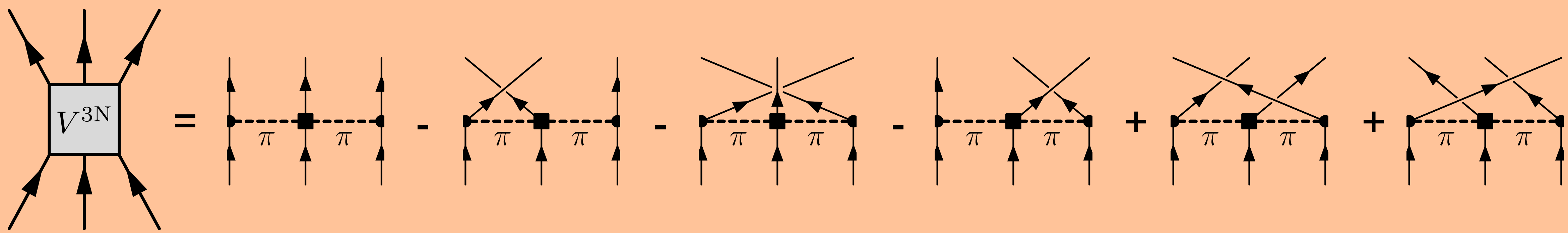

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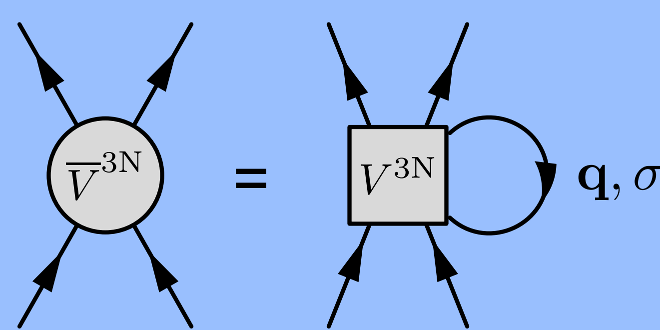
Basic idea: Sum one particle over occupied states in the Fermi sea

$$\overline{V}^{3\mathrm{N}} = \sum_{\mathbf{q},\sigma} V^{3\mathrm{N}} n(k_\mathrm{F} - q)$$



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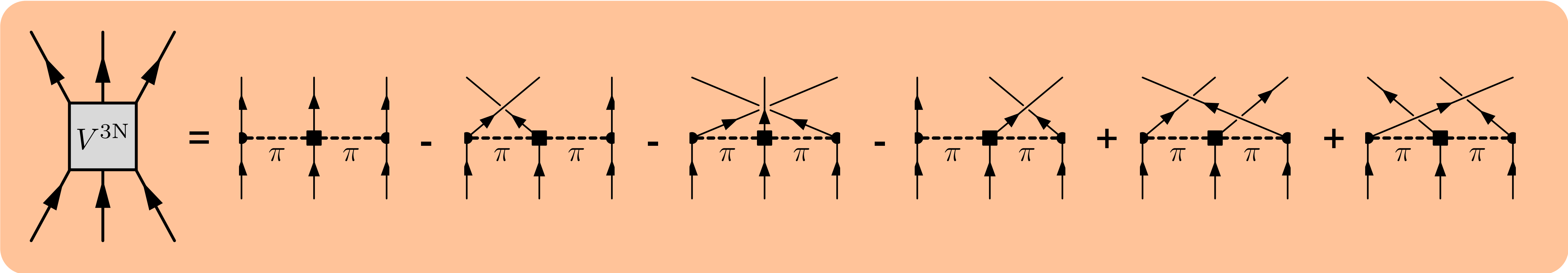

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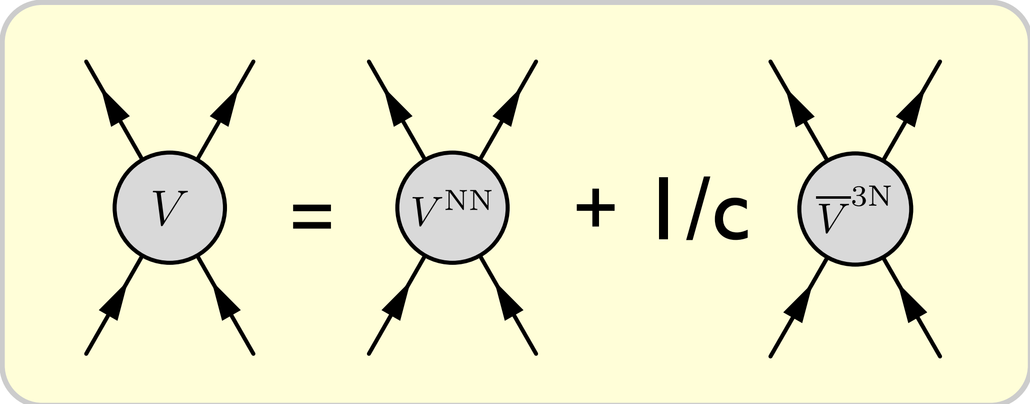
Provides 3N corrections to free space NN interaction:



Operator form of $\overline{V}^{3\mathrm{N}}$ in neutron matter

general momentum dependence : $\overline{V}^{3\mathrm{N}} = \overline{V}^{3\mathrm{N}}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

P-dependence only weak, evaluate for $\mathbf{P} = 0$:

$$\overline{V}^{3\mathrm{N}}_{P=0} = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \left[-\frac{4c_1 m_\pi^2}{f_\pi^2} A(\mathbf{k}, \mathbf{k}') + \frac{2c_3}{f_\pi^2} B(\mathbf{k}, \mathbf{k}')\right]$$

$$\begin{aligned}
B(\mathbf{k}, \mathbf{k}') = \\
& -\frac{1}{3}\left\{\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^4}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2} + 2B_1^s(\mathbf{k},\mathbf{k}') - B_1^s(\mathbf{k},-\mathbf{k}') - (B_2^s(\mathbf{k},\mathbf{k}') + B_2^s(\mathbf{k}',\mathbf{k}))\right\} \\
& +\frac{1}{3}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\left\{\frac{2}{3}\frac{\rho(k,k')(\mathbf{k}-\mathbf{k}')^4}{((\mathbf{k}-\mathbf{k}')^2+m_\pi^2)^2} + \frac{1}{3}\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^4}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2}\right. \\
& \qquad \left. + B_1^s(\mathbf{k},-\mathbf{k}') - \frac{1}{3}\left[B_2^s(\mathbf{k},\mathbf{k}') + B_2^s(\mathbf{k}',\mathbf{k})\right] - \frac{2}{3}\left[B_2^s(\mathbf{k},-\mathbf{k}') + B_2^s(\mathbf{k}',-\mathbf{k})\right]\right\} \\
& +\frac{2}{3}\left[\frac{\rho(k,k')(\mathbf{k}+\mathbf{k}')^2 S_{12}(\mathbf{k}+\mathbf{k}')}{((\mathbf{k}+\mathbf{k}')^2+m_\pi^2)^2} - \frac{\rho(k,k')(\mathbf{k}-\mathbf{k}')^2 S_{12}(\mathbf{k}-\mathbf{k}')}{((\mathbf{k}-\mathbf{k}')^2+m_\pi^2)^2}\right] \\
& +\frac{2}{3}\sigma^a\sigma'^b\left[B_{ab}^t(\mathbf{k},\mathbf{k}') - B_{ab}^t(\mathbf{k},-\mathbf{k}') + B_{ab}^t(\mathbf{k}',\mathbf{k}) - B_{ab}^t(\mathbf{k}',-\mathbf{k})\right] \\
& +\frac{1}{3}i\,(\sigma^a + \sigma'^a)\left[B_a^v(\mathbf{k},\mathbf{k}') - B_a^v(\mathbf{k},-\mathbf{k}')\right]
\end{aligned}$$

Operator form of $\overline{V}^{3\mathrm{N}}$ in neutron matter

general momentum dependence : $\overline{V}^{3\mathrm{N}} = \overline{V}^{3\mathrm{N}}(\mathbf{k}, \mathbf{k}', \mathbf{P})$

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$$B_1^s(\mathbf{k}, \mathbf{k}')$$

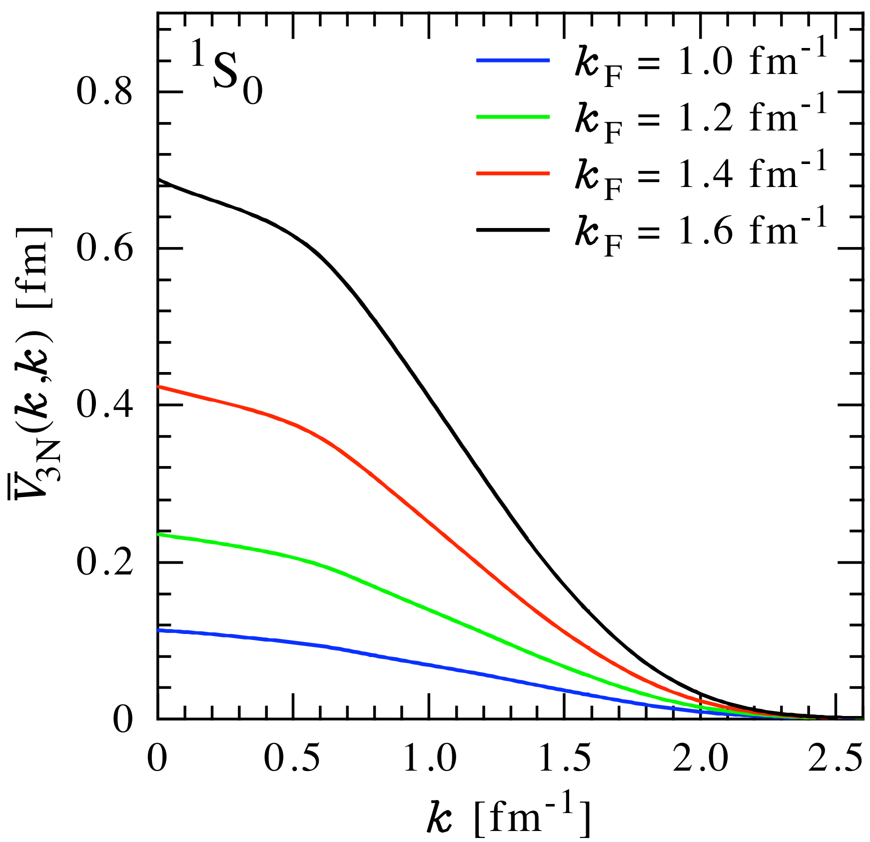
$$B_1^s(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} n(q) f_R(\Lambda_{3\mathrm{N}}, q, k_1) f_R(\Lambda_{3\mathrm{N}}, q, k_2) \cdot \frac{((\mathbf{k}_1 + \mathbf{q}) \cdot (\mathbf{k}_2 + \mathbf{q}))^2}{((\mathbf{k}_1 + \mathbf{q})^2 + m_\pi^2)((\mathbf{k}_2 + \mathbf{q})^2 + m_\pi^2)}$$

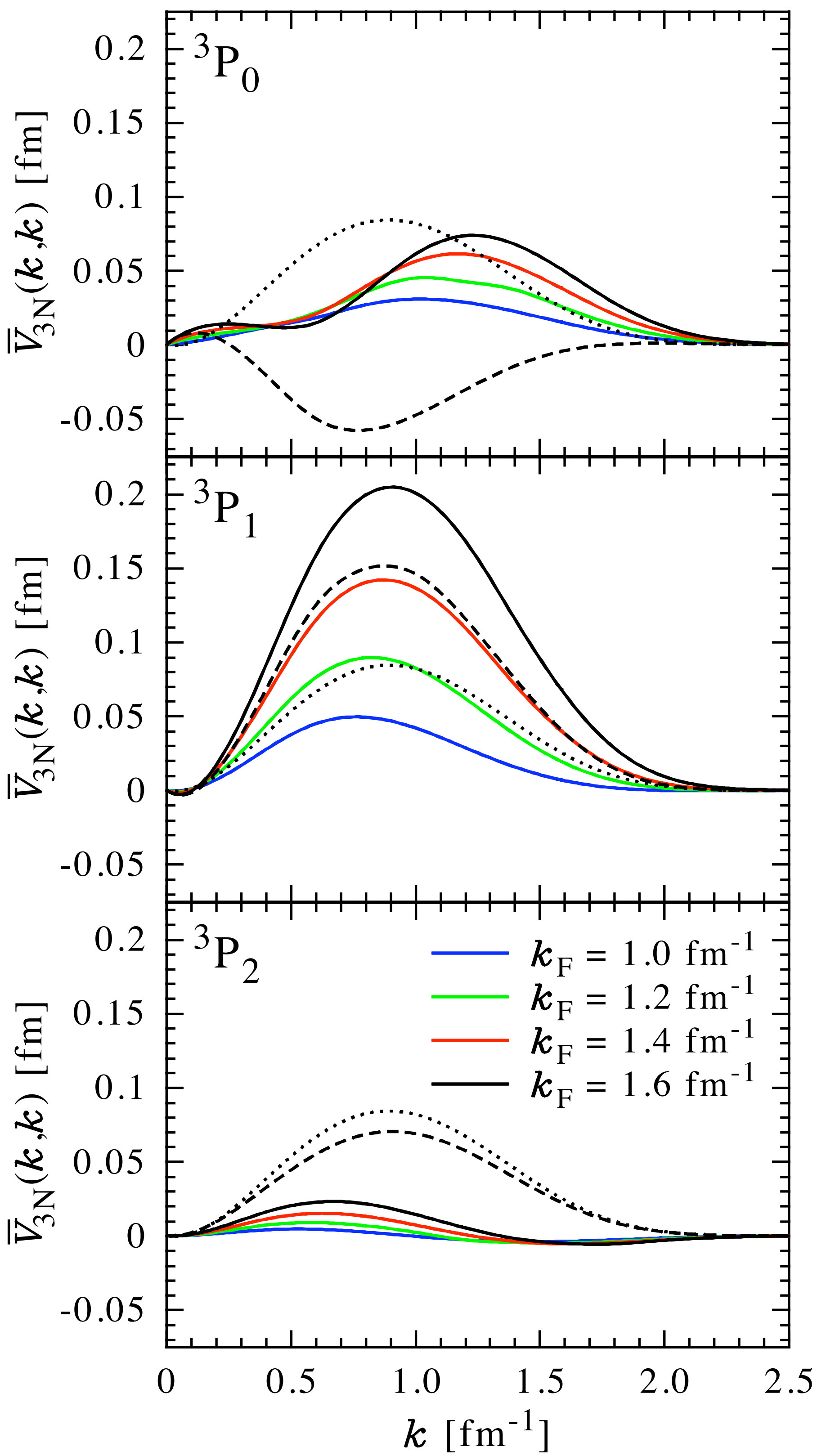
- neglect P-dependence in the following, set P=0
- in fixed-P approximation $\overline{V}^{3\mathrm{N}}$ matrix elements have the same form

like genuine free-space NN matrix elements

- straightforward to incorporate in existing many-body schemes

Partial wave matrix elements $(\Lambda_{3N} = 2.0\,\mathrm{fm}^{-1})$

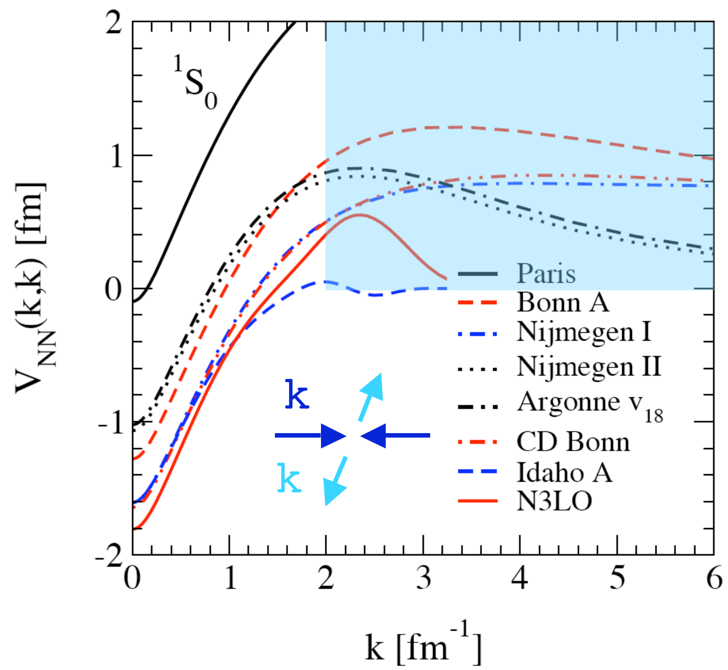


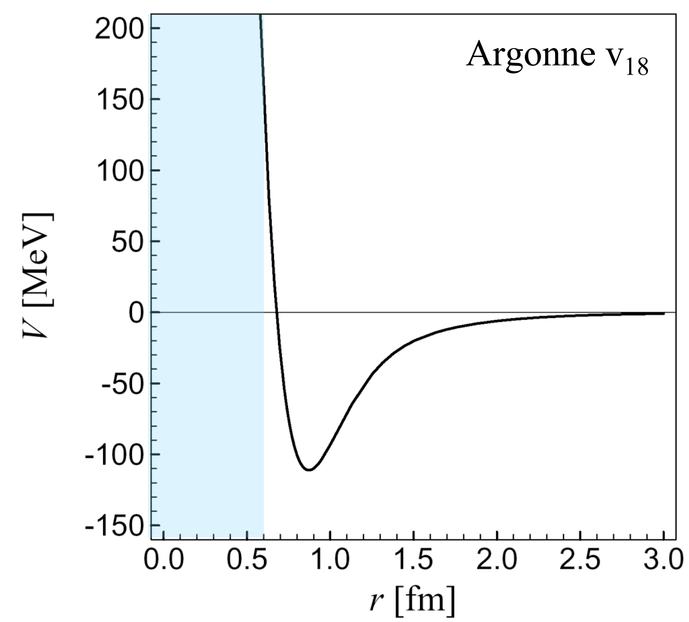


- non-trivial density dependence
- $\overline{V_{3N}}(k, k'; ^1S_0) \sim k_F^4 \sim \rho^{4/3}$
- dominant central contributions
- non-central tensor and spin-orbit components

KH and A.Schwenk arXiv:0911.0483

Low-momentum interactions from the RG





- fundamental ingredient of microscopic nuclear many-body calculations
- interaction constructed from experimental phase shifts and scattering lengths, interaction no physical observable!
- well constrained long-distance part, ill-defined short distance part
- hard core leads to technical problems in many-body calculations, non-perturbative

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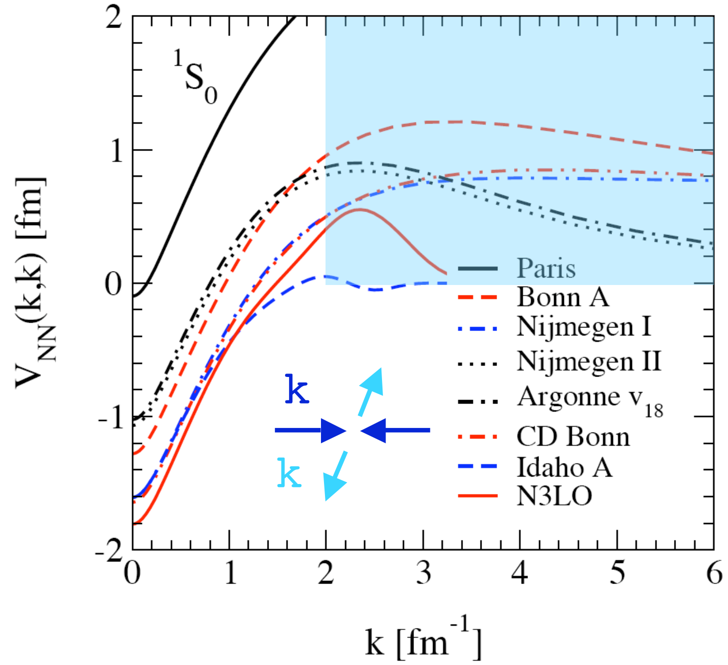
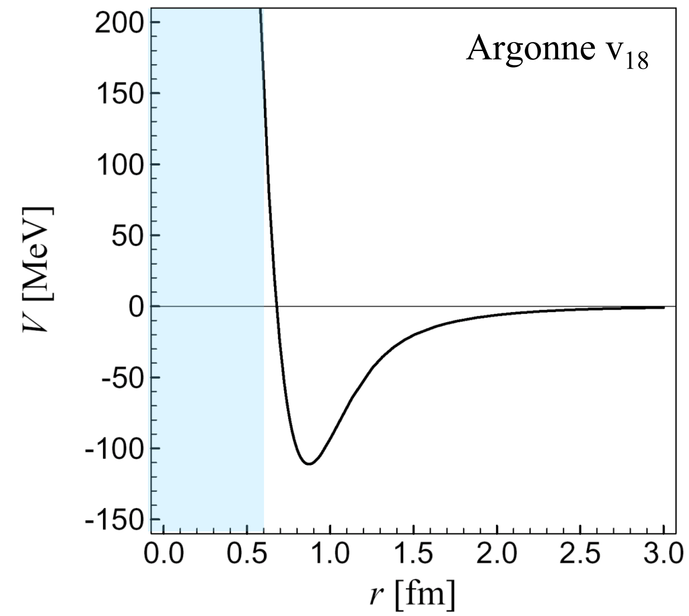
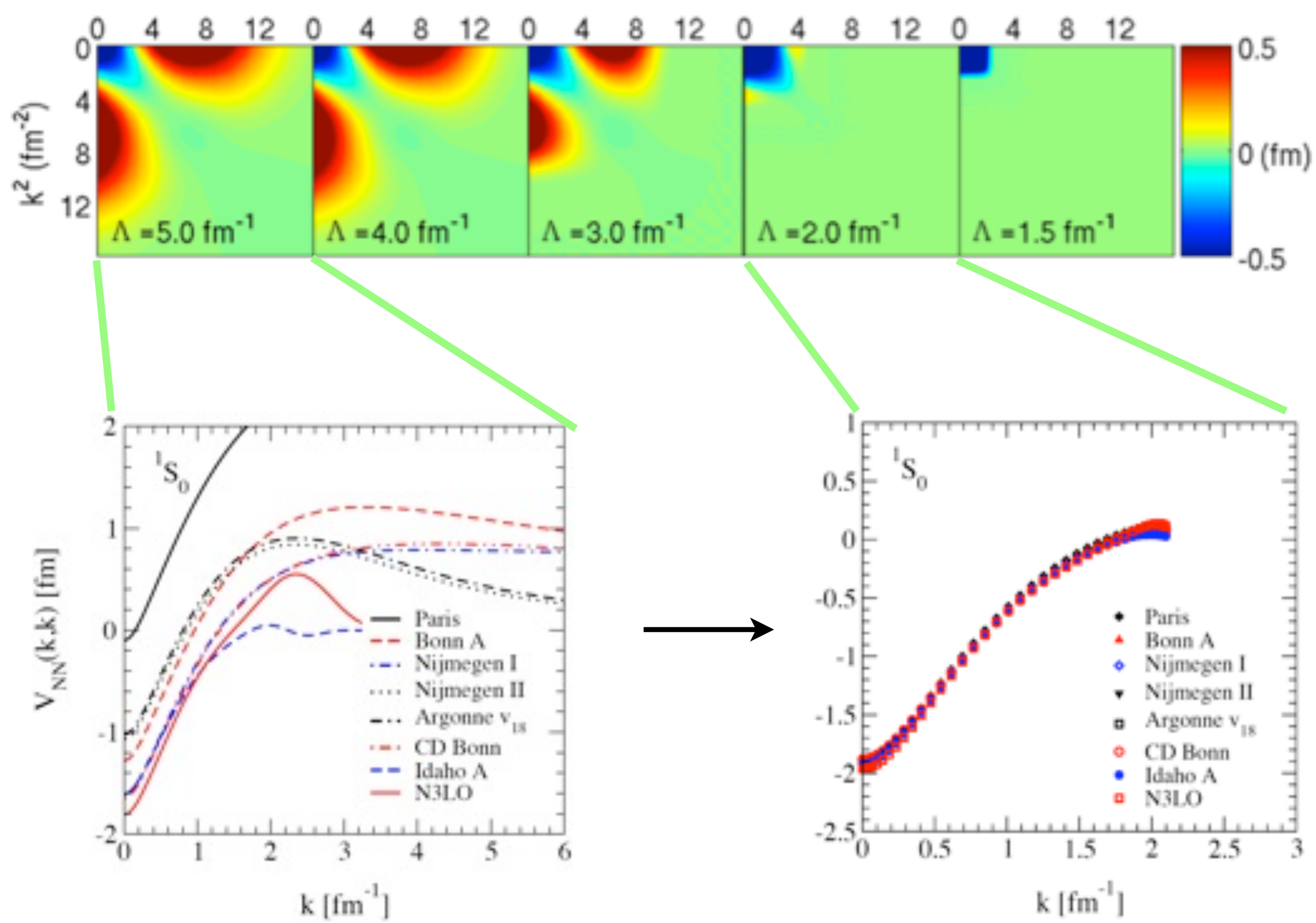






RG evolution decouples low and high momenta

↓

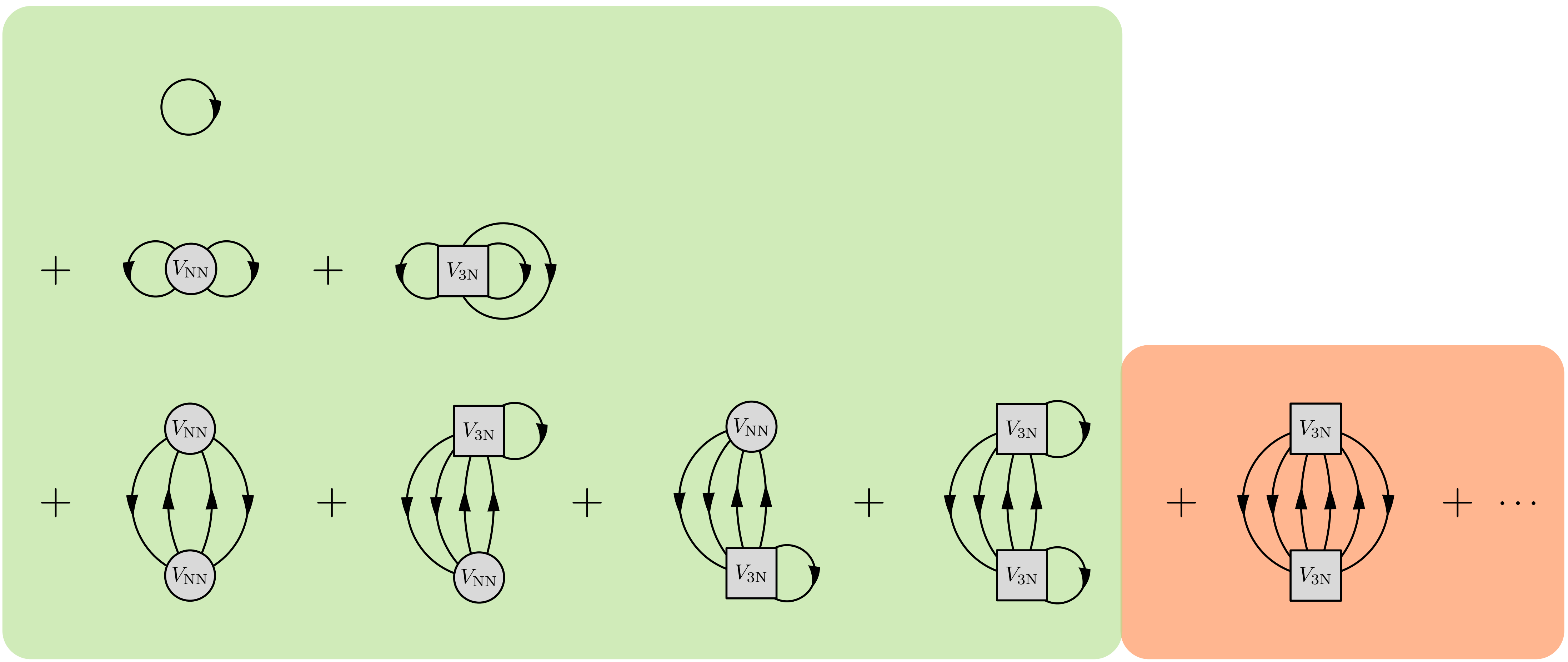
universal interaction for low momenta $V_{low\ k}(\Lambda)$

Equation of state (EOS):
Many-body perturbation theory

central quantity of interest: energy per particle E/N

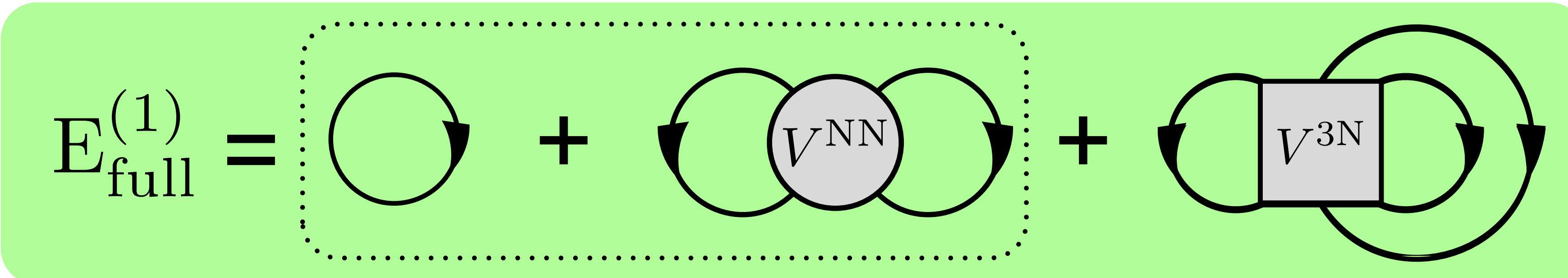
$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + \ldots\,.$$

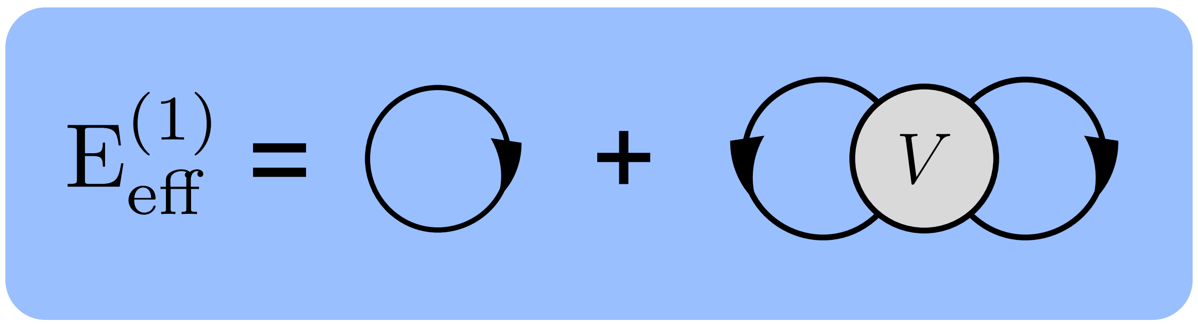
$E =$



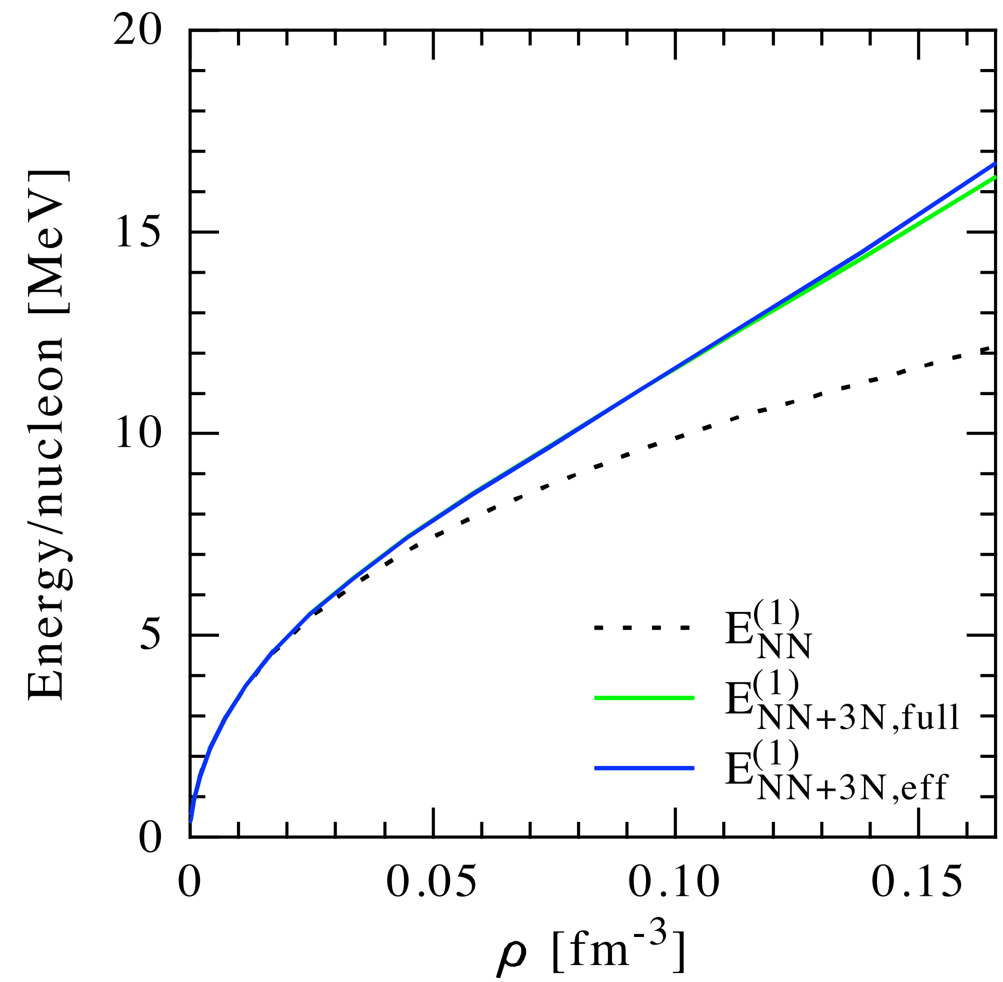
- for low momentum interactions no resummation of diagrams necessary
- self-consistent single-particle propagators $\longrightarrow$ thermodynamic consistency

Neutron matter:
EOS (first order), Test of fixed-P approximation





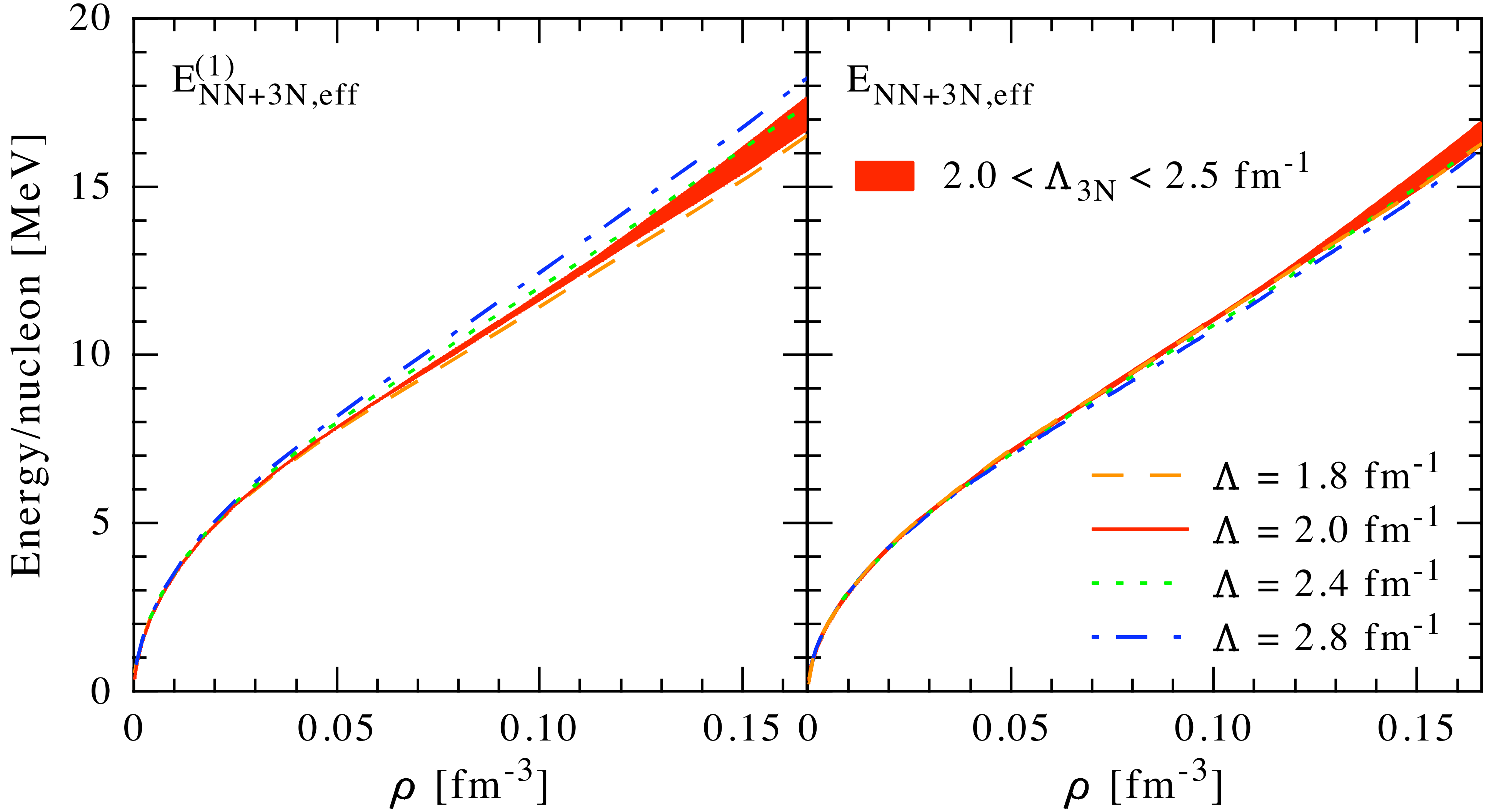
relative difference of
3N contributions only ~3%



KH and A.Schwenk arXiv:0911.0483

P-independent effective NN interaction is a very good approximation

Neutron matter: EOS (second order)



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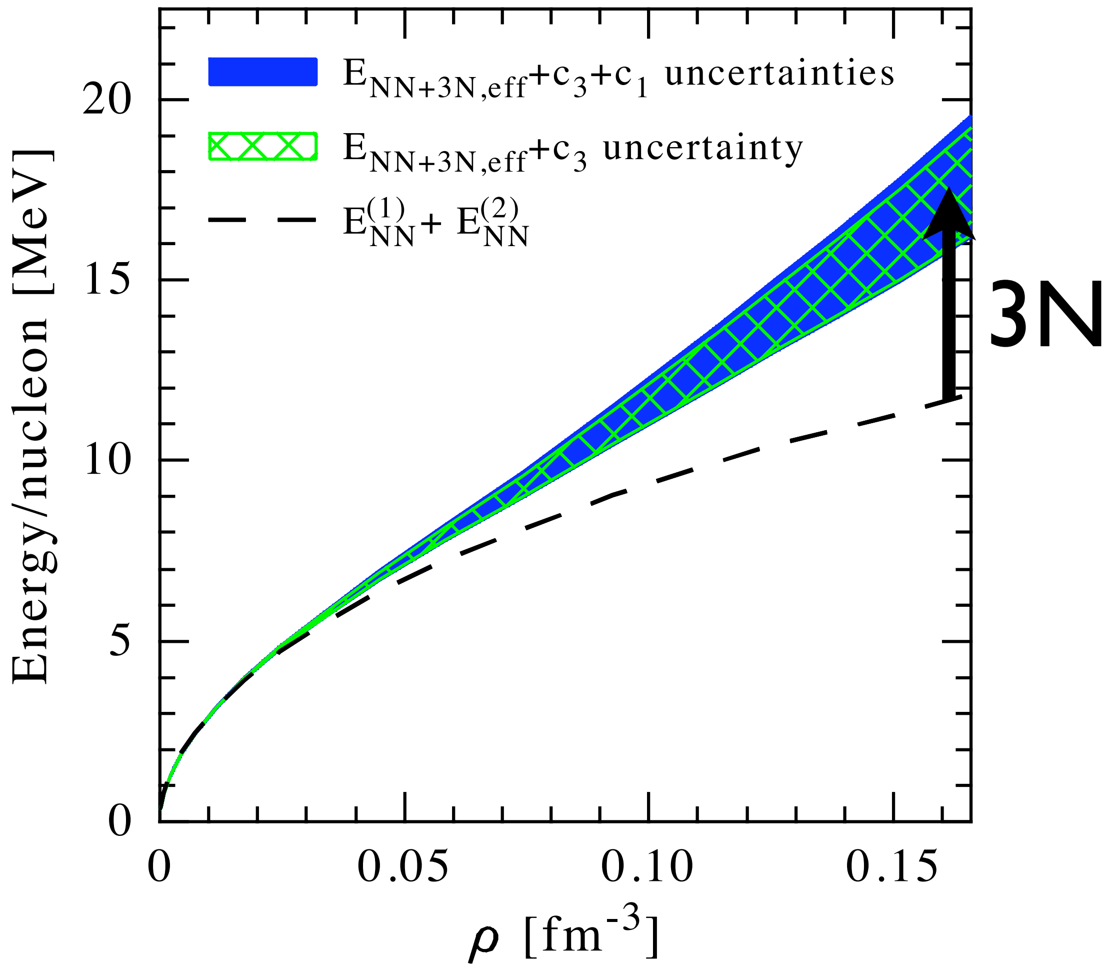
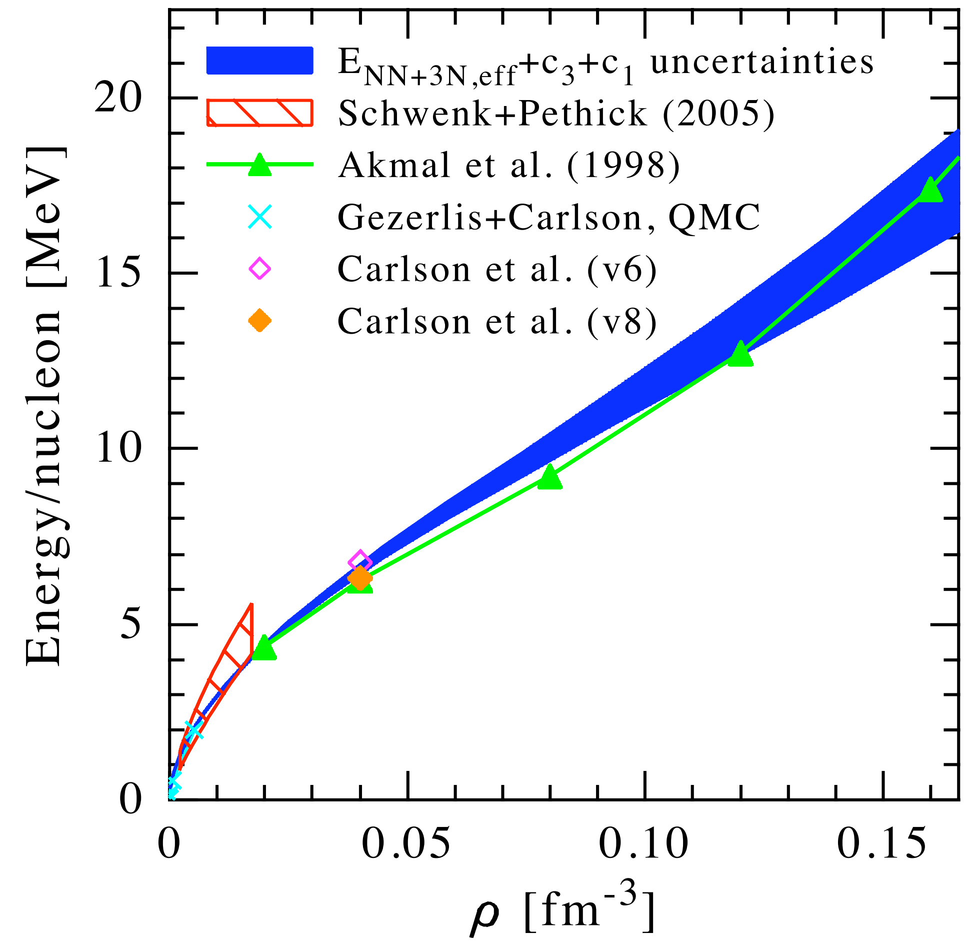
- reduced cutoff dependence at 2nd order
- self-energy effects small
- system is perturbative for low-momentum interactions

Neutron matter:
EOS (second order)





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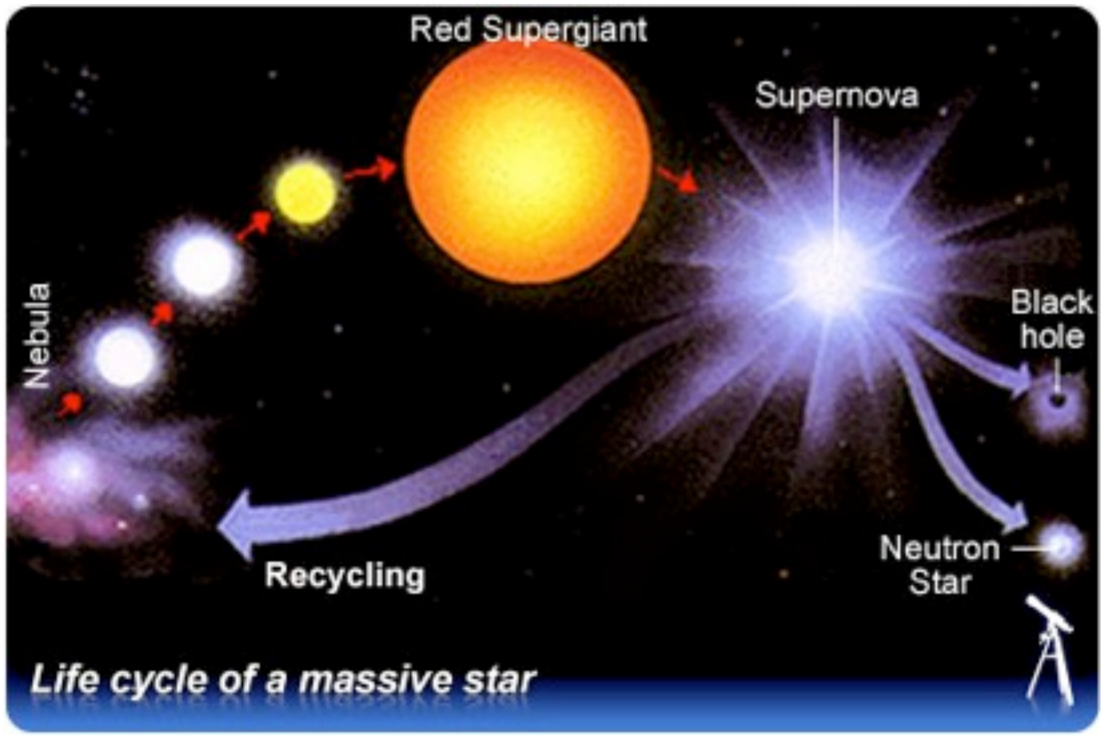
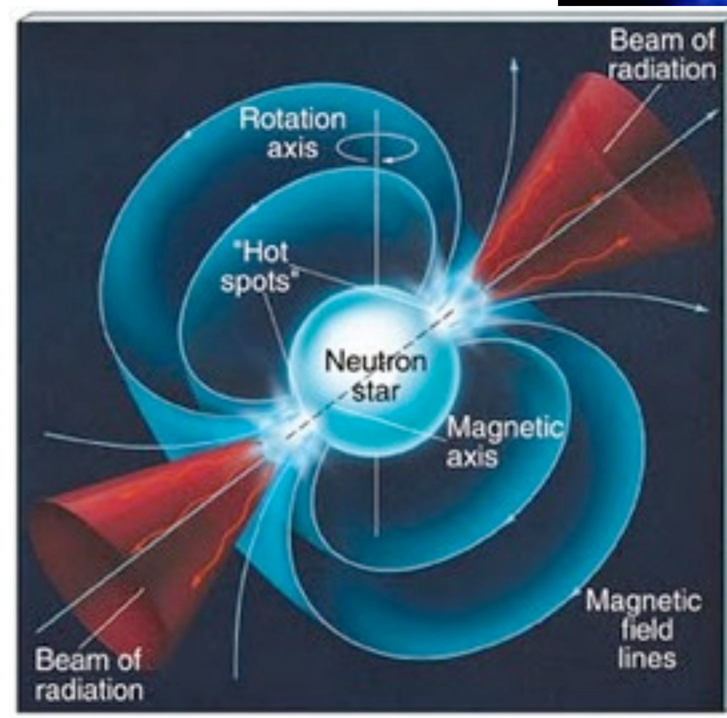
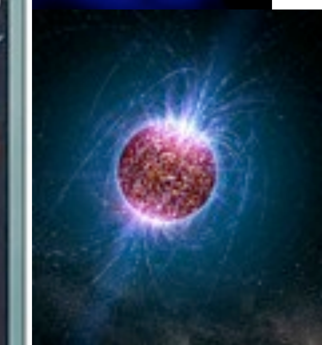
$$c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.5}_{-1.0}\,,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

- energy sensitive to c_3 variations
- uncertainty due to coupling constants much larger than cutoff variation

Basics about neutron stars



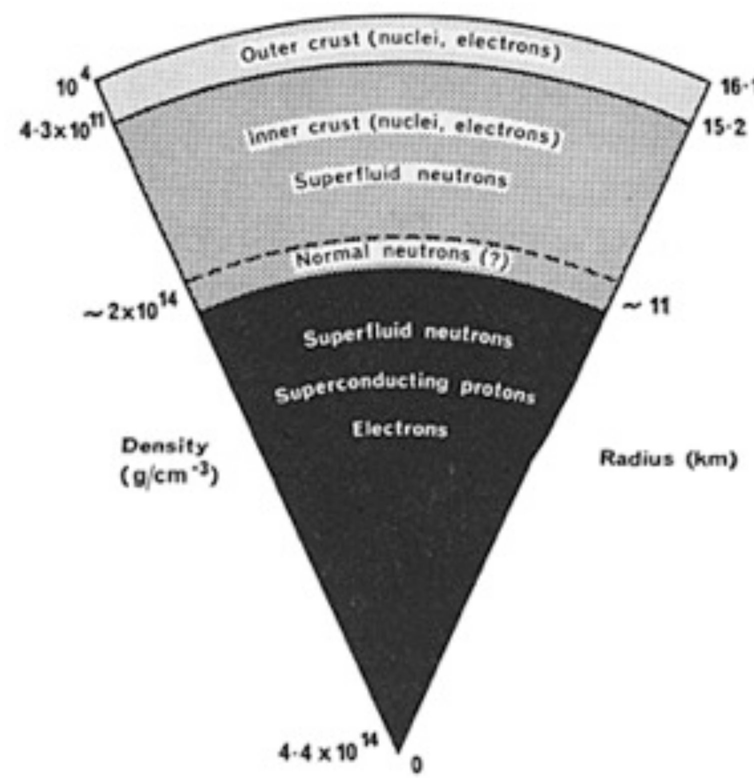




Structure of a neutron star is determined by Tolman-Oppenheimer-Volkov equation:

$$\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}\left[1+\frac{P}{\epsilon c^2}\right]\left[1+\frac{4\pi r^3 P}{Mc^2}\right]\left[1-\frac{2GM}{c^2 r}\right]^{-1}$$

crucial ingredient: energy density $\epsilon = \epsilon(P)$

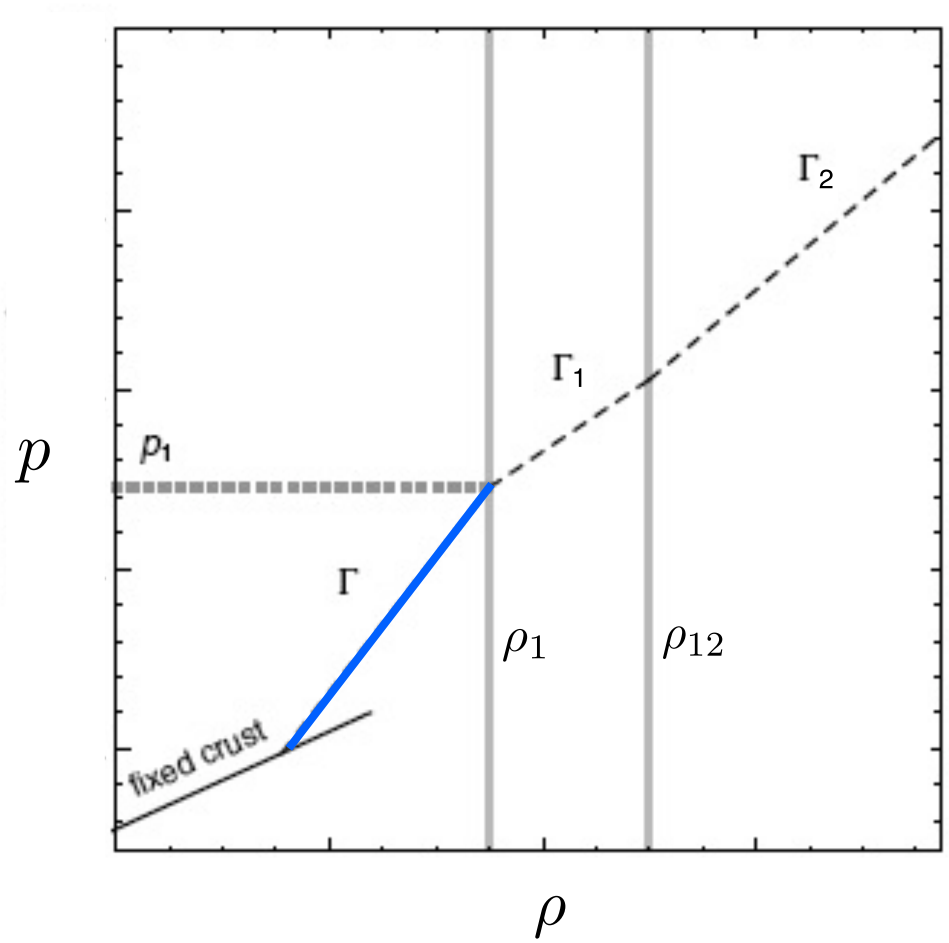


Neutron star radius

Problem: Solution of TOV equation requires EOS up to very high densities. Radius of a typical NS ($M \sim 1.6\ M_\odot$) theoretically not well constrained.

But: Radius of NS is only relatively insensitive to high density region!

parametrize piecewise EOS:



Schwenk, Lattimer, Pethick, KH in preparation

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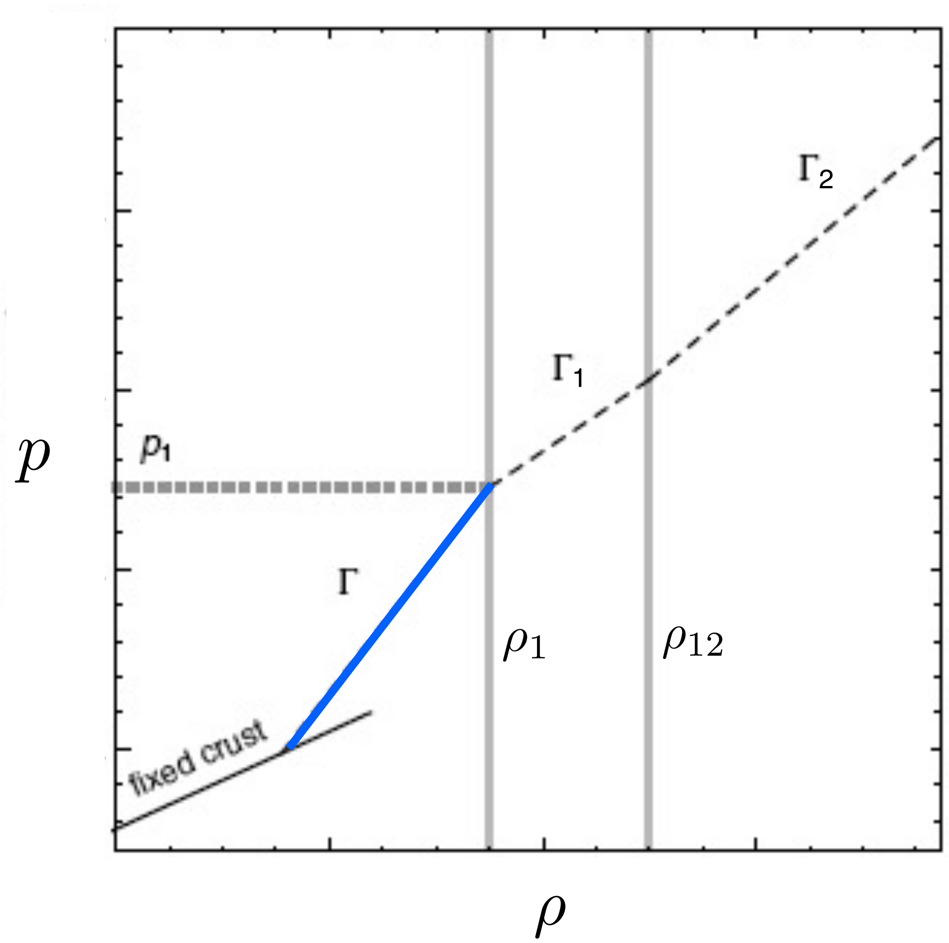
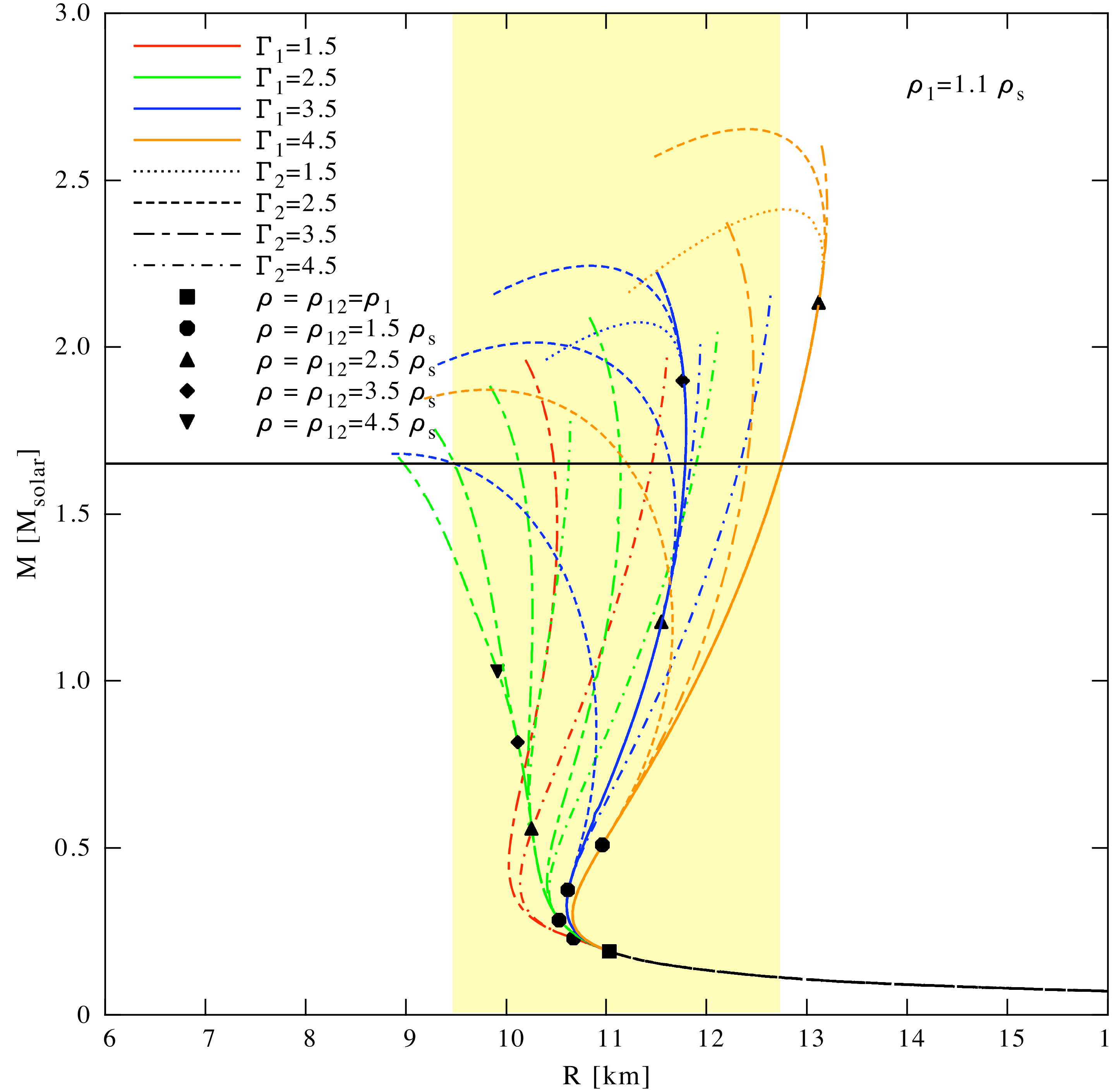


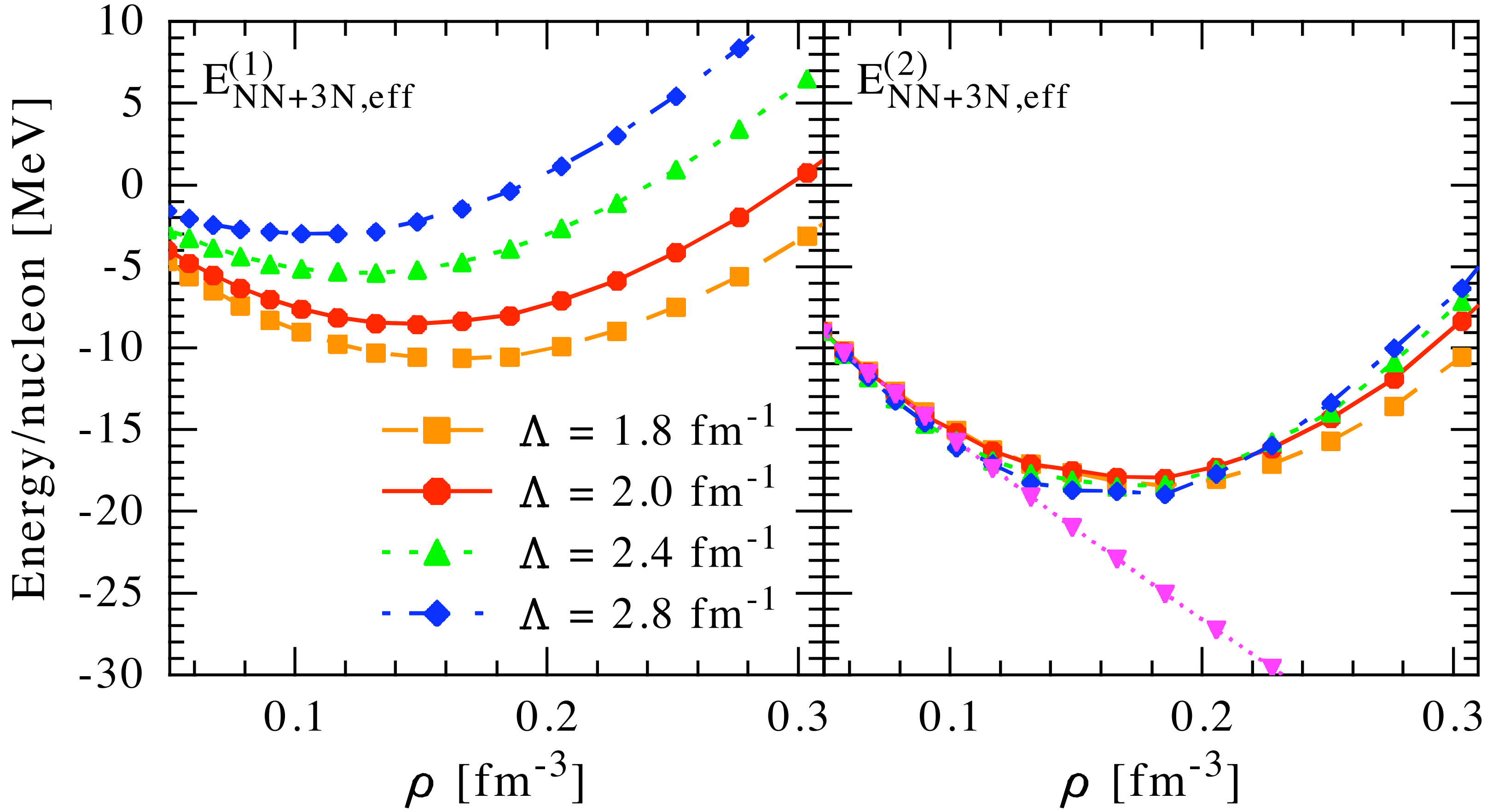


radius constrained to about

$$11\,\mathrm{km} \pm 1.5\,\mathrm{km}$$

Schwenk, Lattimer, Pethick, KH in preparation

Symmetric nuclear matter



Bogner, Furnstahl, Schwenk, Nogga, KH in preparation

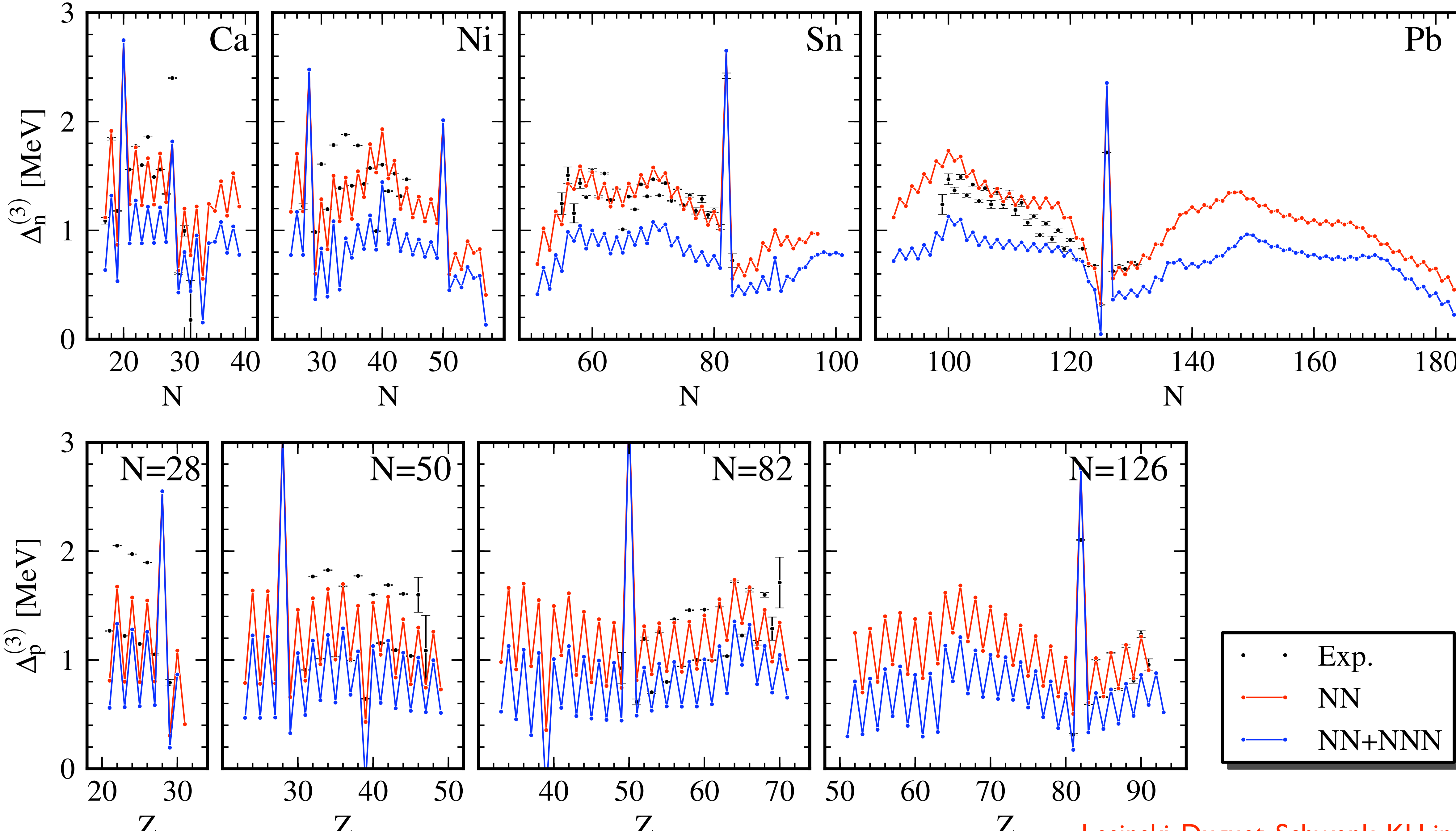
- 3N forces crucial for saturation
- cutoff dependence at 2nd order significantly reduced
- self-energy effects significant, self-consistency of approximation crucial
- saturation density and binding energy consistent with experiment for coupling constants fitted to $E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV}$ and $r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$
- 3rd order pp and hh contributions small

Pairing gap in semi-magic nuclei

Three-body mass difference:

$$\Delta^{(3)}(N) = \frac{(-1)^N}{2}\left[E(N+1) - 2E(N) + E(N-1)\right]$$

repulsive 3N contributions lead to suppression of the pairing gap



Lesinski, Duguet, Schwenk, KH in preparation

Conclusions and Outlook

• derivation of density dependent effective NN interactions from 3N interactions in the zero P-approximation
• effective NN interaction efficient to use and accounts for 3N effects in nucleonic matter to good approximation
• neutron matter from low-momentum interactions more perturbative than nuclear matter due to large 3N and tensor contributions
• saturation properties of SNM consistent with experiment
• many-body constraints to c_i couplings?

• generalization and application of $\overline{V}_{\rm 3N}$ to finite nuclei
• microscopic constraints of non-empirical EDFs