Effective Field Theory and Efimov Physics

H.-W. Hammer

Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics

Universität Bonn

Bethe Center for Theoretical Physics

Bundesministerium für Bildung und Forschung

Deutsche Forschungsgemeinschaft

DFG

Collaborators: E. Braaten, D. Canham, K. Helfrich, D. Kang, S. Kreuzer, D. Lee, L. Platter, R. Springer, ...

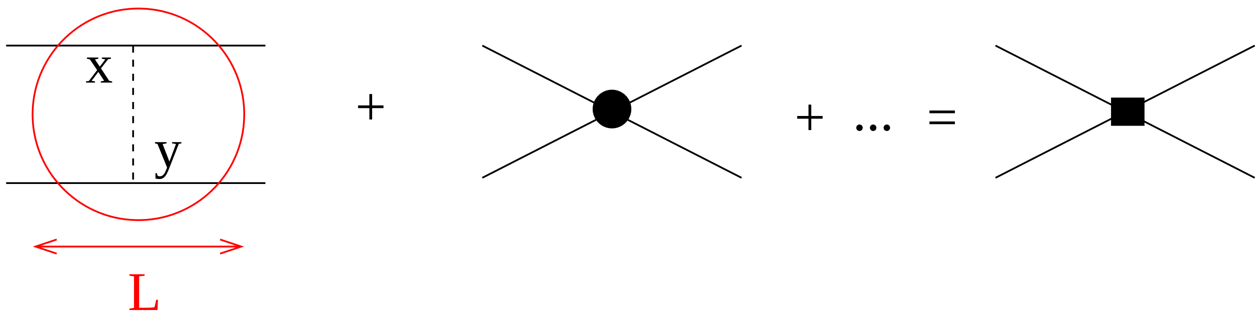
Agenda

- Introduction
- Resonant Interactions and Efimov Physics
- Effective Field Theory for Large Scattering Length
- Applications
  - Ultracold atoms
  - Hadronic Molecules
- Summary and Outlook

Effective Field Theory

universitätbonn

- Limited resolution in low-energy processes

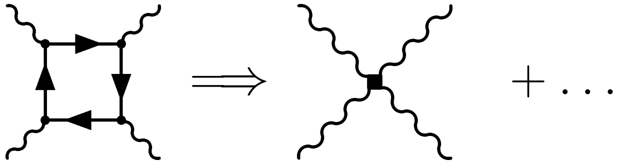


- Typical momentum: $p = \hbar/L \quad \Rightarrow$ resolution: $|\mathbf{x} - \mathbf{y}| \gtrsim L$
- Short-distance physics not resolved

  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

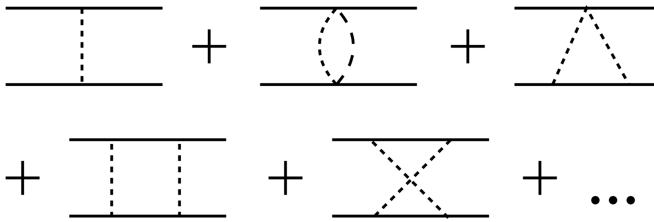
  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



Low-Energy Universality

- Renormalization Group: Systems with very different fundamental interactions can behave similarly at low energies

  $\Rightarrow$ Universal properties

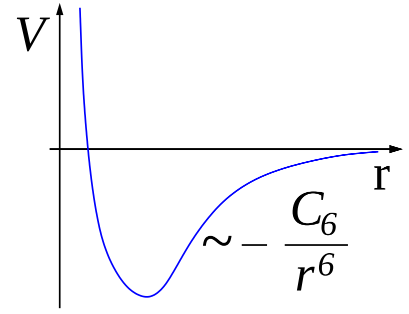


- Nuclear Physics:

  Nucleons and pions

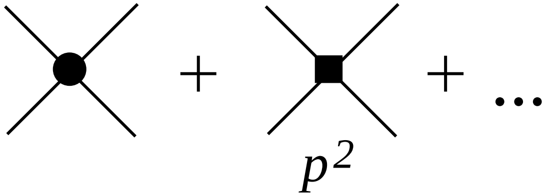
- Atomic Physics:

  Born-Oppenheimer plus van der Waals



$\Rightarrow$ At sufficiently low energy:

contact interactions



Resonant Interactions

- Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
- Natural expansion parameter: $\ell/|a|$, $k\ell$,...

$$a > 0 \quad \Longrightarrow \quad B_d = \frac{1}{2\mu a^2} + \mathcal{O}(\ell/a)$$

- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ variable scattering length
- Nuclear physics: S-wave NN-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{6}He $\Longrightarrow$ P-wave universality?
- Particle physics:
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = m_{D^0} + m_{D^{0*}} - m_X = (0.3 \pm 0.4) \text{ MeV}$$

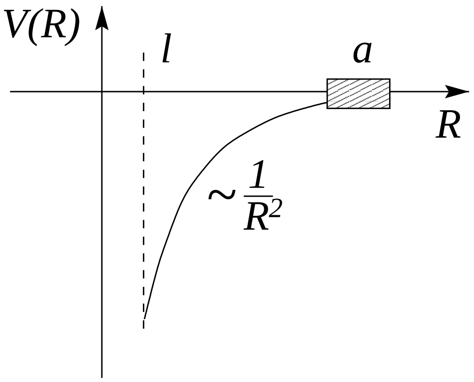
Efimov Physics

universitätbonn

(V. Efimov, Phys. Lett. **33B** (1970) 563)

- Three-body system with large scattering length a
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

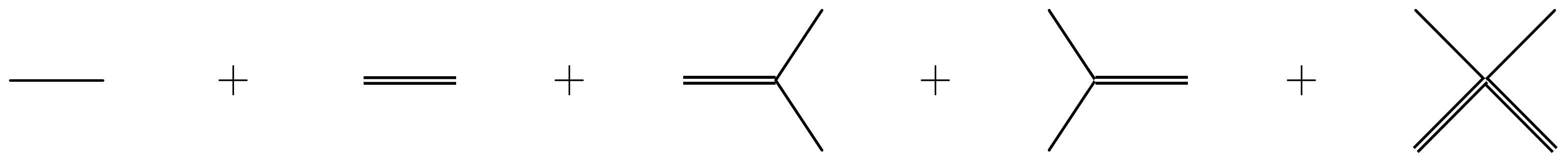
$\Longrightarrow$ dependence of observables on 3-body parameter (and a)

- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction

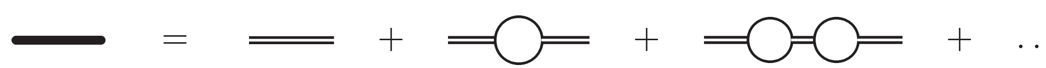
Two-Body System in EFT

- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

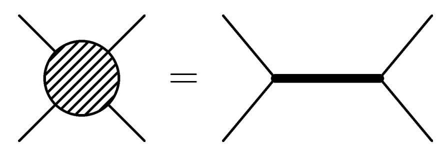
$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$



- Interacting dimeron propagator $\longrightarrow$ sum bubbles



- Two-body amplitude $\mathcal{T}_2(k,k)$:

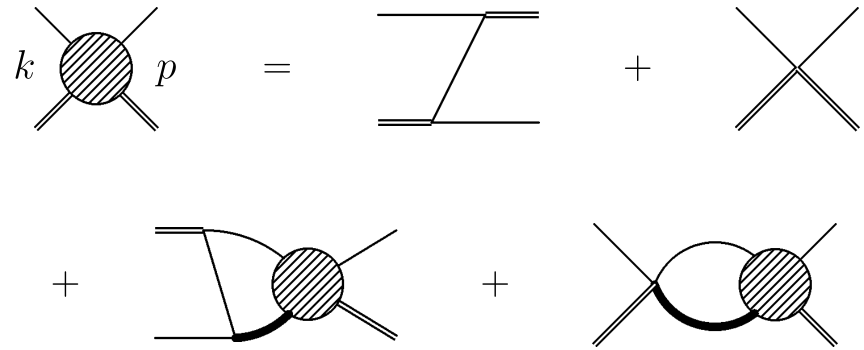


$$\propto \Big[\underbrace{8\pi/g_2 + 2\Lambda/\pi}_{1/a} + ik \Big]^{-1} + \ldots$$

- Matching: $g_2 \longleftarrow a, B_d$
- RG fixed points of $g_2(\Lambda)$: $a = 0$ and $a = \infty$
- Higher order corrections: perturbation theory

Three-Body System in EFT

- Three-body equation :

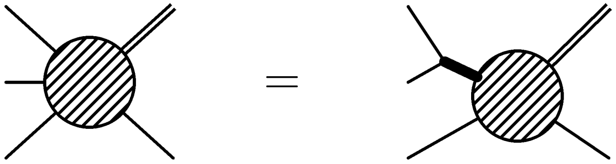


$$\textcolor{red}{\mathcal{T}_3(k,p)} = M(k,p) + \frac{4}{\pi}\int_0^{\textcolor{red}{\Lambda}} dq\, q^2\, M(q,p) D_d(q)\, \textcolor{red}{\mathcal{T}_3(k,q)}$$

with $$M(k,p) = \underbrace{F(k,p)}_{\text{1-atom exchange}} \quad \underbrace{-\frac{g_3}{9 g_2^2}}_{\textcolor{red}{H(\Lambda)}/\Lambda^2}$$

($g_3 = 0$, $\Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian '57)

- Three-body recombination:



Renormalization

universitätbonn

- Observables are independent of regulator/cutoff Λ

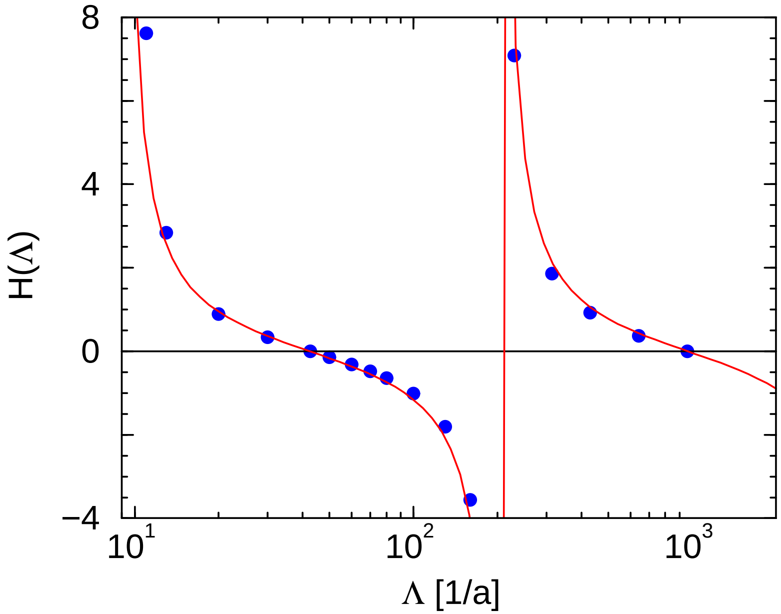
$\Rightarrow$ Running coupling $H(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup



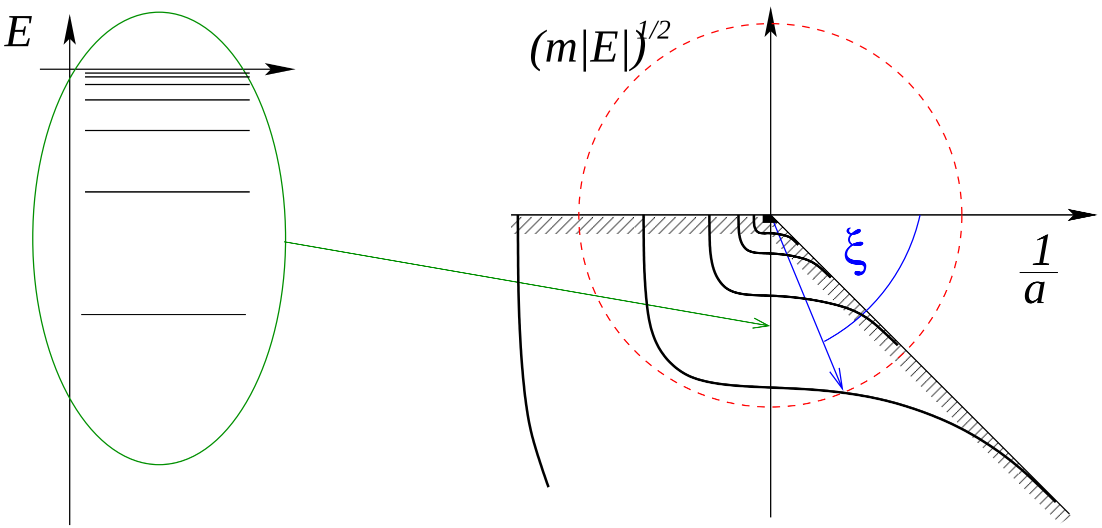
$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow B_t,\, K_3, \ldots \longrightarrow \kappa_*, a_*, a'_*$

Limit Cycle: Efimov Effect

universitätbonn

- Universal spectrum of three-body states
  (V. Efimov, Phys. Lett. **33B** (1970) 563)



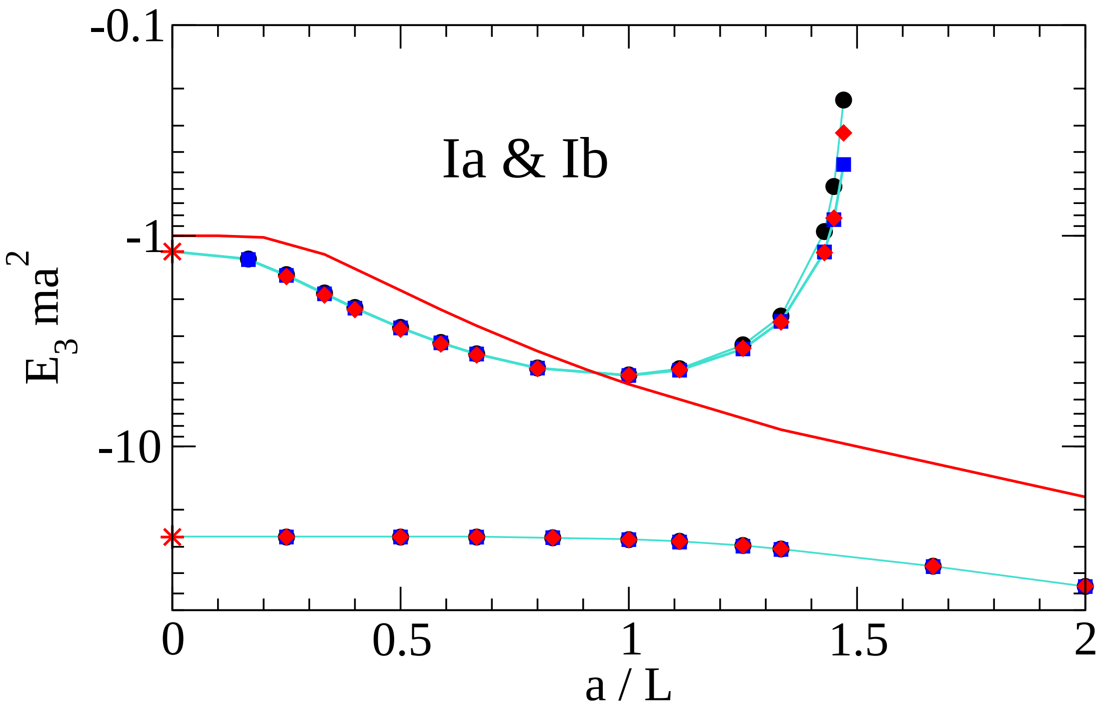
  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum für $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a\to 0} 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length

Efimov Effect in Finite Volume

- Modification of spectrum by cubic box ($V = L^3$)
  - Box provides infrared cutoff $1/L$ $\Rightarrow$ calculable in EFT
  - Box breaks rotational invariance $\Rightarrow$ partial wave mixing
  - 3-momenta quantized $\vec{p} = \vec{n}\,(2\pi/L)$ $\Rightarrow$ 3d sum equation



Kreuzer, HWH, Eur. Phys. J. A **43**, 229 (2010)

Efimov Effect in Finite Volume

- Higher partial waves:
  - few percent effect from $l = 4$ for $L \gtrsim 2R$
  - very small for threshold state
- Indications for universal scaling of finite volume corrections

$\Longrightarrow \quad L_{10\%}$ scales with $1/\sqrt{B_3} \sim R_3$

- First results for 3 nucleons $\quad \Rightarrow \quad$ Lattice QCD

$\Longrightarrow \quad \Delta B_t / B_t \approx 300\%$ for $L \approx 3$ fm

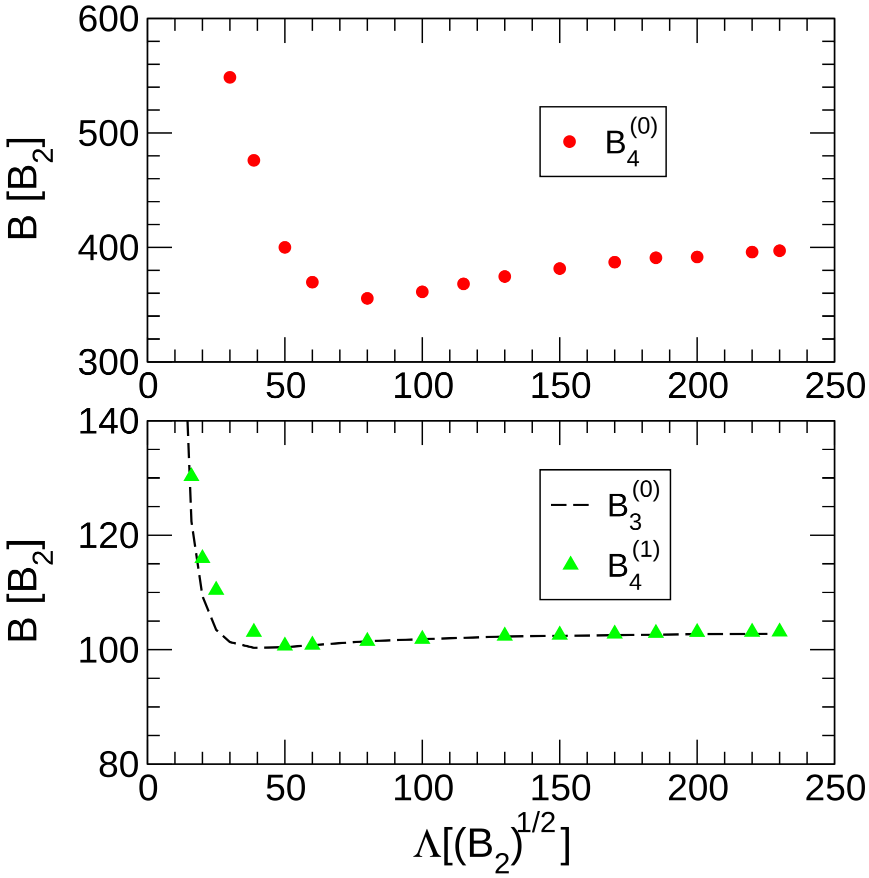
- Limit cycle in finite volume

(Kreuzer, HWH, in progress)

4-Body System

universitätbonn

- Extension to 4-body system in effective QM approach
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)



- 3- and 4-body observables are correlated (Tjon line, ...)

More on the 4-Body System

universitätbonn

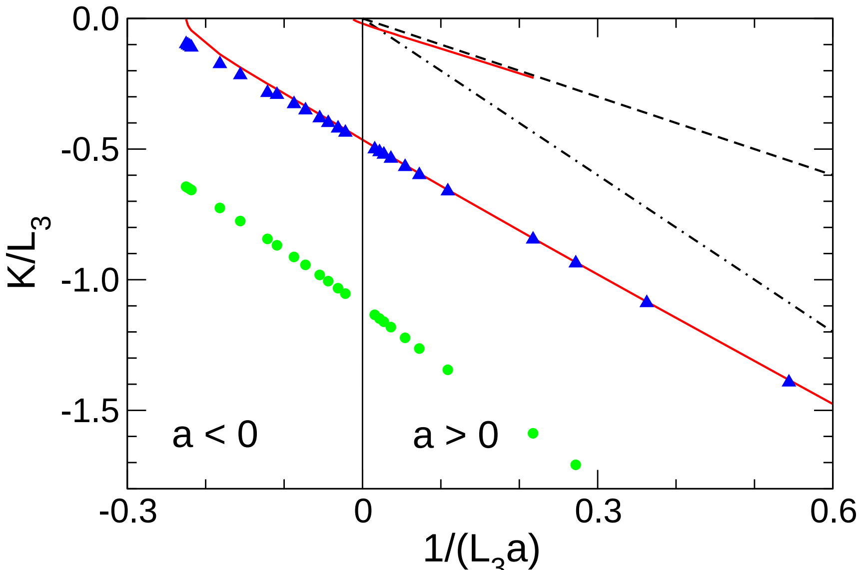
- Universal properties of 4-body system with large a
  - Bound state spectrum, scattering observables, ...
- “Efimov-plot”: 4-body bound state spectrum as function of $1/a$

$$K = \text{sign}(E)\sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



- Improved theoretical decription and signature in Cs loss data

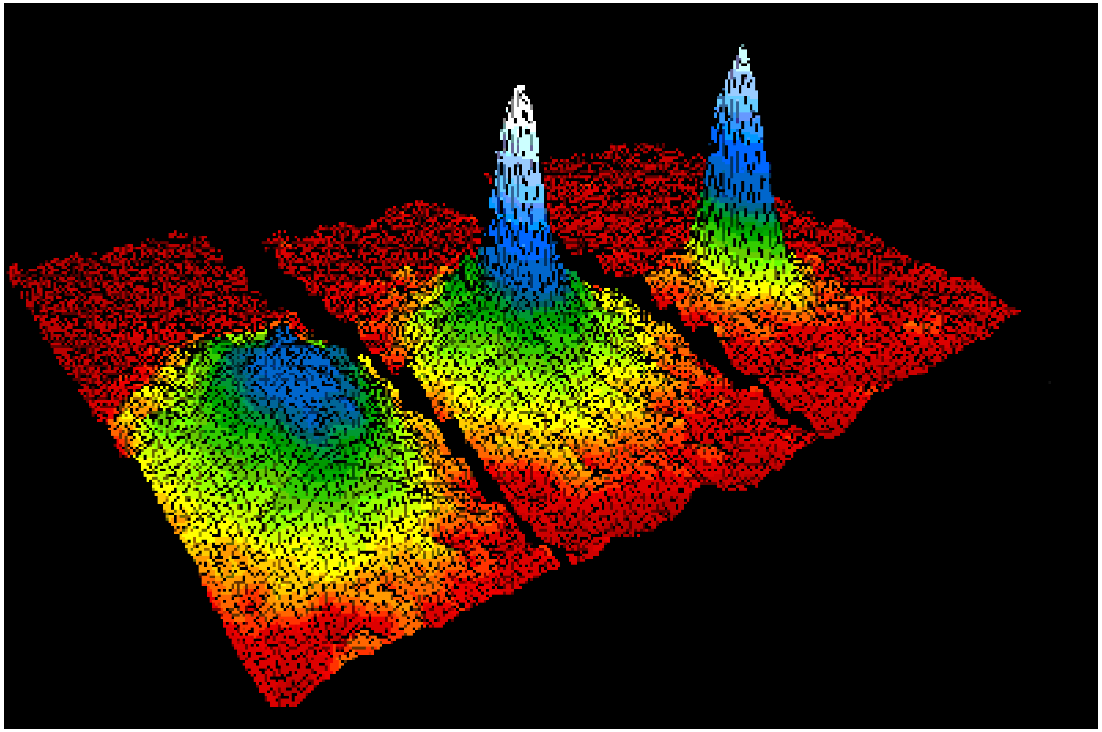
von Stecher, D’Incao, Greene, Nature Physics **5** (2009) 417

Ferlaino, Knoop, Berninger, Harm, D’Incao, Nägerl, Grimm, PRL **102** (2009) 140401

- Four-body recombination (Wang, Esry; Mehta et al.)

Efimov Physics in Cold Atoms

- Velocity distribution ($T =$ 400 nK, 200 nK, 50 nK)

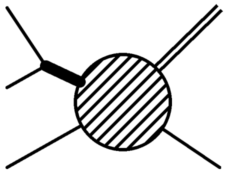


(Source: http://jilawww.colorado.edu/bec/)

- Few-body loss rates provide window on Efimov physics
- Variable scattering length via Feshbach resonances

Three-Body Recombination

universitätbonn



- Recombination into weakly-bound dimer:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- Scattering length dependence for $a > 0$:

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

$$K_3 \approx 201.3 \sin^2\left[s_0 \ln(a\kappa_*) + 1.16\right] \frac{\hbar a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

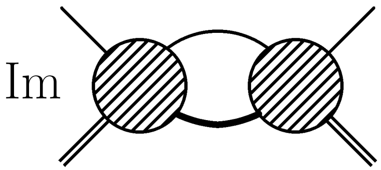
- Modification from deep dimers: Efimov states aquire width

  $\Longrightarrow \quad \kappa_* \rightarrow \kappa_* \exp(i\eta_*/s_0)$

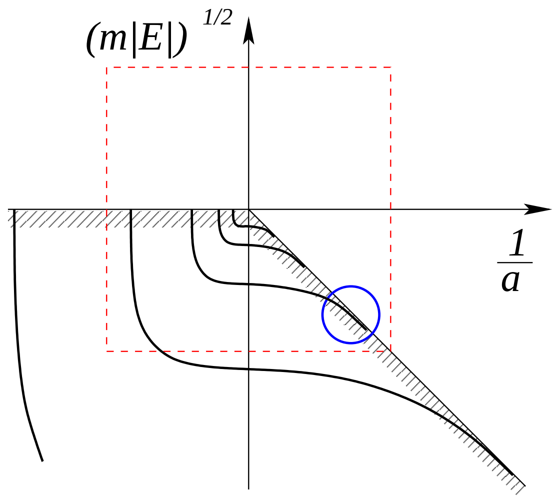
- Recombination into deep dimers $\Longrightarrow$ Efimov resonances
- Evidence for Efimov trimers in ^{133}Cs, ^{6}Li, ^{7}Li, ^{39}K

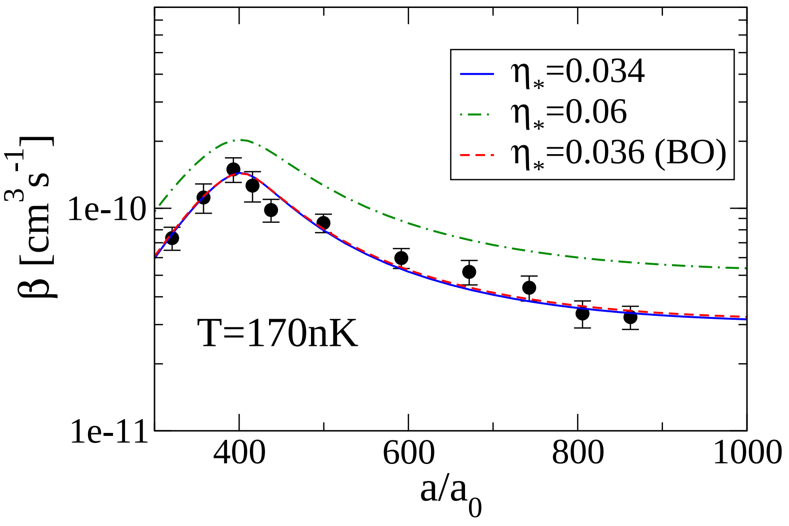
Dimer Relaxation in ^{133}Cs

- Dimer Relaxation: a + d $\rightarrow$ a + D (energetic)
- Relaxation constant: $\dot{n}_A = \dot{n}_D = -\beta\, n_A\, n_D$



- Recent experiment: Knoop et al. (Innsbruck), Nature Physics **5** (2009) 227
- Finite temperature $T \sim T_c$: Bose-Einstein average ?





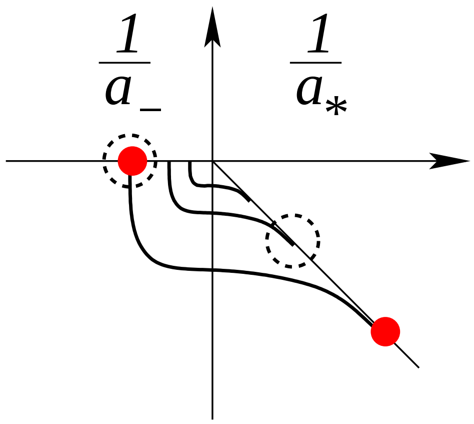
Helfrich, HWH, EPL **86** (2009) 53003

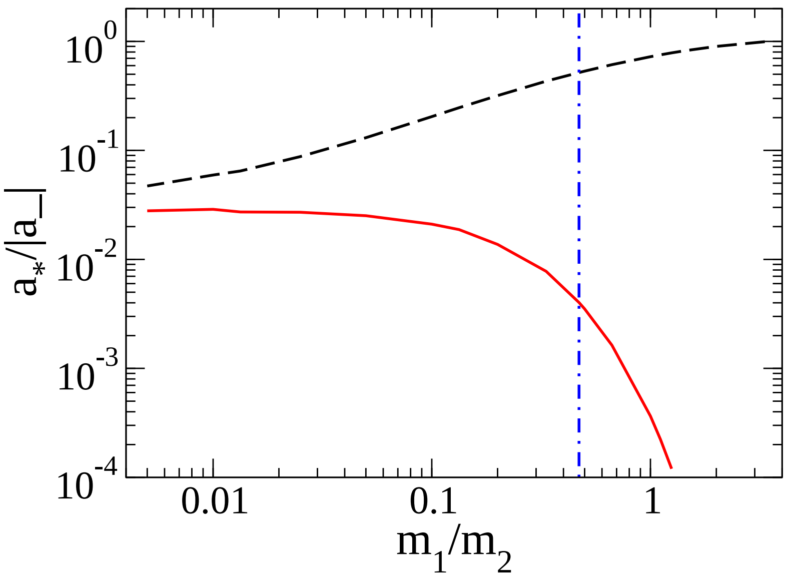
Heteronuclear Efimov Effect

- Recent experiment with heteronuclear mixture of ^{41}K and ^{87}Rb atoms (Barontini et al. (Florence), Phys. Rev. Lett. **103** (2009) 043201)

  $\Rightarrow$ Connected K-Rb-Rb resonances for $a > 0$ and $a < 0$

- Ratio of resonance positions: $a_*/|a_-|$





Helfrich, HWH, Petrov, arXiv:1001.4371

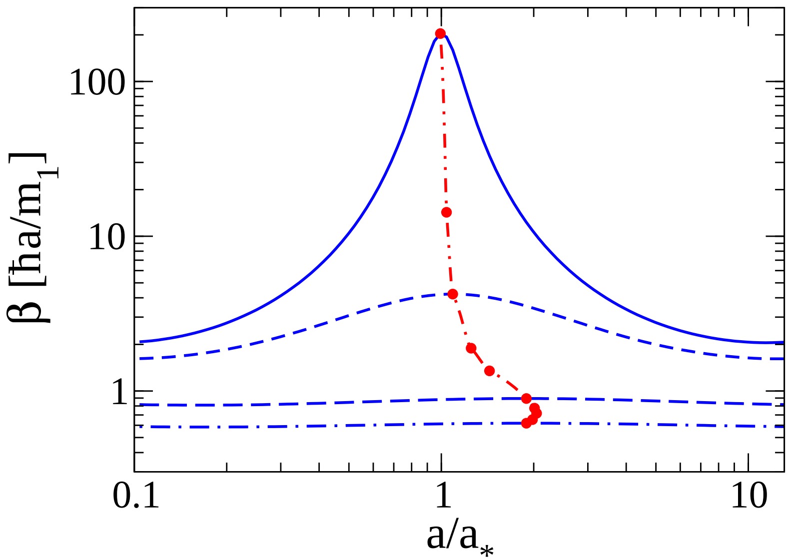
- K-Rb-Rb: $m_1/m_2 = 41/87 \approx 0.47 \quad \Rightarrow \quad \exp(\pi/s_0) \approx 128$

  $a_*/|a_-|$: 2.7 (Exp) $\Leftrightarrow$ 0.52 (Th)

Heteronuclear Efimov Effect

- Identification of atom-dimer resonance in purely atomic sample through rescattering processes? (cf. Barontini et al., 2009)
- Calculate loss rate above threshold: $E/B_d = -1, -0.95, -0.5, 0$

$$\eta_* = 0.1$$



Helfrich, HWH, Petrov, arXiv:1001.4371

- Should not exclude other interpretations of data
- Analytical and numerical results for mass dependence of recombination and relaxation rates (cf. D'Incao, Esry, 2006)

P-Wave Universality?

universitätbonn

- Universal properties for resonant P-wave interactions?

  $\Longrightarrow$ not in the usual sense
- Interactions corresponding to a_1 and r_1 required for consistent renormalization (Bertulani, HWH, van Kolck, 2002)
- Two relevant directions near RG fixed point (Barford, Birse, 2003)
- Causal wave propagation requires $(2L + d \geq 5)$

  (HWH, Lee, Phys. Lett. B **681**, 500 (2009))

$$r_{L,d} \leq -2\Gamma(L + \frac{d}{2} - 2)\Gamma(L + \frac{d}{2} - 1)/\pi \times (2\Lambda)^{2L+d-4}$$

  - $d = 3$, $L = 0 \quad \Longrightarrow \quad r_{0,3} \leq 2/\Lambda$

    (Phillips, Cohen, Phys. Lett. B **390**, 7 (1997))
  - $d = 3$, $L \geq 1 \quad \Longrightarrow \quad r_{0,3} \leq -|\text{const.}|\Lambda^{2L-1}$

$\Rightarrow$ effective range cannot be tuned to zero

Exotic Charmonium Mesons

universität bonn

- Many new $c\bar{c}$-mesons at B-factories: X, Y, Z
  - Challenge for understanding of QCD
  - Large scattering length physics important
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)

$$m_X = (3871.55 \pm 0.20)\ \text{MeV} \qquad \Gamma < 2.3\ \text{MeV} \qquad J^{PC} = 1^{++}$$

- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel
- Nature of $X(3872)$?
  - $D^0 D^{0*}$-molecule? (cf. Tornquist, 1991)
  - Tetraquark
  - Charmonium Hybrid
  - ...

Nature of $X(3872)$

- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound D^0-$\bar{D}^{0*}$-molecule

  $\Longrightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\,,\qquad B_X = (0.26 \pm 0.41)$ MeV

  $\Longrightarrow$ universal properties (cf. Braaten et al., 2003-2008, ...)

  - Explains isospin violation in decays of $X(3872) \Rightarrow$ superposition of $I = 1$ and $I = 0$
  - Different masses due to different line shapes in decay channels
- EFT with explicit pions: short distance contributions dominate (Fleming, Kusunoki, Mehen, van Kolck, 2007)

  $\Longrightarrow$ EFT for large scattering length is applicable
- Large scattering length determines interaction of $X(3872)$ with D^0 and D^{0*}

Interactions of $X(3872)$

- Large scattering length determines interaction of $X(3872)$ with D^0 and D^{0*}
- Efimov effect?

  $\Rightarrow$ occurs if 2 out of 3 pairs have resonant interactions
- $X(3872)$: only 3 out of 6 pairs have resonant interactions

  $\Rightarrow$ no Efimov effect (Braaten, Kusunoki, 2003)

  $\Rightarrow$ no X-D^0- and X-D^{0*}-molecules

  $\Rightarrow$ no three-body interaction at leading order

Interactions of $X(3872)$

universitätbonn

- Large scattering length determines interaction of $X(3872)$ with D^0 and D^{0*}
- Efimov effect?

  $\Rightarrow$ occurs if 2 out of 3 pairs have resonant interactions
- $X(3872)$: only 3 out of 6 pairs have resonant interactions

  $\Rightarrow$ no Efimov effect (Braaten, Kusunoki, 2003)

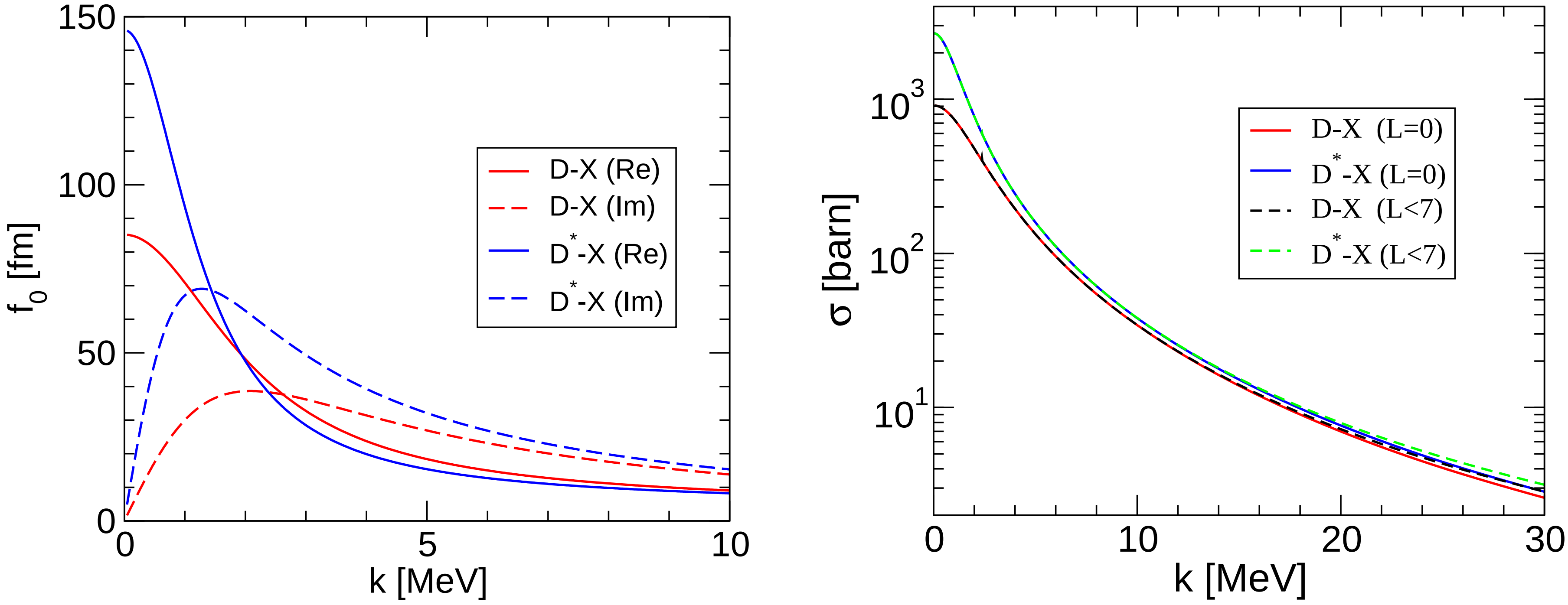
  $\Rightarrow$ no X-D^0- and X-D^{0*}-molecules

  $\Rightarrow$ no three-body interaction at leading order
- But: parameter-free prediction of X-D^0-, X-D^{0*}-scattering
- Low-energy parameters: $B_X = (0.26 \pm 0.41)$ MeV

  $\Rightarrow$ Scattering length in the X channel: $a = (8.8^{+\infty}_{-3.3})$ fm

X-D- and X-D^*-Scattering

- Predictions for scattering amplitude/cross section



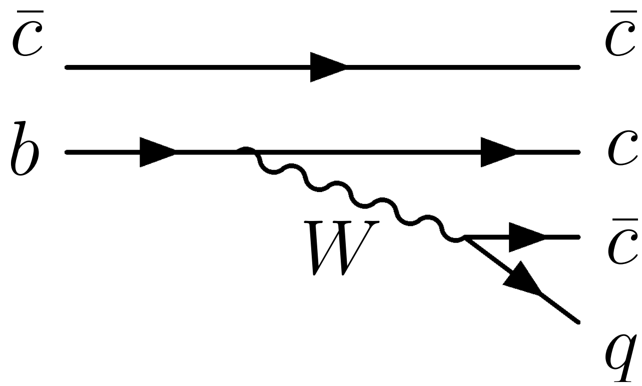
Canham, HWH, Springer, Phys. Rev. D **80**, 014009 (2009)

- Three-body scattering lengths

$$a_{D^0X} = a_{\bar{D}^0X} = -9.7a\,, \quad \text{and} \quad a_{D^{*0}X} = a_{\bar{D}^{*0}X} = -16.6\,a$$

Experimental Observation ?

- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0$, $\bar{D}^{*0}$
- Rare events in $B\bar{B}$ production ($B \to X$, $\bar{B} \to D, D^*$)
- Final state interaction of D, D^* mesons in B_c-decays
- Example: quark-level B_c decay yielding three charmed/anticharmed quarks in final state



- Process may be accessible at the LHC

Summary and Outlook

universitätbonn

- Effective field theory for large scattering length
  - Limit cycle in three-body system $\Leftrightarrow$ Efimov physics
  - Universal correlations (Phillips, Tjon line,...)
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Halo nuclei
  - Scattering properties of the $X(3872)$
- Future directions:
  - Cold atoms: heteronuclear systems, $N \geq 4$, 2d-systems, ...
  - Halo nuclei: reactions, external currents, ...
  - Hadronic molecules: universal properties, three-body molecules?
  - Three-nucleon system on the lattice: finite volume corrections, limit cycle in “deformed” QCD?