Pairing in Neutron Matter and in Cold Atomic Systems

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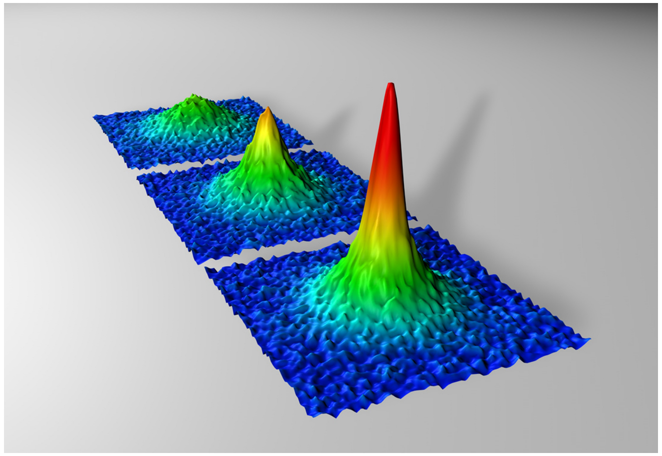
Fermionic superfluidity

- Conventional superconductors: $T_c \sim 10\ K,\ \Delta/E_F \sim 10^{-5}$
- Liquid ^{3}He: $T_c \sim 10^{-3}\ K,\ \Delta/E_F \sim 10^{-3}$
- High-temperature superconductors: $T_c \sim 10^{2}\ K,\ \Delta/E_F \sim 10^{-2}$

Cold atoms

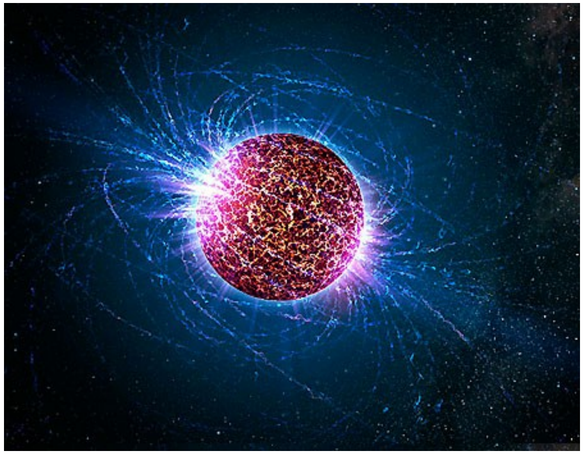
Credit: NIST/University of Colorado



$T_c \sim 10^{-6}\ K,\ \Delta/E_F \sim 0.5$

Neutron matter

Credit: Casey Reed, Penn State University



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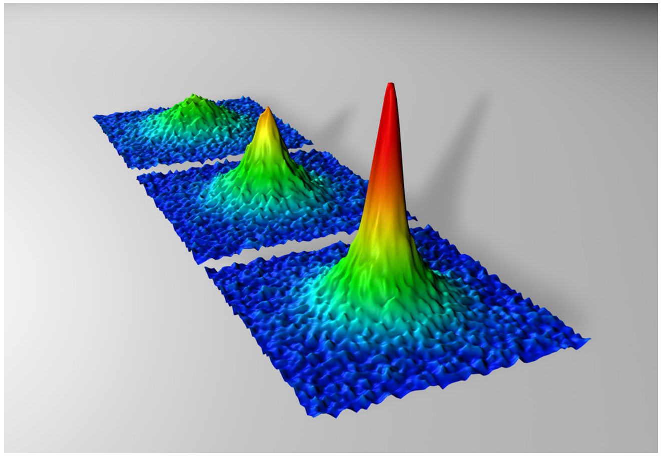
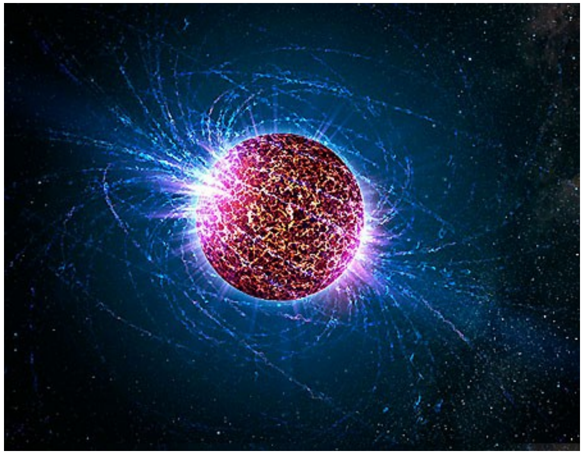


$T_c \sim 10^{-6}$ K $\Delta/E_F \sim 0.5$

Strongly Paired Fermions

Neutron matter

Credit: Casey Reed, Penn State University



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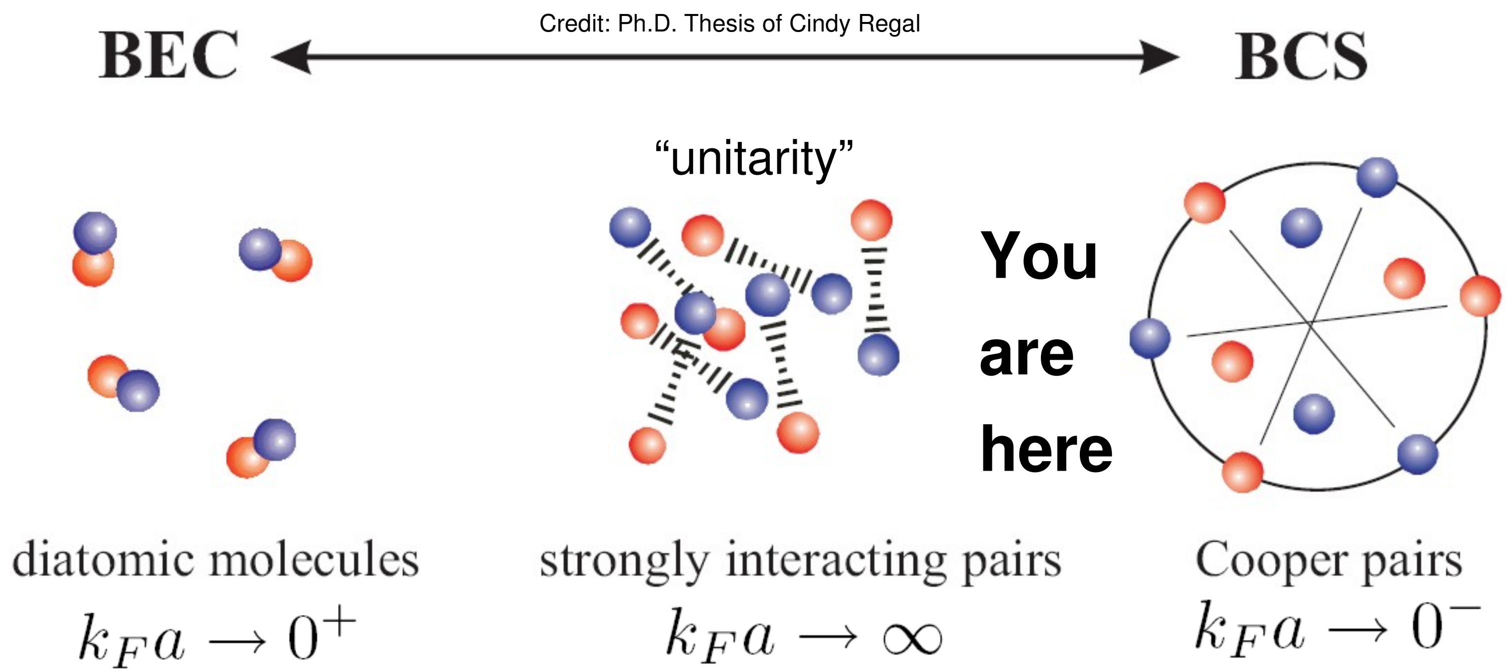
Motivation: BCS-BEC crossover in cold atoms



At unitarity: universal behavior

- Ground-state energy $\frac{E}{N} = \xi E_{FG}$
- Pairing gap $\Delta = \eta E_F$

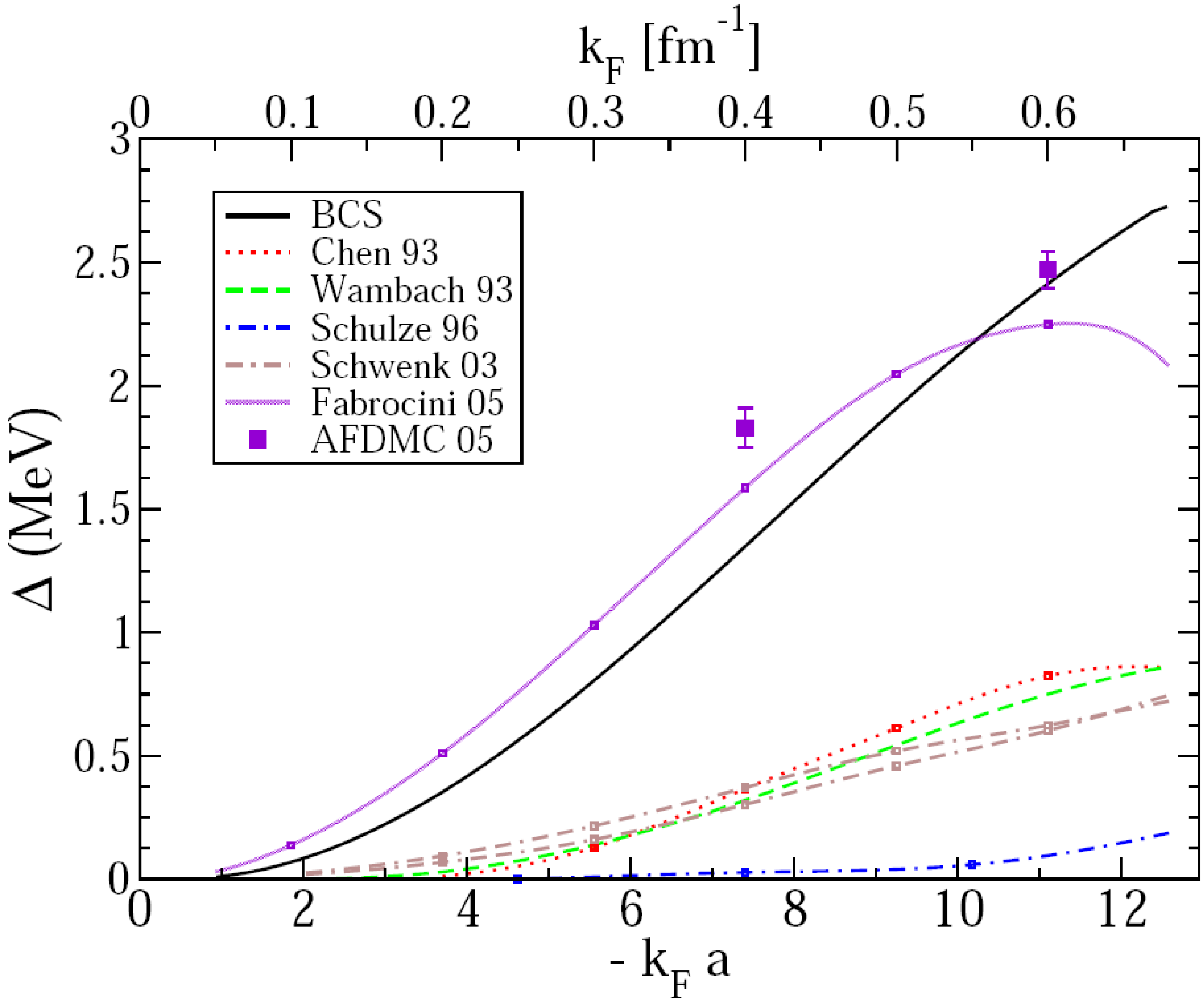
Away from unitarity

Energy and gap as a function of Fermi momentum (k_F) times scattering length (a)

Experimental situation

Within direct experimental reach, in contradistinction to neutron matter

Motivation: $^1S_0$ neutron matter pairing gap



"No consensus"

Only a selection of extant calculations is shown

Weak coupling: what are we more or less sure of?

Equation of State

- Exact for normal gas at $|k_F a| << 1$
- Derived by Lee-Yang(1957) / Galitskii (1958)

$$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} a k_F + \frac{4}{21\pi^2} (11 - 2\ln 2) (a k_F)^2$$

Pairing gap

BCS: bare interaction

(not exact even at $|k_F a| << 1$)

$$\Delta_{BCS}(k_F) \overset{k_F \to 0}{\longrightarrow} \Delta^0_{BCS}(k_F) = \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2 a k_F}\right)$$

Gorkov/Melik-Barkhudarov: screening

(exact only at $|k_F a| << 1$)

$$\Delta_{GMB}(k_F) \overset{k_F \to 0}{\longrightarrow} \Delta^0_{GMB}(k_F) = \frac{1}{(4e)^{1/3}} \frac{8}{e^2} \frac{\hbar^2 k_F^2}{2m} \exp\left(\frac{\pi}{2 a k_F}\right)$$

See also: H.-J. Schulze, A. Polls, and A. Ramos, Phys. Rev. C 63, 044310 (2001)

Mean-field theory: finite-size effects

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k}|V|\mathbf{k}'\rangle \frac{\Delta(\mathbf{k}')}{2\sqrt{\epsilon(\mathbf{k}')^2 + \Delta(\mathbf{k}')^2}} \qquad A = \sum_{\mathbf{k}} \left[1 - \frac{\epsilon(\mathbf{k})}{\sqrt{\epsilon(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}\right]$$

Green's Function Monte Carlo

Propagation in imaginary time

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

$$= \lim_{\tau \to \infty} \sum_i \alpha_i e^{-(E_i - E_T)\tau} \Psi_i \longrightarrow \alpha_0 e^{-(E_0 - E_T)\tau} \Psi_0$$

Jastrow-BCS wave function

$$\Psi_V(\mathbf{R}) = \prod_{i,j'} f(r_{ij'}) \Phi_{BCS}(\mathbf{R})$$

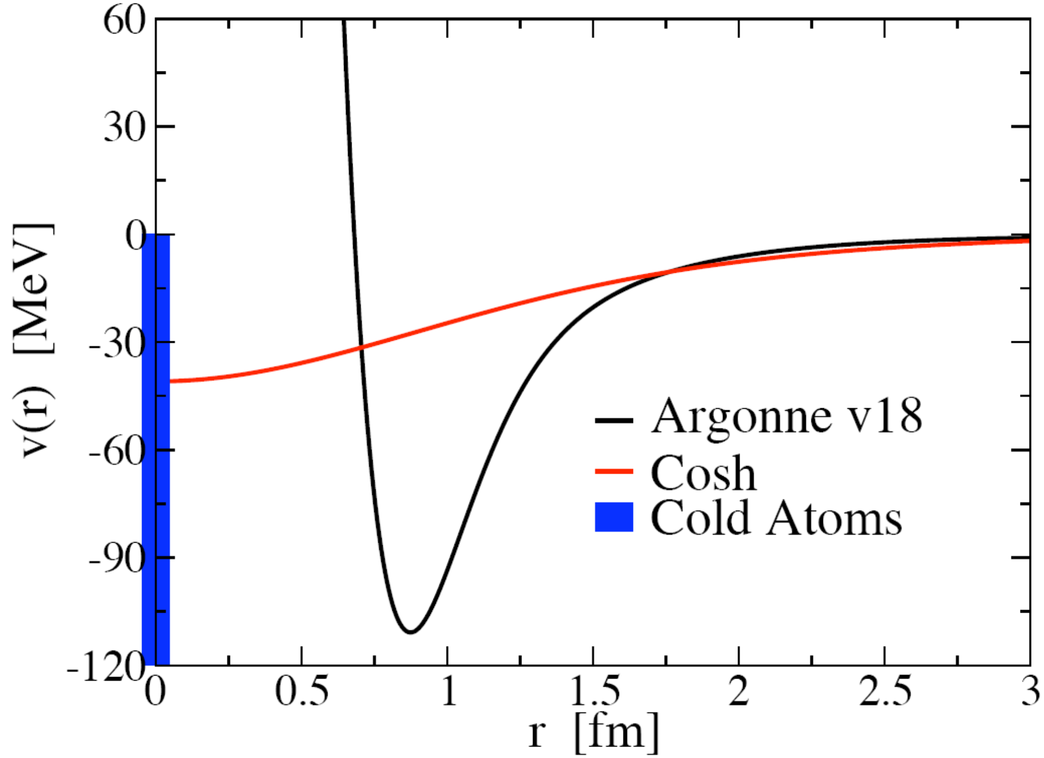
Fixed-node approximation

Allows us to variationally optimize the parameters

Even-odd staggering

$$\Delta = E(N+1) - (E(N) + E(N+2))/2$$

$$\mathcal{H} = \sum_{k=1}^{A}\left(-\frac{\hbar^2}{2m_k}\nabla_k^2\right) + \sum_{i<j'} v(r_{ij'})$$



Neutron Matter

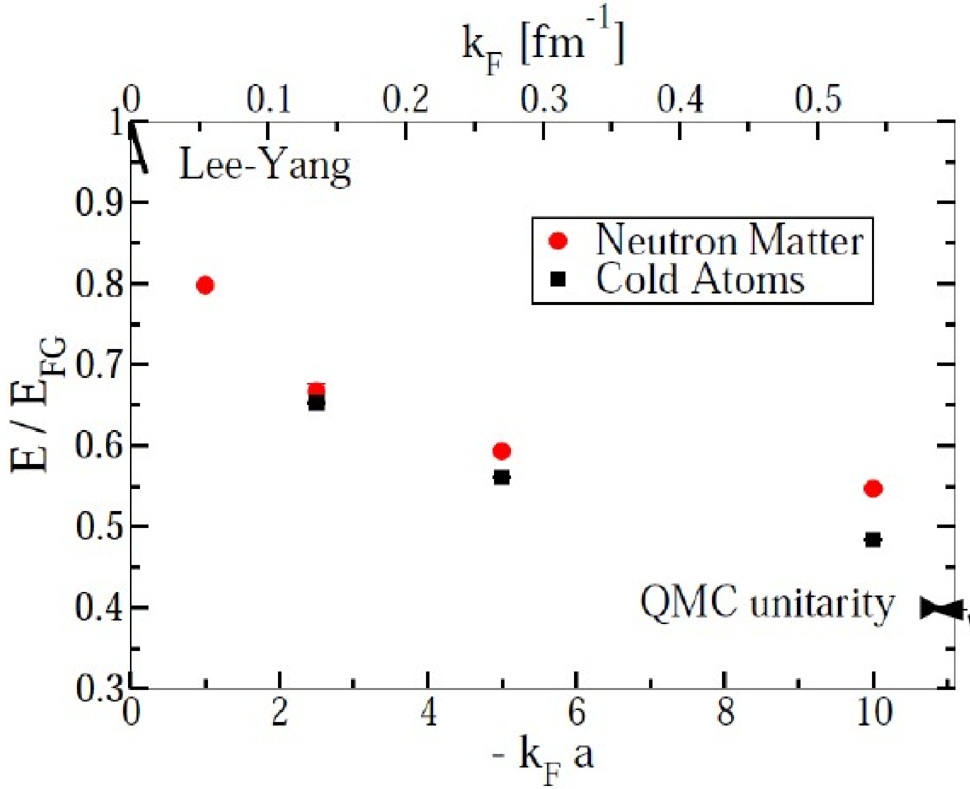
$^1S_0$ channel of AV18 – later AV4

a = -18.5 fm, r_e = 2.7 fm

Cold Atoms

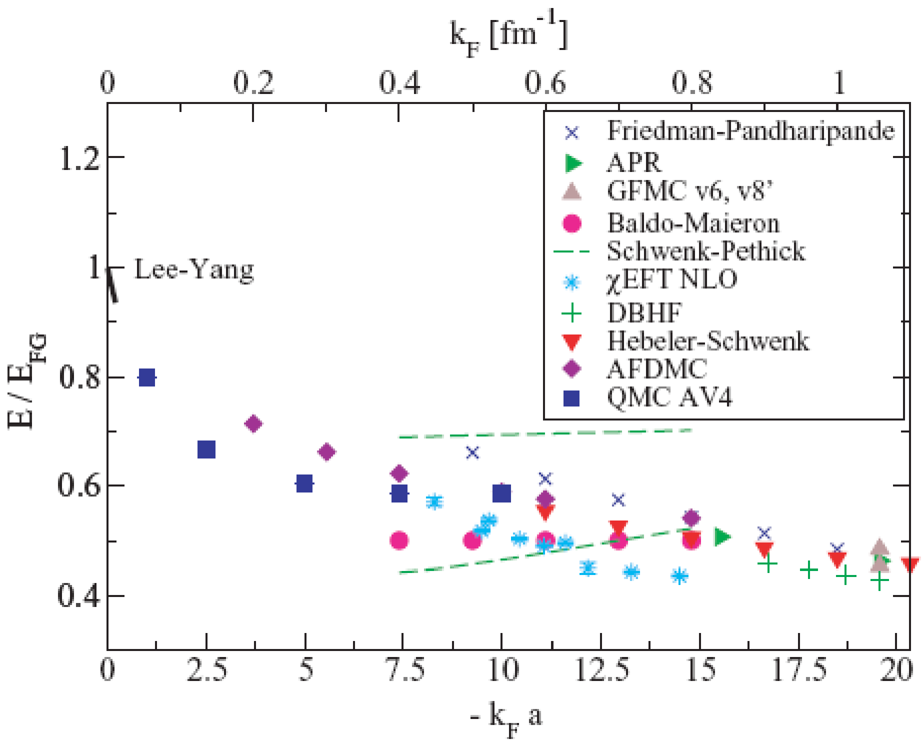
modified Pöschl-Teller potential

a = tunable, r_e =tunable/infinitesimal



A. Gezerlis and J. Carlson, Phys. Rev. C 77, 032801(R) (2008)

neutron matter



A. Gezerlis and J. Carlson, Phys. Rev. C, 81, 025803 (2010).

Duke experiments

at unitarity

L. Luo and J. E. Thomas,

J. Low. Temp. Phys., 154, 1 (2009)

Finite effective range

Relevant at large densities

Equation of state

Qualitatively established

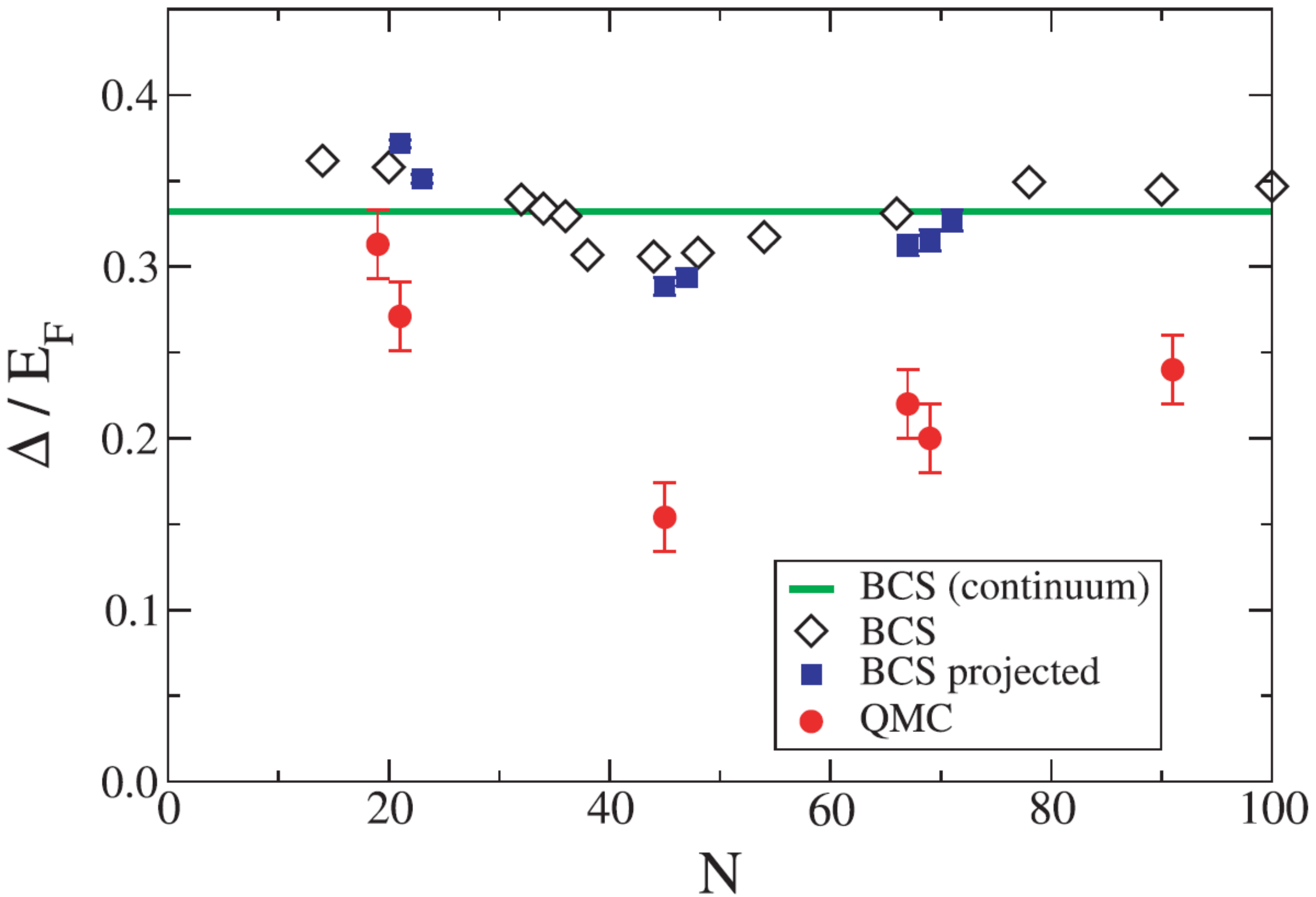
Results: finite-size scaling for the gap at $k_F a = -10$

Neutron r_e = finite

Non-negligible finite-size effects: estimate the trend by comparing to the corresponding mean-field theory results

(energy exhibits considerably smaller finite-size effects)

neutron matter



A. Gezerlis and J. Carlson, Phys. Rev. C 77, 032801(R) (2008)

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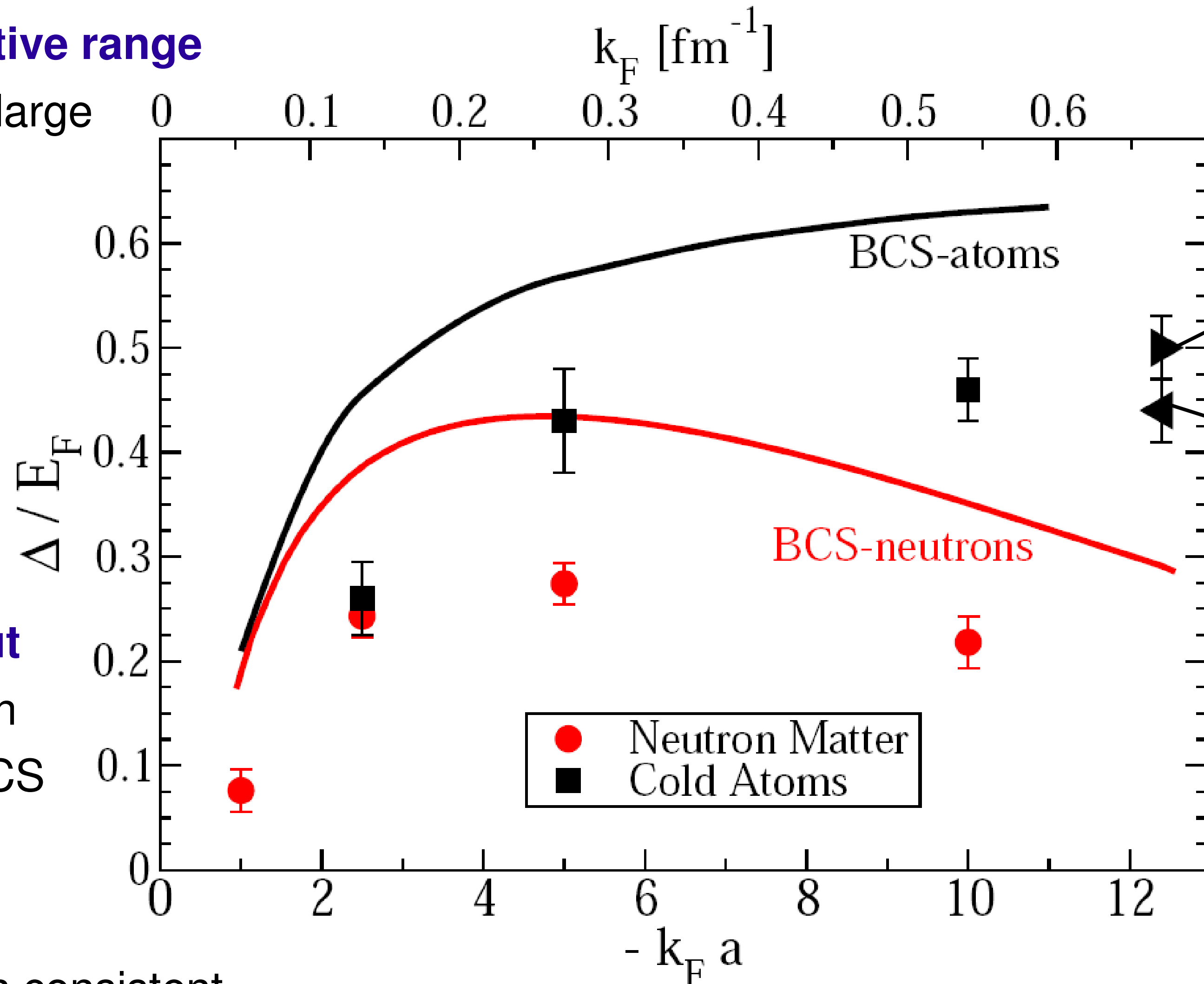


A. Gezerlis and J. Carlson, Phys. Rev. C 77, 032801(R) (2008)

QMC at unitarity

Two independent MIT experiments at unitarity

J. Carlson and S. Reddy, Phys. Rev. Lett. 100, 150403 (2008)

A. Schirotzek *et al*, Phys. Rev. Lett. 101, 140403 (2008)

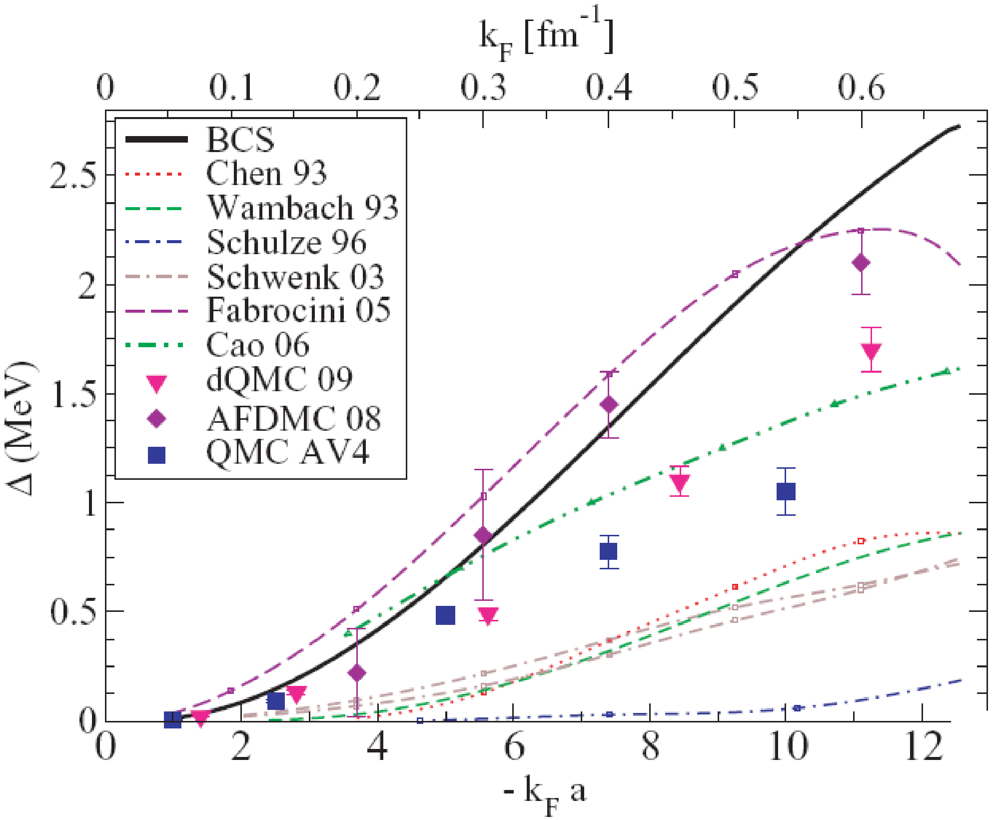
Throughout

Suppression w.r.t. the BCS results

At $k_F a = -1$

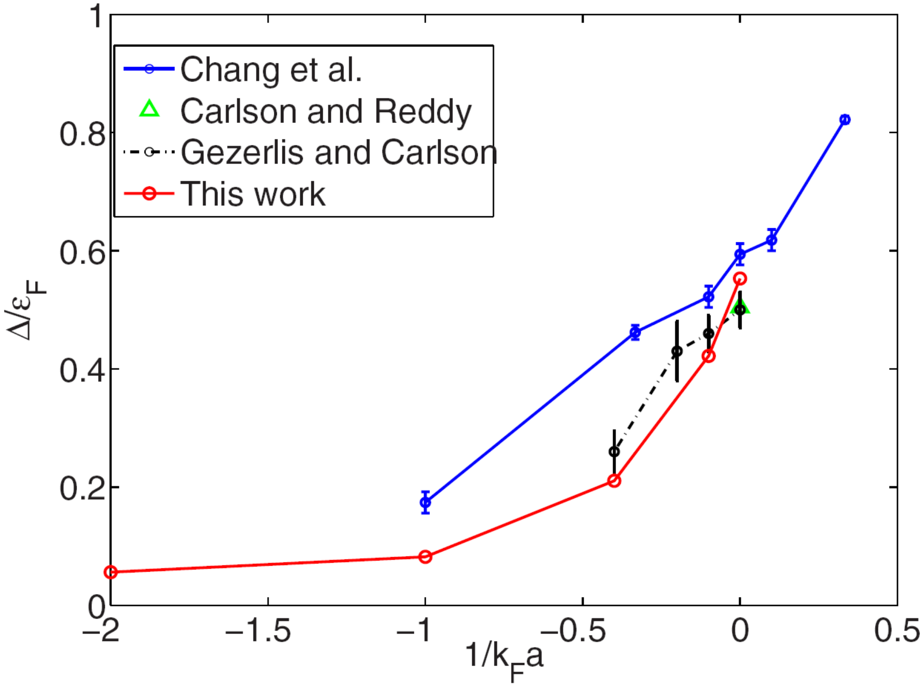
Suppression consistent with the Gorkov result

neutron matter



A. Gezerlis and J. Carlson, Phys. Rev. C, 81, 025803 (2010).

cold atoms



A. Bulgac, J. E. Drut, and P. Magierski, Phys. Rev. A 78, 023625 (2008)

Neutron-star crust consequences

- Negligible neutron contribution to the specific heat consistent with cooling of transients: E. F. Brown and A. Cumming arXiv:0901.3115.
- Young neutron star cooling curves depend on the magnitude of the gap: D. Page, J. M. Lattimer, M. Prakash, A. W. Steiner arXiv:0906.1621.
- Superfluid-phonon heat conduction mechanism viable: D. Aguilera, V. Cirigliano, J. Pons, S. Reddy, R. Sharma Phys. Rev. Lett. 102, 091101 (2009)

Variations on a theme

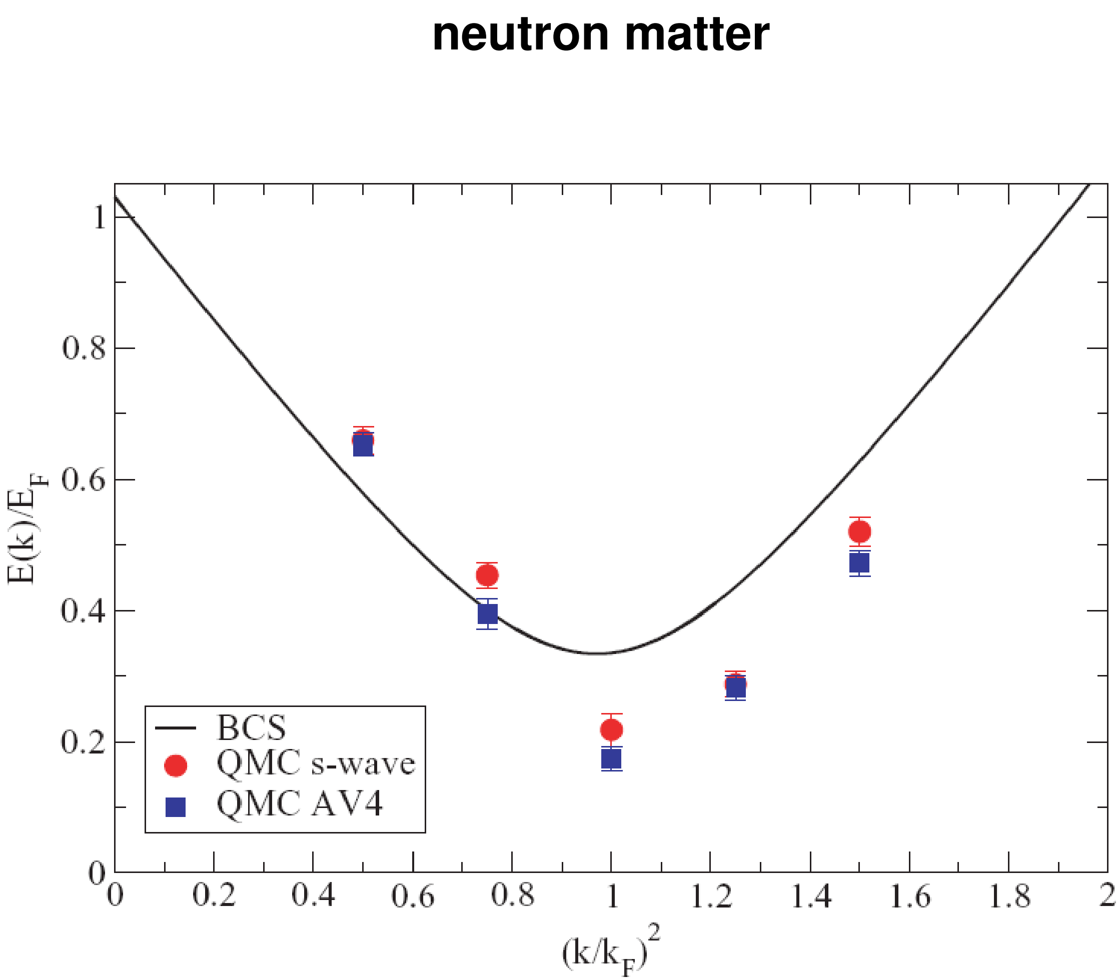
Up to this point in my talk

- Degeneracy = 2: spin-up or spin-down neutrons or atoms
- Only unpolarized matter: $N_\uparrow = N_\downarrow$
- Only equation of state and pairing gap

I will now start adding extra particles

- Spin-up or spin-down neutrons
- Spin-up or spin-down cold atoms (^{6}Li or ^{40}K)
- Heavy or light atoms with interspecies interactions (^{6}Li *and* ^{40}K)
- A third hyperfine state of ^{6}Li

Neutron matter: single-particle dispersion



A. Gezerlis and J. Carlson, Phys. Rev. C, 81, 025803 (2010).

34 up + 33 down neutrons

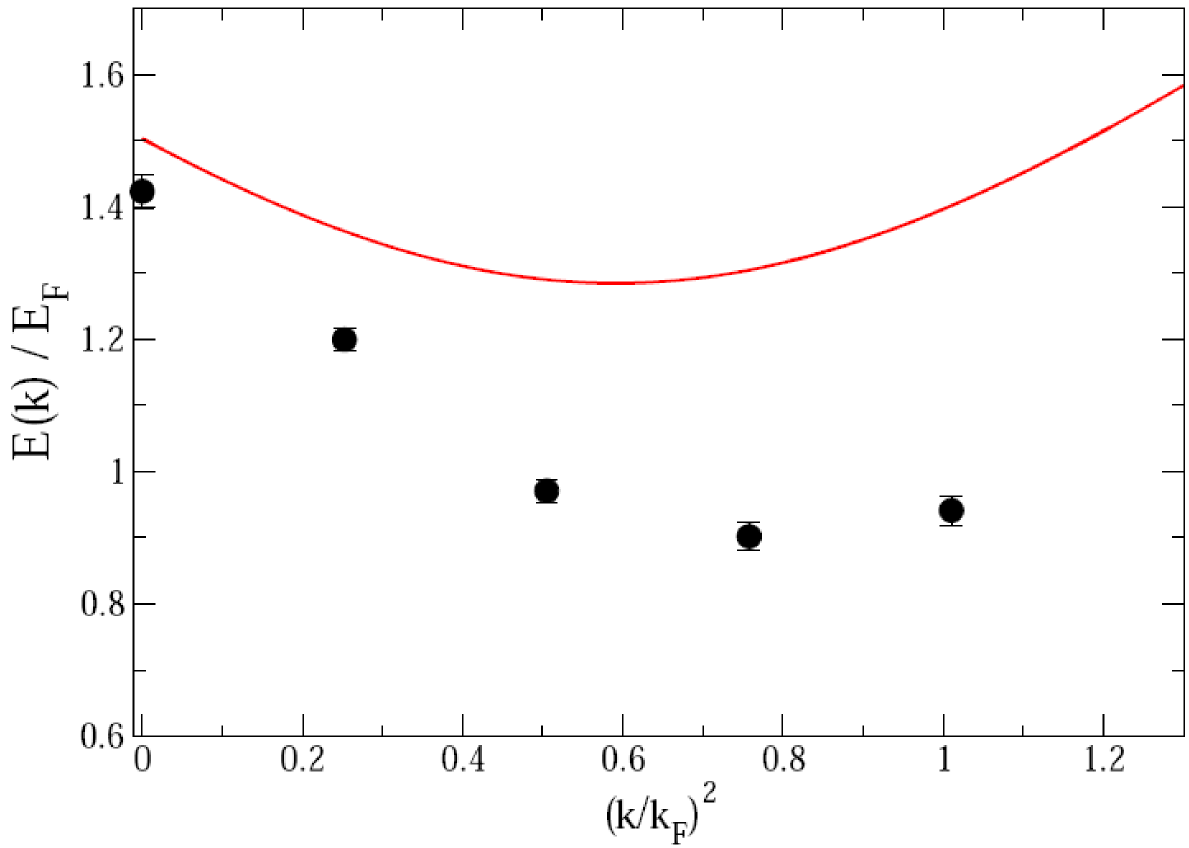
$$E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}$$

Quasi-particle spectrum

- Calculated by adding an extra particle with momentum k
- Analogous to cold atom spectra, which can be probed in RF response experiments

Cold atoms: 2 species, equal masses

34 up + 33 down cold atoms



J. Carlson and S. Reddy, Phys. Rev. Lett. 95, 060401(2005).

Equal masses: M/m = 1.0

Two hyperfine states of the same atom, say ^{6}Li.

$$E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}$$

Quasi-particle spectrum

- Calculated by adding an extra particle with momentum k
- Minimum at ~ 0.8 k_F.

34 heavy + 33 light

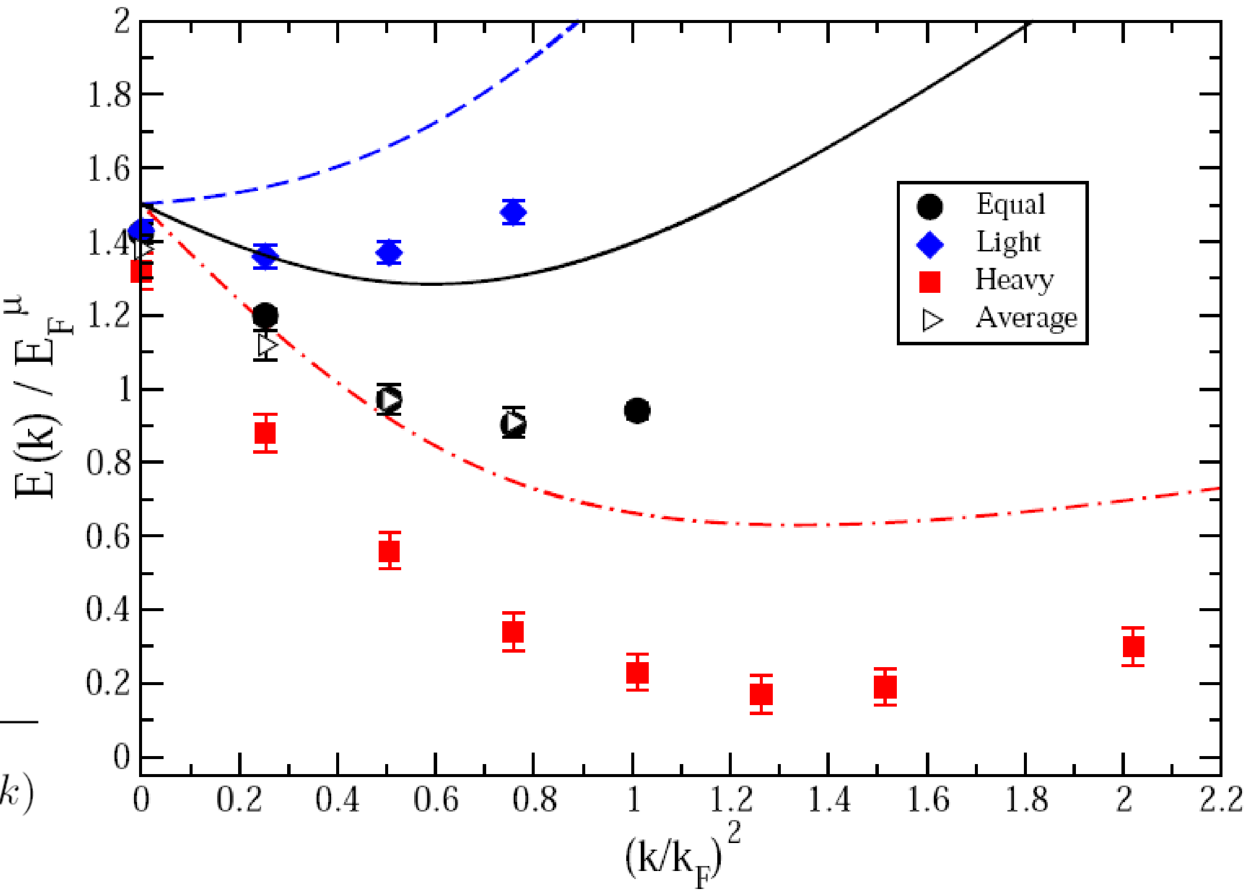
and

34 light + 33 heavy

Unequal masses: M/m = 6.7

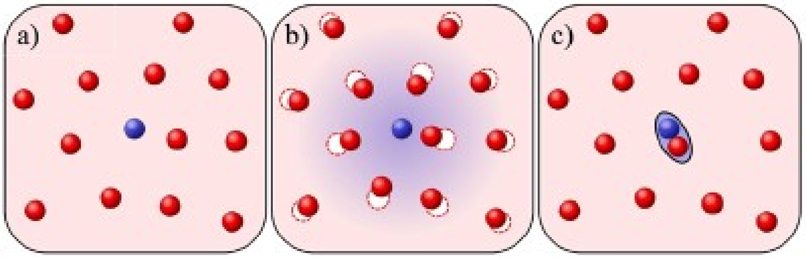
Hyperfine states of two different atoms, in this case ^{6}Li and ^{40}K:

$$E_{h(l)}(k) = \frac{\xi_{h(l)}(k) - \xi_{l(h)}(k)}{2} + \sqrt{\left(\frac{\xi_h(k) + \xi_l(k)}{2}\right)^2 + \Delta^2(k)}$$



A. Gezerlis, S. Gandolfi, K. E. Schmidt, and J. Carlson, Phys. Rev. Lett. 103, 060403 (2009).

Credit: see next slide



33 up + 1 down cold atoms

$$\frac{E_N(n_\uparrow, x)}{N_\uparrow} = \frac{3}{5} E_F^\uparrow \left(1 - Ax + \frac{m}{m^*} x^{5/3} + \cdots \right)$$

where $x = n_\downarrow / n_\uparrow$ **and** $E_{F\uparrow} = (\hbar^2/2m)(6\pi^2 n_\uparrow)^{2/3}$

Trento FN-DMC gives $A = 0.99(1)$ **and** $\frac{m^\star}{m} = 1.09(2)$ **at unitarity**

S. Pilati and S. Giorgini, Phys. Rev. Lett. 100, 030401(2008).

Now within experimental reach!

Selected for a Viewpoint in *Physics*
PRL **102,** 230402 (2009) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2009

Observation of Fermi Polarons in a Tunable Fermi Liquid of Ultracold Atoms

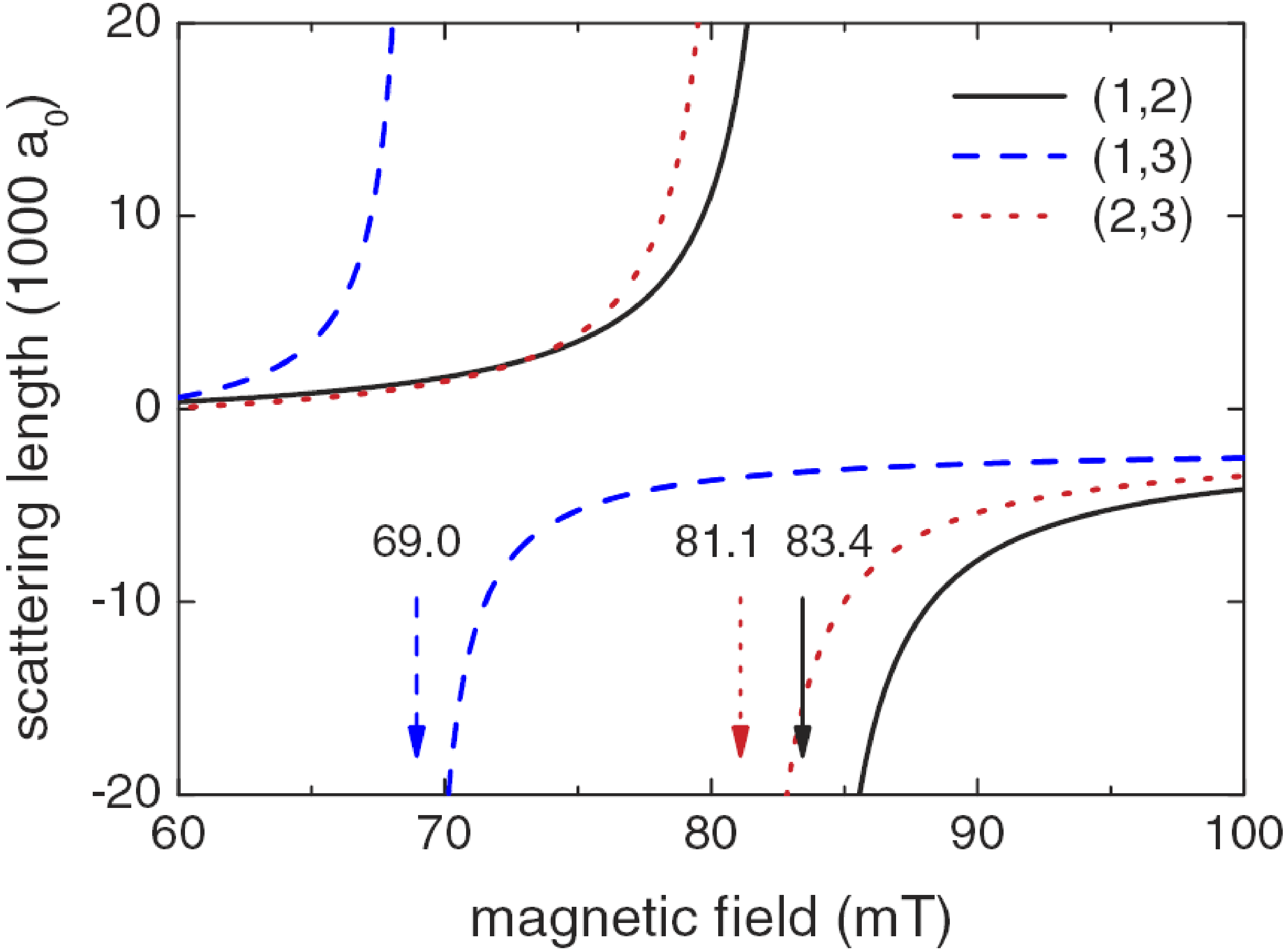
André Schirotzek, Cheng-Hsun Wu, Ariel Sommer, and Martin W. Zwierlein
Department of Physics, MIT-Harvard Center for Ultracold Atoms, and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 17 February 2009; revised manuscript received 9 April 2009; published 8 June 2009)

We have observed Fermi polarons, dressed spin-down impurities in a spin-up Fermi sea of ultracold atoms. The polaron manifests itself as a narrow peak in the impurities' rf spectrum that emerges from a broad incoherent background. We determine the polaron energy and the quasiparticle residue for various interaction strengths around a Feshbach resonance. At a critical interaction, we observe the transition from polaronic to molecular binding. Here, the imbalanced Fermi liquid undergoes a phase transition into a Bose liquid, coexisting with a Fermi sea.

DOI: 10.1103/PhysRevLett.102.230402 PACS numbers: 05.30.Fk, 03.75.Ss, 32.30.Bv, 67.60.Fp

Cold atoms: 3 species - polarons

Next step – connection with few-body physics: 33 |1> + 33 |2> + 1|3> for ^{6}Li



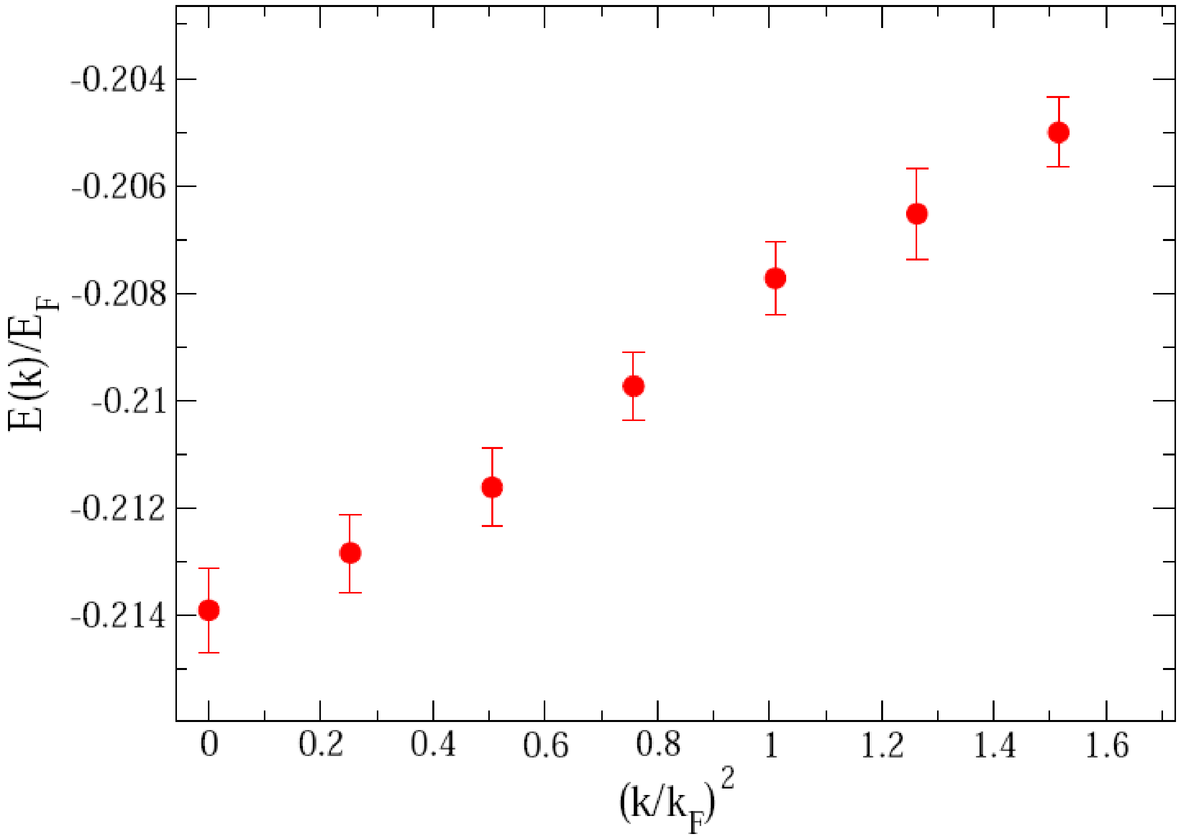
M. Bartenstein, A. Altmeyer, S. Riedl, R. Geursen, S. Jochim, C. Chin, J. Hecker Denschlag, and R. Grimm, Phys. Rev. Lett. 94, 103201(2005).

(I am) interested in

- $k_F a_{12} = \infty$
- $k_F a_{13} \sim -5$
- $k_F a_{23} \sim -1$

Cold atoms: 3 species - polarons

33 |1> + 33 |2> + 1|3> for ^{6}Li

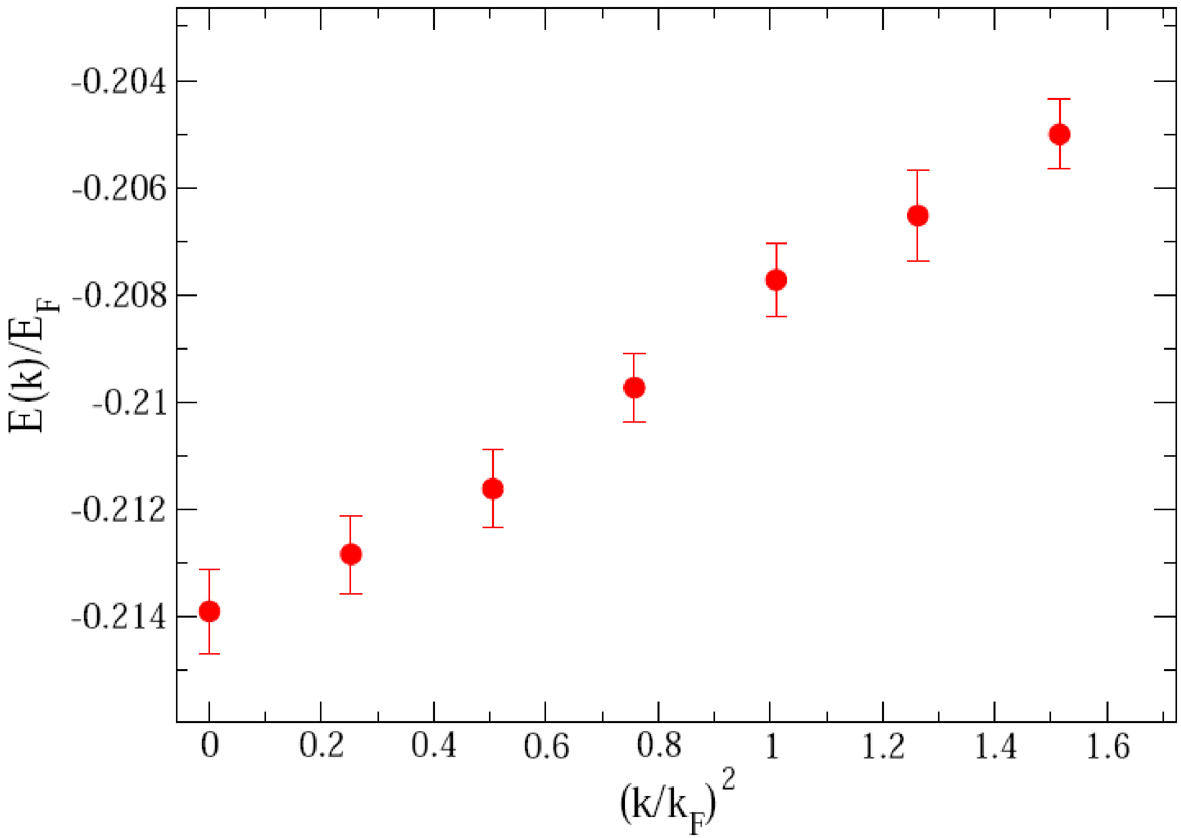


A. Gezerlis and A. Bulgac, *under preparation*

$r_e \approx \frac{r_0}{12}$ **(finite but small)**

$$\frac{4}{3}\pi r_0^3 = \frac{1}{n}$$

Total energy negative:
E_3 **is very large**



Energy of
33 |1> + 33 |2> + 1|3>
essentially is energy of
32 |1> + 32 |2> + E_3

Dispersion is quadratic:

$$\frac{m^\star}{m} \approx 3$$

A. Gezerlis and A. Bulgac, *under preparation*

Conclusions

- Both cold atoms and neutron matter have been tackled using QMC
- The cold atom superfluid gap prediction has been verified experimentally
- More complicated systems (heavy-light, 3-species, ...) are now being studied

The Present Future

- Effects of the ion lattice on neutron matter
- Highly asymmetric nuclear matter
- 3-species many-body system for more weakly bound trimer