Three-body Resonances and Applications in Nuclear Astrophysics

Three-body Resonances

- ✔ Hyperspherical Adiabatic Expansion Method
- ✔ Resonances and the Complex Scaling Method
- ✔ Decay of three-body resonances and Energy Distributions

Applications in Nuclear Astrophysics

- ✔ Three-body radiative capture
  - Sequential
  - Direct
- ✔ A=5 and A=8 gaps
  - Production of ^{12}C, ^{6}He, ^{9}Be

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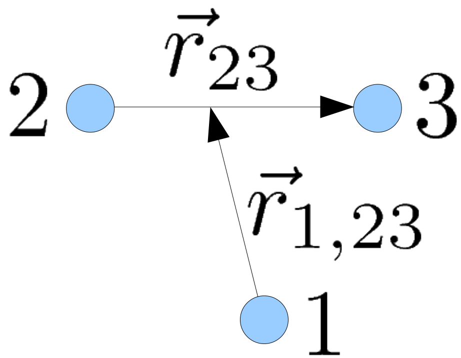

$$\left[\frac{p_{23}^2}{2\mu_{23}} + \frac{p_{1,23}^2}{2\mu_{1,23}} + V_{12}(r_{12}) + V_{13}(r_{13}) + V_{23}(r_{23})\right]\Psi = E\Psi$$

$$\frac{p_{23}^2}{2\mu_{23}} + \frac{p_{1,23}^2}{2\mu_{1,23}} = \frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{\hat{\Lambda}^2(\alpha,\Omega_x,\Omega_y)}{\rho^2}\right]$$

$$\Psi(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Jacobi coordinates

$$\vec{x}_1 = \sqrt{\frac{\mu_{23}}{m}}\vec{r}_{23}$$

$$\vec{y}_1 = \sqrt{\frac{\mu_{1,23}}{m}}\vec{r}_{1,23}$$

→

Hyperspheric coordinates

$$\rho^2 = x_1^2 + y_1^2$$

$$\alpha_1 = \arctan(x_1/y_1)$$

$$\Omega_{x_1}, \Omega_{y_1}$$

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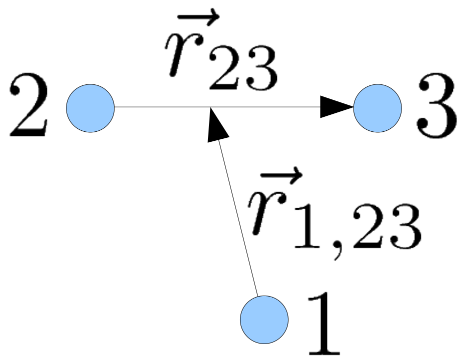

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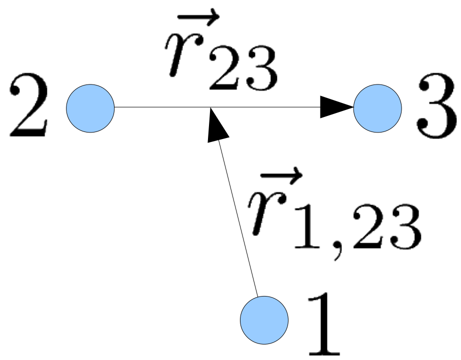

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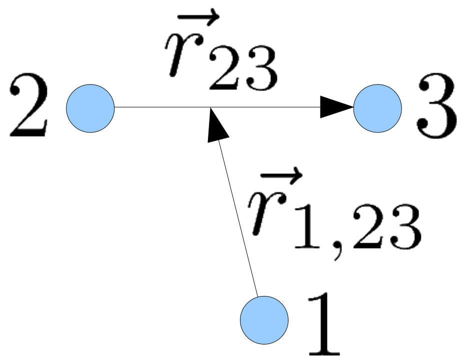

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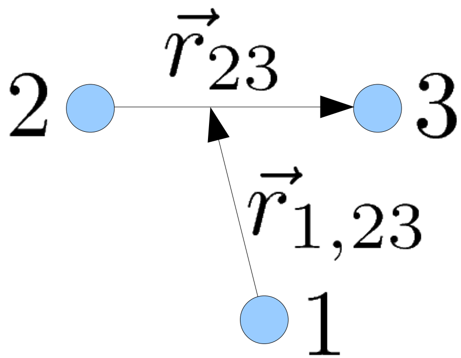

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Adiabatic assumption: The hyperangles change much faster than ρ.
We can solve the angular part for fixed values of ρ.

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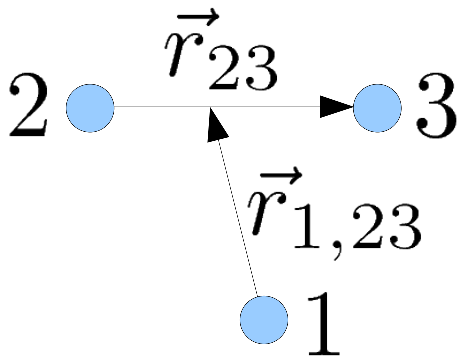

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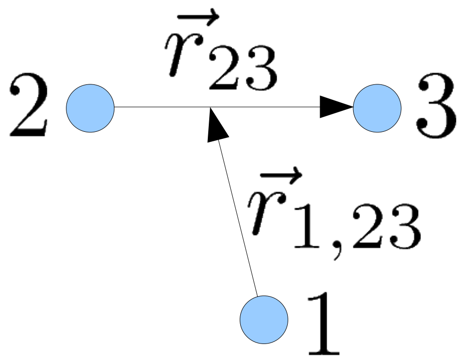

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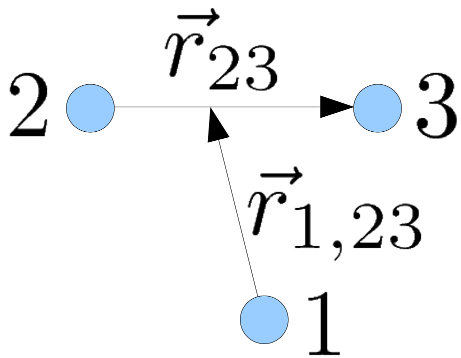

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$$\Phi_n(\rho,\Omega) = \sum_q C_q^{(n)}\mathcal{Y}_q(\Omega)$$

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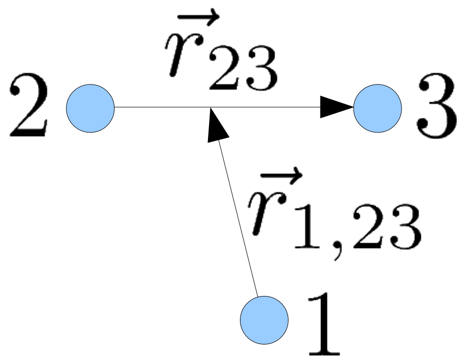

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Step 2→ $$\left[\frac{\partial^2}{\partial\rho^2} + \frac{2mE}{\hbar^2} - \frac{1}{\rho^2}\left(\lambda_n(\rho) + \frac{15}{4}\right)\right] f_n(\rho) + \sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho} + Q_{nn'}(\rho)\right] f_{n'}(\rho) = 0$$

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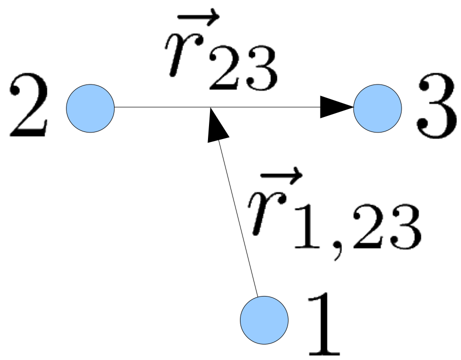

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$$P_{nn'}(\rho) = \langle\Phi_n(\rho,\Omega)|\frac{\partial}{\partial\rho}|\Phi_{n'}(\rho,\Omega)\rangle \qquad Q_{nn'}(\rho) = \langle\Phi_n(\rho,\Omega)|\frac{\partial^2}{\partial\rho^2}|\Phi_{n'}(\rho,\Omega)\rangle$$

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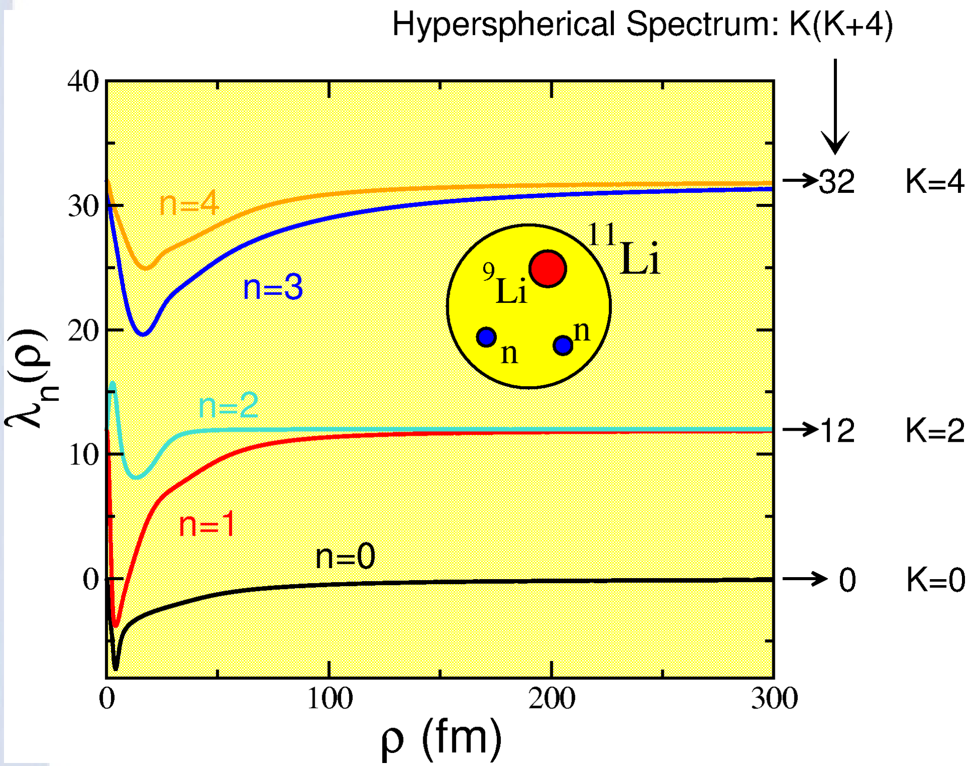

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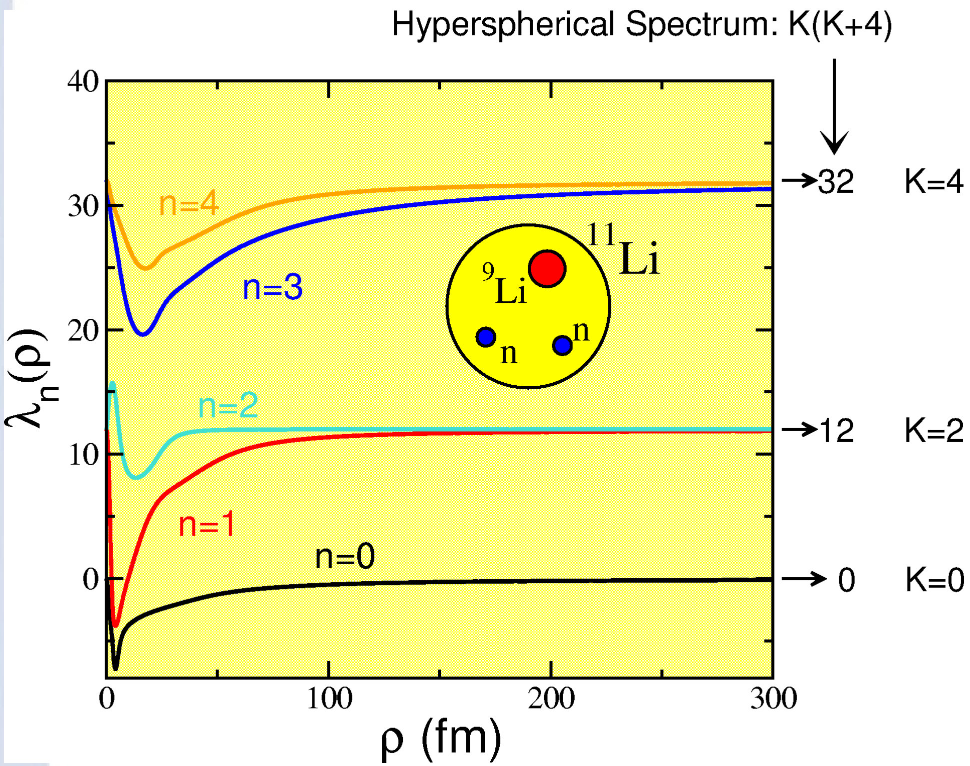

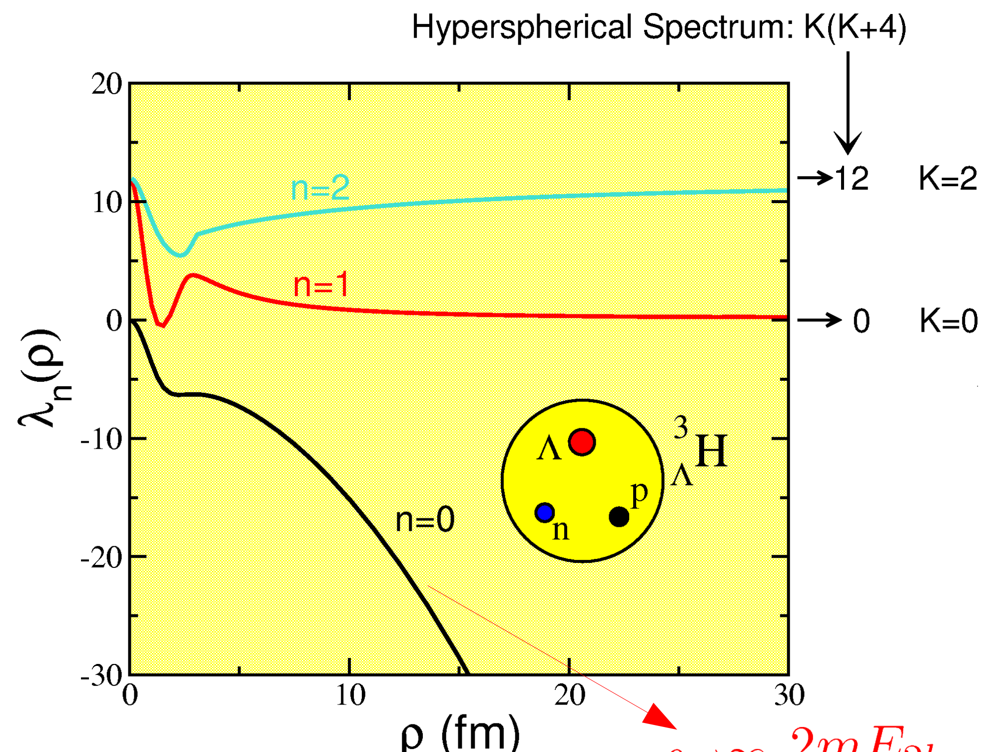

$$\lambda_0(\rho) \xrightarrow{\rho\to\infty} \frac{2mE_{2b}}{\hbar^2}\rho^2$$

Step 1→ $$\hat{\Lambda}^2\Phi_n(\rho,\Omega) + \frac{2m\rho^2}{\hbar^2}(V_{12}+V_{13}+V_{23})\Phi_n(\rho,\Omega) = \lambda_n(\rho)\Phi_n(\rho,\Omega)$$

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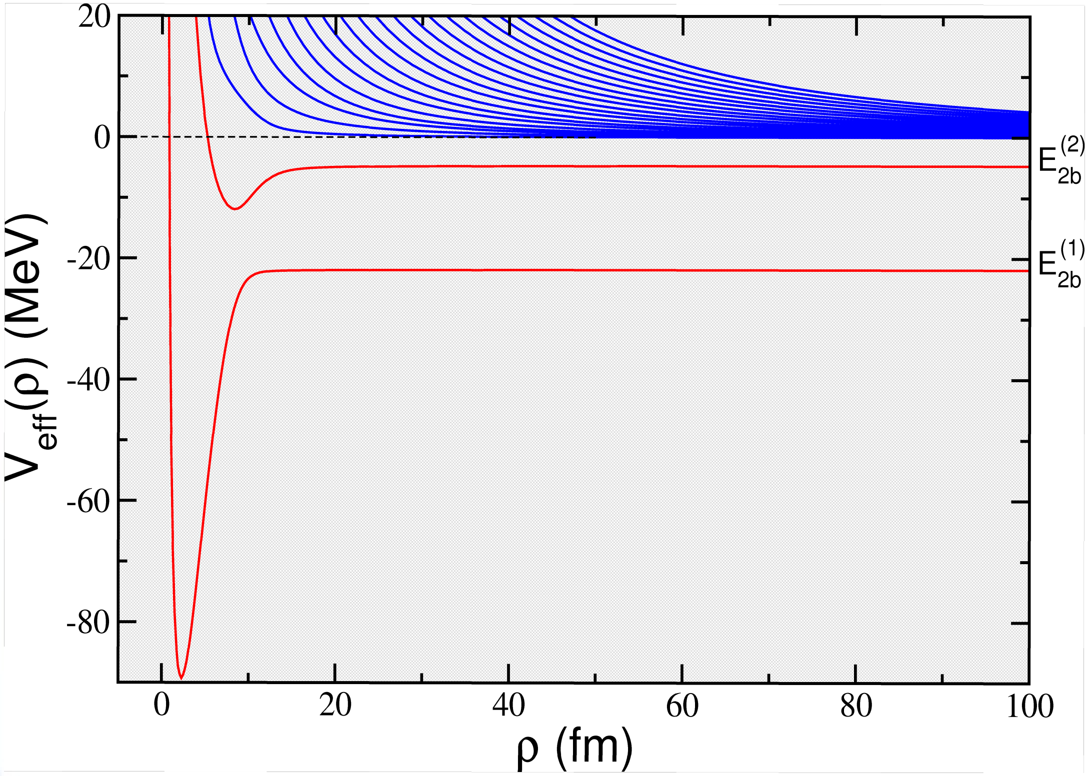

$$\Psi(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho)\Phi_n(\rho,\Omega)$$

Bound States

$$f_n(\rho) \overset{\rho\to\infty}{\longrightarrow} \exp(-\kappa\rho)$$

$$\kappa^2 = \frac{2m|E|}{\hbar^2}$$

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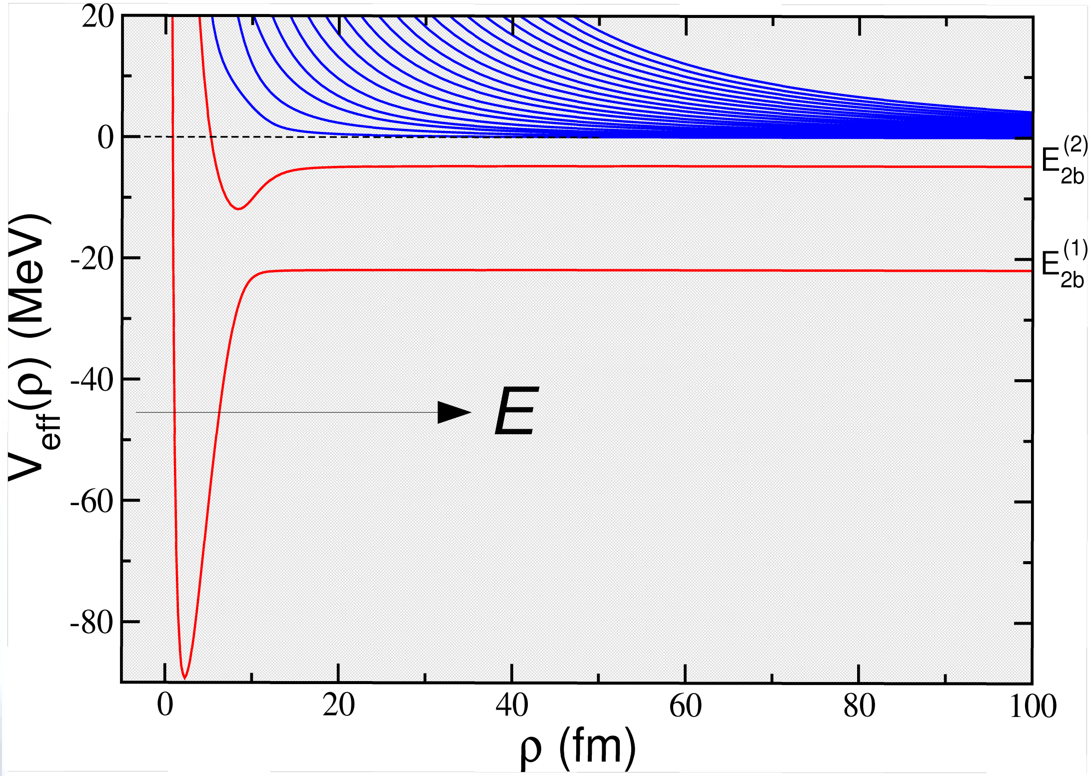

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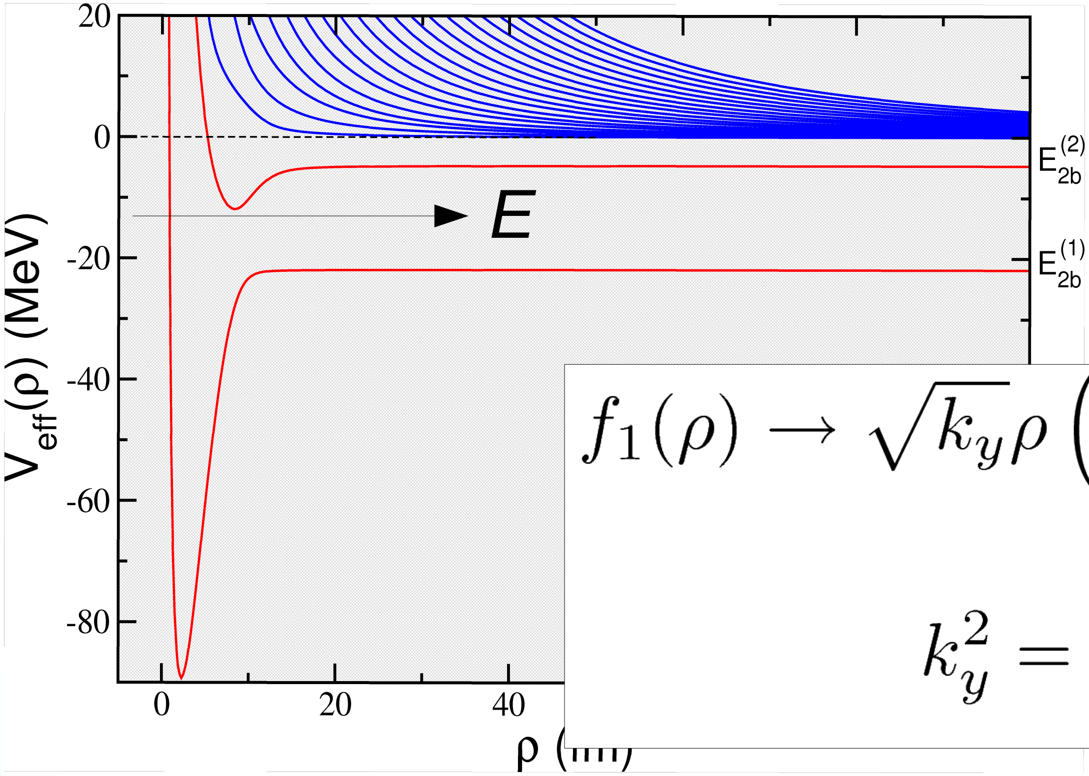

Scattering States

$$f_1(\rho) \to \sqrt{k_y\rho}\left(h^{(2)}_{\ell_y}(k_y\rho) + S h^{(1)}_{\ell_y}(k_y\rho)\right)$$

$$k_y^2 = \frac{2m(E - E^{(1)}_{2b})}{\hbar^2}$$

Step 1→ $$\hat{\Lambda}^2\Phi_n(\rho,\Omega) + \frac{2m\rho^2}{\hbar^2}(V_{12}+V_{13}+V_{23})\Phi_n(\rho,\Omega) = \lambda_n(\rho)\Phi_n(\rho,\Omega)$$

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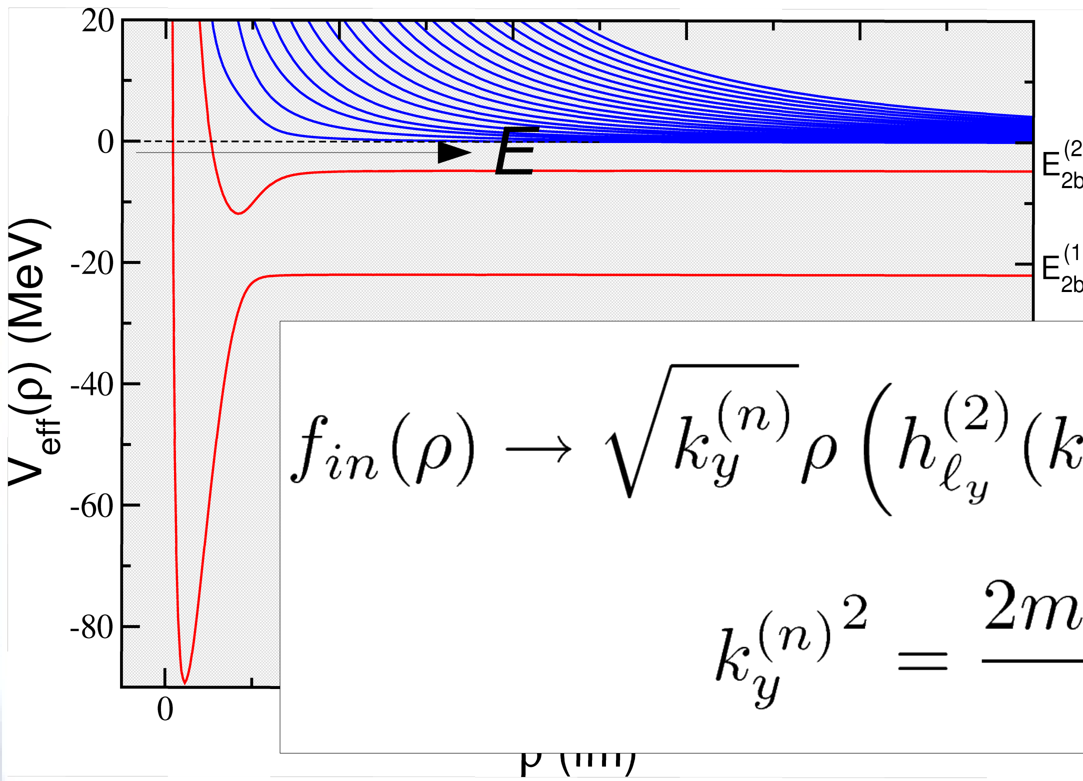
$$\Psi_i(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}} \sum_n f_{in}(\rho)\Phi_n(\rho,\Omega)$$



Scattering States

$$f_{in}(\rho) \to \sqrt{k_y^{(n)}}\rho\left(h_{\ell_y}^{(2)}(k_y^{(n)}\rho)\delta_{i,n} + S_{i,n}h_{\ell_y}^{(1)}(k_y^{(n)}\rho)\right)$$

$${k_y^{(n)}}^2 = \frac{2m(E - E_{2b}^{(n)})}{\hbar^2}$$

Step 1→ $$\hat{\Lambda}^2\Phi_n(\rho,\Omega) + \frac{2m\rho^2}{\hbar^2}(V_{12}+V_{13}+V_{23})\Phi_n(\rho,\Omega) = \lambda_n(\rho)\Phi_n(\rho,\Omega)$$

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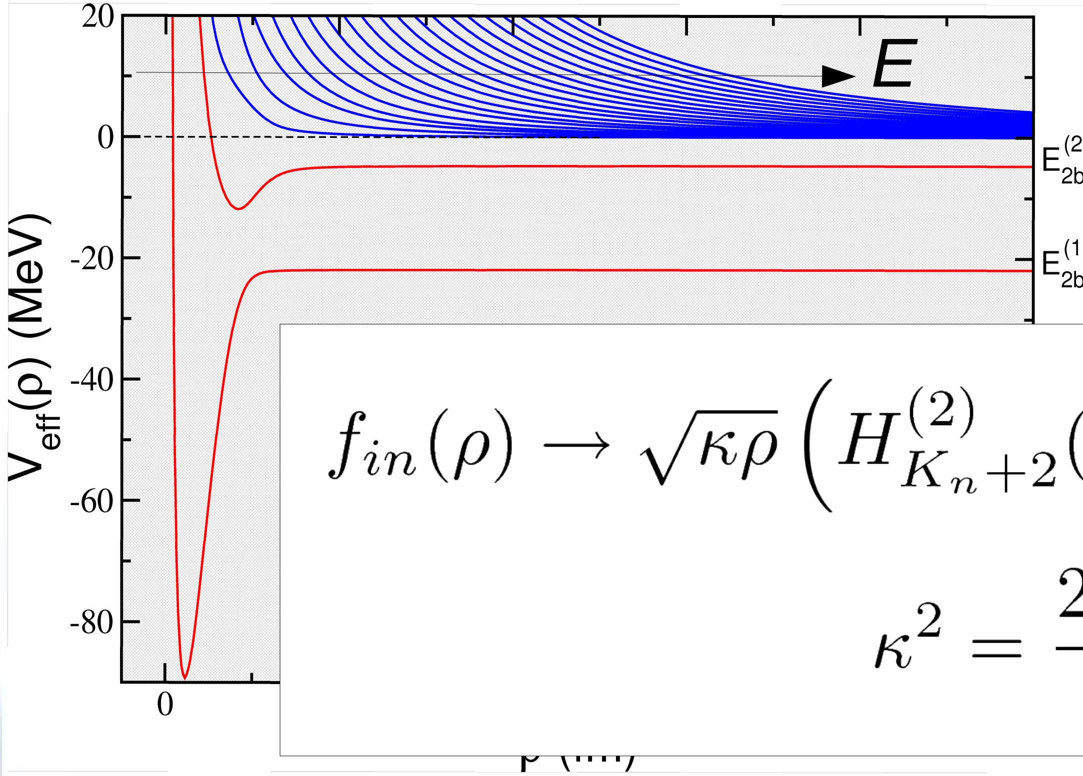
Three-body Resonances

✔ Resonances and The Complex Scaling Method



$$\Psi_i(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}} \sum_n f_{in}(\rho)\Phi_n(\rho,\Omega)$$

Scattering States

$$f_{in}(\rho) \rightarrow \sqrt{\kappa\rho}\left(H^{(2)}_{K_n+2}(\kappa\rho)\delta_{i,n} + S_{i,n}H^{(1)}_{K_n+2}(\kappa\rho)\right)$$

$$\kappa^2 = \frac{2mE}{\hbar^2}$$

Step 1→ $$\hat{\Lambda}^2\Phi_n(\rho,\Omega) + \frac{2m\rho^2}{\hbar^2}(V_{12}+V_{13}+V_{23})\Phi_n(\rho,\Omega) = \lambda_n(\rho)\Phi_n(\rho,\Omega)$$

Step 2→ $$\left[\frac{\partial^2}{\partial\rho^2} + \frac{2mE}{\hbar^2} - \frac{1}{\rho^2}\left(\lambda_n(\rho) + \frac{15}{4}\right)\right] f_n(\rho) + \sum_{n'}\left[2P_{nn'}(\rho)\frac{\partial}{\partial\rho} + Q_{nn'}(\rho)\right] f_{n'}(\rho) = 0$$

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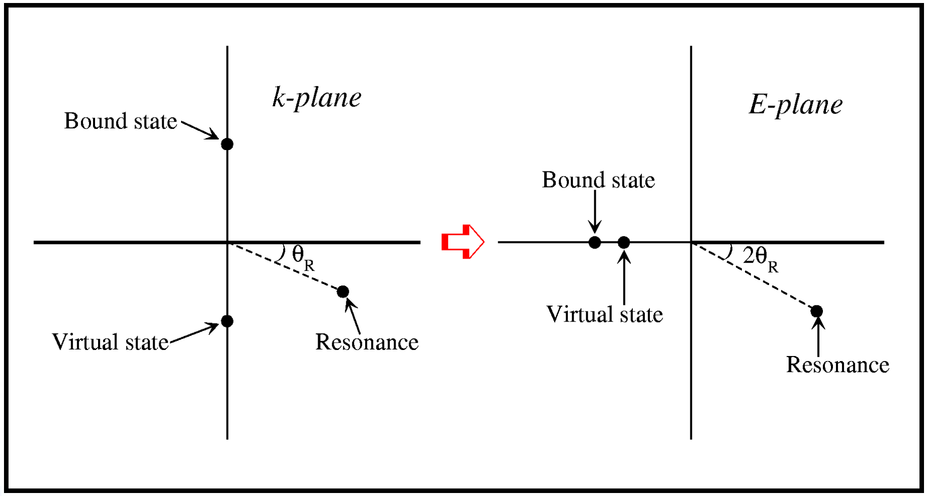

Complex Energy Method

$$E = |E| e^{-2i\theta_R}; \kappa = |\kappa| e^{-i\theta_R}$$

Resonances appear as poles of the S-matrix in the lower half of the momentum plane

$$f_{in}(\rho) \to \sqrt{\kappa\rho}\left(H^{(2)}_{K_n+2}(\kappa\rho)\delta_{i,n} + S_{i,n} H^{(1)}_{K_n+2}(\kappa\rho)\right)$$

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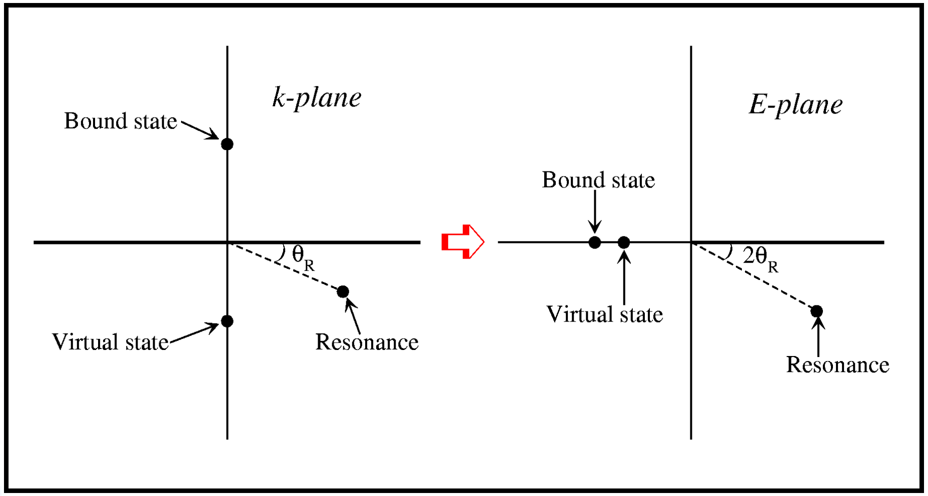

Complex Energy Method

$$E = |E|e^{-2i\theta_R}; \kappa = |\kappa|e^{-i\theta_R}$$

Resonances appear as poles of the S-matrix in the lower half of the momentum plane

The resonance wave functions diverge

$$f_n(\rho) \rightarrow \sqrt{\kappa\rho}H^{(1)}_{K_n+2}(\kappa\rho) \rightarrow e^{|\kappa|\rho\sin\theta_R}e^{i|\kappa|\rho\cos\theta_R}$$

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Complex Scaling Method: All the radial coordinates are rotated into the complex plane

$$\left.\begin{array}{ccc} x & \longrightarrow & xe^{i\theta} \\ y & \longrightarrow & ye^{i\theta} \end{array}\right\} \Rightarrow \left\{ \begin{array}{ll} \rho \longrightarrow \rho e^{i\theta} & \\ \alpha, \Omega_x, \Omega_y & \text{unchanged} \end{array}\right.$$

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As soon as $\theta > \theta_R$ the resonance wave function dies exponentially

As a bound state!!!

$$f_n(\rho e^{i\theta}) \rightarrow \sqrt{\kappa \rho e^{i\theta}} H^{(1)}_{K_n+2}(\kappa \rho e^{i\theta}) \rightarrow e^{-|\kappa|\rho \sin(\theta-\theta_R)}$$

Step 1→
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After complex scaling the resonances can be computed as "bound states" with complex energy

$$E = E_R - i\frac{\Gamma_R}{2}$$

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Bound states

$$f_n(\rho) \stackrel{\rho\rightarrow\infty}{\longrightarrow} \exp\left(-\kappa\rho\right)$$

$$\Big\downarrow\ \theta < \pi/2$$

$$f_n(\rho e^{i\theta}) \stackrel{\rho\rightarrow\infty}{\longrightarrow} \exp\left(-\kappa\rho\cos\theta\right)$$

Continuum states

$$f_n(\rho e^{i\theta}) \stackrel{\rho\rightarrow\infty}{\longrightarrow} \sin(\kappa\rho e^{i\theta} + \delta)$$

$$\Big\downarrow\ f_n(\rho_{max}) = 0$$

$$\kappa_n \approx e^{-i\theta} n\pi/\rho_{max}$$

$$E_n \approx e^{-2i\theta}(n\pi/\rho_{max})^2/2m$$

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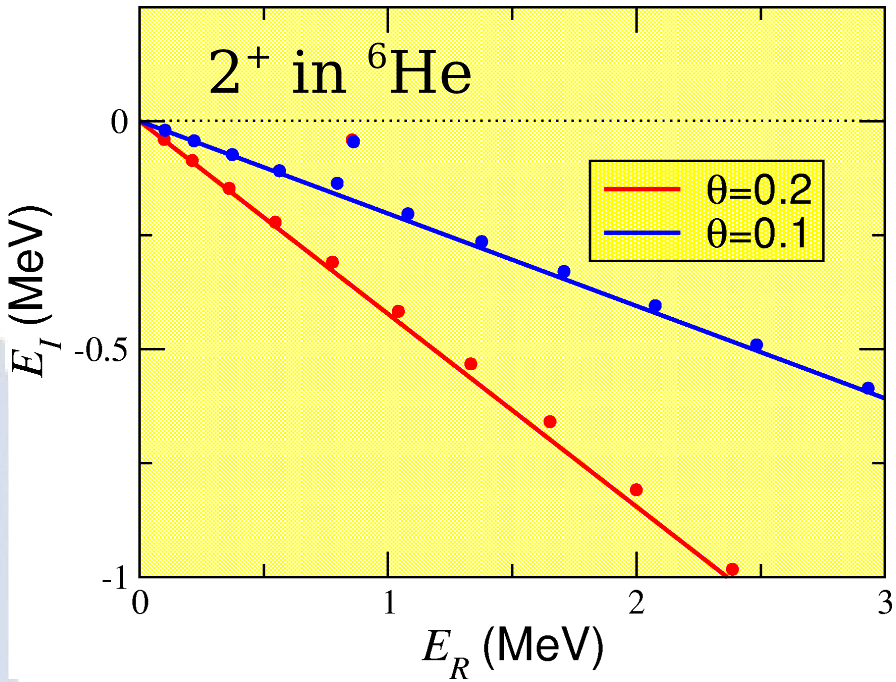

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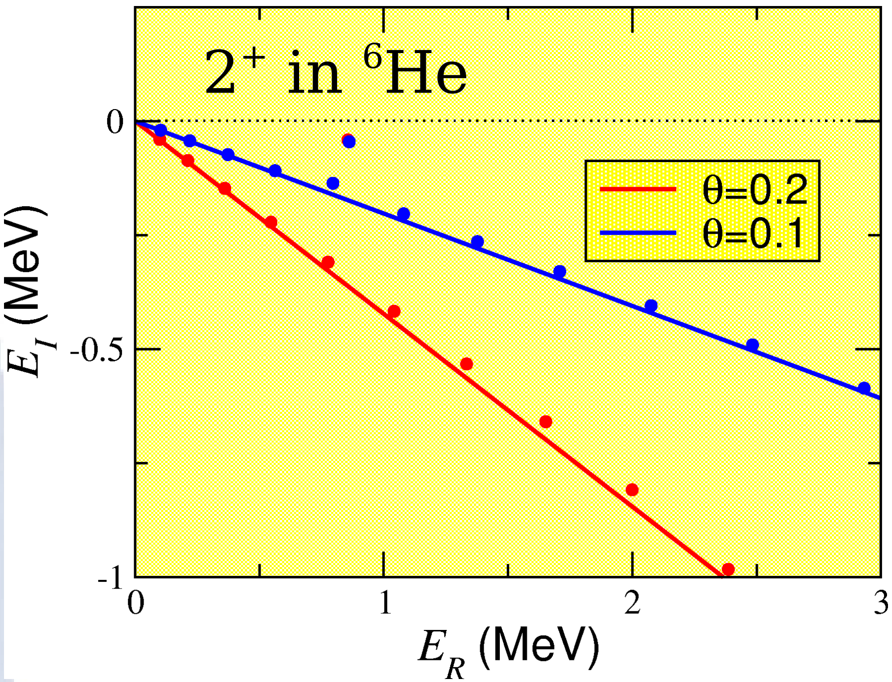
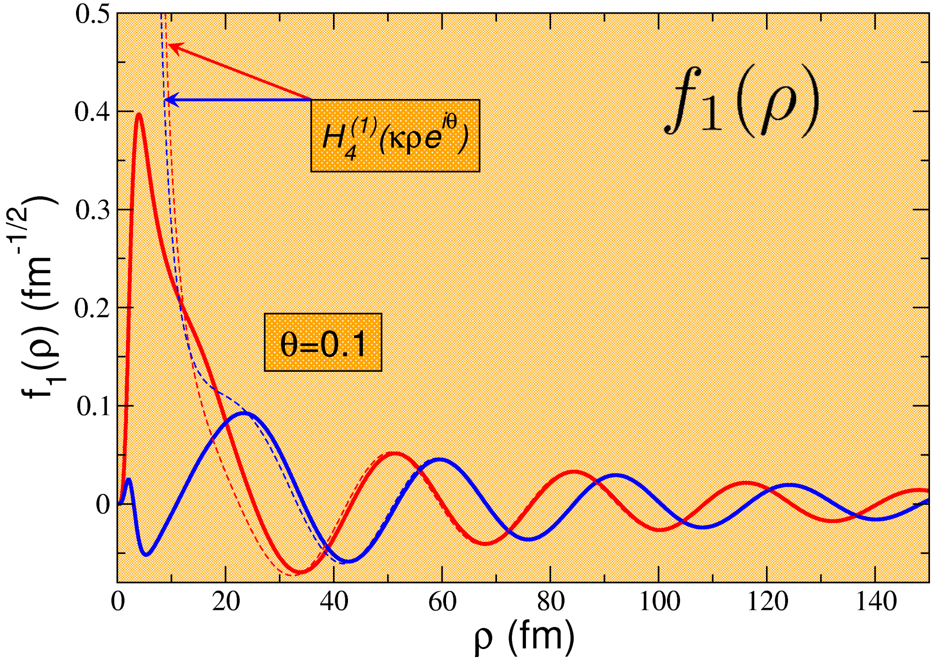


$$\Psi = \frac{1}{\rho^{5/2}} \left(f_1 \Phi_1 + f_2 \Phi_2 + f_3 \Phi_3 \cdots \right)$$



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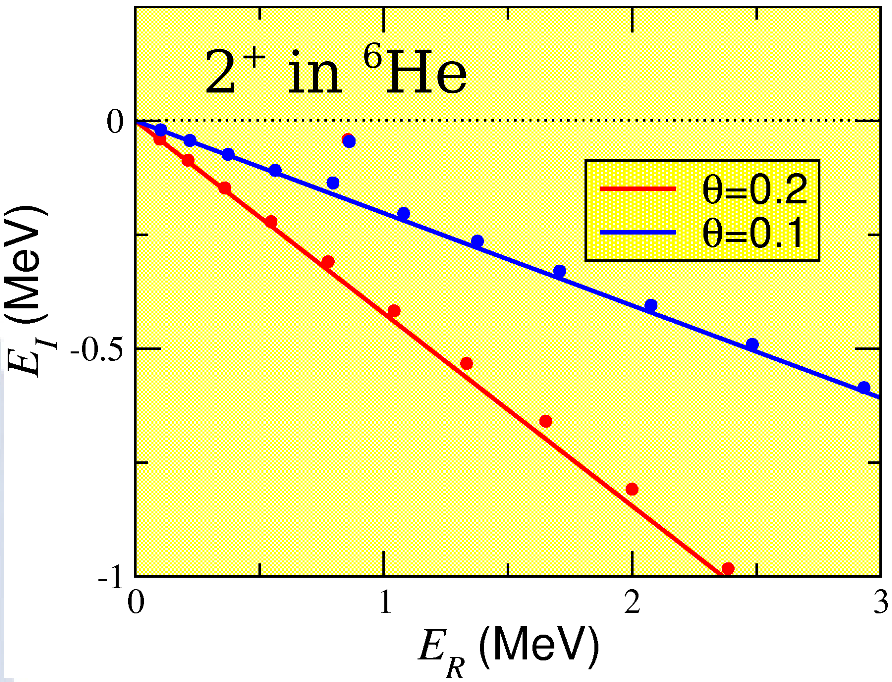

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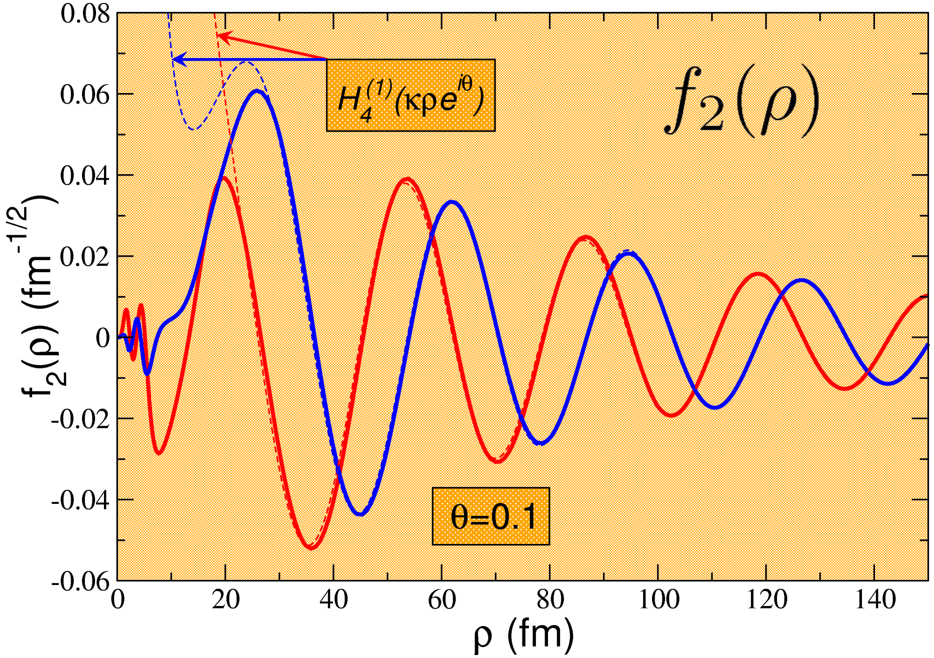

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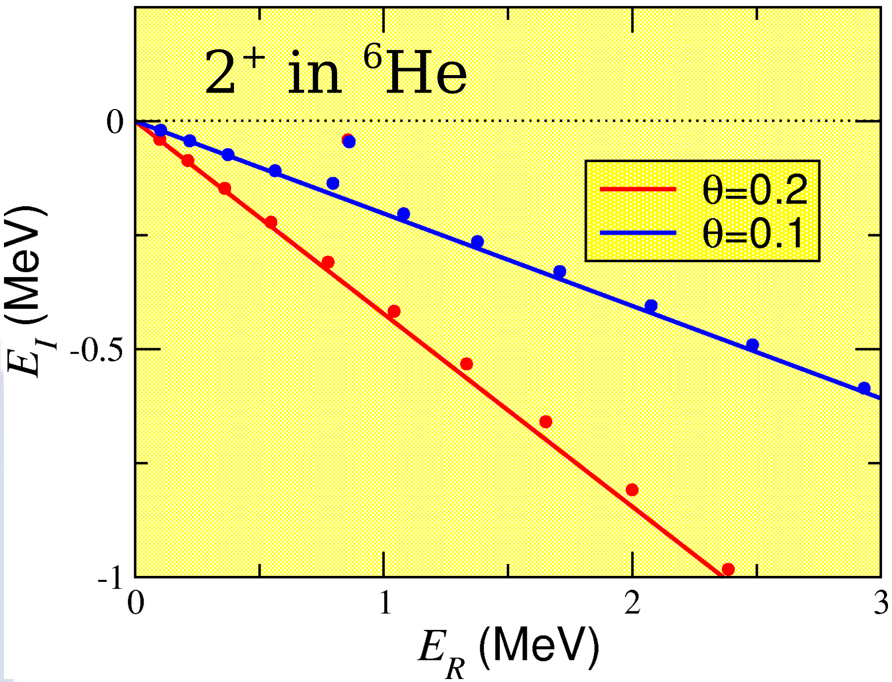

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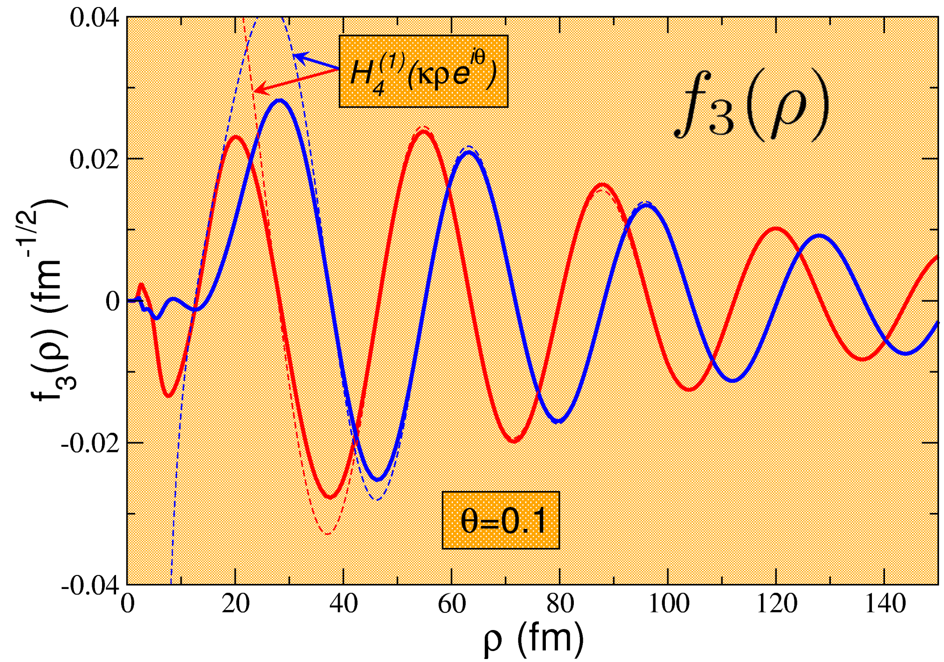

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✔ Decay of three-body resonances and Energy Distributions

$$\Psi(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho)\Phi_n(\rho,\Omega) = \frac{1}{\rho^{5/2}} \sum_n f_n(\rho) \sum_q C_q^{(n)}(\rho)\mathcal{Y}_q(\Omega)$$

The large distance part of the wave function contains the information about how the three-body resonance decays

$$\Psi(\vec{x},\vec{y}) \stackrel{\rho\to\infty}{\longrightarrow} \frac{1}{\rho^{5/2}} e^{i\kappa\rho} \sum_n \sum_q D_q^{(n)} \mathcal{Y}_q(\Omega)$$

The asymptotic coefficients can be obtained from the complex rotated wave function

It can be proved that ....

$$\Psi(\vec{k}_x,\vec{k}_y) \propto f(\kappa) \sum_n \sum_q D_q^{(n)} \mathcal{Y}_q(\Omega_\kappa)$$

D.V. Fedorov et al., FBS 34 (2003) 33

Hyperspheric coordinates

$$\kappa^2 = k_x^2 + k_y^2$$

$$\alpha_\kappa = \arctan(k_x/k_y)$$

$$\Omega_{k_x}, \Omega_{k_y}$$

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$$|F(\Omega_\kappa)|^2 = \left| \sum_n \sum_q D_q^{(n)} \mathcal{Y}_q(\Omega_\kappa) \right|^2$$

It contains the information about the energy distribution between the three particles after the decay

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D.V. Fedorov et al., FBS 34 (2003) 33

$$|F(\Omega_\kappa)|^2 = \left| \sum_n \sum_q D_q^{(n)} \mathcal{Y}_q(\Omega_\kappa) \right|^2$$

$$P(\alpha_\kappa) = \int d\Omega_{k_x} d\Omega_{k_y} |F(\Omega_\kappa)|^2$$

$$\sin^2\alpha_\kappa = k_x^2/\kappa^2$$

$$\cos^2\alpha_\kappa = k_y^2/\kappa^2$$

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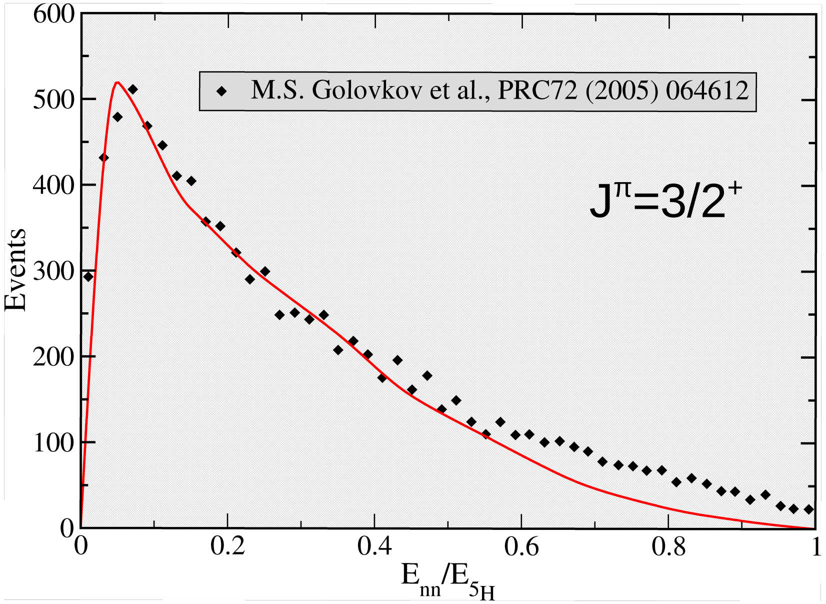
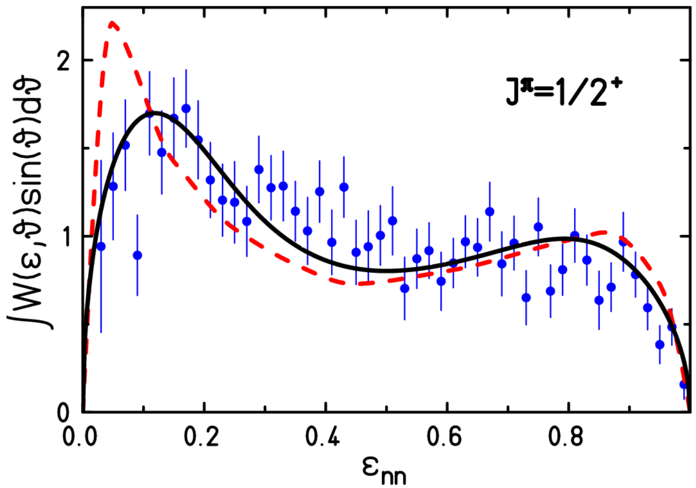
$$^5H \equiv {}^3H + n + n$$

Theory: R. de Diego *et al.*, NPA 786 (2007) 71
Exp: M. Meisner *et al.*, PRL 91 (2003) 162504





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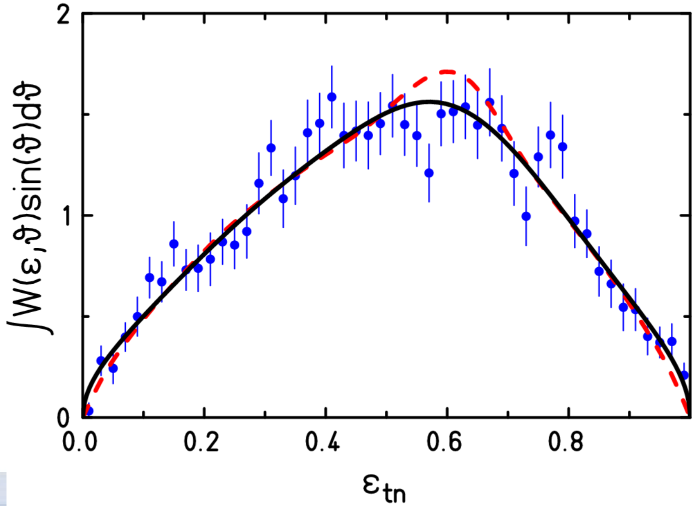

$$P(\alpha_\kappa) = \int d\Omega_{k_x} d\Omega_{k_y} \, |F(\Omega_\kappa)|^2$$

$$\sin^2 \alpha_\kappa = k_x^2 / \kappa^2$$

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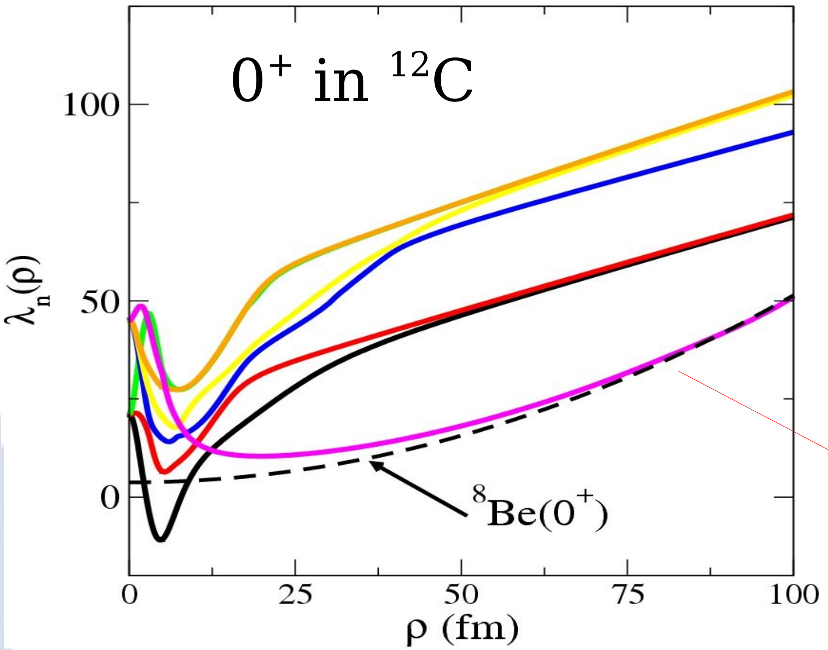
Three-body Resonances

✔ Decay of three-body resonances and Energy Distributions



The content in this adiabatic channel represents the amount of sequential decay

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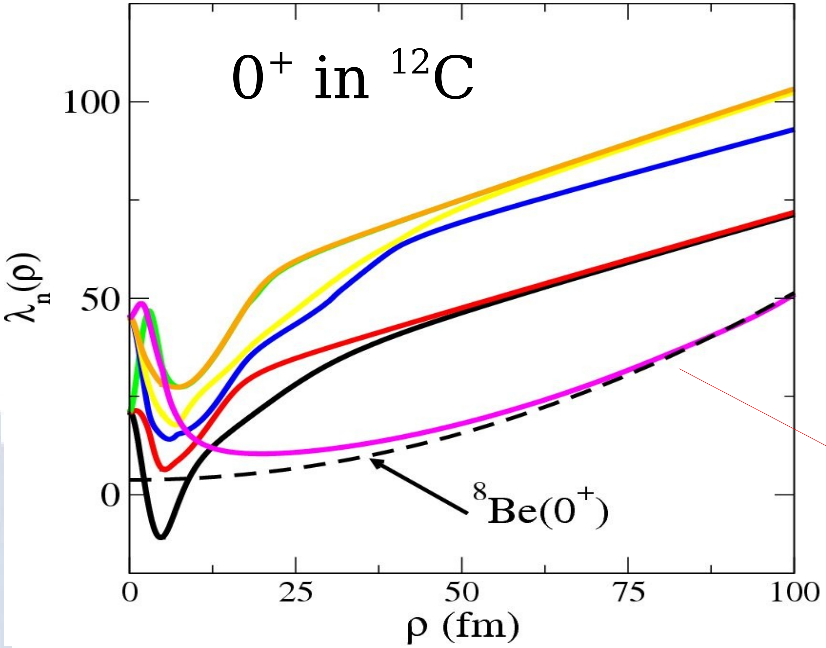

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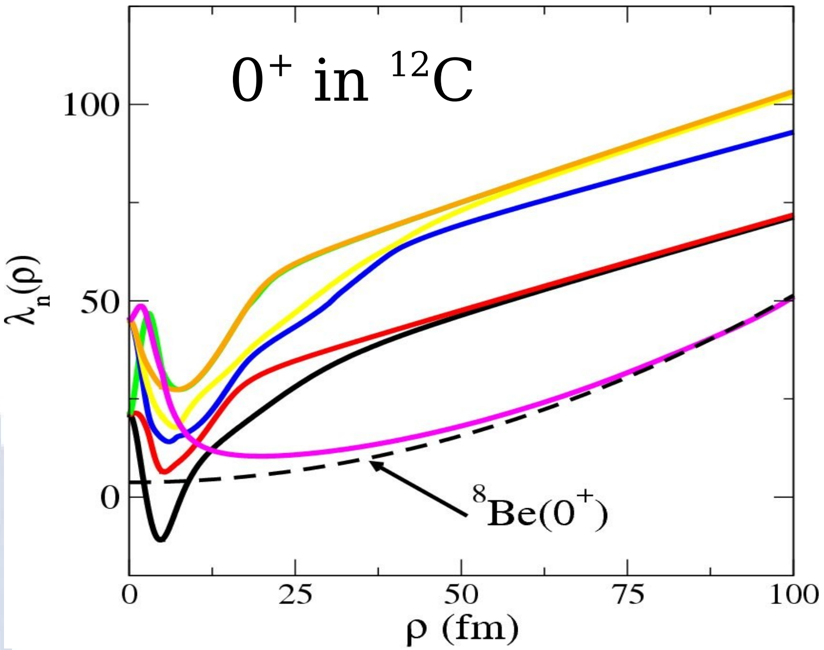

J^π	$E_{R,exp}$	$\Gamma_{R,exp}$	$E_{R,th}$	$\Gamma_{R,th}$	Sequential (%)	Direct (%)
0^+	−7.25	0.0	−7.25	0.0	—	—
	0.38	0.0085	0.38	0.0625	95	5
	4.3	3490	3.95	1000	59	41
1^-	3.57	315	3.61	475	70	30
3^-	2.37	34	2.33	68	96	4
2^+	−2.88	0.0	−3.04	0.0	—	—
			1.38	132	97	3
	3.88	430	4.48	1086	15	85
	6.3	1700	6.49	2250	4	96
4^+			3.25	396	92	8
	6.81	258	6.83	606	20	80
6^+			7.13	1267	5	95

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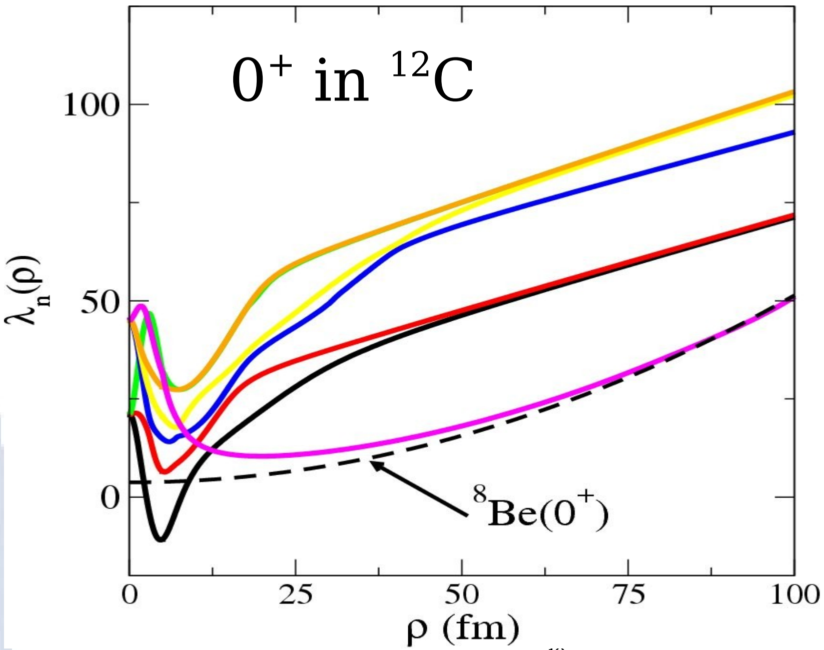

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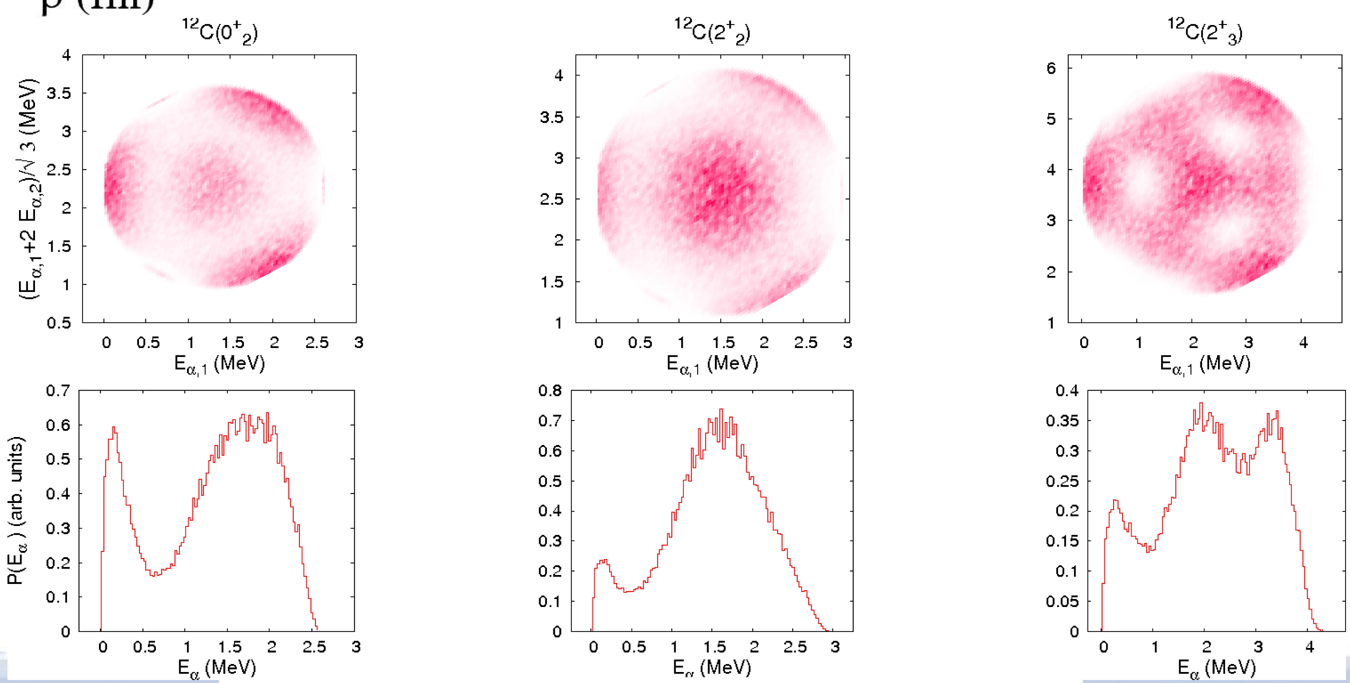

 R. Álvarez-Rodriguez et al., PRC 77 (2008) 064305

Applications in Nuclear Astrophysics

✔ Three-body radiative capture. The A=5 and A=8 gaps.

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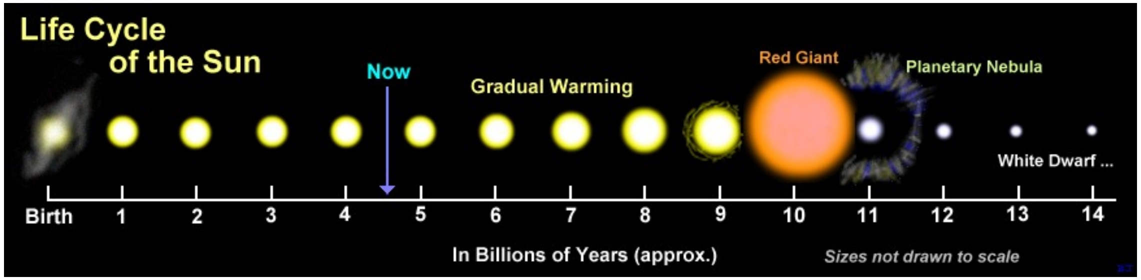

In the early stages of the life cycle the source of energy is the hydrogen nuclei

The **pp-chain** transforms four protons into ^{4}He

The core accumulates ^{4}He

When the hydrogen fuel is exhausted the nuclear reactions in the core stop

The gravitational collapse of the core raises the temperature

The fusion of the external layers begin: Red giant phase

Production of heavier nuclei requires to skip the A=5 and A=8 gaps

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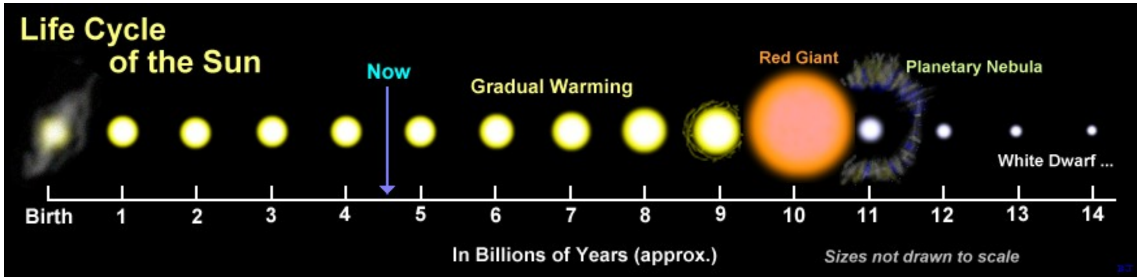

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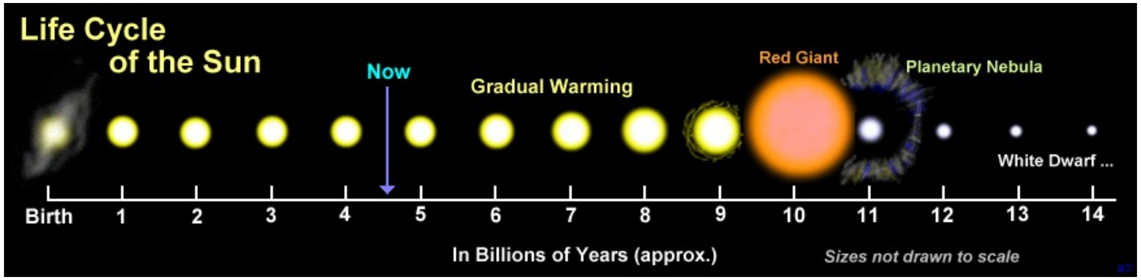

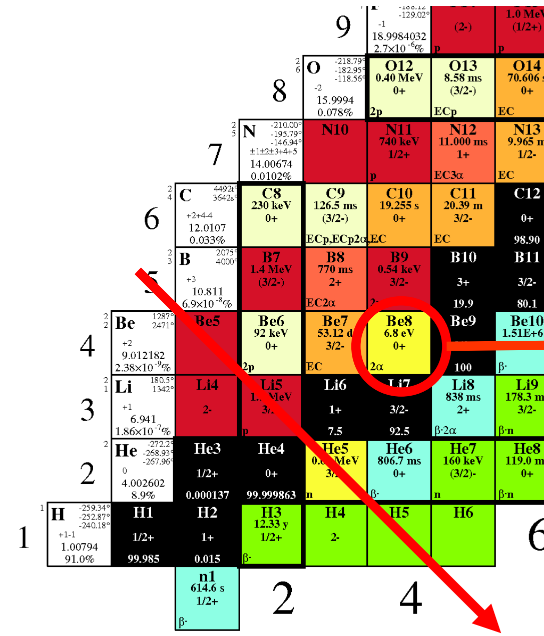

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All the A=5 nuclei are particle unstable

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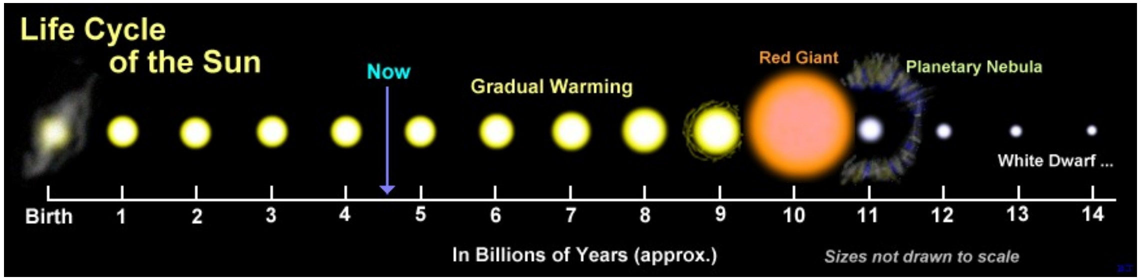

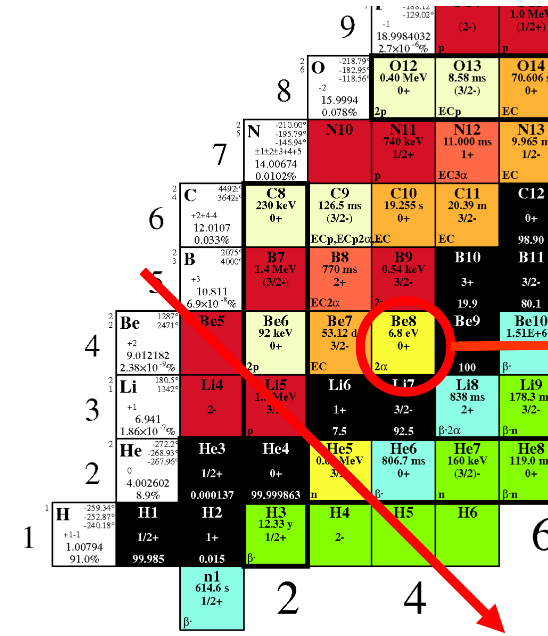

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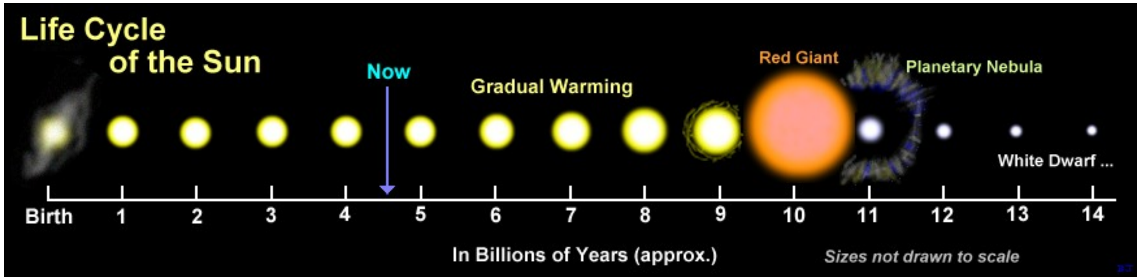

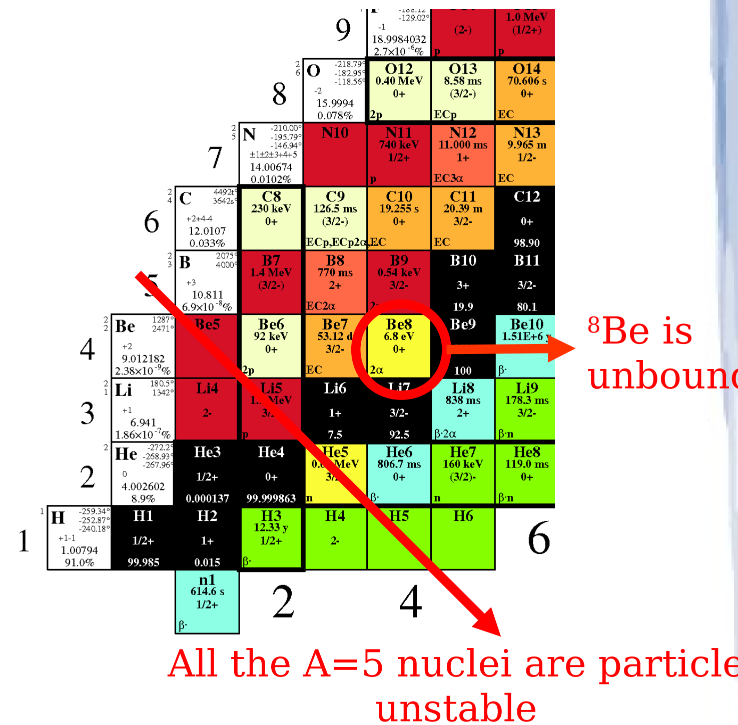

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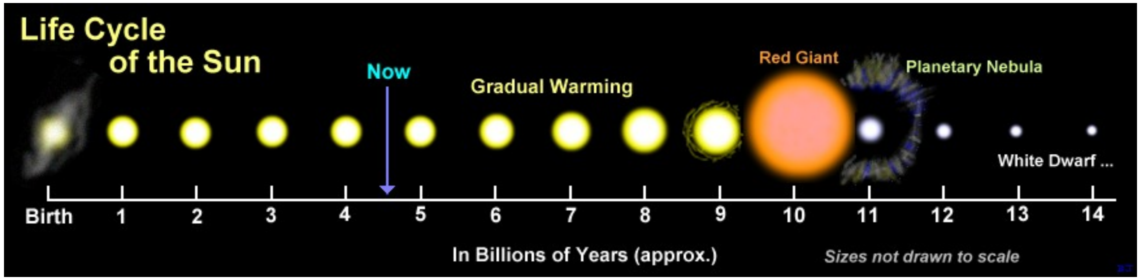

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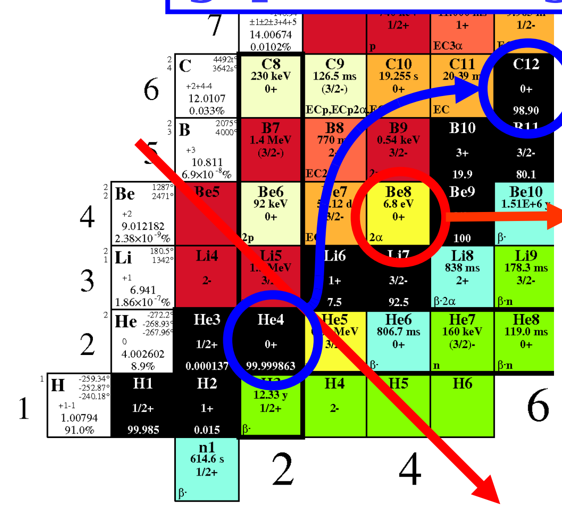
The A=5 and A=8 gaps are bridged!!!



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When T~10^8 K
$\alpha+\alpha+\alpha \longrightarrow {}^{12}C+\gamma$ is relevant!!!

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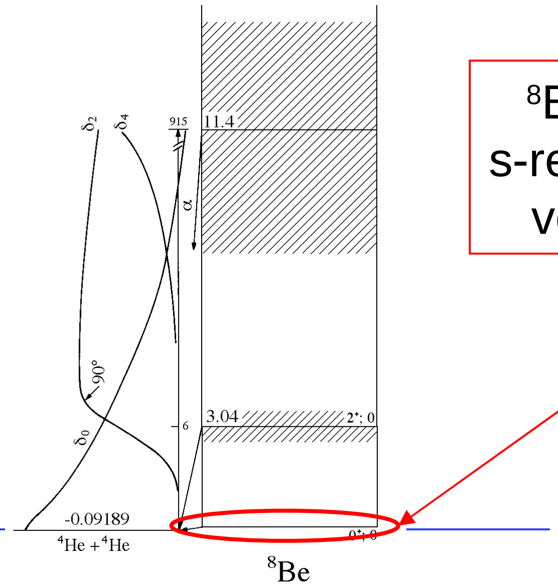
The α+α+α⟶^{12}C+γ reaction



^{8}Be has a very low lying s-resonance (~92 keV) and very narrow (a few eV).

This fact suggests to understand the triple α reaction as two consecutive two-body processes.
Sequential process

Applications in Nuclear Astrophysics

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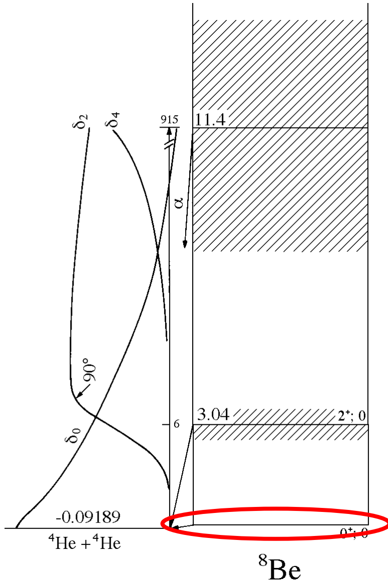

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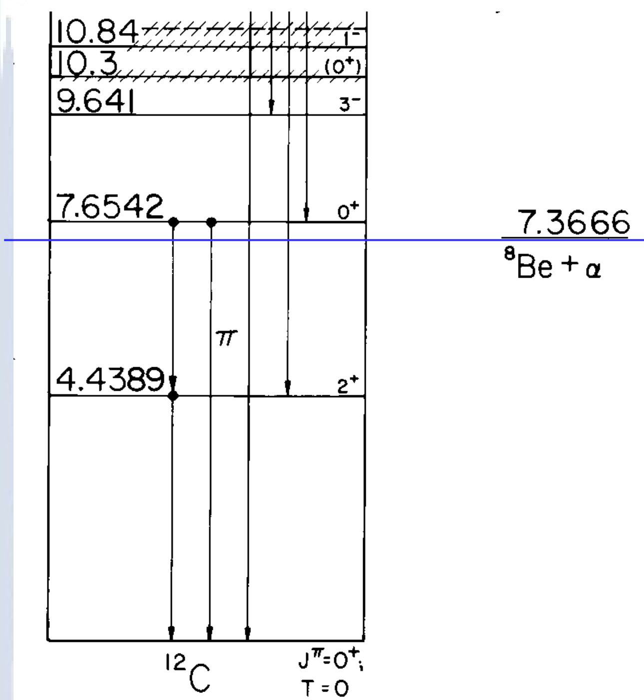

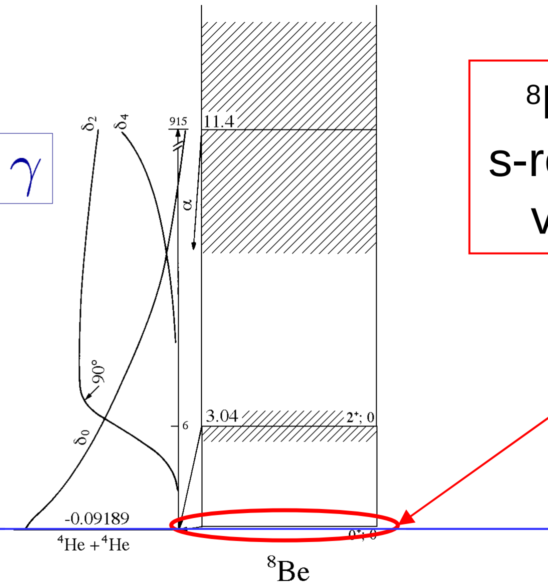

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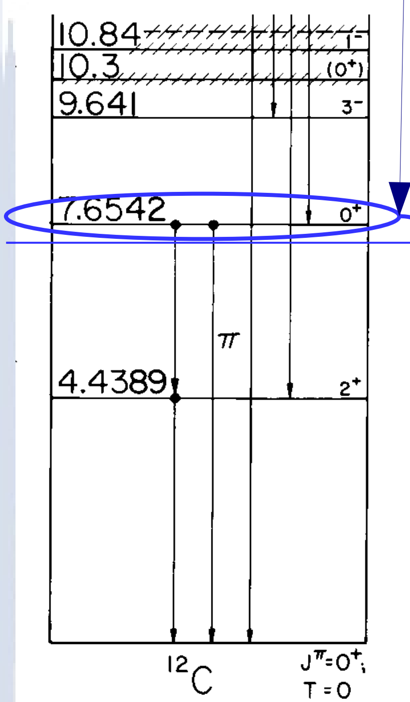

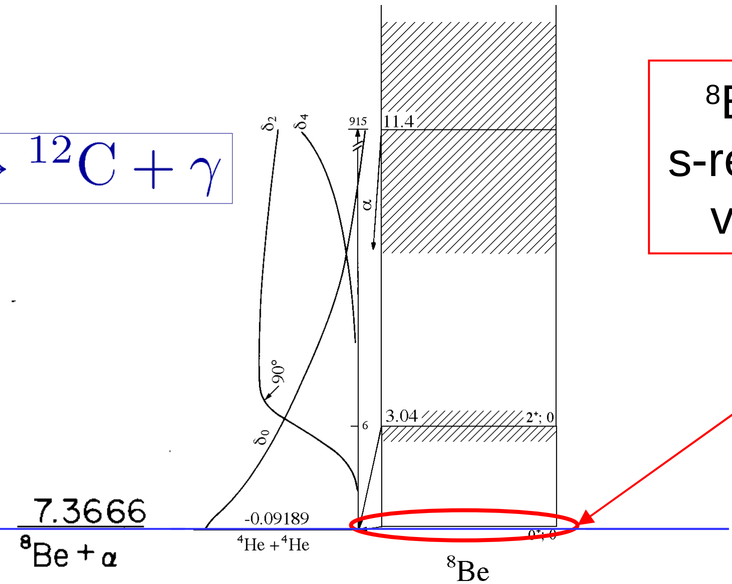

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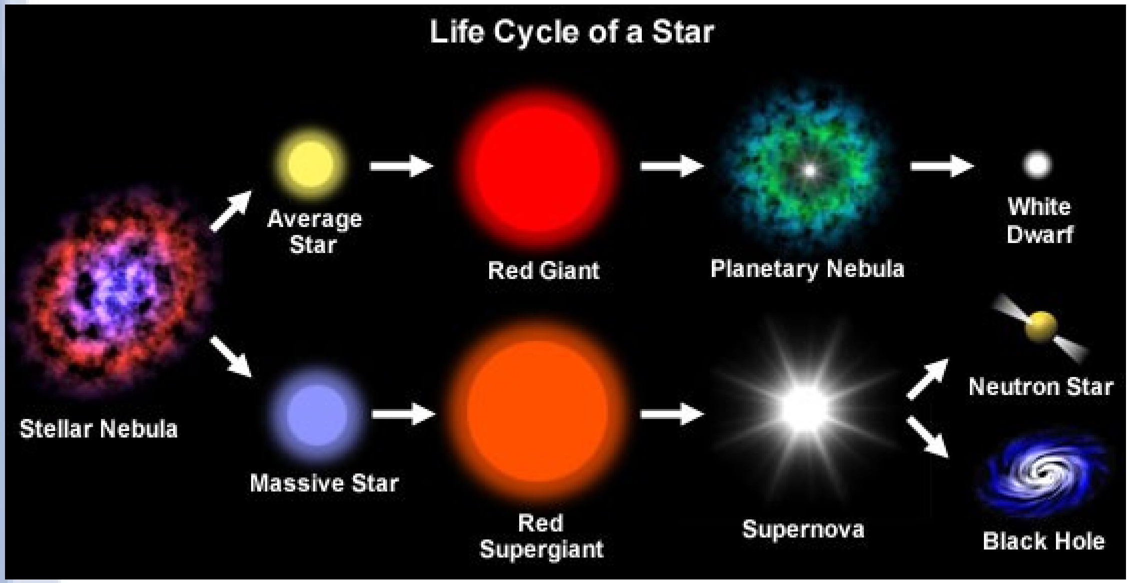

Stars with $M \sim 10 M_{\odot} \Rightarrow$ **Hot bubble** as a remnant of the supernova explosion

Hot bubble: rapidly expanding matter with a significant **neutron excess** and **T ~ 7-10 GK**

Ideal site for the **r-process** to take place

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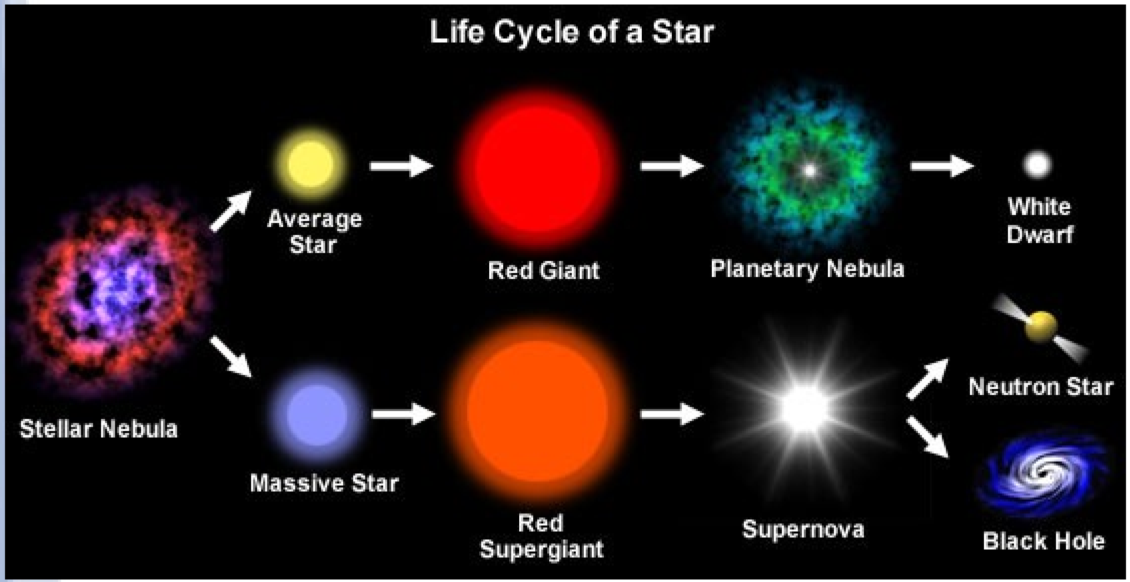

In this scenario **other reactions can play a role in the bridging of the A=5 and A=8 gaps**

$$\alpha + \alpha + n \rightarrow {}^{9}\mathrm{Be} + \gamma$$

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Stars with $M \sim 10 M_{\odot}$ ⇨ **Hot bubble** as a remnant of the supernova explosion

^{5}He (α+n) has a rather broad p-resonance

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A method **including sequential and direct capture** is desirable!!!

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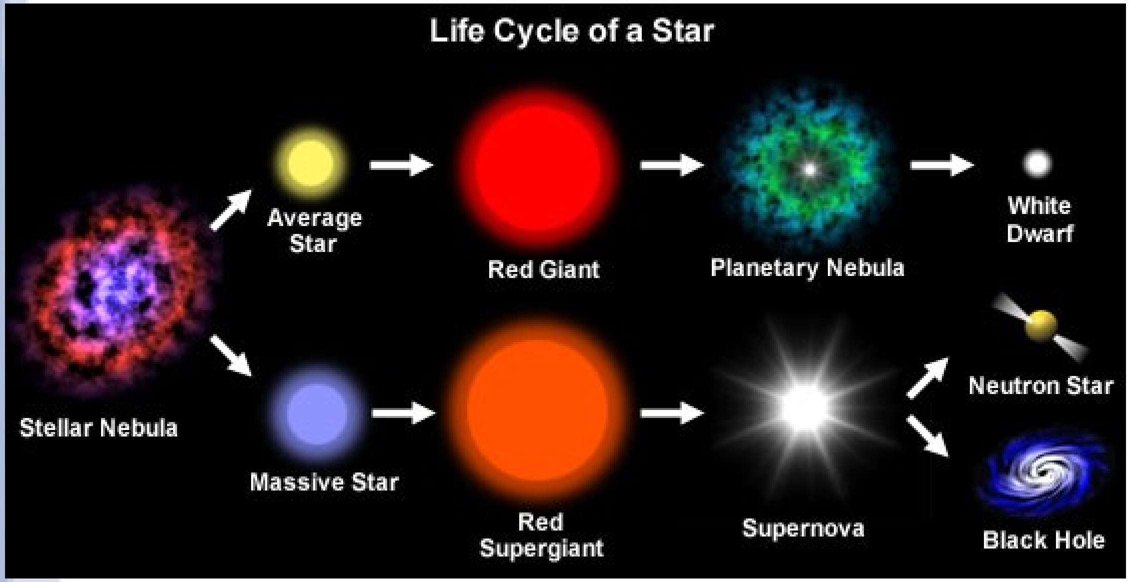

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What is the production rate for the different reactions in the stellar medium??

How many reactions per unit time and per unit volume??

$$a+b+c\to d+\gamma$$ Radiative capture process

$$P_{abc}(\rho,T)=n_a n_b n_c\frac{\hbar^3}{c^2}\left(\frac{m_a+m_b+m_c}{m_a m_b m_c}\right)^{3/2}\frac{2\pi}{(K_BT)^3}e^{-\frac{Q}{K_BT}}\int_{|Q|}^{\infty}E^2\sigma_{\gamma,d}(E)e^{-\frac{E}{K_BT}}dE$$

d is a bound state of $a, b,$ and c

$$Q=m_d-m_a-m_b-m_c$$

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$$\sigma_{\gamma,d}=\sum_\lambda\left(\sigma_{\gamma,d}^{E\lambda}+\sigma_{\gamma,d}^{M\lambda}\right)$$

$$\sigma_{\gamma,d}^{E\lambda}(E_\gamma)=\frac{\alpha(2\pi)^3\hbar c(\lambda+1)}{\lambda\left[(2\lambda+1)!!\right]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{d\mathcal{B}(E\lambda)}{dE_\gamma}$$

$$\mathcal{B}(E\lambda,I_i\to nI_f)=\sum_{\mu,M_f}|\langle nI_f,M_f|\mathcal{M}_\mu(E\lambda)|I_i,M_i\rangle|^2;\ \mathcal{M}_\mu(E\lambda)=e\sum_i Z_i r_i^\lambda Y_{\lambda,\mu}(\hat{r}_i)$$

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Three-body continuum wave function

Three-body bound state wave function

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e.g., for ^{6}He, the electric dipole contribution requires the continuum 1^- states

The continuum spectrum is discretized by solving the equations with a box boundary condition.

$$\mathcal{B}(E\lambda, I_i \rightarrow nI_f)=\sum_{\mu,M_f}|\langle nI_f,M_f|\mathcal{M}_\mu(E\lambda)|I_i,M_i\rangle|^2;\ \mathcal{M}_\mu(E\lambda)=e\sum_i Z_i r_i^\lambda Y_{\lambda,\mu}(\hat{r}_i)$$

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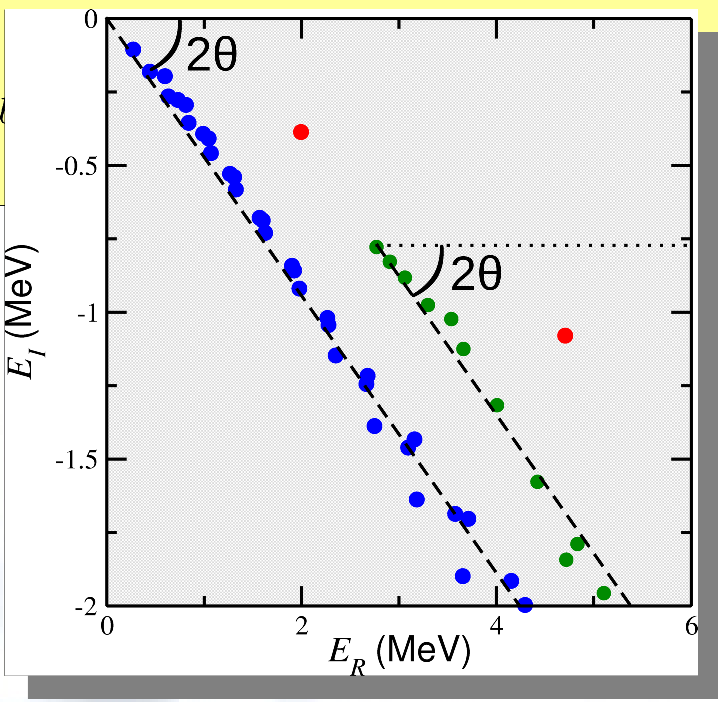
Radiative capture process

$$P_{a\ldots} \quad \ldots \frac{\ldots + m_c}{\ldots m_c}\Big)^{3/2} \frac{2\pi}{(K_BT)^3} e^{-\frac{Q}{K_BT}} \int_{|Q|}^{\infty} E^2 \sigma_{\gamma,d}(E) e^{-\frac{E}{K_BT}} dE$$



$$\mathcal{B}(E\lambda, I_i \rightarrow nI_f) = \sum_{\mu, M_f} |\langle nI_f, M_f | \mathcal{M}_\mu(E\lambda) | I_i, M_i \rangle|^2$$

Applications in Nuclear Astrophysics

✔ Three-body radiative capture. The A=5 and A=8 gaps.

$$\alpha + \alpha + \alpha \rightarrow {}^{12}\mathrm{C} + \gamma$$

$$\alpha + n + n \rightarrow {}^{6}\mathrm{He} + \gamma$$

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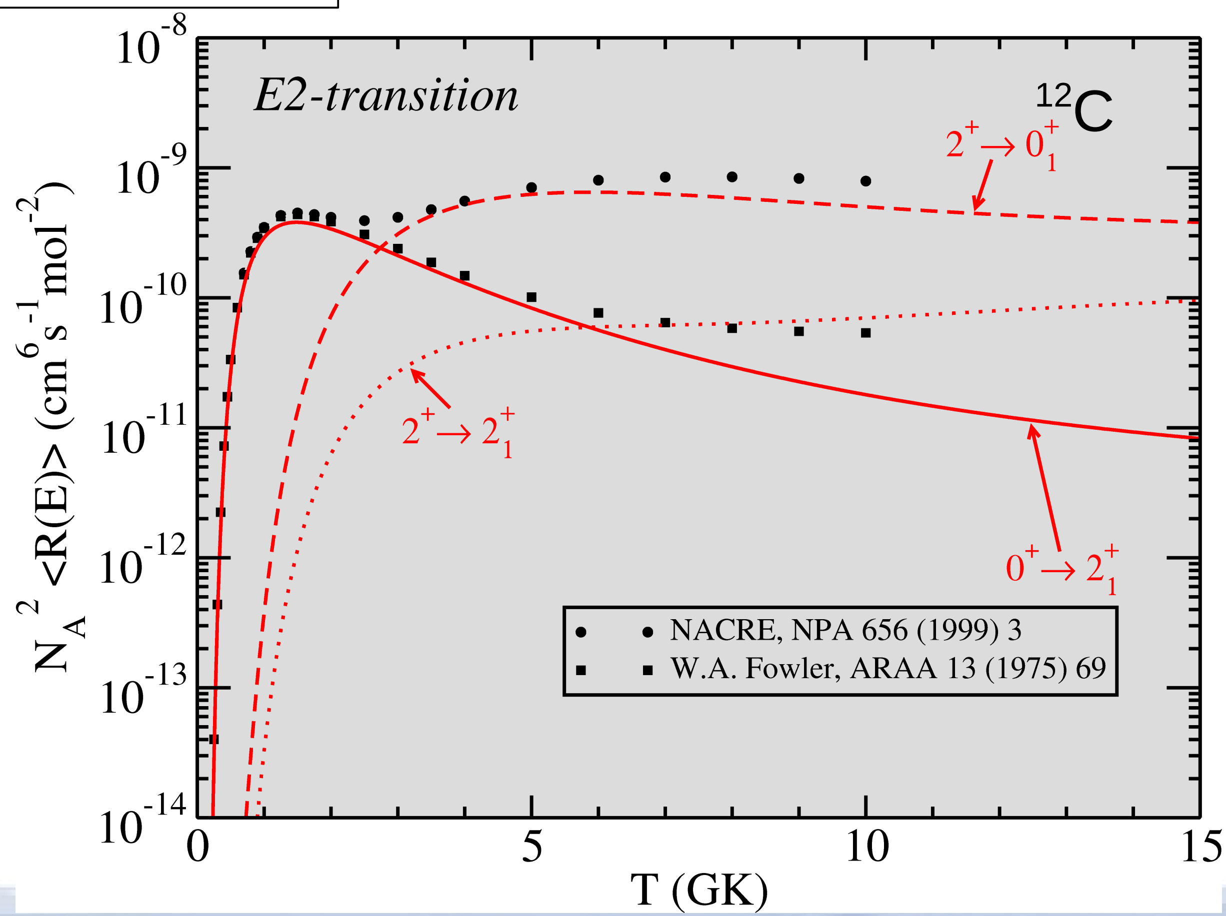
What is the production rate for the different reactions in the stellar medium??

$$P(\rho, T) = n_a n_b n_c R(T)$$



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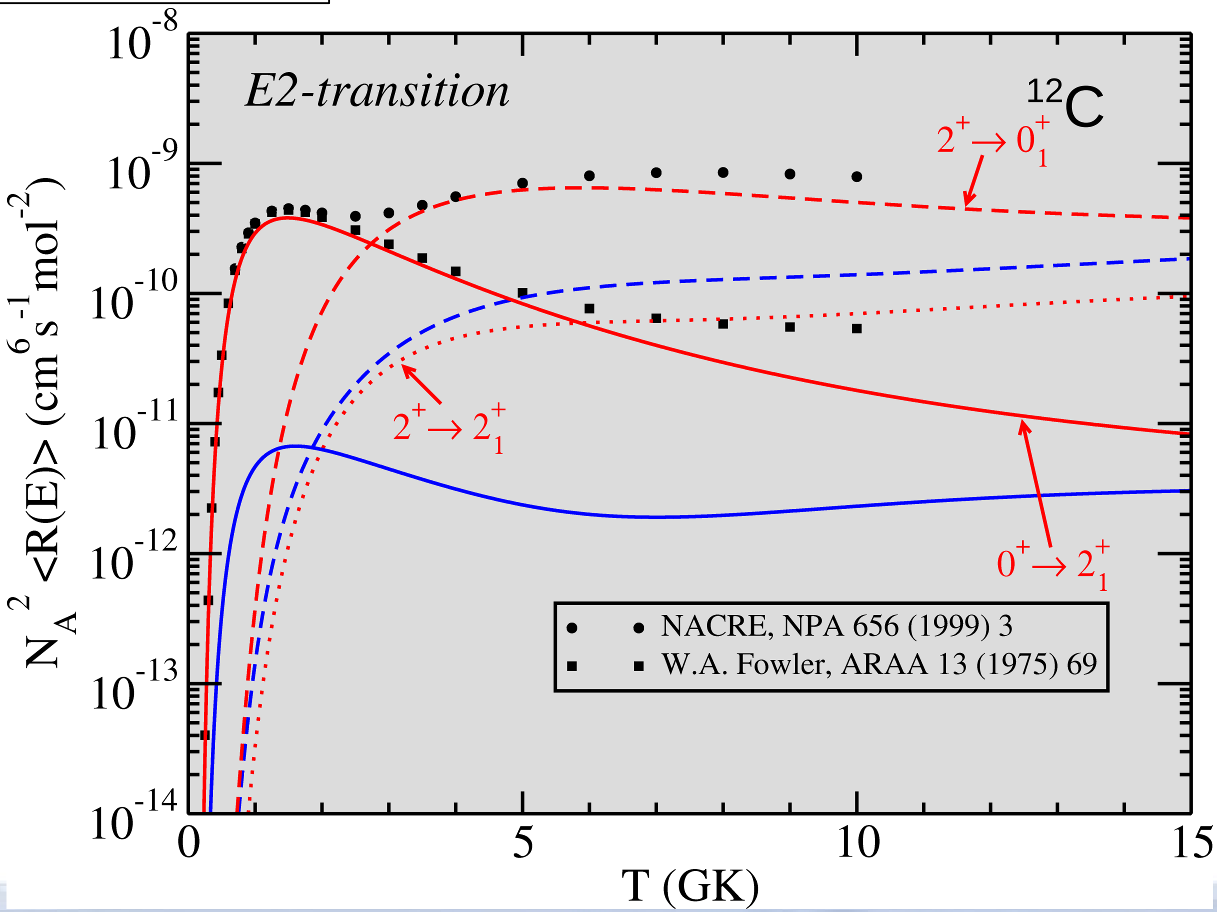

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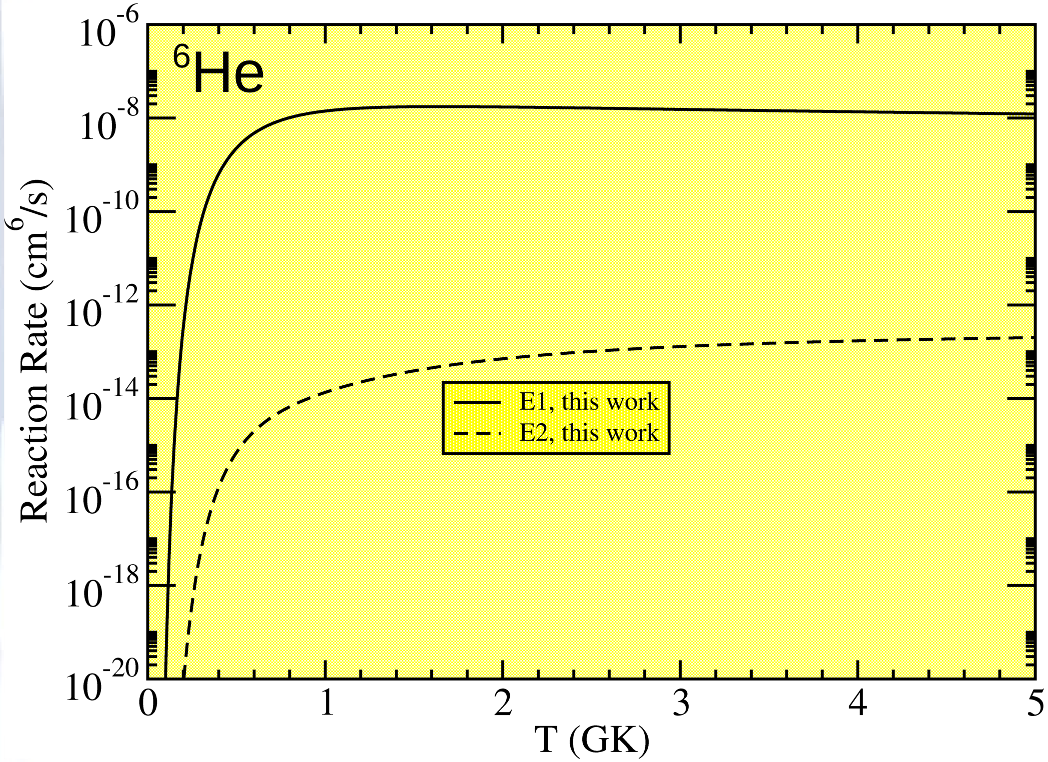

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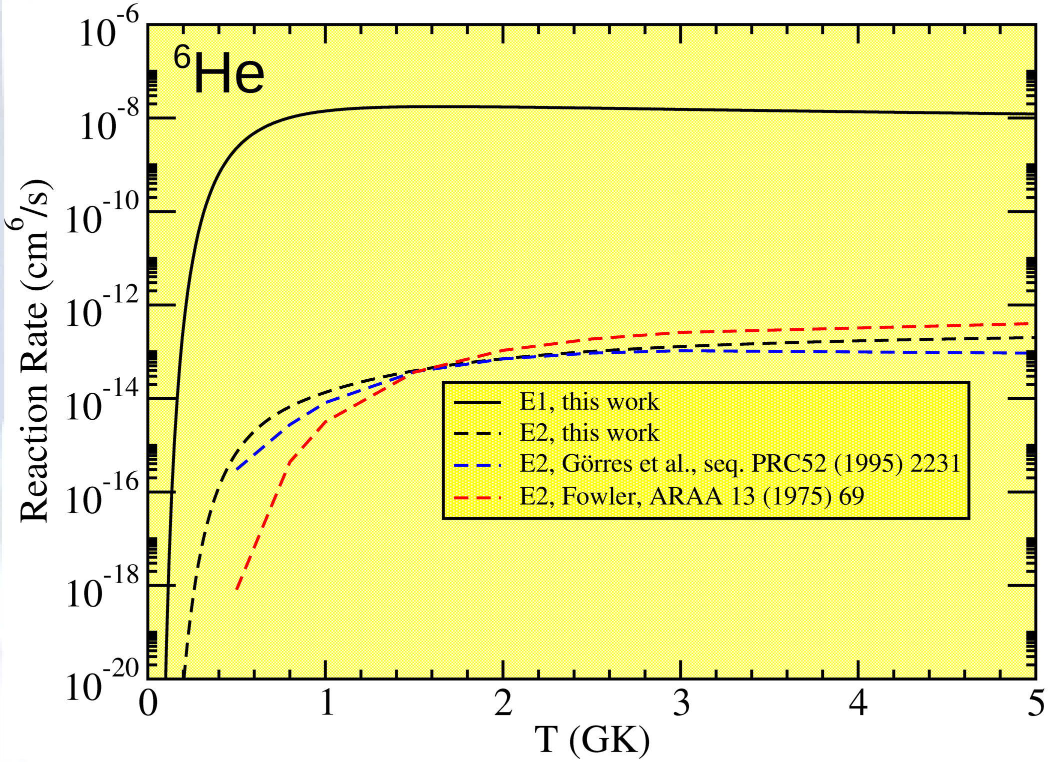

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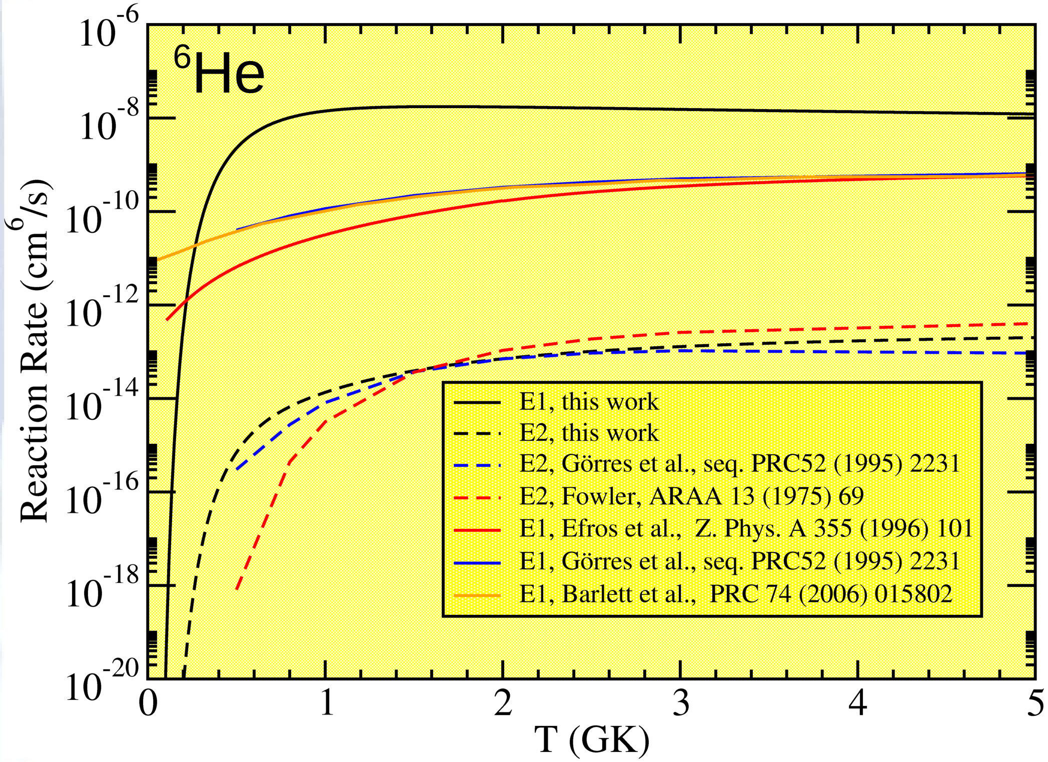

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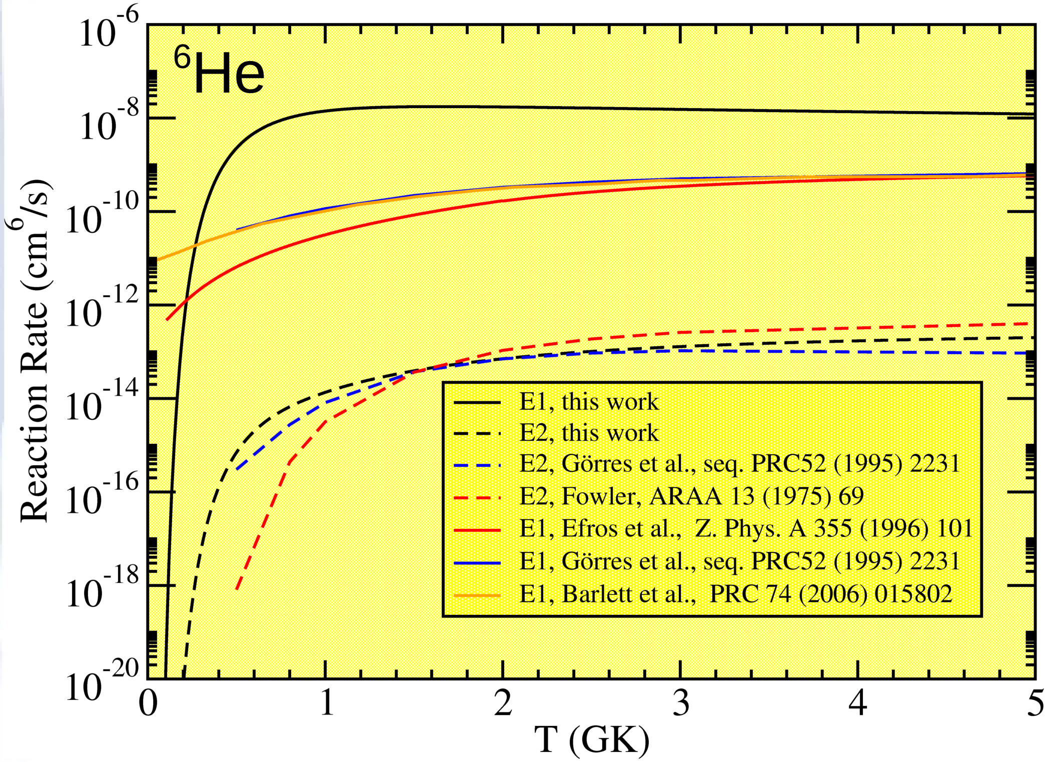

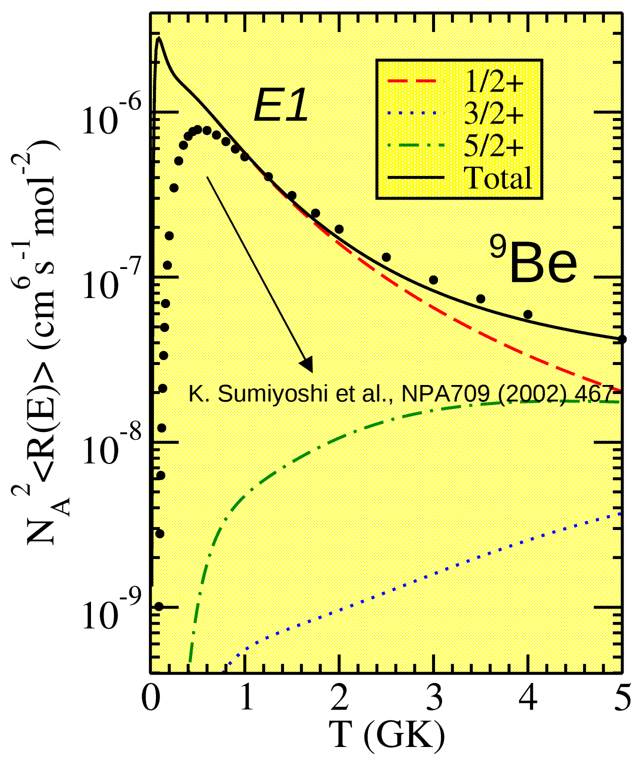

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What is the production rate for the different reactions in the stellar medium??

$$P(\rho, T) = n_a n_b n_c R(T)$$

$$n_i = \rho N_A \frac{X_i}{A_i} \quad \begin{cases} \rho \rightarrow \text{mass density} \\ X_i = \dfrac{N_i m_i}{\sum_j N_j m_j}; Y_i = \dfrac{N_i}{\sum_j N_j} \end{cases}$$

Applications in Nuclear Astrophysics

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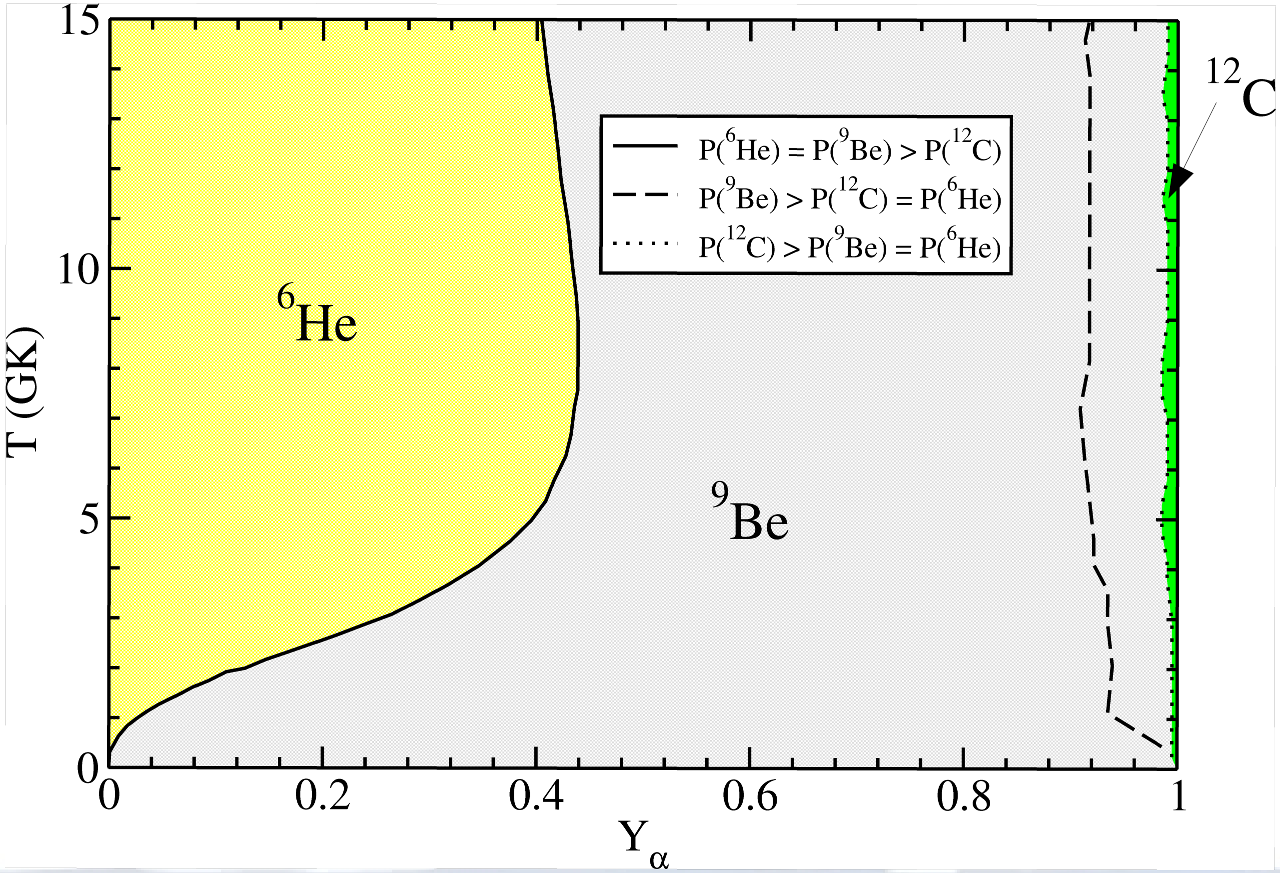
What is the production rate for the different reactions in the stellar medium??

$$P(E)=n_a n_b n_c R(E);\, n_i=\rho N_A \frac{X_i}{A_i}$$



Three-body Resonances

✔ Hyperspherical Adiabatic Expansion Method

✗**Bound states:** Fast convergence in terms of adiabatic channels

✗**Scattering states:** Clean distinction between different open channels

Also fast convergence when combined with integral relations

✗**Resonances:** As for bound states when combined with complex scaling

Three-body Resonances and Applications in Nuclear Astrophysics

Three-body Resonances

- ✔ Hyperspherical Adiabatic Expansion Method
- ✔ Resonances and the Complex Scaling Method
- ✔ Decay of three-body resonances and Energy Distributions

Applications in Nuclear Astrophysics

- ✔ Three-body radiative capture
  - Sequential
  - Direct
- ✔ A=5 and A=8 gaps
  - Production of ^{12}C, ^{6}He, ^{9}Be

THE END

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