Momentum-space techniques for reactions involving weakly bound nuclei

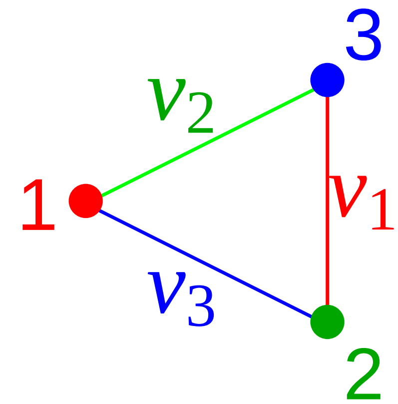
A. Deltuva

Centro de Física Nuclear da Universidade de Lisboa

Few-body scattering

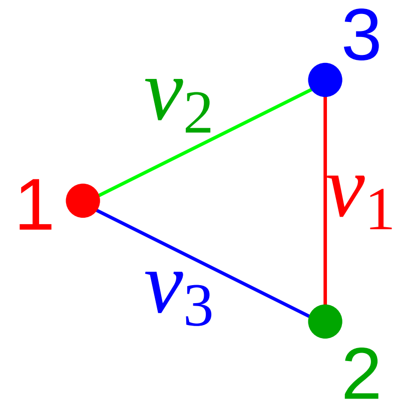
- Three-body scattering equations
- Inclusion of the Coulomb interaction
- Three-nucleon system
- Three-body direct nuclear reactions
- Four-nucleon scattering
- Extension to atomic physics

Three-body system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-body system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

- Faddeev equations

$$(E - H_0 - v_{\alpha})|\psi_{\alpha}\rangle = v_{\alpha} \sum_{\sigma} \bar{\delta}_{\alpha\sigma} |\psi_{\sigma}\rangle$$

$$|\Psi\rangle = \sum_{\alpha} |\psi_{\alpha}\rangle$$

Alt-Grassberger-Sandhas equations

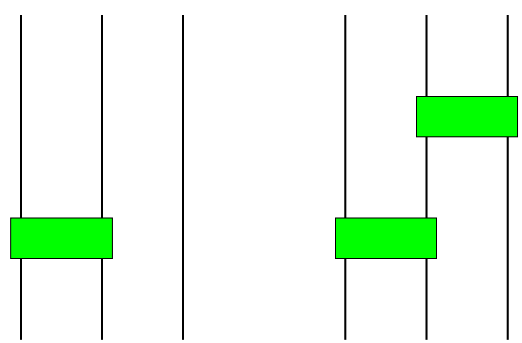
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\sigma \bar{\delta}_{\beta\sigma} T_\sigma G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_\sigma T_\sigma G_0 U_{\sigma\alpha}$$

$$T_\sigma = v_\sigma + v_\sigma G_0 T_\sigma$$

$$G_0 = (E + i0 - H_0)^{-1}$$

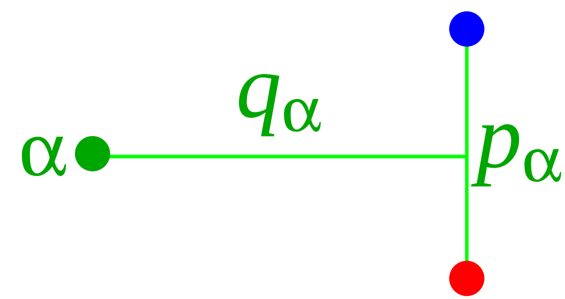
channel states $(E - H_0 - v_\alpha)|\phi_\alpha\rangle = 0$



...

AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$



- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Gaussian integration, spline interpolation, Padé summation

Inclusion of Coulomb: screening

$$w_R(r) = \frac{\alpha_e}{r}\, e^{-\left(\frac{r}{R}\right)^n}$$

- standard scattering theory

$$v_\sigma \longrightarrow v_\sigma + w_{\sigma R}: \quad T_\sigma,\, U_{\beta\alpha} \longrightarrow T_\sigma^{(R)},\, U_{\beta\alpha}^{(R)}$$

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- standard scattering theory

  $$v_\sigma \longrightarrow v_\sigma + w_{\sigma R}: \quad T_\sigma,\, U_{\beta\alpha} \longrightarrow T_\sigma^{(R)},\, U_{\beta\alpha}^{(R)}$$

- nature: Coulomb is screened at large distances
- large R:
  physical observables insensitive to screening,
  screened and full Coulomb physically indistinguishable

Screening and renormalization

J. R. Taylor, Nuovo Cimento **B23**, *313 (1974),*
V. G. Gorshkov, Sov. Phys.-JETP **13**, *1037 (1961)*:

Screening and renormalization

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Renormalization of the on-shell screened Coulomb transition matrix $T_R = w_R + w_R G_0 T_R$ and wave function

$$T_R z_R^{-1} \xrightarrow[R\to\infty]{} T_C \quad \text{as distribution}$$

$$(1 + G_0 T_R)|\mathbf{p}\rangle z_R^{-\frac{1}{2}} \xrightarrow[R\to\infty]{} |\psi_C^{(+)}(\mathbf{p})\rangle$$

in the limit $R \to \infty$ yields Coulomb amplitude
and Coulomb wave function

$$z_R \xrightarrow[R\to\infty]{} \exp(-2i(\sigma_L - \eta_{LR})) \xrightarrow[R\to\infty]{} \exp(-2i\alpha_e M/p\,[\ln(2pR) - C/n])$$

Two-particle scattering

transition matrix

$$T^{(R)} = v + w_R + (v + w_R) G_0 T^{(R)}$$

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with long-range and Coulomb-distorted short-range parts

$$T^{(R)} = T_R + (1 + T_R G_0) \tilde{T}^{(R)} (1 + G_0 T_R)$$

$$\tilde{T}^{(R)} = v + v G_R \tilde{T}^{(R)}$$

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Renormalized amplitude:

$$T^{(R)} z_R^{-1} \xrightarrow[R \to \infty]{} T = T_C + \langle \psi_C^{(-)} | \tilde{T}^{(C)} | \psi_C^{(+)} \rangle$$

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$$= T_C + \lim_{R\to\infty} z_R^{-\frac{1}{2}} [T^{(R)} - T_R] z_R^{-\frac{1}{2}}$$

short-range part: fast convergence with R

Three-particle scattering

long-range part

$$T^{\text{c.m.}}_{\alpha R} = W^{\text{c.m.}}_{\alpha R} + W^{\text{c.m.}}_{\alpha R} G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}$$

Three-particle scattering

Split into long-range part

$$T^{\mathrm{c.m.}}_{\alpha R} = W^{\mathrm{c.m.}}_{\alpha R} + W^{\mathrm{c.m.}}_{\alpha R} G^{(R)}_{\alpha} T^{\mathrm{c.m.}}_{\alpha R}$$

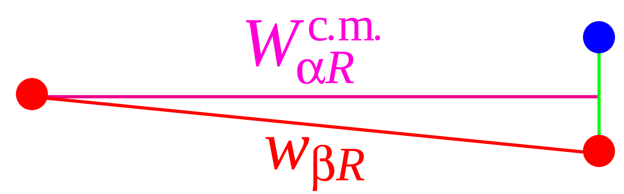
and Coulomb-distorted short-range part

$$U^{(R)}_{\beta\alpha} = \delta_{\beta\alpha} T^{\mathrm{c.m.}}_{\alpha R} + [U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{\mathrm{c.m.}}_{\alpha R}]$$

$$[U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{\mathrm{c.m.}}_{\alpha R}] = [1 + T^{\mathrm{c.m.}}_{\beta R} G^{(R)}_{\beta}] \tilde{U}^{(R)}_{\beta\alpha} [1 + G^{(R)}_{\alpha} T^{\mathrm{c.m.}}_{\alpha R}]$$

$$U^{(R)}_{0\alpha} = [1 + T_{\rho R} G_0] \tilde{U}^{(R)}_{0\alpha} [1 + G^{(R)}_{\alpha} T^{\mathrm{c.m.}}_{\alpha R}]$$ [ρ is neutral]

Three-particle scattering



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$$U^{(R)}_{0\alpha} = [1 + T_{\rho R} G_0] \tilde{U}^{(R)}_{0\alpha} [1 + G^{(R)}_{\alpha} T^{\text{c.m.}}_{\alpha R}] \quad [\rho \text{ is neutral}]$$

Renormalized amplitudes:

$$U_{\beta\alpha} = \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha C} + \lim_{R\to\infty} Z_{R\,f}^{-\frac{1}{2}} [U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha R}] Z_{R\,i}^{-\frac{1}{2}}$$

$$U_{0\alpha} = \lim_{R\to\infty} z_R^{-\frac{1}{2}} U^{(R)}_{0\alpha} Z_{R\,i}^{-\frac{1}{2}}$$

fast convergence with R

Practical realization

- Calculation of short-range part using standard scattering theory (Faddeev/AGS) for nuclear + screened Coulomb interaction

$$U_{\beta\alpha}^{(R)} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\sigma \bar{\delta}_{\beta\sigma} T_\sigma^{(R)} G_0 U_{\sigma\alpha}^{(R)}$$

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+ renormalization

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- Additional difficulties:
quasi-singular nature of screened Coulomb potential
slow partial-wave convergence

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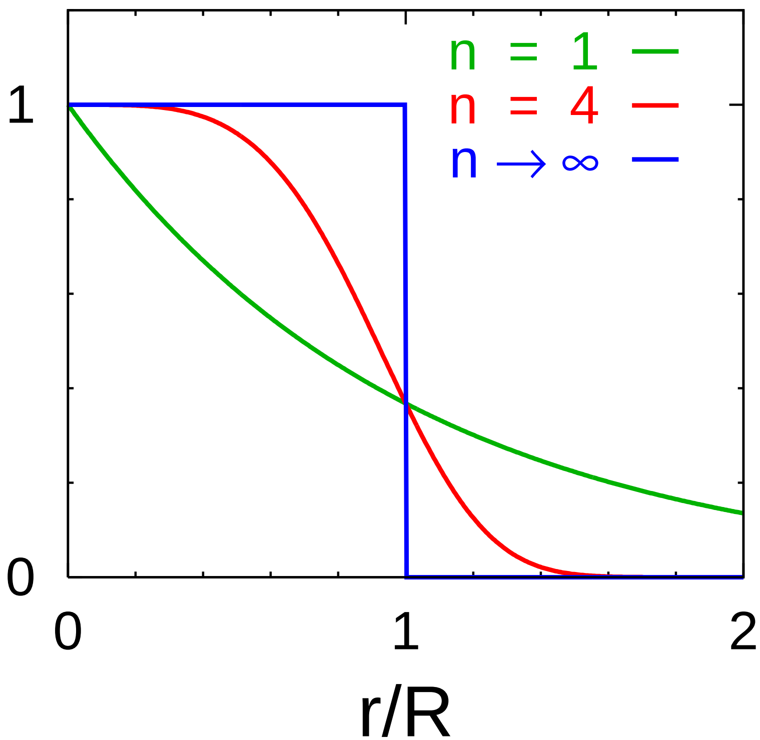
- Success of the method depends strongly on the choice of screening function

Screened Coulomb potential

$$\frac{w_R(r)}{w(r)} = e^{-\left(\frac{r}{R}\right)^{\mathrm{n}}}$$



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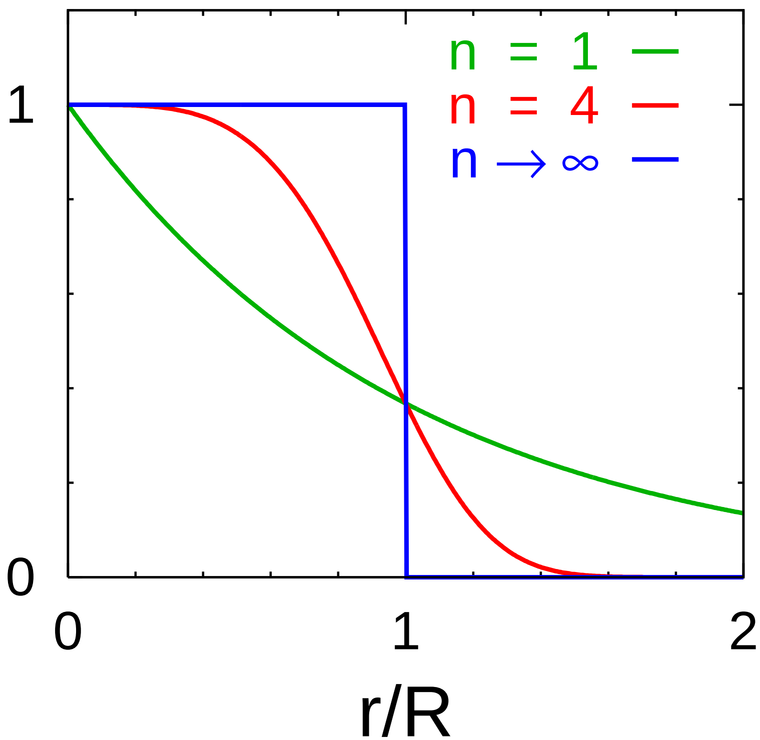

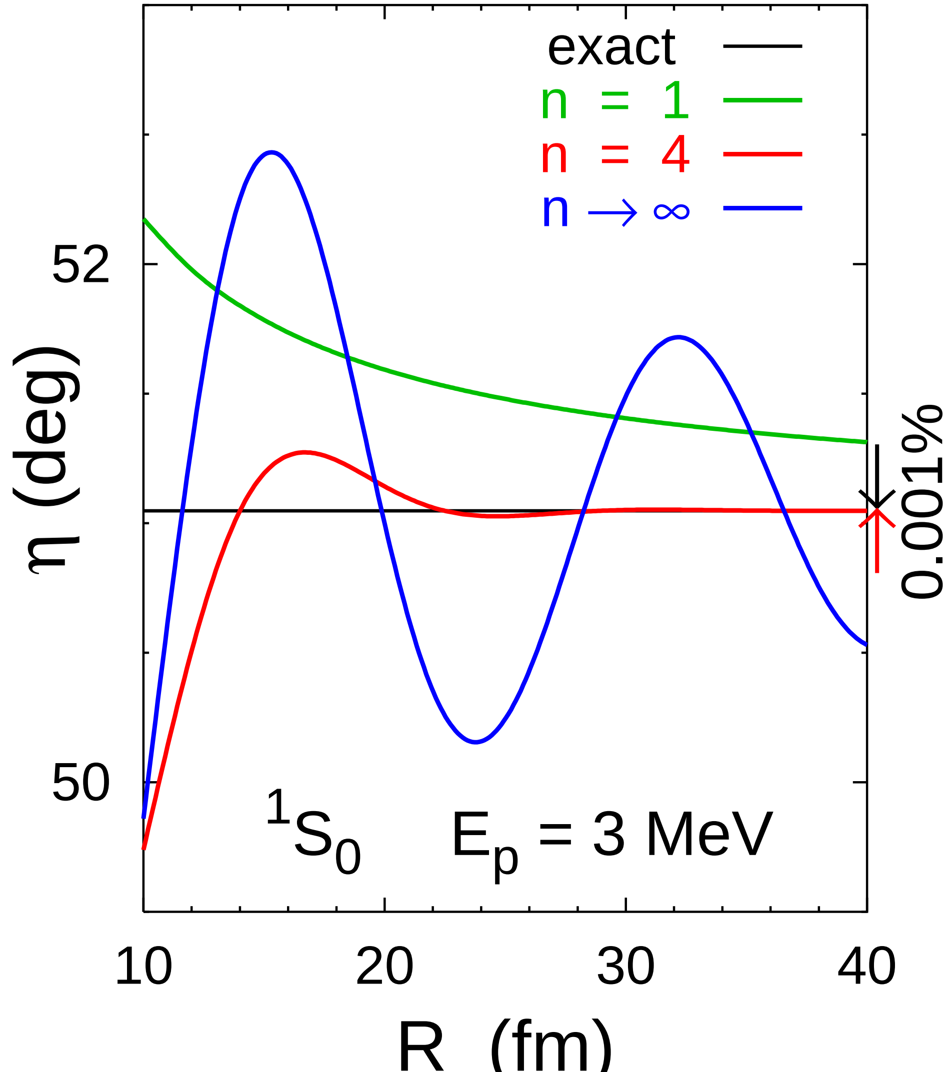

optimal choice: $3 \leq n \leq 8$

Limits of practical applicability

$p \longrightarrow 0$:
$\kappa = \alpha M/p$, $\sigma_L = \arg\Gamma(1+L+i\kappa)$, and z_R diverge,
renormalization procedure ill-defined

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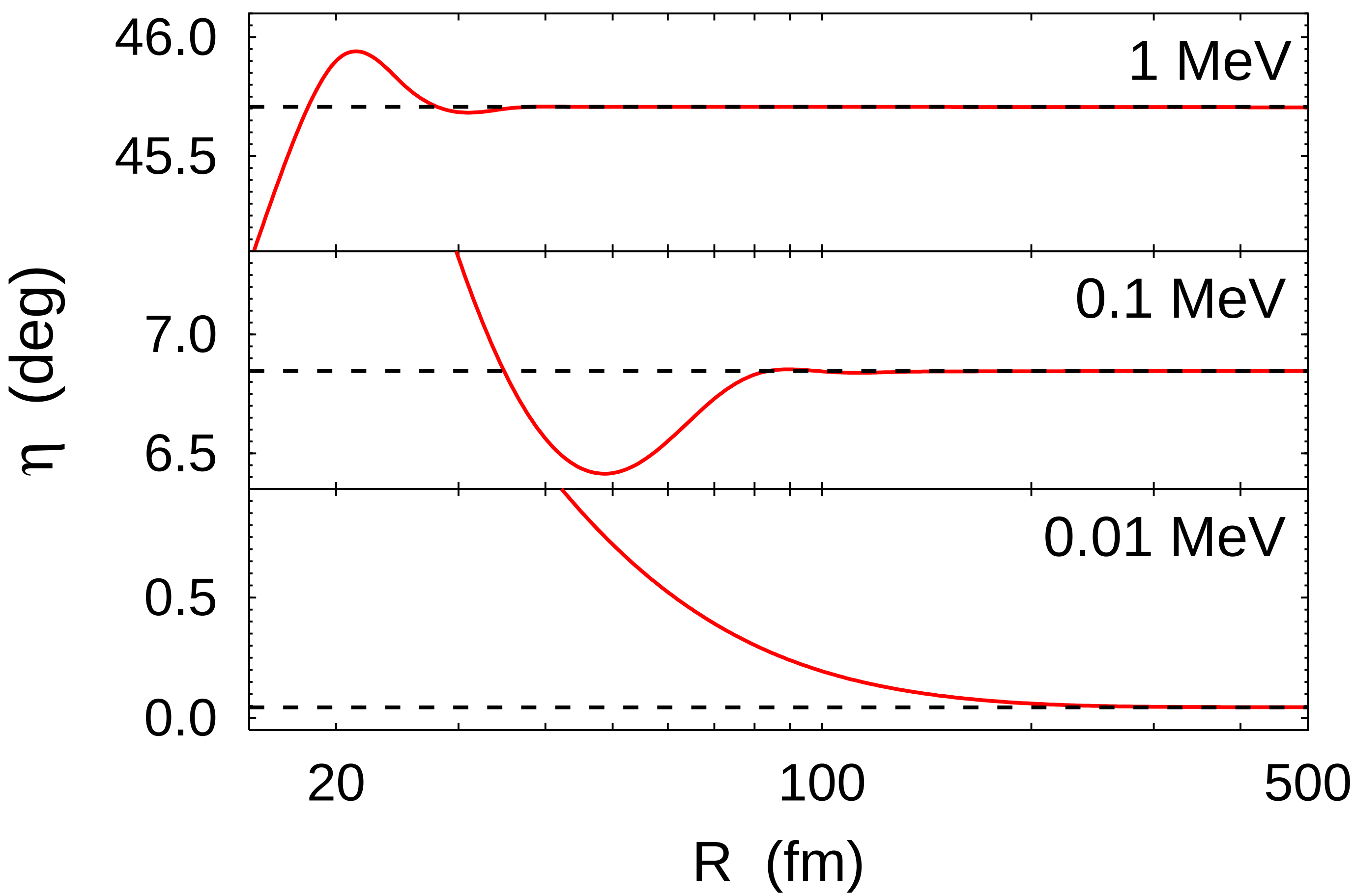
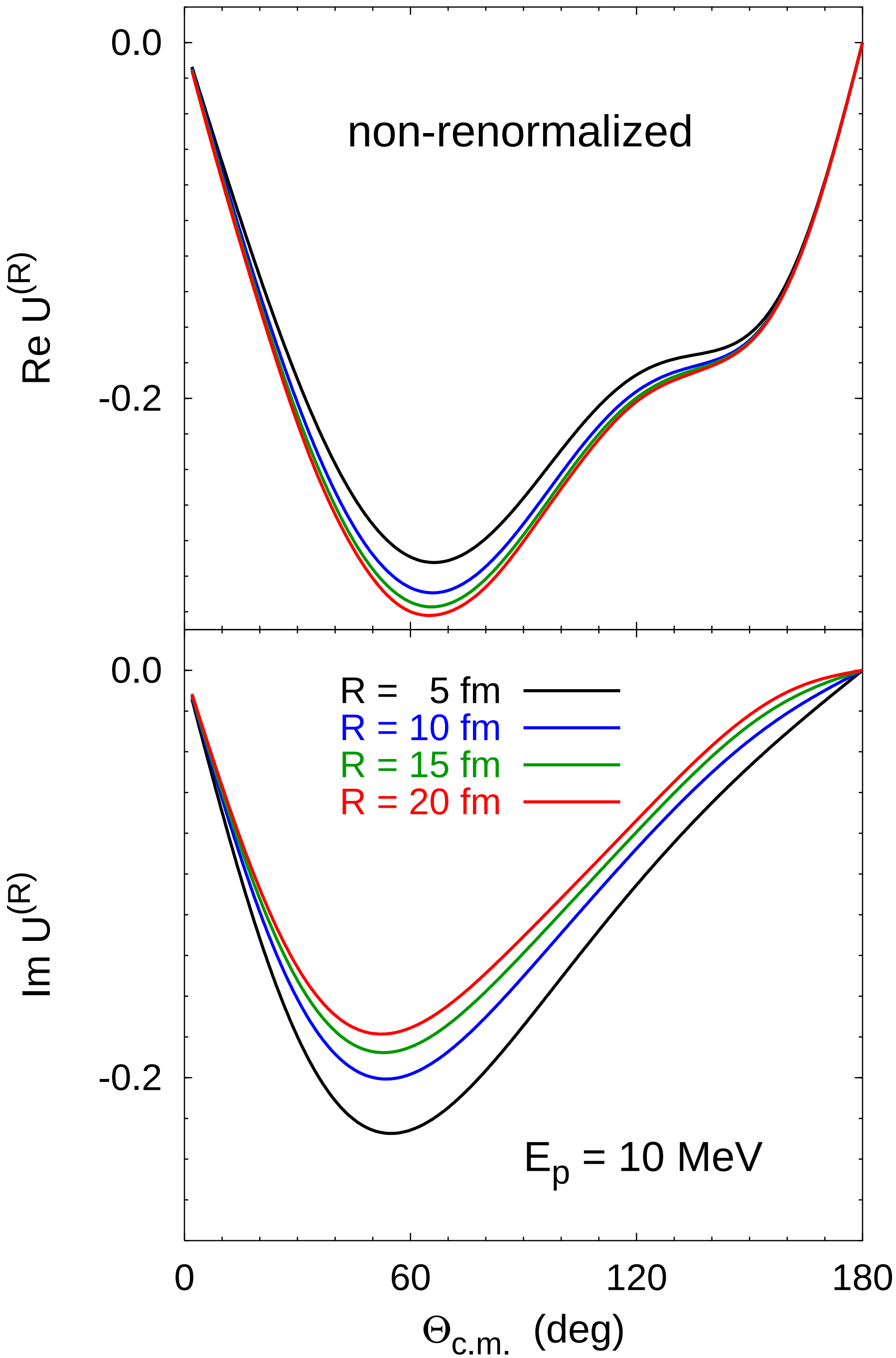
renormalization procedure ill-defined

$\Rightarrow$ slow convergence with R at low relative energies



pd elastic amplitude: convergence with *R*



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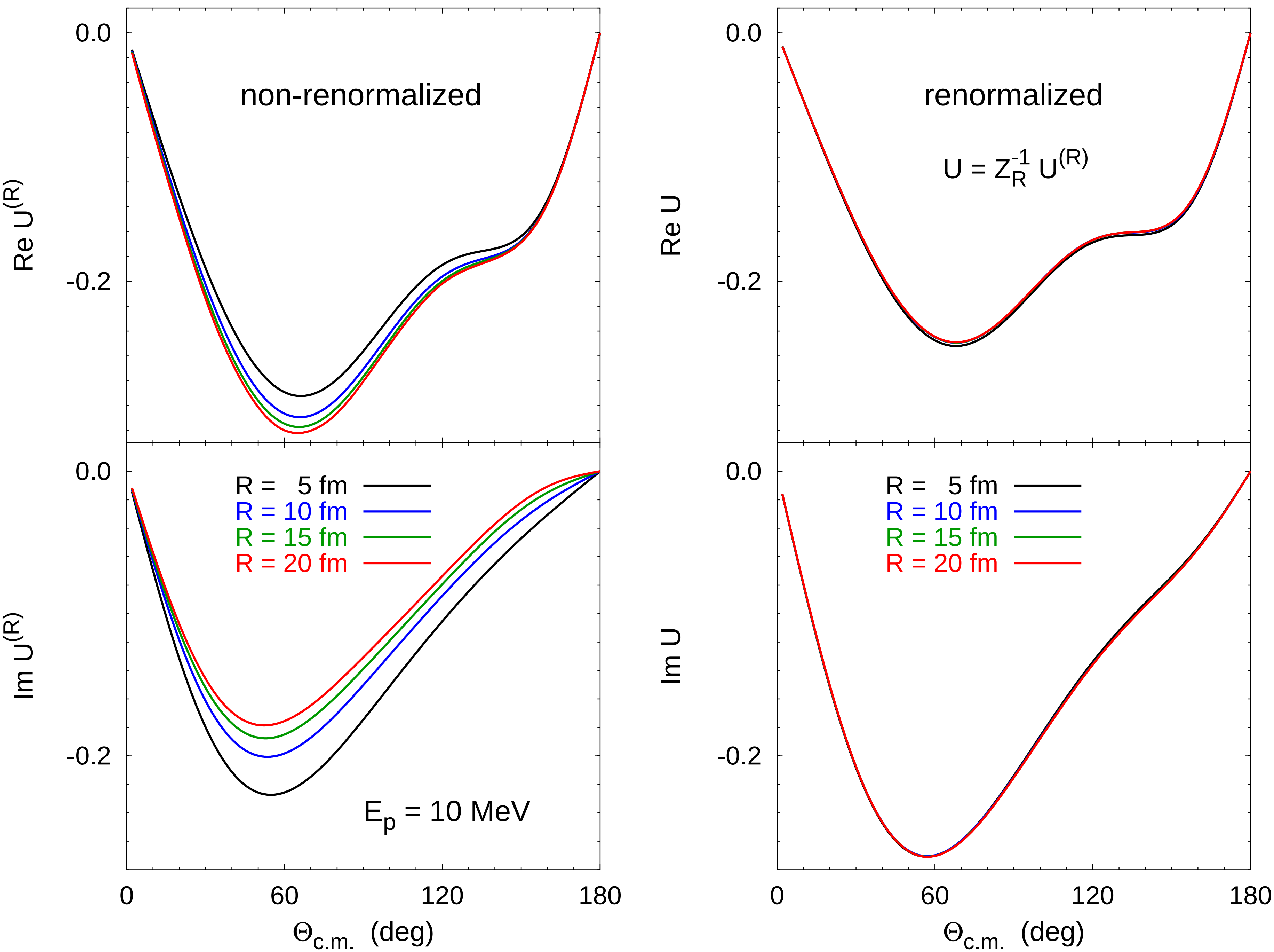
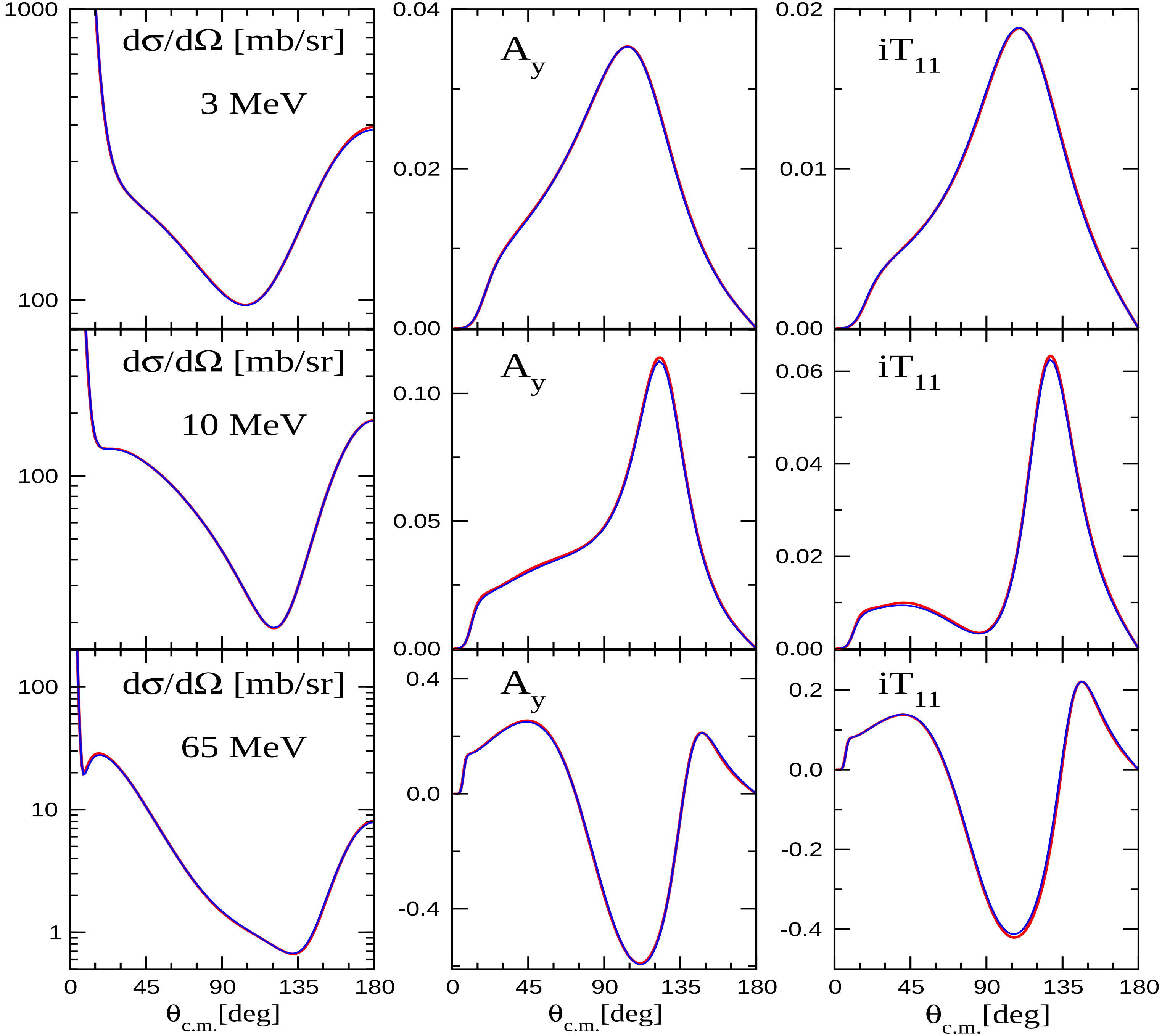


Comparison with configuration-space results

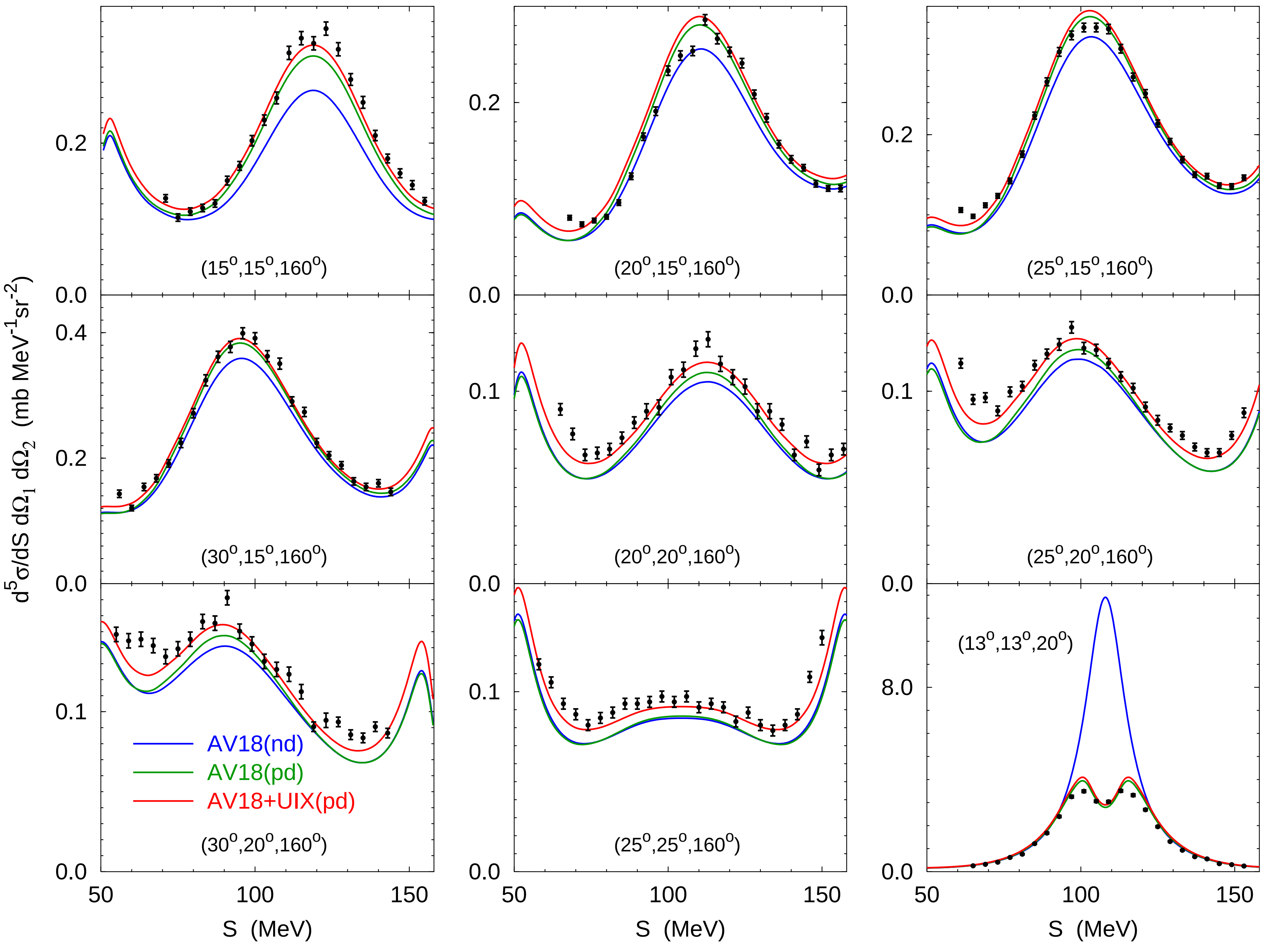


p-*d* elastic scattering:

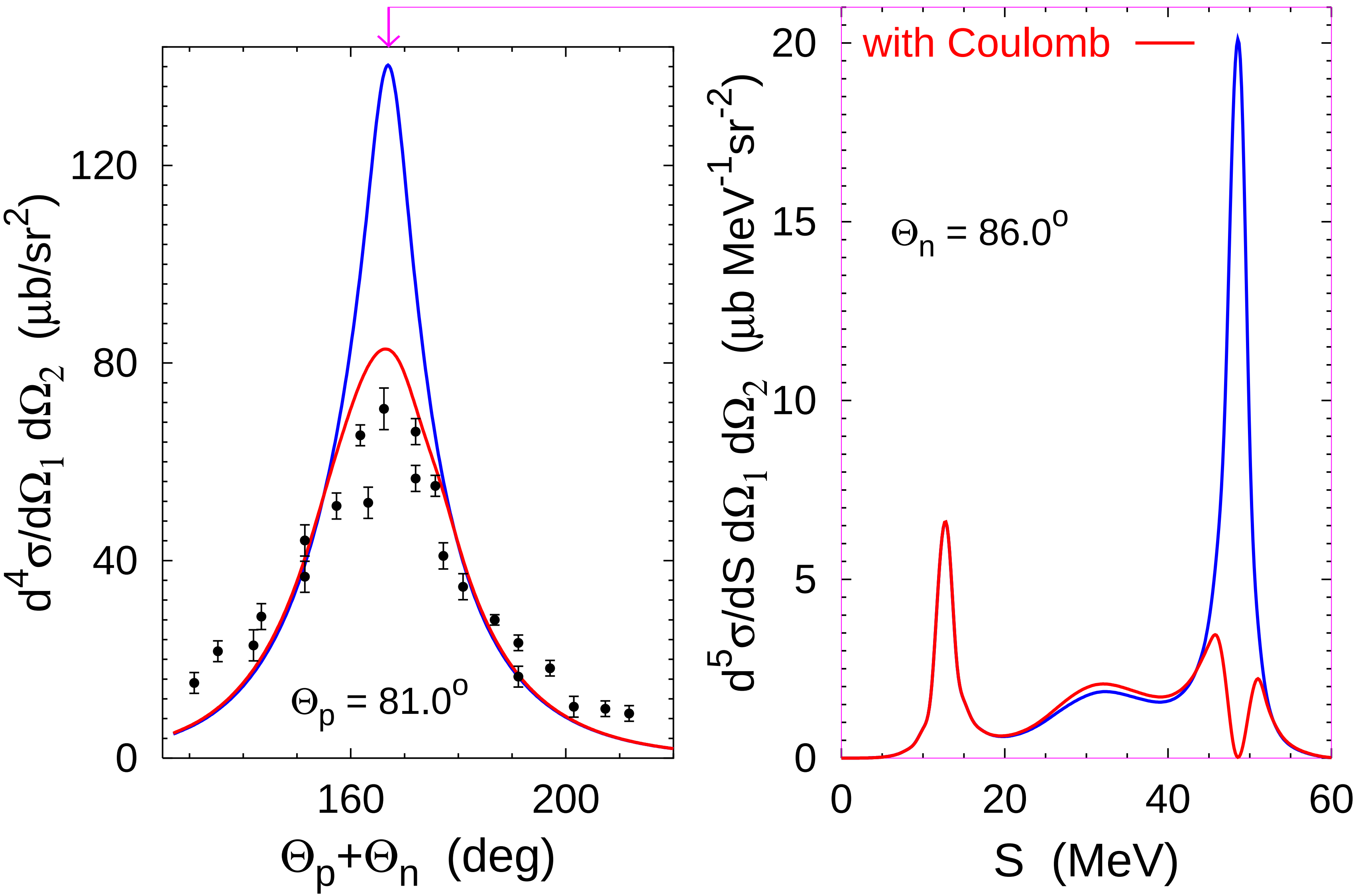
Kohn variational principle (Pisa)

screening and renormalization

Coulomb vs 3NF: ^{1}H(d,pp)n at $E_d = 130$ MeV



^{3}He$(\gamma, pn)p$ at $E_\gamma = 55$ MeV



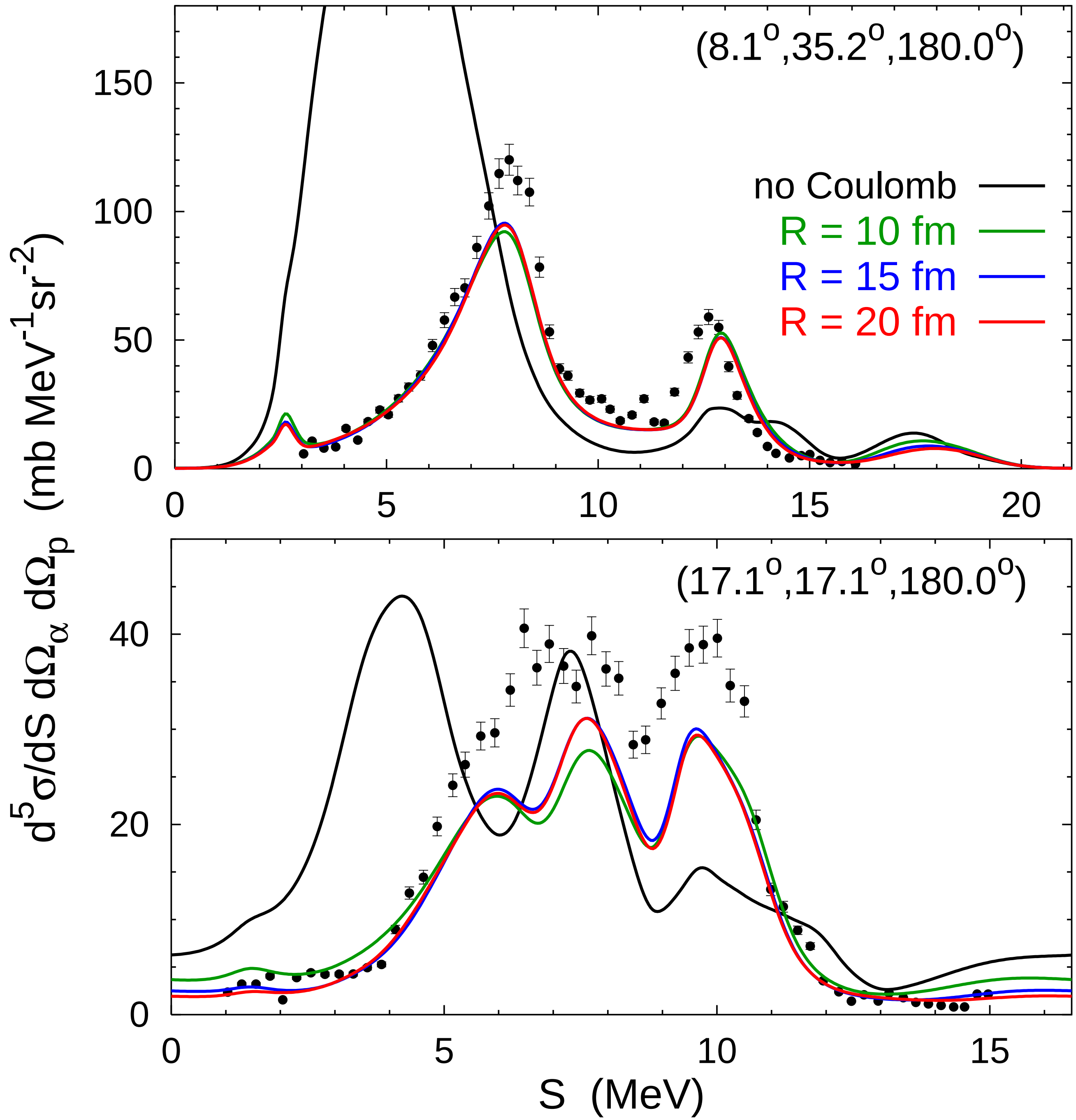
Application to 3-body nuclear reactions

$$\left.\begin{array}{l} p+(nA) \\ d+A \end{array}\right\} \rightarrow \left\{\begin{array}{l} n+(pA) \\ p+(nA) \\ d+A \\ p+n+A \end{array}\right.$$

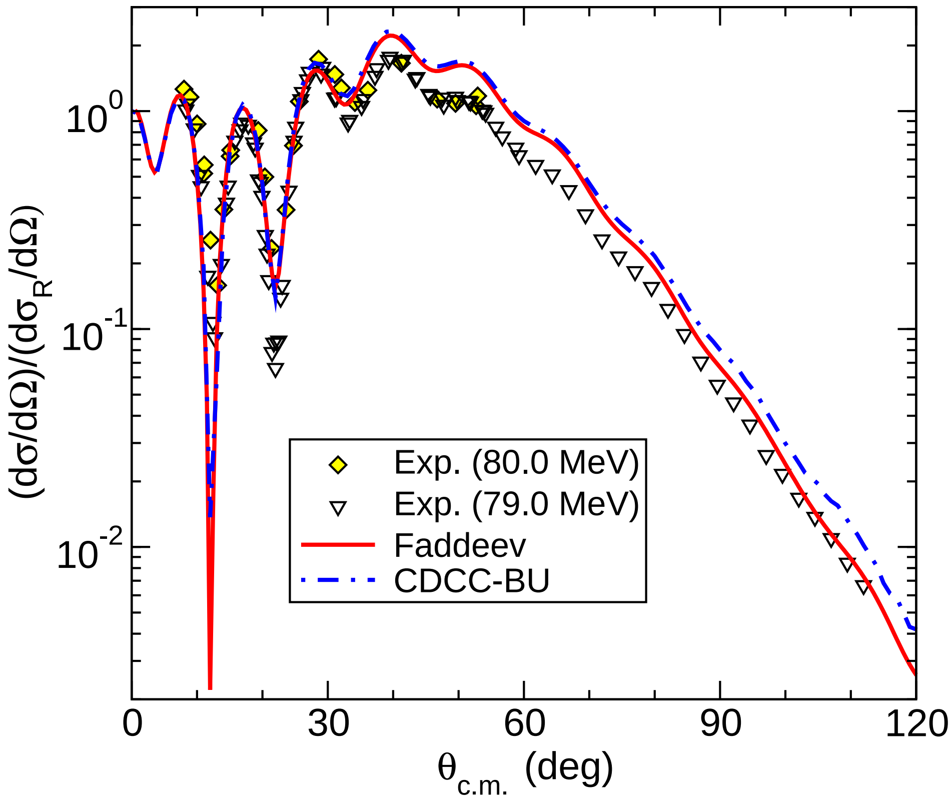
with $A = {}^{4}\mathrm{He},\ {}^{10}\mathrm{Be},\ {}^{12}\mathrm{C},\ {}^{14}\mathrm{C},\ {}^{16}\mathrm{O},\ {}^{28}\mathrm{Si},\ {}^{40}\mathrm{Ca},\ {}^{58}\mathrm{Ni},\ \ldots$

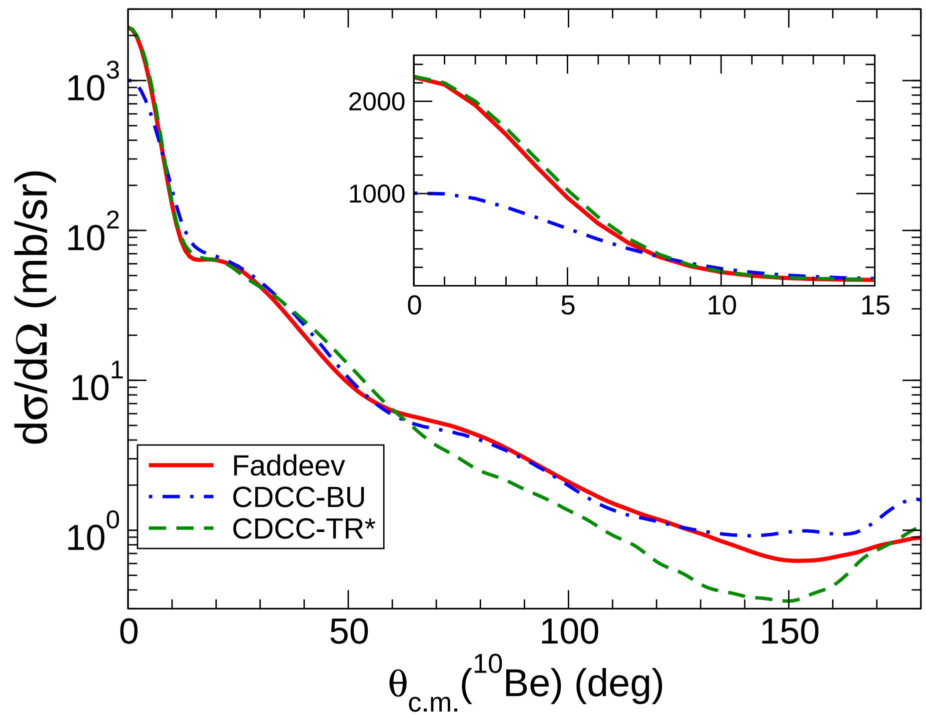
- Validity test of approximate nuclear reaction methods: CDCC, DWBA, Glauber, ...
- Novel dynamic input: nonlocal potentials, ...

α-d breakup at $E_\alpha = 15$ MeV: convergence with R

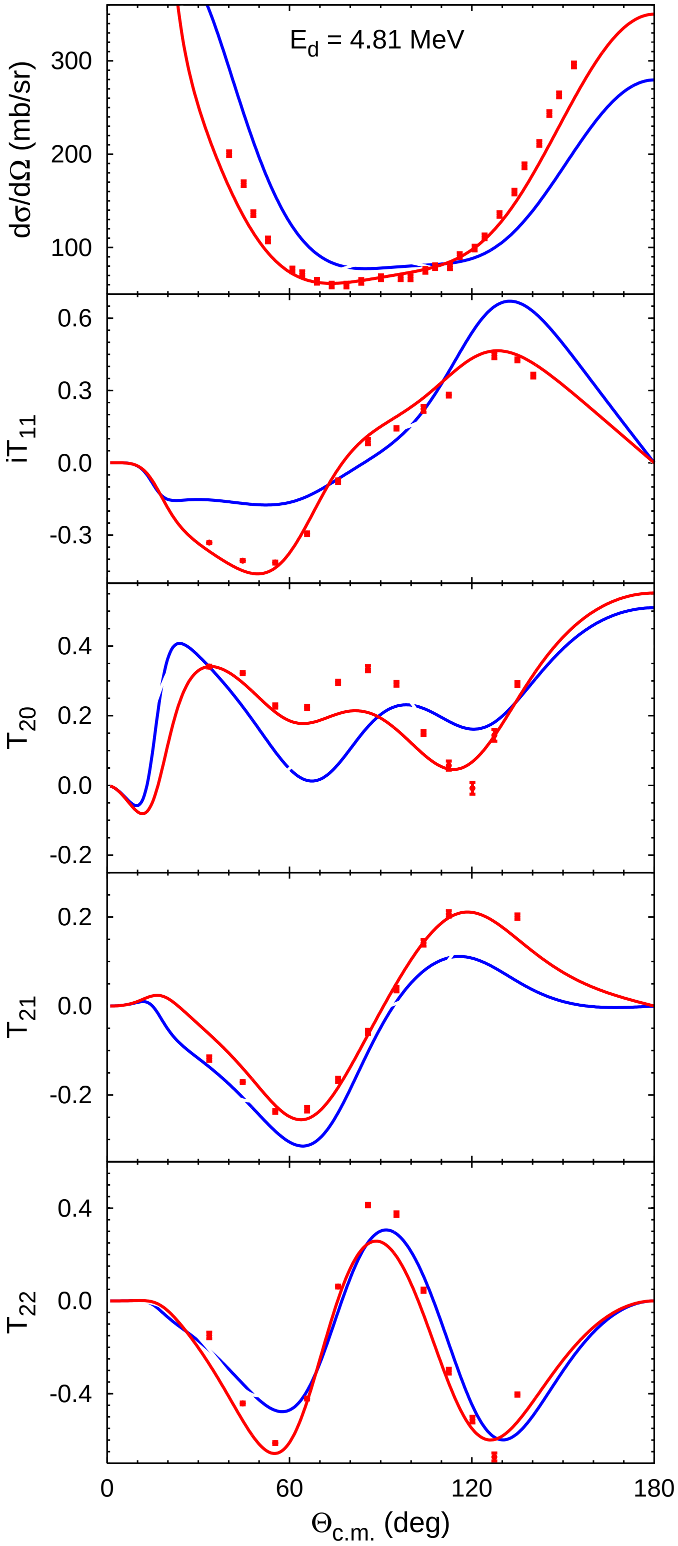


CDCC test: ^{58}Ni$(d,d)^{58}$Ni and ^{1}H$(^{11}$Be$,^{10}$Be$)np$





CDCC: A. M. Moro and F. M. Nunes



$\vec{d} + \alpha$ elastic scattering

N-α S-wave:

$V_{N\alpha} \rightarrow V_{N\alpha} + |b\rangle\Gamma\langle b|$

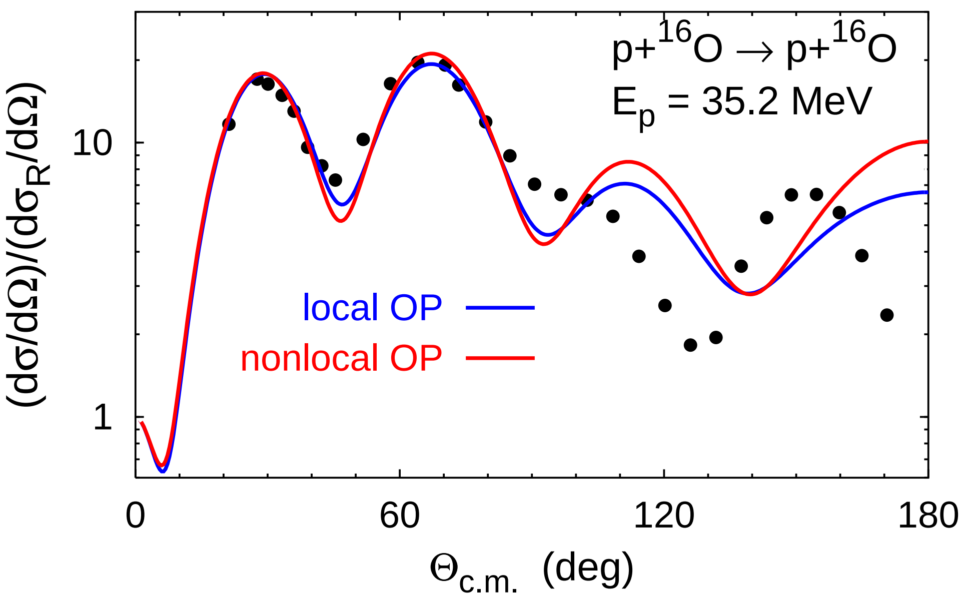
Pauli forbidden bound state projected out

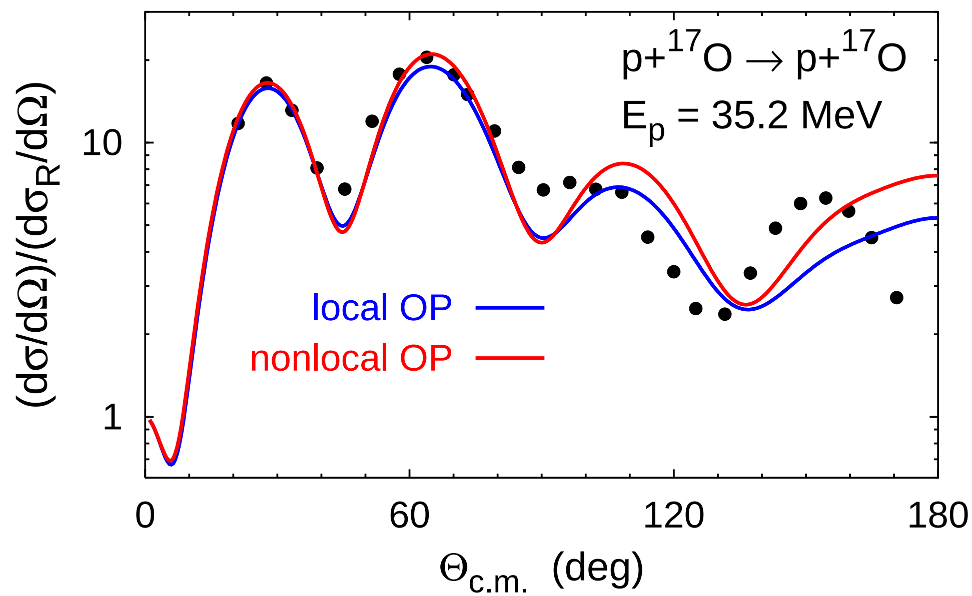
$V_{N\alpha}$ local repulsive

Nonlocal optical potential: proton elastic scattering

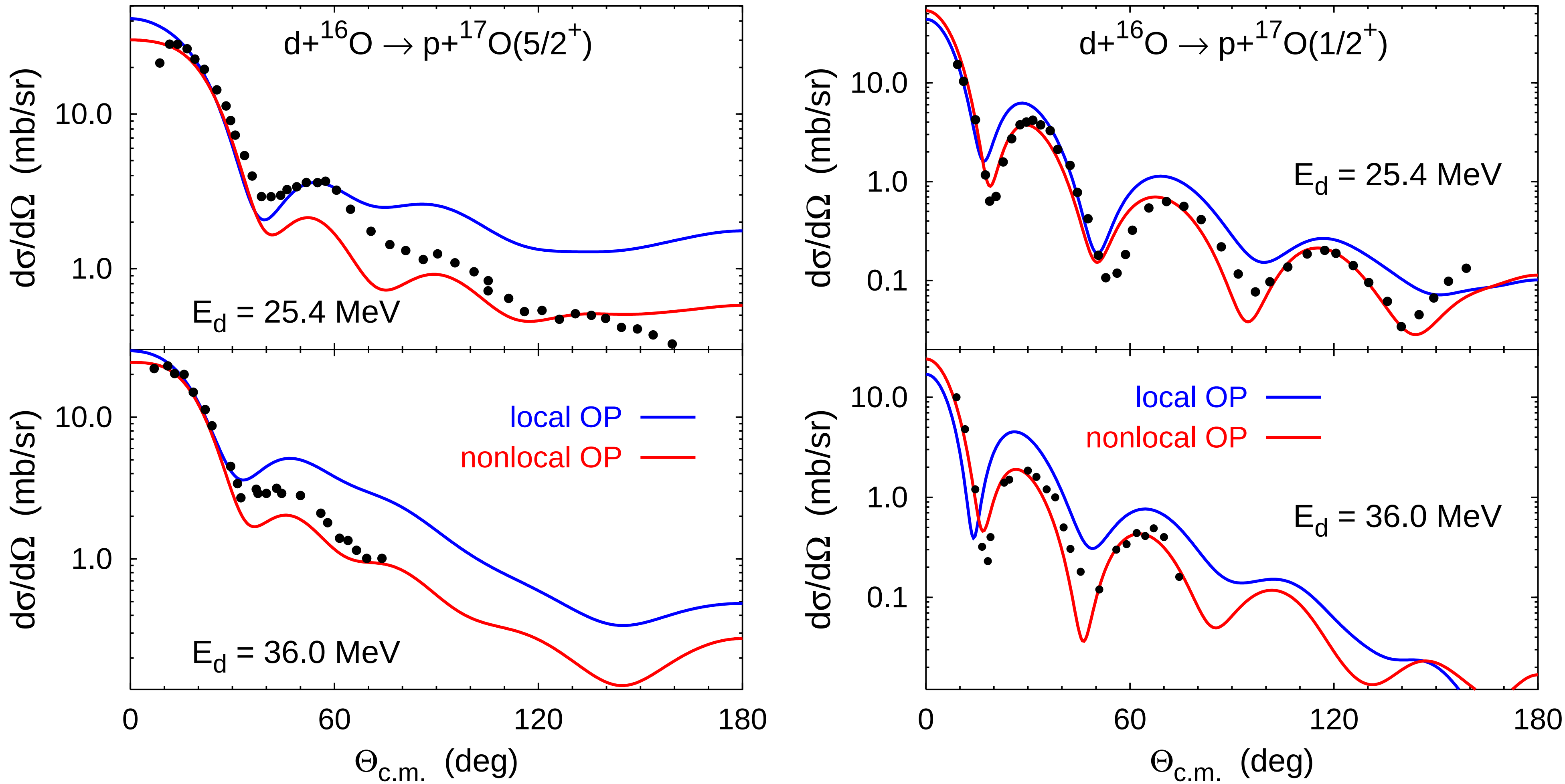
$$V_N(\mathbf{r}',\mathbf{r}) \sim e^{-(\mathbf{r}'-\mathbf{r})^2/\beta^2} V((\mathbf{r}'+\mathbf{r})/2)$$

[M. M. Giannini *et al.*,
Ann. Phys. (NY) 102, 458 (1976) & 124, 208 (1980)]





Nonlocal OP: transfer reactions $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}$



4N scattering: symmetrized AGS equations

two-cluster 1+3 and 2+2 transition operators

$$\mathcal{U}_{11} = -(G_0 T G_0)^{-1} P_{34} - P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{11} + U_2 G_0 T G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 T G_0)^{-1} (1 - P_{34}) + (1 - P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} - P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{12} + U_2 G_0 T G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 - P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{12}$$

$$U_j = P_j G_0^{-1} + P_j T G_0 U_j$$

$$P_1 = P = P_{12} P_{23} + P_{13} P_{23}$$

$$P_2 = \tilde{P} = P_{13} P_{24}$$

$$T = v + v G_0 T$$

scattering amplitude $\quad \mathcal{T}_{fi} = S_{fi} \langle \mathbf{p}_f \phi_f | \mathcal{U}_{fi} | \mathbf{p}_i \phi_i \rangle$

$$|\phi_j\rangle = G_0 T P_j |\phi_j\rangle$$

Screening and renormalization in 4N scattering

$$v \longrightarrow v + w_R$$

$$T,\, U_j,\, \mathcal{U}_{fi},\, \mathcal{T}_{fi} \longrightarrow T^{(R)},\, U_j^{(R)},\, \mathcal{U}_{fi}^{(R)},\, \mathcal{T}_{fi}^{(R)}$$

isolate long-range interaction
and Coulomb distortion between c.m. of two clusters

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$$T,\ U_j,\ \mathcal{U}_{fi},\ \mathcal{T}_{fi} \longrightarrow T^{(R)},\ U_j^{(R)},\ \mathcal{U}_{fi}^{(R)},\ \mathcal{T}_{fi}^{(R)}$$

isolate long-range interaction
and Coulomb distortion between c.m. of two clusters

Renormalization:

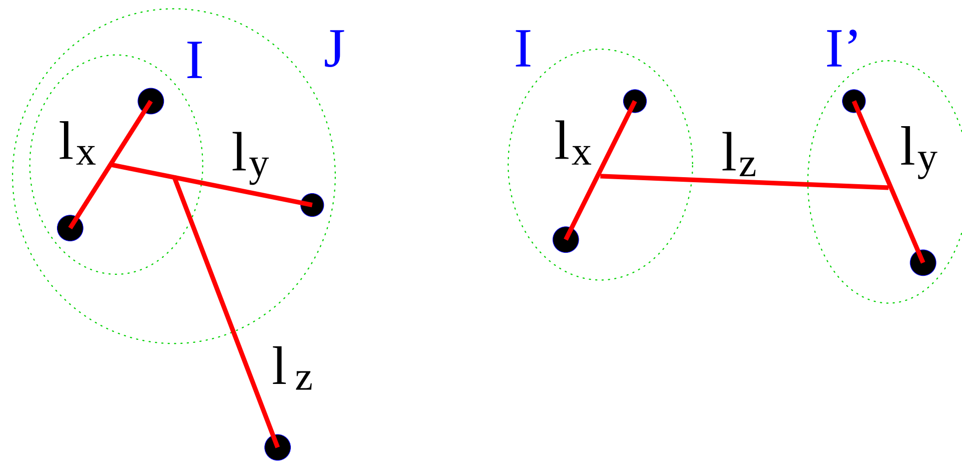
$$\mathcal{T}_{fi} = \lim_{R\to\infty} Z_{Rf}^{-\frac{1}{2}} \mathcal{T}_{fi}^{(R)} Z_{Ri}^{-\frac{1}{2}}$$

$$= \delta_{fi} T_{Ci}^{\text{c.m.}} + \lim_{R\to\infty} Z_{Rf}^{-\frac{1}{2}} [\mathcal{T}_{fi}^{(R)} - \delta_{fi} T_{Ri}^{\text{c.m.}}] Z_{Ri}^{-\frac{1}{2}}$$

Coulomb-distorted short-range part: fast convergence with R

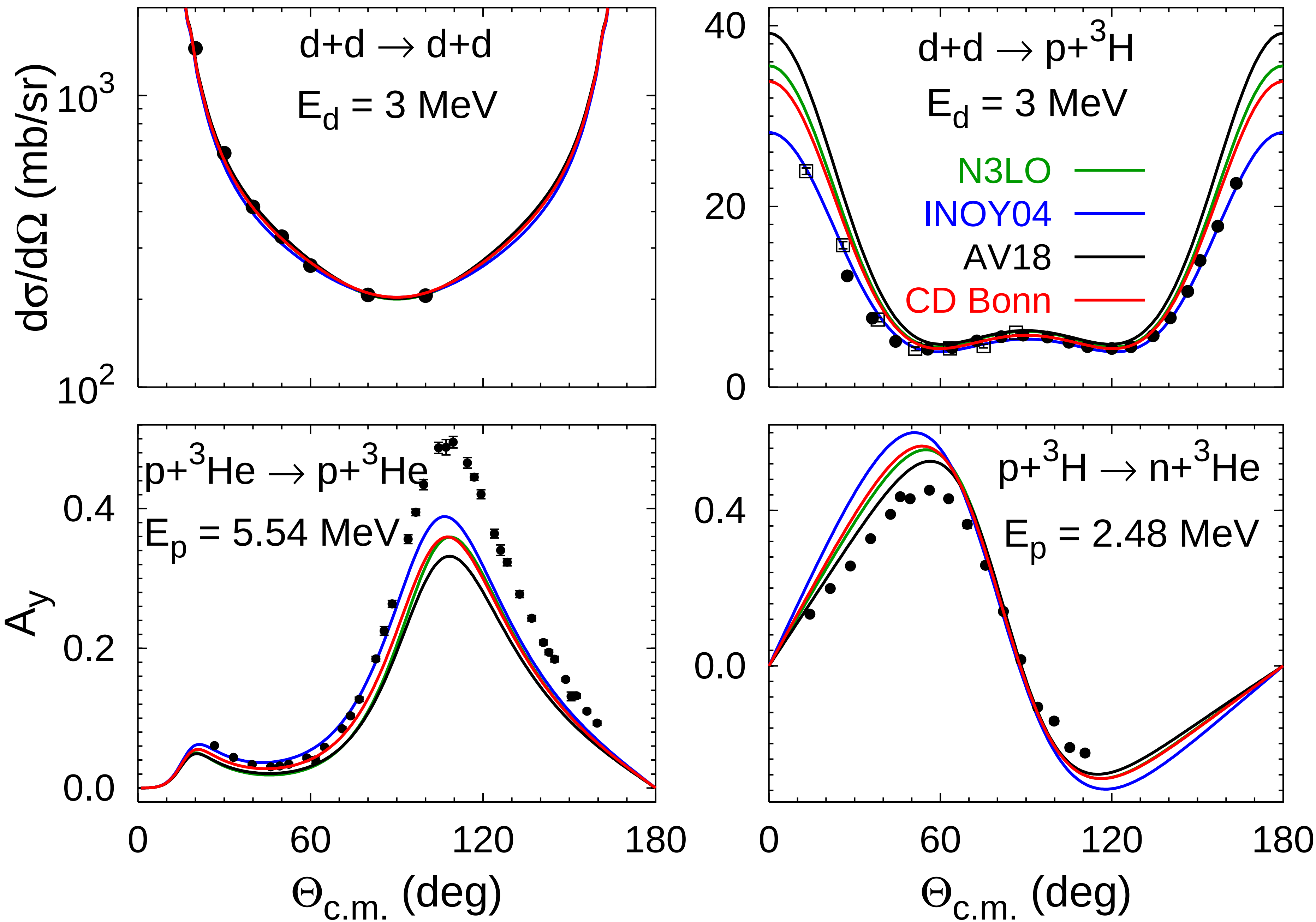
Practical realization

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} - P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{12} + U_2 G_0 T G_0 \, \mathcal{U}_{22}$$



- momentum-space partial-wave basis (up to 30000 partial waves)
- set of coupled integral equations in 3 variables
- integrable singularities in the kernel
- Gaussian integration, spline interpolation, double Padé summation

4N elastic, transfer, and charge exchange reactions



Atomic systems

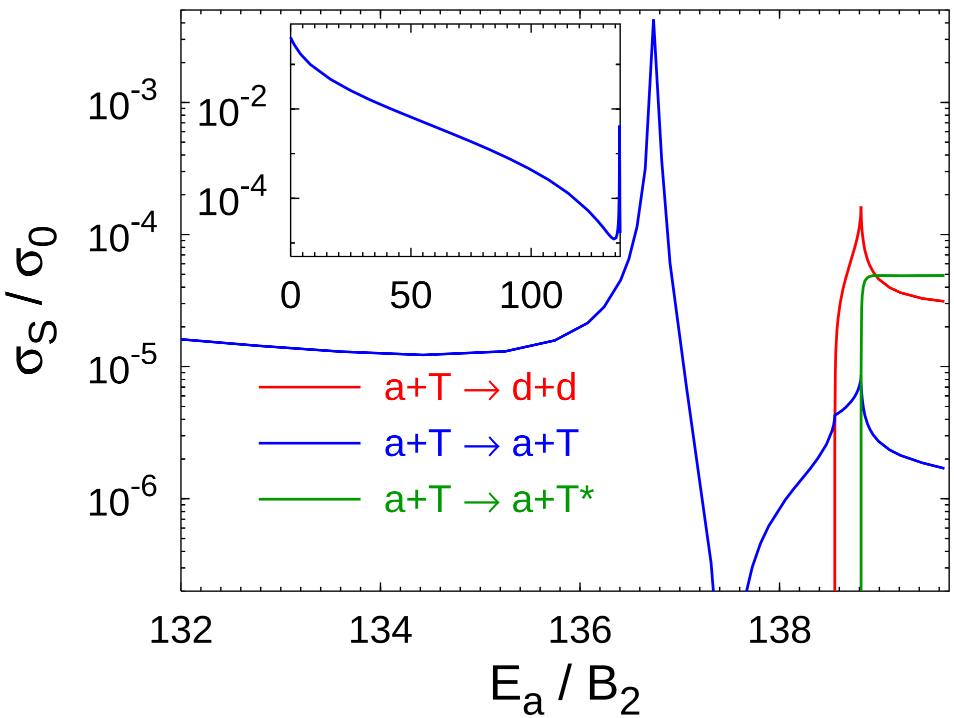
- Realistic interactions very repulsive at short distances
- Cutoff: ^{4}He trimer BE and atom-dimer scattering in agreement with r-space results by R. Lazauskas

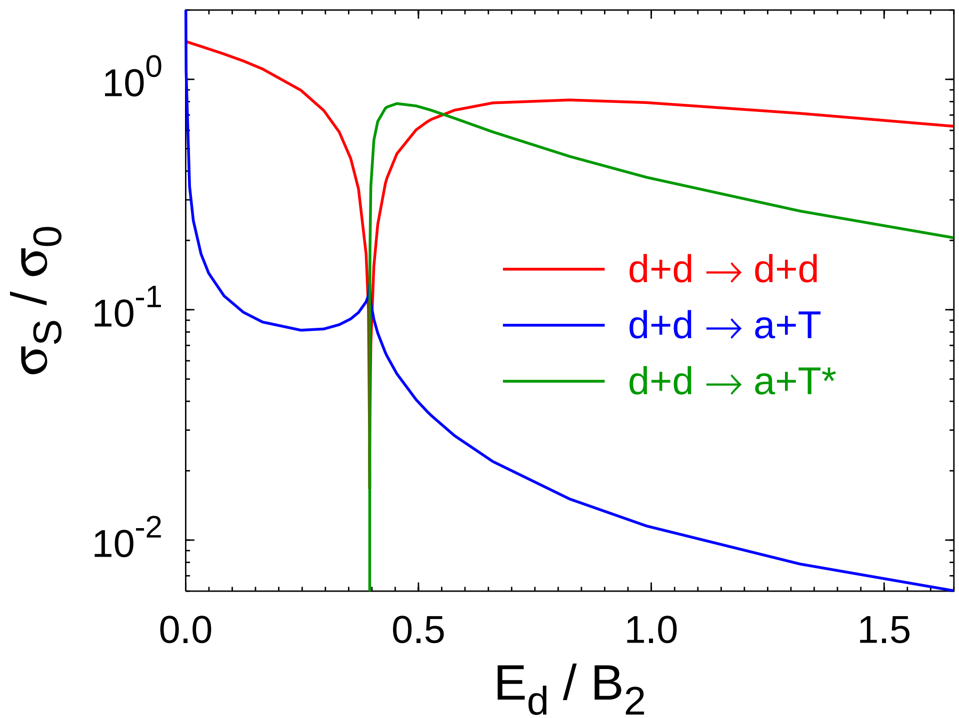
Four-boson scattering

Effective separable potentials between ^{4}He atoms:

$B_2 = 1.213$ mK, $B_3 = 128.43$ mK, $B_{3*} = 2.186$ mK,

$a_0 = 104$ Å, $\sigma_0 = 4\pi a_0^2$





Summary

- Faddev/AGS equations in momentum space
- Coulomb interaction: screening and renormalization

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- Coulomb interaction: screening and renormalization

- hadronic and electromagnetic 3N reactions
- 3-body nuclear reactions
- low energy 4N scattering

- atomic systems