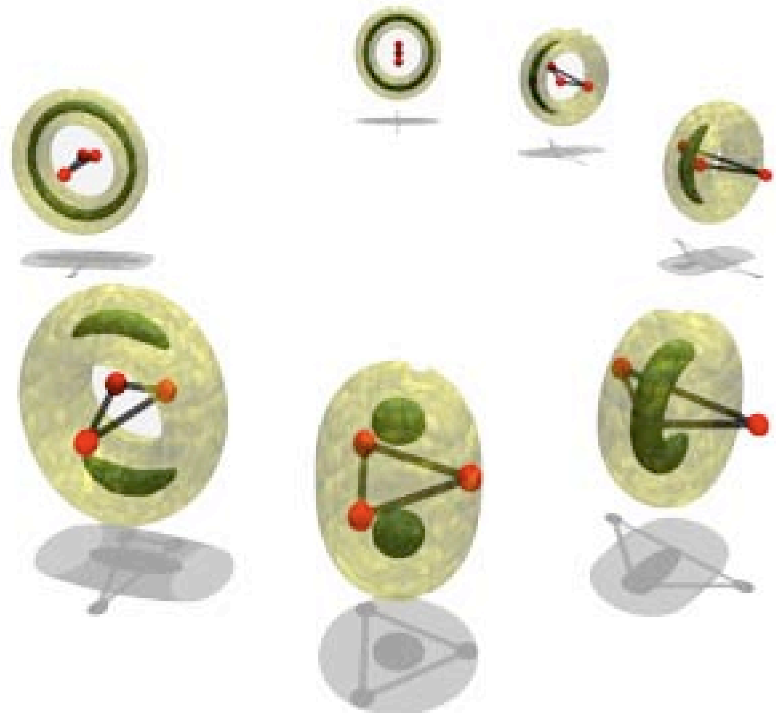
The Hyperspherical Approach for Atomic Few-Body Systems

NIST University of Colorado
National Science Foundation

Jose P. D'Incao
JILA, University of Colorado at Boulder and NIST



JILA
NIST
University of Colorado

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["Weakly bound systems in atomic and nuclear physics"]
Institute for Nuclear Theory, University of Washington,
Seattle, WA (March 2010)

Weakly bound systems in Ultracold Quantum Gases ?

How Cold is Ultracold ?

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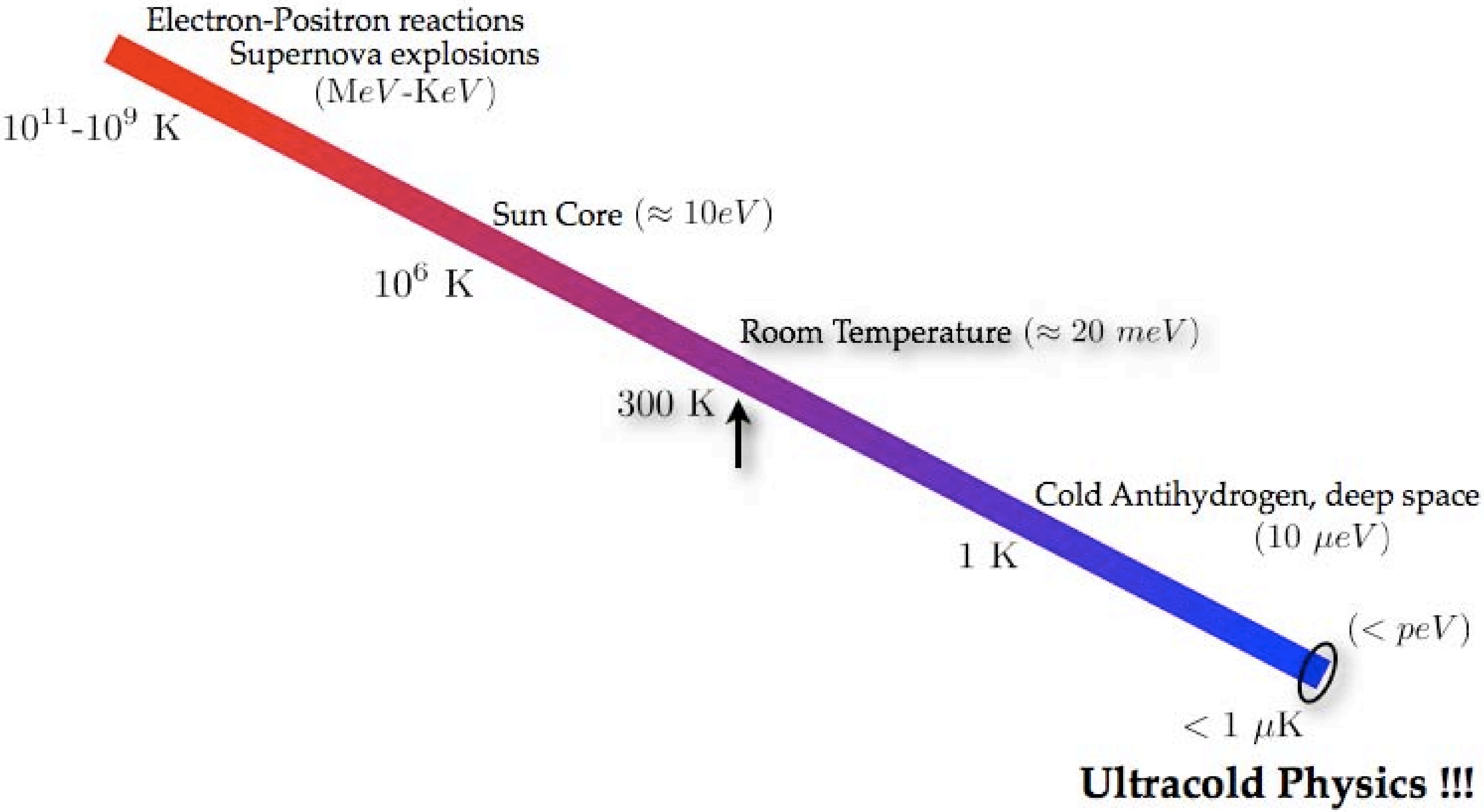

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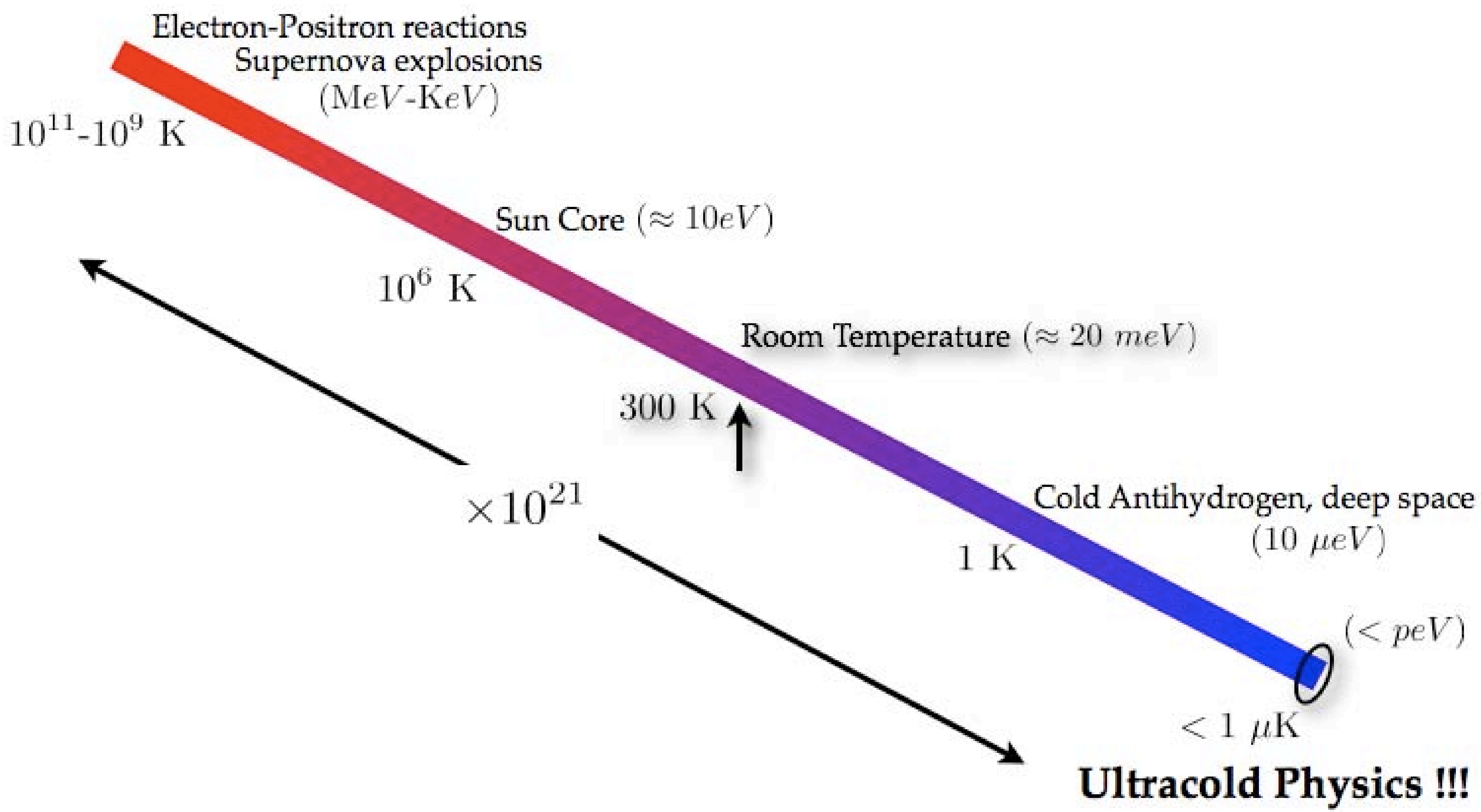
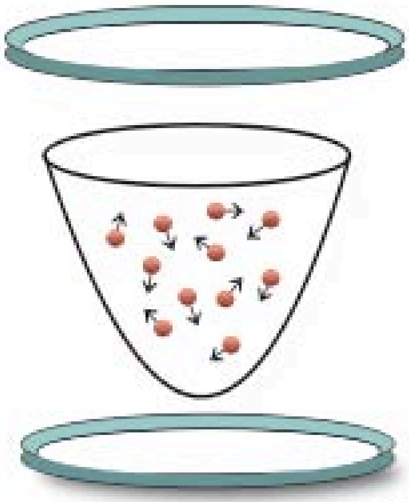


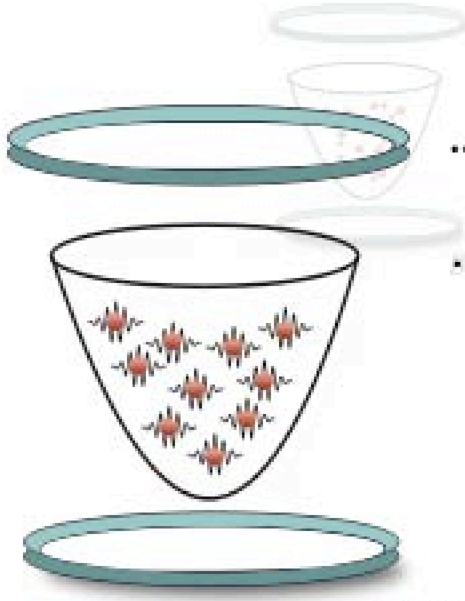
What is the big deal in ultracold physics ?

Ultracold Physics



... "High" T
(thermal gas)

Ultracold Physics



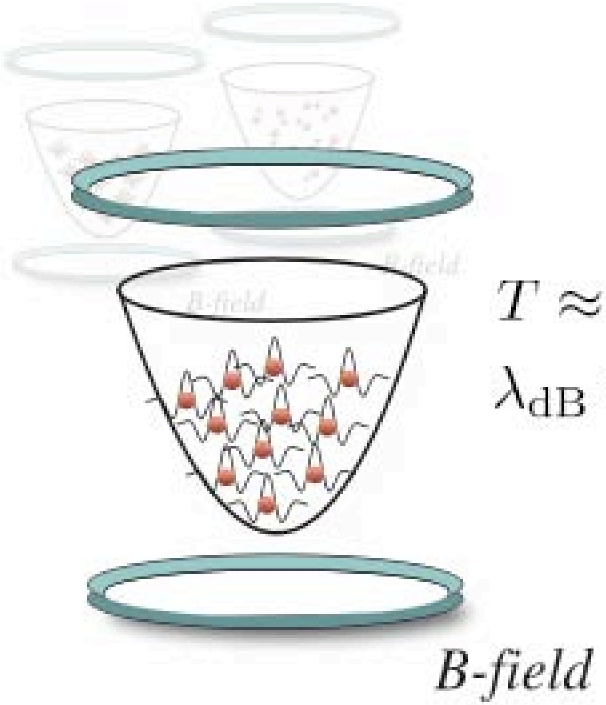
B-field

... evaporative cooling ...

... Low T

$$\lambda_{\mathrm{dB}} \propto \frac{1}{T^{1/2}}$$

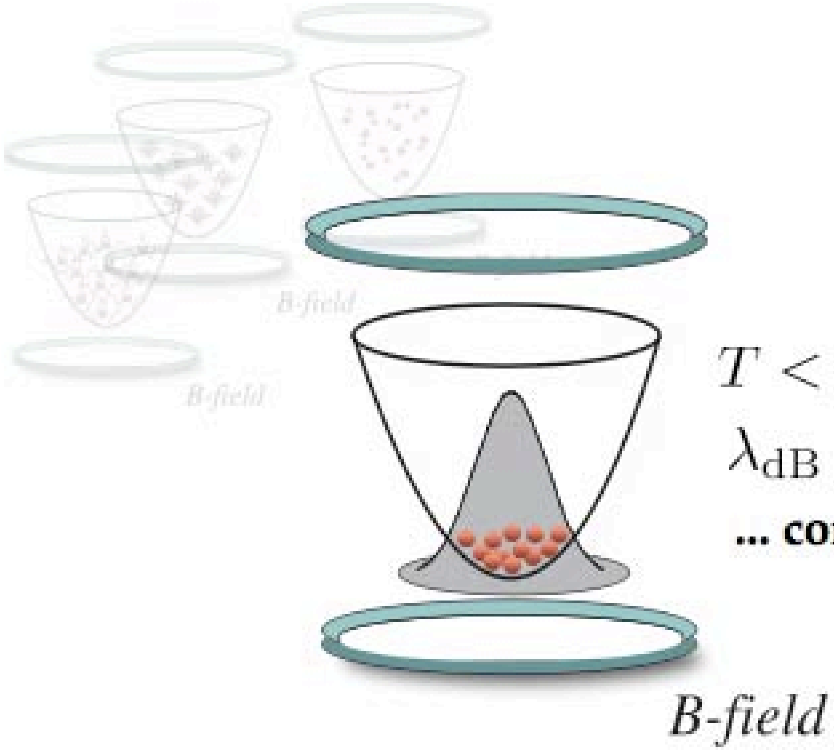
Ultracold Physics



$T \approx T_c$

$\lambda_{\mathrm{dB}} \approx$ Int. atomic sep.

Ultracold Physics

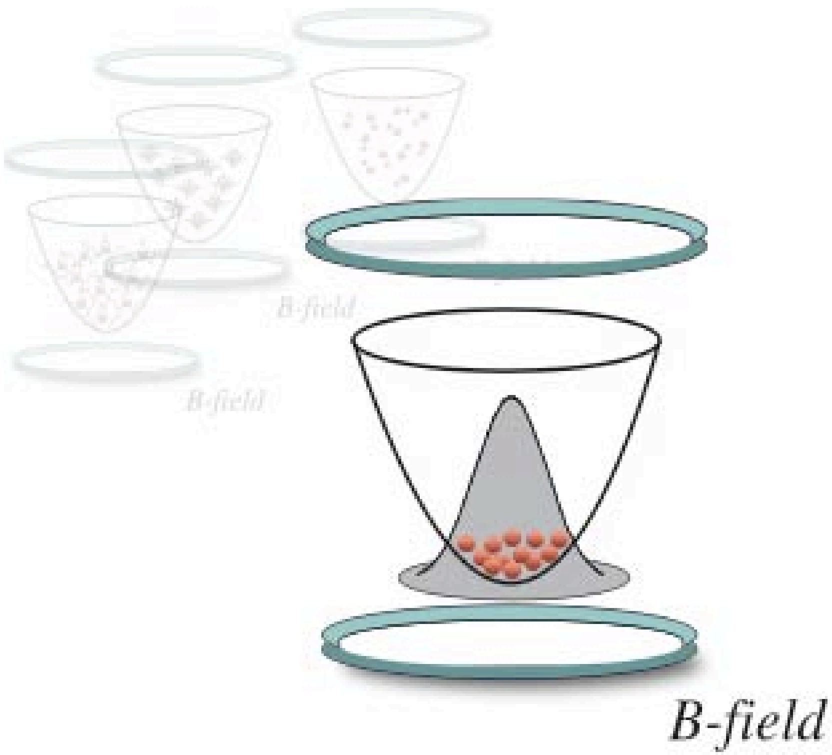


$T < T_c$

$\lambda_{\mathrm{dB}} > \mathrm{Int.\ atomic\ sep.}$

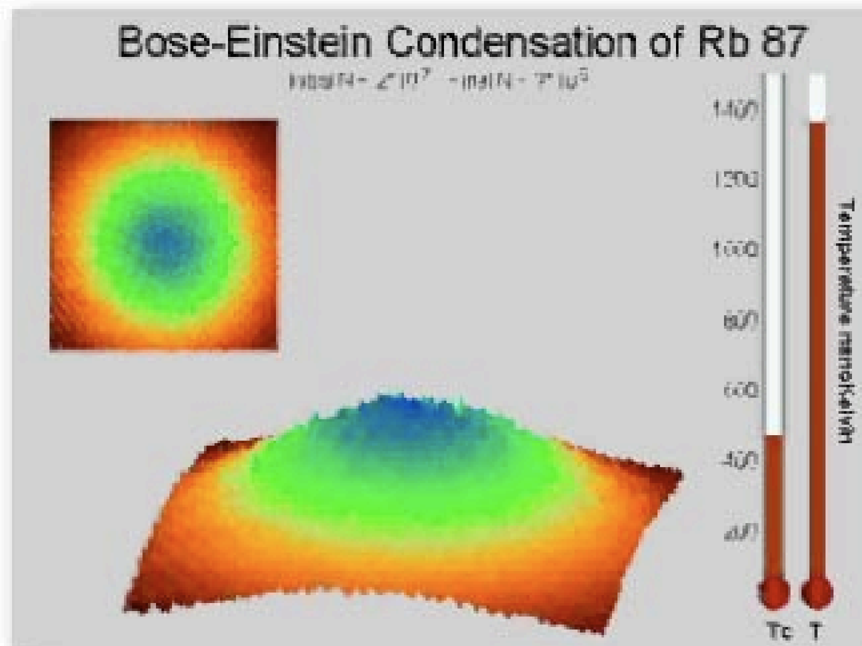
... condensation !!!

Ultracold Physics



Nobel Prize (2001) 

Cornell, Weiman (JILA) and Ketterle (MIT)



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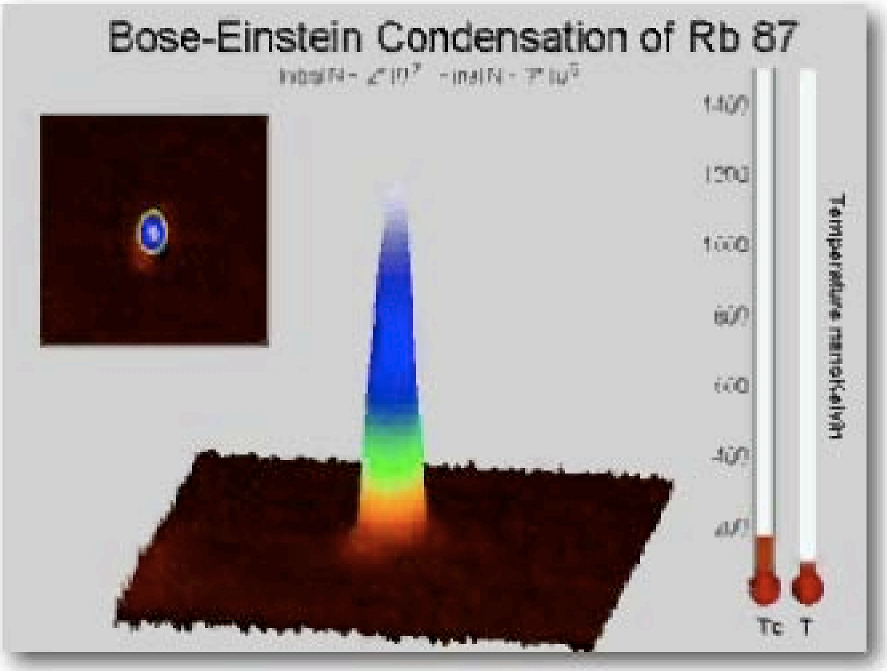

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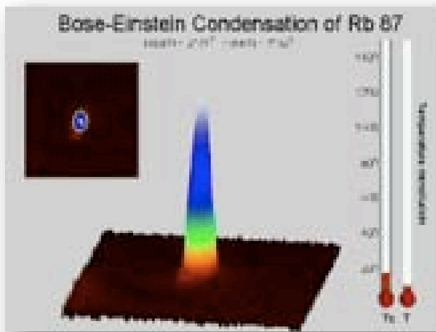
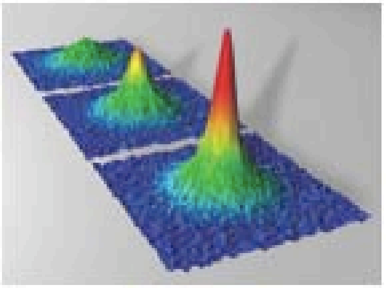
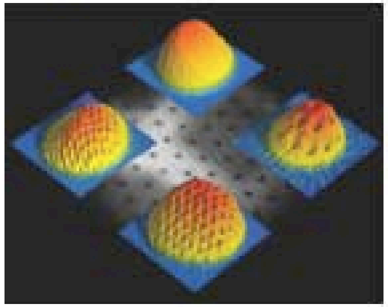


Fermi Gases: BEC-BCS



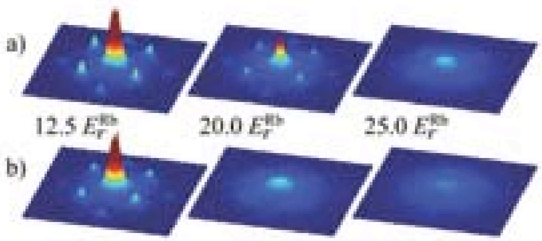
D. S. Jin (JILA)

Superfluidity



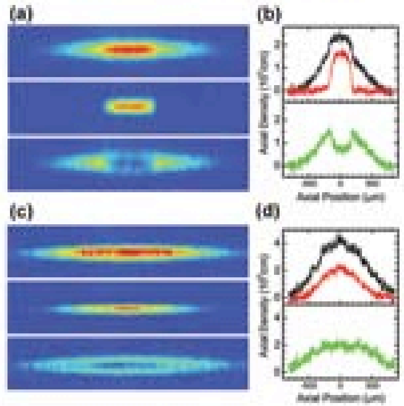
Ketterle (MIT)

Bose-Fermi



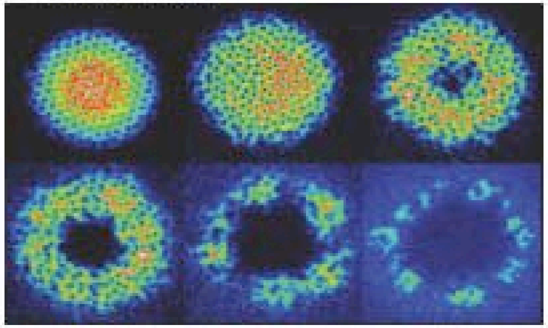
Sengstock (Hamburg)

Fermi gases: pop. imbalance



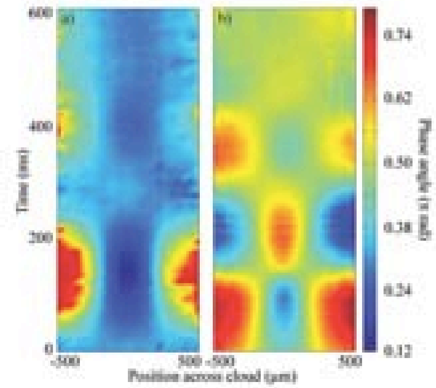
Hulet (Rice)

Giant vortex



Cornell (JILA)

Spin waves



Cornell (JILA)

Feshbach Resonances: Quantum control

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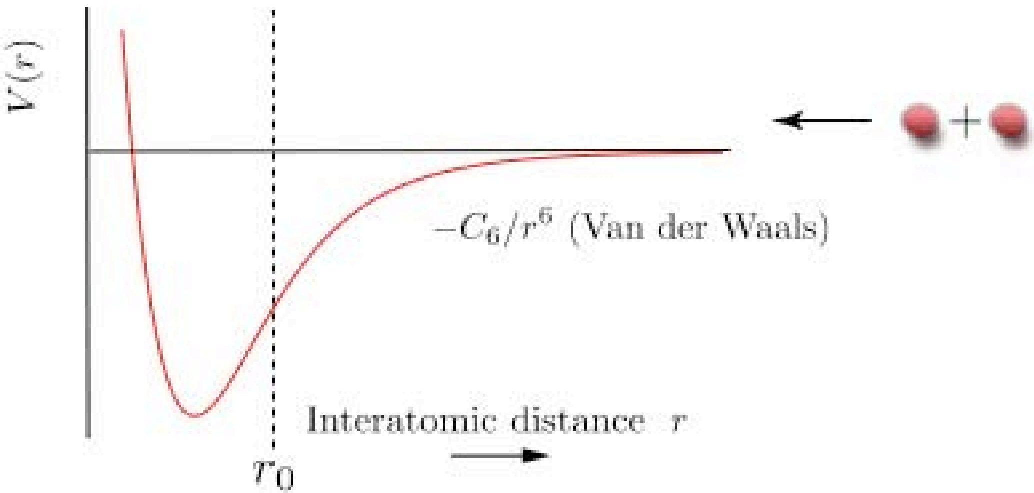
Two-body physics ... at ultracold temperatures



(r_0 : characteristic size)

s-wave scattering length :

$$a = -\lim_{k \to 0} \frac{\tan \delta}{k}$$

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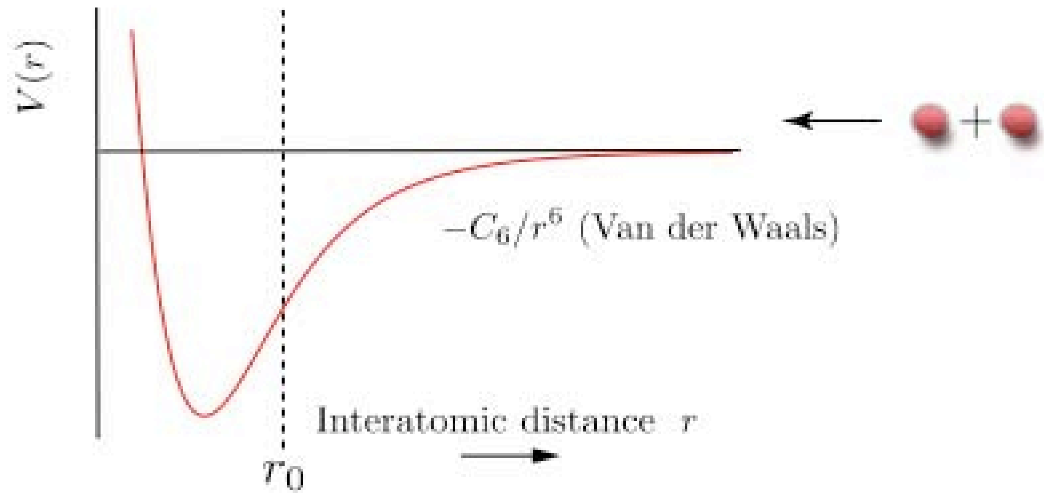

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Elastic crossection

$$\sigma \propto a^2$$

(strength of the interatomic interaction)

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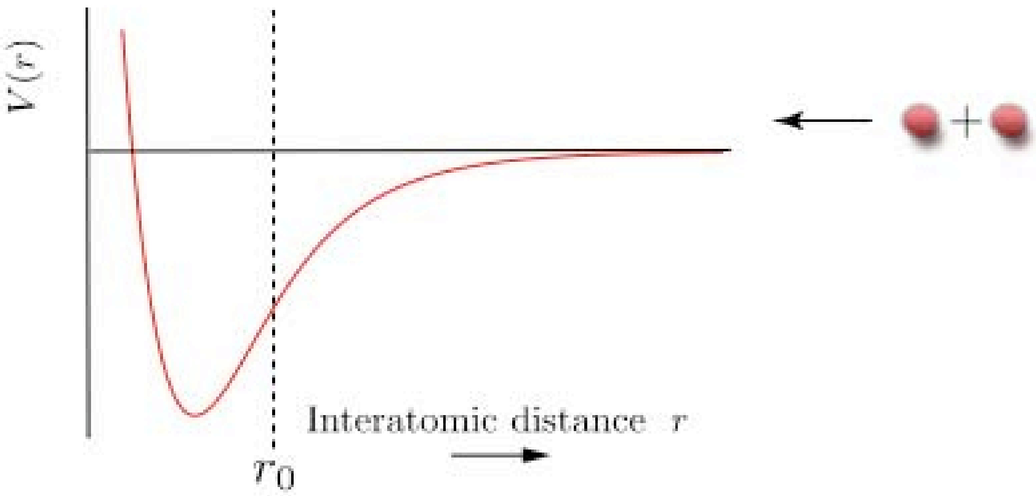

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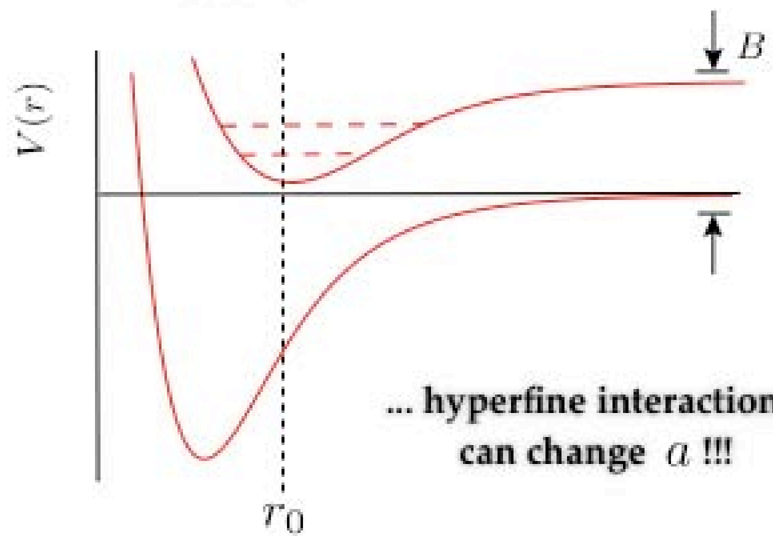

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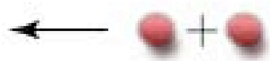
... **hyperfine interactions:**
can change a **!!!**



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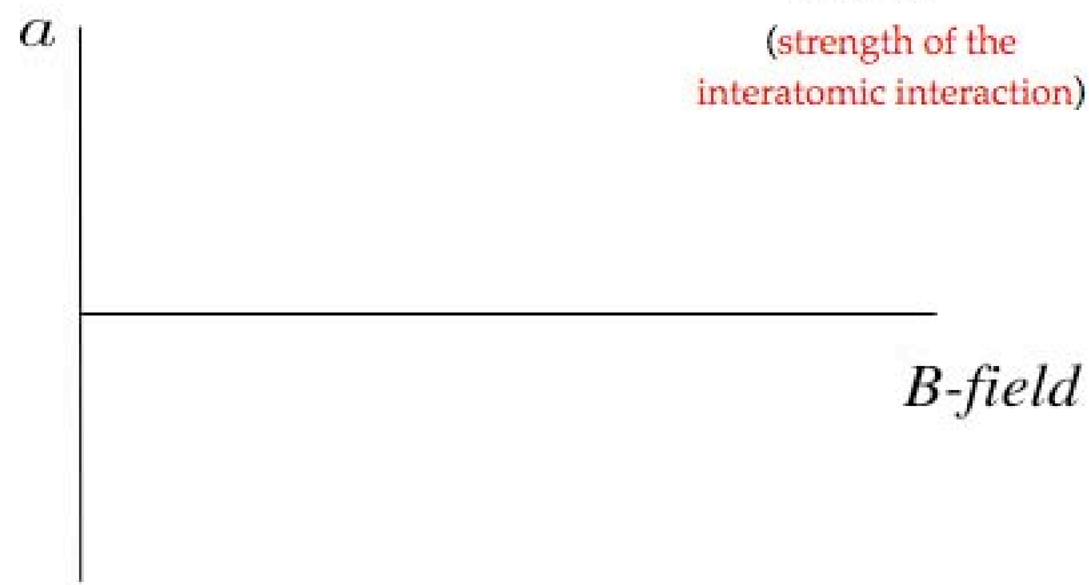

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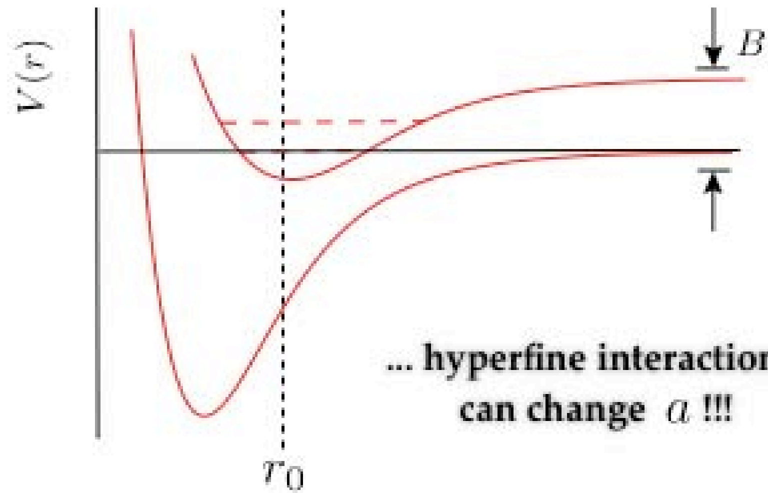

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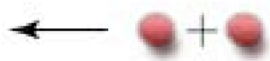

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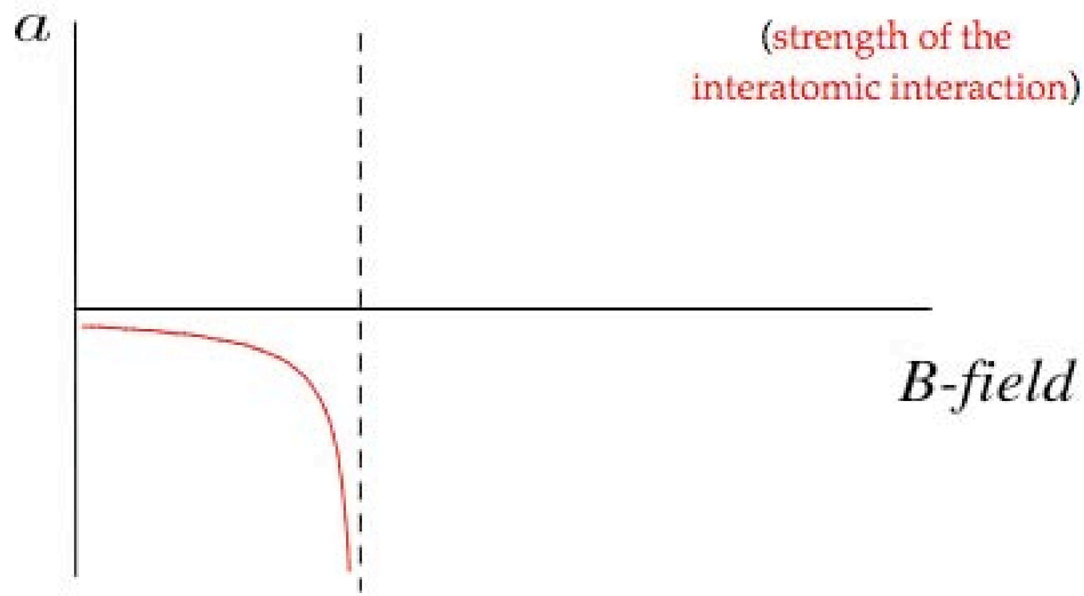

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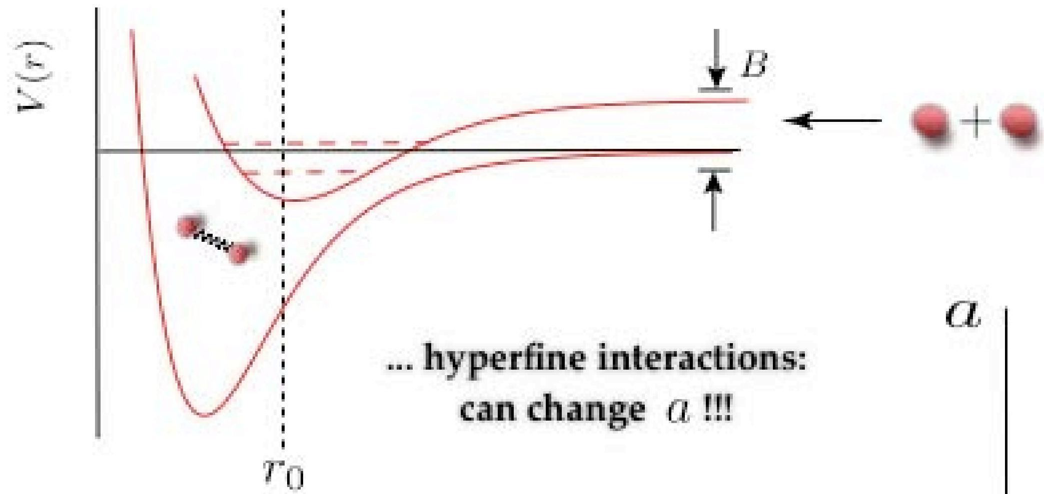

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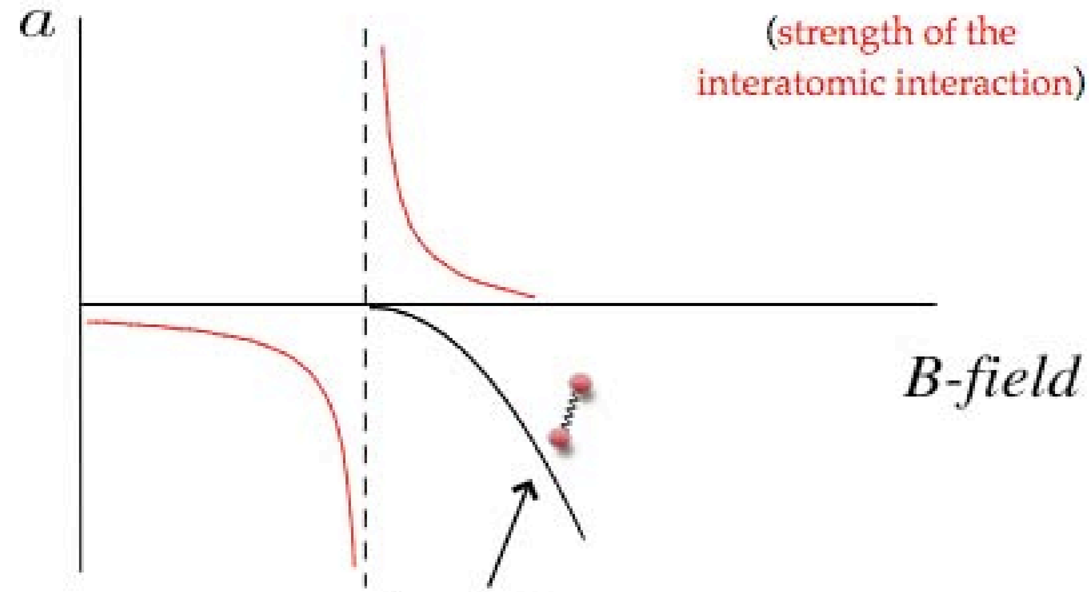

Feshbach Resonances: Quantum control

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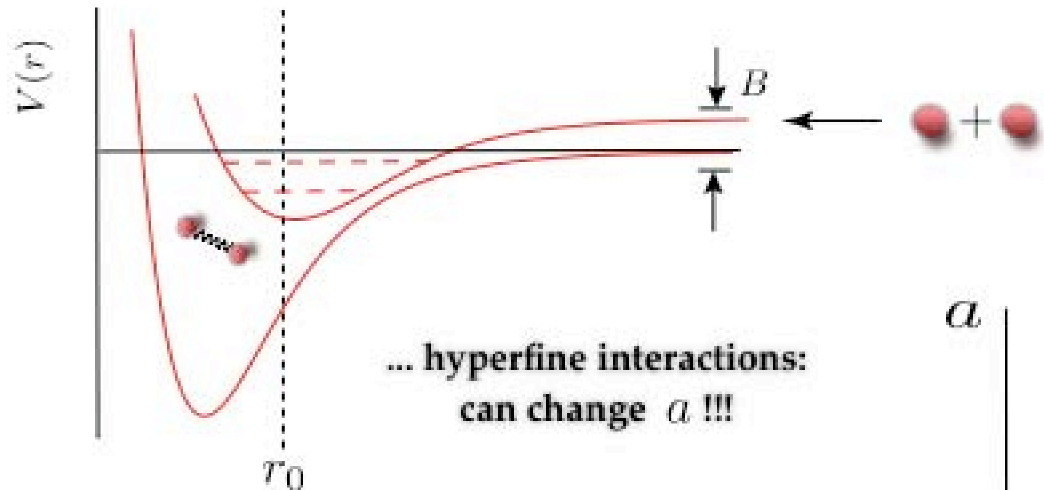

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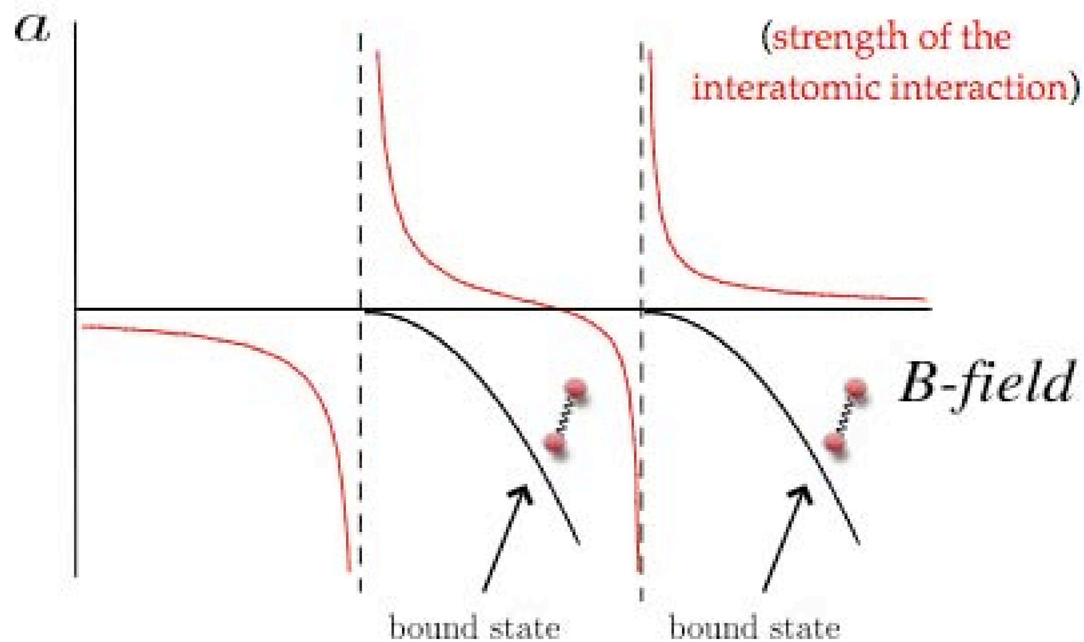

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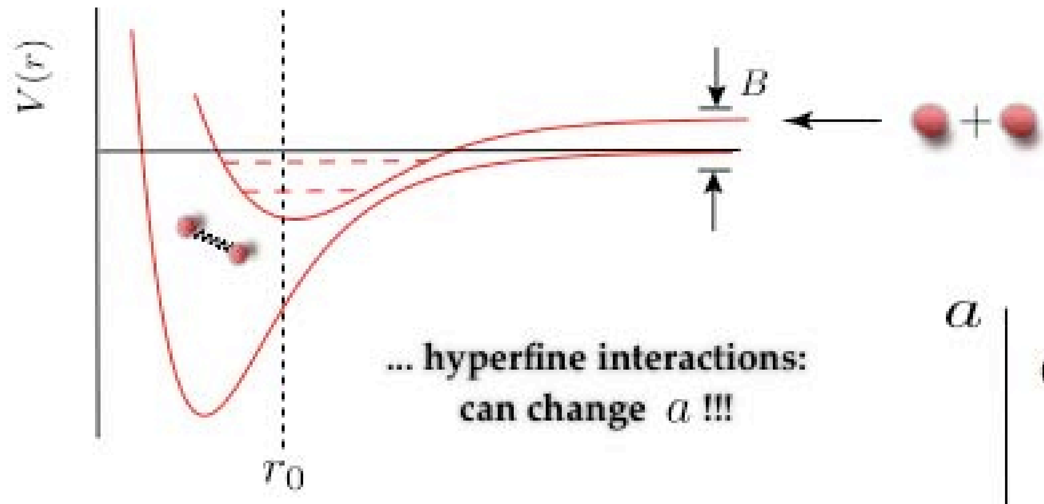

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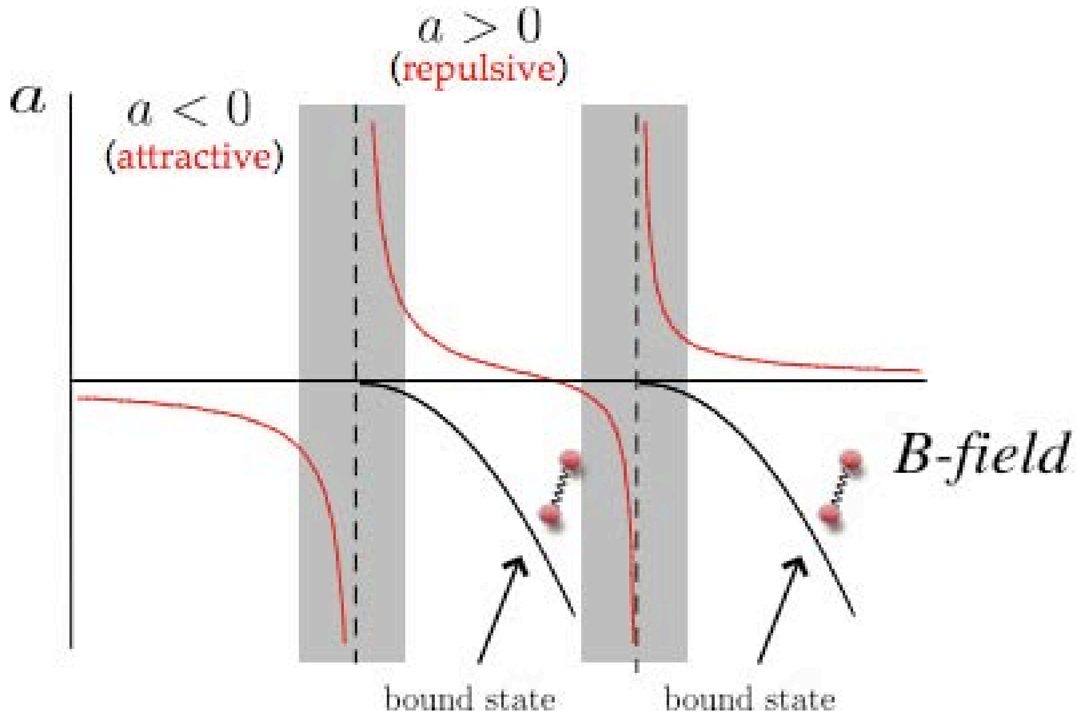
Strongly correlated regime :
$|a| \gg r_0$



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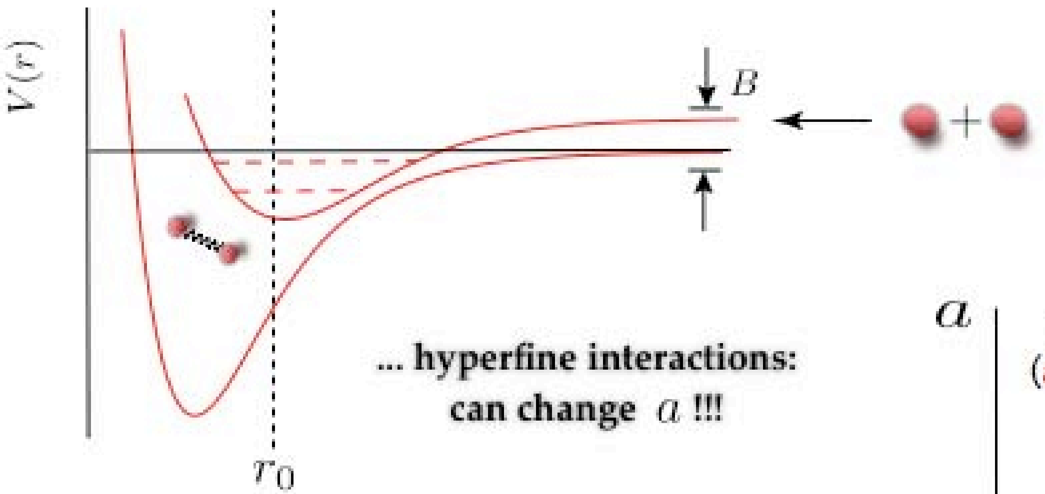

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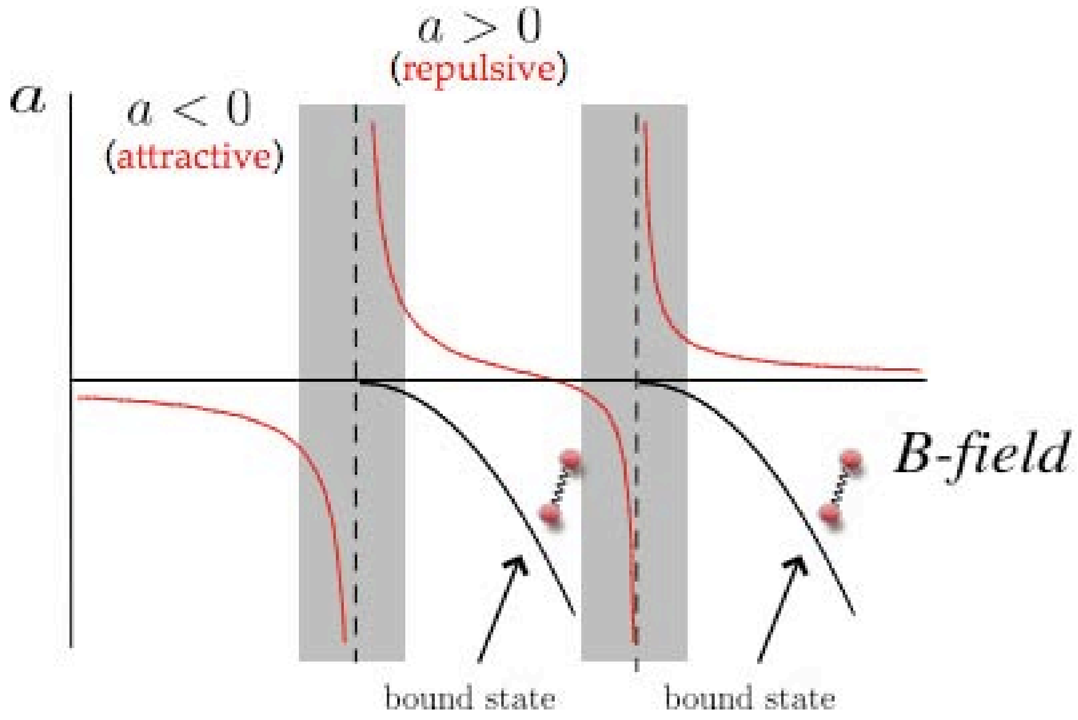
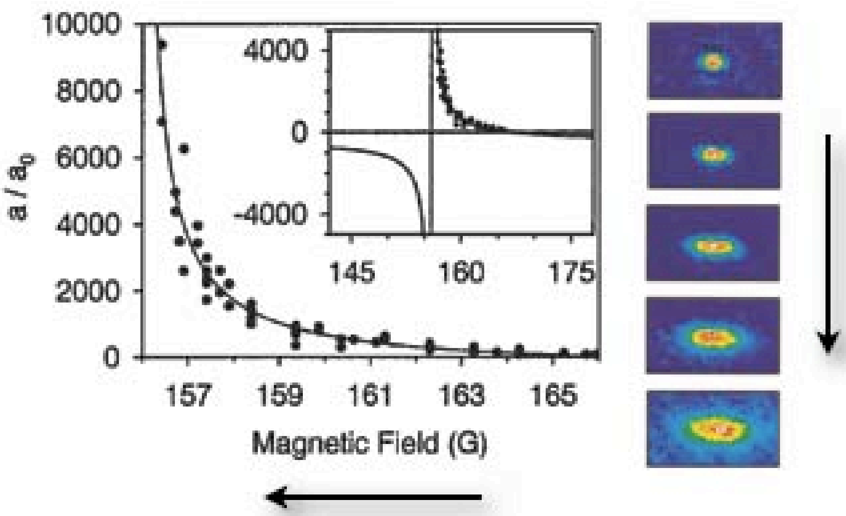


Cornish *et al.*, PRL (2000)



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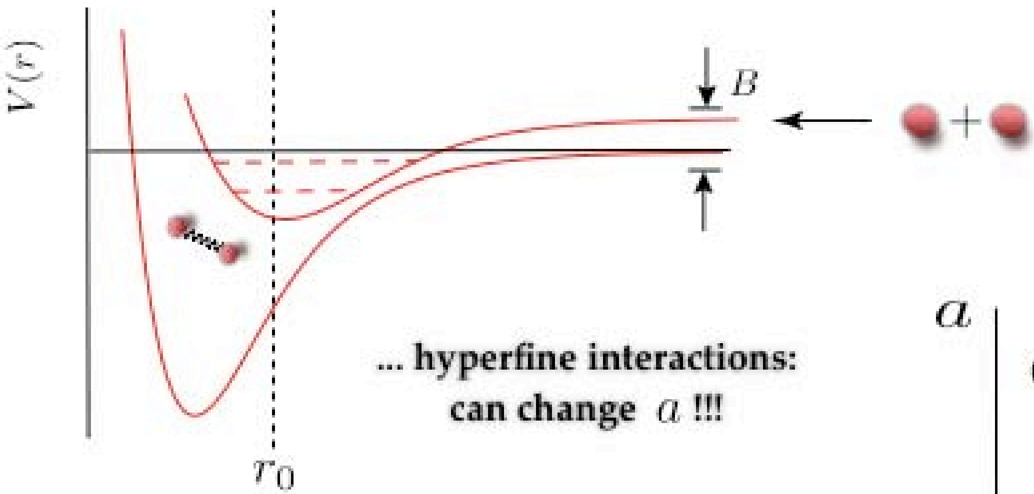

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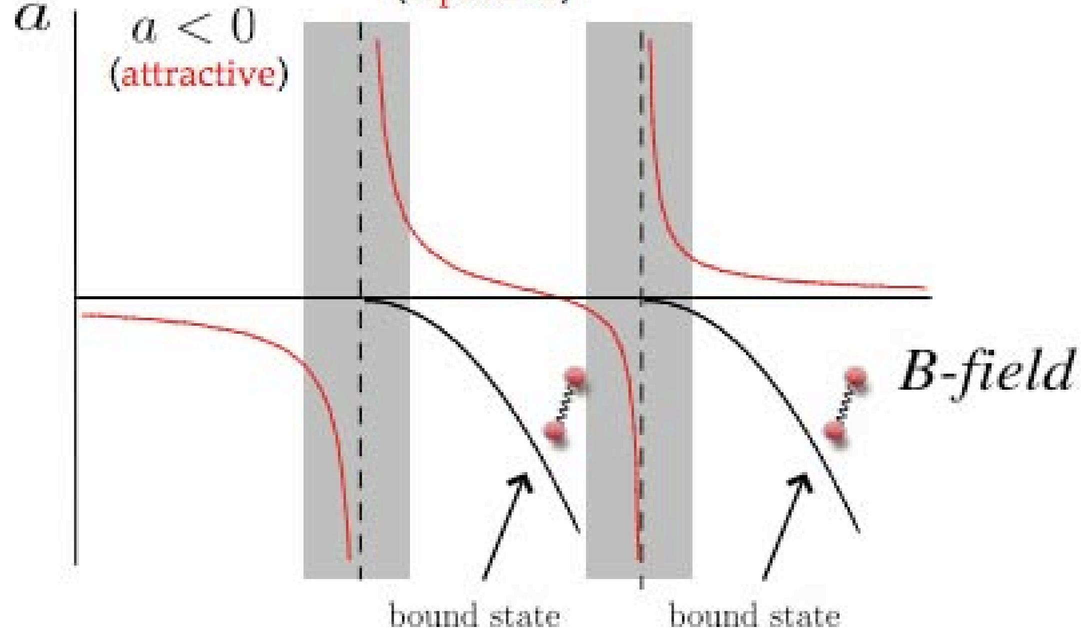
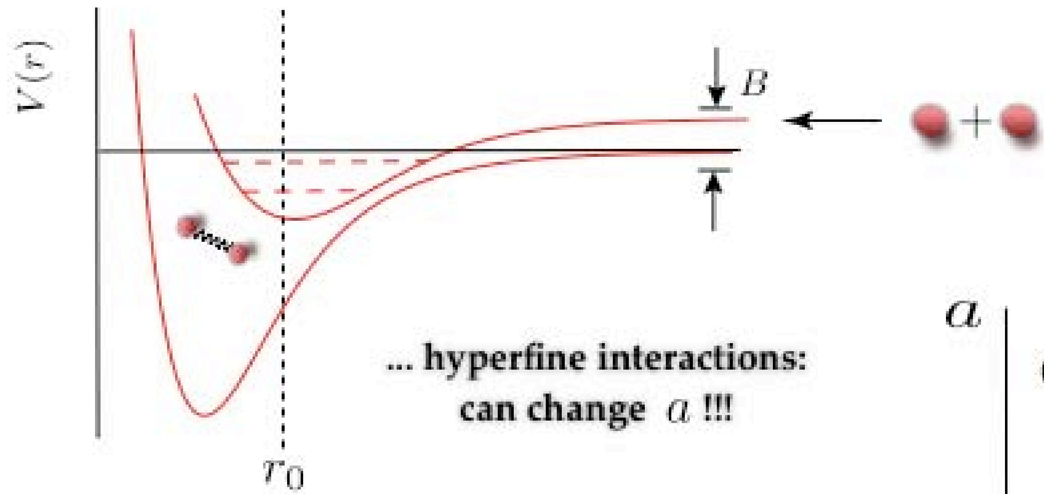


- **Universality :**
  - **properties depends *only* on a**
  - **allows use of potential models**
- **Many interesting many-body effects ...**

Feshbach Resonances: Quantum control

Two-body physics ... at ultracold temperatures



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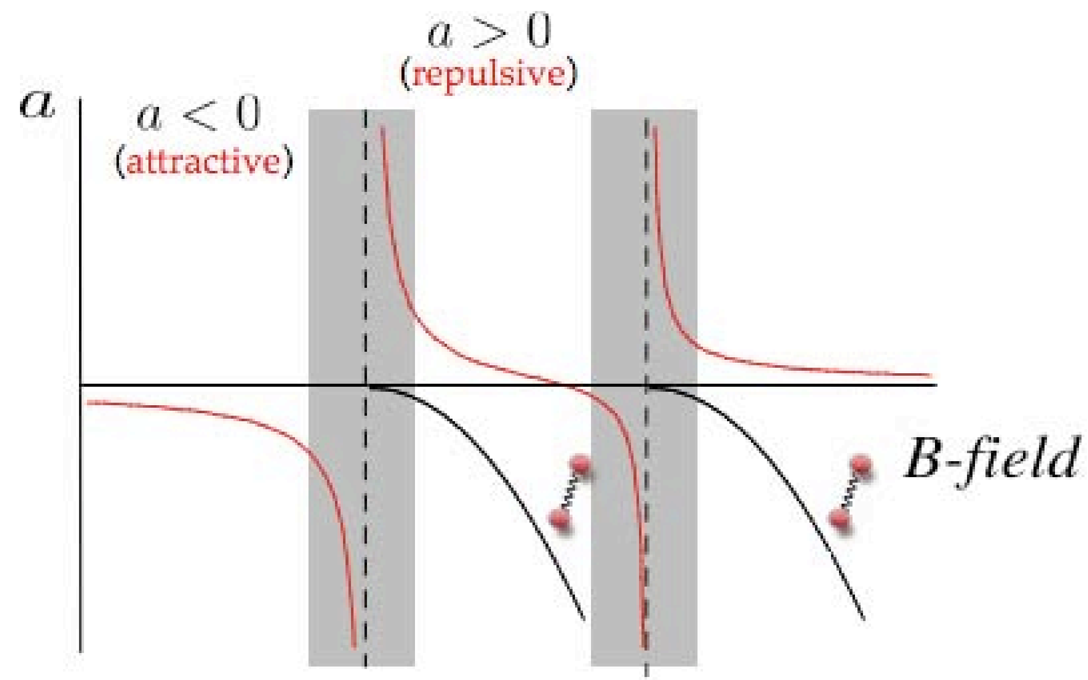

Two-body weakly bound states !!!

- **Universality :**
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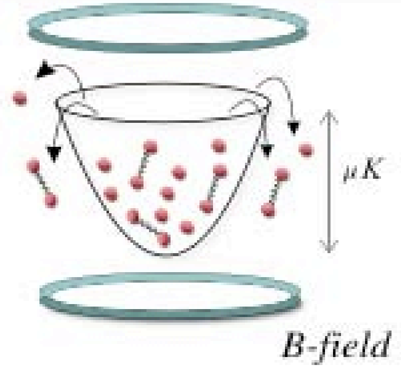
- **Many interesting many-body effects ...**

Few-body physics in Ultracold Gases
(Why do we care ?)

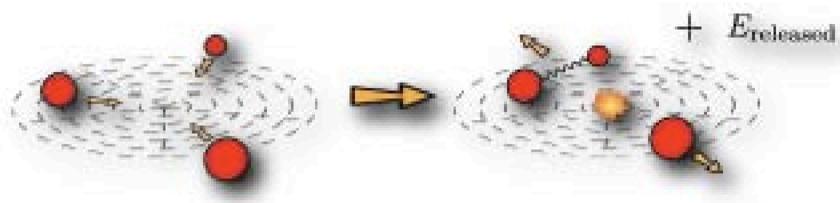
Few-body Physics in Ultracold Gases

- *a* dependence of 3-body scattering processes

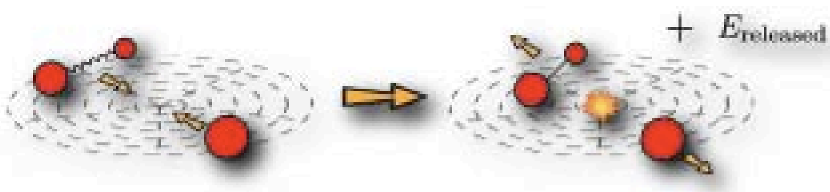
Losses
(Lifetime / Stability)



(Three-body Recombination)



(Vibrational Relaxation)



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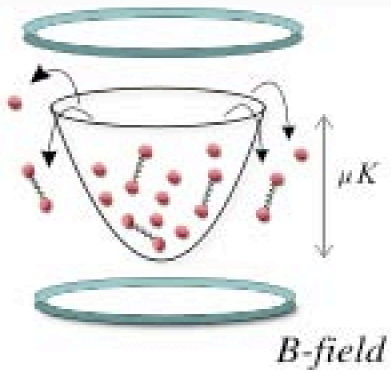

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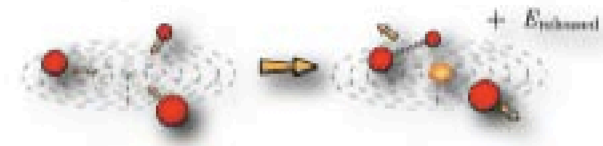

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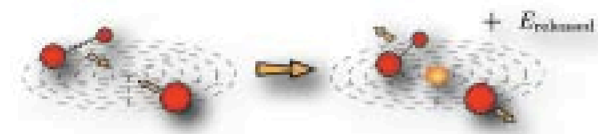


(Atom-dimer scattering length)

• Control of few-body correlations

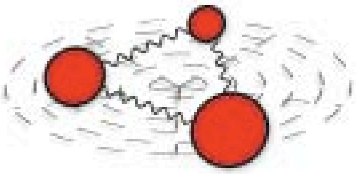
Elastic parameters
(strength of interactions, equation of state, ...)

(Dimer-dimer scattering length)

Efimov Physics (~1970): Nuclear Physics

Vitaly Efimov

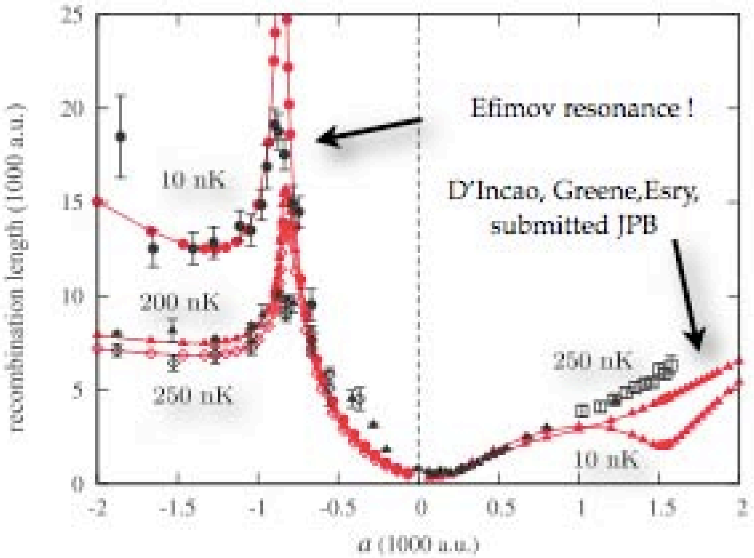
☑ Weakly bound trimers



(no two-body states :
infinitely many three-body states)

Ultracold gases (Cs Exp. - Innsbruck)

Kraemer, *et al.*, Nature (2006)



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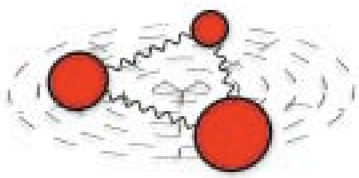

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... afterwards ...

☑ Long-lived weakly bound dimers

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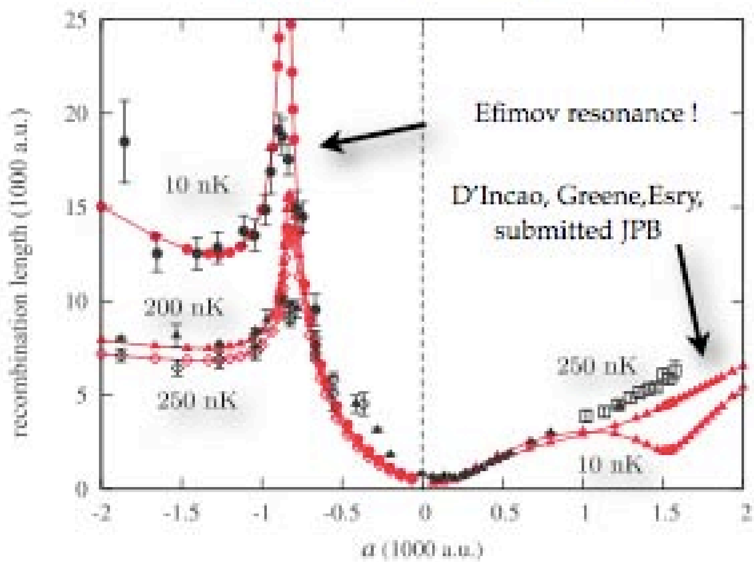

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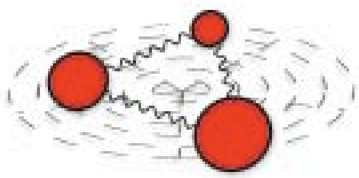

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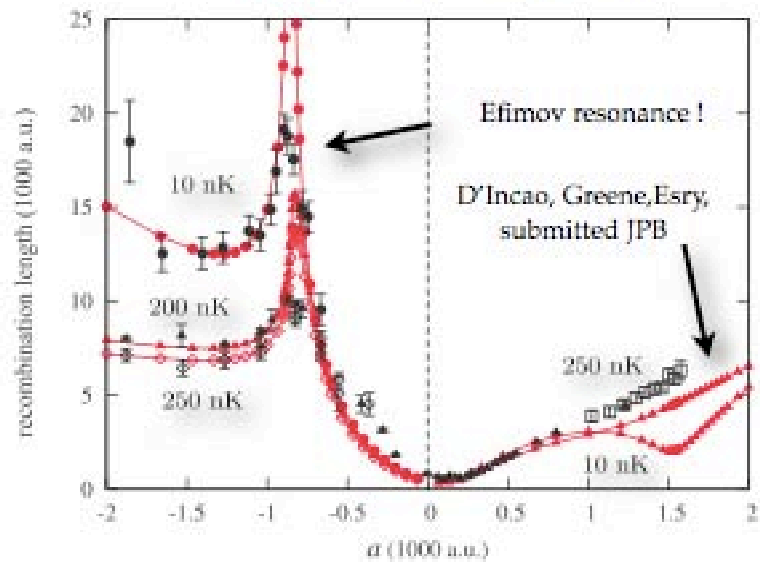

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☑ Control of the few-body interactions

Efimov Physics (~1970): **Nuclear Physics**

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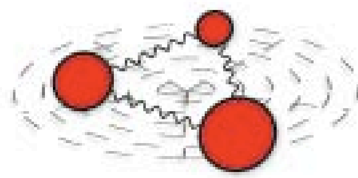
- ☑ Weakly bound trimers



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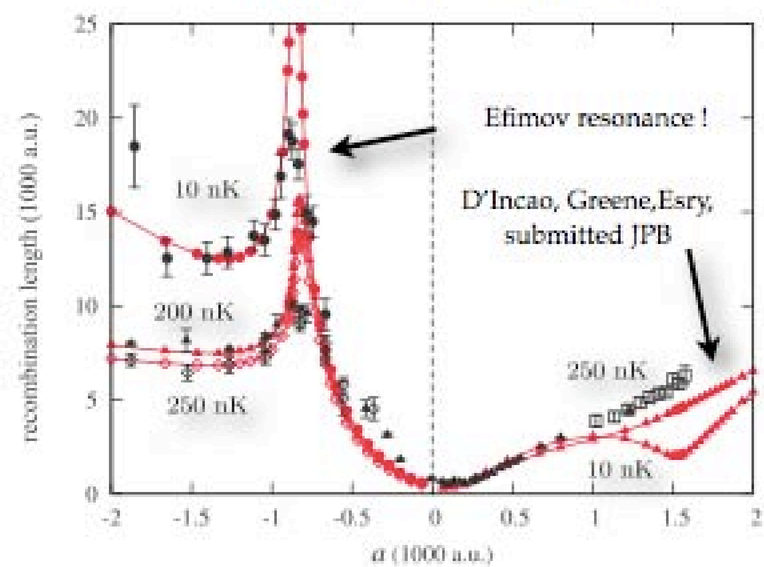


... afterwards ...

- ☑ Long-lived weakly bound dimers
- ☑ Control of the few-body interactions
- ☑ Scattering length dependence on 3-body collision rates →

Ultracold Gases

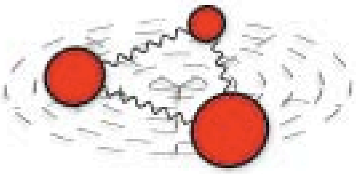
(can change *a* !!!)

Few-body Physics in Ultracold Gases

Efimov Physics (~1970): **Nuclear Physics**

Vitaly Efimov

- Weakly bound trimers



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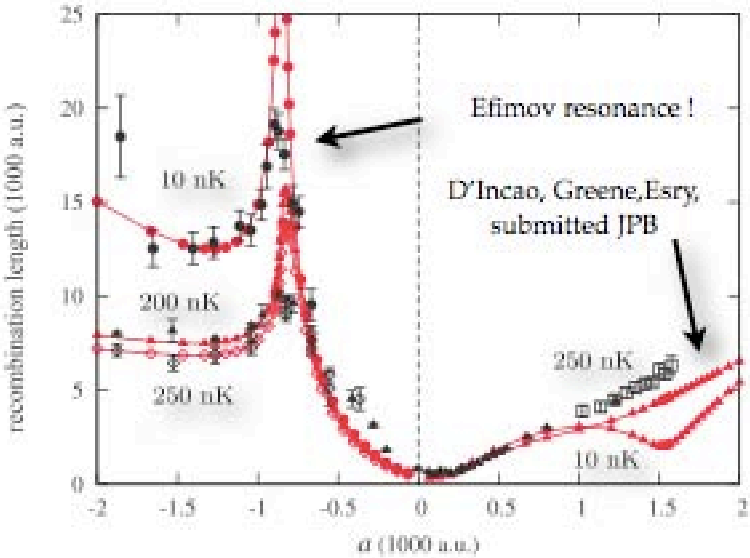

... afterwards ...

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- Scattering length dependence on 3-body collision rates

A new (exp) research venue:

Innsbruck (Grimm)
Rice (Hulet)
LENS (Minardi, Modugno)
Heidelberg (Jochim)
PenState (O'Hara)
Israel (Khaykovich)

Toolkit for Exploring Few-body Physics ...

For N particles ...

$$\hat{H} = -\frac{1}{2\mu}\nabla_T^2 + \sum_{i<j} V(r_{ij})$$

... angles + set of non-compact coordinates $r_{ij} \rightarrow [0, \infty]$

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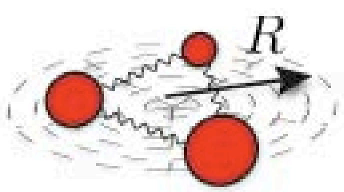
... angles + set of non-compact coordinates $r_{ij} \to [0, \infty]$

... the hyperspherical way !!!

$$\hat{H} = -\frac{1}{2\mu}\frac{d^2}{d^2R} + \frac{\Lambda^2(\Omega)}{2\mu R^2} + V(R,\Omega)$$



hyperradius R : overall size (collective motion)

$R \to [0, \infty]$

hyperangles $\{\Omega\}$: internal motion

$\{\Omega\} \to [0, \propto \pi]$

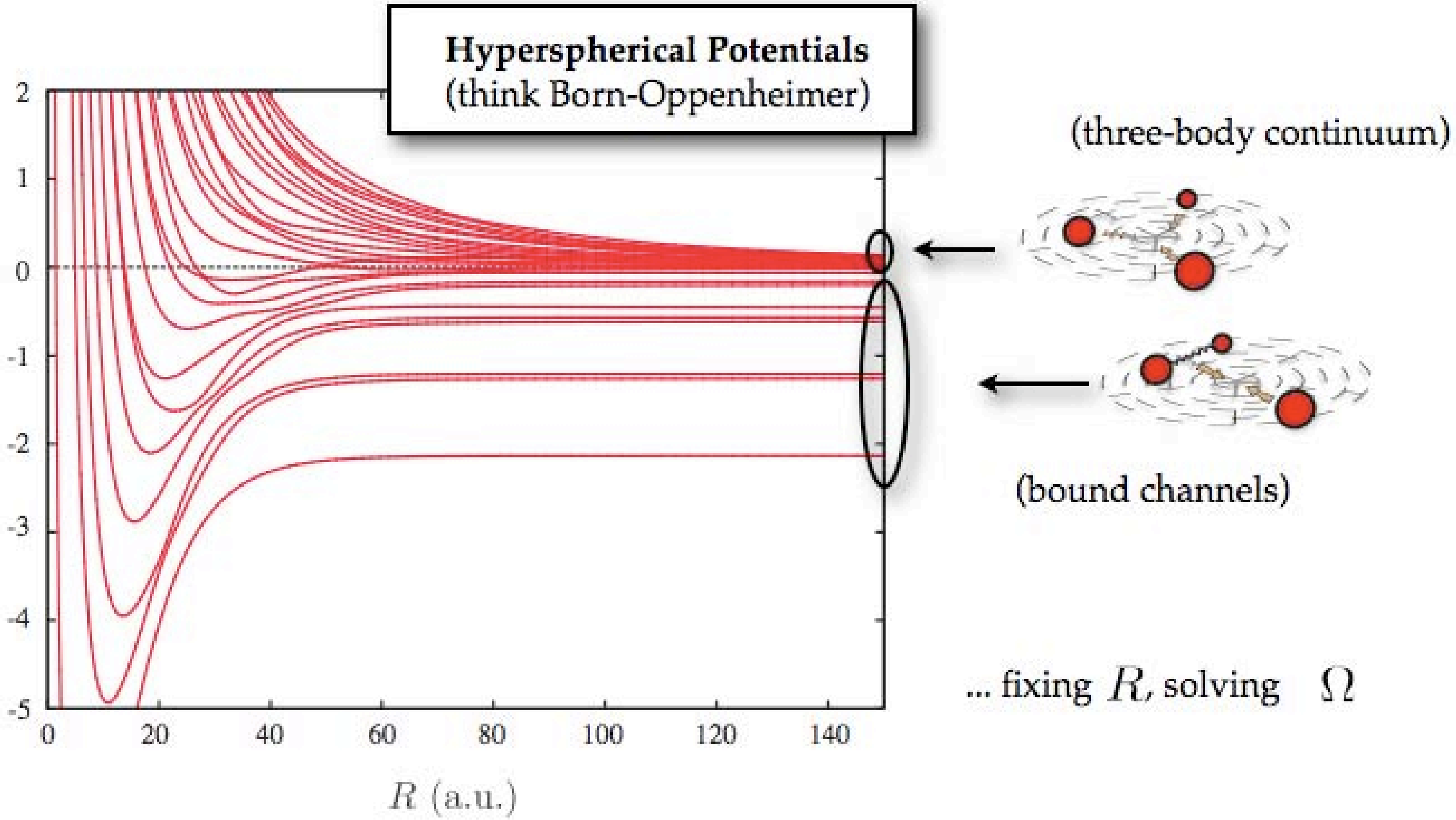
Toolkit for Exploring Few-body Physics ...

For N particles ...

$$\hat{H} = -\frac{1}{2\mu}\nabla_T^2 + \sum_{i<j} V(r_{ij})$$

... the hyperspherical way !!!

$$\hat{H} = -\frac{1}{2\mu}\frac{d^2}{d^2R} + \frac{\Lambda^2(\Omega)}{2\mu R^2} + V(R,\Omega)$$



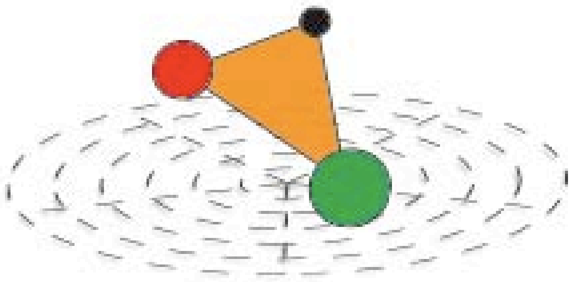
Toolkit for Exploring Few-body Physics ...

<u>*N*-body Problem :</u>

$3N$: coordinates
-3 : CM motion
-3 : Rigid body rotations

<u>3*N*-6 : Internal coordinates</u>
(Hyperradius + 3*N*-7 hyperangles)

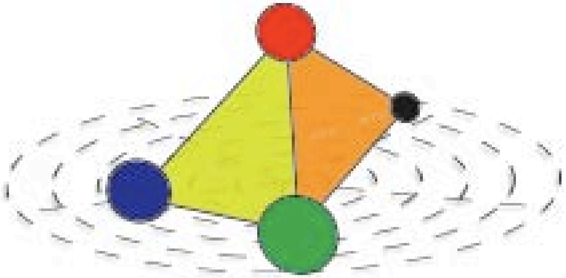
Three-body Problem



$$\hat{H}_{ad}\Phi_\nu(R;\Omega) = U_\nu(R)\Phi_\nu(R;\Omega)$$

$\Omega \equiv \{\theta, \varphi\}$: <u>2D PDE</u>

Four-body Problem



$$\hat{H}_{ad}\Phi_\nu(R;\Omega) = U_\nu(R)\Phi_\nu(R;\Omega)$$

$\Omega \equiv \{\theta_1, \theta_2, \phi_1, \phi_2, \phi_3\}$: <u>5D PDE</u>

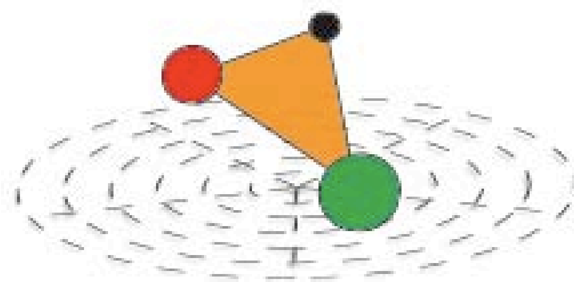
Toolkit for Exploring Few-body Physics ...

<u>*N*-body Problem :</u>

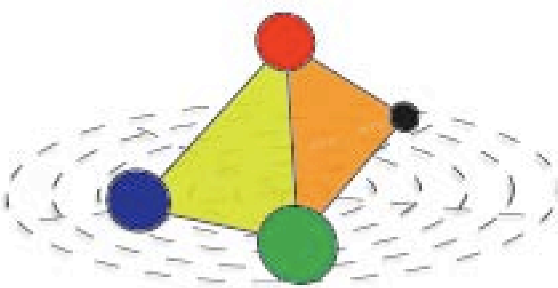
$3N$: coordinates
-3 : CM motion
-3 : Rigid body rotations

<u>$3N$-6 : Internal coordinates</u>
(Hyperradius + $3N$-7 hyperangles)

Three-body Problem



Four-body Problem



- **Democratic hyperangles**
  (Smith-Whiten, Johnson, Kupperman, Aquilante)
- **Correlated Gaussian + Hypersherical representation**

☑ Fragmentation thresholds

☑ Symmetrization is simpler

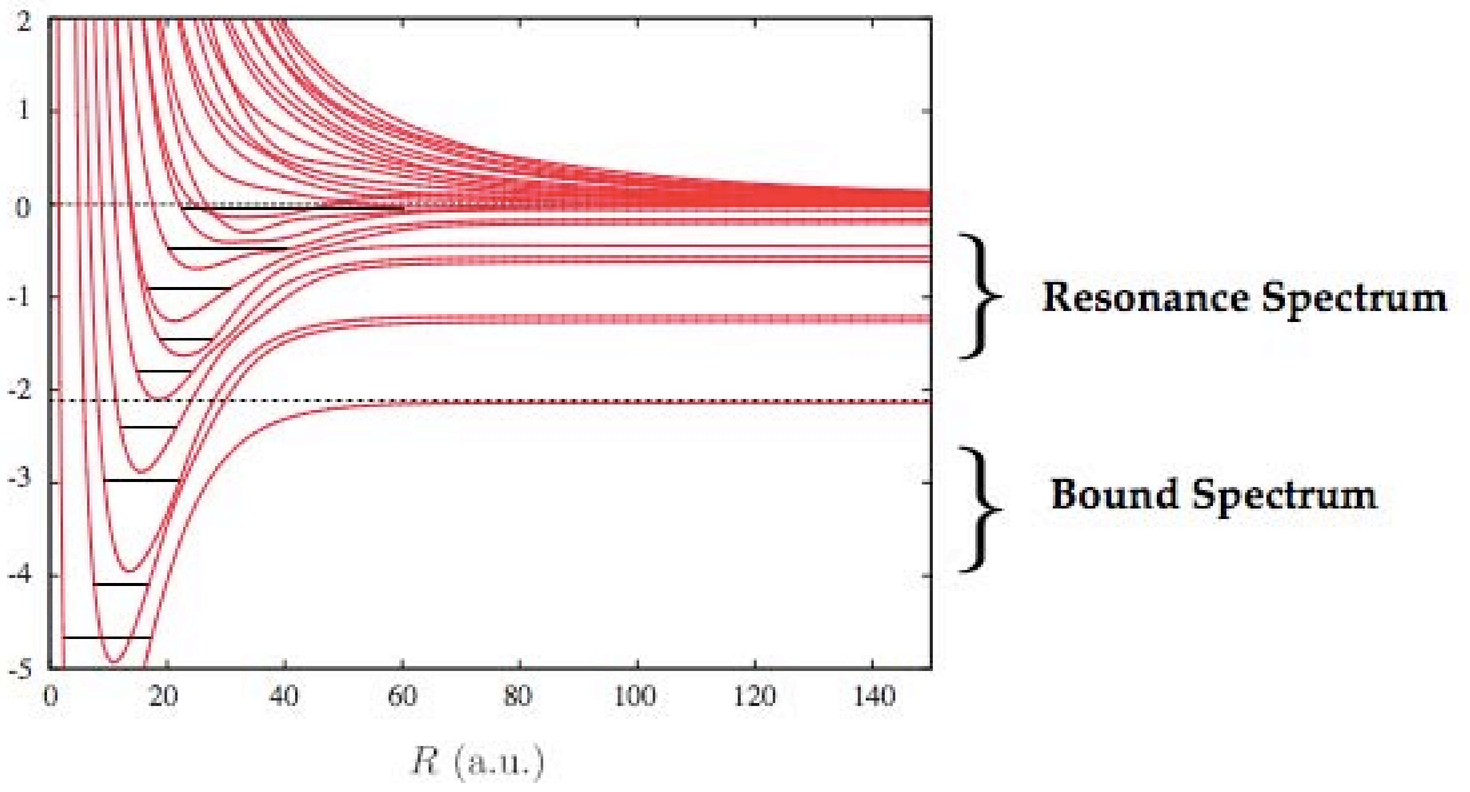
<u>Problem</u>: numerically challenging
(as *N* increases)

Toolkit for Exploring Few-body Physics ...

Bound and Scattering properties

(Hyperradial Schrodinger Equation)

$$\left[-\frac{1}{2\mu}\frac{d^2}{dR^2}+U_\nu(R)-E\right]F_\nu(R)+\sum_{\nu'}W_{\nu\nu'}(R)F_{\nu'}(R)=0$$



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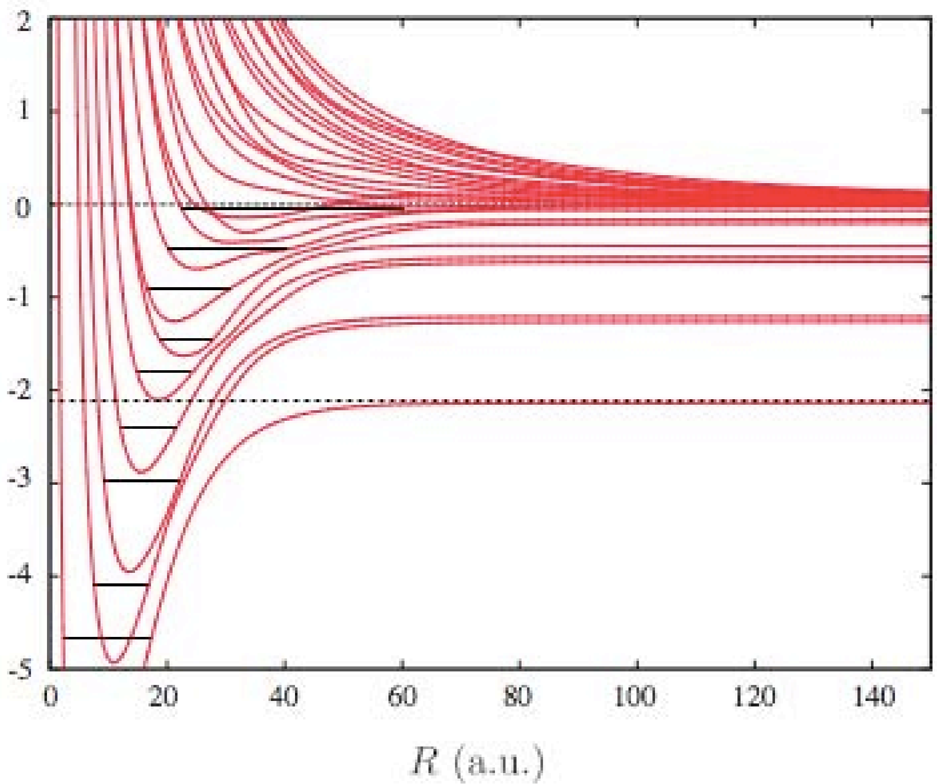
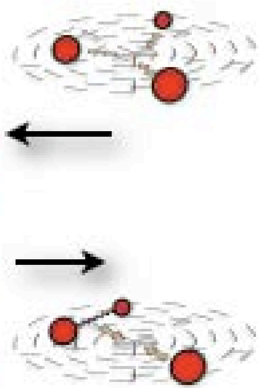


Ultracold Few-body Collisions



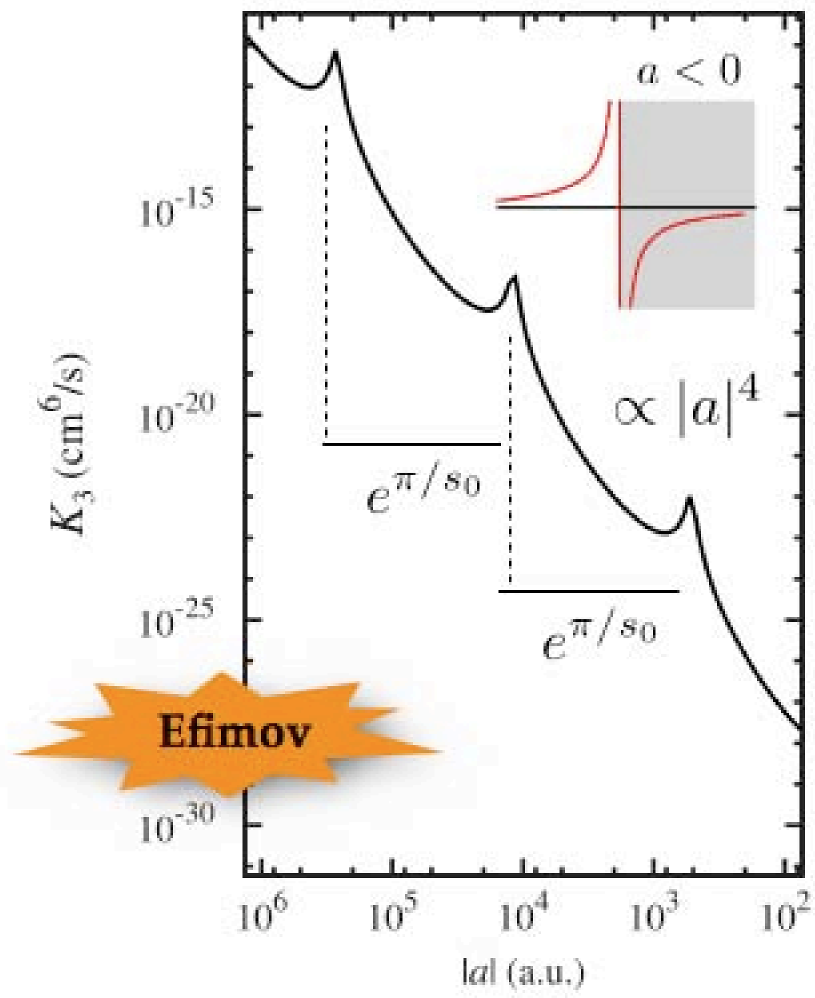
$W_{\nu\nu'}(R)$: non-adiabatic couplings (drive inelastic transitions)

Typical length & energy scales:

- Van der Waals length: $r_0 \approx 100a_0$
- Temperature: $T \approx 100\text{nK}$

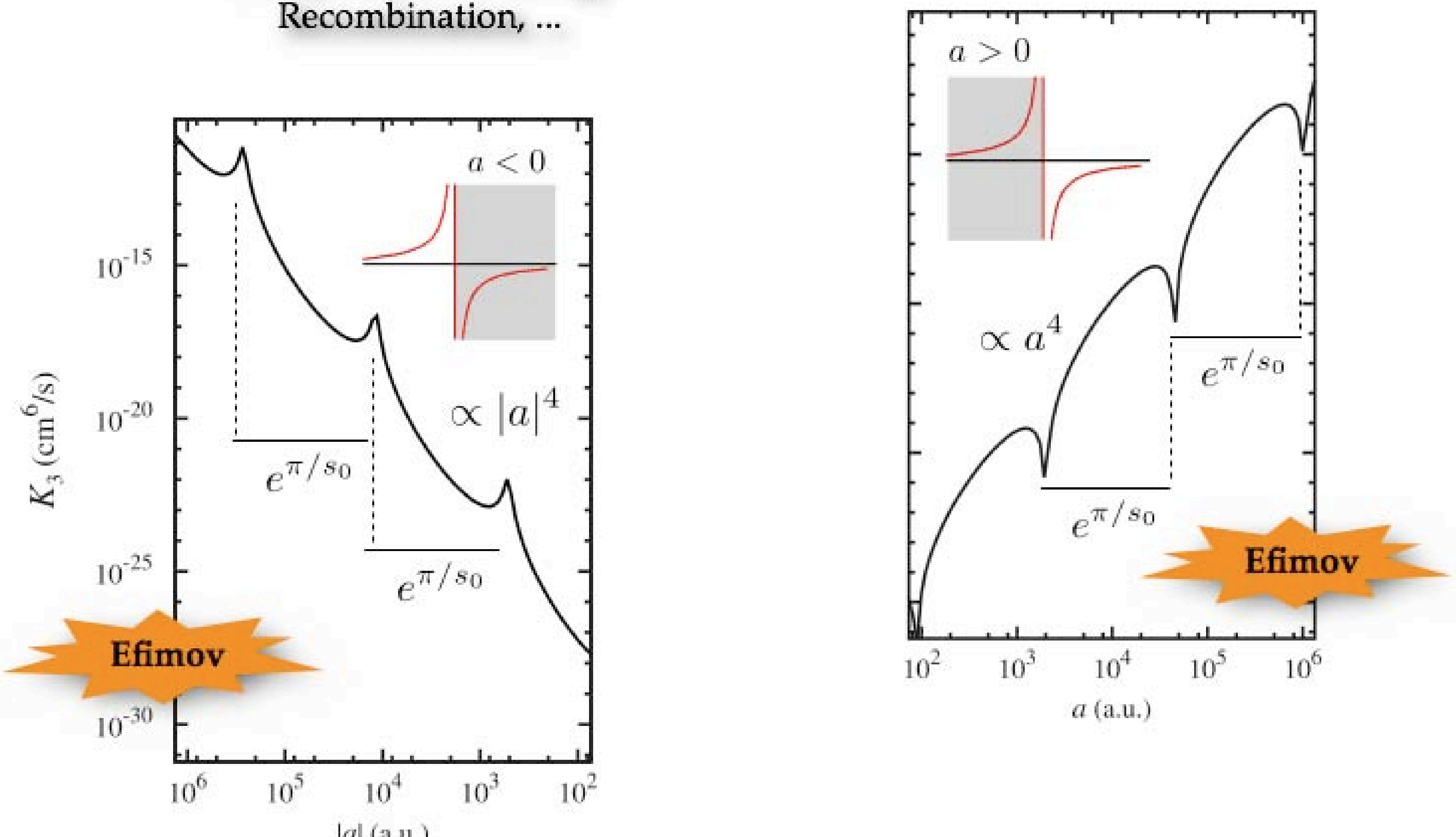
$\rightarrow$ Solve Schrödinger equation for $R \approx 10^6 a_0$ (0.05mm!!!)

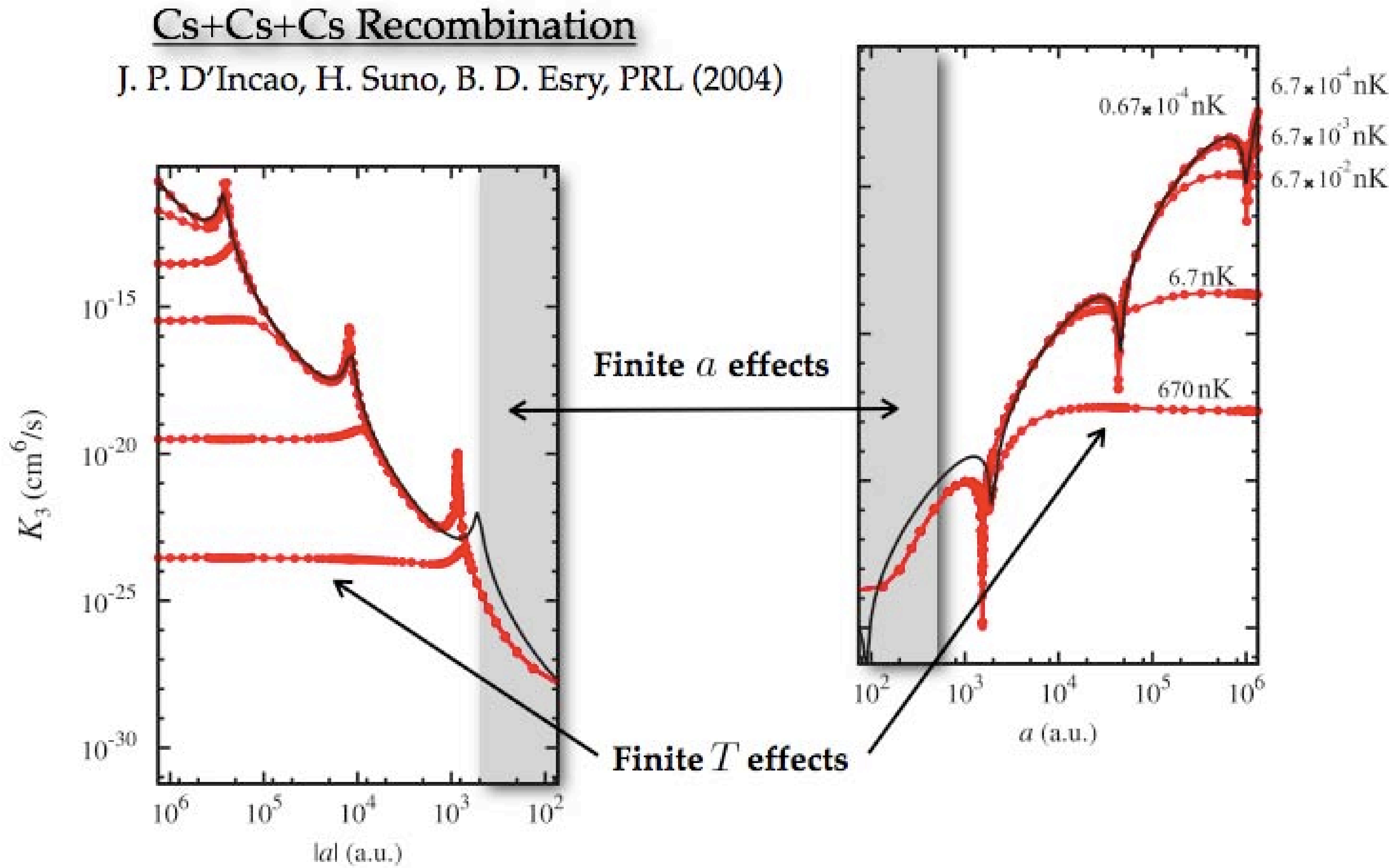
Finite range & Finite temperature effects



Finite range & Finite temperature effects

Analytical, Zero-Energy Recombination, ...





Cs+Cs+Cs Recombination
J. P. D'Incao, H. Suno, B. D. Esry, PRL (2004)
K_3 (cm^6/s)
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^6 10^5 10^4 10^3 10^2
$|a|$ (a.u.)
Finite a effects
Finite T effects
0.67×10^{-4}nK
6.7×10^{-4}nK
6.7×10^{-3}nK
6.7×10^{-2}nK
6.7nK
670nK
10^2 10^3 10^4 10^5 10^6
a (a.u.)

Universal Properties ... from the Hyperspherical view point

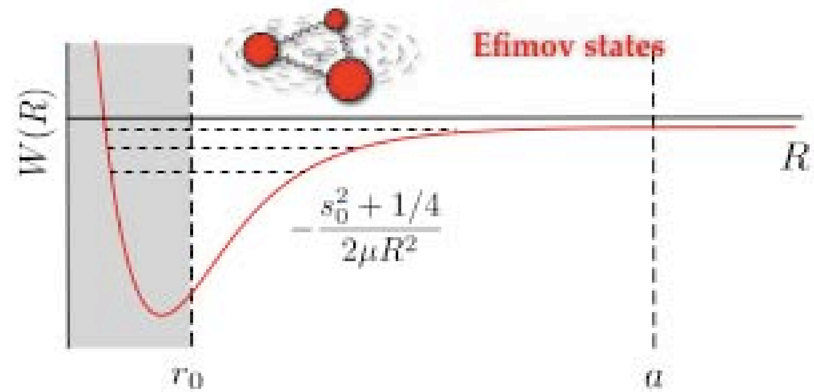
Efimov Physics: Universality

Efimov Physics: Universality

<u>Efimov Physics</u> = appearance of an ***attractive*** or ***repulsive*** three-body effective interaction ... $(|a| \gg r_0)$

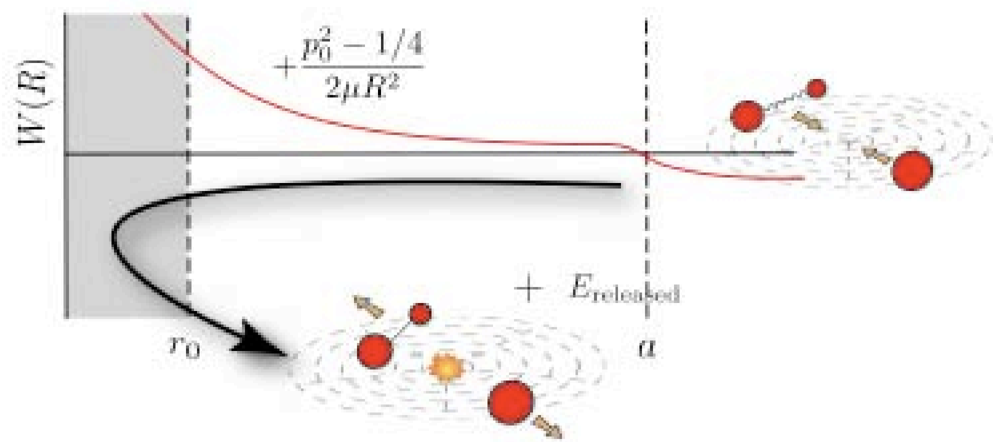
<u>Classification</u>: *all* three-body systems (with s-wave int.) fall into one of the two categories !!!

(Attractive)



$$BBB,\ BBF,\ FFf\ (\delta < 0.0735),\ \ldots$$

(Repulsive)



$$FFF',\ BFF,\ FFf\ (\delta < 0.0735),\ \ldots$$

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Efimov Physics: Universality

Efimov Physics = appearance of an *attractive* or *repulsive* three-body effective interaction ... $(|a| \gg r_0)$

Near threshold behavior $(\mathrm{E} \lesssim 1/2\mu \mathrm{a}^2)$:
valid for all s – wave systems !!!

		Relaxation			Recombination		
	J^π	E	$a>0$	$a<0$	E	$a>0$	$a<0$
BBB	0^+	const	$a^{\triangle}$	?	const	$a^{4\triangle}$	$\|a\|^{4\triangle}$
	1^-	k^2	?	?	k^6	?	?
	2^+	k^4	?	?	k^4	$a^{8\blacktriangle}$	?
BBB'	0^+	const	?	?	const	?	?
	1^-	k^2	?	?	k^2	?	?
	2^+	k^4	?	?	k^4	?	?
FFF'	0^+	const	$a^{-3.332\blacklozenge}$	?	k^4	?	?
	1^-	k^2	?	?	k^2	$a^{6\lozenge}$	?
	2^+	k^4	?	?	k^4	?	?

E-dep. Esry, Greene, and Suno (2002); △Fedichev, Reynolds, and G. V. Shlyapnikov (1996); Esry, Greene, and Burke, (19 Nielsen and Macek (1999); Bedaque, Braaten, and Hammer (2000); Braaten and Hammer (2001,2004); ▲D'Incao, Suno, nd Esry, (2004); ◊Petrov (2003); ♦Petrov, Salomon, and Shlyapnikov (2004).

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	J^π	E	$a>0$	$a<0$	E	$a>0$	$a<0$
BBB	0^+	**const**	$\boldsymbol{a}^{\triangle}$	**const**	**const**	$\boldsymbol{a}^{4\triangle}$	$\boldsymbol{\vert a\vert}^{4\triangle}$
	1^-	k^2	$a^{-2.728}$	const	k^6	a^{10}	$\vert a\vert^{4.272}$
	2^+	k^4	$a^{-0.647}$	const	k^4	$a^{8\blacktriangle}$	$\vert a\vert^{2.353}$
BBB'	0^+	**const**	$\boldsymbol{a}$	**const**	**const**	$\boldsymbol{a}^4$	$\boldsymbol{\vert a\vert}^4$
	1^-	k^2	$a^{-1.558}$	const	k^2	a^6	$\vert a\vert^{1.443}$
	2^+	k^4	$a^{-0.815}$	const	k^4	a^8	$\vert a\vert^{2.815}$
FFF'	0^+	**const**	$\boldsymbol{a}^{-3.332\blacklozenge}$	**const**	k^4	a^8	$\vert a\vert^{3.668}$
	1^-	k^2	$a^{-0.546}$	const	$\boldsymbol{k}^2$	$\boldsymbol{a}^{6\lozenge}$	$\boldsymbol{\vert a\vert}^{2.455}$
	2^+	k^4	$a^{-1.210}$	const	k^4	a^8	$\vert a\vert^{1.790}$

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Heteronuclear systems: J. P. D'Incao, B. D. Esry, PRA(R) (2006)

Multiple species: J. P. D'Incao, B. D. Esry, PRL (2008)

Overlaping resonances: J. P. D'Incao, B. D. Esry, PRL (2009)

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... but that is not all !!!

<u>Heteronuclear systems</u>: J. P. D'Incao, B. D. Esry, PRA(R) (2006)

<u>Multiple species</u>: J. P. D'Incao, B. D. Esry, PRL (2008)

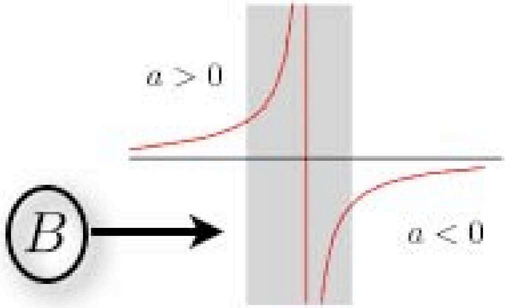
<u>Overlaping resonances</u>: J. P. D'Incao, B. D. Esry, PRL (2009)

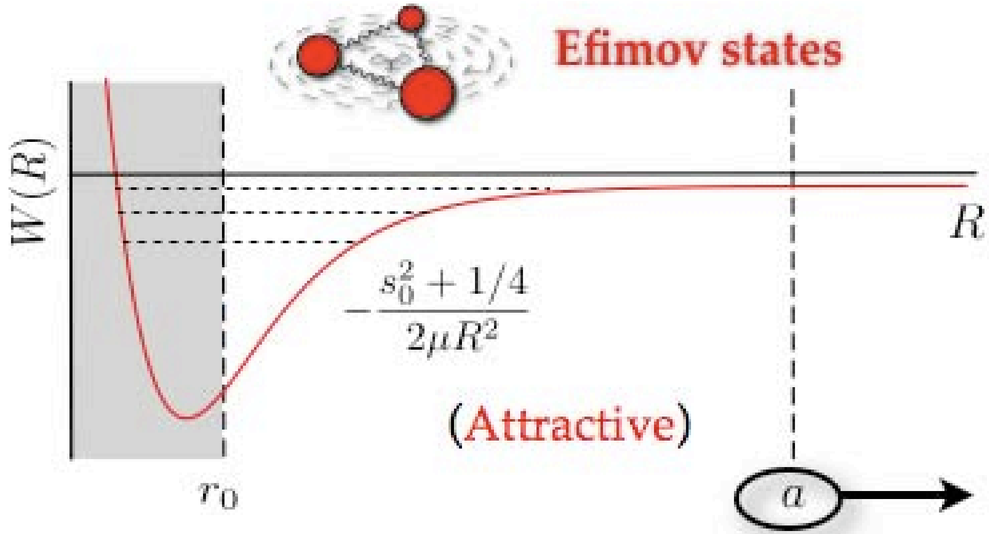
How does Efimov Physics affect Three-body Collisions ?

Intuitive Picture ...

Intuitive Picture ...

... roughly speaking ...





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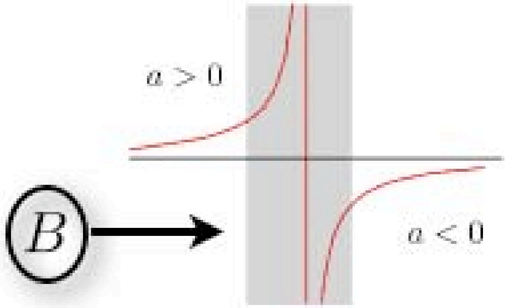
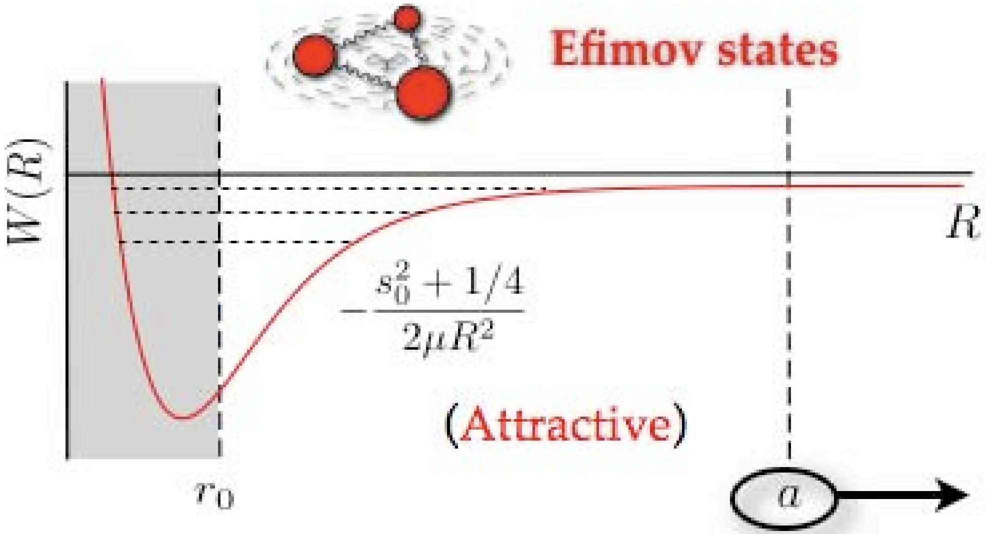
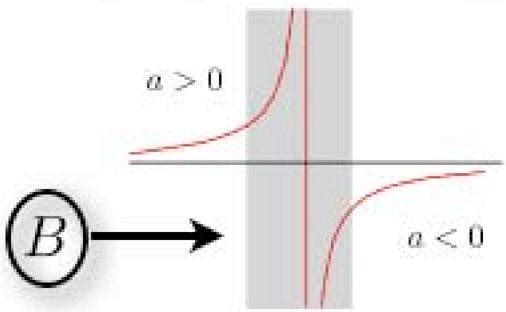


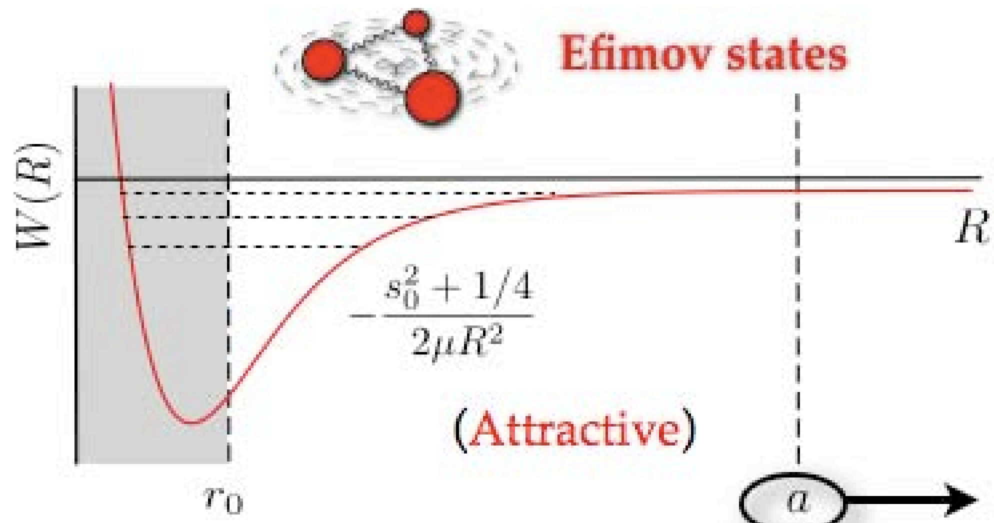


$a \rightarrow a \times e^{\pi/s_0}$ A new Efimov state appears ...

Intuitive Picture ...

... roughly speaking ...



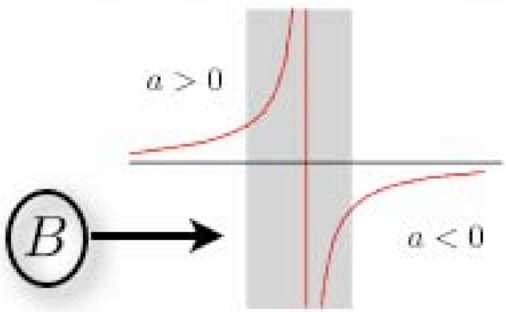


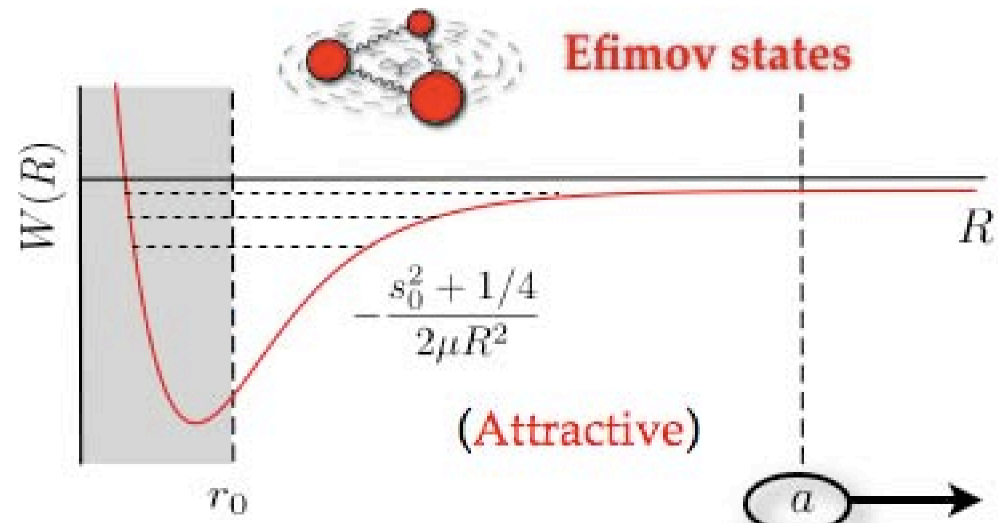
$a \to a \times e^{\pi/s_0}$ A new Efimov state appears ...

Geometrically spaced minima/peaks !!!

Intuitive Picture ...

... roughly speaking ...





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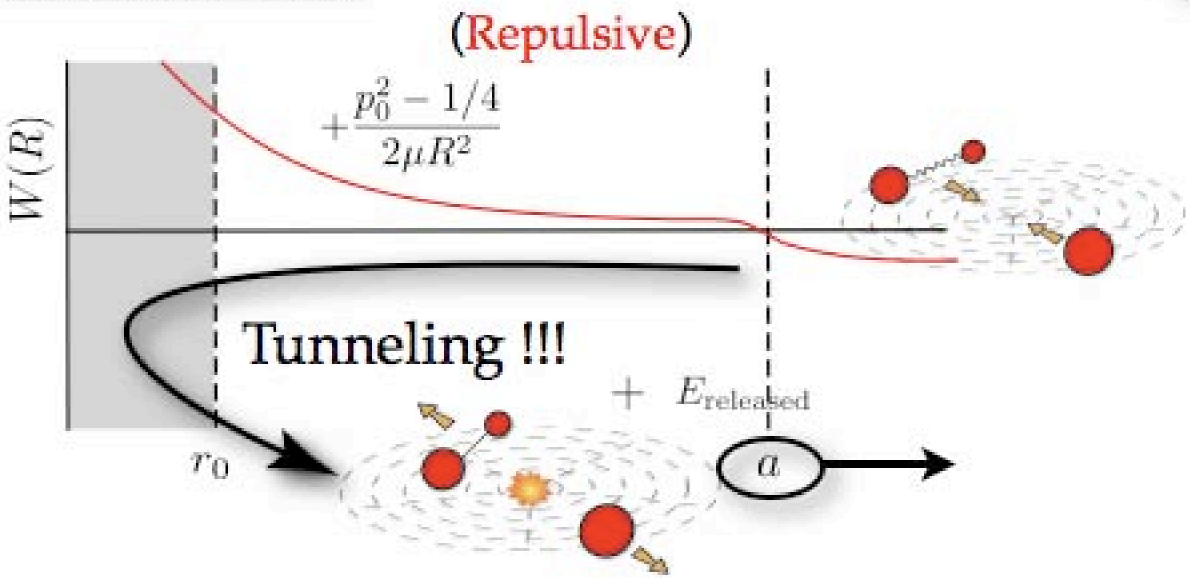
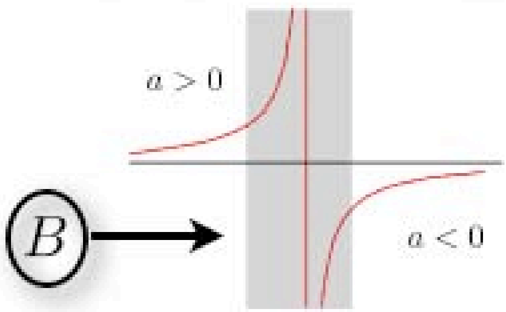
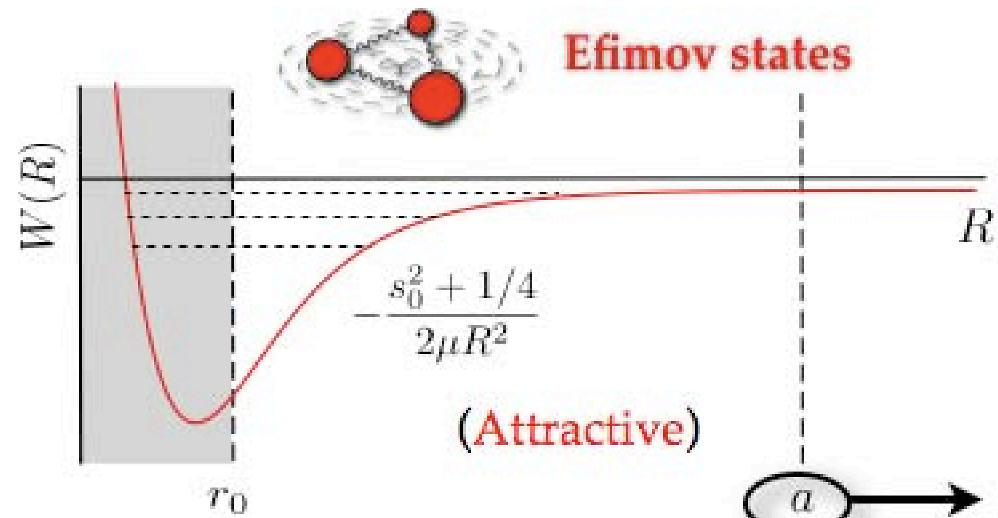
No Efimov states



... roughly speaking ...



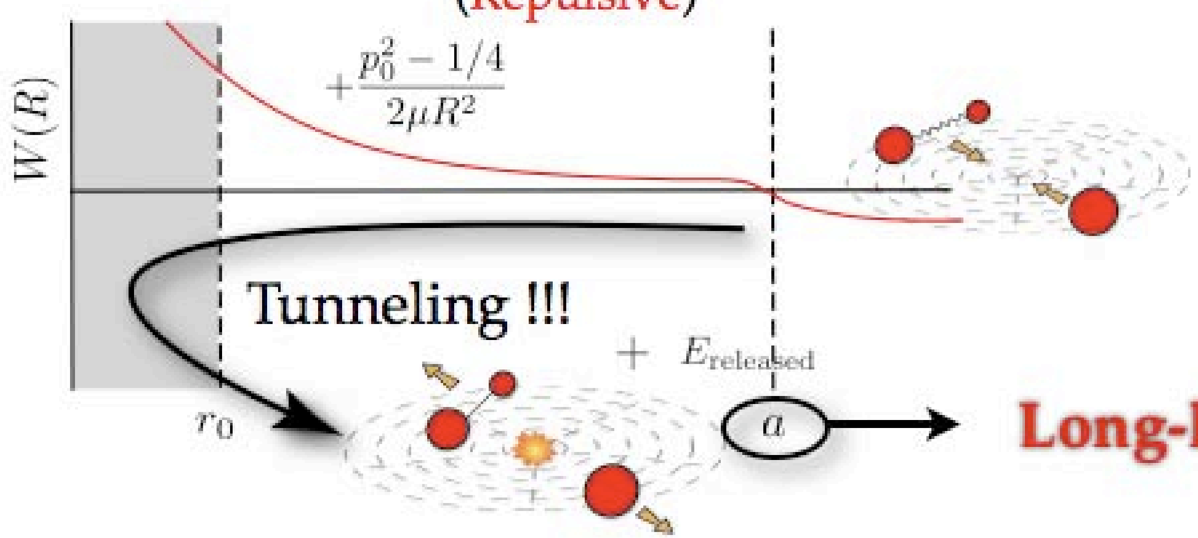


$a \to a \times e^{\pi/s_0}$ A new Efimov state appears ...

Geometrically spaced minima/peaks !!!

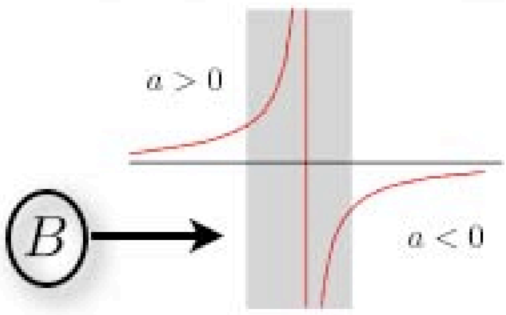
No Efimov states

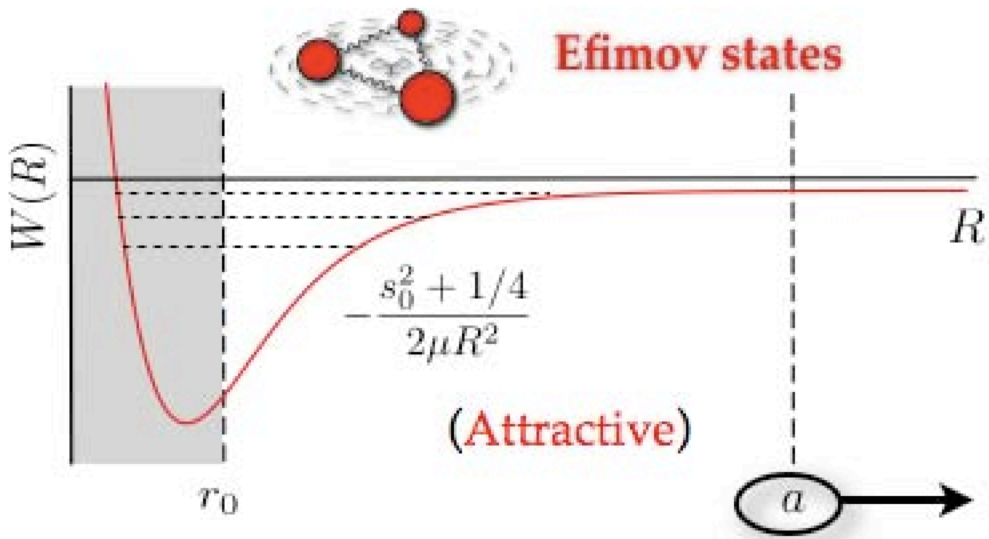
(Repulsive)



Long-lived molecules !!!

... roughly speaking ...





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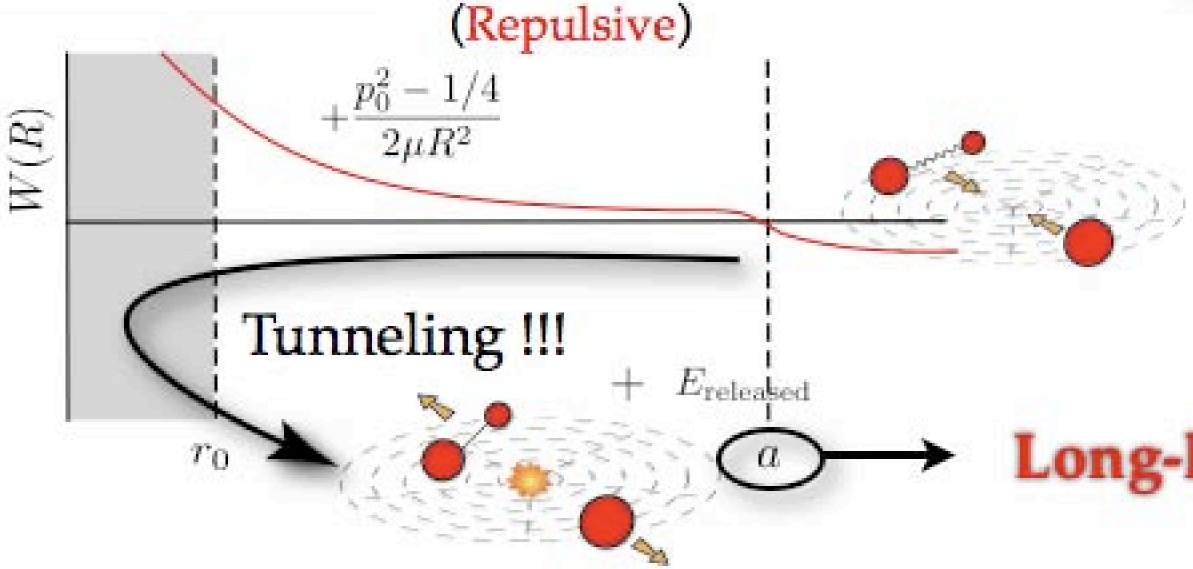


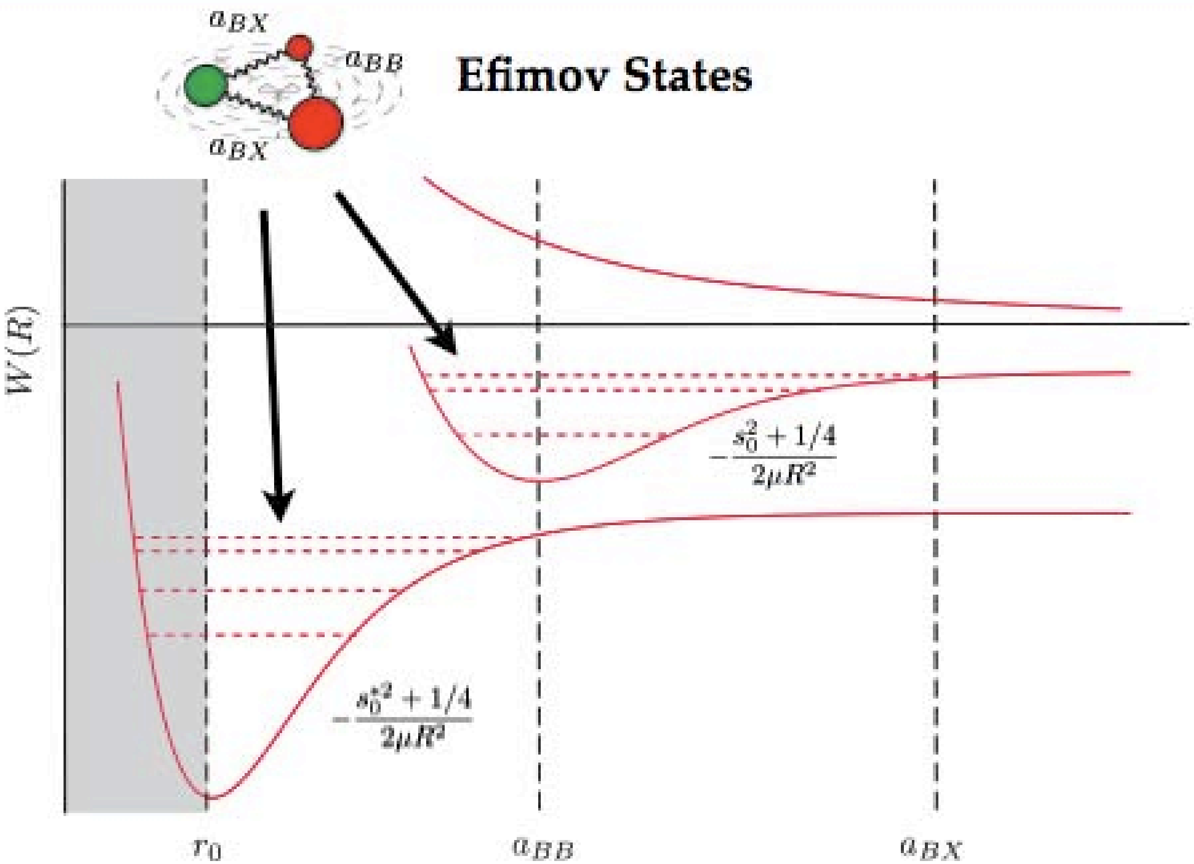
Signatures of Efimov Physics !!!

Long-lived molecules !!!

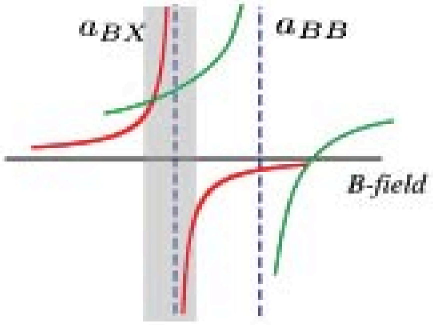
Three-body Collisions

Three-body Collisions

Efimov States



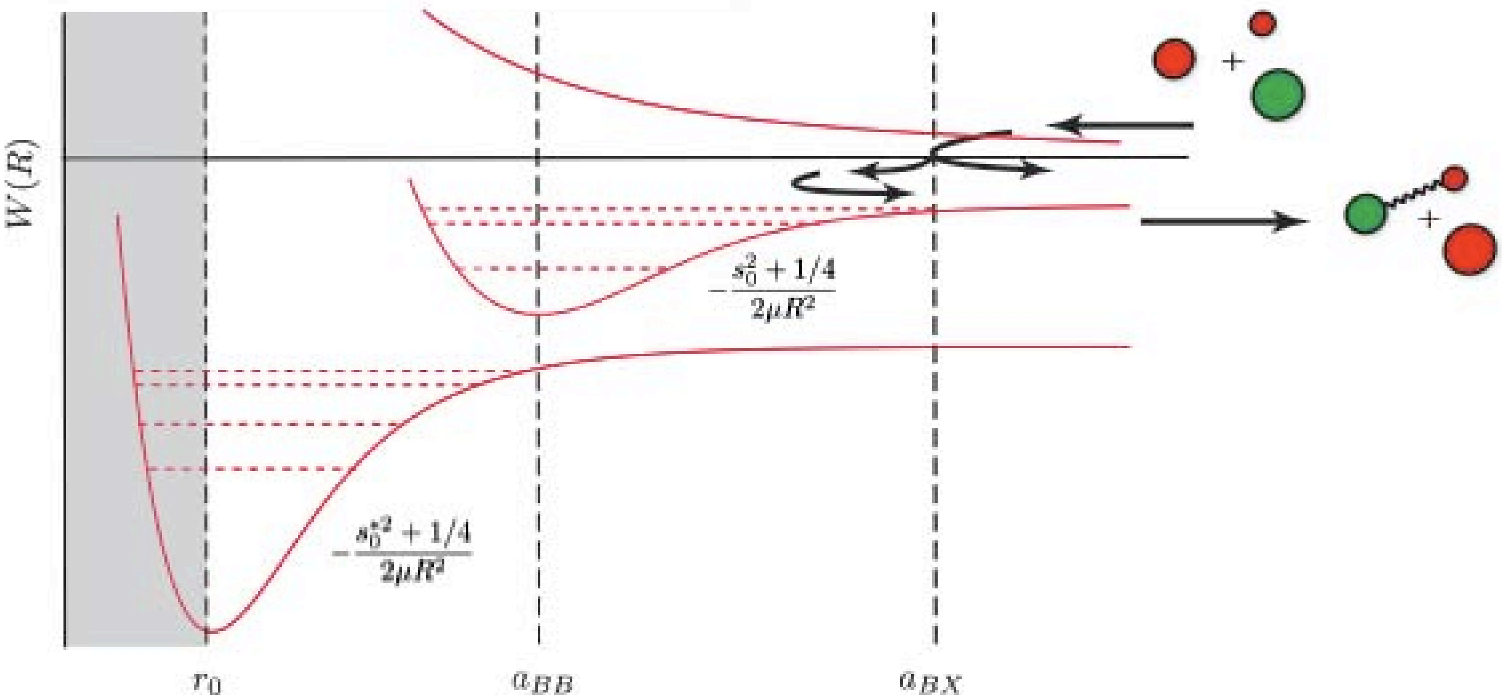
Overlapping resonances



D'Incao & Esry, PRL (2009)

Three-body Collisions

Three-body recombination



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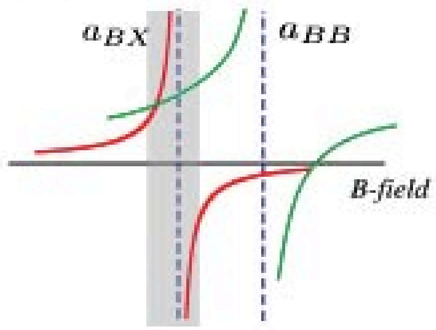

D'Incao & Esry, PRL (2009)

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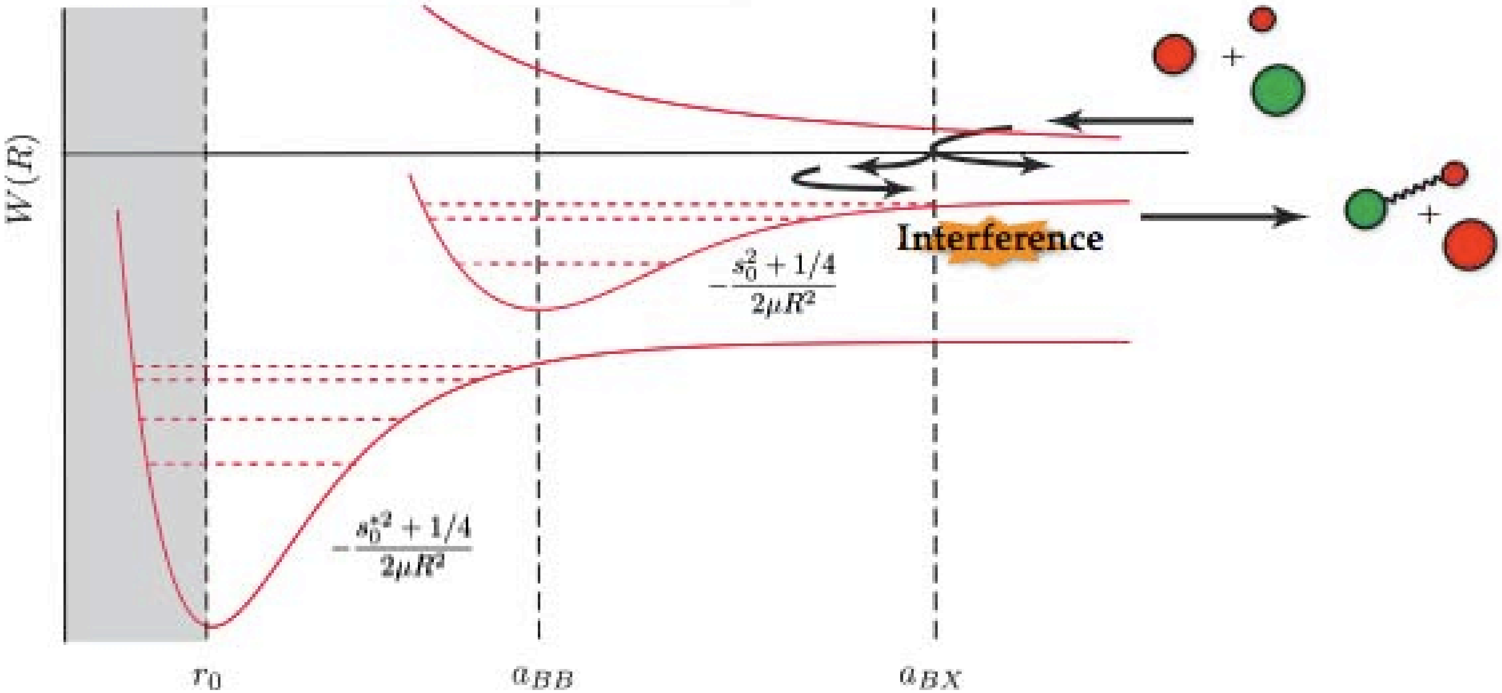
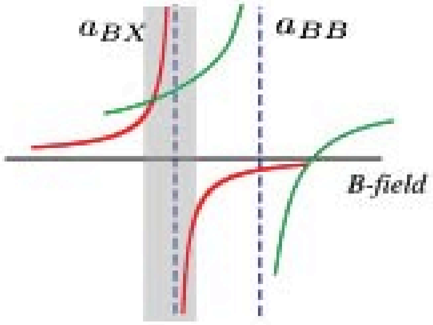


Overlapping resonances



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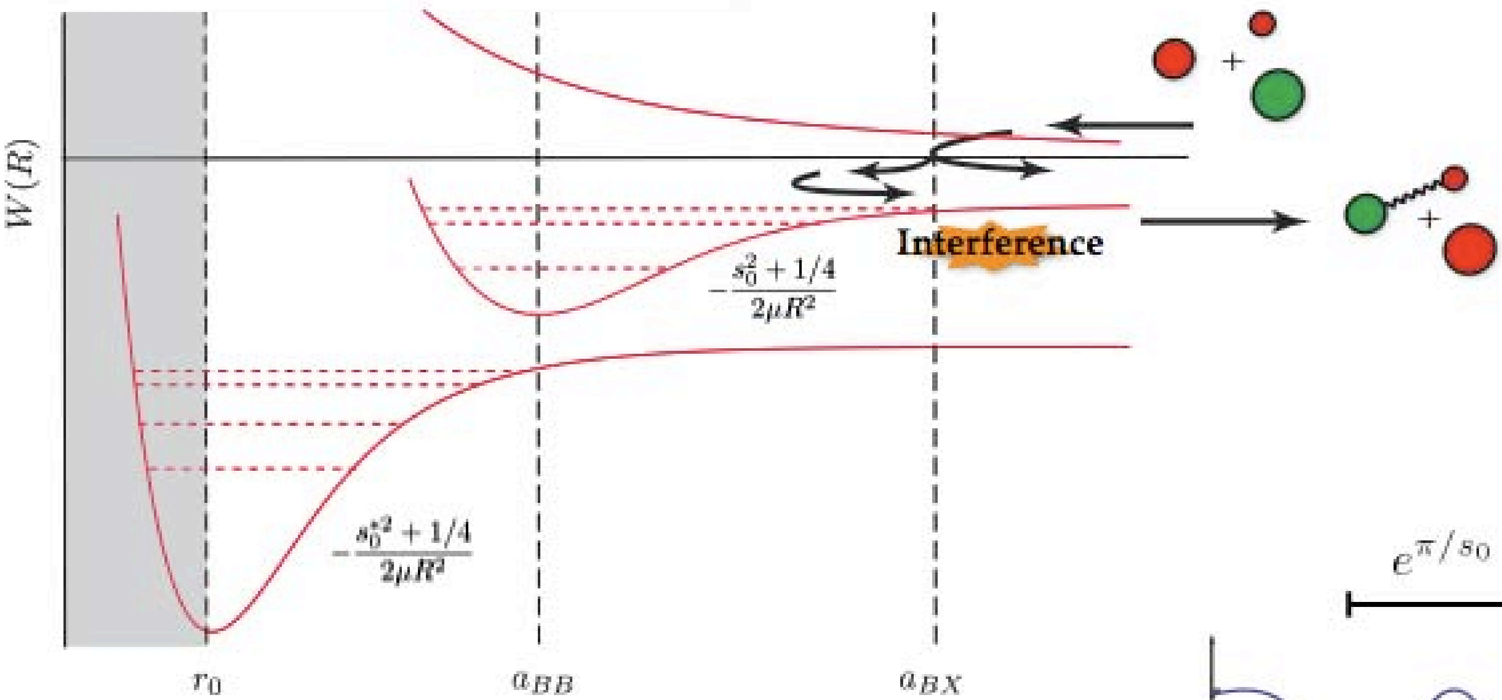
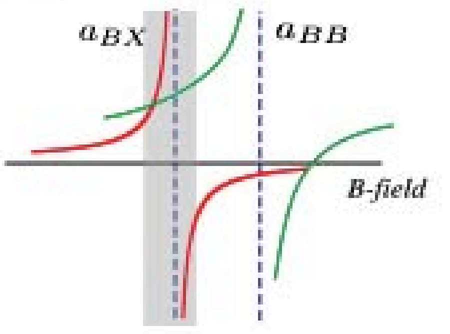
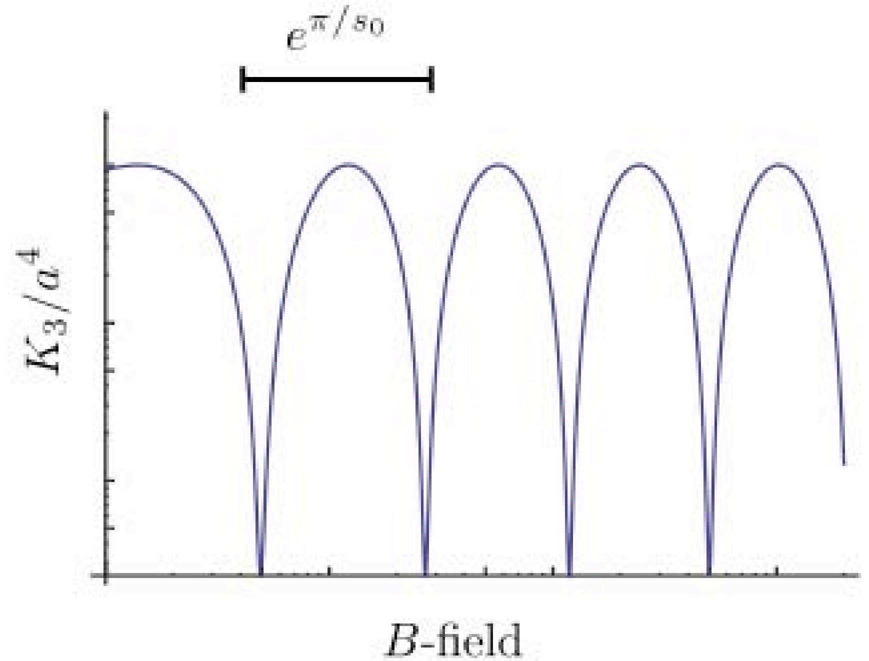


Overlapping resonances

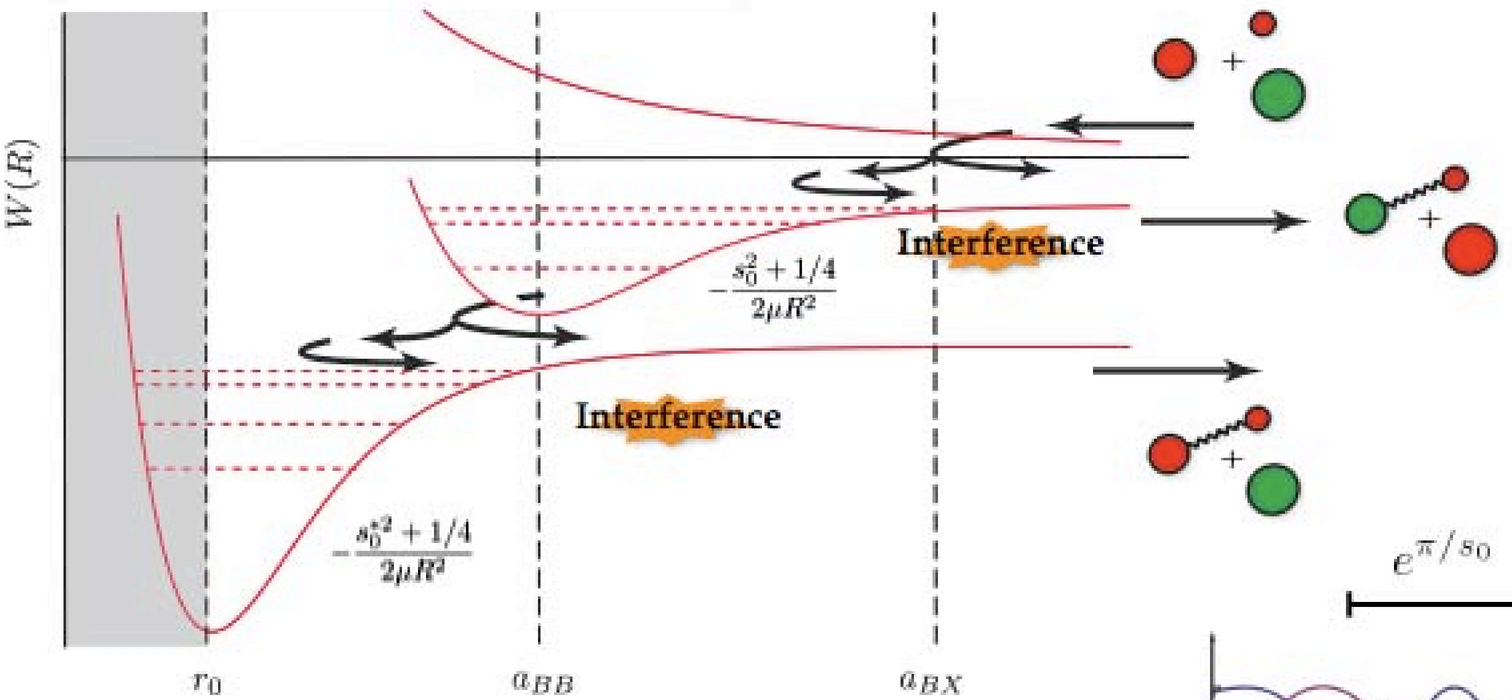


D'Incao & Esry, PRL (2009)

$$K_3^{BX+B} \propto \sin^2[s_0 \ln(a_{BX}/a_{BB}) + \Phi] a_{BX}^4$$



Three-body recombination



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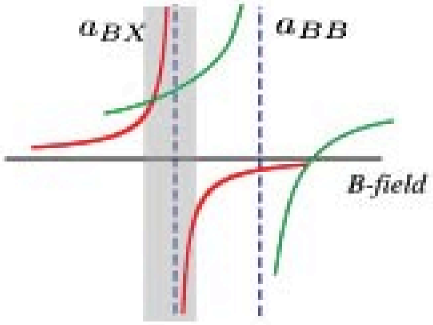
Overlapping resonances



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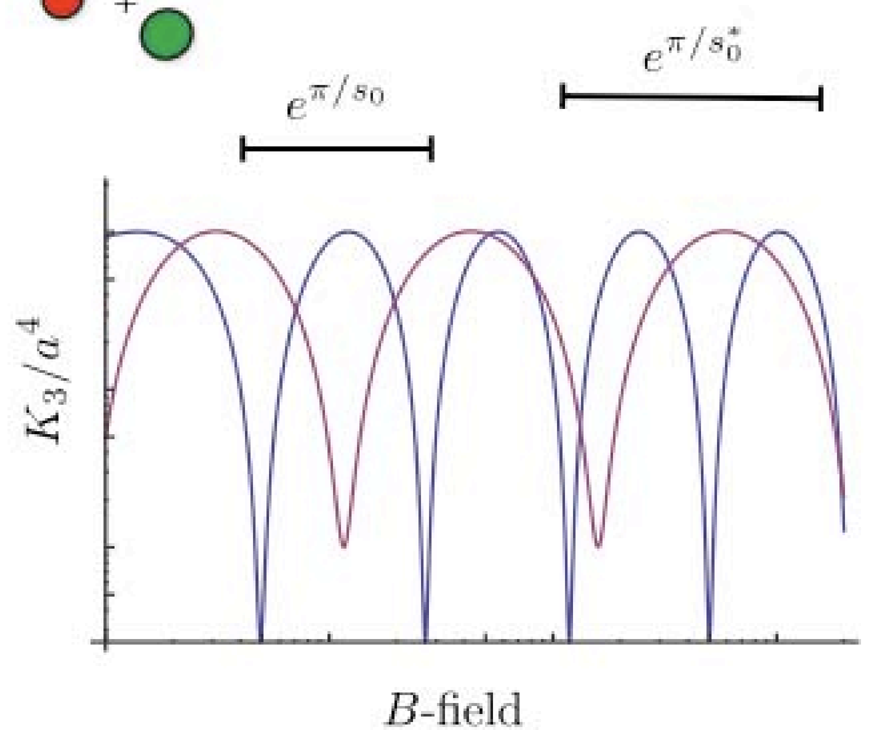

Three-body Collisions

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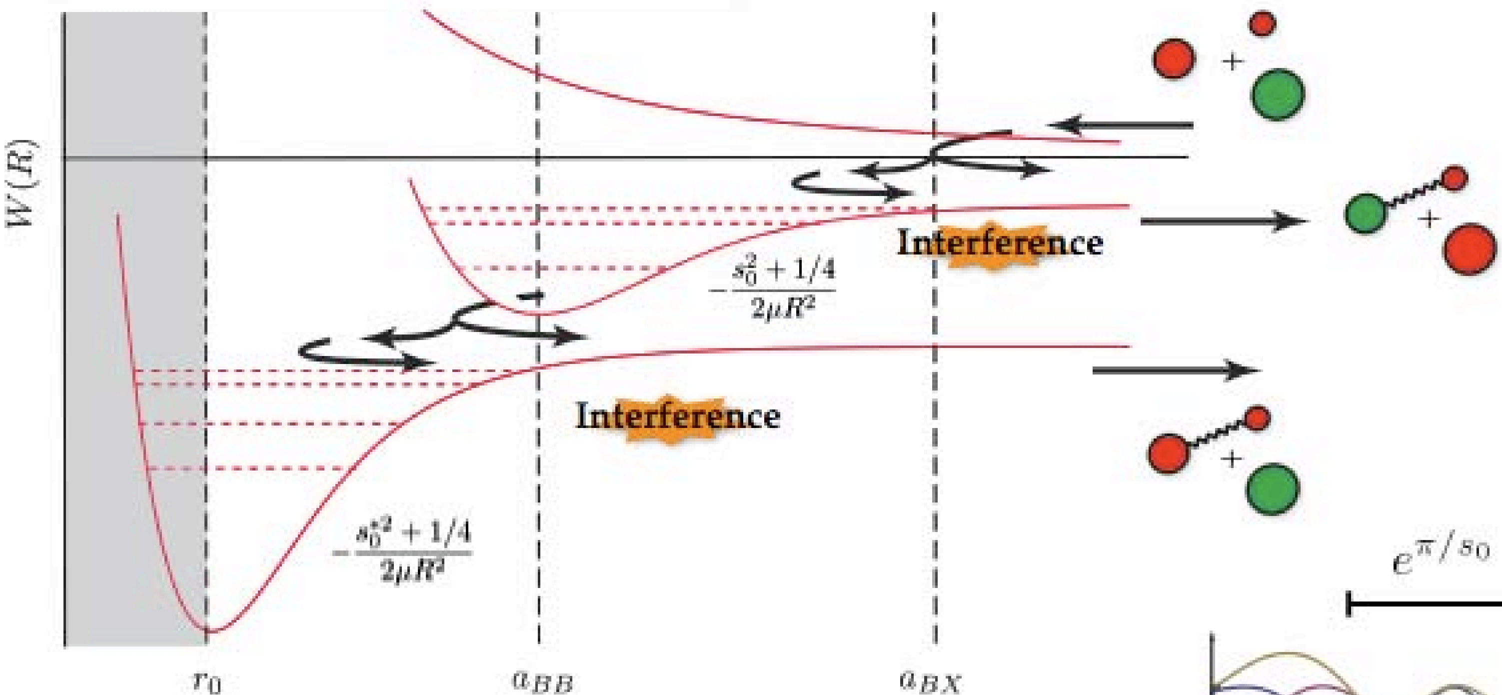

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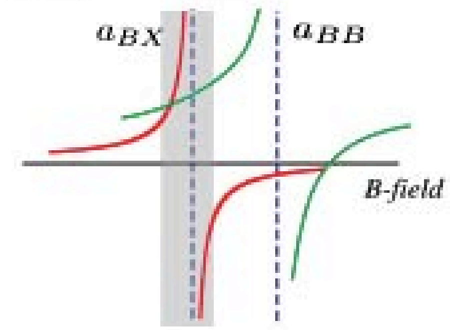

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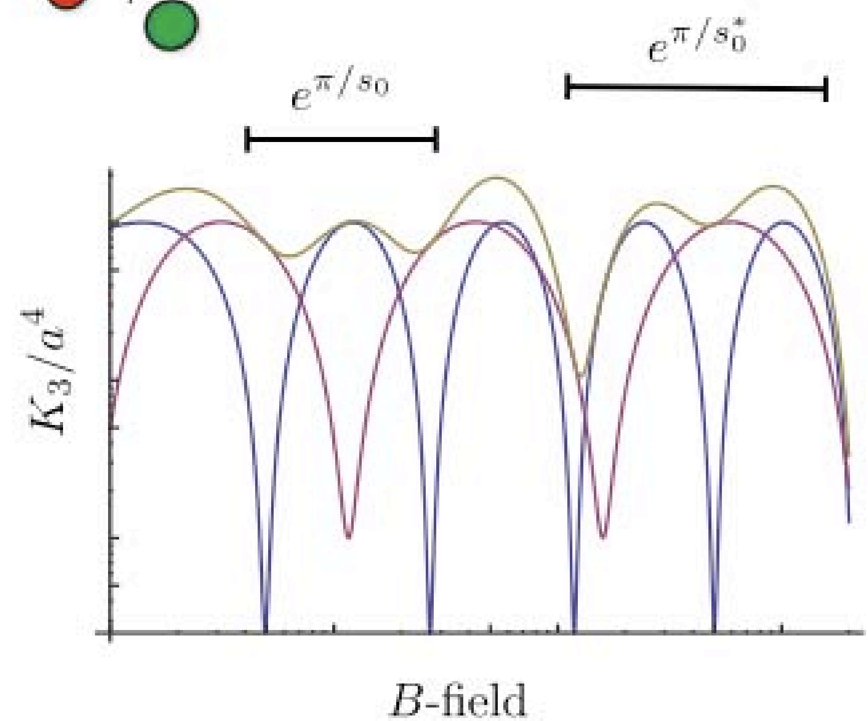
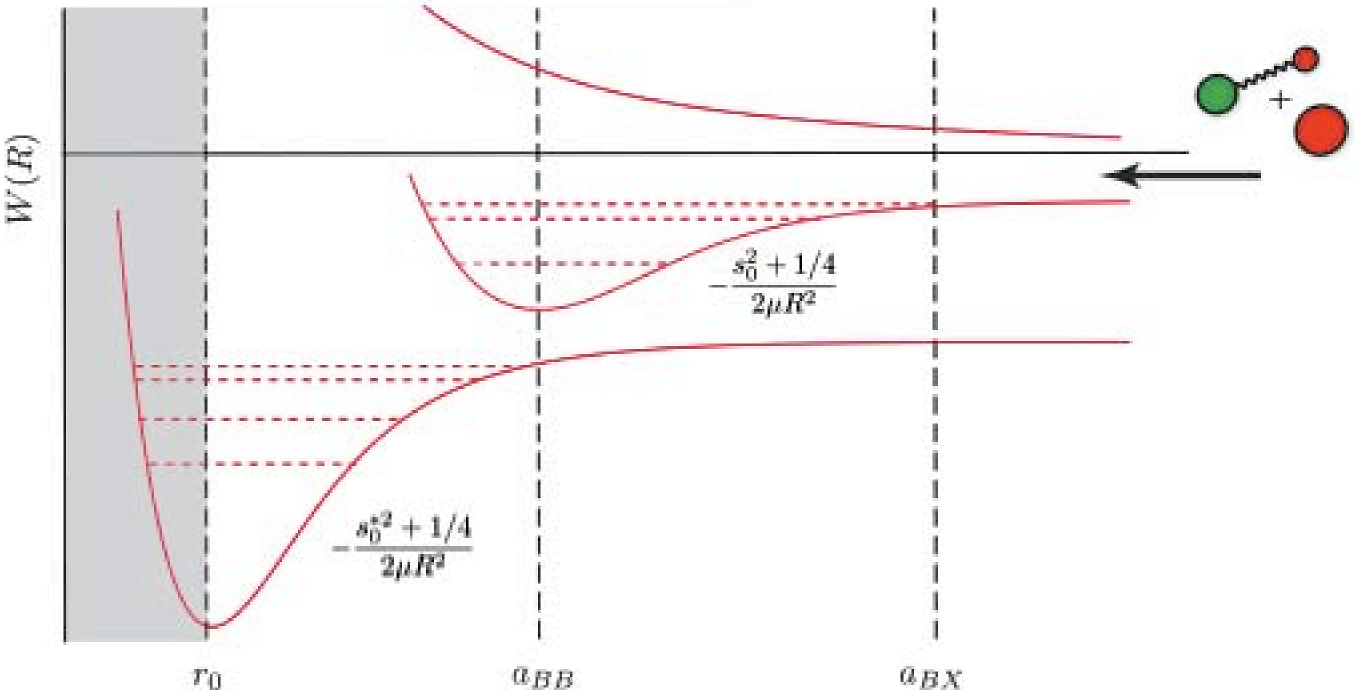
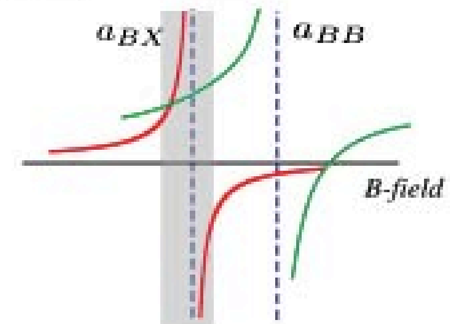


Three-body Collisions

Atom-dimer Collisions



Overlapping resonances



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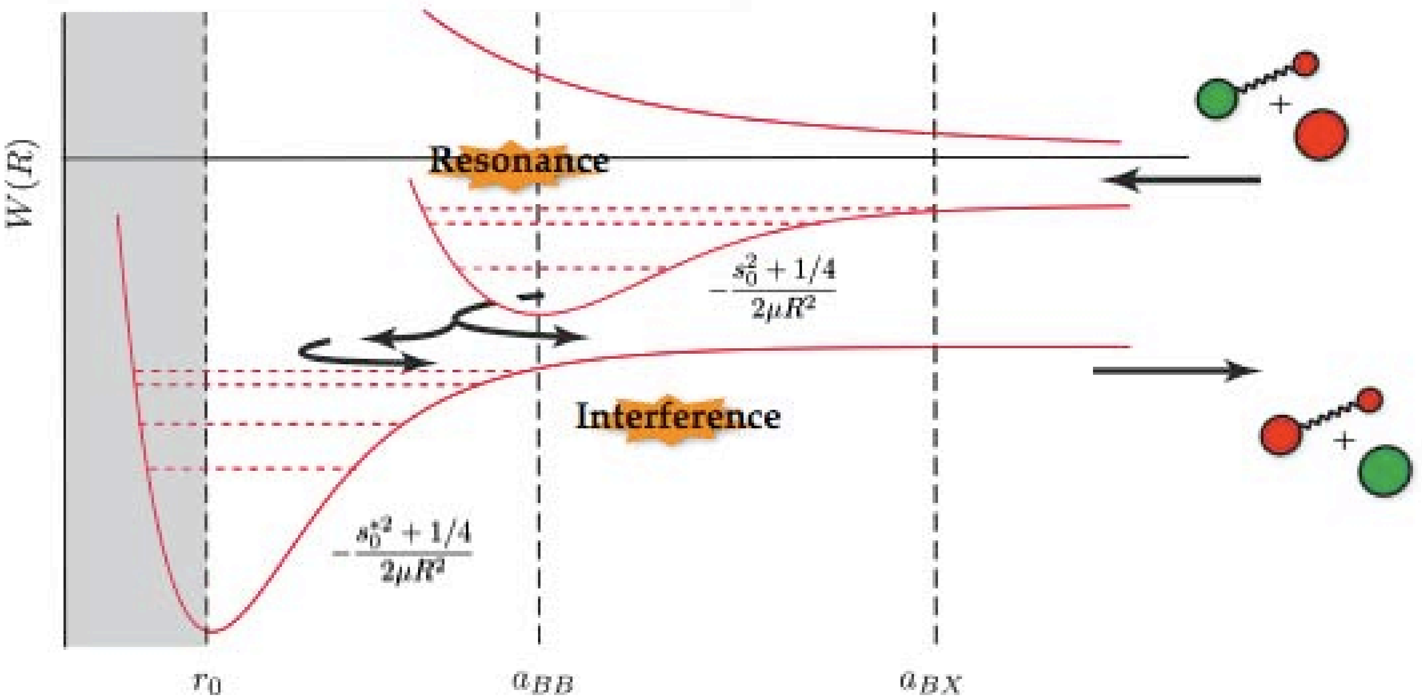

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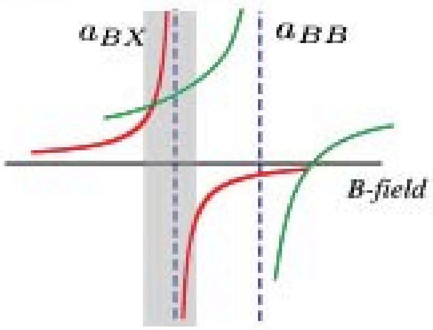

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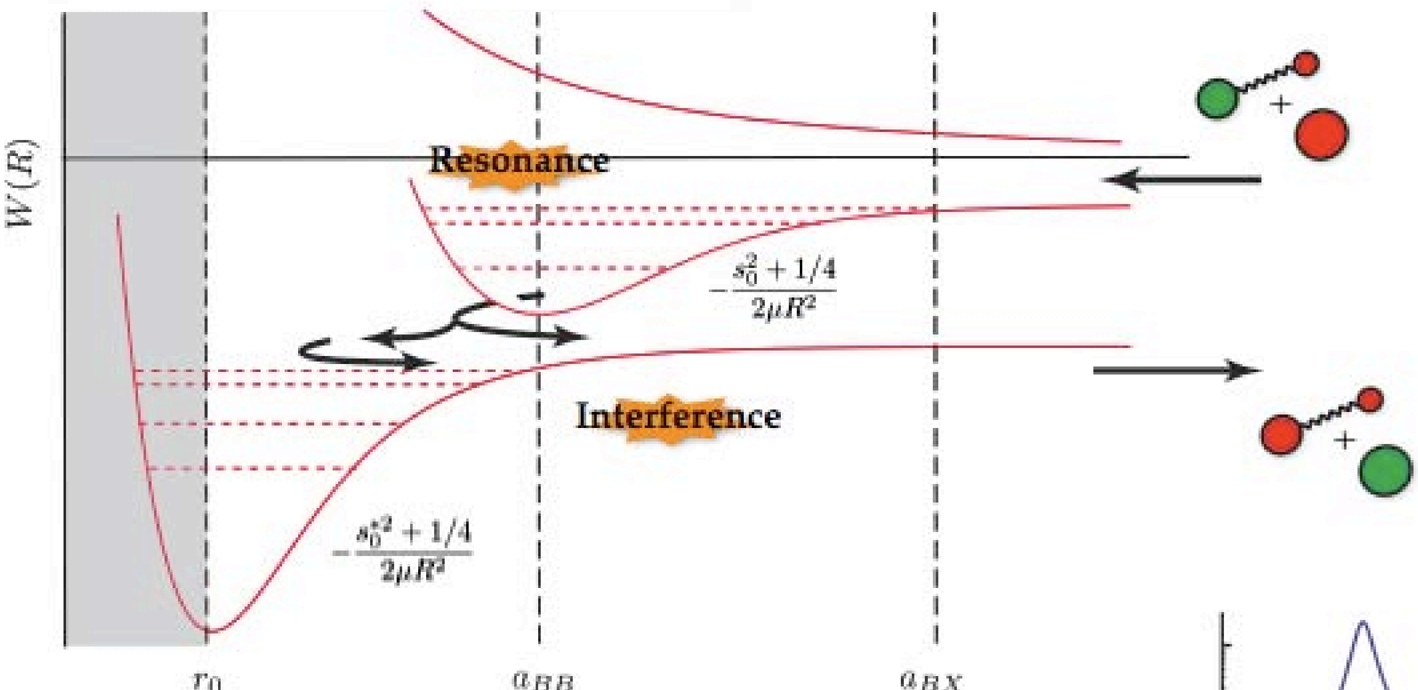

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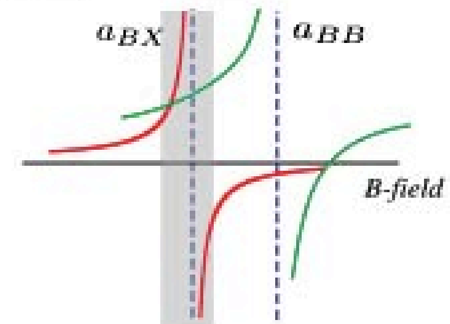

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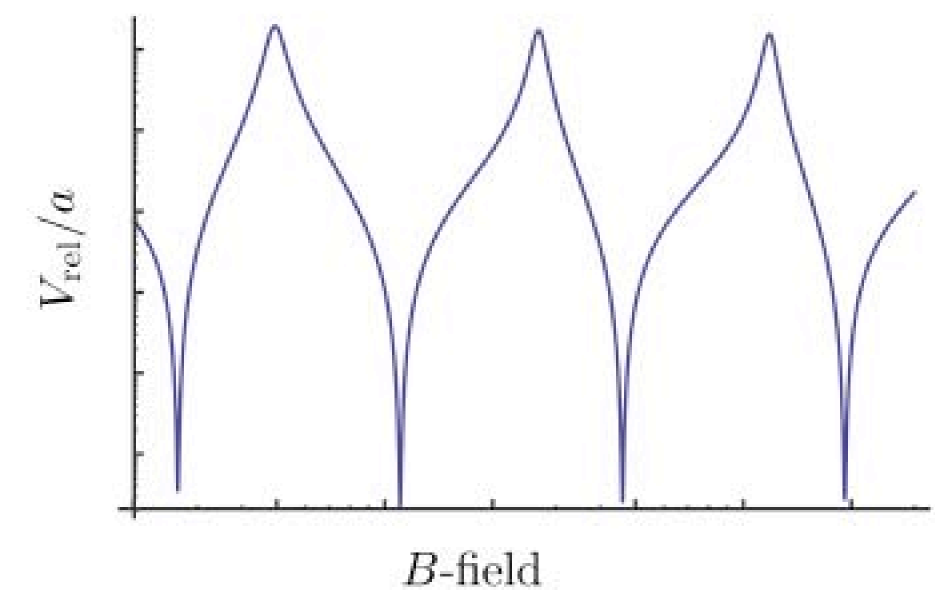

Three-body Collisions

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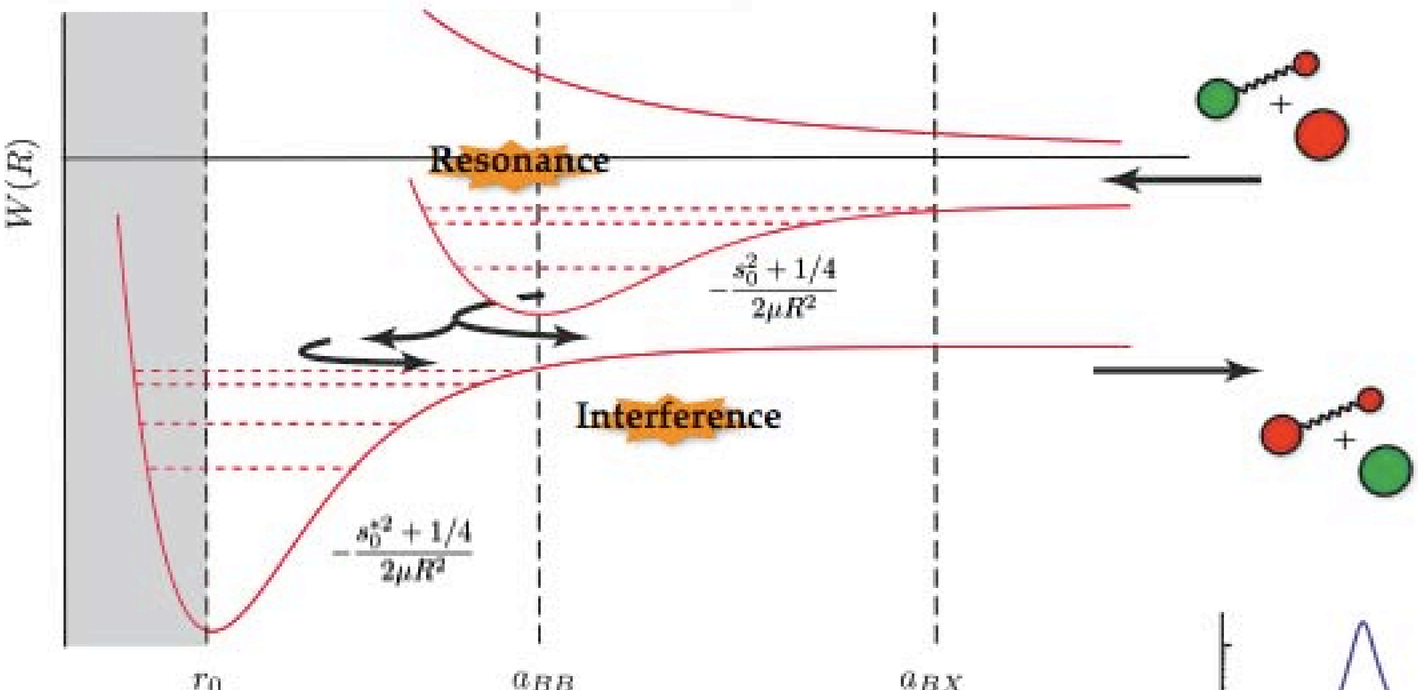

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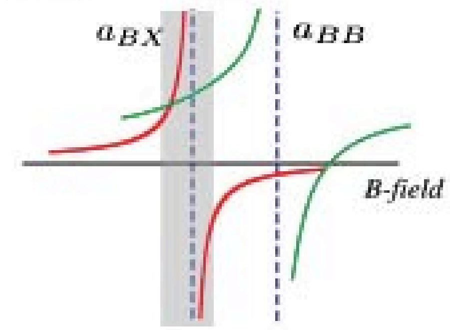

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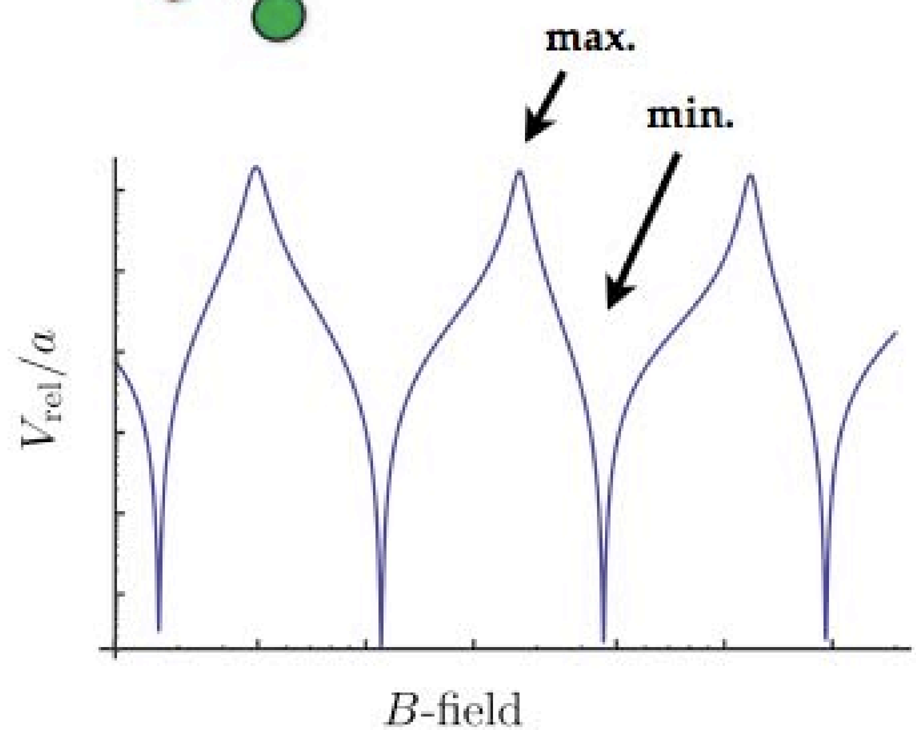
Exchange Reactions



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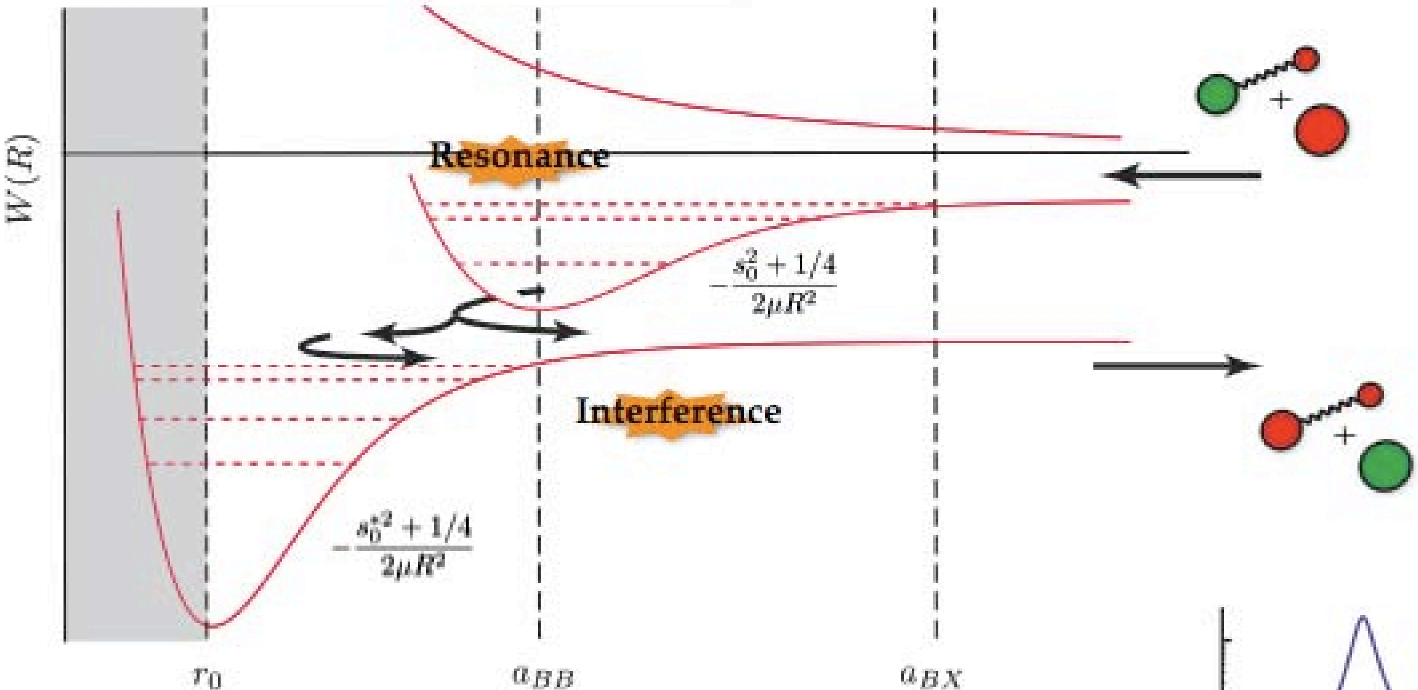

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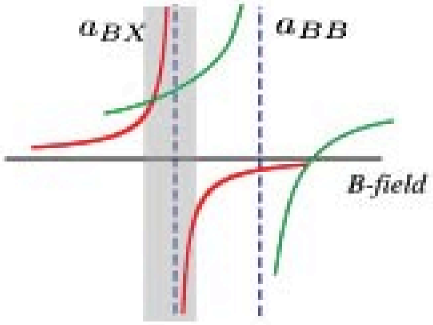

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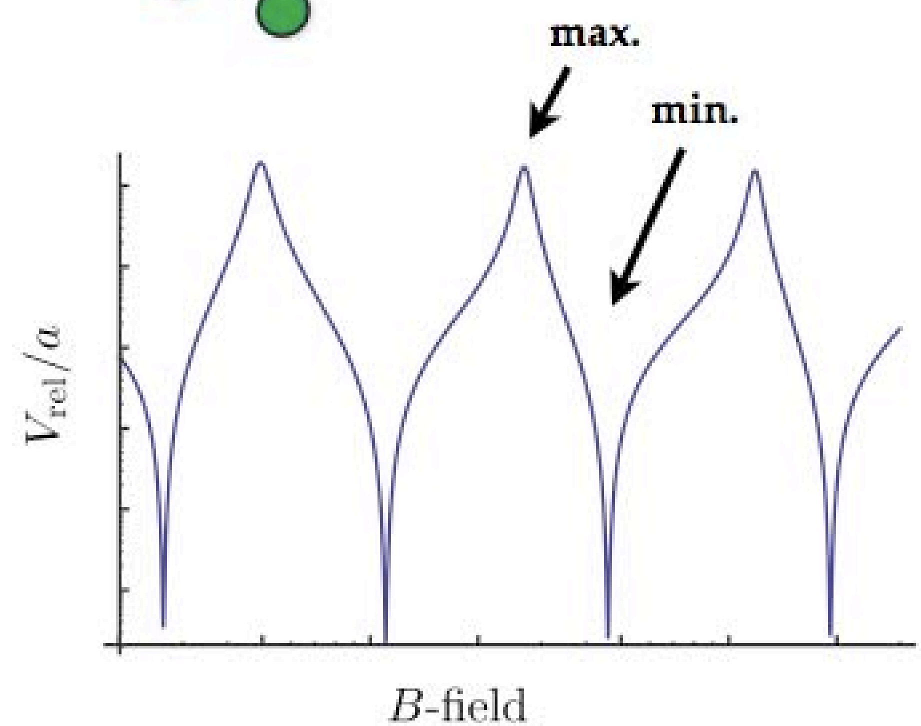
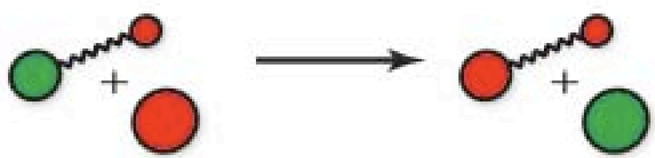


Exchange Reactions



... Efimov Chemistry !?

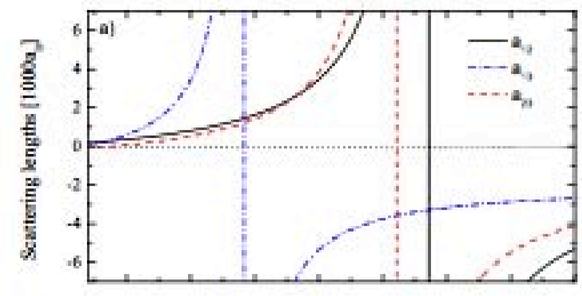
Atom-dimer Exchange reactions

Atom-dimer Exchange reactions

Exchange Reactions

Atom-Dimer Scattering in a Three-Component Fermi Gas

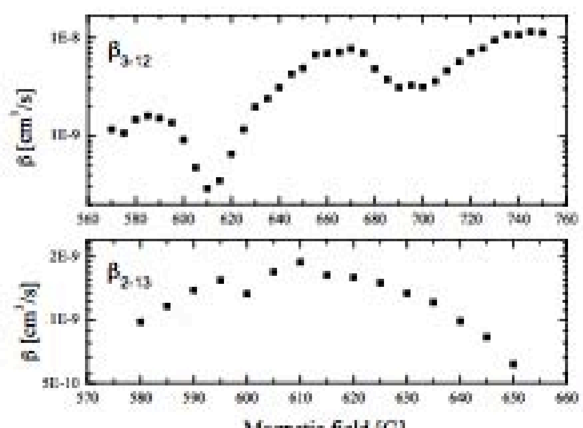
T. Lompe,[1,2,*] T. B. Ottenstein,[1,2] F. Serwane,[1,2] K. Viering,[3] A. N. Wenz,[1,2] G. Zürn,[1,2] and S. Jochim[1,2]
[1]*Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, Germany*
[2]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[3]*Department of Physics, The University of Texas at Austin, Austin, TX 78712*
(Dated: March 2, 2010)

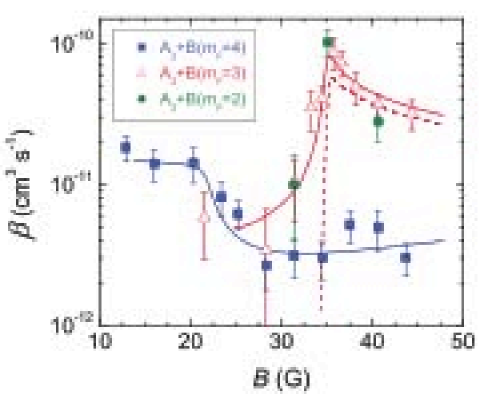


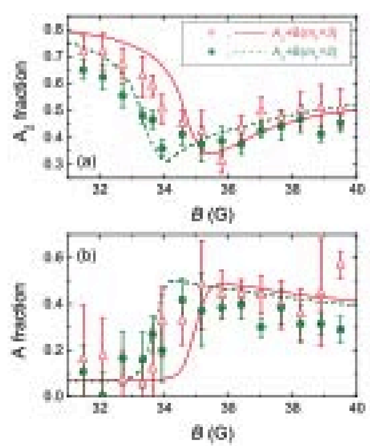
Selected for a Viewpoint in *Physics*
PRL **104**, 053201 (2010) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2010

Magnetically Controlled Exchange Process in an Ultracold Atom-Dimer Mixture

S. Knoop,[1,*] F. Ferlaino,[1] M. Berninger,[1] M. Mark,[1,†] H.-C. Nägerl,[1] R. Grimm,[1,2] J. P. D'Incao,[3] and B. D. Esry[4]
[1]*Institut für Experimentalphysik and Zentrum für Quantenphysik, Universität Innsbruck, 6020 Innsbruck, Austria*
[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
[3]*JILA, University of Colorado and NIST, Boulder, Colorado 80309-0440, USA*
[4]*Department of Physics, Kansas State University, Manhattan, Kansas 66506, USA*
(Received 10 November 2009; published 1 February 2010)







How does Efimov Physics extend to More bodies ?

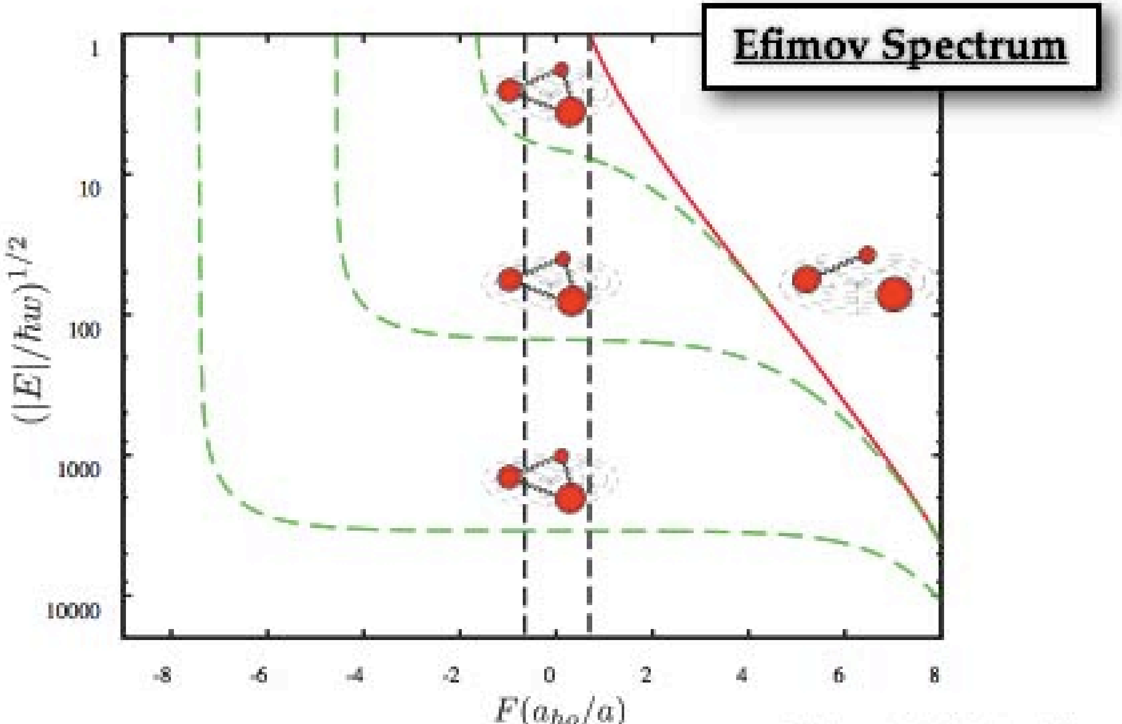
Some Questions ...

Some Questions ...

- **Geometrical scaling for *N*>3 ?**

$$E_n = E_{n-1}(e^{-\pi/s_0})^2$$

$$e^{\pi/s_0} \approx 22.7$$



$F(x) = \mathrm{sign}(x)\ln(1+|x|)$

Some Questions ...

- **Geometrical scaling for *N*>3 ?**

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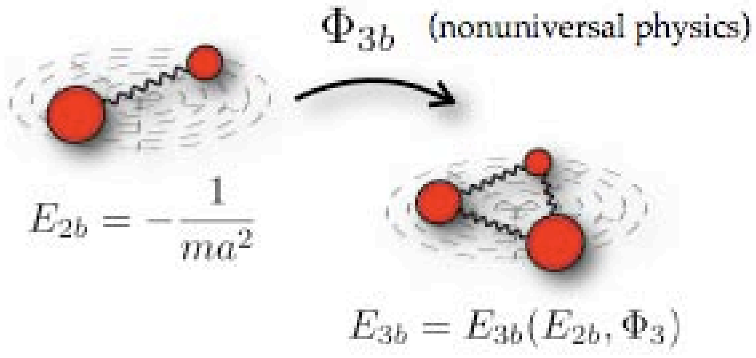
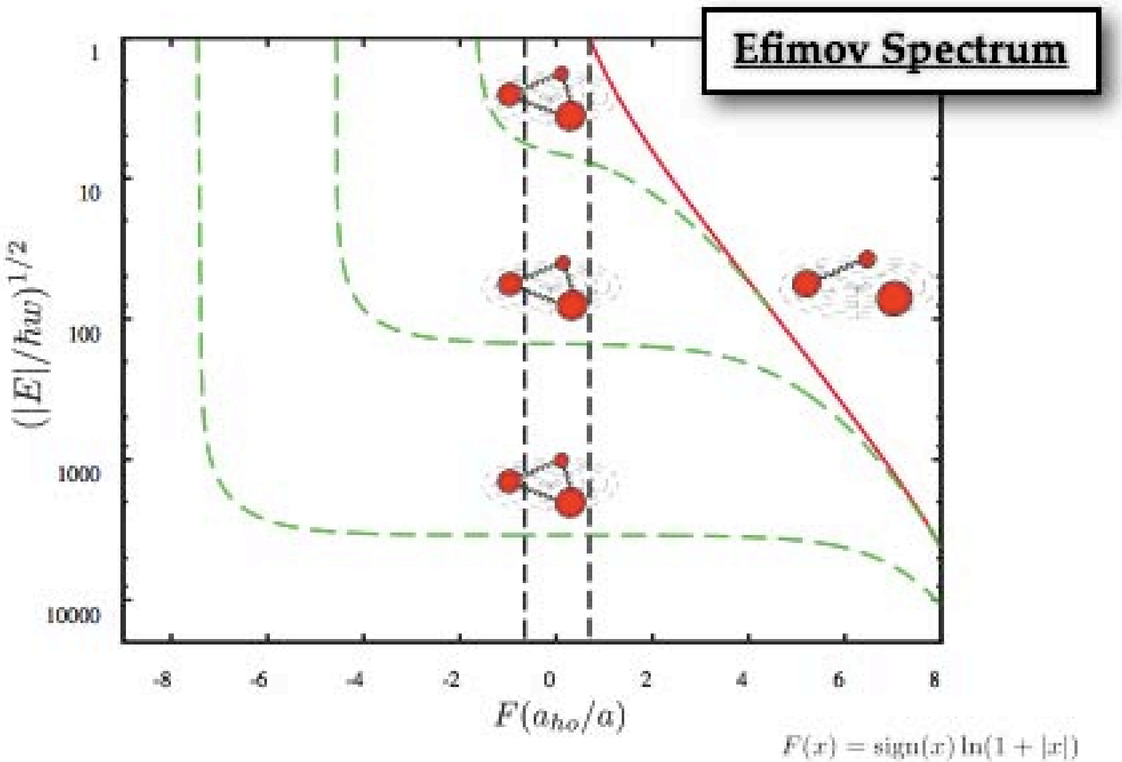
- **Universal properties ?**





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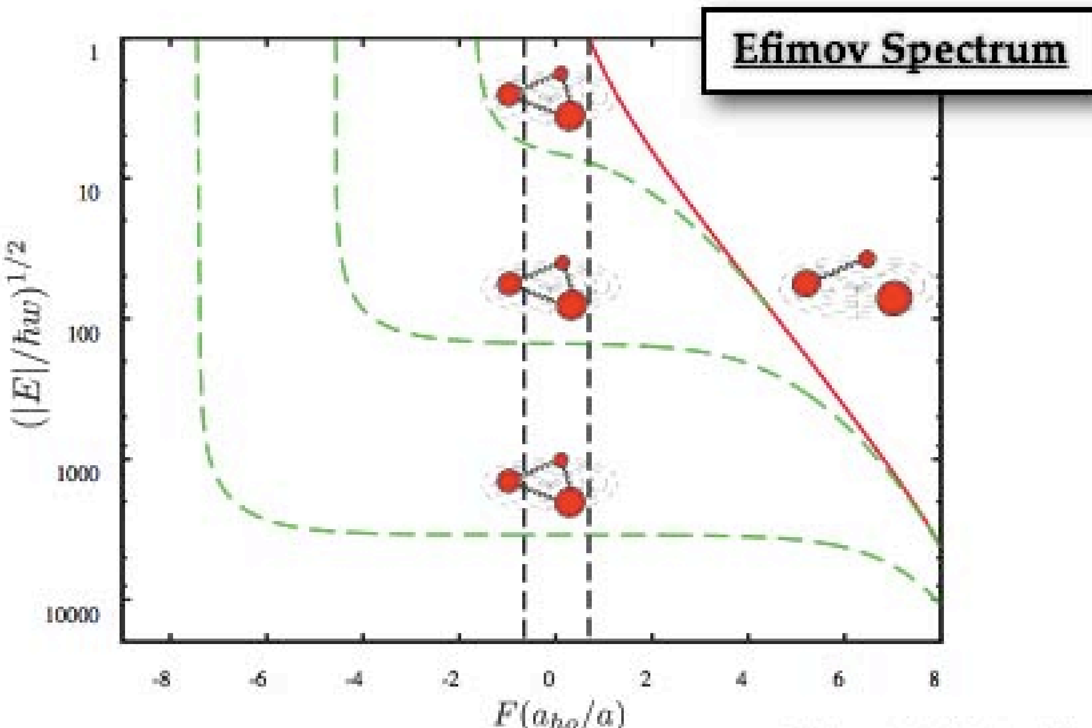

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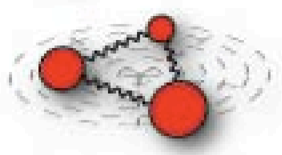
- **Universal properties ?**

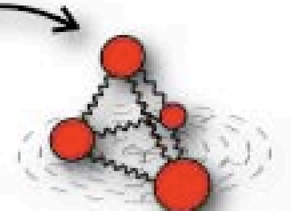
Φ_{3b} (nonuniversal physics)

$$E_{2b} = -\frac{1}{ma^2}$$



Φ_{4b} ?

$$E_{3b} = E_{3b}(E_{2b}, \Phi_3)$$



$$E_{4b} \stackrel{?}{=} E_{4b}(E_{3b}, \Phi_4)$$

Some Questions ...

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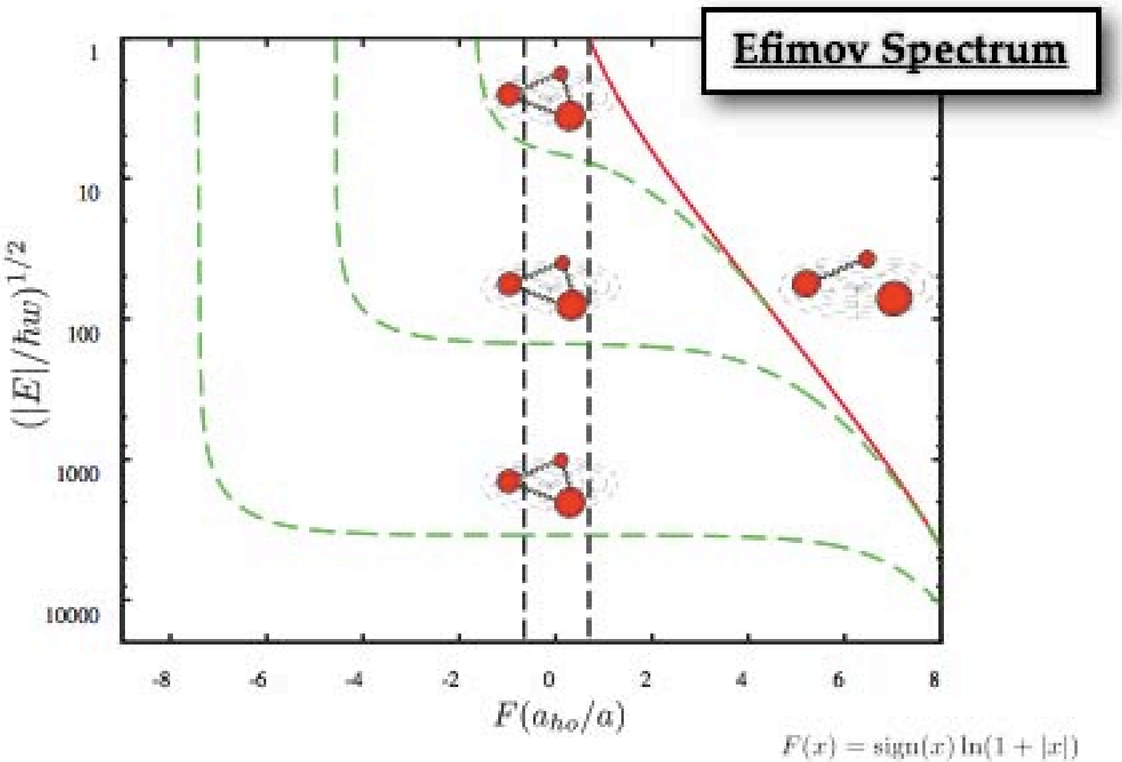

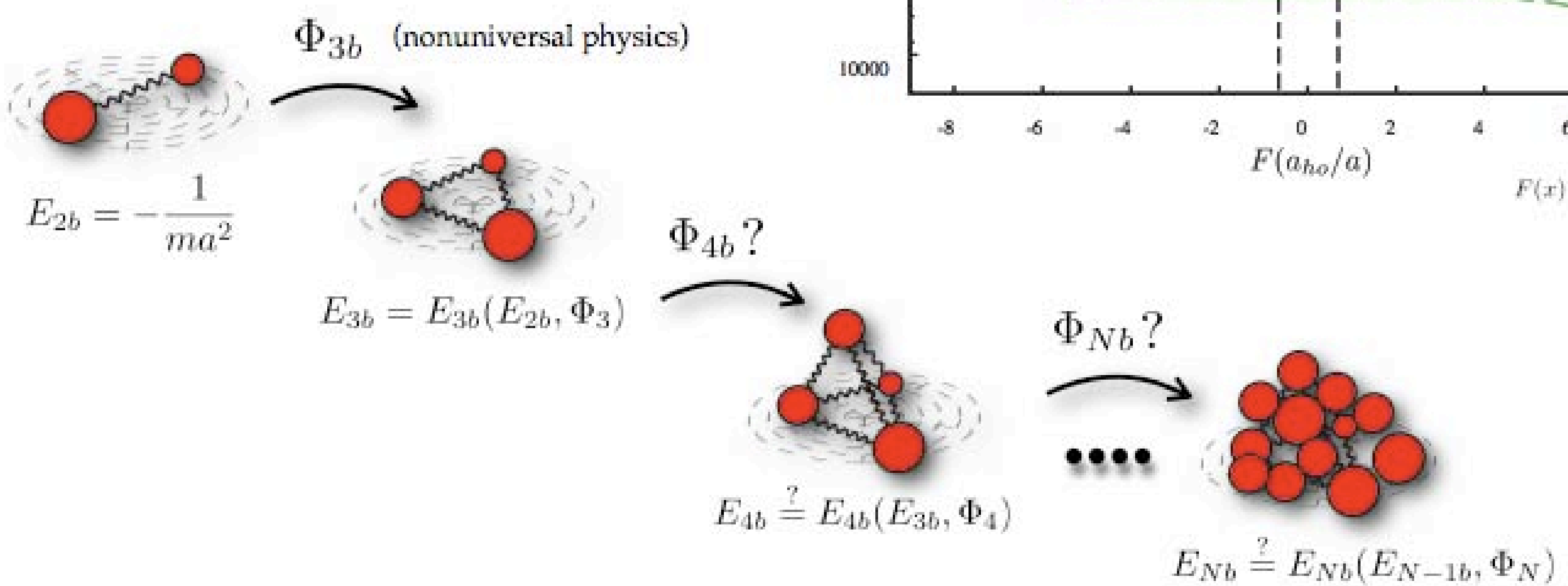

Universal Four-boson States

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Universal properties of the properties of the four-body system with large scattering lengths,
Hammer & Platter, *Eur. Phys. J. A* **32**, 113 (2007)

Signatures of universal four-body phenomena and its relation to the Efimov effect
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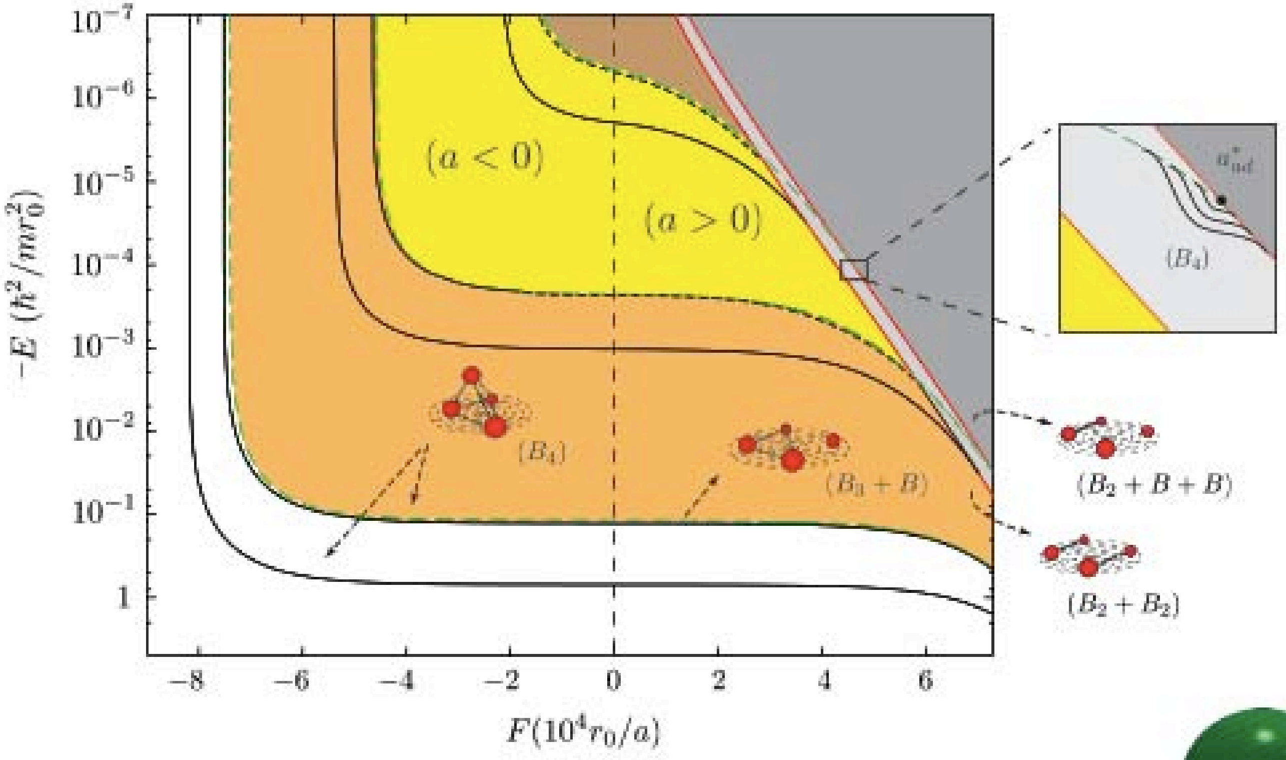
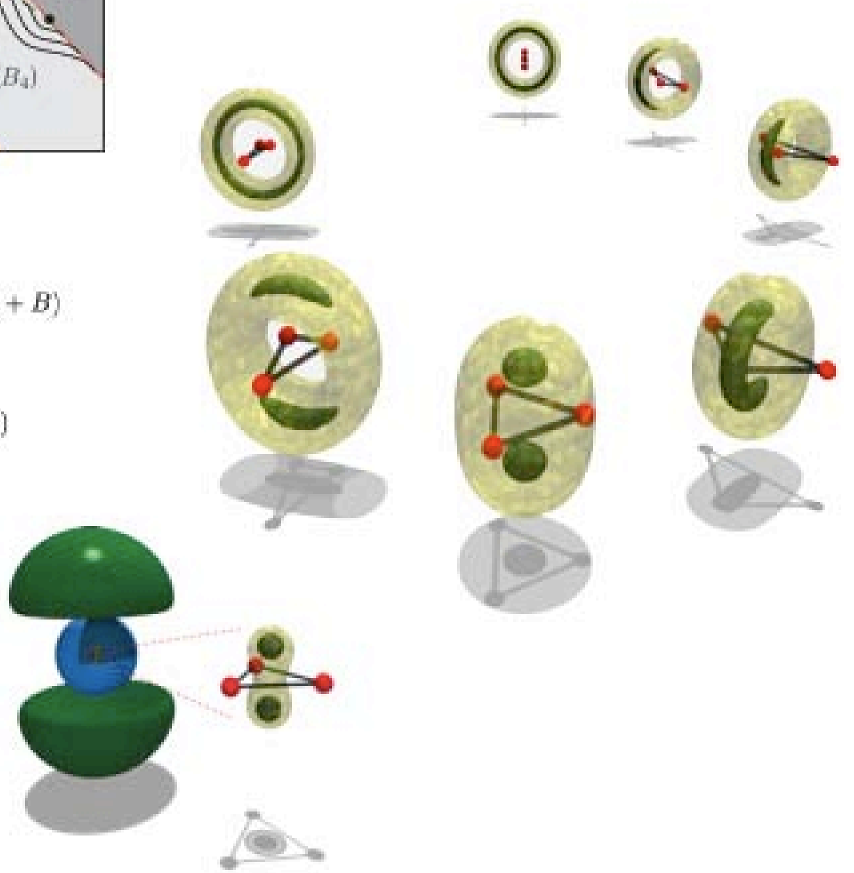


- Two four-boson states for each Efimov trimer



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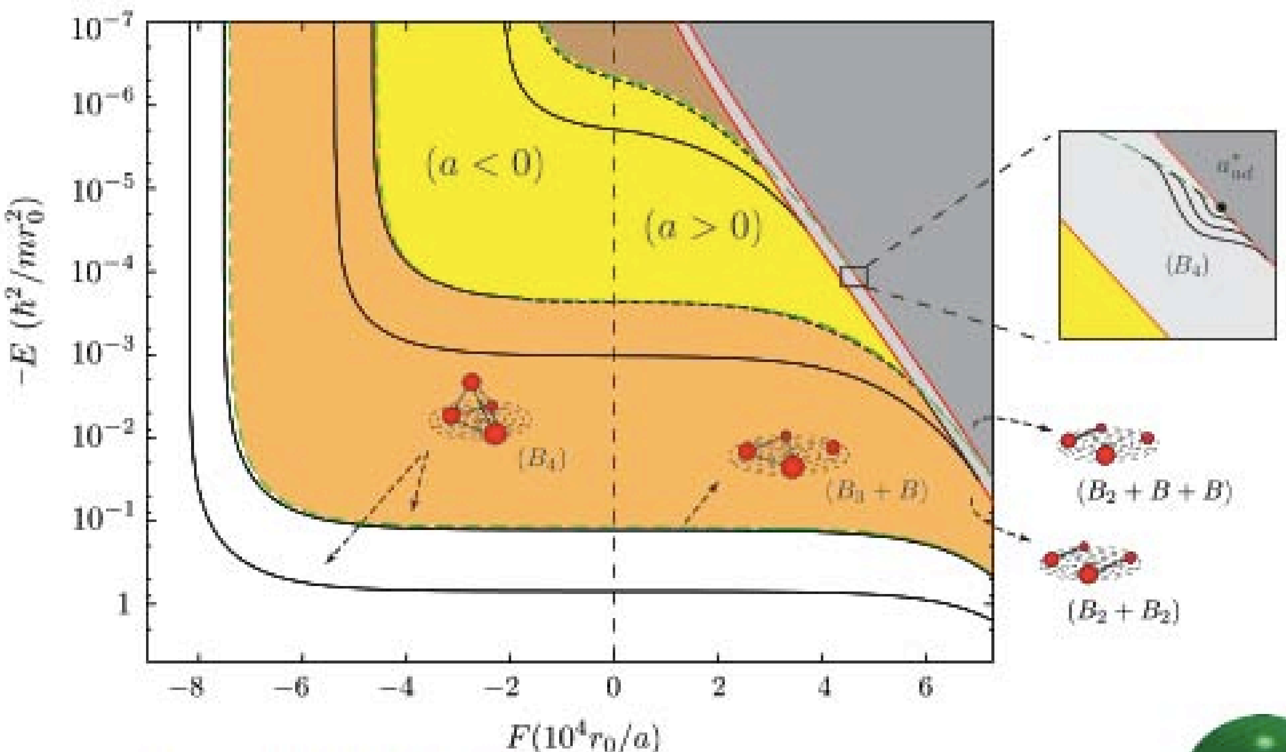

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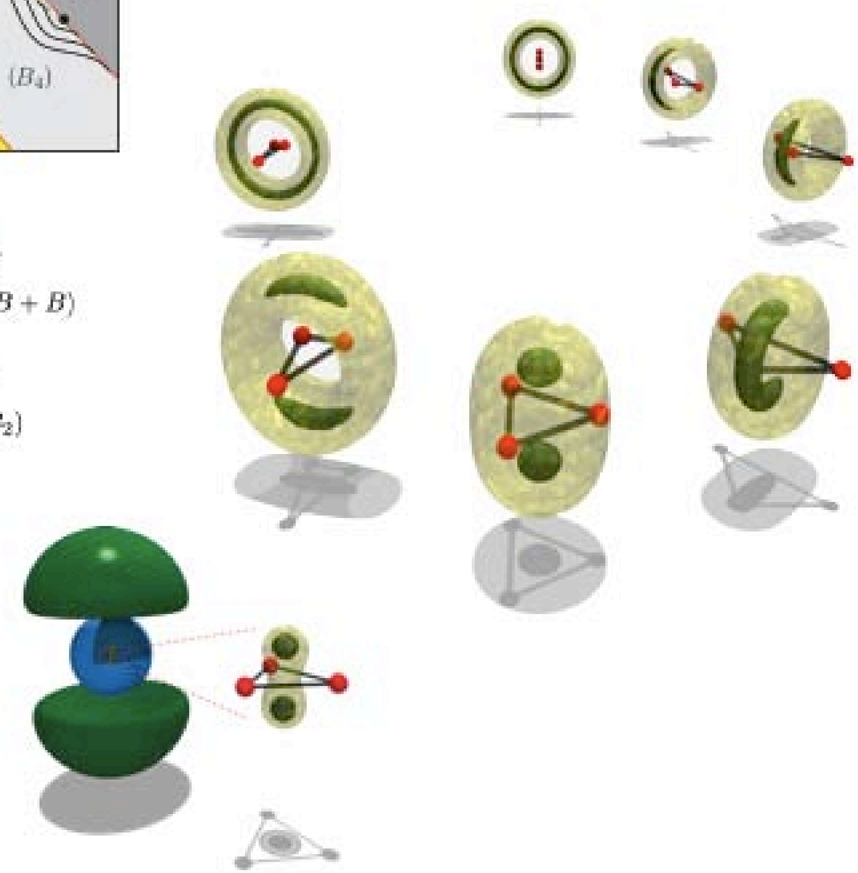

- **at unitarity (1/|a|=0) :**

$$E_{4b}^{(n,m)} = c_m E_{3b}^{(n)} \qquad m = 1,2$$
$$n = 1,2,...,\infty$$
$$(c_1 \approx 4.58,\ c_2 \approx 1.01)$$

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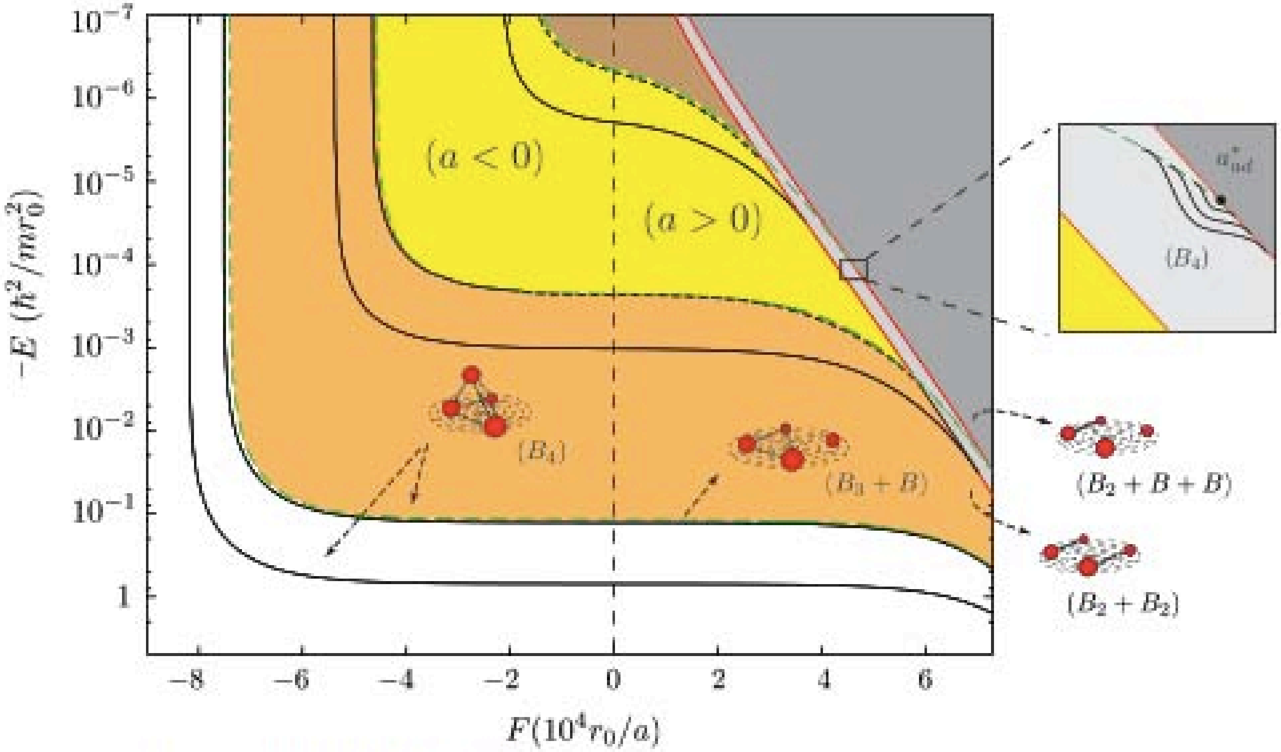

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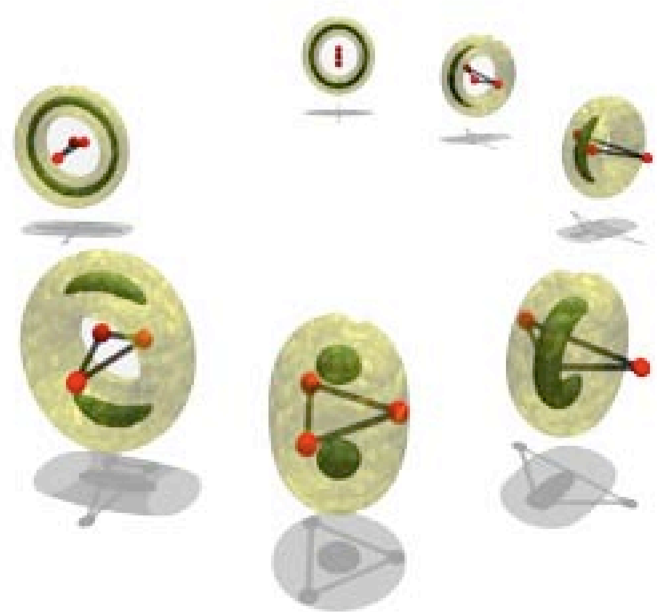

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... geometric scaling: e^{π/s_0} ✓

Why : “Universal Four-boson states ?

Hyperspherical four-bosons potentials (1/|a|=0)

[from von Stecher, D'Incao, and Greene, *Nat. Phys.* (2009)]

Hypersperical four-bosons potentials (1/|a|=0)

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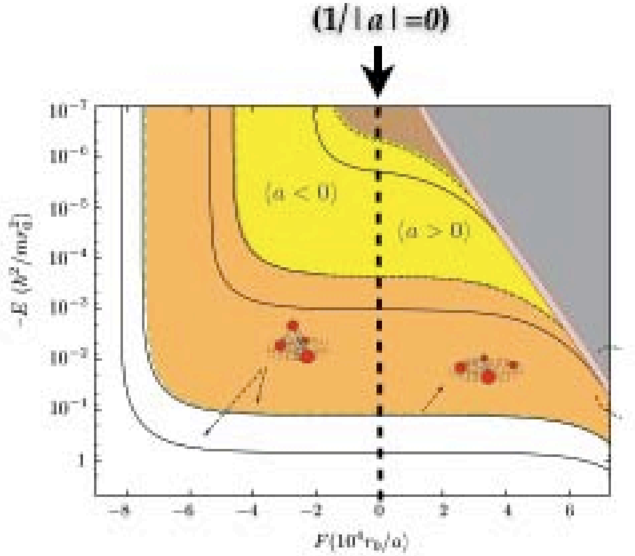

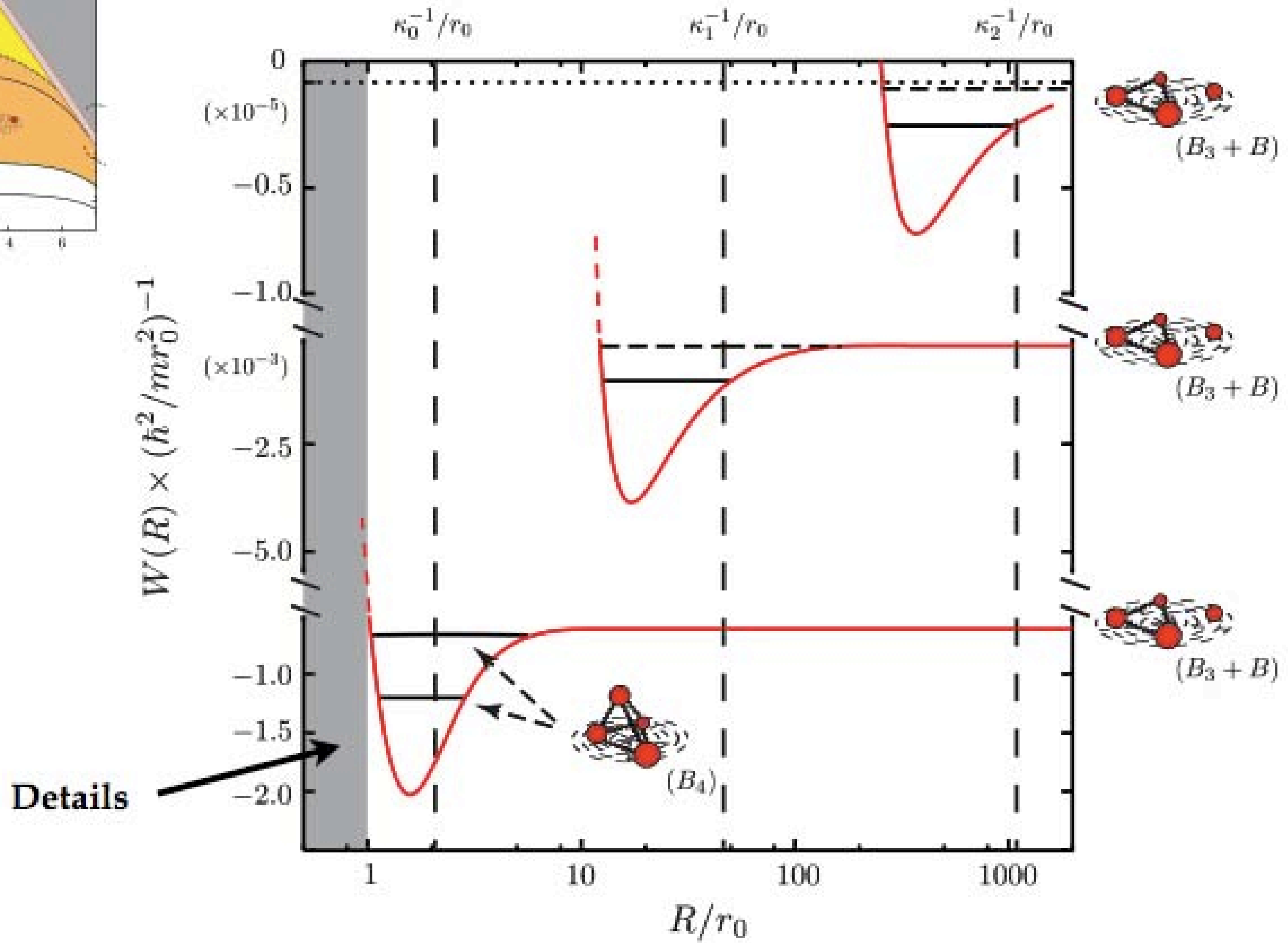

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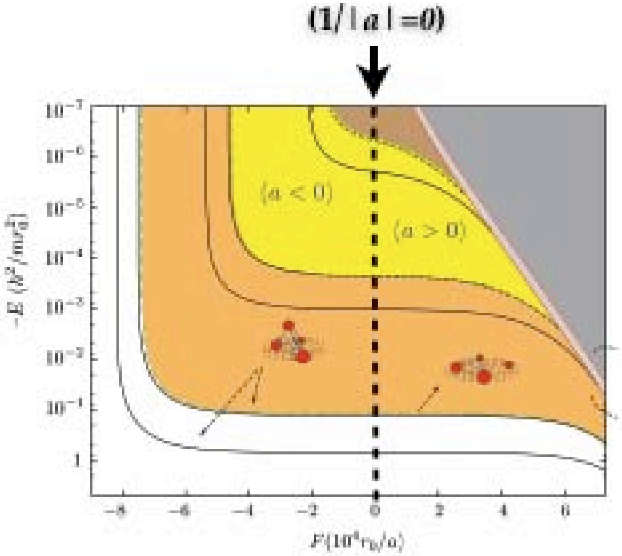

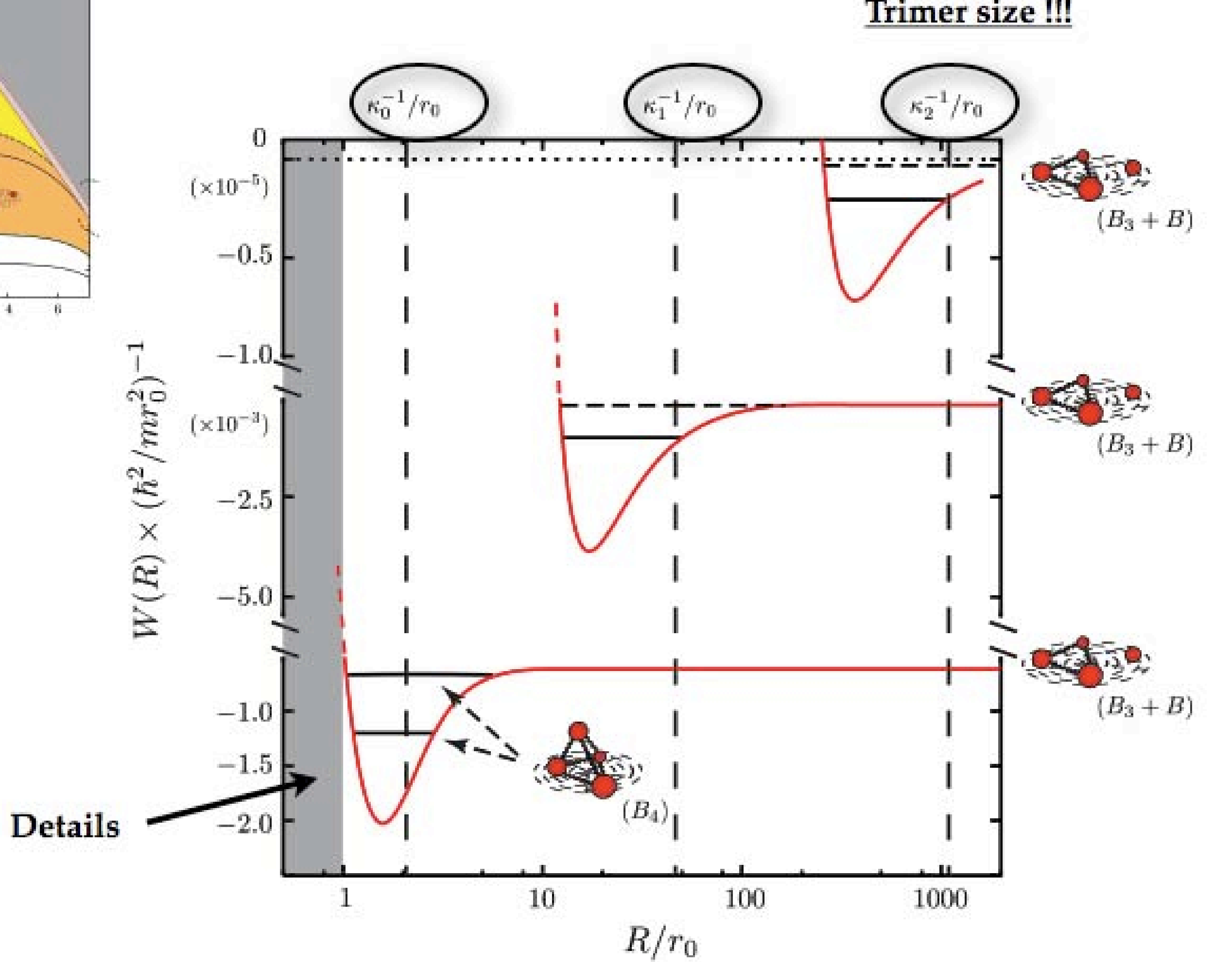

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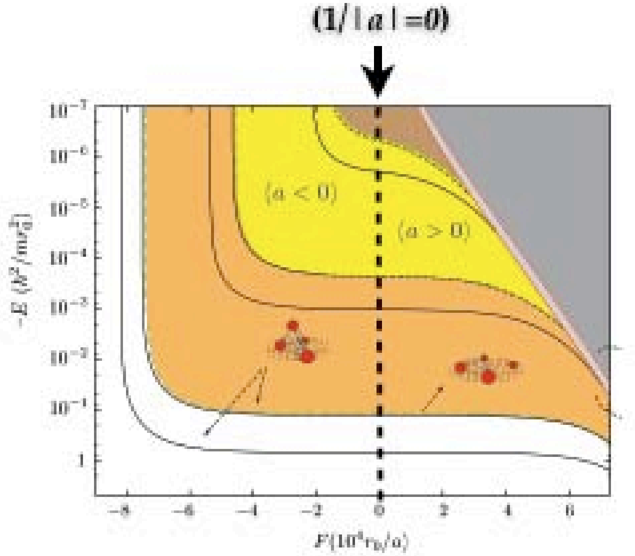

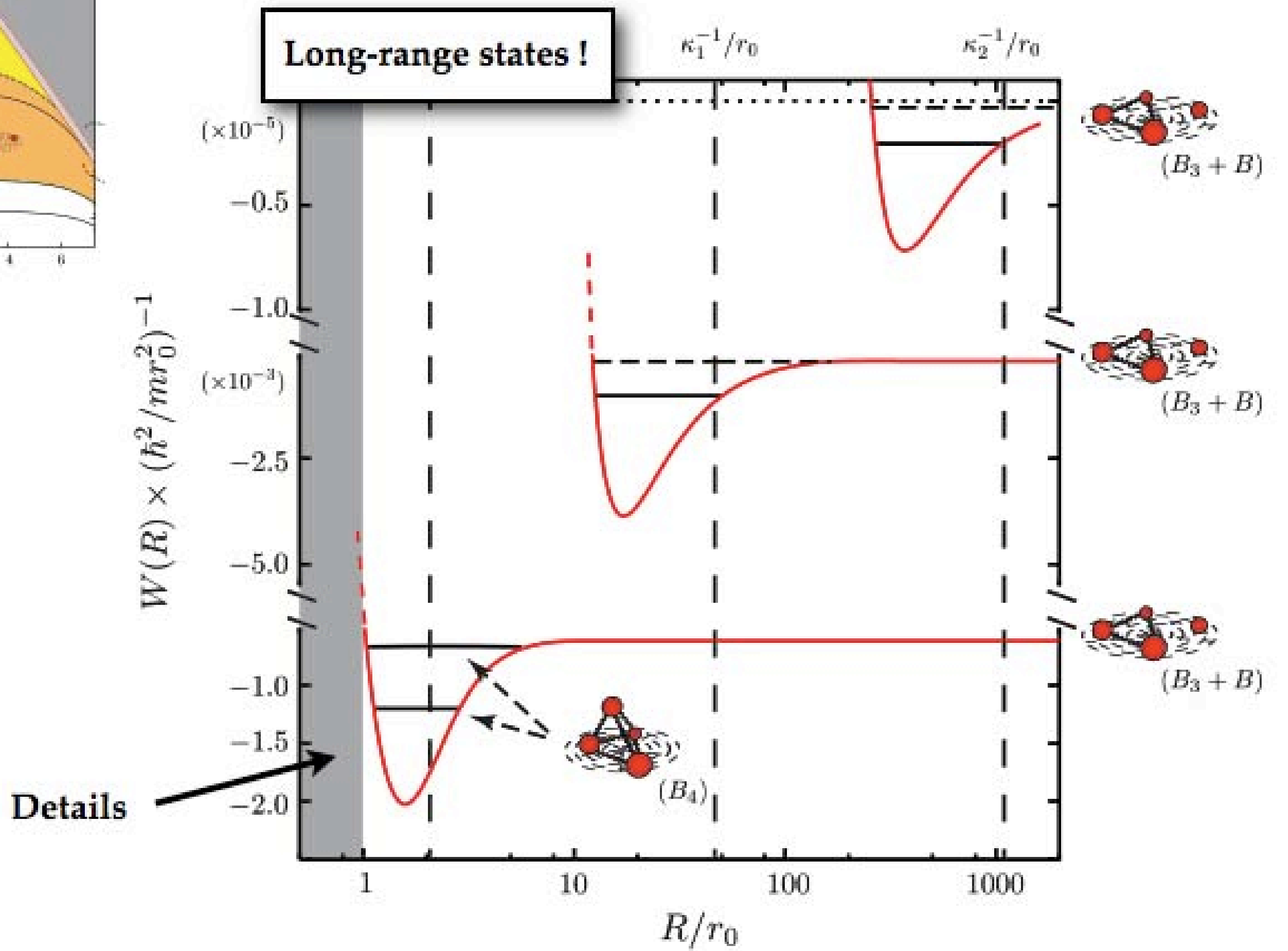

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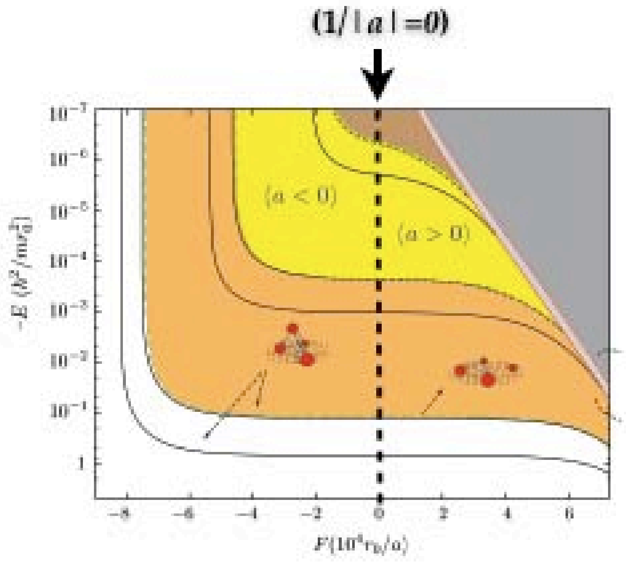

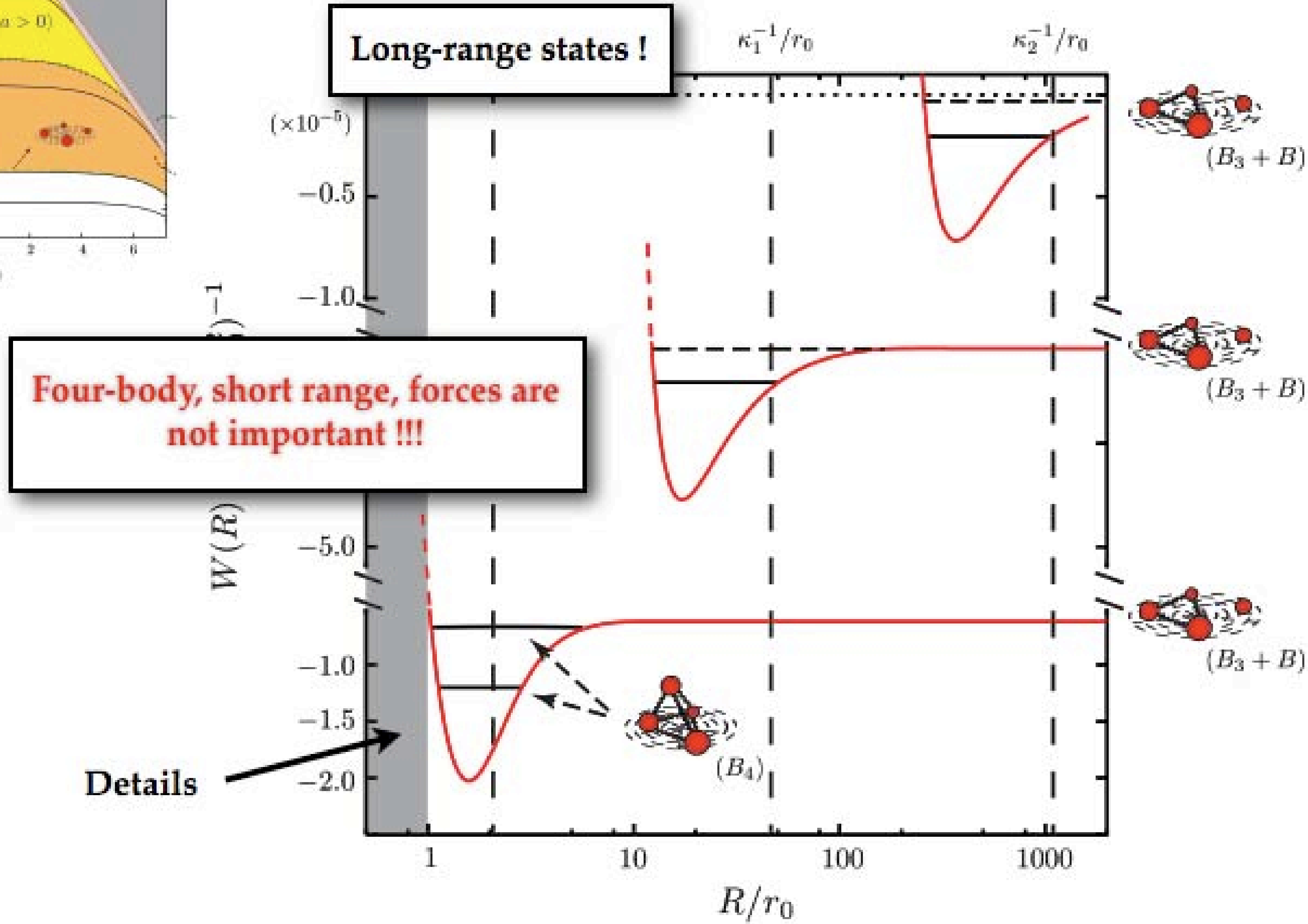

How about ultracold four-body collisions ?

Four-boson Universal Resonant Phenomena

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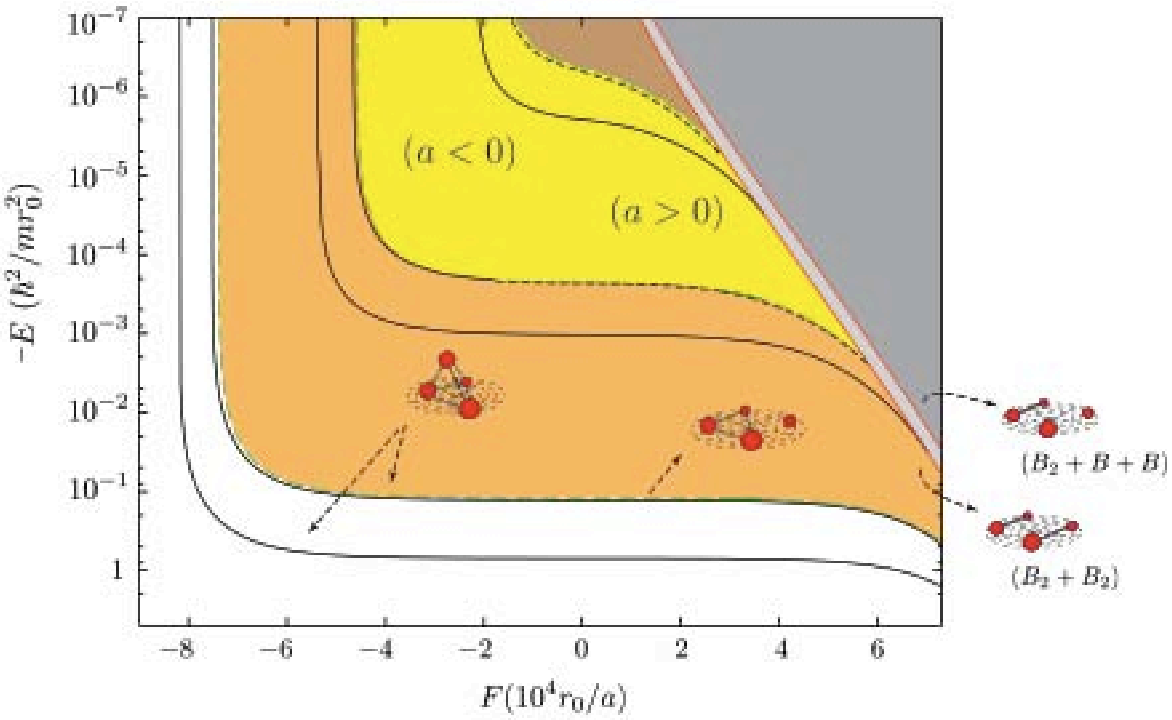
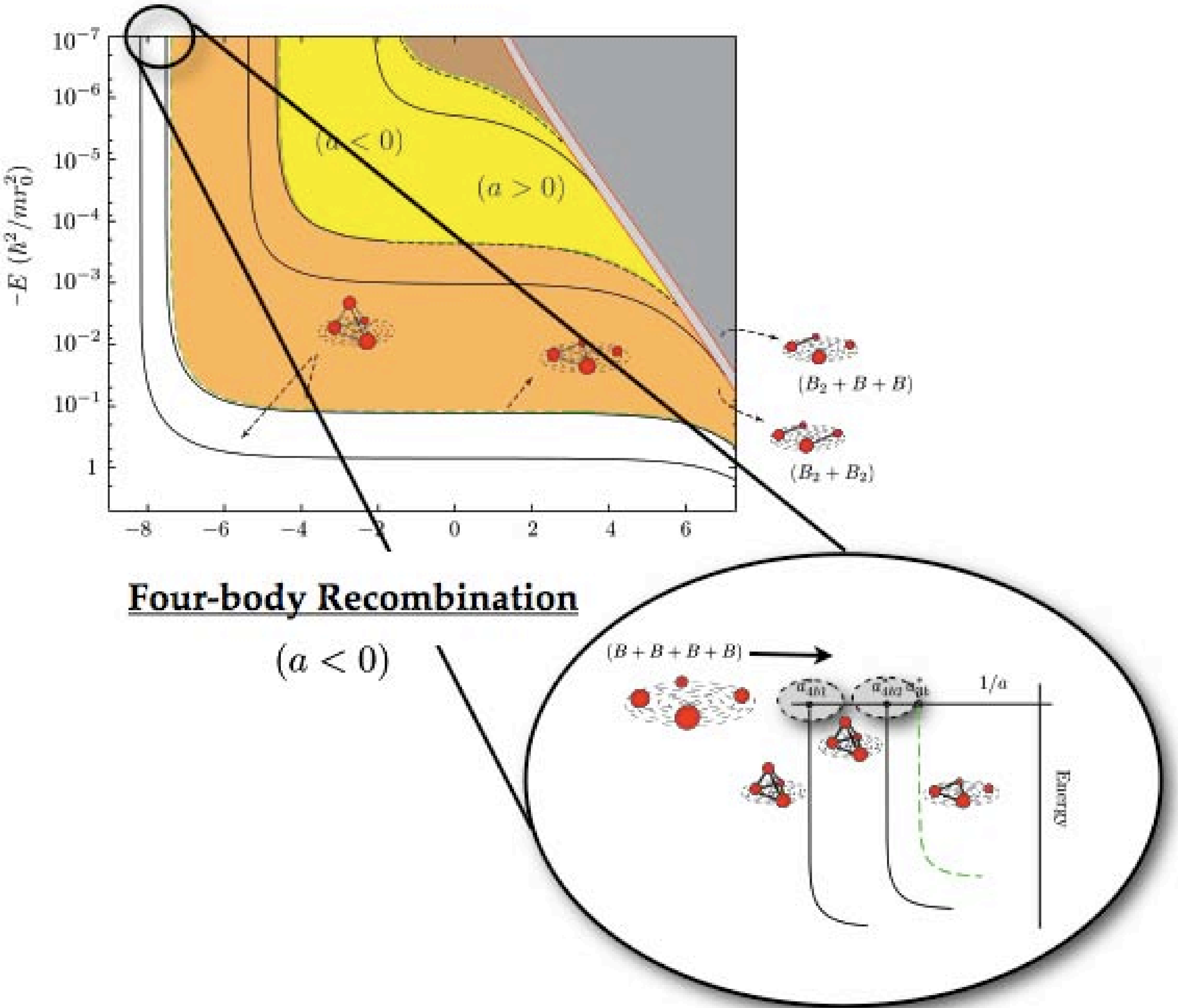


Bound state at collision threshold --> resonance !!!

Four-boson Universal Resonant Phenomena



Four-boson Recombination

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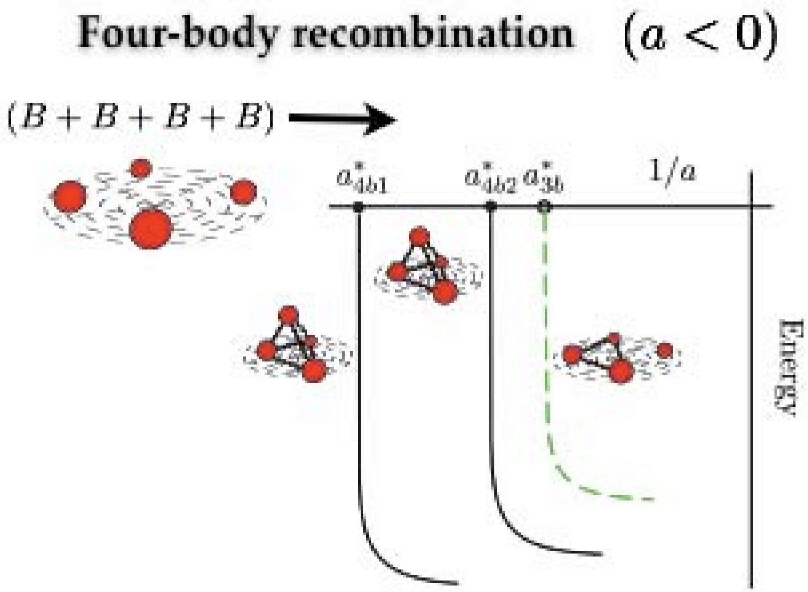
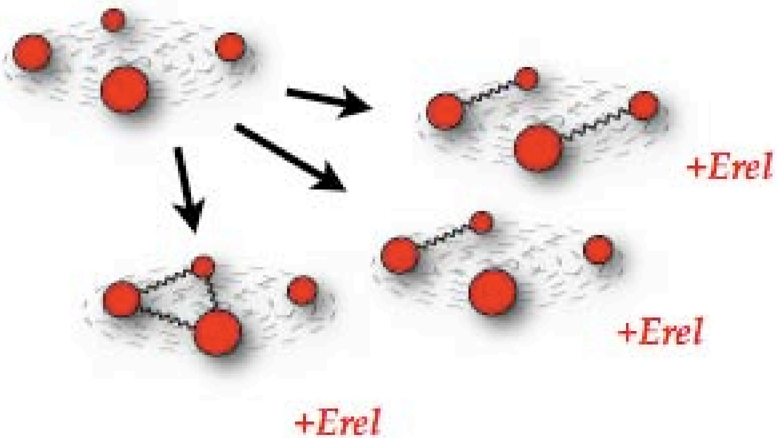


Universal resonances associated with the Efimov trimer,
von Stecher, D'Incao, and Greene, Nature Physics (2009)

$$a^*_{4b,1} = 0.43\ a^*_{3b} \qquad a^*_{4b,2} = 0.90\ a^*_{3b}$$

Four-boson Recombination

Four-body recombination $(a < 0)$



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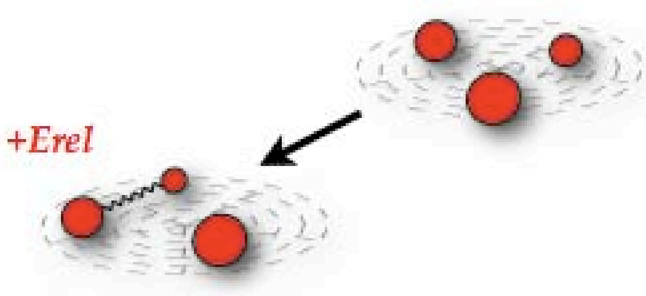

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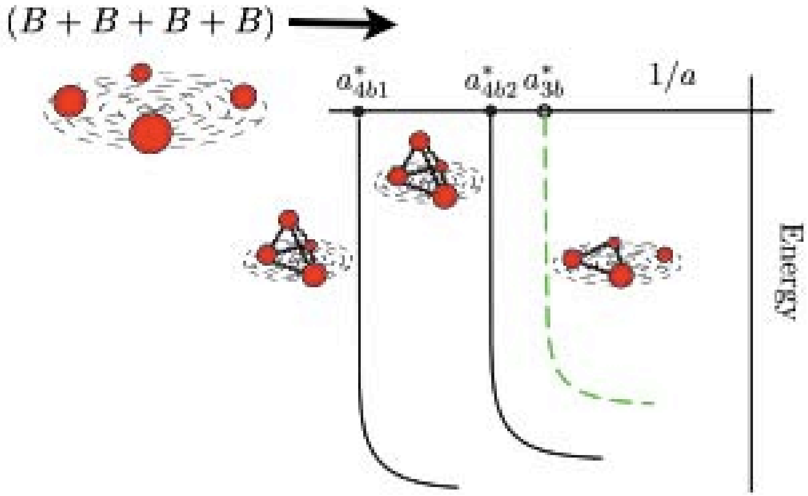
$a^*_{4b,1} = 0.43\ a^*_{3b} \qquad a^*_{4b,2} = 0.90\ a^*_{3b}$

- Is *K4* observable ? $(na^3 \ll 1)$

$$\frac{d}{dt}n(t) = -\frac{K_3}{2}n(t)^3 - \frac{K_4}{6}n(t)^4$$

Four-boson Recombination

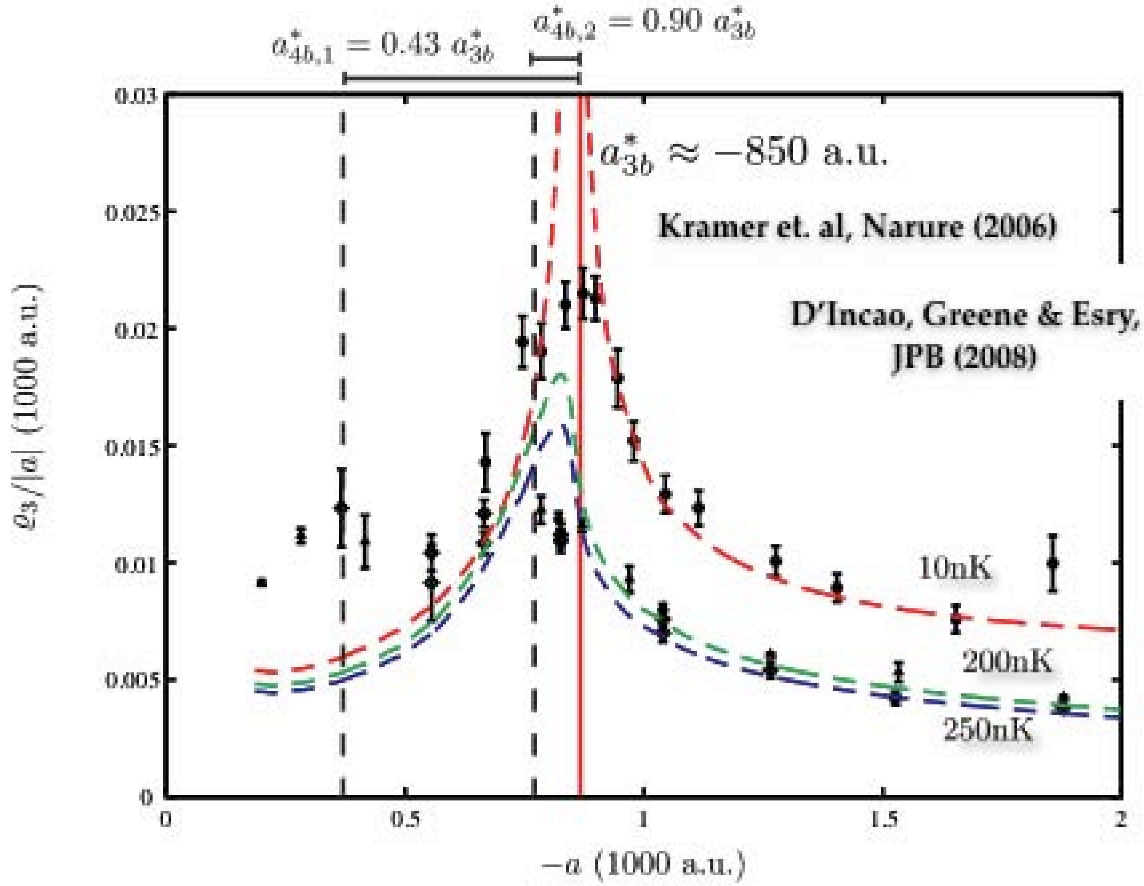
Four-body recombination $(a < 0)$



Universal resonances associated with the Efimov trimer,
von Stecher, D'Incao, and Greene, Nature Physics (2009)

$$a^*_{4b,1} = 0.43\ a^*_{3b} \qquad a^*_{4b,2} = 0.90\ a^*_{3b}$$

four-body physics !?



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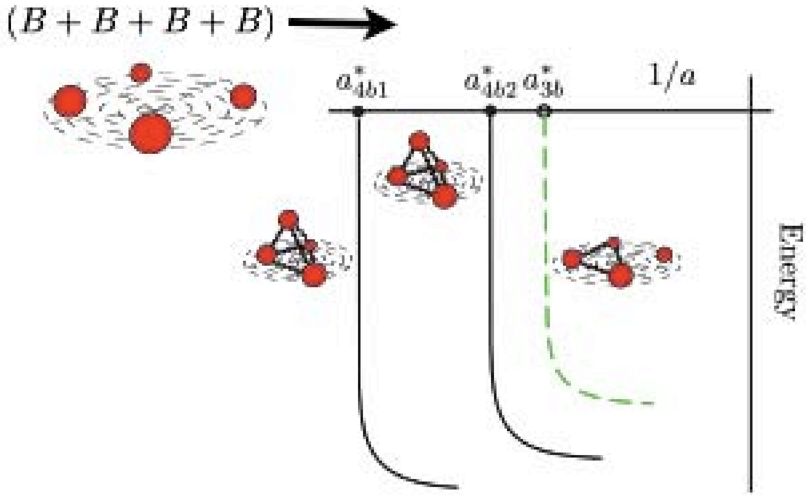

Three- and four-body recombination ...

$$K_3^{\text{eff}}(a,t) = K_3(a) + n(t)K_4(a)$$

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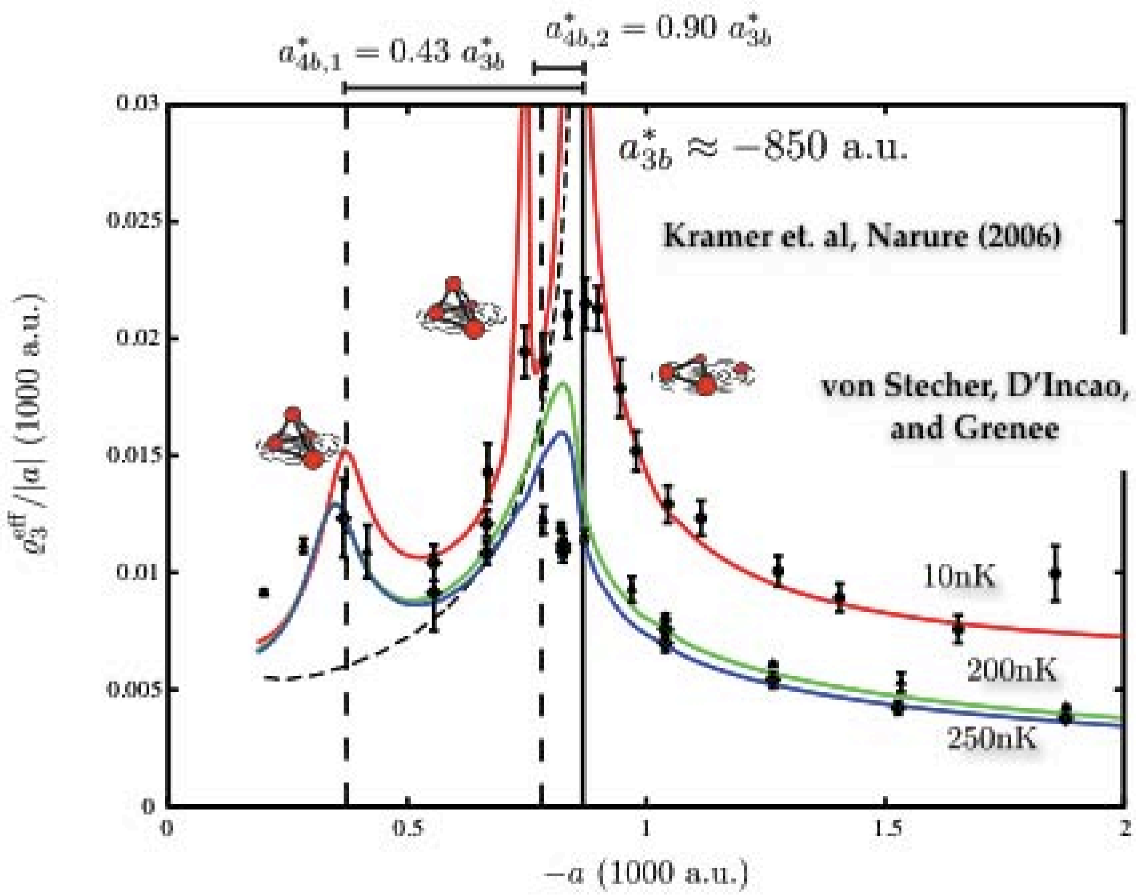

Four-boson Recombination

Innsbruck - 133Cs

Selected for a Viewpoint in *Physics*

PRL **102,** 140401 (2009) PHYSICAL REVIEW LETTERS week ending 10 APRIL 2009

Evidence for Universal Four-Body States Tied to an Efimov Trimer

F. Ferlaino,[1] S. Knoop,[1] M. Berninger,[1] W. Harm,[1] J. P. D'Incao,[2,3] H.-C. Nägerl,[1] and R. Grimm[1,2]

[1]*Institut für Experimentalphysik and Zentrum für Quantenphysik, Universität Innsbruck, 6020 Innsbruck, Austria*

[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*

[3]*JILA, University of Colorado and NIST, Boulder, Colorado 80309-0440, USA*

(Received 6 March 2009; published 6 April 2009)

We report on the measurement of four-body recombination rate coefficients in an atomic gas. Our results obtained with an ultracold sample of cesium atoms at negative scattering lengths show a resonant enhancement of losses and provide strong evidence for the existence of a pair of four-body states, which is strictly connected to Efimov trimers via universal relations. Our findings confirm recent theoretical predictions and demonstrate the enrichment of the Efimov scenario when a fourth particle is added to the generic three-body problem.

DOI: 10.1103/PhysRevLett.102.140401 PACS numbers: 03.75.−b, 21.45.−v, 34.50.Cx, 67.85.−d

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...ne, Nature Physics (2009)

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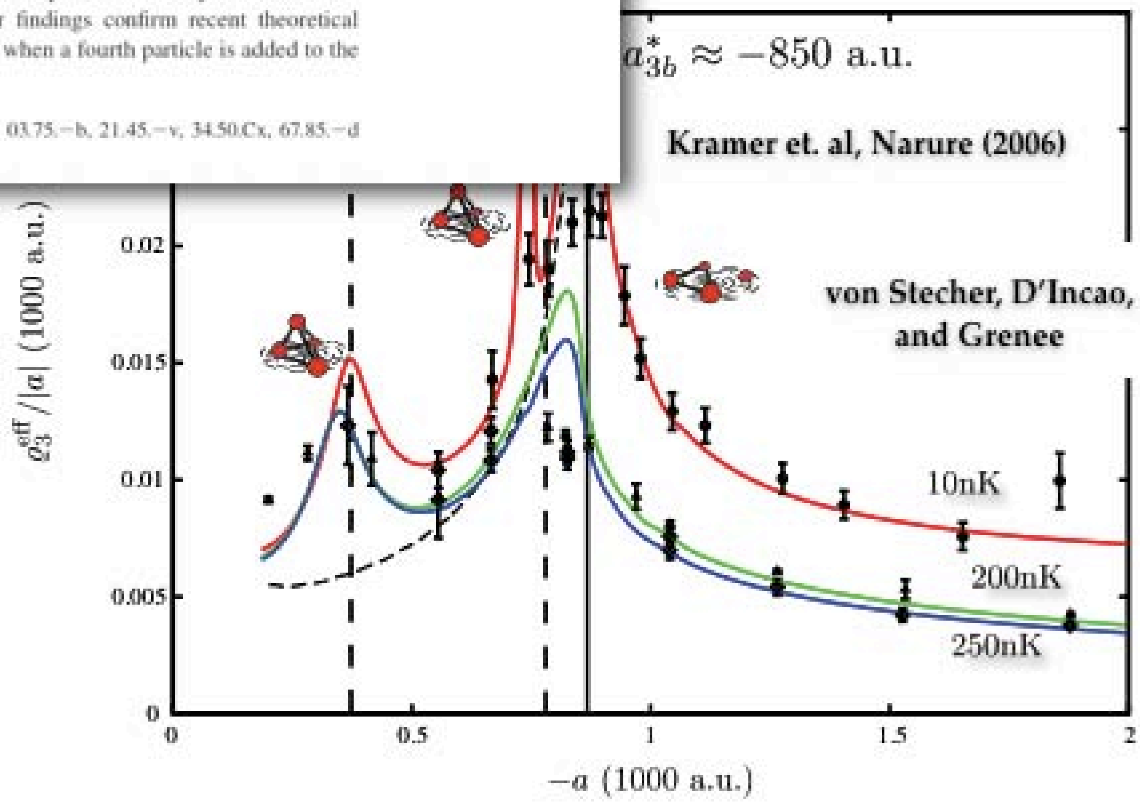

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LENS -39K

ARTICLES
PUBLISHED ONLINE: 13 JULY 2009 | DOI: 10.1038/NPHYS1334
nature physics

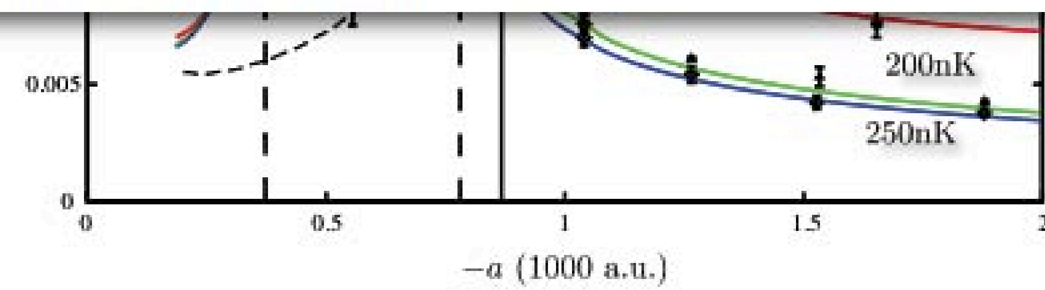
Observation of an Efimov spectrum in an atomic system

M. Zaccanti[1]*, B. Deissler[1], C. D'Errico[1], M. Fattori[1,2], M. Jona-Lasinio[1], S. Müller[3], G. Roati[1], M. Inguscio[1] and G. Modugno[1]

In 1970, Vitaly Efimov predicted that three quantum particles subjected to a resonant pair-wise interaction can join into an infinite number of loosely bound states, even if each pair of particles cannot bind. The properties of these aggregates, such as the peculiar geometric scaling of their energy spectrum, are universal, that is, independent of the microscopic details of their components. Despite an extensive search in many different physical systems, including atoms, molecules and nuclei, the characteristic spectrum of Efimov trimer states has not been observed so far. Here, we report on the discovery of two bound trimer states of potassium atoms very close to the Efimov scenario, which we reveal by studying three-particle collisions in an ultracold gas. Our observation provides the first evidence of an Efimov spectrum and enables a direct test of its scaling behaviour, providing potentially general insights into the physics of few-body systems.

Three- and four-body recomb

$K_3^{\text{eff}}(a,t) = K_3(a) + n(t)I$



Four-boson Recombination

Innsbruck - 133Cs

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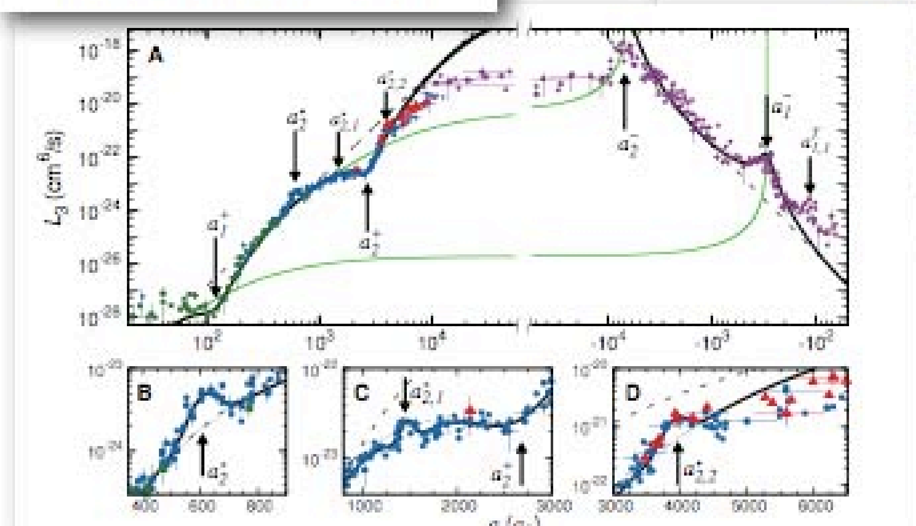
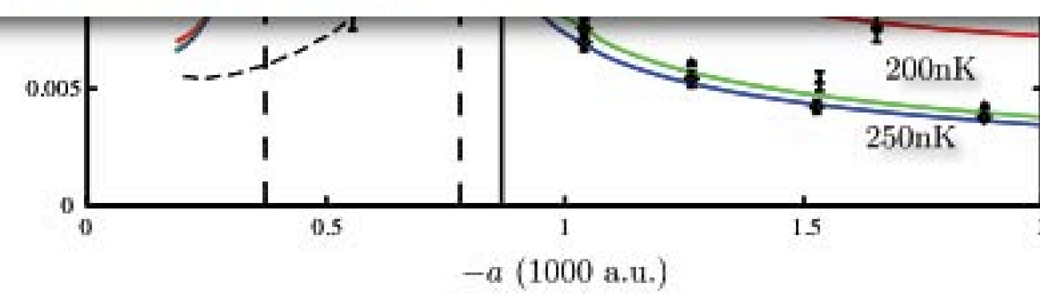
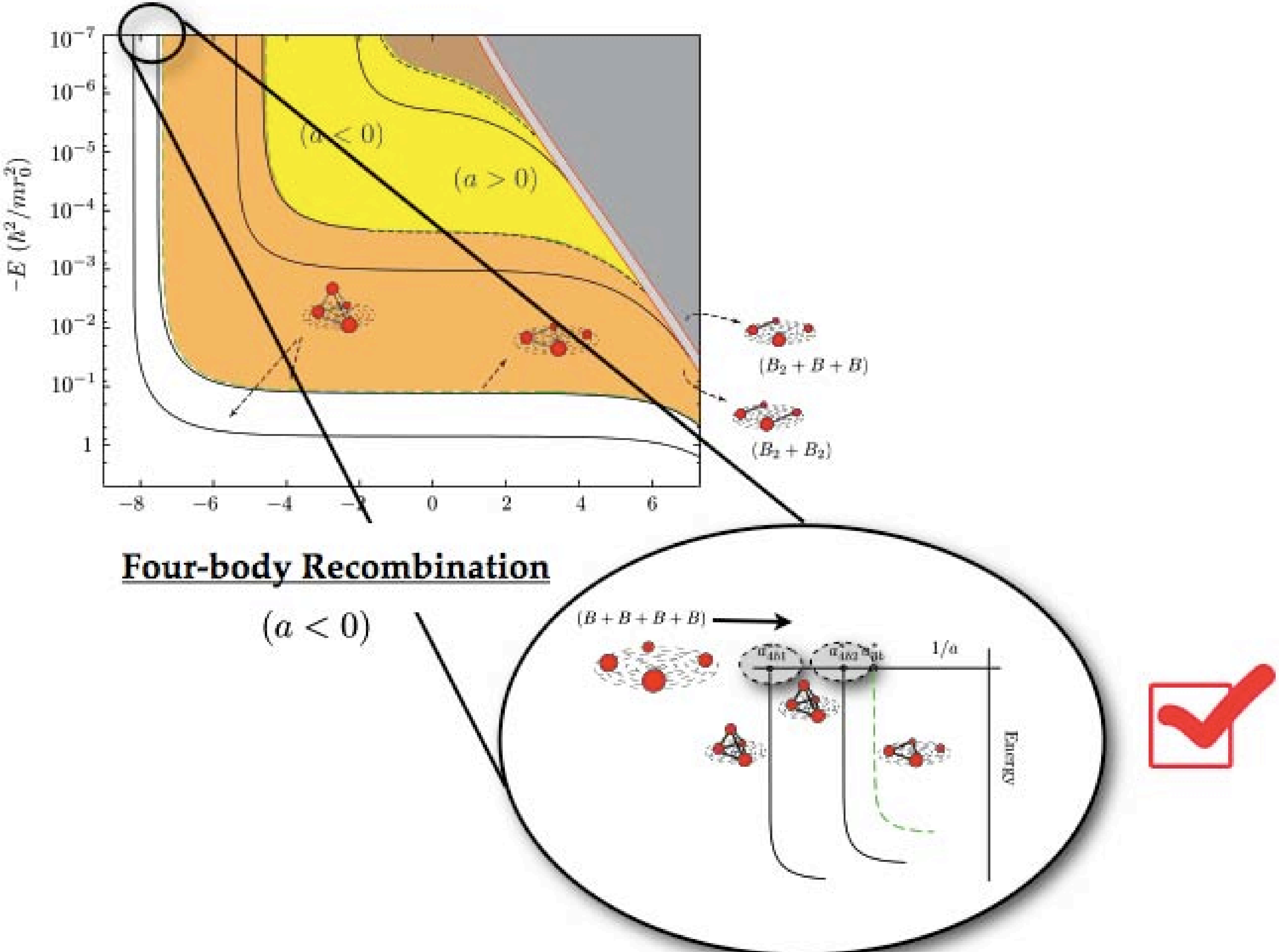
Rice-Hulet - 7Li





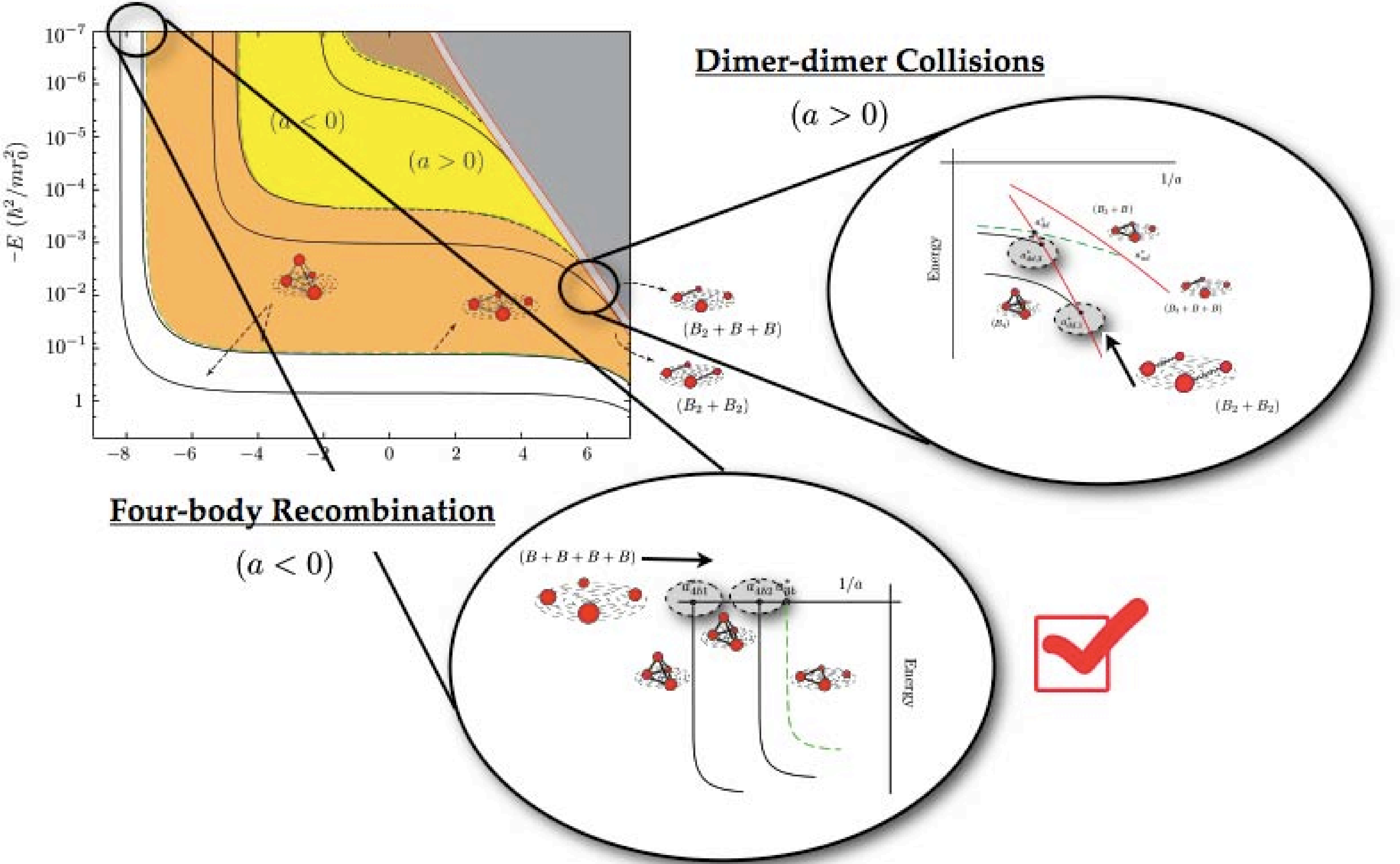
Four-boson Universal Resonant Phenomena



Four-body Recombination

$(a < 0)$

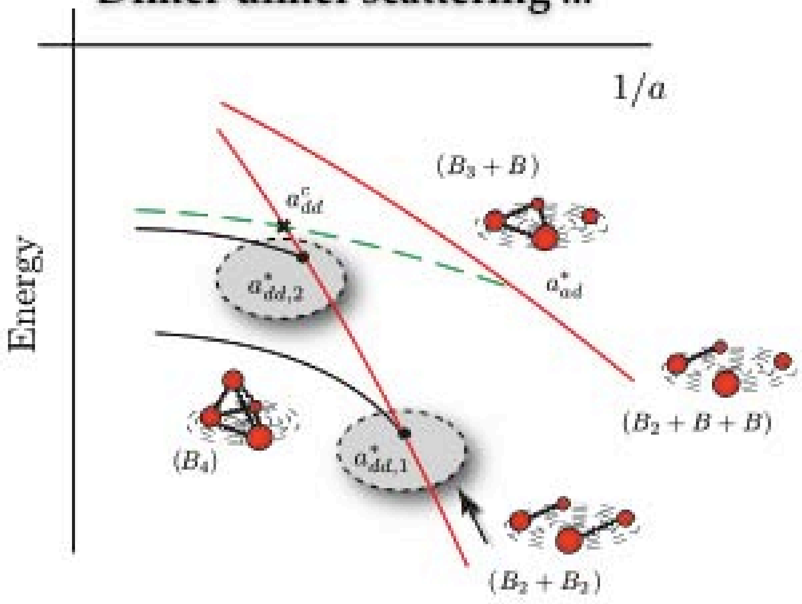
Four-boson Universal Resonant Phenomena



Universal Dimer-dimer resonances

Universal Dimer-dimer resonances

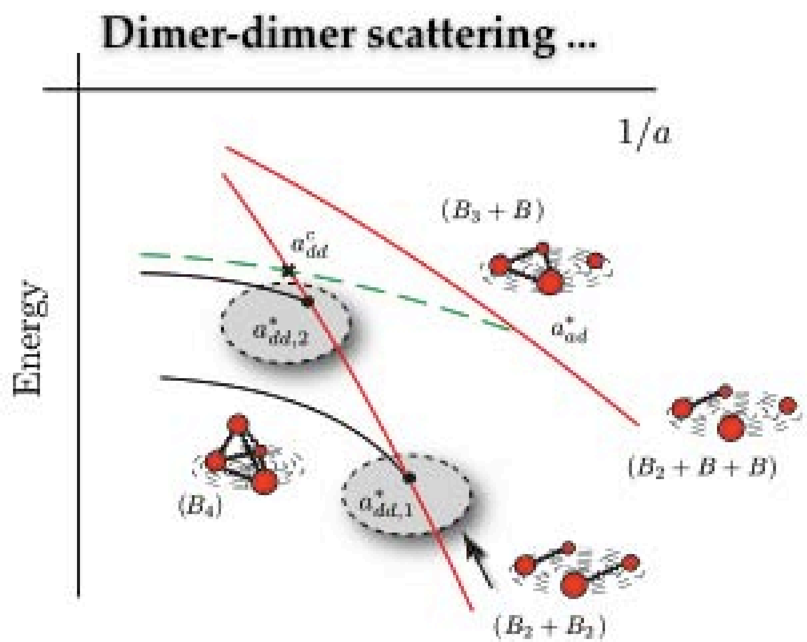
Dimer-dimer scattering ...



Dimer-dimer resonances associated with four-body universal states: D'Incao, von Stecher, Greene PRL (2009)

$$a^*_{dd,1} \approx 2.37 a^*_{ad} \qquad a^*_{dd,2} \approx 6.6 a^*_{ad}$$

Universal Dimer-dimer resonances



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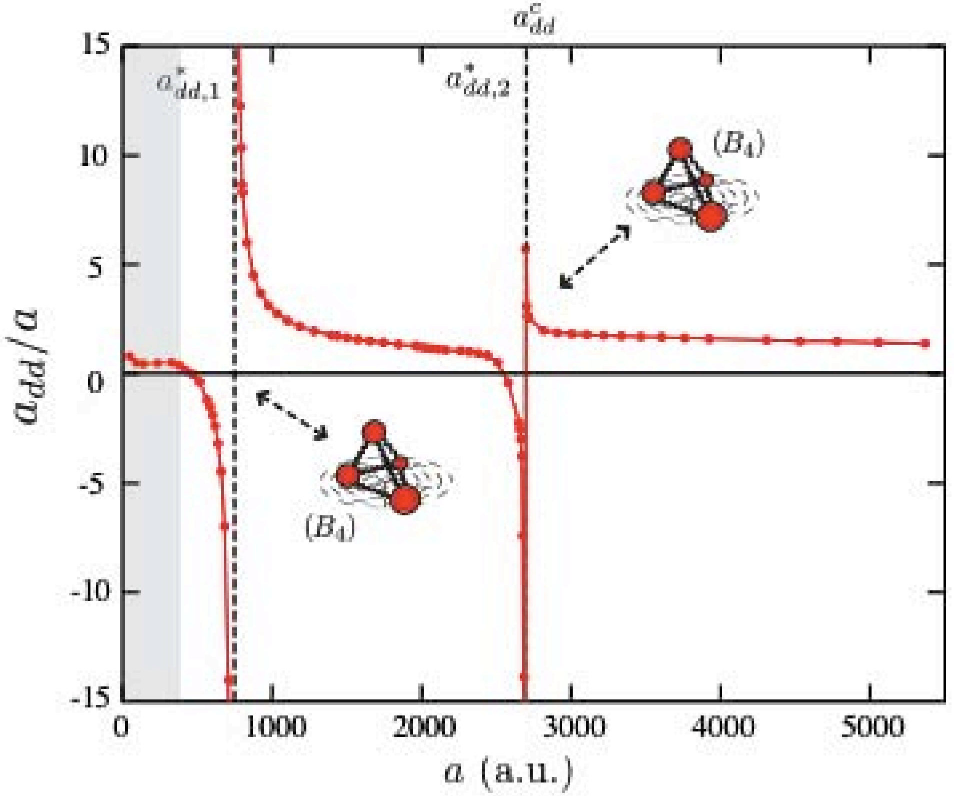


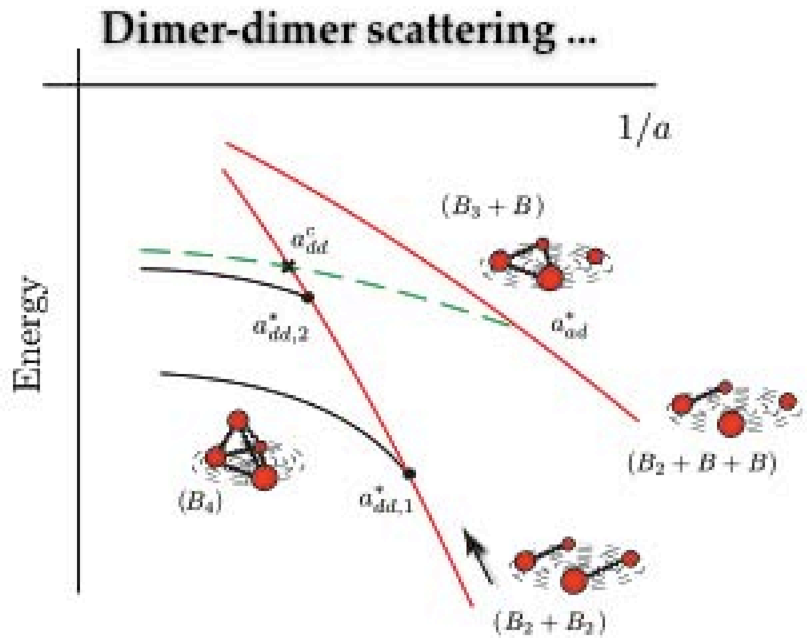
Control of the dimer-dimer interactions

$a_{dd} > 0$: repulsive

$a_{dd} < 0$: attractive

... for fermions $a_{dd} \approx 0.6a \ (a_{dd} > 0)$

Universal Dimer-dimer resonances



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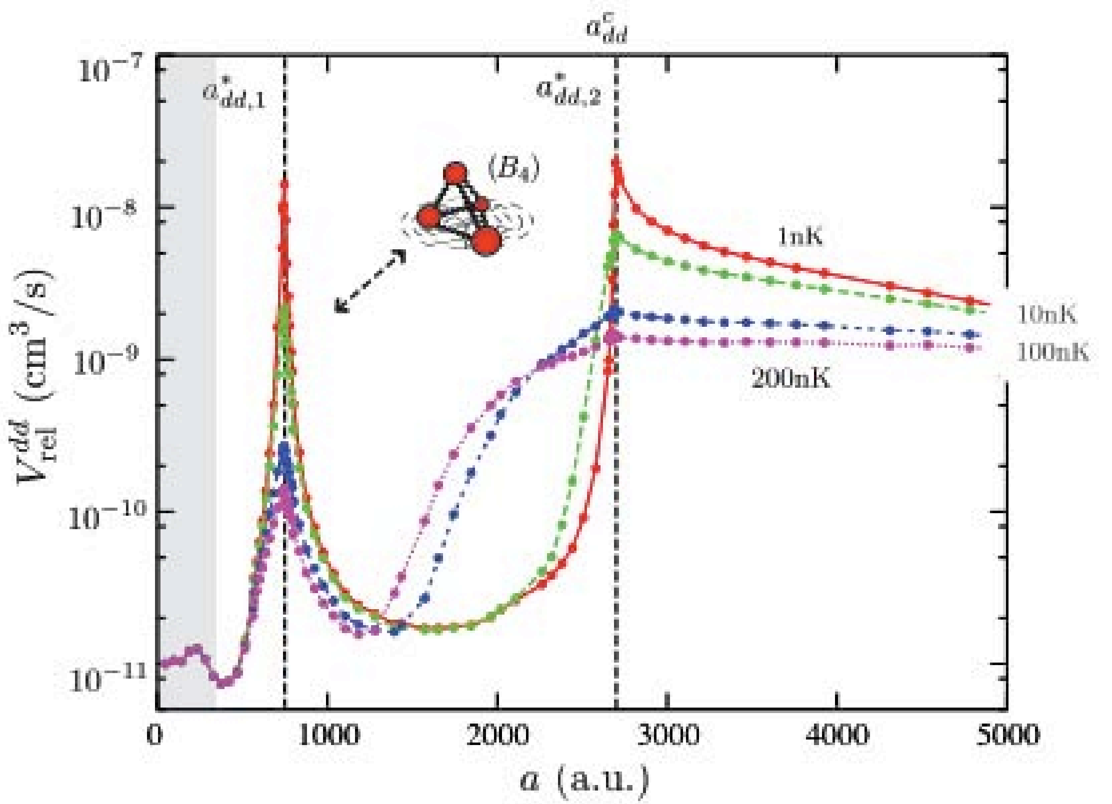

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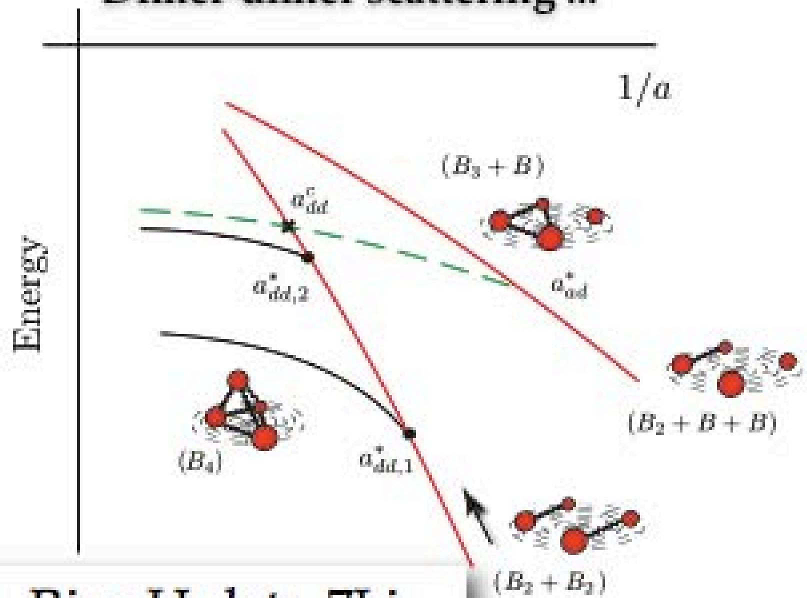
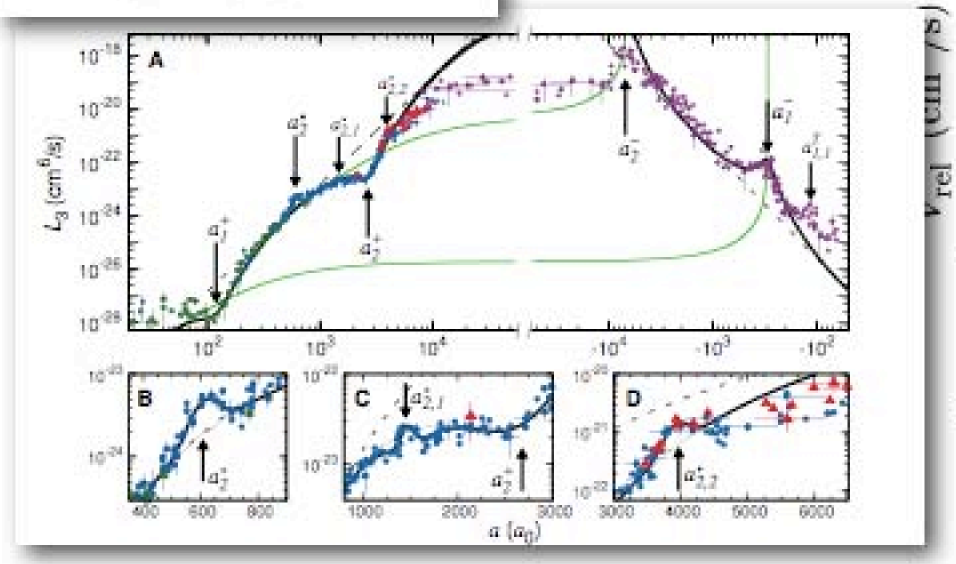


Rice-Hulet - 7Li



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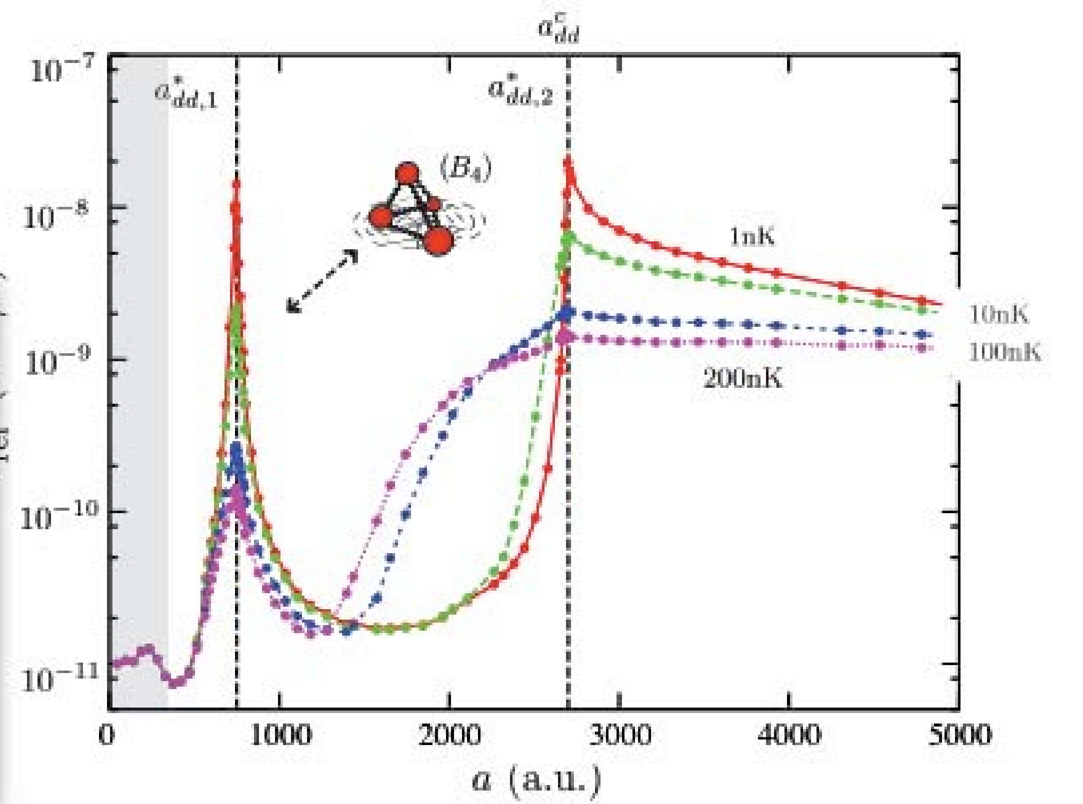

Universal Dimer-dimer resonances

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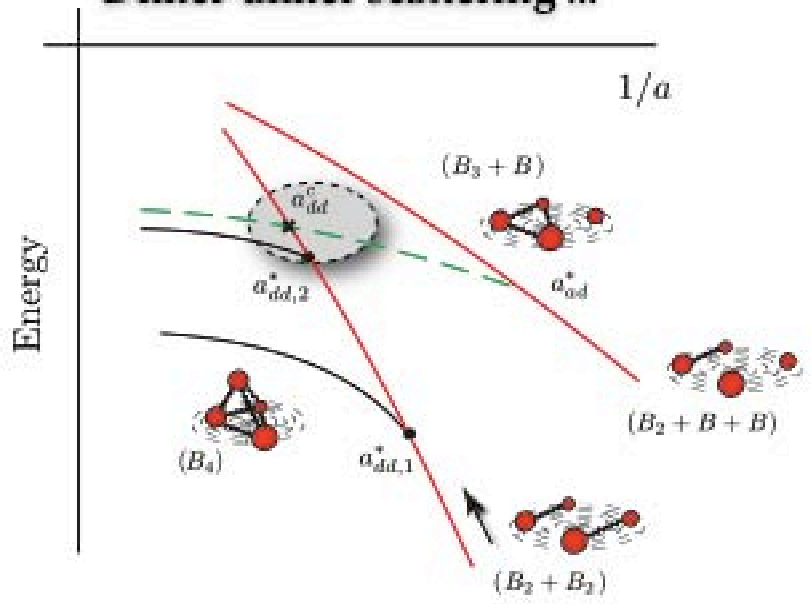

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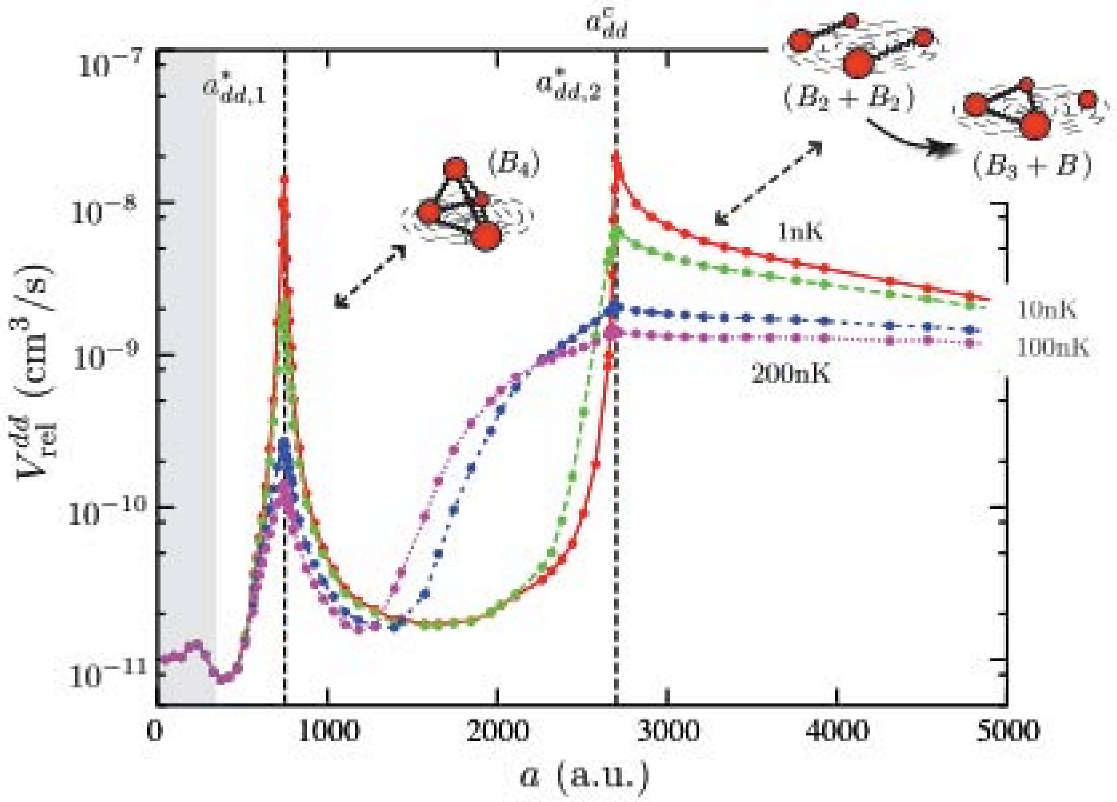
Trimer formation !!!



Rearrangement reaction ...

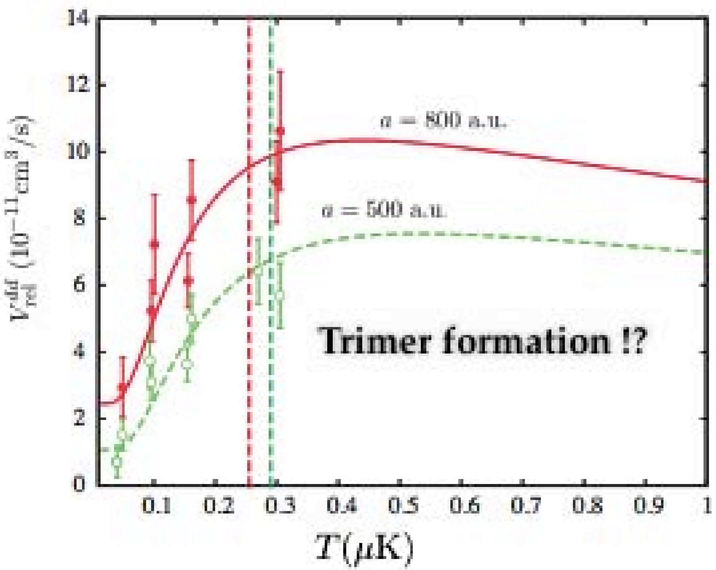
$$B_2 + B_2 \rightarrow B_3 + B + E_{rel}(\approx 0)$$

- high efficiency (98%)
- negligible Erel: trimers remain trapped !
- clear signature: reappearance of atoms !

Universal Dimer-dimer resonances

Are there Efimov trimers amog us ?

Ferlaino, et. al, PRL 2008



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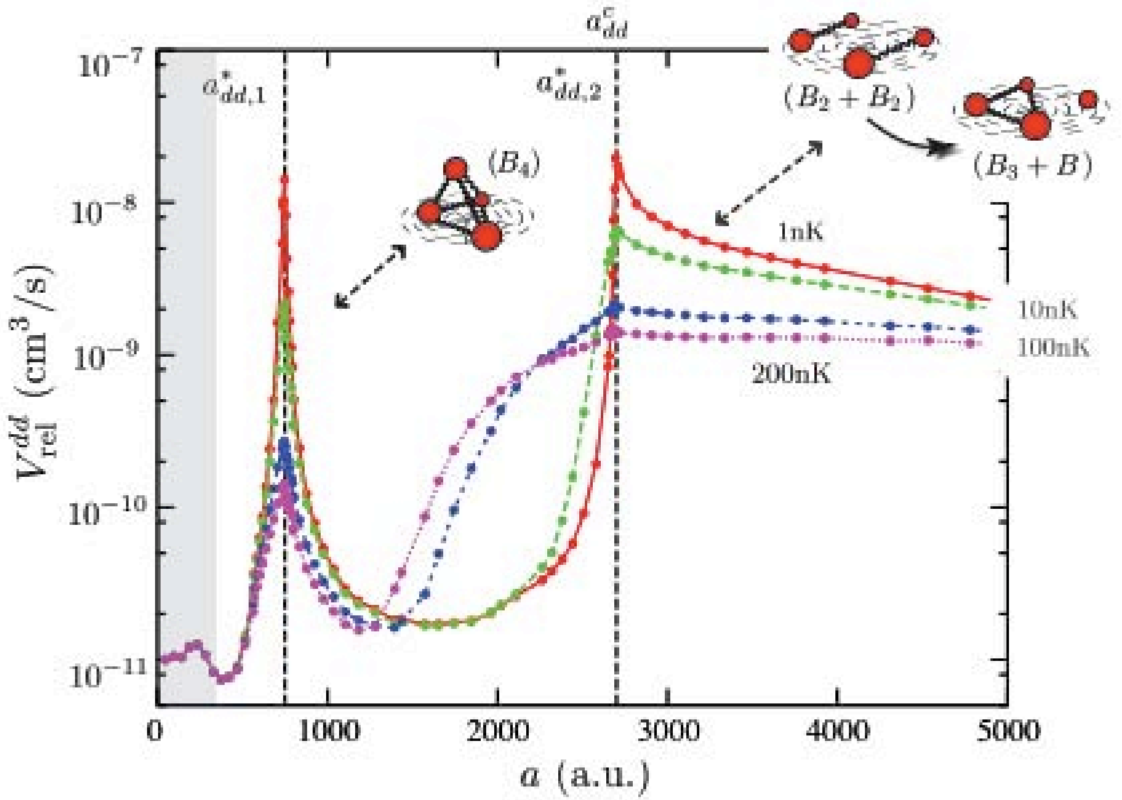


Summary

Summary

- The hyperspherical representation offers a "simple" and conceptual picture of properties of weakly bound few-body systems

- Few-body physics have a practical and fundamental importance :
  - losses/lifetime/stability
  - Efimov, quantum phases

- Developed a physical understanding of few-body systems, supported by models and numerical calculations:
  - scattering length,
  - temperature effects,
  - masses,
  - particle symmetry, etc ...

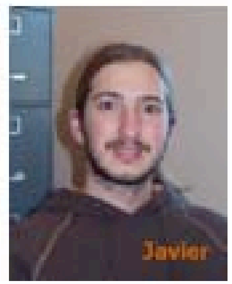
Acknowledgements ...

Acknowledgements ...

Chris H. Greene (JILA)

Brett D. Esry (KSU)

Yujun Wang



Javier von Stecher

Nirav Mehta

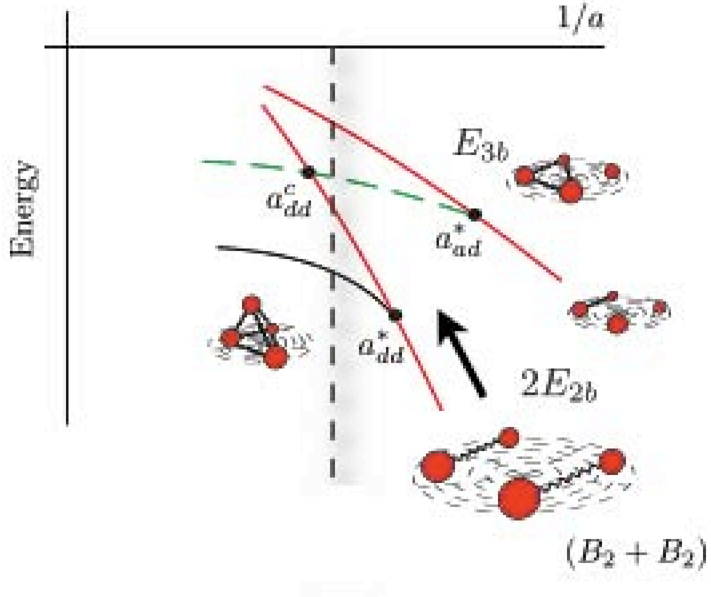
Seth Rittenhouse

Rudi Grimm (Innsbruck)

Dimer-dimer (inelastic) collisions

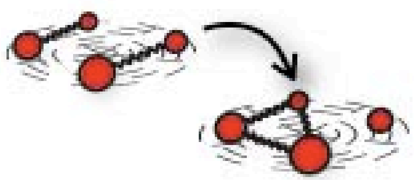
Dimer-dimer (inelastic) collisions

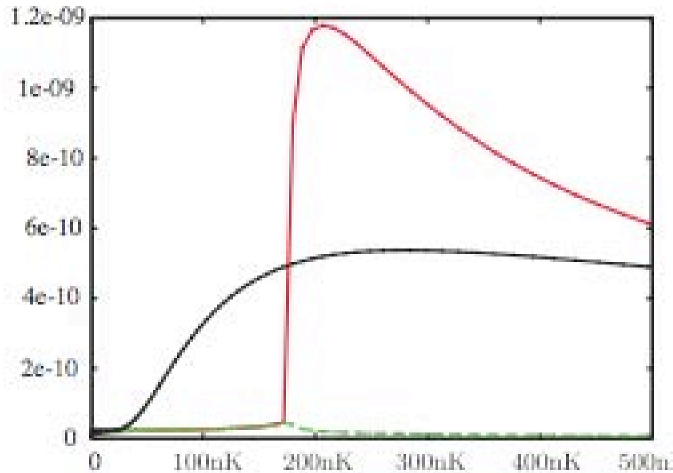
... energy dependence !?



Trimer formation !!!

(threshold effects)





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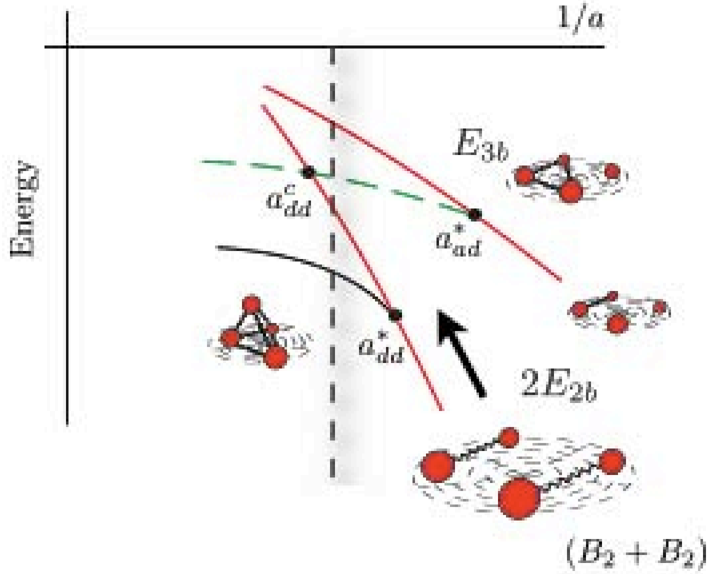

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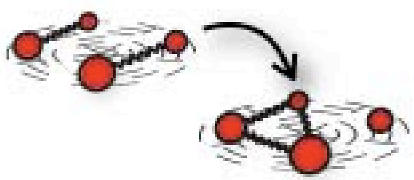
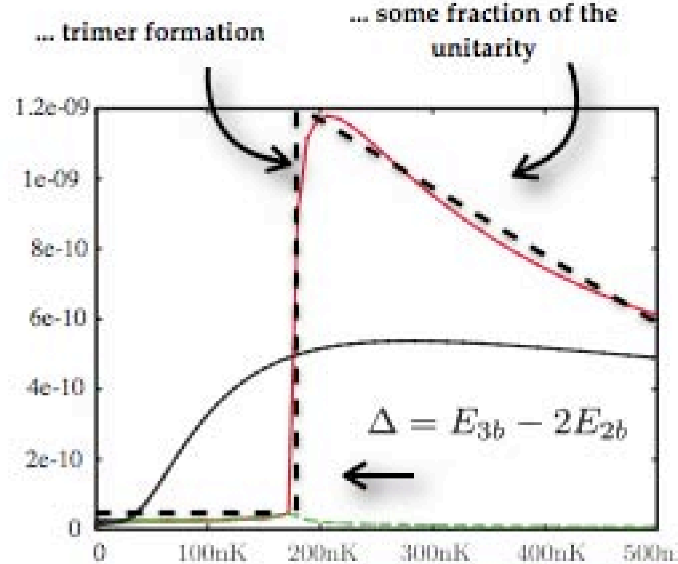


(... a simple model)



Dimer-dimer (inelastic) collisions

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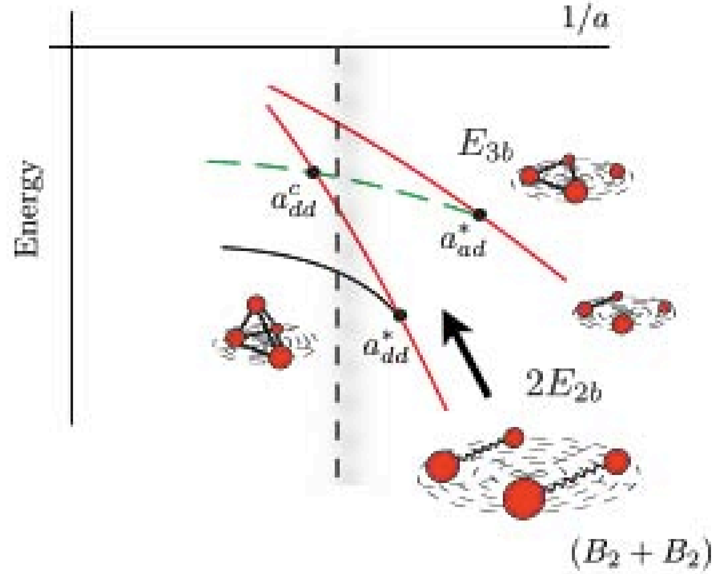

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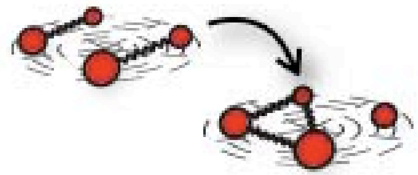

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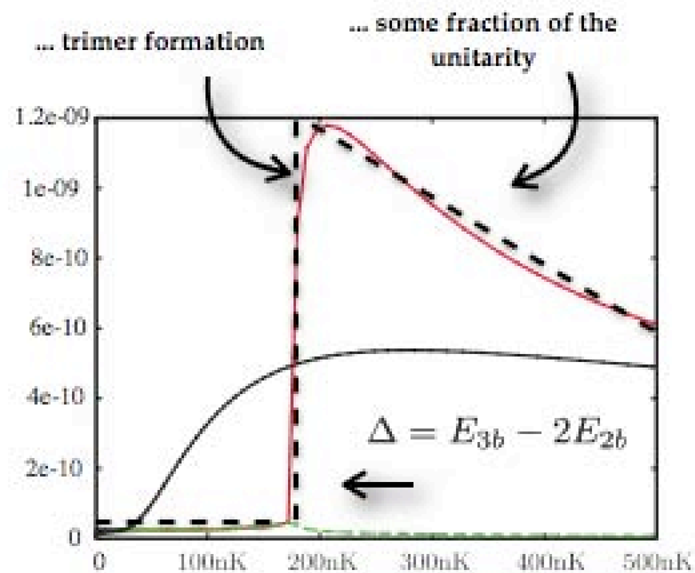


$$V_{\rm rel}^{(total)}(T) = V_{\rm rel}^{(c)}(T) + V_{\rm rel}^{(o)}(T)$$

$$V_{\rm rel}^{(c)}(T) = C_\eta \left[\mathrm{Erf}\left(\sqrt{\frac{\Delta}{k_b T}}\right) - \frac{2}{\sqrt{\pi}} \left(\frac{\Delta}{k_b T}\right)^{1/2} e^{-\Delta/k_b T} \right]$$

$$V_{\rm rel}^{(o)}(T) = F_\eta \frac{8\sqrt{2\pi}}{m^{3/2}(k_b T)^{1/2}} e^{-\Delta/k_b T}$$

(... maybe too many par. ?)

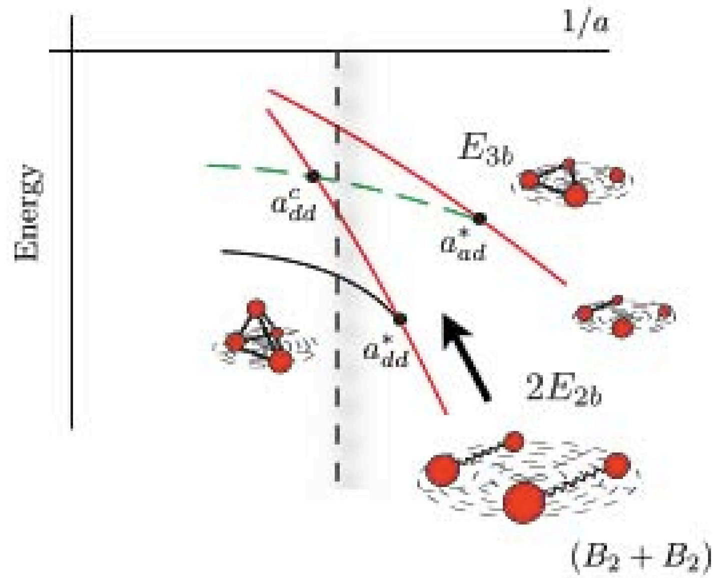
C_η : unknown constant

Δ : trimer energy $\Delta = E_{3b} - 2E_{2b}$

F_η : fraction of the unitrity

Dimer-dimer (inelastic) collisions

... energy dependence !?



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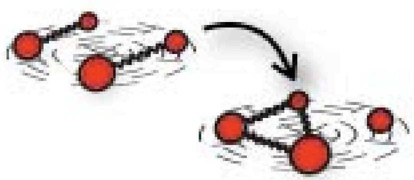
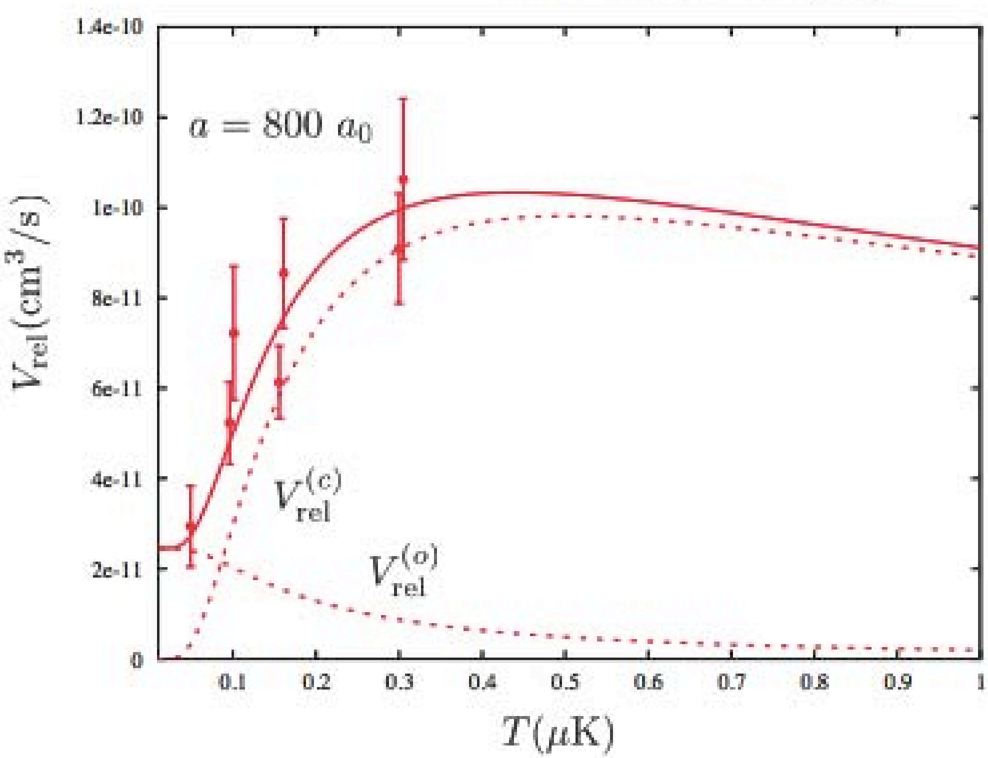


F. Ferlaino *et. al* PRL (2008)



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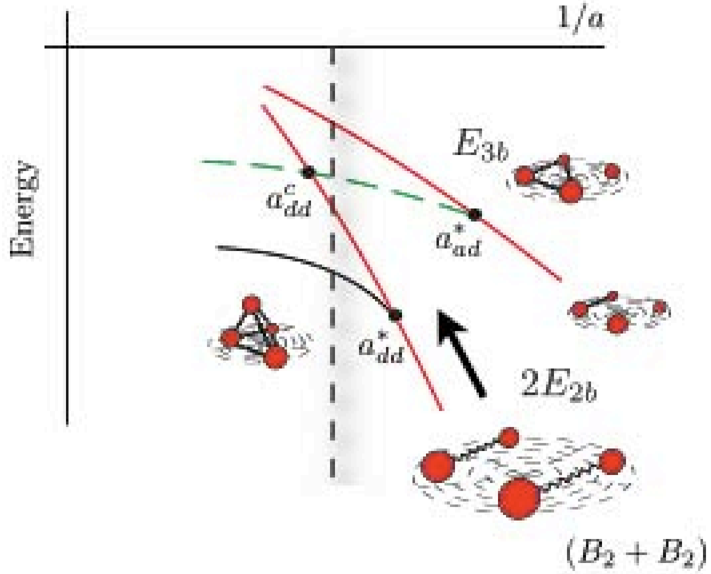

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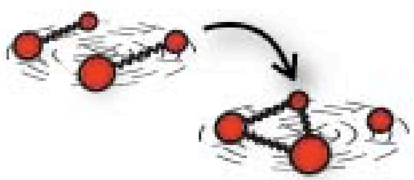

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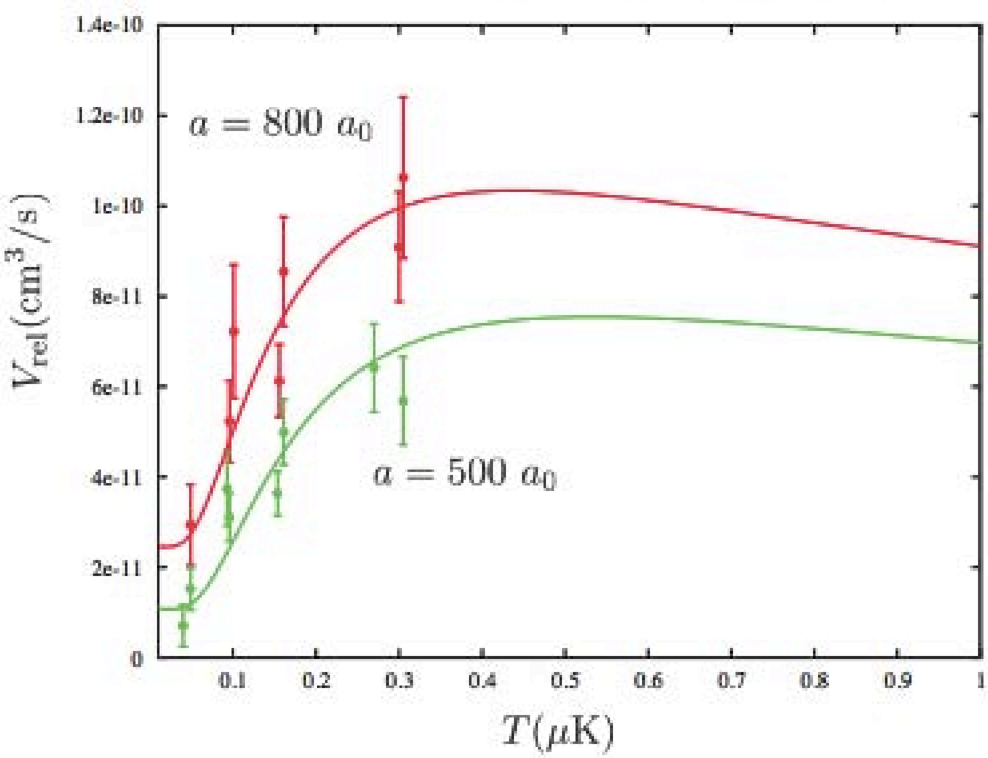

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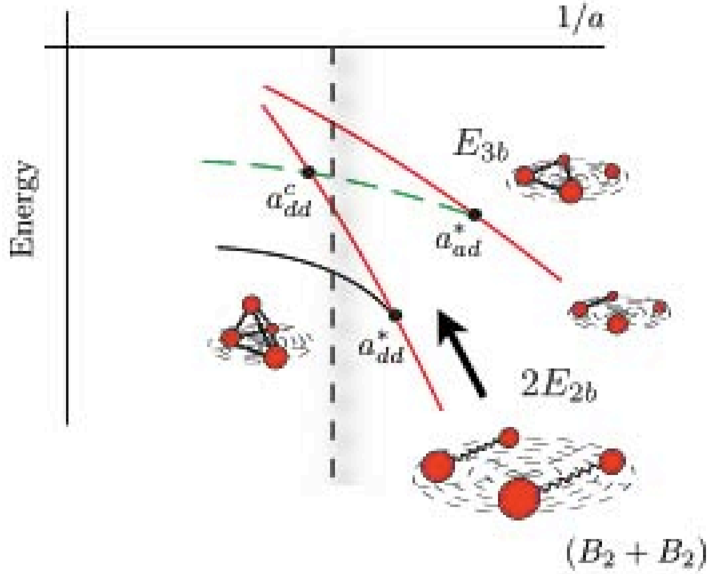

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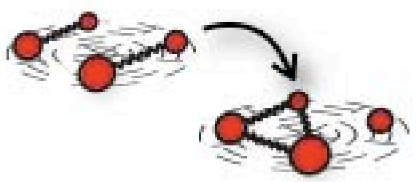

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