Hyperspherical Harmonics for Weakly Bound Systems

Nir Barnea

The Hebrew University, Jerusalem, Israel

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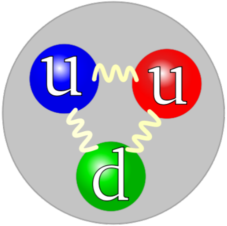
Outline

Nucleon-nucleon interaction

Background

- The underling theory for nuclear physics is QCD.



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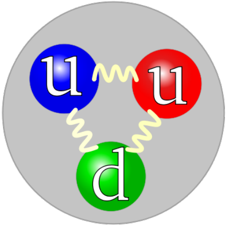

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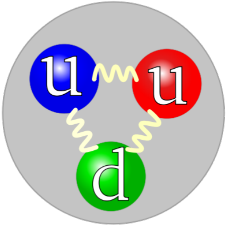

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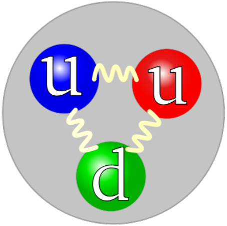

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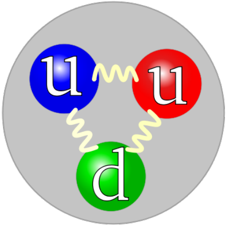
- Effective Field Theory provides a consistent way to derive the nuclear interaction.



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Characteristics of realistic NN forces

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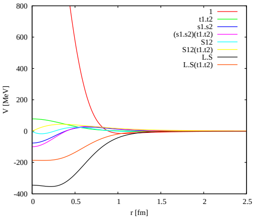

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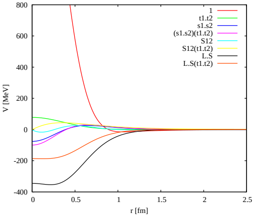

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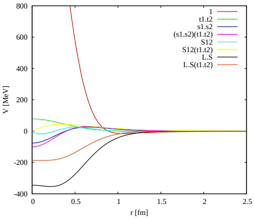

χ^2/datum for the reproduction of the 1999 *np* database

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
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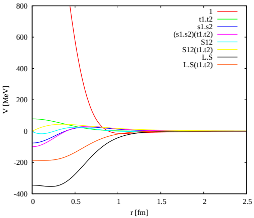

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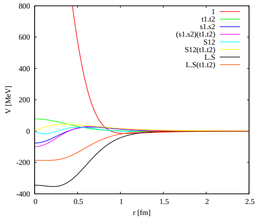

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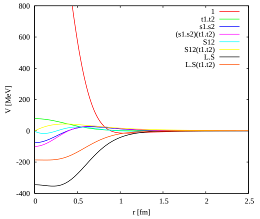

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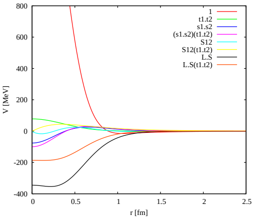

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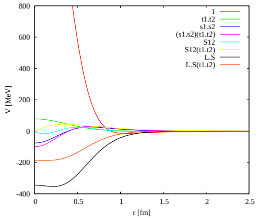
- Non-central.
- EFT potentials are naturally formulated in momentum space.



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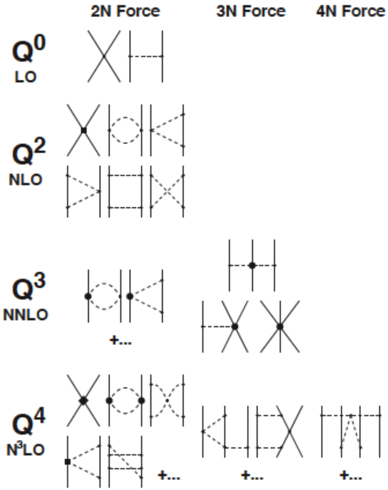
The EFT expansion



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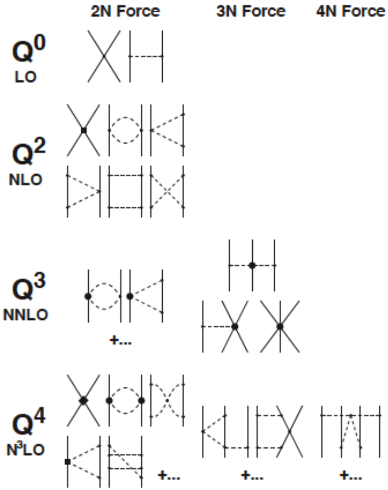

The V8 potential

$$V_{NN}(r_{ij}) = \sum_p V_p(r_{ij})\hat{O}_p(ij)$$

Central terms $p = 1, \ldots, 4$

$$1, (\sigma_i \cdot \sigma_j), (\tau_i \cdot \tau_j), (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

LS coupling $p = 5, 6$

$$LS, LS(\tau_i \cdot \tau_j)$$

Tensor force $p = 7, 8$

$$S_{ij}, S_{ij}(\tau_i \cdot \tau_j)$$

The Hyperspherical Harmonics

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x_1 &= \rho \cos(\alpha) \cos(\beta) \cos(\delta) \\
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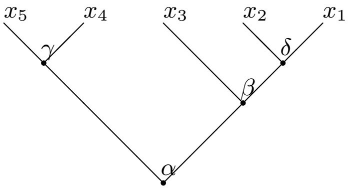
The "Tree" diagram



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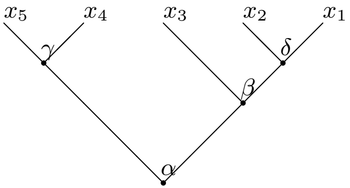

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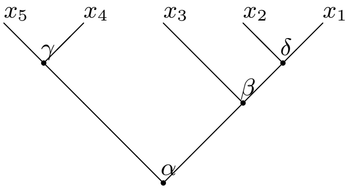

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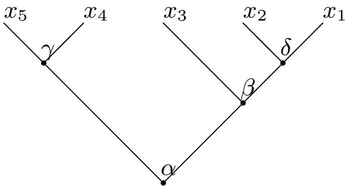

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5. All the rest of us ...

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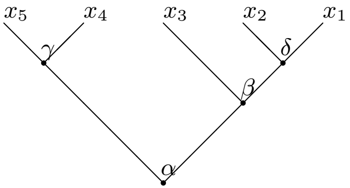

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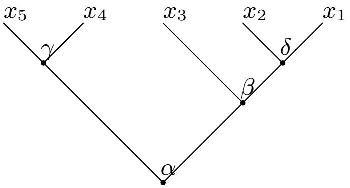

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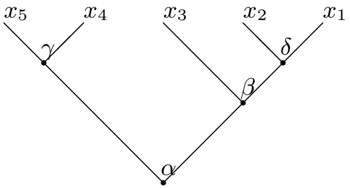

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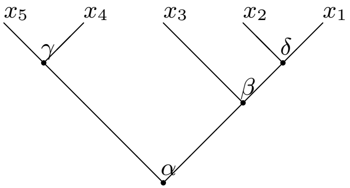

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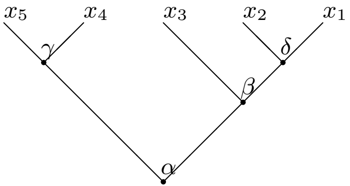

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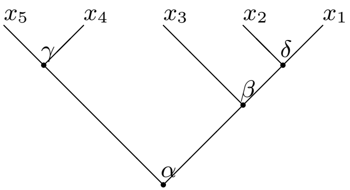

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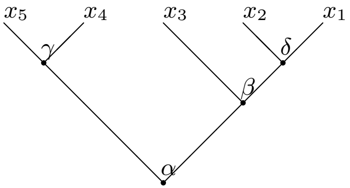

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7. Each junction adds a factor
   $N \cos^{K_R}(\theta) \sin^{K_L}(\theta) P^{(\alpha_R,\alpha_L)}_{(K-K_R-K_L)/2}(\cos(2\theta))$

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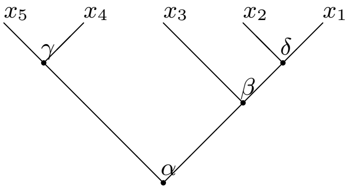

Removing the center of mass - The Jacobi coordinates

The normalized equal mass Jacobi coordinates

$$
\begin{aligned}
\boldsymbol{\eta}_1 &= \sqrt{\frac{1}{2}}\left(\boldsymbol{r}_2 - \boldsymbol{r}_1\right) \\
\boldsymbol{\eta}_2 &= \sqrt{\frac{2}{3}}\left(\boldsymbol{r}_3 - \frac{1}{2}(\boldsymbol{r}_2 + \boldsymbol{r}_3)\right) \\
& \dots \\
\boldsymbol{\eta}_{N-2} &= \sqrt{\frac{N-2}{N-1}}\left(\boldsymbol{r}_{N-2} - \frac{1}{N-2}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \cdots + \boldsymbol{r}_{N-3})\right) \\
\boldsymbol{\eta}_{N-1} &= \sqrt{\frac{N-1}{N}}\left(\boldsymbol{r}_{N-1} - \frac{1}{N-1}(\boldsymbol{r}_1 + \boldsymbol{r}_2 + \cdots + \boldsymbol{r}_{N-1})\right)
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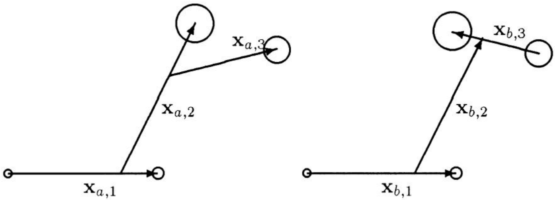
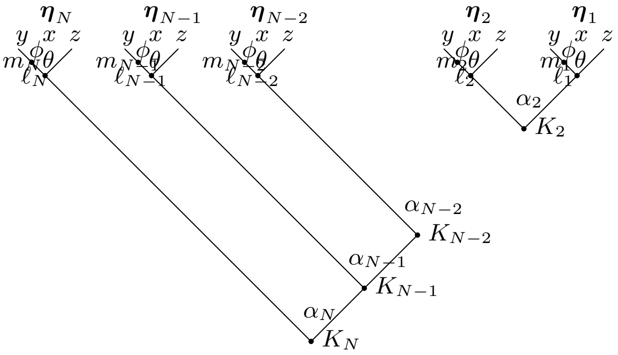


The common "Tree"



$$\mathcal{Y}_{[K]} = \Big[\prod_{j=1}^{N} Y_{\ell_j,\, m_j}(\hat{\eta}_j)\Big]$$
$$\times \Big[\prod_{j=2}^{N} \mathcal{N}_{j,K_j}^{\ell_j,K_{j-1}} (\sin\alpha_j)^{\ell_j} (\cos\alpha_j)^{K_{j-1}} P_{\mu_j}^{(\ell_j+\frac{1}{2},K_{j-1}+\frac{3j-5}{2})}(\cos(2\alpha_j))\Big]$$

The Merits of the HH expansion

- A complete set of basis functions.

$$\sum_{[K]} \mathcal{Y}^*_{[K]}(\Omega')\mathcal{Y}_{[K]}(\Omega)\frac{\delta(\rho-\rho')}{\rho^{D-1}} = \prod_{i=1}^{N} \delta(\boldsymbol{\eta}_i - \boldsymbol{\eta}'_i)$$

- Easy transformation between configuration and momentum space

$$e^{i\sum \boldsymbol{\eta}_j \boldsymbol{q}_j} = \frac{(2\pi)^{D/2}}{(Q\rho)^{D/2-1}} \sum_{[K]} i^K \mathcal{Y}^*_{[K]}(\Omega_q)\mathcal{Y}_{[K]}(\Omega) J_{K+D/2-1}(Q\rho)$$

- Good asymptotics.
- With appropriate choice of Jacobi coordinates and states clusterization can be "easily" treated.

The HH expansion in 4 steps

1. Remove the center of mass

$$\vec{r}_1, \vec{r}_2, \ldots \vec{r}_A \longrightarrow \vec{R}_{c.m.}, \vec{\eta}_1 \vec{\eta}_2 \ldots \vec{\eta}_{A-1}$$

2. Introduce hyperspherical coordinates

$$\vec{\eta}_1 \vec{\eta}_2 \ldots \vec{\eta}_{A-1} \longrightarrow \rho = \sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_{A-1}^2}, \Omega$$

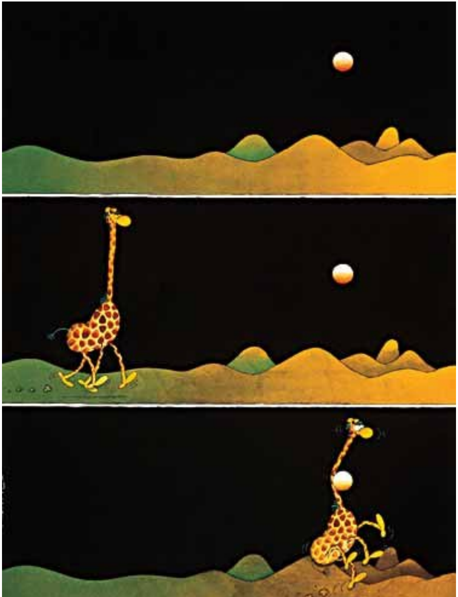
3. Expand the wave function using hyperspherical harmonics

$$\Psi(\rho, \Omega) = \sum_{K \leq K_max} R_{[K]}(\rho) \mathcal{Y}_{[K]}(\Omega)$$

4. Solve the Schrödinger equation

$$H = -\frac{1}{2}\left(\frac{\partial^2}{\partial \rho^2} + \frac{3A-4}{\rho}\frac{\partial}{\partial \rho} - \frac{\hat{K}^2}{\rho^2}\right) + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

Not so fast !!!



There are two Major obstacles

1. The HH basis has no good permutational symmetry. (Anti)Symmetrization must be enforced.
2. For some nuclear forces the convergence of the HH expansion is notoriously slow and must be accelerated.

Strategies

1. Apply the anti-symmetrization to the HH basis

$$\hat{A} = \sum_{g \in S_A} sign(g)\hat{g}$$

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2. Do it recursively in steps

$$\hat{A} = \Big(1 - (1,A) - (2,A) - \ldots (A-1,A)\Big) \\ \cdots \\ \times\Big(1 - (1,3) - (2,3)\Big) \times \Big(1 - (1,2)\Big)$$

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3. Generate HH states from HO Slater determinant.

$$det(HO) = e^{-\frac{1}{2}\rho^2}\rho^K \hat{A}\Big(\mathcal{Y}(\Omega)\mathcal{X}(s_i, t_i)\Big)$$

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4. Use the group of kinematic rotations $\boldsymbol{\eta}_i \longrightarrow \boldsymbol{\eta}'_i = \hat{g}\boldsymbol{\eta}_i$

(Anti) Symmetrization via Kinematic rotations

Using the group of kinematic rotations $\boldsymbol{\eta}'_i = \hat{g}\boldsymbol{\eta}_i$, the HH symmetrization can be carried out in two steps

$$HH \longrightarrow O_{(A-1)}$$

and

$$O_{(A-1)} \longrightarrow S_A$$

In short we use the following group-subgroup chain

$$\begin{array}{ccccccc} O_{3(A-1)} & \subset & O_3 \otimes O_{(A-1)} & \subset & O_3 \otimes S_A \\ K & & LM \quad \boldsymbol{\Lambda_{A-1}} & & LM \quad Y_A \end{array}$$

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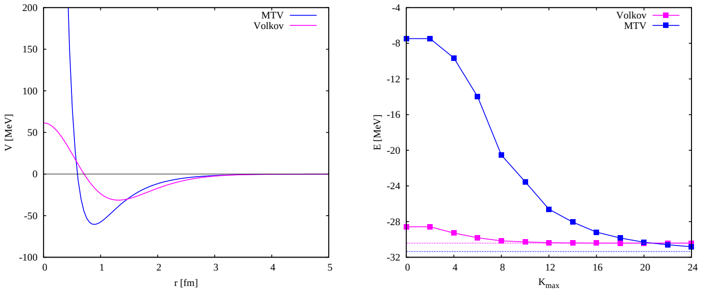
6-body system: A comparison between direct symmetrization and symmetrization through the kinematical group O(A-1)

K	HH $\rightarrow S_A$	HH $\rightarrow O_{(A-1)}$	$O_{(A-1)} \rightarrow S_A$	Ratio
2	198	56	108	1.21
4	11308	728	1528	5.01
6	516647	8771	16511	20.44
8	10^7	84700	127544	47.12

Convergence - Statement of the problem



- For potentials with Coulomb type singularities the HH expansion of Ψ converge as K_{max}^{-2}
- For Gaussian potentials Ψ converge as $e^{-cK_{max}}$.
- Actually. The **problem** is not the slow convergence rate but rather the fast growth in the number of HH states.

Convergence

1. Correlations: CHH, PHH, CFHHM ...

$$\Psi(\rho,\Omega) = F(r_{ij}) \sum_{K \leq K_max} R_{[K]}(\rho)\mathcal{Y}_{[K]}(\Omega)$$

2. Basis Reduction: The potential basis (Fabre de-la Ripple), The Pisa Group, Efros.
For a Bose-system this expansion may take the following form

$$\Psi(\rho,\Omega) = \sum_{ij}\sum_{[K]} R^{(2)}_{[K]}(\rho)\mathcal{Y}_{[K]}(\Omega_{ij}) + \sum_{ij,kl}\sum_{[K]} R^{(2,2)}_{[K]}(\rho)\mathcal{Y}_{[K]}(\Omega_{ij,kl}) + \ldots$$

3. Effective interaction for the HH expansion. Replace the bare potential by an effective one:

$$V^{(2)} \longrightarrow V^{(2)}_{eff}$$

$$V^{(3)} \longrightarrow V^{(3)}_{eff}$$

The Effective Interaction Hyperspherical Harmonics (EIHH)

The $V_{eff}^{(2)}$ is derived from a "2-body" Hamiltonian

$$H_2(\rho) = \frac{1}{2m}\frac{\hat{K}^2}{\rho^2} + V(\vec{r} = \sqrt{2}\rho\sin\alpha_N \cdot \hat{\eta}_N)\ ,$$

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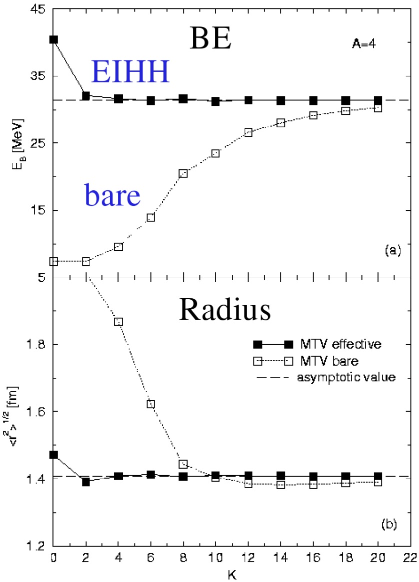
Finally the effective interaction is given by

$$V_{2\,eff}(\rho) = H_{2\,eff}(\rho) - \frac{1}{2m}\frac{\hat{K}^2}{\rho^2}$$

4-body ground-state - Bare vs Effective



4-body ground-state

Convergence of the **EIHH** method for ^{4}He binding energy E_b [MeV] and root mean square matter radius $\langle r^2 \rangle^{\frac{1}{2}}$ [fm] with AV18 and AV18+UIX potentials.

K_{max}	AV18 E_b	AV18 $\langle r^2 \rangle^{\frac{1}{2}}$	AV18+UIX E_b	AV18+UIX $\langle r^2 \rangle^{\frac{1}{2}}$
6	25.312	1.506	26.23	1.456
8	25.000	1.509	27.63	1.428
10	24.443	1.520	27.861	1.428
12	24.492	1.518	28.261	1.427
14	24.350	1.518	28.324	1.428
16	24.315	1.518	28.397	1.430
18	24.273	1.518	28.396	1.431
20	24.268	1.518	28.418	1.432

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FY [Nogga]	24.25		28.50	
FY [Lazauskas]	24.22	1.516		
HH [Viviani]	24.21	1.512	28.46	1.428
GFMC [Wiringa]			28.34	1.44

BENCHMARK for ^{4}He ground state with AV8' potential

H. Kamada *et al.*, PRC **64** 044001 (2001)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.25	-128.13	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

The effective 3-body force

Convergence of ^{6}Li ground state with the AV8' NN potential

K_{max}	B.E $V^{(2)}eff$	B.E. ($K_Q^{(3)} = 16$)	B.E. ($K_Q^{(3)} = 20$)
2	39.80	38.83	
4	33.43	30.37	30.27
6	31.02	31.11	30.94
8	31.13	31.17	30.94
10	30.23	31.22	30.88

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NCSM [Navratil]	30.30		
GFMC [Pieper]	29.70(5)		

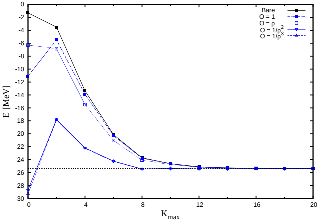
EIHH for Non-local interactions

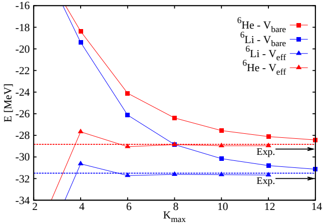
Local potential: $V^{(2)}_{eff}(\rho)$ was derived using the fact that V is diagonal in configuration space.
Non-local potential: (or in general) we can construct EI from a "2-body" Hamiltonian of the form:

$$\langle n|H_2|n\rangle = \frac{1}{2m}\hat{K}^2\langle n|\frac{1}{\rho^2}|n\rangle + \langle n|V_{A,A-1}|n\rangle$$

In this case

$$\langle n|V^{(2)}_{eff}|n'\rangle = \delta_{n,n'}\langle n|H^{(2)}_{eff} - \frac{1}{2m}\frac{\hat{K}^2}{\rho^2}|n\rangle + (1-\delta_{n,n'})\langle n|V^{(2)}|n'\rangle$$





Reactions

The nuclear response function

$$R(\omega) = \int d\psi_f |\langle \psi_i \mid \hat{O} \mid \psi_f \rangle|^2 \delta(E_f - E_i - \omega)$$

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$$L(\sigma) = \int d\psi_f \langle \psi_i \mid \hat{O} \mid \psi_f \rangle \frac{1}{E_f - E_i - \sigma - i\Gamma} \frac{1}{E_f - E_i - \sigma + i\Gamma} \langle \psi_f \mid \hat{O} \mid \psi_i \rangle$$

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Or

$$L(\sigma) = \int d\psi_f \langle \psi_i \mid \hat{O} \frac{1}{H - E_i - \sigma - i\Gamma} \mid \psi_f \rangle \langle \psi_f \mid \frac{1}{H - E_i - \sigma + i\Gamma} \hat{O} \mid \psi_i \rangle$$

The Lorentz Integral transform (LIT) method

Using closure

$$L(\sigma) = \langle \psi_i \mid \hat{O} \frac{1}{H - E_i - \sigma - i\Gamma} \frac{1}{H - E_i - \sigma + i\Gamma} \hat{O} \mid \psi_i \rangle = \langle \tilde{\psi} \mid \tilde{\psi} \rangle$$

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5. $R(\omega)$ is obtained trough inversion of the transform, $L(\sigma)$.

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1. The LIT equation is just the Schrödinger equation with a source.
2. The only solution to the homogeneous equation is the trivial $\tilde{\psi}=0$ solution.
3. Since the source is localized $\tilde{\psi} \longrightarrow 0$ as $r \longrightarrow \infty$.
4. The LIT equation can be solved using bound state methods !!!
5. $R(\omega)$ is obtained trough inversion of the transform, $L(\sigma)$.

The Lorentz Integral transform (LIT) method

Using closure

$$L(\sigma) = \langle\psi_i \mid \hat{O}\frac{1}{H - E_i - \sigma - i\Gamma}\frac{1}{H - E_i - \sigma + i\Gamma}\hat{O} \mid \psi_i\rangle = \langle\tilde{\psi} \mid \tilde{\psi}\rangle$$

$\mid \tilde{\psi}\rangle$ is the solution of the Schrödinger like equation

$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi}\rangle = \hat{O} \mid \psi_0\rangle.$$

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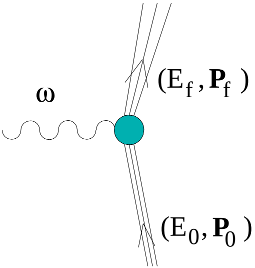
V. Efros, W. Leidemann, and G .Orlandini, PLB **238**, 130 (1994).

Photoabsorption of Nuclei



Real Photon

$$|\mathbf{q}| = \omega$$

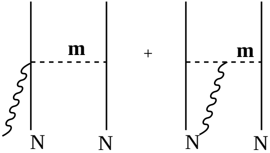
$$\sigma(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

$$R(\omega) = \sum\!\!\!\!\!\!\int_f |\langle \Psi_f | \mathrm{E1} | \Psi_0 \rangle|^2 \, \delta(E_f - E_0 - \omega)$$

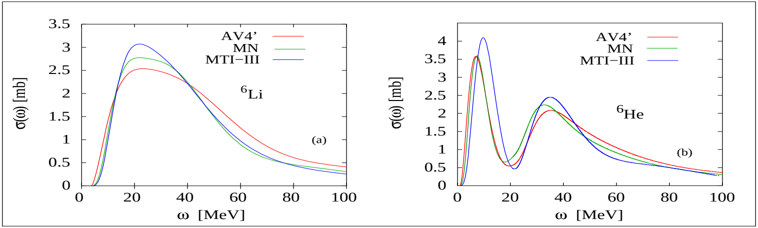
$$\mathrm{E1} = \sum_i^A (\mathbf{r}_i - \mathbf{R}_{\mathrm{CM}}) \frac{1 + \tau_i^3}{2}$$

The MEC are implicitly included in the E1-response via the Siegert theorem

Alternatively....

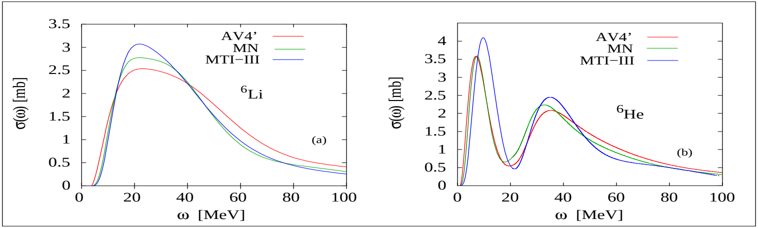


Six-body Photoabsorption



Bacca, Marchisio, Barnea, Leidemann, Orlandini, PRL **89** (2002)

Six-body Photoabsorption



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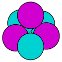
Giant Dipole Mode



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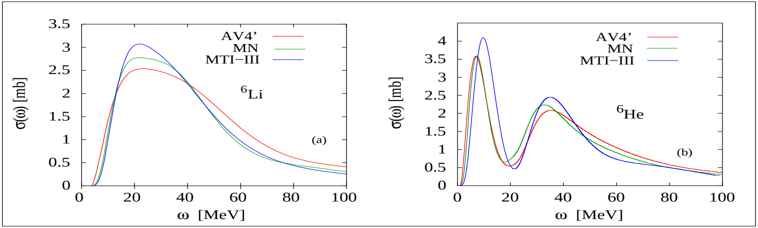

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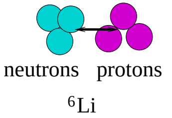

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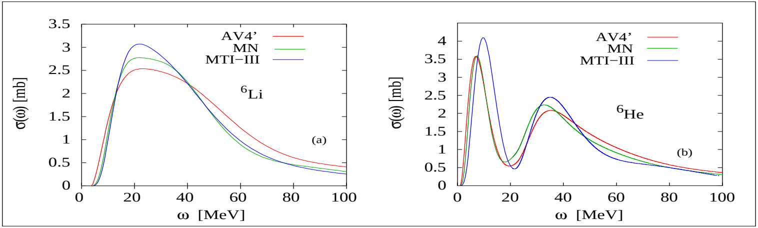

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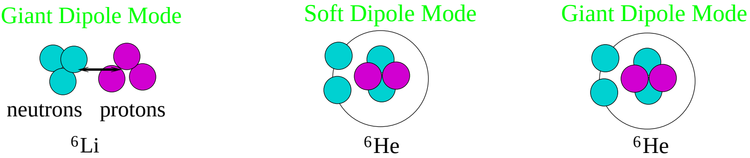

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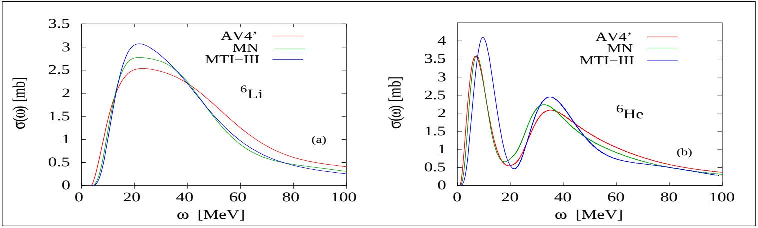

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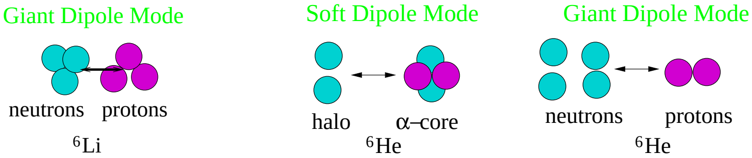

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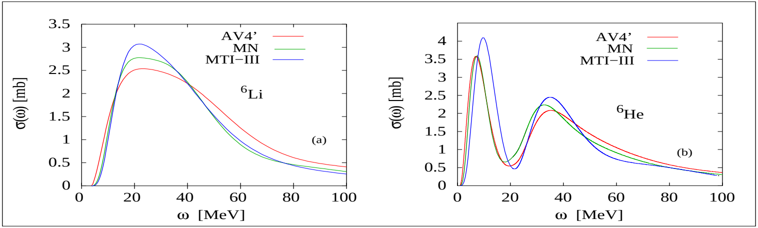

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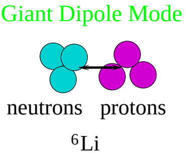
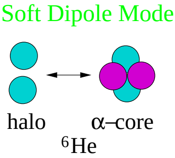
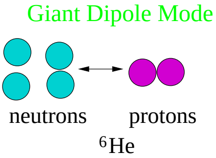
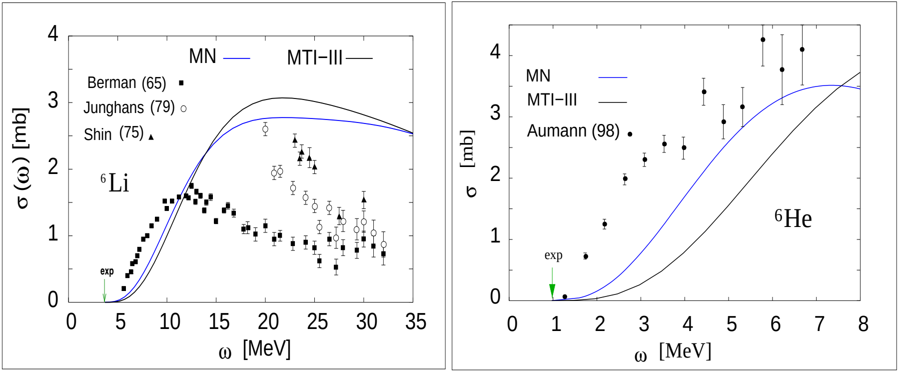






AV4'	$\sqrt{< r^2 >}$ fm	$\sqrt{< r_p^2 >}$ fm
	2.41(5)	1.80(4)
Exp	PRL **93** (2004) 142501	1.912 ± 0.018

Six-body Photoabsorption - Comparison with experiment



S. Bacca, N. Barnea, W. Leidemann, and G. Orlandini, PRC **69**, 057001 (2004)

- MTI-III and MN are dominant S-wave potentials (L-even) interactions

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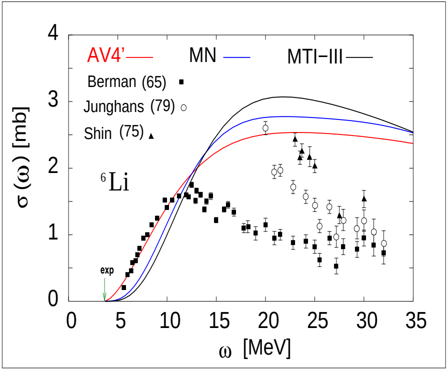

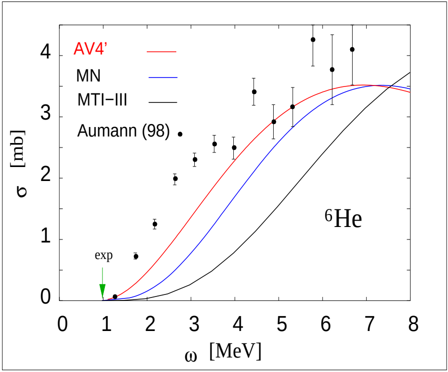

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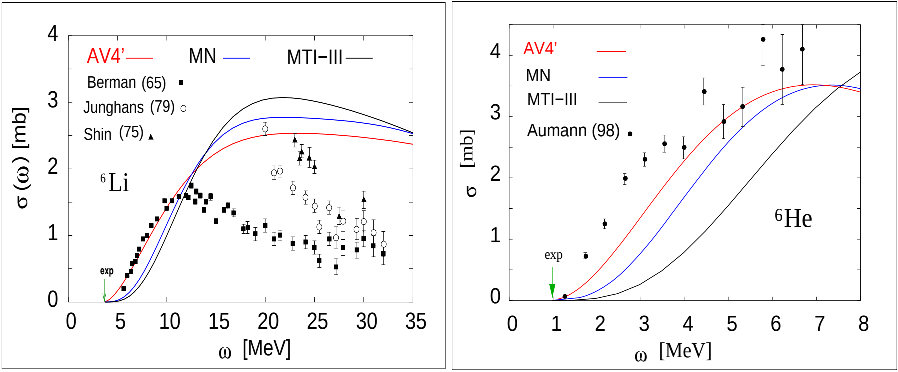

S. Bacca, N. Barnea, W. Leidemann, and G. Orlandini, PRC **69**, 057001 (2004)

- MTI-III and MN are dominant S-wave potentials (L-even) interactions
- AV4' contains also (L-odd) interactions
  Dominant S- and P-wave Potential

Summary and Conclusions

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Thanks, and enjoy the workshop !!!

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