RG study of IR structure in gauge theories - Confinement and IR fixed points

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New Applications of the Renormalization Group Method

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At some finite T it undergoes a phase transition to a deconfined phase exhibiting new phenomena. For up to temperatures several times the deconfinement T it appears to behave like a strongly coupled low viscosity fluid; the dynamics evolution of the transition apparently brings about (effective) thermalization within surprisingly short time scales (0.5 fm/c).

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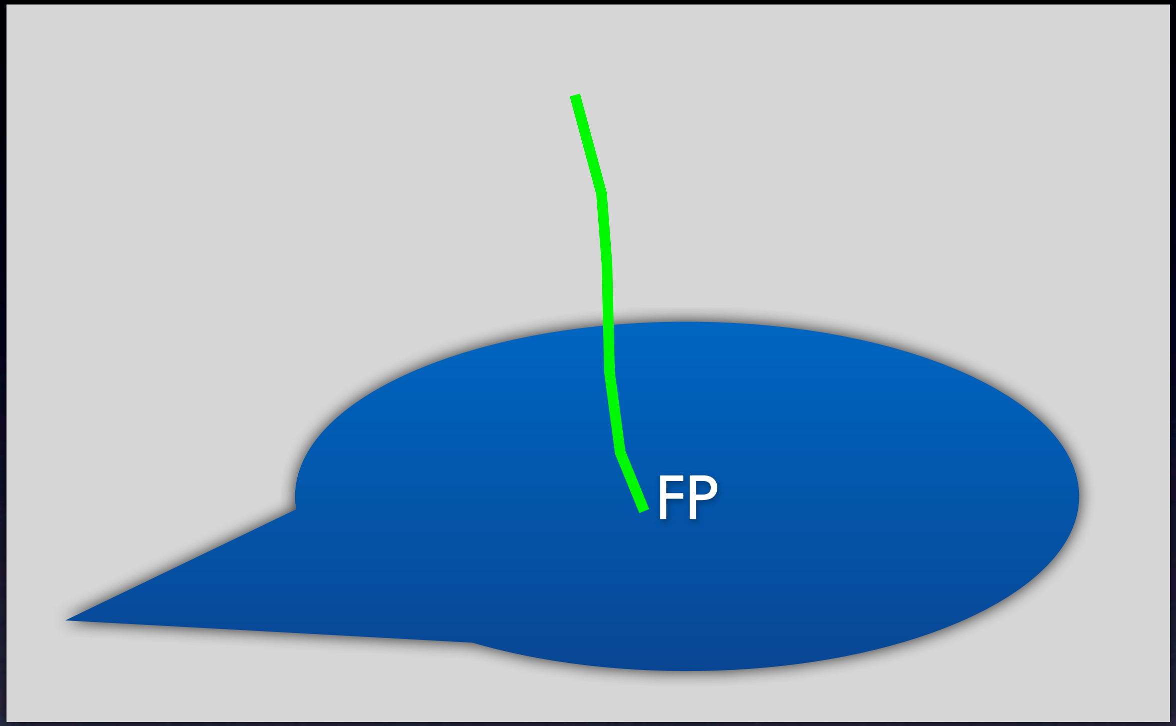

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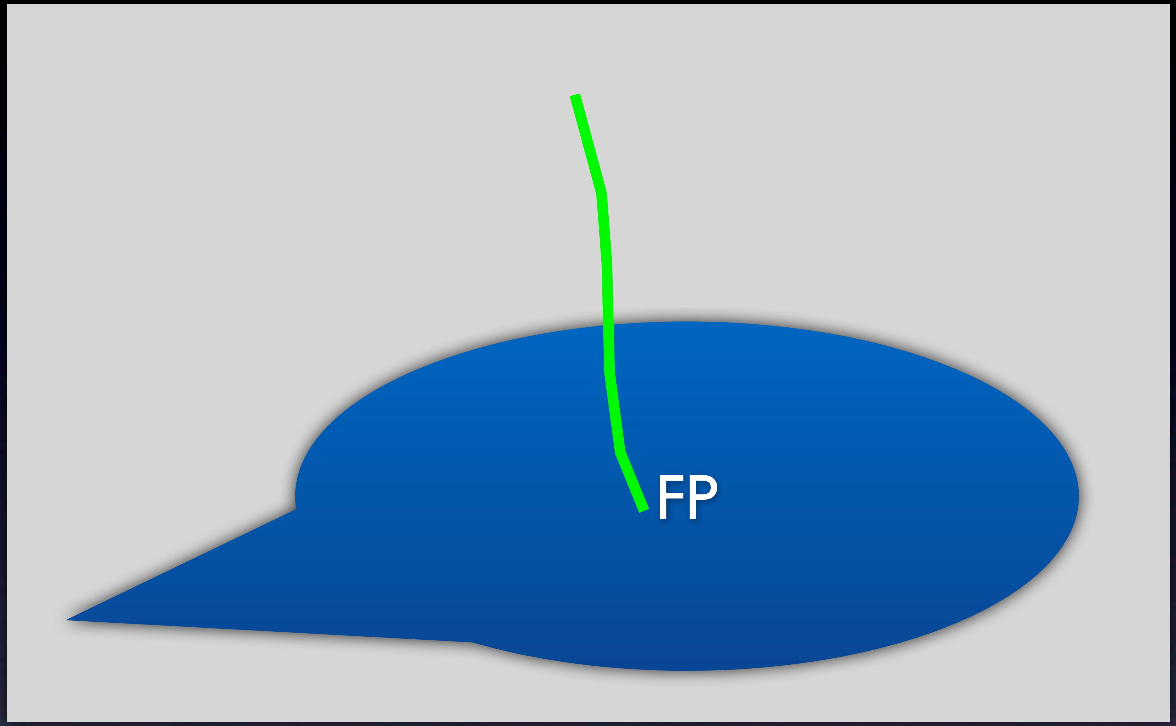

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This means that QCD with massless fermions is a theory with no parameters : 'the perfect theory'

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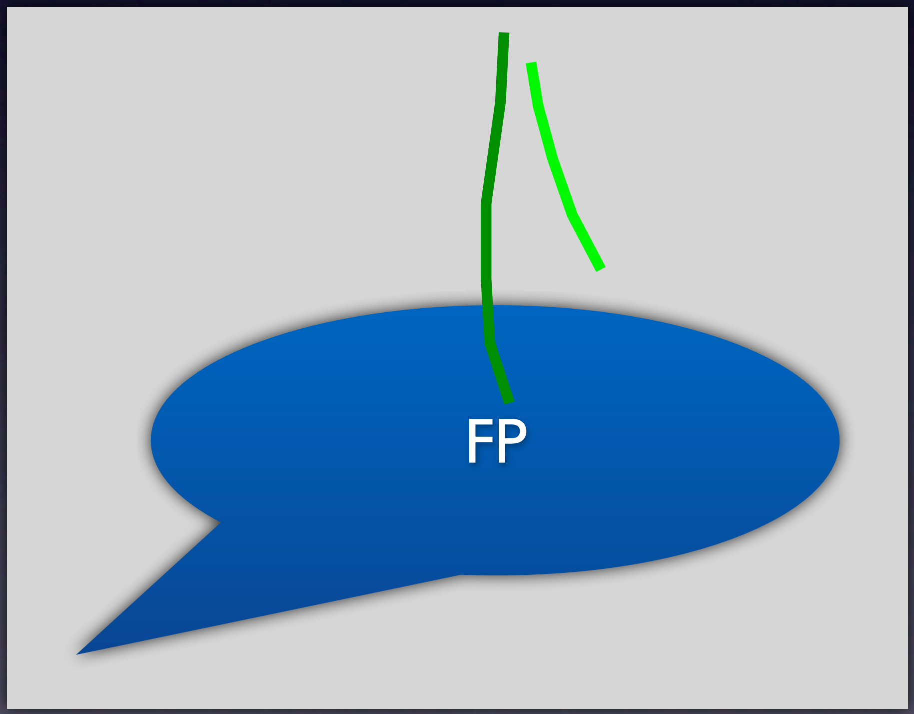
The natural framework for dealing such problems is a Wilsonian RG blocking procedure bridging the different scale regimes.

The history of actual RG blocking implementation in LGT is somewhat patchy.

This is probably due to the success of direct MC simulations of PF's and expectations in pure gauge theories -- much more challenging in the presence of fermions (Grassmann variables) and the non-local fermion determinant.

Major unresolved problems: finite chemical potential, non-equilibrium (real-time) dynamical evolution.

Various RG blocking implementations:



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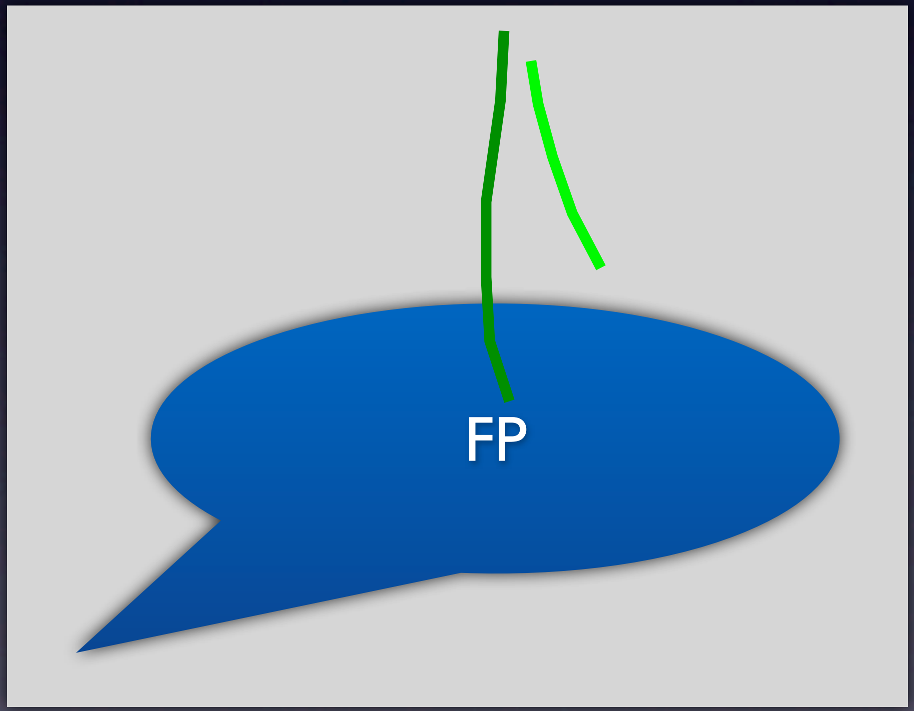
- Wilson (early)



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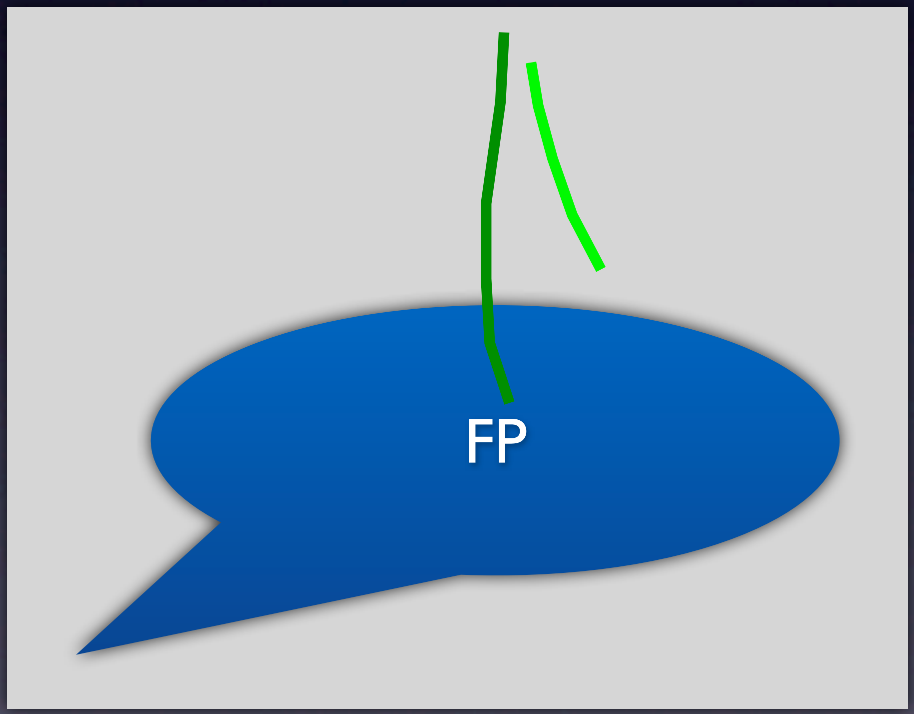

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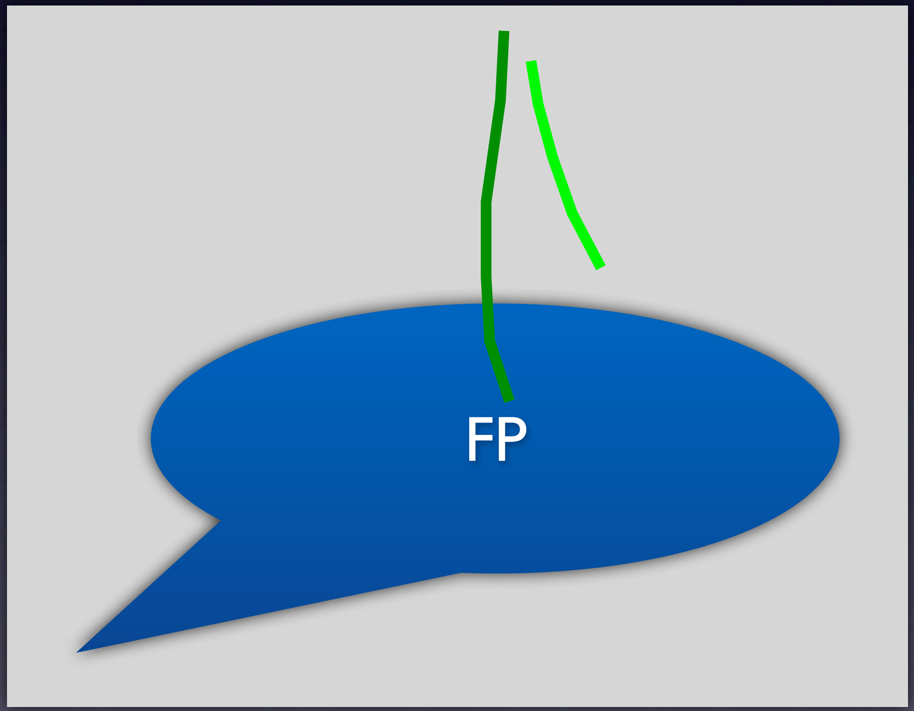

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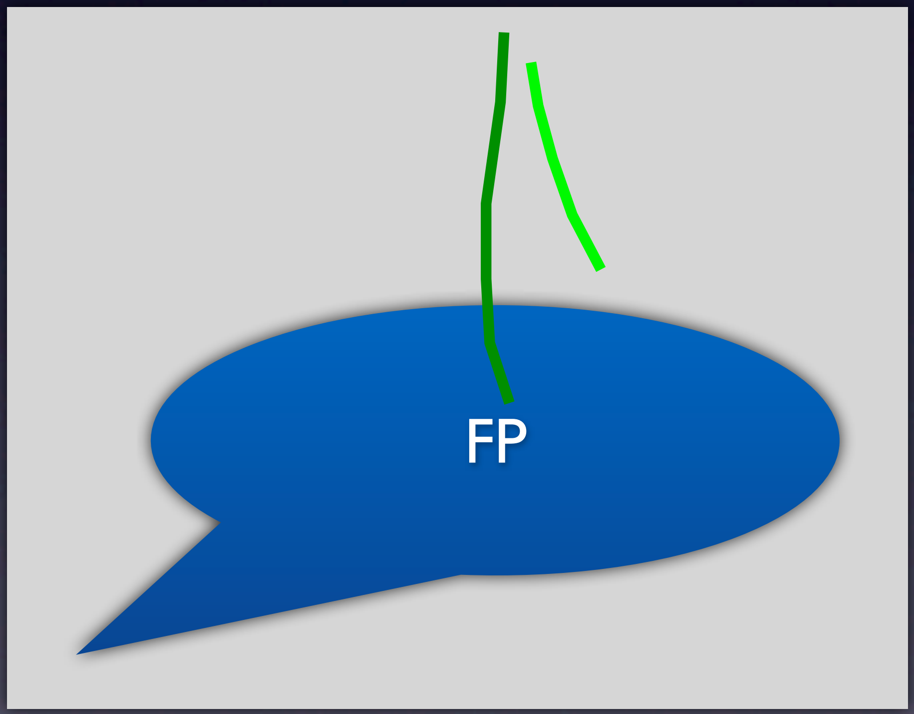
Find action along Wilsonian renormalized trajectory



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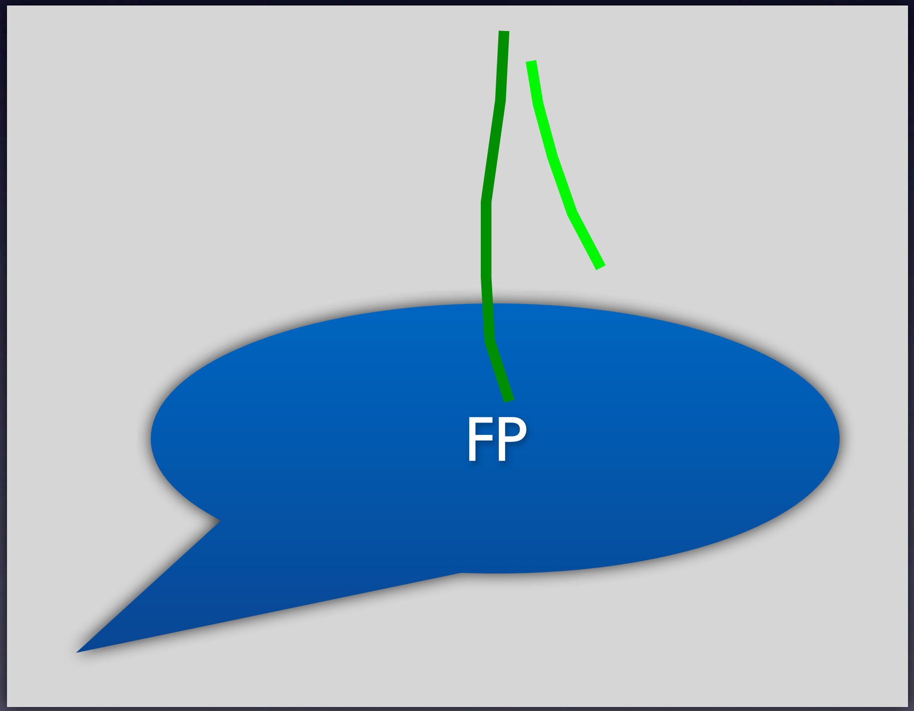

- MCRG (more modest)

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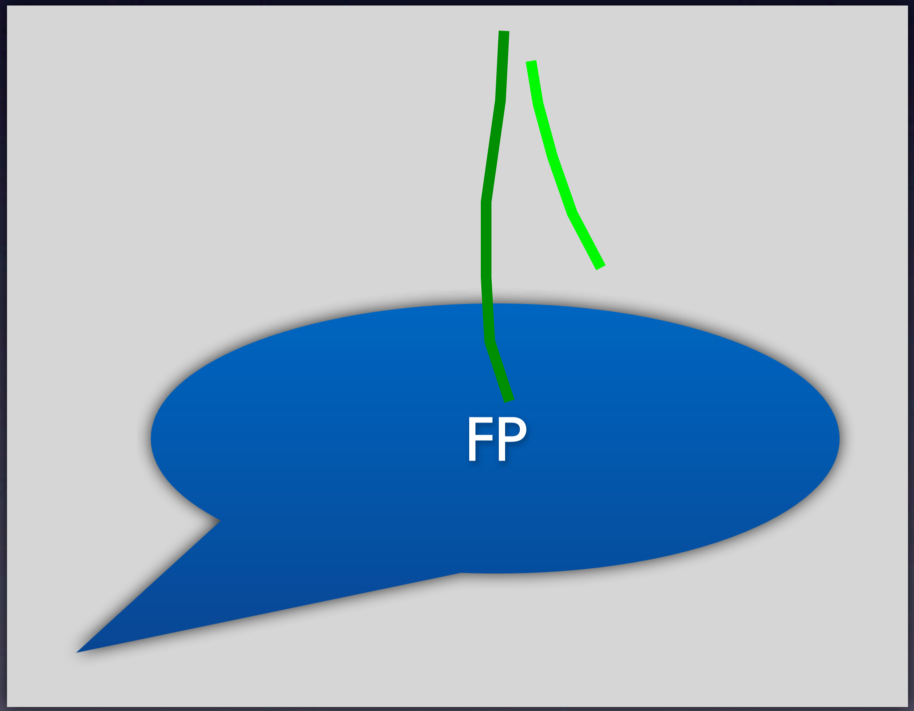

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• Do not attempt to construct general RG effective action suitable for any observable.

• Employ approximate but easily explicitly computable RG decimation procedures that can provide bounds on judicially chosen observables (free energies, ...)

• Use the bounds to constrain the corresponding exact quantities and hence derive statements about their behavior.

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It turns out to be very effective for pure gauge theories.

Adding fermions, though, presents, as usual, a challenge of another order of magnitude.

Start with plaquette action at spacing a, for example Wilson action:

$$A_p(U) = \frac{\beta}{2}\,\mathrm{Re}\,\mathrm{tr}U_p$$

Character expansion of the exponential of plaquette action:

$$\begin{aligned} e^{A_p(U)} &= \sum_j d_j\, F_j(\beta, a)\, \chi_j(U) \\ &= F_0 \Big[1 + \sum_{j\neq 0} d_j\, c_j(\beta)\, \chi_j(U) \Big] \end{aligned}$$

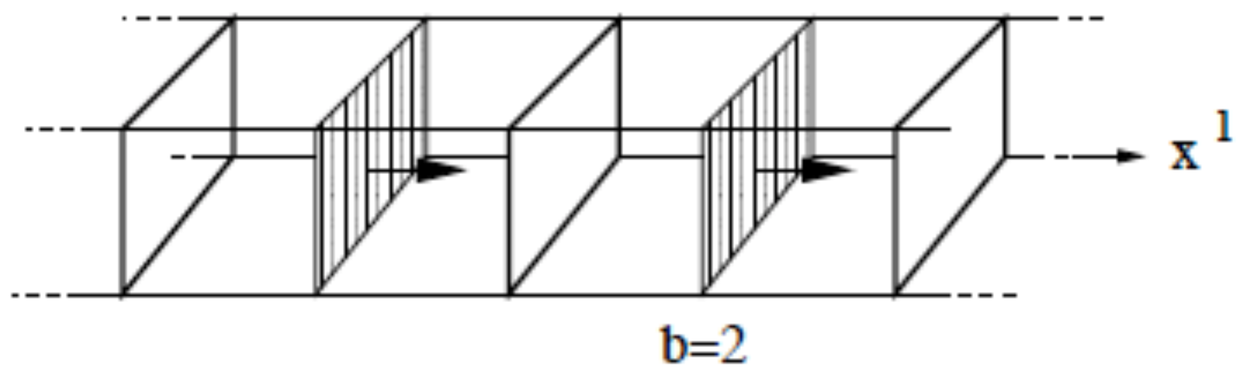
SU(2): $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, \quad d_j = (2j+1)$.

Partition Function (PF) on lattice Λ

$$Z_\Lambda(\beta) = \int dU_\Lambda \prod_p \Big[1 + \sum_{j\neq 0} d_j\, c_j(\beta)\, \chi_j(U) \Big] \equiv Z_\Lambda\Big(\{ c_j(\beta) \} \Big)$$

‘Potential moving’ RG schemes

‘Plaquette moving’:



‘Weaken’, i.e. decrease β, c_j of $-$plaquettes (shaded);
‘Strengthen’, i.e. increase β, c_j of $+$plaquettes (heavy).
Apply this procedure successively in all directions in each hypercube of side length *ba*.

Limiting case (MK): vanishing strength for interior plaquettes -- complete ‘move’ to boundary.

The RG decimation procedure can be summarized as set of decimation rules for each successive step:

$$a \longrightarrow ba \longrightarrow b^2 a \longrightarrow \cdots \longrightarrow b^n a$$
$$\Lambda \longrightarrow \Lambda^{(1)} \longrightarrow \Lambda^{(2)} \longrightarrow \ldots \longrightarrow \Lambda^{(n)}$$

from lattice $\Lambda^{(m)}$ of spacing ab^m to lattice $\Lambda^{(m+1)}$ of spacing ab^{m+1}

The rules give explicit expressions for the computation of the character expansion coefficients at the m-th step given those at the (m-1)-th step:

$$\begin{aligned} F_0(m) &= F_0(\zeta, r, b, \{c_i(m-1)\}) \\ c_j(m) &= c_j(\zeta, r, b, \{c_i(m-1)\}) \end{aligned}$$

Parameters $\zeta, \ r, \ldots$ control the amount by which undecimated plaquettes are 'renormalized' to compensate for the decimated ones.

After each decimation step:

$$Z_{\Lambda^{(m-1)}}\Big(\{c_j(m-1)\}\Big) \to F_0(m)^{|\Lambda^{(m)}|}\, Z_{\Lambda^{(m)}}\Big(\{c_j(m)\}\Big)$$

with resulting action on the m-th lattice

$$\begin{aligned} \exp A_p(m) &= \Big[1+\sum_{j\neq 0} d_j\, c_j(m)\, \chi_j(U)\Big] \\ &= \exp\Big[\sum_j \beta_j(m)\, \chi_j(U)\Big] \end{aligned}$$

Both positive and negative couplings generally occur in action but coefficients $c_j(m) \geq 0$ if reflection positivity is maintained by decimation rules.

The resulting PF at the m-th step can be either an upper or a lower bound on the PF of the (m-1)-th step by appropriate choice of the decimation parameters.

One may then interpolate between the upper and lower bounds by means of interpolating expansion coefficients :

$$c_j(m, \alpha) \;, \qquad 0 < \alpha < 1$$

There is then a value $\alpha^{(m)} = \alpha_\Lambda^{(m)}(\zeta, r, \cdots)$ at which the two PF become equal. Iterating this procedure one obtains an exact representation of the PF on successively decimated lattices:

$$\begin{aligned} Z_\Lambda(\beta) &= Z_\Lambda\Big(\{c_j(\beta)\}\Big) \\ &= \ln F_0(1, \alpha^{(1)})\, Z_{\Lambda^{(1)}}\Big(\{c_j(1, \alpha^{(1)}\}\Big) \\ &= \cdots \\ &= \exp\Big[\sum_{m=1}^{n} \ln F_0(m, \alpha^{(m)})\, |\Lambda|/b^{dm}\Big]\, Z_{\Lambda^{(n)}}\Big(\{c_j(n, \alpha^{(n)})\}\Big) \end{aligned}$$

Choice of interpolation is of course not unique. The values $\alpha^{(m)}$ depend on this choice. Suppose one considers a family of interpolations parametrized by a parameter t. Then $\alpha^{(m)} = \alpha^{(m)}(t)$ and there is 'reparametrization invariance' in the PF representation:

$$Z_\Lambda(\beta) = \exp\Big[\sum_{m=1}^{n} \ln F_0(m, \alpha^{(m)}(t))\,|\Lambda|/b^{dm}\Big]\, Z_{\Lambda^{(n)}}\Big(\{c_j(n, \alpha^{(n)}(t))\}\Big)$$

Changes of interpolation choice, i.e. shifts in t, amount to shifts in the relative size of the accumulated 'bulk contributions' and the PF on the final lattice $\Lambda^{(m)}$ of lattice spacing ab^m

To fix this we need to compare with observables that couple at scales ab^m

Note: Procedure analogous to that in MCRG

Consider then expectation of observable O:

$$\Big\langle \mathcal{O} \Big\rangle_\Lambda = \frac{Z_\Lambda[\mathcal{O}]}{Z_\Lambda}$$

$$Z_\Lambda[\mathcal{O}] = \int dU_\Lambda \prod_p \Big[1 + \sum_{j\neq 0} d_j \, c_j(\beta) \, \chi_j(U) \Big] \, \mathcal{O}(U)$$

such as Wilson loops W[C], connected 2-plaquette correlator, defect order parameters (vortex free energy).

One may now, with some extra work, apply the previous procedure to $Z_\Lambda[\mathcal{O}]$ obtaining representations for it on successively decimated lattices.

Note: Contribution from insertion of $\mathcal{O}$ is of order $|S|/|\Lambda|$ (where $|S|$ 'support' of the observable) relative to bulk free energy per unit lattice volume.

Hence at each step bulk free energy contributions cancel (if need be by small shift in parametrization between numerator and denomiinator.

So

$$\langle \mathcal{O} \rangle = \frac{Z_\Lambda[\mathcal{O}]}{Z_\Lambda} = \cdots = \frac{Z_{\Lambda^{(m)}}[\mathcal{O}]}{Z_{\Lambda^{(m)}}} = \cdots = \frac{Z_{\Lambda^{(n)}}[\mathcal{O}]}{Z_{\Lambda^{(n)}}}$$

This procedure can be applied, with slightly varying technical details, to a variety of appropriate (long distance) observables.

Note:

- Each such representation holds only for the particular expectation considered -- it is not implied that the same specific values of decimation-interpolation parameters give exact representations for a different observable.
- In practice though they may give good approximation.
- The exact values of the interpolating parameters need not be known in order to (rigorously) deduce the observable's behavior as a function of scale provided the upper (lower) bounds are sharp enough -- the latter are easily computable at each step by explicit algebraic rules.

$SU(2)$

- Confining behavior is the result for any initial β since the upper bound

$$c_j(n) \to 0 \qquad \text{for} \qquad n \to \infty$$

for any initial β.

- Fixing the resulting string tension $\kappa(\beta, n)$ implies a relation between n and initial $\beta = 2/g^2$.

- Now zero coupling $g = 0$ is a fixed point of the decimations. This implies that to reach any fixed value of the string tension (some given value of $c_j(n)$'s) requires

$$n \to \infty \qquad \Longleftrightarrow \qquad \beta \to \infty \, .$$

In other words one necessarily has

$$g(a) \to 0 \qquad \text{for} \qquad a \to 0$$

(UV asymptotic freedom) as an essentially qualitative feature of the decimation flow.

$SU(3)$ Ditto

$U(1)$

- Starting with $\beta \geq \beta_0$, where $\beta_0 \sim 1$, the upper bound decimations hit a fixed point. In fact, the location of the fixed point is found to vary with varying starting (large) β, i.e. one gets a line of fixed points.
- For starting $\beta \geq \beta_0$ the decimations run to the strong coupling fixed point.
- This signals free massless behavior for weak coupling (continuum limit); and a transition of the lattice model to the (universal) confined phase at some $\beta \sim 1$.

RG blocking with Fermions

- Practical RG blocking schemes with dynamical fermions presents new set of problems

- Free fermions: Analytical work (Balaban et al, Wiese); more recent work in connection with rooting problem (Shamir)

- Interacting fermions: In connection with perfect action and MCRG

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Fermions being Grassmann variables cannot be dealt with as bosons.

• Grassmann 'integrals' cannot be simulated or approximated as bosonic integrals

• Proofs of bounds as those used in pure gauge theory above no longer hold.

• Any straight integrations over light fermions result in non-localities

The problem:
$$Z_\Lambda = \int dU_\Lambda [d\bar\psi d\psi]_\Lambda \; \exp[\, A_g(U) + \bar\psi K(U)\psi \,]$$
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$$\Lambda = \Lambda^{(1)} \cup (\Lambda \setminus \Lambda^{(1)})$$
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$$\exp[\, A_g^{(1)}(U)\,] = \int DV \; \exp[\, A_g(U,V)\,] \left[\frac{Det K(U,V)}{\int DV \; Det K(U,V)} \right]$$
$$\exp[\, A_f^{(1)}(U, \bar\psi, \psi\,] = \int DV [d\bar\eta d\eta] \exp A_f(U, V, \bar\psi, \bar\eta, \psi, \eta) \int [d\bar\chi d\chi]\, \rho(\bar\eta, \eta, \bar\chi, \chi)$$
$$= \int DV \; [d\bar\chi d\chi] \; \exp \hat A_f(U, V, \bar\psi, \psi, \bar\chi, \chi)$$

$$\exp[\,A_g^{(1)}(U)\,] \;=\; \int DV\;\exp[\,A_g(U,V)\,]\left[\frac{DetK(U,V)}{\int DV\;DetK(U,V)}\right]$$

$$\exp[\,A_f^{(1)}(U,\bar\psi,\psi\,] \;=\; \int DV[d\bar\eta d\eta]\exp A_f(U,V,\bar\psi,\bar\eta,\psi,\eta)\;\int[d\bar\chi d\chi]\,\rho(\bar\eta,\eta,\bar\chi,\chi)$$

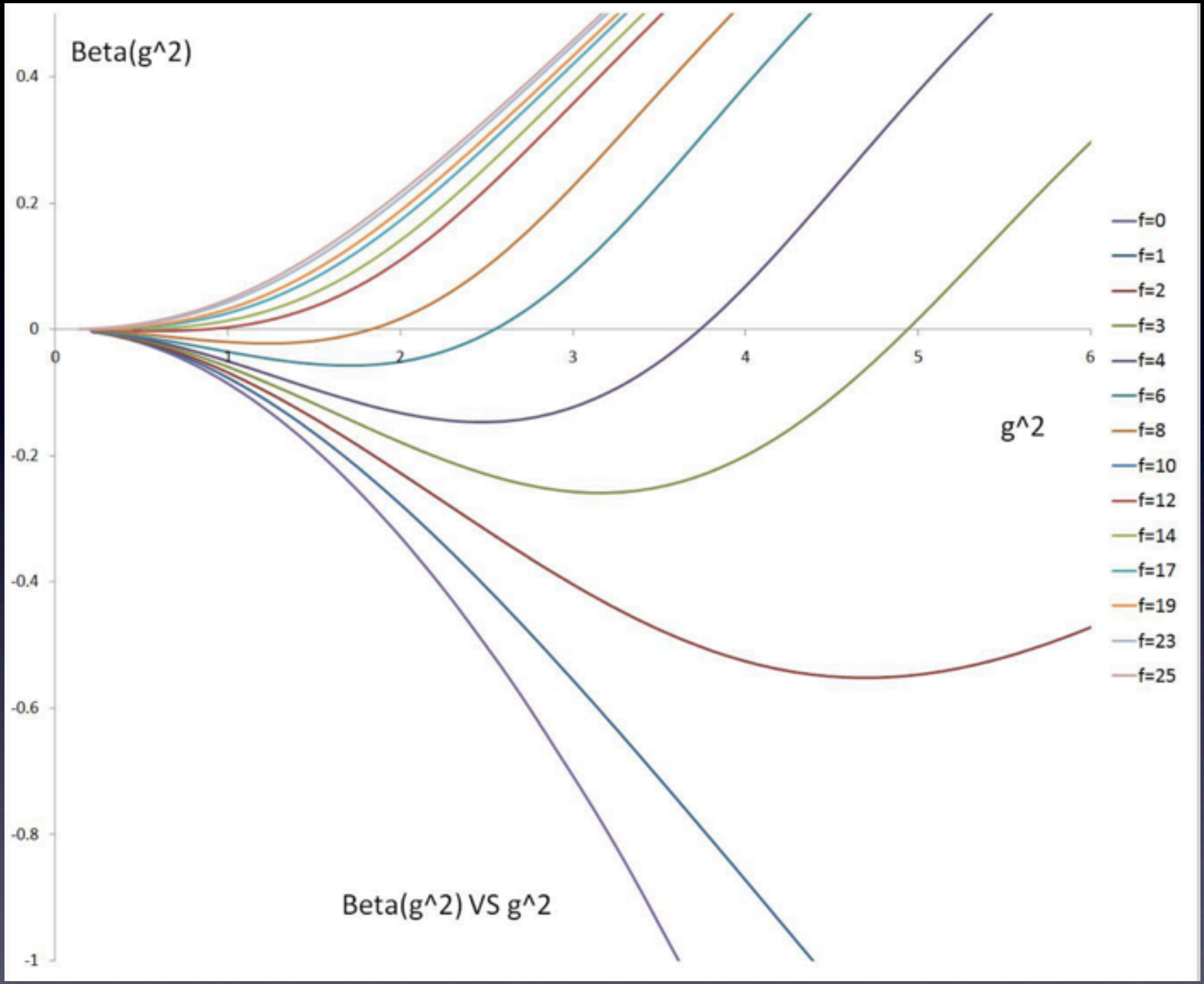
$$=\; \int DV\;[d\bar\chi d\chi]\;\exp\hat{A}_f(U,V,\bar\psi,\psi,\bar\chi,\chi)$$

Use the loop expansion of the determinants

$$DetK(U) = \prod_{\{L\}} det[\,K_L(U)\,]$$

to evaluate the dets ratio -- essentially local!

The blocking factor $\rho(\bar\eta,\eta,\bar\chi,\chi)$ introduces ‘fat’ links for the thinned-out (integrated) fermions.



SU(2) with f flavors of fundamental fermions
(preliminary)

Summary - Outlook

• A framework was developed that utilizes approximate explicitly computable RG transformations to constrain the behavior of observables.
• This allows one to obtain exact representations of PF's and suitable observables in pure gauge systems on progressively coarser lattices.
• RG flow from weak coupling to the strong coupling FP at T=0 in SU(2) and SU(3).
• A fixed point prevents flow from weak to strong coupling in U(1)
• The same method can be applied to 2-dim spin models
• The method cannot be extended in the presence of fermions since the necessary upper and lower bounds cannot be obtained.
• More elaborate schemes are needed to elucidate the modifications to IR structure (e.g. IR FP's) due to sufficient number of fermions