The Wilson-Fisher Fixed Point via ERG

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Abstract

I construct the RG flows of the Wilson action for the real scalar field theory in $D = 3$ using the exact renormalization group. The construction results in a systematic and analytical way of computing the critical exponents at the Wilson-Fisher fixed point.

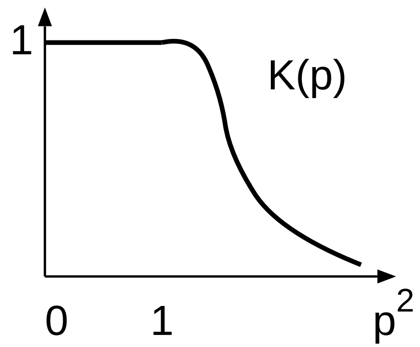
Introduction

1. We consider a real scalar theory in D dimensional Euclid space:

$$S = -\int_p \frac{p^2}{K(p/\Lambda)} \frac{1}{2} \phi(p)\phi(-p) - \int d^D x \left(\frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4 \right)$$

where K is a cutoff function with

$$K(p) \begin{cases} = 1 & (p^2 \le 1) \\ \to 0 & (p^2 \to \infty) \end{cases}$$



Λ plays the role of a momentum cutoff.

We study the correlation functions $\langle \cdots \rangle_S = \int [d\phi] \cdots \mathrm{e}^S / \int [d\phi] \mathrm{e}^S$.

2. Critical behavior — The critical value $m^2_{0,\mathrm{cr}}(\lambda_0)$ depends on λ_0.

- $m_0^2 > m^2_{0,\mathrm{cr}}$ (unbroken phase) — $\langle\, \phi \,\rangle = 0$
- $m_0^2 < m^2_{0,\mathrm{cr}}$ (broken phase) — $\langle\, \phi \,\rangle \neq 0$.

The critical behavior is characterized by 2 critical exponents

$$y_E = \frac{1}{\nu} > 0, \quad \eta > 0$$

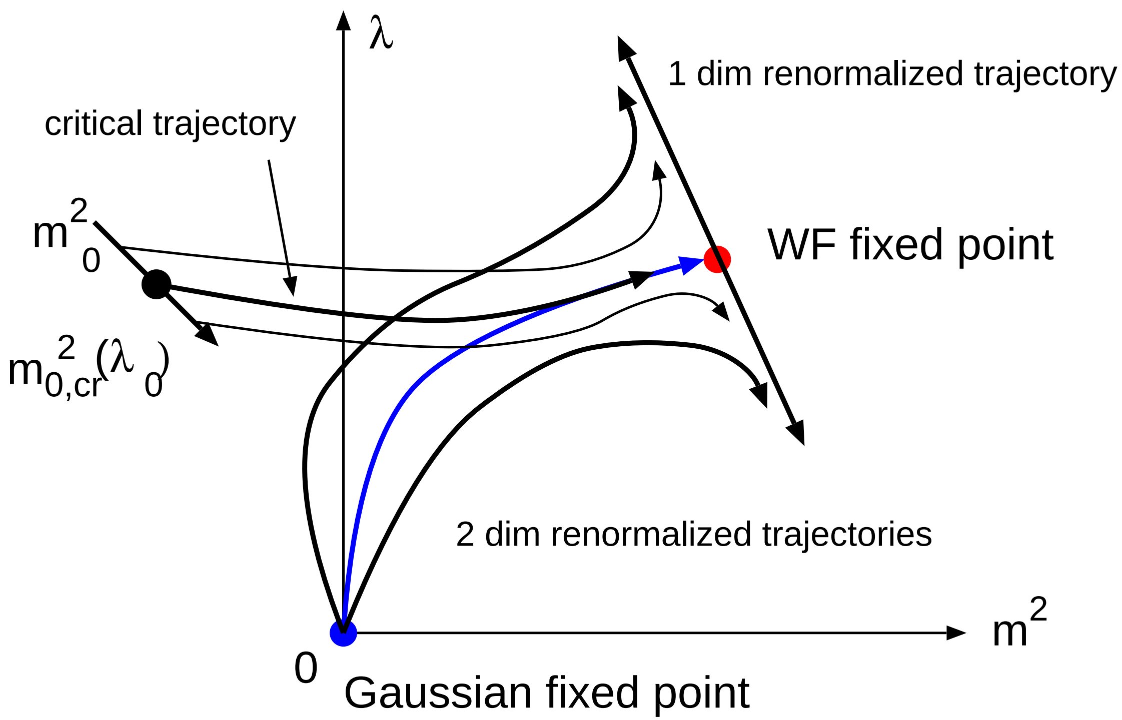
For $m^2 \simeq m^2_{0,\mathrm{cr}}$ the scaling formula holds:

$$\langle\, \phi(r)\phi(0) \,\rangle = \frac{1}{r^{D-2+\eta}} F_\pm(r/\xi) \quad \begin{cases} + \text{ if } m^2 > m^2_{0,\mathrm{cr}} \\ - \text{ if } m^2 < m^2_{0,\mathrm{cr}} \end{cases}$$

where ξ is the correlation length:

$$\xi \propto |m^2 - m^2_{0,\mathrm{cr}}(\lambda)|^{-\frac{1}{y_E}} = |m^2 - m^2_{0,\mathrm{cr}}(\lambda)|^{-\nu}$$

3. We can understand the critical behavior from the RG flows in the space of Wilson actions with a fixed momentum cutoff Λ.



m^2 and λ parametrize the renormalized trajectories out of the GFP.

4. The RG flows are given in the infinite dimensional space of Wilson actions.

5. The flows out of the gaussian fixed point make a two-dimensional subspace, where the RG flows are given by

$$\left\{ \begin{array}{rcl} \frac{d}{dt} m^2 &=& 2m^2 + \beta_m(m^2, \lambda) \\ \frac{d}{dt}\lambda &=& \lambda + \beta(m^2, \lambda) \end{array} \right.$$

with the gaussian fixed point at $m^2 = \lambda = 0$.

6. The Wilson-Fisher fixed point lies on this subspace, and it is given by (m^{*2}, λ^*) satisfying

$$\left\{ \begin{array}{l} 2m^{2*} + \beta_m(m^{2*}, \lambda^*) = 0 \\ \lambda^* + \beta(m^{2*}, \lambda^*) = 0 \end{array} \right.$$

7. The goal is a concrete realization of Wilson's picture for $D = 3$ using the exact renormalization group. Especially,

   - construction of the renormalized trajectories parametrized by m^2, λ
   - analytical calculation of the critical exponents y_E, η
     (Cf. Parisi's method with the Callan-Symanzik equation)

8. The ϵ expansions for the critical exponents with $\epsilon \equiv 4 - D$ [Fisher & Wilson]

$$\begin{cases} y_E &= 2 - \frac{\epsilon}{3} + \cdots \\ \eta &= \frac{\epsilon^2}{54} + \cdots \end{cases}$$

   - high precision results from high order calculations [Brézin, Le Gillou, Zinn-Justin]
   - **Drawback** — The expansions do not work for theories that can be defined only for specific dimensions such as chiral theories or supersymmetric theories.

9. Outline

(a) introduce ERG for the Wilson action
(b) construction of 2-dim renormalized trajectories out of the GFP
(c) analytical formulas for β_m, β in terms of the Wilson action
(d) perturbative calculation of the exponents y_E, η

Brief introduction to ERG

1. The initial action

$$S_{\Lambda_0} = -\frac{1}{2}\int_p \frac{p^2}{K(p/\Lambda_0)}\phi(p)\phi(-p) \underbrace{- \int d^D x \left(\frac{m_0^2}{2}\phi^2 + \frac{\lambda_0}{4!}\phi^4\right)}_{=S_{I,\Lambda_0}}$$

We define the Wilson action by

$$S_\Lambda = -\frac{1}{2}\int_p \frac{p^2}{K\left(p/\Lambda\right)}\phi(p)\phi(-p) + S_{I,\Lambda}$$

where

$$\exp\left[S_{I,\Lambda}[\phi]\right] \equiv \int [d\phi'] \exp\left[-\frac{1}{2}\int_p \frac{p^2}{K(p/\Lambda_0) - K\left(p/\Lambda\right)}\phi'(p)\phi'(-p) + S_{I,\Lambda_0}[\phi+\phi']\right]$$

2. A consequence of the gaussian functional integration:

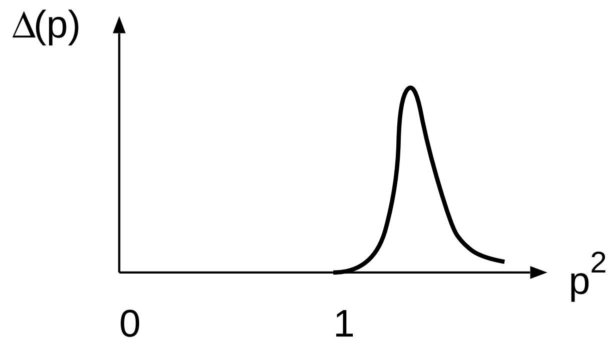
$$\exp\left[S_{I,\Lambda}[\phi]\right] = \int [d\phi'] \exp\left[-\frac{1}{2}\int_p \frac{p^2}{K(p/\Lambda') - K(p/\Lambda)}\phi'(p)\phi'(-p) + S_{I,\Lambda'}[\phi+\phi']\right]$$

3. Equivalently, the Λ dependence is given by the Polchinski equation:

$$-\Lambda\frac{\partial}{\partial\Lambda}S_\Lambda = \int_p \frac{\Delta(p/\Lambda)}{p^2}\left[\frac{p^2}{K(p/\Lambda)}\phi(p)\frac{\delta S_\Lambda}{\delta\phi(p)} + \frac{1}{2}\left\{\frac{\delta S_\Lambda}{\delta\phi(-p)}\frac{\delta S_\Lambda}{\delta\phi(p)} + \frac{\delta^2 S_\Lambda}{\delta\phi(-p)\delta\phi(p)}\right\}\right]$$

where

$$\Delta(p) \equiv -2p^2\frac{d}{dp^2}K(p)$$



4. S_Λ have the same correlation functions as S_{Λ_0}:

$$\begin{cases} \langle\, \phi(p)\phi(-p)\,\rangle \;\equiv\; \frac{1}{K(p/\Lambda)^2} \langle\, \phi(p)\phi(-p)\,\rangle_{S_\Lambda} + \frac{1-1/K(p/\Lambda)}{p^2} \\ \langle\, \phi(p_1)\cdots\phi(p_n)\,\rangle \;\equiv\; \prod_{i=1}^{N} \frac{1}{K(p_i/\Lambda)} \cdot \langle\, \phi(p_1)\cdots\phi(p_N)\,\rangle_{S_\Lambda} \end{cases}$$

are independent of Λ.

Renormalization for $D = 3$

1. The solution S_Λ of the Polchinski equation is determined uniquely by the initial condition at $\Lambda = \Lambda_0$.

2. Renormalization: for $\lim_{\Lambda_0 \to \infty} S_\Lambda$ to exist, m_0^2 must be given an appropriate Λ_0 dependence.

3. Notation:

$$S_\Lambda = \sum_{n=1}^{\infty} \int_{p_1, \cdots, p_{2n}} \delta(p_1 + \cdots p_{2n})\, u_{2n}(\Lambda; p_1, \cdots, p_{2n}) \phi(p_1) \cdots \phi(p_{2n})$$

4. Equivalently, $S_\Lambda(m^2, \lambda; \mu)$ for the renormalized theory can be constructed by the following conditions:

   (a) Conditions at $\Lambda = \mu$:

$$\begin{cases} u_2(\Lambda = \mu; p, -p) & = & -m^2 - p^2 + \mathrm{O}(p^4) \\ u_4(\Lambda = \mu; p_i = 0) & = & -\lambda \end{cases}$$

   (b) Asymtptotic conditions for $\Lambda \to \infty$:

$$\begin{cases} u_2(\Lambda; p, -p) & \stackrel{\Lambda\to\infty}{\longrightarrow} & \text{linear in } p^2 \\ u_4(\Lambda; p_1, p_2, p_3, p_4) & \stackrel{\Lambda\to\infty}{\longrightarrow} & \text{independent of } p_i \\ u_{2n\ge 6}(\Lambda; p_1, \cdots, p_{2n}) & \stackrel{\Lambda\to\infty}{\longrightarrow} & 0 \end{cases}$$

5. S_Λ is now parametrized by m^2, λ, and a renormalization scale μ.

6. The μ dependence of S_Λ is given by the RG equation:

$$-\mu \frac{\partial S_\Lambda}{\partial \mu} = \beta \mathcal{O}_\lambda + \beta_m \mathcal{O}_m + \gamma \mathcal{N}$$

$$\left\{ \begin{array}{ccl} \mathcal{O}_m & \equiv & -\partial_{m^2} S_\Lambda \\ \mathcal{O}_\lambda & \equiv & -\partial_\lambda S_\Lambda \\ \mathcal{N} & \equiv & -\int_p \left[\phi(p) \frac{\delta S_\Lambda}{\delta \phi(p)} + \frac{K(p/\Lambda)(1-K(p/\Lambda))}{p^2} \left\{ \frac{\delta S_\Lambda}{\delta \phi(p)} \frac{\delta S_\Lambda}{\delta \phi(-p)} + \frac{\delta^2 S_\Lambda}{\delta \phi(p) \delta \phi(-p)} \right\} \right] \end{array} \right.$$

β, β_m, η are functions of m^2, λ, μ. $\mathcal{N}$ counts the number of ϕ insertions:

$$\langle \mathcal{N} \phi(p_1) \cdots \phi(p_n) \rangle = n \langle \phi(p_1) \cdots \phi(p_n) \rangle$$

7. This is equivalent to the RG equation:

$$\left(-\mu \frac{\partial}{\partial \mu} + \beta \partial_\lambda + \beta_m \partial_{m^2} \right) \langle \phi(p_1) \cdots \phi(p_n) \rangle = n\gamma \langle \phi(p_1) \cdots \phi(p_n) \rangle$$

Universality

1. How does S_Λ depend on the arbitrary choice of $K(p)$?

2. An infinitesimal change $\delta K(p)$ of the cutoff function changes S_Λ by

$$\begin{aligned}\delta S_\Lambda &= \frac{\delta z_\Lambda}{2}\mathcal{N} - \int_p \left[\frac{\delta K(p/\Lambda)}{K(p/\Lambda)} \phi(p) \frac{\delta S_\Lambda}{\delta \phi(p)} \right. \\ &\quad \left. + \frac{1}{p^2} \delta K(p/\Lambda) \frac{1}{2} \left\{ \frac{\delta S_\Lambda}{\delta \phi(p)} \frac{\delta S_\Lambda}{\delta \phi(-p)} + \frac{\delta^2 S_\Lambda}{\delta \phi(p) \delta \phi(-p)} \right\} \right]\end{aligned}$$

where δz_Λ is determined so that δS_Λ satisfies the normalization condition

$$\frac{\partial}{\partial p^2} \delta u_2(\Lambda = \mu; p, -p)\Big|_{p^2=0} = 0$$

3. Equivalence:

$$\frac{1}{K(p)^2} \langle\, \phi(p)\phi(-p)\,\rangle_{S(t)} + \frac{1-1/K(p)}{p^2}$$
$$= (1-\delta z(t)) \left[\frac{1}{(K+\delta K)(p)^2} \langle\, \phi(p)\phi(-p)\,\rangle_{(S+\delta S)(t)} + \frac{1-1/(K+\delta K)(p)}{p^2}\right]$$

$$\prod_{i=1}^{n} \frac{1}{K(p_i)} \cdot \langle\, \phi(p_1)\cdots\phi(p_n)\,\rangle_{S(t)}$$
$$= \left(1-\frac{n}{2}\delta z(t)\right) \prod_{i=1}^{n} \frac{1}{(K+\delta K)(p_i)} \cdot \langle\, \phi(p_1)\cdots\phi(p_n)\,\rangle_{(S+\delta S)(t)}$$

4. δK is absorbed by the change of m^2, λ, and normalization of ϕ.

$$\left\{ \begin{array}{rcl} \delta m^2 &=& -\delta u_2(\Lambda=\mu; p=0) \\ \delta\lambda &=& -\delta u_4(\Lambda=\mu; p_i=0) \end{array} \right.$$

Construction of Wilson's RG flows

It takes 3 steps to construct Wilson's RG flows.

Step 1: Combine Polchinski's equation and the RG equation:

$$
\begin{aligned}
&\left(-\Lambda\frac{\partial}{\partial\Lambda} - \mu\frac{\partial}{\partial\mu} + \beta\partial_\lambda + \beta_m\partial_{m^2}\right) S_\Lambda \\
&= \int_p \Bigg[\left(\frac{\Delta(p/\Lambda)}{K\left(p/\Lambda\right)} - \gamma\right) \phi(p) \frac{\delta S_\Lambda}{\delta\phi(p)} \\
&+ \frac{1}{p^2}\left(\Delta(p/\Lambda) - 2\gamma K\left(p/\Lambda\right)\left(1 - K\left(p/\Lambda\right)\right)\right) \frac{1}{2}\left\{\frac{\delta S_\Lambda}{\delta\phi(p)}\frac{\delta S_\Lambda}{\delta\phi(-p)} + \frac{\delta^2 S_\Lambda}{\delta\phi(p)\delta\phi(-p)}\right\}\Bigg]
\end{aligned}
$$

Step 2: rescaling by dimensional analysis — m^2 has dimension 2, λ has 1, and $\phi(p)$ has $-\frac{D+2}{2}$:

$$\left(\Lambda\frac{\partial}{\partial\Lambda} + \mu\frac{\partial}{\partial\mu} + 2m^2\partial_{m^2} + \lambda\partial_\lambda\right) S_\Lambda = \int_p \left(p_\mu \frac{\partial\phi(p)}{\partial p_\mu} + \frac{D+2}{2}\phi(p)\right)\frac{\delta S_\Lambda}{\delta\phi(p)}$$

This amounts to rescaling that restores the original Λ.

Combining Step 1 & 2,

$$\begin{aligned}&\left((\lambda+\beta)\partial_\lambda + (2m^2+\beta_m)\partial_{m^2}\right) S_\Lambda \\ &= \int_p \Bigg[\left\{ p_\mu \frac{\partial\phi(p)}{\partial p_\mu} + \left(\frac{D+2}{2} - \gamma + \frac{\Delta(p/\Lambda)}{K\left(p/\Lambda\right)}\right)\phi(p)\right\}\frac{\delta S_\Lambda}{\delta\phi(p)} \\ &\qquad + \frac{\Delta(p/\Lambda) - 2\gamma K(1-K)}{p^2}\frac{1}{2}\left\{\frac{\delta S_\Lambda}{\delta\phi(p)}\frac{\delta S_\Lambda}{\delta\phi(-p)} + \frac{\delta^2 S_\Lambda}{\delta\phi(p)\delta\phi(-p)}\right\}\Bigg]\end{aligned}$$

Step 3: Set $\Lambda = \mu$ (fixed) and write $S_\Lambda = S(m^2, \lambda)$.

Finally, we obtain

$$\begin{aligned}
&\left((\lambda+\beta)\partial_\lambda + (2m^2+\beta_m)\partial_{m^2}\right) S(m^2,\lambda) \qquad (1)\\
&= \int_p \Bigg[\left\{ p_\nu \frac{\partial\phi(p)}{\partial p_\nu} + \left(\frac{D+2}{2} - \gamma + \frac{\Delta(p/\mu)}{K(p/\mu)}\right)\phi(p)\right\} \frac{\delta S}{\delta\phi(p)}\\
&+ \frac{1}{p^2}\left(\Delta(p/\mu) - 2\gamma K(1-K)\right)\frac{1}{2}\left\{\frac{\delta S}{\delta\phi(p)}\frac{\delta S}{\delta\phi(-p)} + \frac{\delta^2 S}{\delta\phi(p)\delta\phi(-p)}\right\}\Bigg]
\end{aligned}$$

From now on we can set $\mu = 1$.

The RG flow is given by

$$\begin{cases} \frac{d}{dt}m^2 &= 2m^2 + \beta_m(m^2, \lambda) \\ \frac{d}{dt}\lambda &= \lambda + \beta(m^2, \lambda) \end{cases}$$

and we obtain the scaling formula

$$\begin{aligned} & \left\langle \phi(p_1 \mathrm{e}^{\Delta t}) \cdots \phi(p_n \mathrm{e}^{\Delta t}) \right\rangle_{m^2 \mathrm{e}^{2\Delta t}(1+\Delta t \cdot \beta_m),\, \lambda \mathrm{e}^{\Delta t}(1+\Delta t \cdot \beta)} \\ = \quad & \mathrm{e}^{\Delta t \left\{ D + n\left(-\frac{D+2}{2} + \gamma\right)\right\}} \cdot \left\langle \phi(p_1) \cdots \phi(p_n) \right\rangle_{m^2, \lambda} \end{aligned}$$

Determination of β, β_m in terms of S

1. Expansion

$$S(m^2, \lambda)[\phi] = \sum_{n=1}^{\infty} \int_{p_1, \cdots, p_{2n}} \delta(p_1 + \cdots + p_{2n}) \cdot u_{2n}(m^2, \lambda; p_1, \cdots, p_{2n})\, \phi(p_1) \cdots \phi(p_{2n})$$

where u_2, u_4 are normalized by

$$\begin{cases} u_2(m^2, \lambda; p, -p) &= -m^2 - p^2 + \cdots \\ u_4(m^2, \lambda; p_i = 0) &= -\lambda \end{cases} \tag{2}$$

2. Substituting the normalization conditions (2) into (1), we obtain

$$
\begin{aligned}
-\beta_m &= \frac{1}{2}\int_q \frac{1}{q^2}\left(\Delta(q) - \eta K(q)(1-K(q))\right) u_4(m^2,\lambda;q,-q,0,0) \\
-2\gamma &= \frac{1}{2}\frac{\partial}{\partial p^2}\int_q \frac{1}{q^2}\left(\Delta(q) - \eta K(q)(1-K(q))\right) u_4(m^2,\lambda;q,-q,p,-p)\Big|_{p^2=0} \\
-\beta - 4\lambda\gamma &= \frac{1}{2}\int_q \frac{1}{q^2}\left(\Delta(q) - \eta K(q)(1-K(q))\right) u_6(m^2,\lambda;q,-q,0,0,0,0)
\end{aligned}
$$

These determine β_m, β, γ in terms of $S(m^2, \lambda)$.

Perturbative calculations

1. In practice, a mass independent scheme is more convenient:

$$\left\{ \begin{array}{rcl} \frac{\partial}{\partial m^2} u_2(m^2, \lambda; p = 0)\Big|_{m^2=0} &=& -1 \\ \frac{\partial}{\partial p^2} u_2(m^2, \lambda; p, -p)\Big|_{m^2=p^2=0} &=& -1 \\ u_4(m^2, \lambda; p_i = 0)\Big|_{m^2=0} &=& -\lambda \end{array} \right.$$

This gives

$$\left\{ \begin{array}{rcl} \beta_m &=& C(\lambda) + \beta_m(\lambda) m^2 \\ \beta &=& \beta(\lambda) \end{array} \right.$$

2. RG flow equations

$$\begin{cases} \frac{d}{dt}m^2 &= (2+\beta_m(\lambda))m^2 + C(\lambda) \\ \frac{d}{dt}\lambda &= \lambda + \beta(\lambda) \end{cases}$$

3. Wilson-Fisher fixed point (m^{2*}, λ^*)

$$\begin{cases} (2+\beta_m(\lambda^*))m^{2*} + C(\lambda^*) = 0 \\ \lambda^* + \beta(\lambda^*) = 0 \end{cases}$$

4. Critical exponents:

$$\begin{cases} y_E &= 2+\beta_m(\lambda^*) \\ \eta &= 2\gamma(\lambda^*) \end{cases}$$

5. Lowest non-trivial loop expansions

$$\begin{cases} \beta_m &= \frac{\lambda}{2}\int_q \Delta(q)\left(\frac{1}{q^2} - \frac{m^2}{q^2}\right) \\ \beta &= -3\lambda^2 \int_q \frac{\Delta(q)(1-K(q))}{q^4} \\ 2\gamma &= -\frac{\lambda^2}{2}\frac{\partial}{\partial p^2}\int_{q,r} \frac{\Delta(q)}{q^2}\frac{1-K(r)}{r^2}\frac{1-K(q+r+p)}{(q+r+p)^2}\Big|_{p^2=0} \end{cases}$$

6. Fixed point

$$\lambda^* = \frac{1}{3\int_q \frac{\Delta(1-K)}{q^4}}, \quad m^{2*} = \frac{-\frac{\lambda^*}{2}\int_q \frac{\Delta}{q^2}}{2 - \lambda^* \int_q \frac{\Delta}{q^4}}$$

gives

$$\begin{cases} y_E &= 2 + \beta_m(\lambda^*) = 2 - \frac{1}{2}\lambda^* \int_q \frac{\Delta}{q^4} \\ \eta &= -\frac{1}{2}\lambda^{*2}\frac{\partial}{\partial p^2}\int_{q,r} \frac{\Delta(q)}{q^2}\frac{1-K(r)}{r^2}\frac{1-K(q+r+p)}{(q+r+p)^2}\Big|_{p^2=0} \end{cases}$$

7. Wegner-Houghton limit — $K(p) = \theta(1 - p^2)$

$$\begin{cases} y_E &= 2 - \frac{1}{3} = \frac{5}{3} \\ \eta &= \frac{2\ln 2 - 1}{54} = \frac{0.38629...}{54} = 0.007154.... \end{cases}$$

η comes out too small.

Application to the WZ model in $D = 3$

1. The bare action

$$S_{\Lambda_0} = -\frac{1}{2}\int_p \frac{1}{K(p/\Lambda_0)} \big(p^2\phi(p)\phi(-p) + \bar\chi(-p)\vec\sigma\cdot i\vec p\chi(p) + F(p)F(-p)\big)$$
$$- \int d^3x \left(\frac{g}{2}\left(\phi^2 iF + \bar\chi\chi\phi\right) - \frac{gv^2}{2} iF\right)$$

where $\bar\chi \equiv \chi^T\sigma_y$.

2. Classical analysis: scalar potential $\propto (\phi^2 - v^2)^2$

(a) $v^2 > 0$ — $\mathbf{Z_2}$ broken, SUSY exact
(b) $v^2 < 0$ — $\mathbf{Z_2}$ unbroken, SUSY broken

$$\mathbf{Z_2} : \phi(x,y,z) \to -\phi(x,-y,z),\ \chi(x,y,z) \to \sigma_y\chi(x,-y,z)$$

3. Critical exponents:

(a) Scale dimension of $g^*(v^2 - v^{2*})$:

$$1 + \frac{1}{2} \frac{3 \int_q \frac{\Delta(1-K)^2}{q^4} + \int_q \frac{\Delta(1-K)}{q^4}}{3 \int_q \frac{\Delta(1-K)^2}{q^4} + \frac{3}{2} \int_q \frac{\Delta(1-K)}{q^4}} \overset{\mathrm{WH}}{=} 1 + \frac{3}{7} = 1.428...$$

(b) Anomalous dimension

$$\eta = \frac{1}{2} \frac{\int_q \frac{\Delta(1-K)}{q^4}}{3 \int_q \frac{\Delta(1-K)^2}{q^4} + \frac{3}{2} \int_q \frac{\Delta(1-K)}{q^4}} \overset{\mathrm{WH}}{=} \frac{1}{7} = 0.142...$$

Conclusions

1. Wilson's RG flows can be constructed concretely using ERG.

2. Fixed points and critical exponents can be calculated by loop expansions.

3. ERG is applicable to dimension specific theories such as chiral and supersymmetric theories.

4. The nature of expansions is unclear due to the absence of an obvious expansion parameter.

5. "Optimization" should help for better numerical accuracy.

Appendix: Redefinition of β_m, β, γ

$$
\left\{
\begin{array}{lcl}
\beta_m &=& C_1\lambda + C_2\lambda^2 + \cdots + m^2(B_1\lambda + B_2\lambda^2 + \cdots) \\
\beta &=& A_1\lambda^2 + A_2\lambda^3 + \cdots \\
\gamma &=& D_2\lambda^2 + D_3\lambda^3 + \cdots
\end{array}
\right.
$$

By redefining

$$
\left\{
\begin{array}{lcl}
m^{2'} &\equiv& c_1\lambda + c_2\lambda^2 + \cdots + m^2(1 + b_1\lambda + b_2\lambda^2 + \cdots) \\
\lambda' &\equiv& \lambda + a_1\lambda^2 + a_2\lambda^3 + \cdots \\
\phi' &\equiv& (1 + d_1\lambda + d_2\lambda^2 + \cdots)\phi
\end{array}
\right.
$$

we can make

$$
\left\{
\begin{array}{lcl}
\frac{d}{dt}m^{2'} &=& 2m^{2'} - \frac{1}{24}\frac{\lambda^{2'}}{(2\pi)^2} \\
\frac{d}{dt}\lambda' &=& \lambda' \\
\gamma' &=& 0
\end{array}
\right.
$$