RG for nuclear forces and nuclear structure

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TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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EMMI

INT RG Workshop, Feb. 23, 2010 - schwenk@triumf.ca

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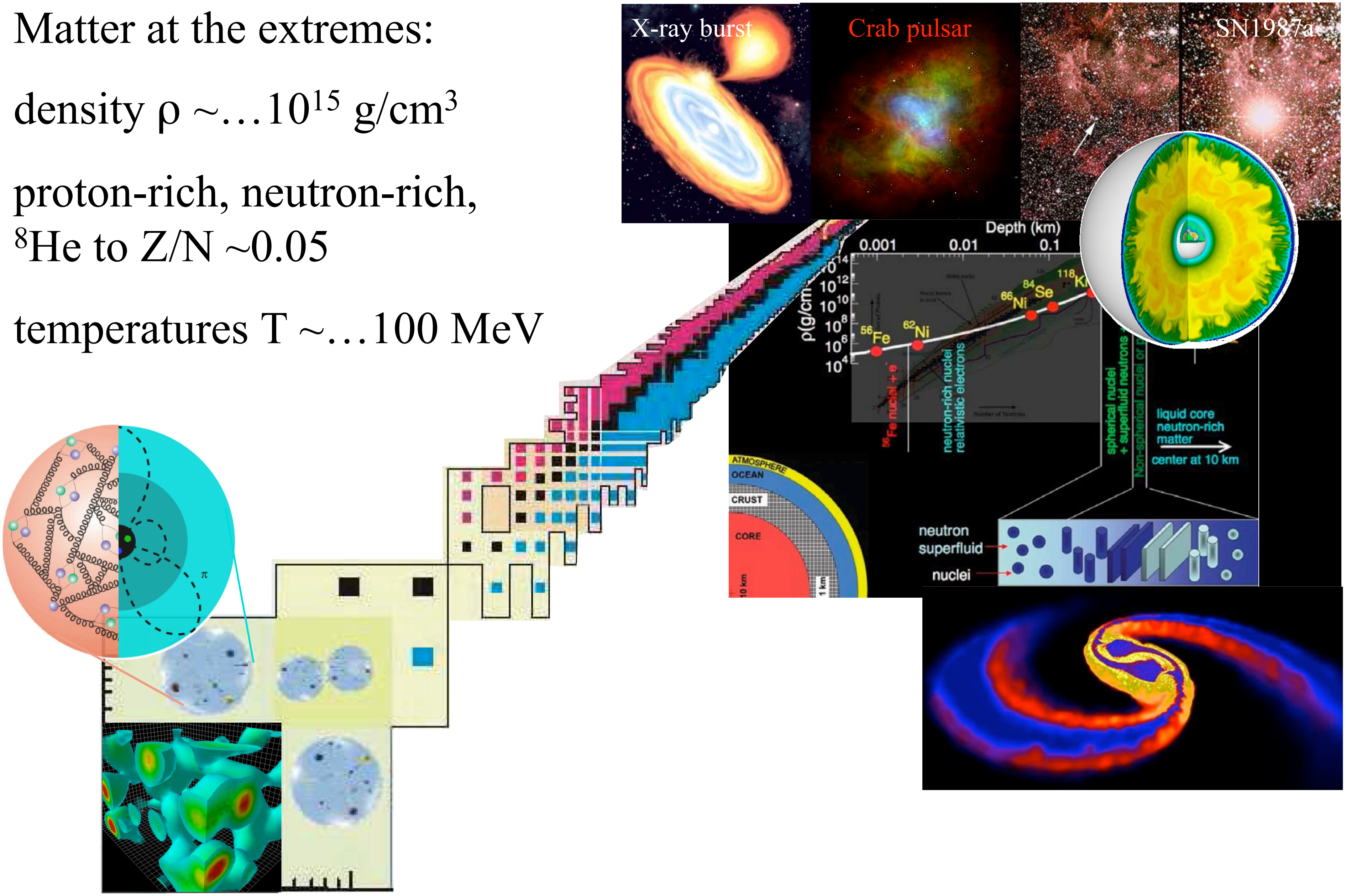
Strong interaction physics in the lab and cosmos

Matter at the extremes:

density ρ ~…10^{15} g/cm^3

proton-rich, neutron-rich,
^{8}He to Z/N ~0.05

temperatures T ~…100 MeV



Outline

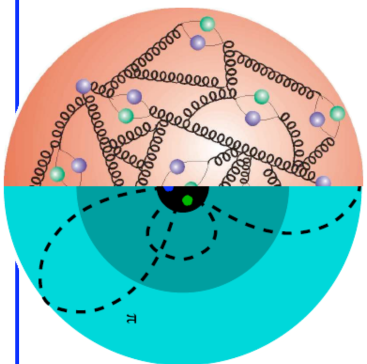
RG evolution for nuclear forces

Applications to neutron-rich nuclei and neutron stars

RG for many-body problems

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons

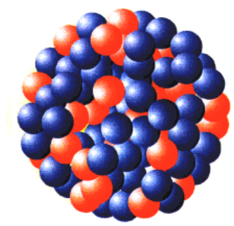


Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/Λ-dependent

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{\mathrm{3N}}(\Lambda) + V_{\mathrm{4N}}(\Lambda) + \ldots$$

$\Lambda_{\mathrm{chiral}}$

momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory (EFT)



neutrons and protons interacting via pion exchanges
and shorter-range contact interactions

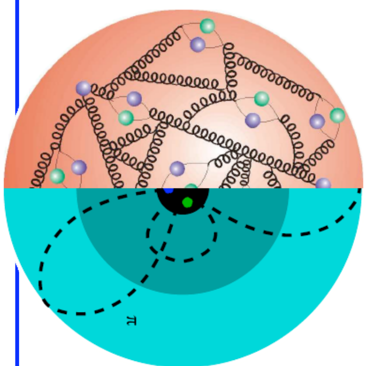
typical momenta in nuclei $\sim m_\pi$

$\Lambda_{\mathrm{pionless}}$

$Q << m_\pi$

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons



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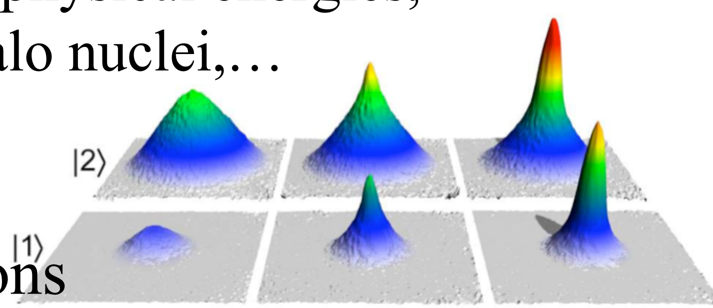
$\Lambda_{\rm chiral}$
momenta $Q \sim \lambda^{-1} \sim m_\pi$

universal properties of neutrons and cold atoms,
reactions at astrophysical energies,
loosely-bound halo nuclei,…

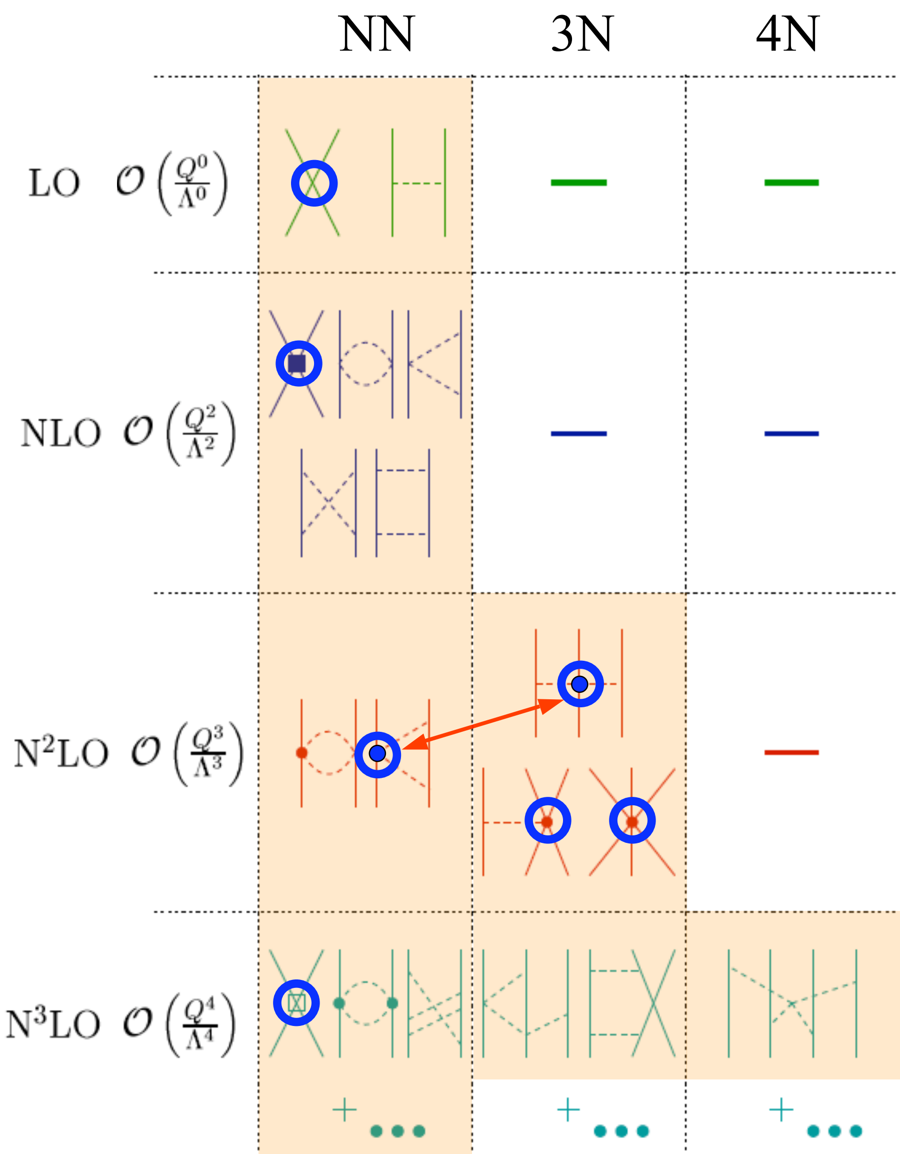


$\Lambda_{\rm pionless}$
$Q << m_\pi$: pionless effective field theory

large scattering length physics and corrections

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



limited resolution at low energies, can expand in powers Q/Λ_b

include long-range pion physics

details at short-distance not resolved

capture in few short-range couplings, fit to experiment once, Λ-dependent

systematic: can work to desired accuracy and obtain error estimates from truncation order and Λ variation

can connect to lattice QCD

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner,…

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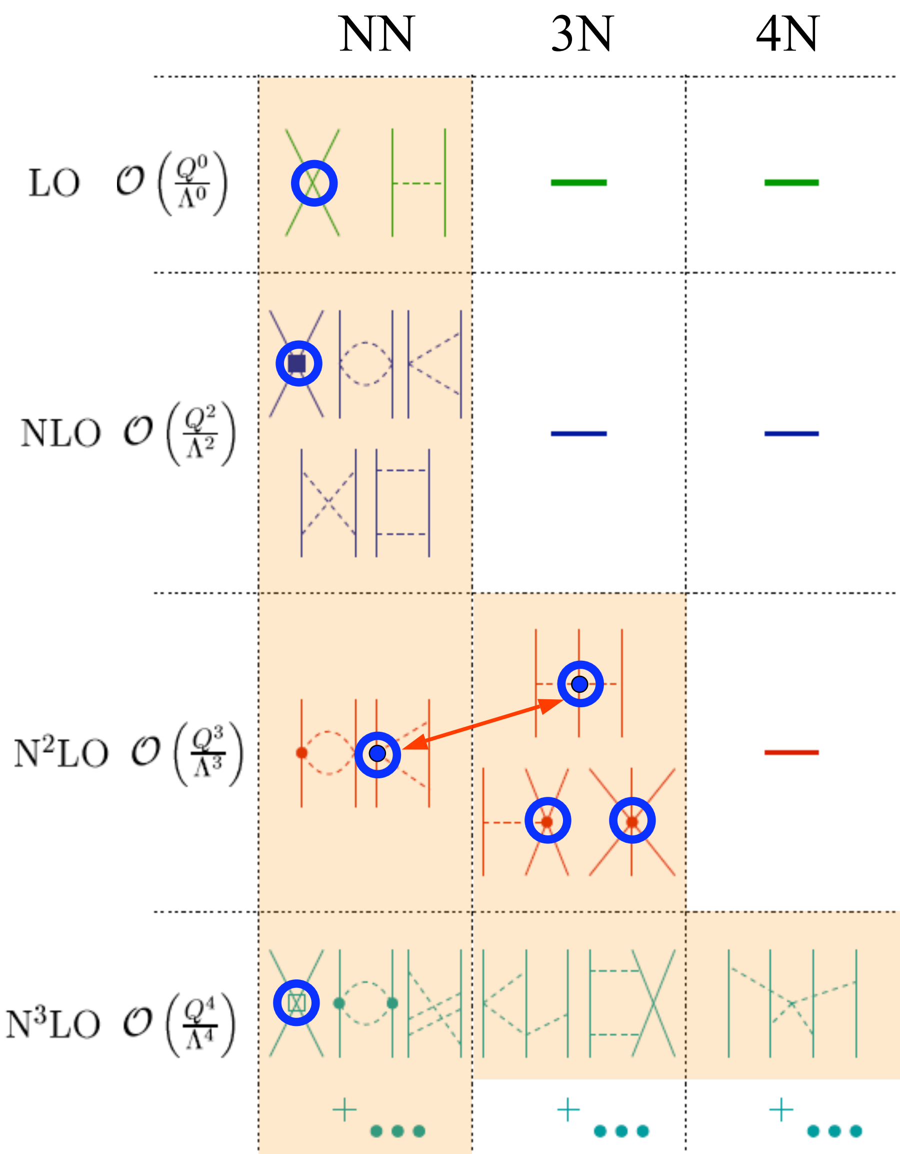
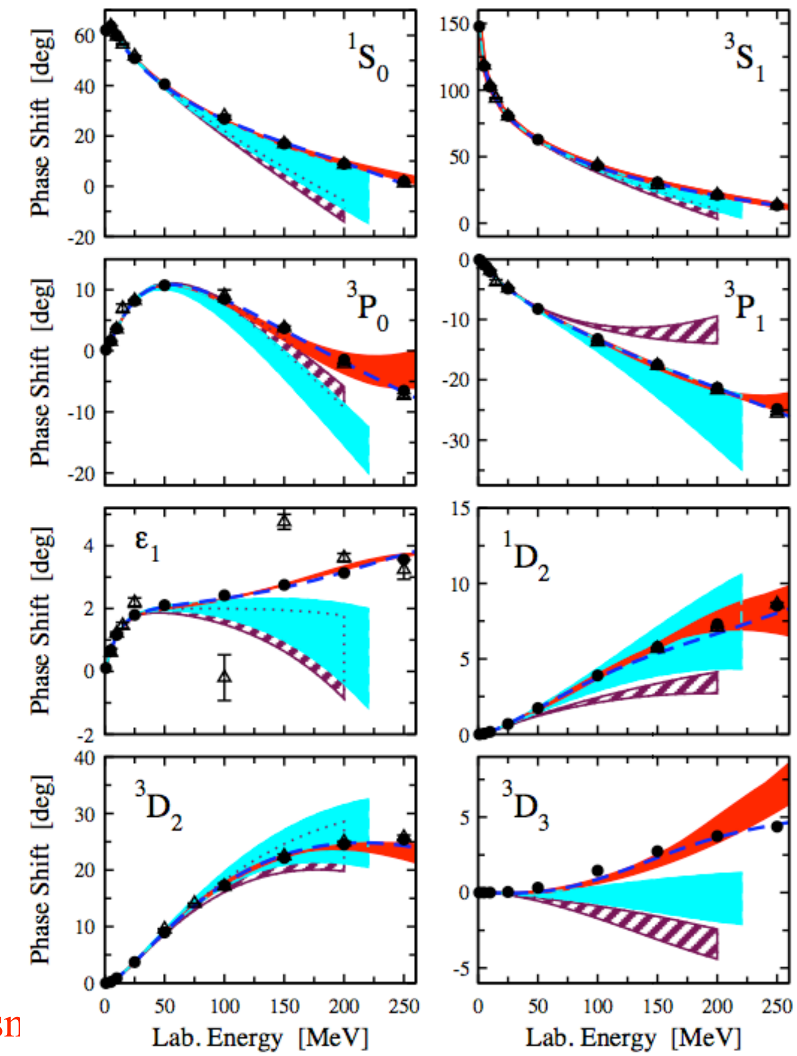


accurate reproduction of low-energy NN scattering at N^{3}LO



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissn

Nuclear forces and the Renormalization Group

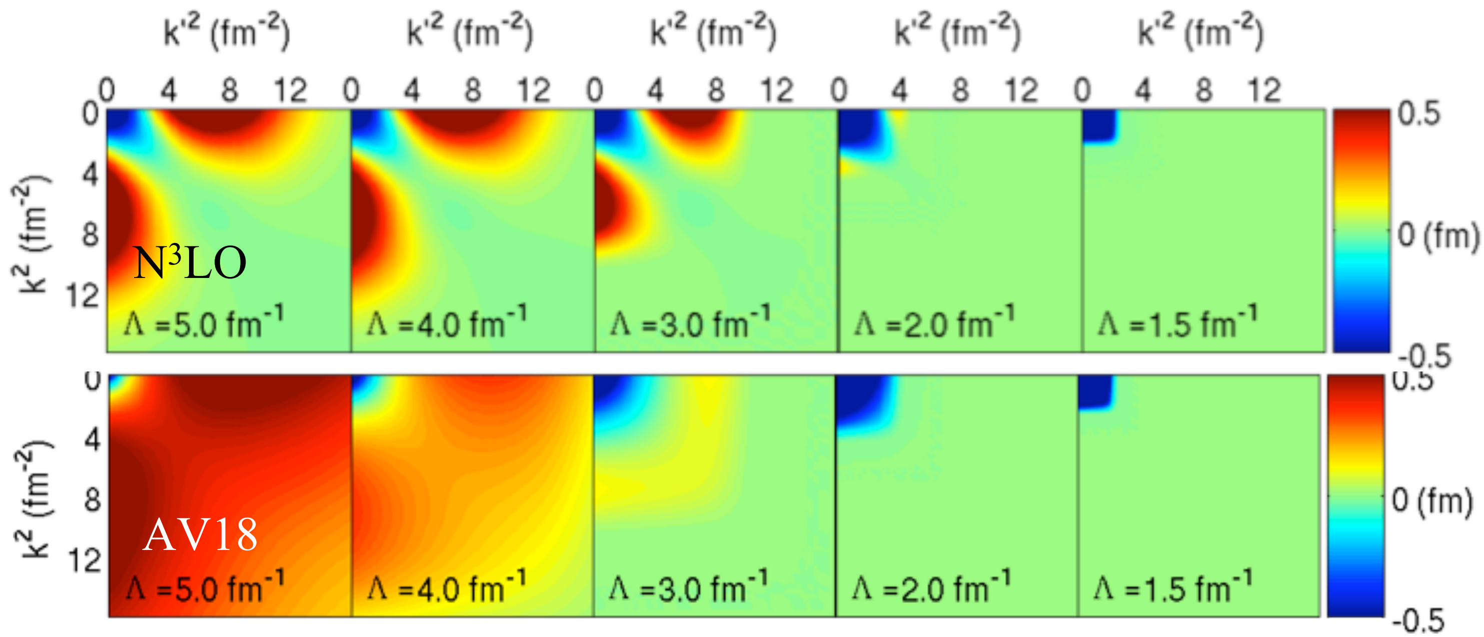
RG evolution to lower resolution/cutoffs

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

exact RG for NN interactions

Bogner, Kuo, AS, Furnstahl, …

$$\frac{d}{d\Lambda} V^{\Lambda}_{{\rm low}\,k}(k',k) = \frac{2}{\pi} \frac{V^{\Lambda}_{{\rm low}\,k}(k',\Lambda) T^{\Lambda}(\Lambda,k;\Lambda^2)}{1-(k/\Lambda)^2}$$

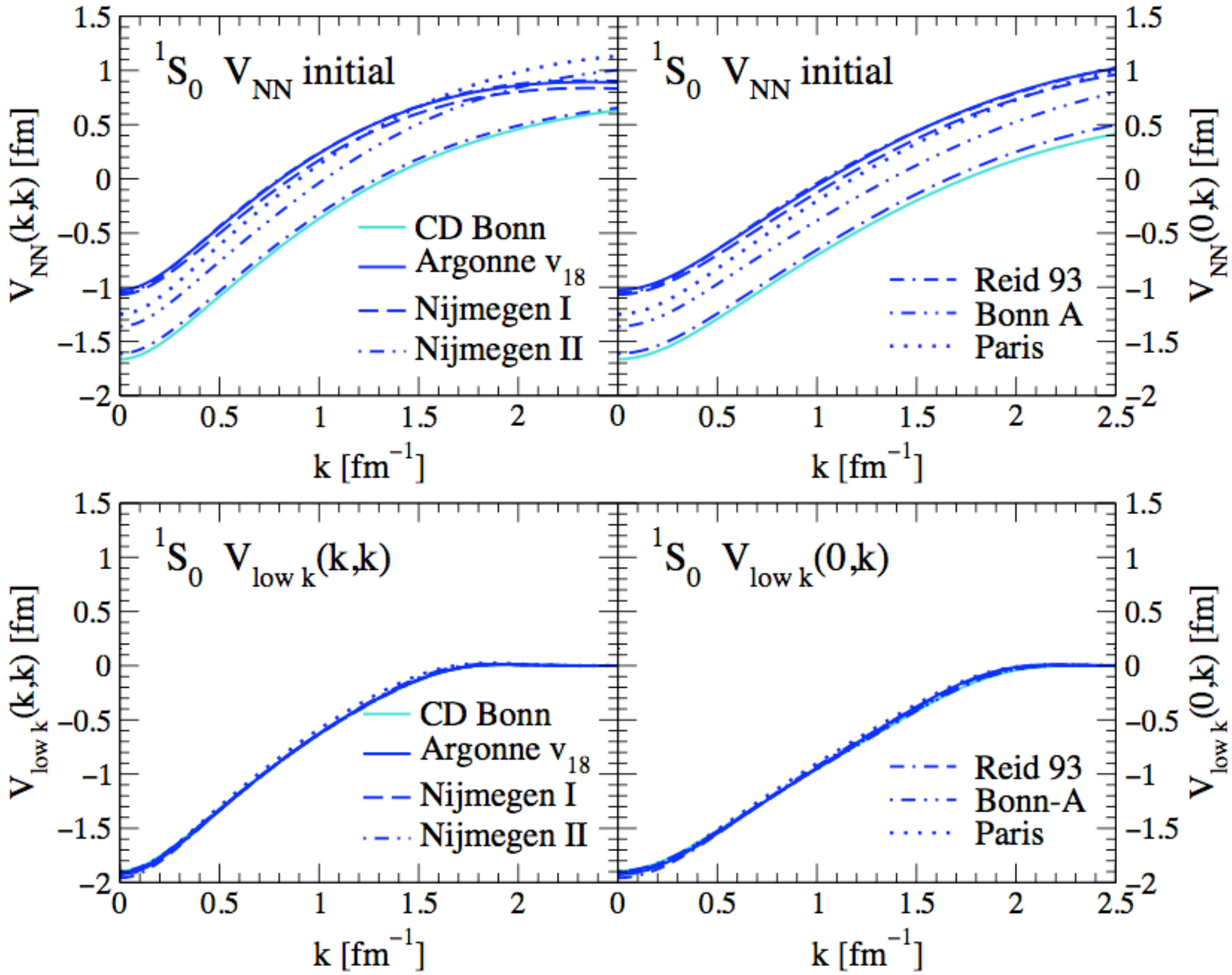


low-momentum interactions $V_{{\rm low}\,k}(\Lambda)$ with sharp or smooth regulators

decouples low-momentum physics from high momenta

(red=short-range repulsion and short-range tensor parts)

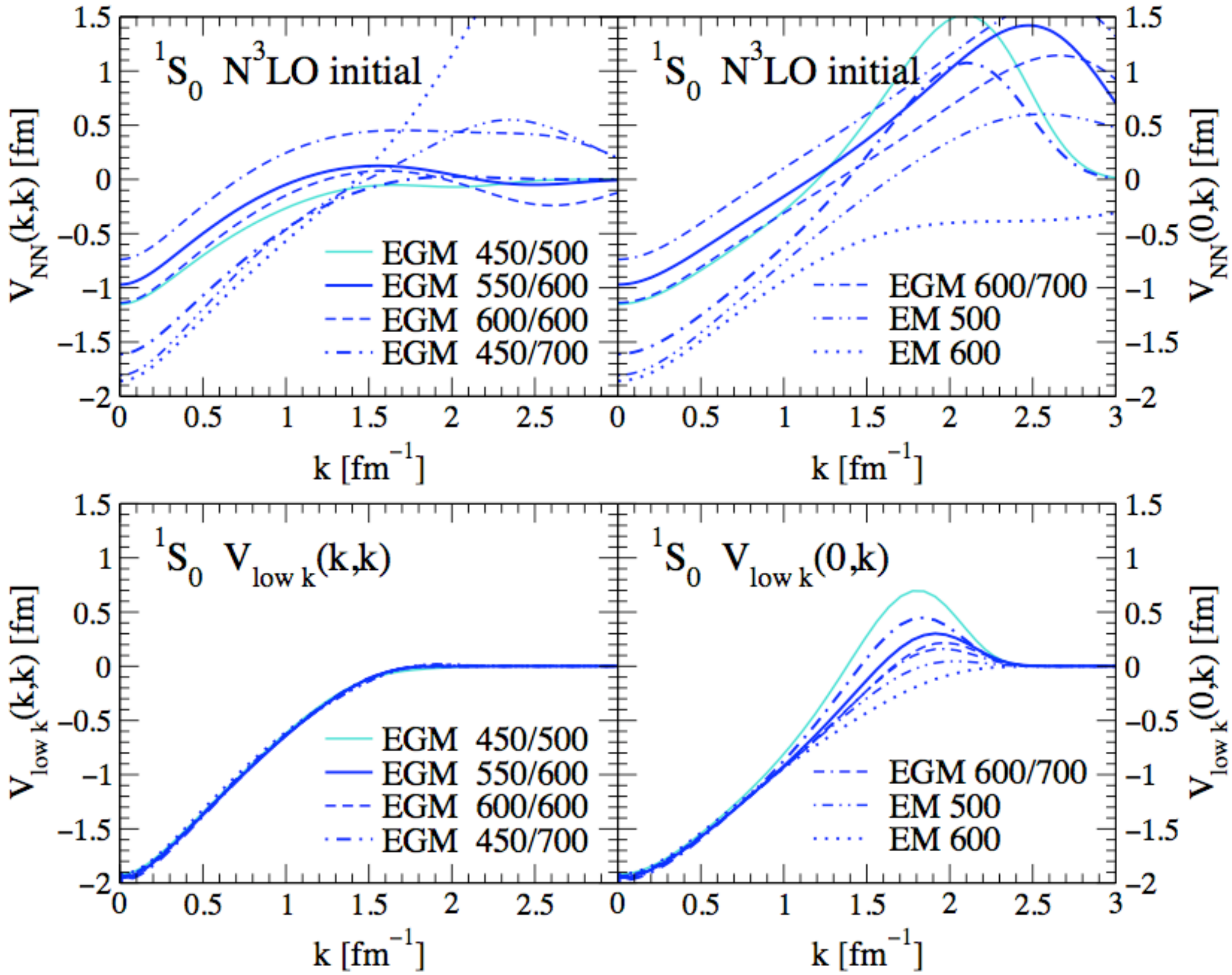
Low-momentum universality



≈ universality from different phenomenological potentials

RG preserves NN observables and long-range parts

Low-momentum universality



≈ universality from different chiral N^3LO potentials

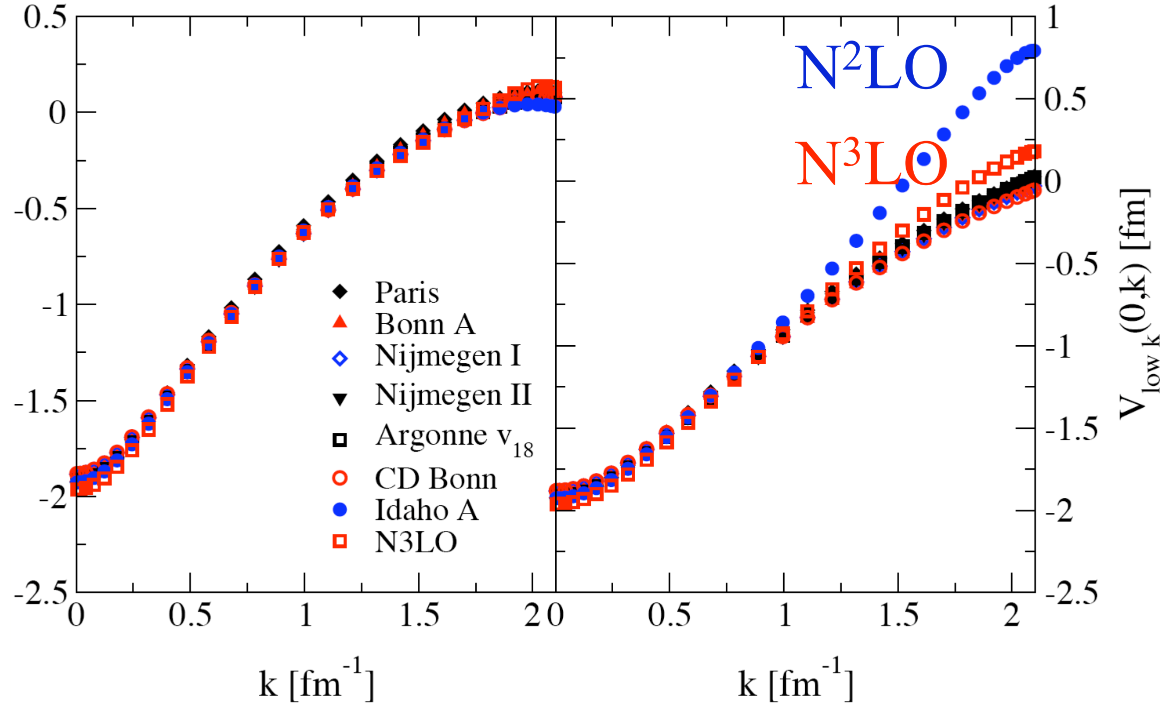
RG preserves NN observables and long-range parts

What drives this universality? RG basis?

Chiral EFT and RG

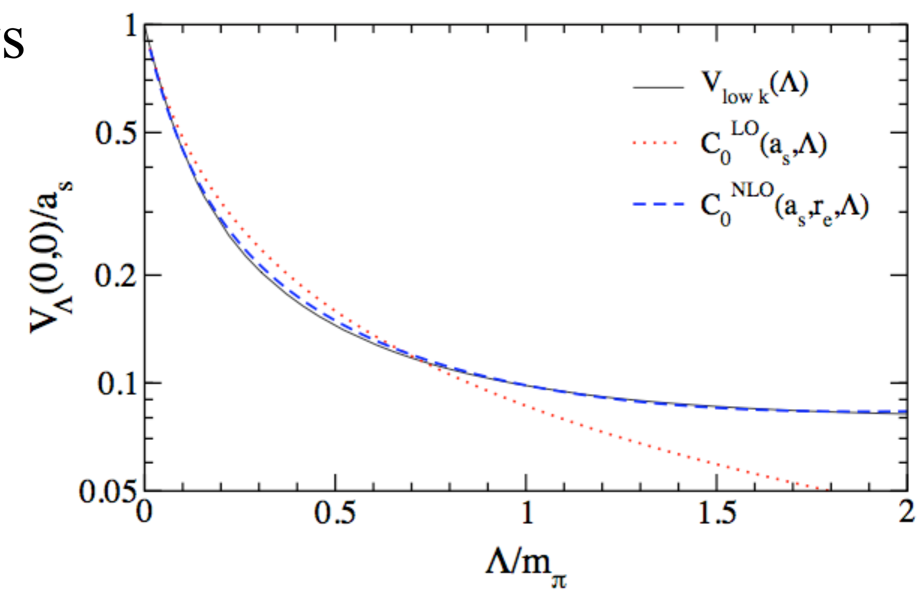
Collapse to $V_{\text{low }k}$ band for higher orders in chiral EFT (should revisit this)

RG generates higher-order contact interactions



Evolution of $V_{\text{low }k}(0,0;\Lambda)$ follows contact interaction $c_0(\Lambda)$ at NLO

EFT connections?



Similarity RG

unitary transformations to band-diagonal $V_{srg}(\lambda)$ from flow equations

Glazek, Wilson (1993), Wegner (1994)

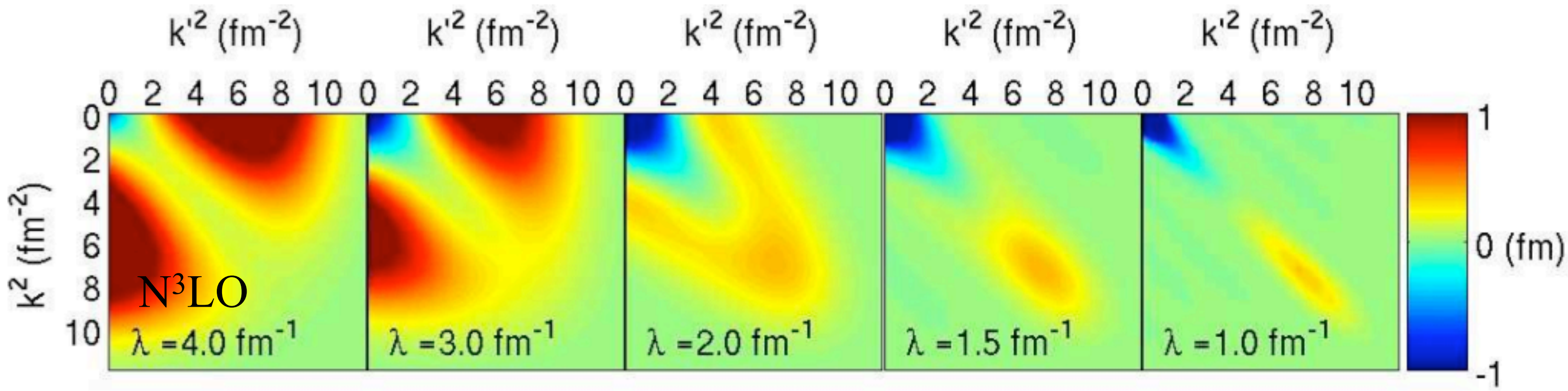
$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

evolution driven towards nonzero part of generator G_s

with flow operator $G_s = T_{rel}$ and resolution $\lambda = s^{-1/4}$

Bogner, Furnstahl, Perry,…

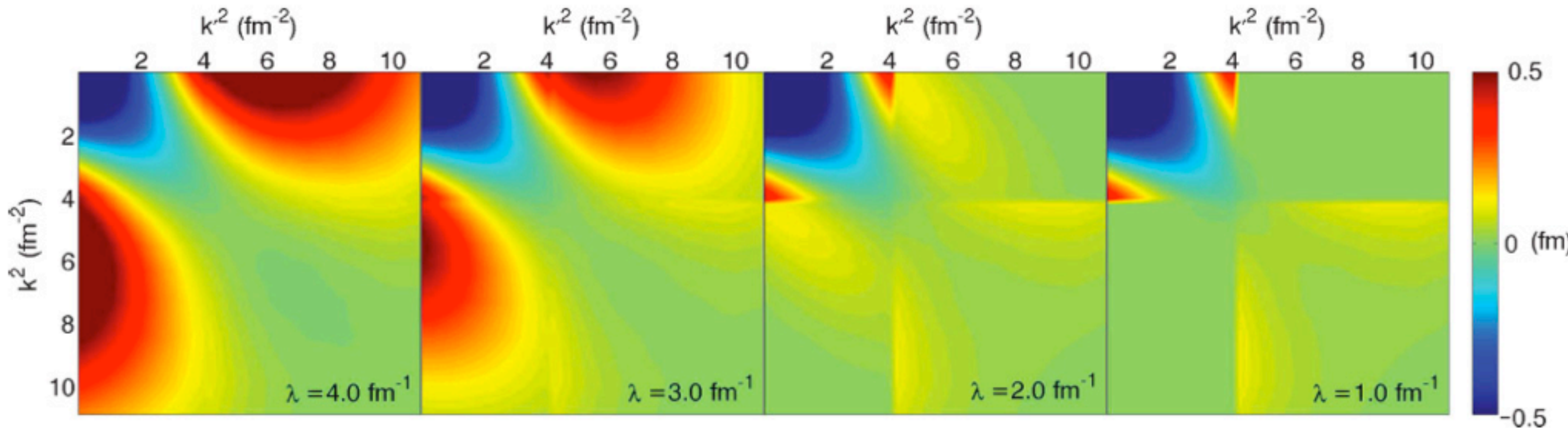
$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2\, V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k')$$



SRG decouples high momenta with similar low-momentum universality

Block diagonalization using SRG

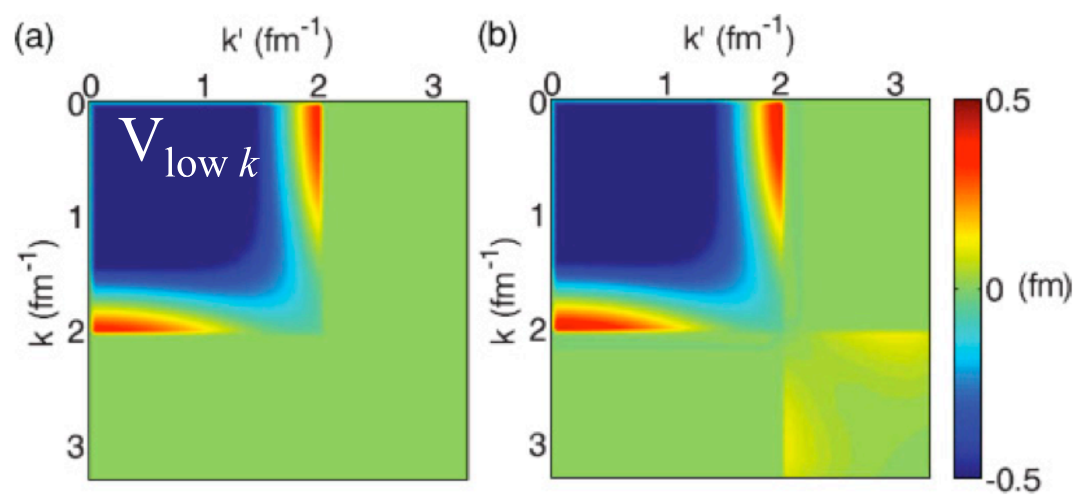
with block-diagonal flow operator Bogner et al. (2008) $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



low-momentum blocks very similar to $V_{\text{low }k}$

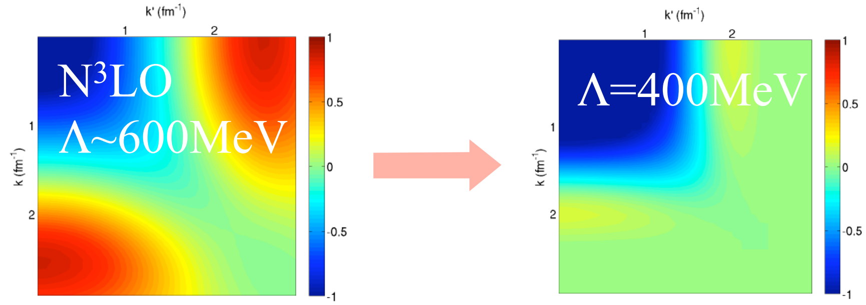
formal equivalence?
SRG is exact at second-order
in the (tree-level) potential

SRG connections to EFT?



Advantages of low-momentum interactions for nuclei

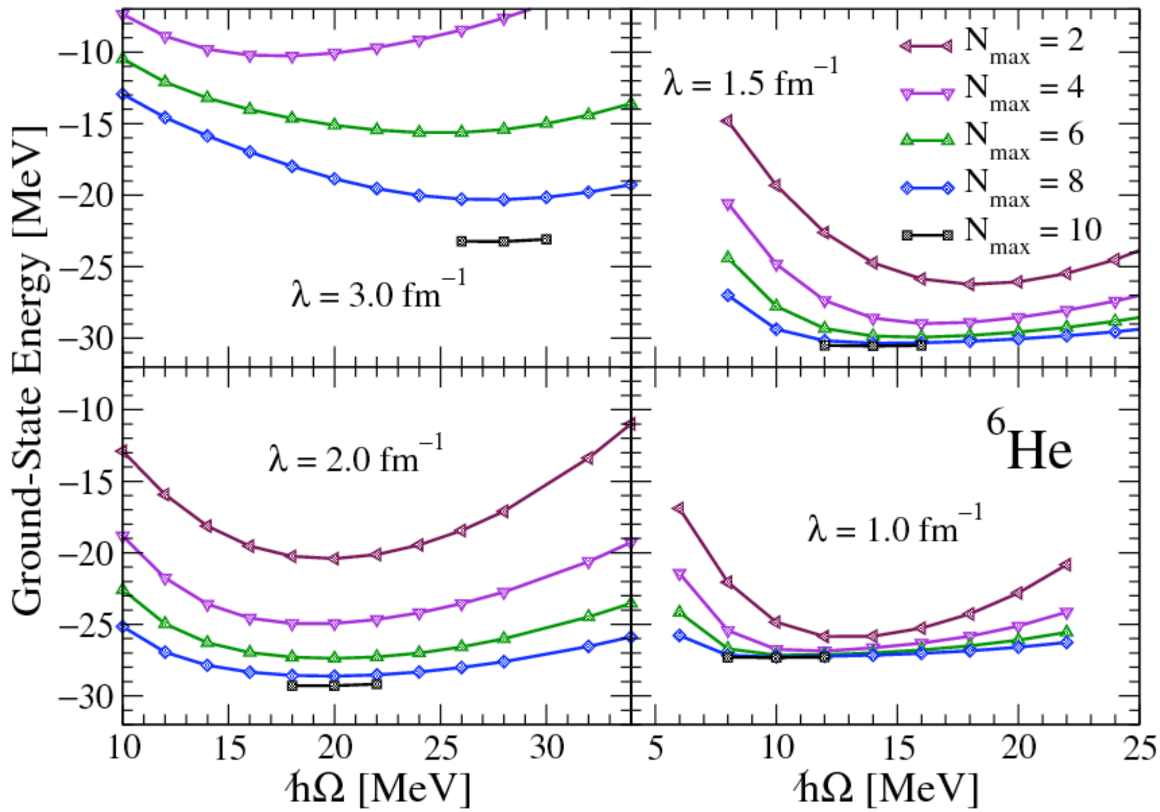
high momenta/large cutoffs lead to slow convergence for nuclei



lower cutoffs need smaller basis

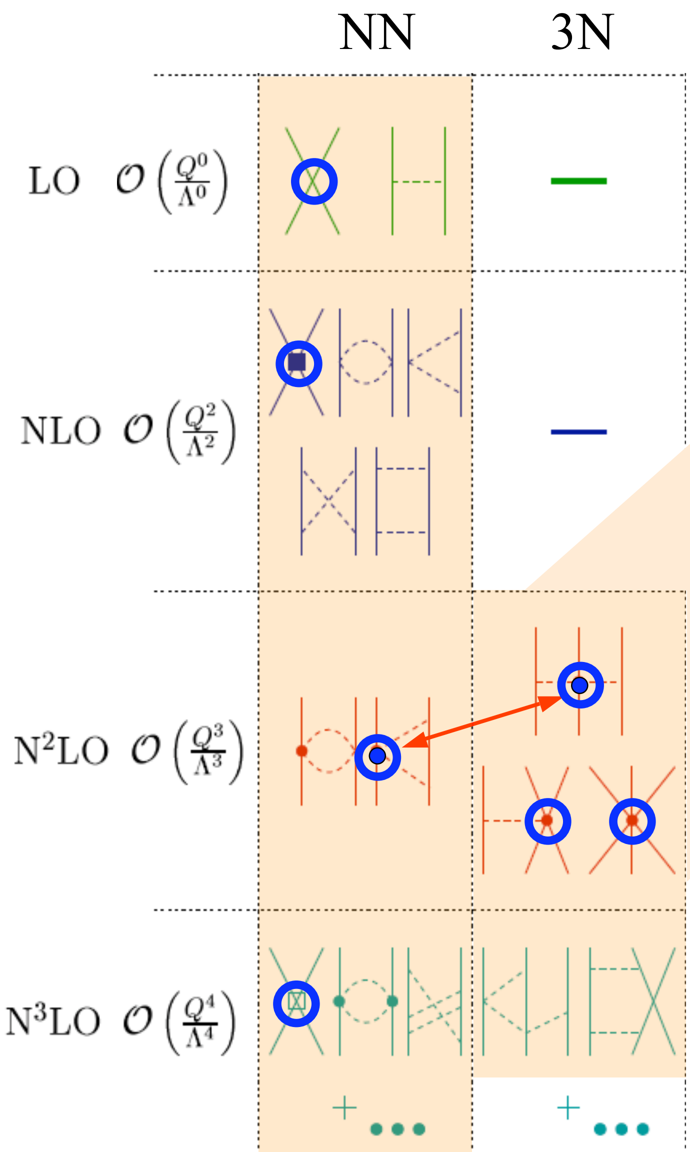
improved convergence for nuclei Bogner et al. (2008)

10^3 states for N_{max}=2 vs.
10^7 states for N_{max}=10



Chiral EFT for 3N forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV

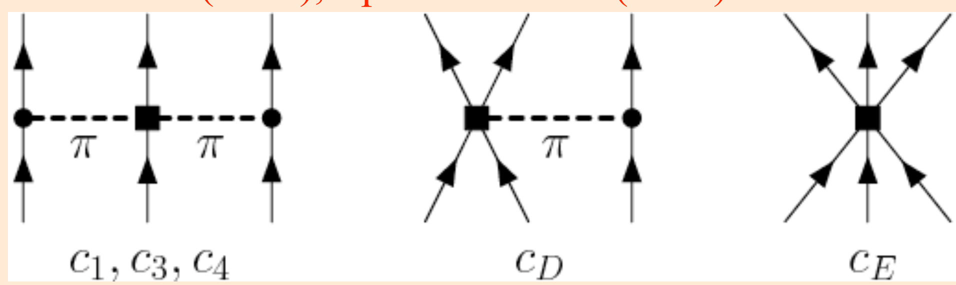


consistent NN-3N interactions

3N,4N: only 2 new couplings to N^3LO

leading 3N: N^2LO

van Kolck (1994), Epelbaum et al. (2002)



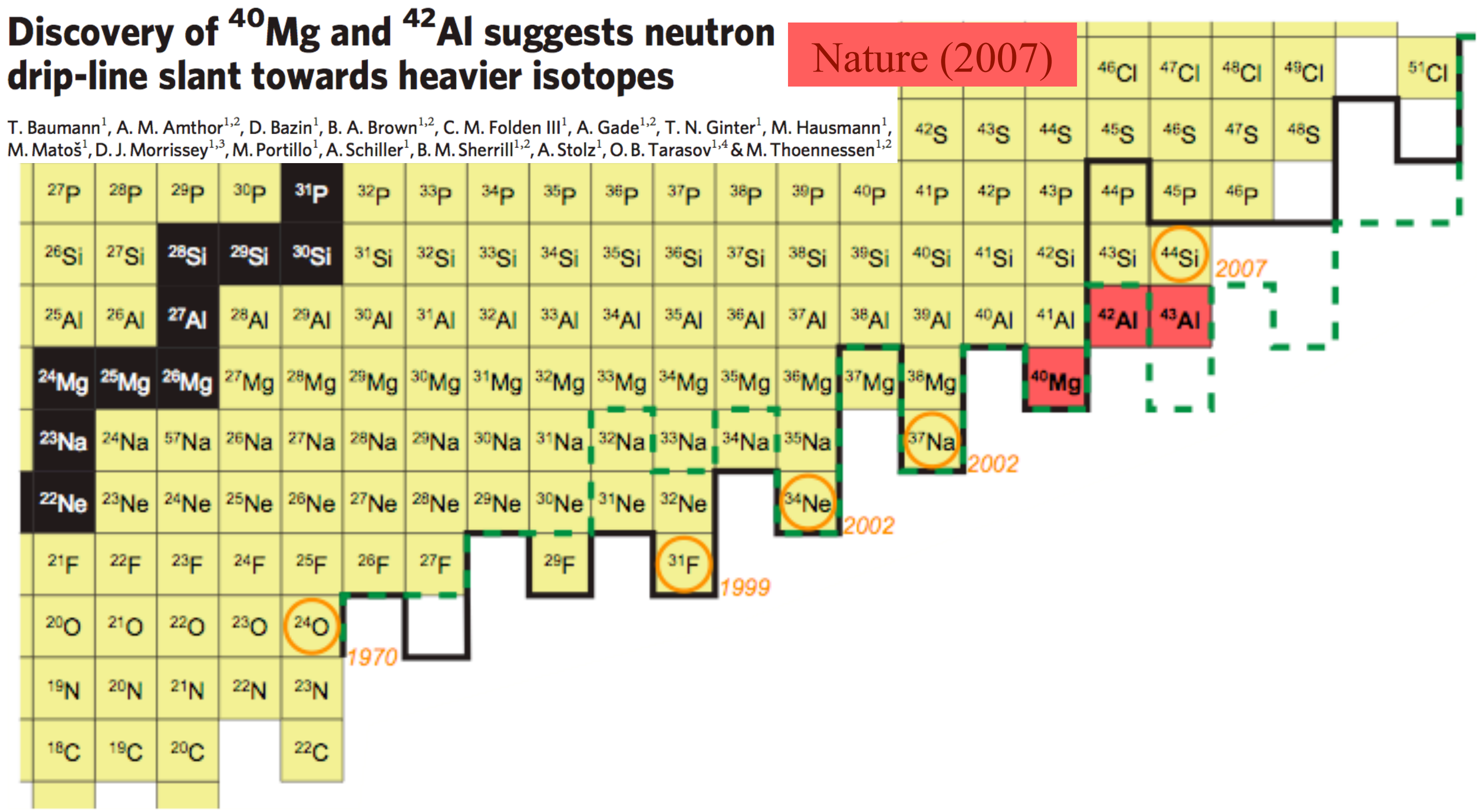
c_i from πN and NN from Meissner (2007)

$$c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.2}_{-1.0}\,,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

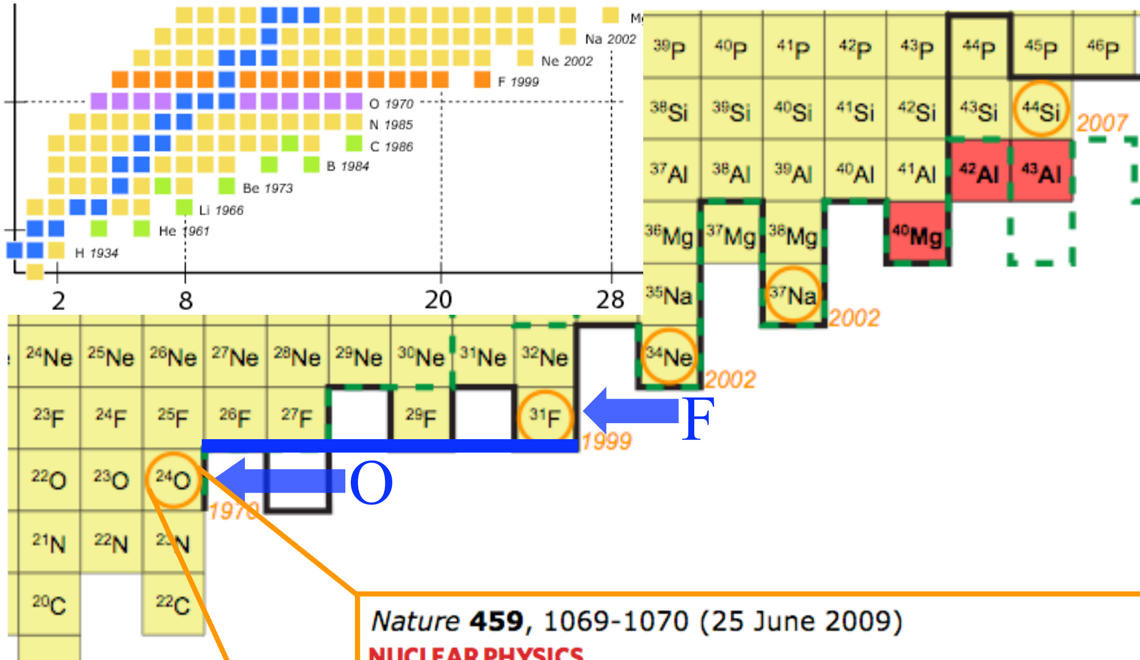
single-Δ excitation = particular c_i

c_D, c_E fit to ^{3}H binding energy and ^{4}He radius (or ^{3}H beta decay half-life)

Towards the limits of existence - the neutron drip-line



The oxygen anomaly



Nature **459**, 1069-1070 (25 June 2009)

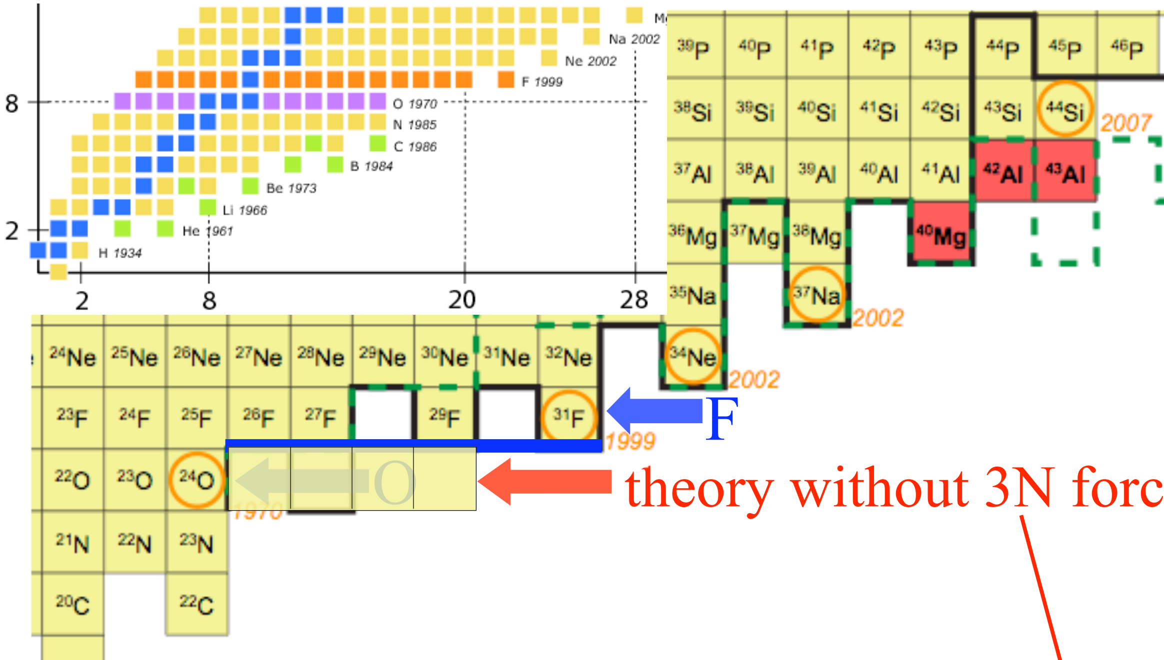
NUCLEAR PHYSICS

Unexpected doubly magic nucleus

Robert V. F. Janssens

Nuclei with a 'magic' number of both protons and neutrons, dubbed doubly magic, are particularly stable. The oxygen isotope ^{24}O has been found to be one such nucleus — yet it lies just at the limit of stability.

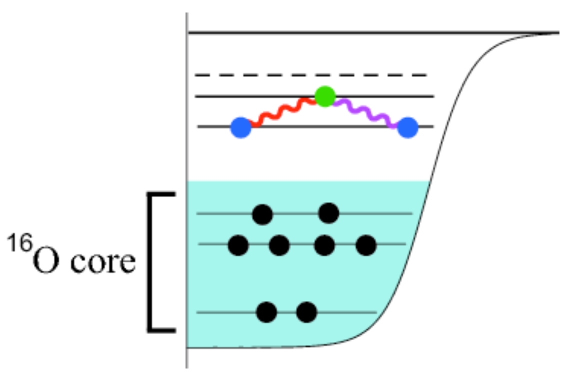
The oxygen anomaly - not reproduced without 3N forces



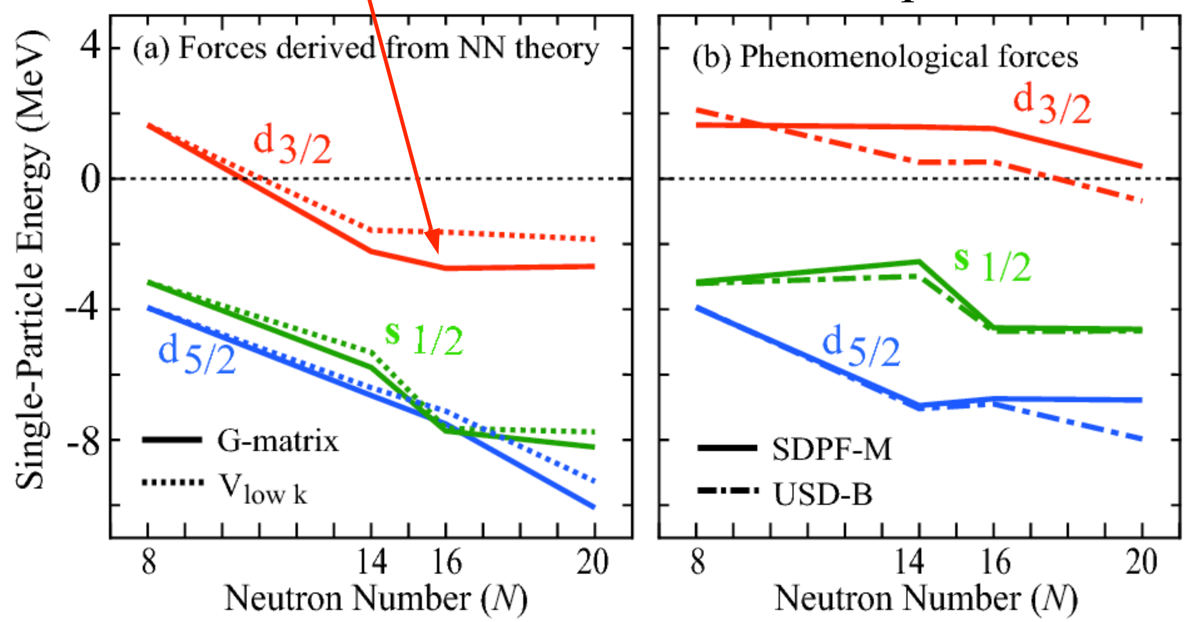
theory without 3N forces

many-body theory based on two-nucleon forces:

drip-line incorrect at ^{28}O



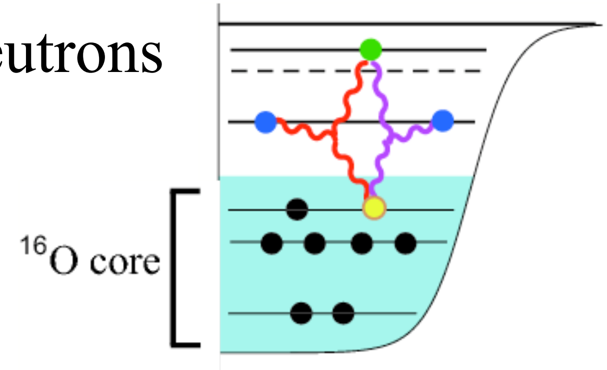
fit to experiment



The oxygen anomaly - impact of 3N forces

include normal-ordered 2-body part of 3N forces (enhanced by core A)

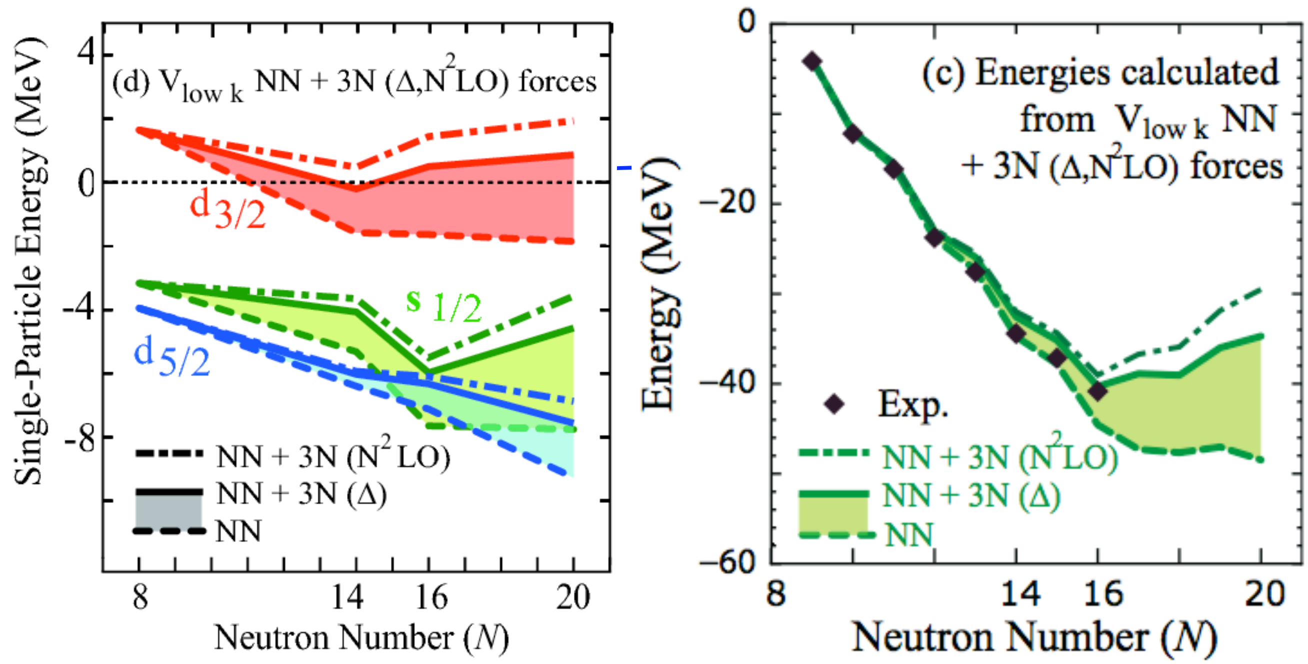
leads to repulsive interactions between valence neutrons
(repulsive based on the Pauli principle)



$d_{3/2}$ orbital remains unbound

first microscopic explanation of the oxygen anomaly

Otsuka, Suzuki, Holt, AS, Akaishi (2009)



Weinberg eigenvalue diagnostic

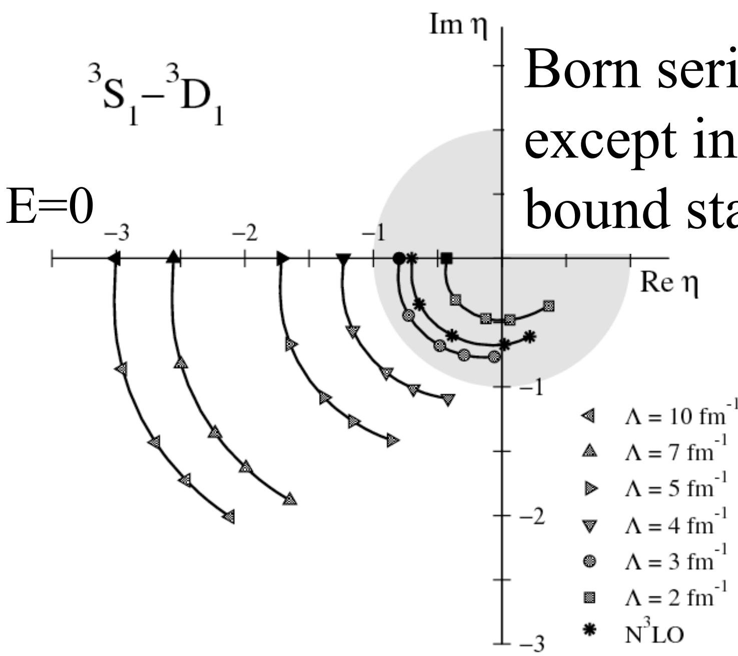
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = \left(1 + \eta_\nu(z) + \eta_\nu(z)^2 + \ldots\right) V\,|\Psi_\nu(z)\rangle$

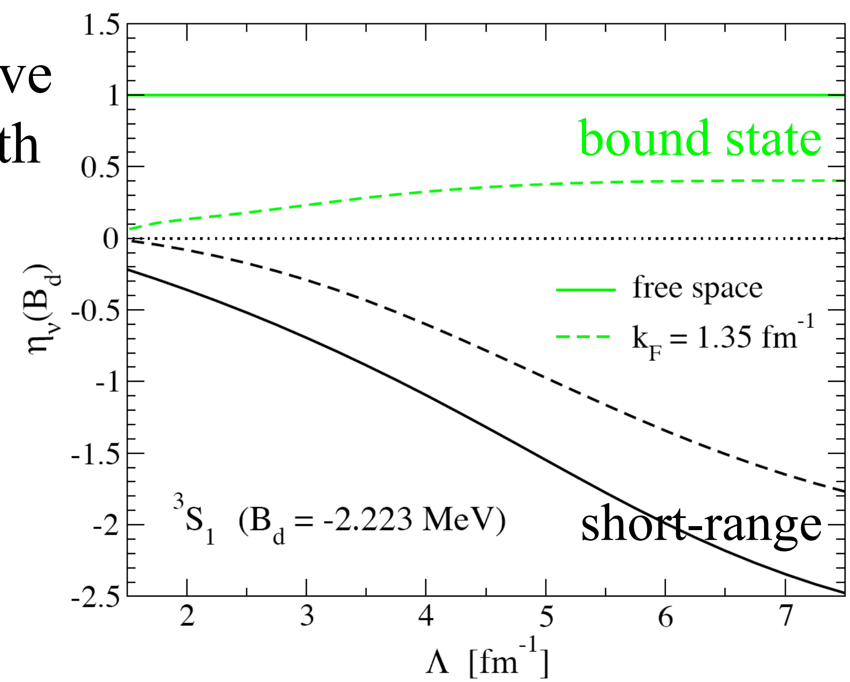
can write as Schrödinger equation $\left(H_0 + \dfrac{1}{\eta_\nu(z)} V\right)|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

high momenta/large cutoffs lead to flipped-potential bound states of -λV for small λ/large η → strong coupling to high momenta/short range and Born series always nonperturbative

RG evolution decouples high momenta (short-range repulsion and tensor parts)

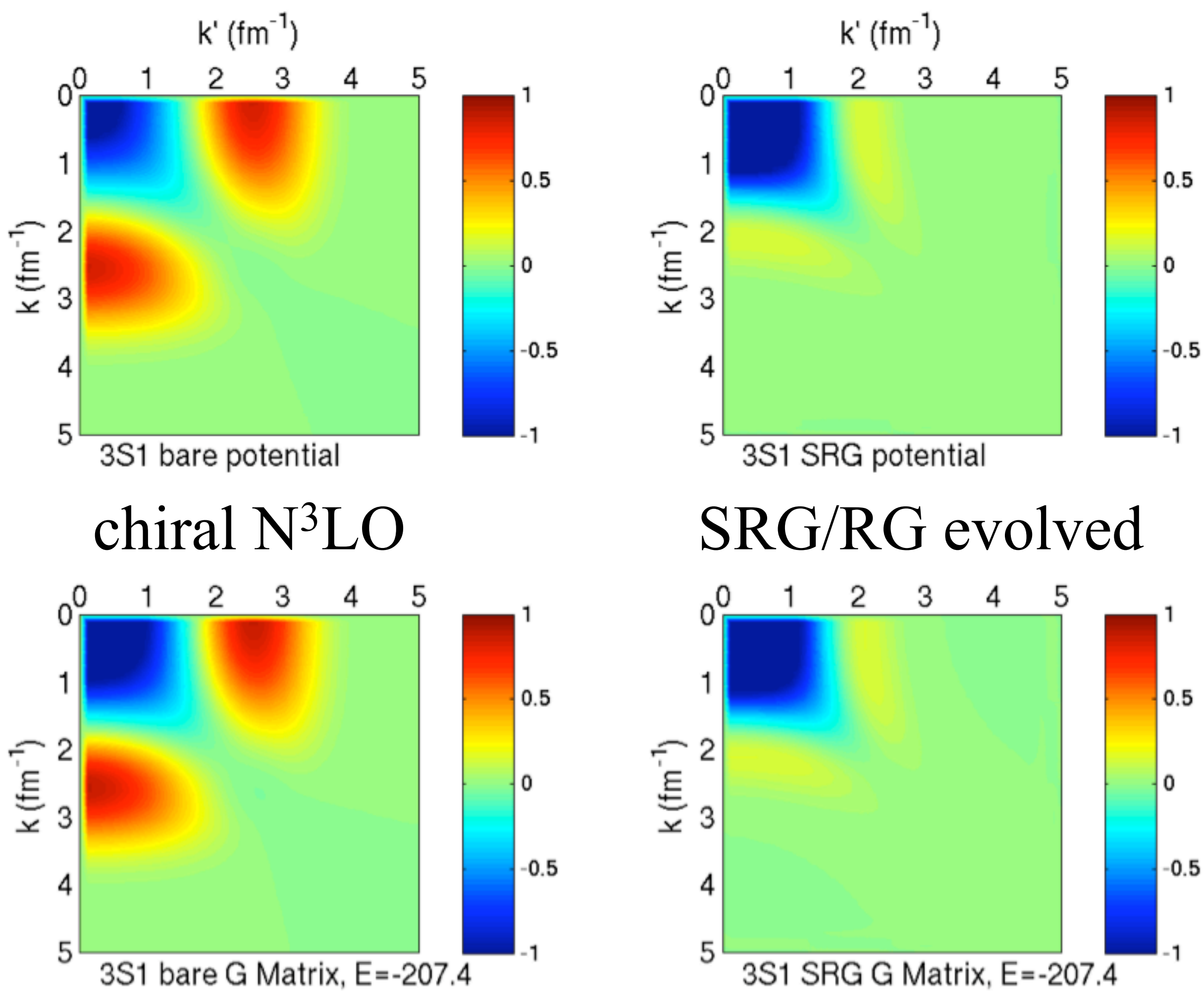


Born series perturbative except in channels with bound states



Is nuclear matter perturbative with chiral EFT and RG?

conventional Bethe-Brueckner-Goldstone expansion (sums ladders):
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling



chiral N^{3}LO   SRG/RG evolved

G matrix

conventional G-matrix approach does not solve off-diagonal coupling

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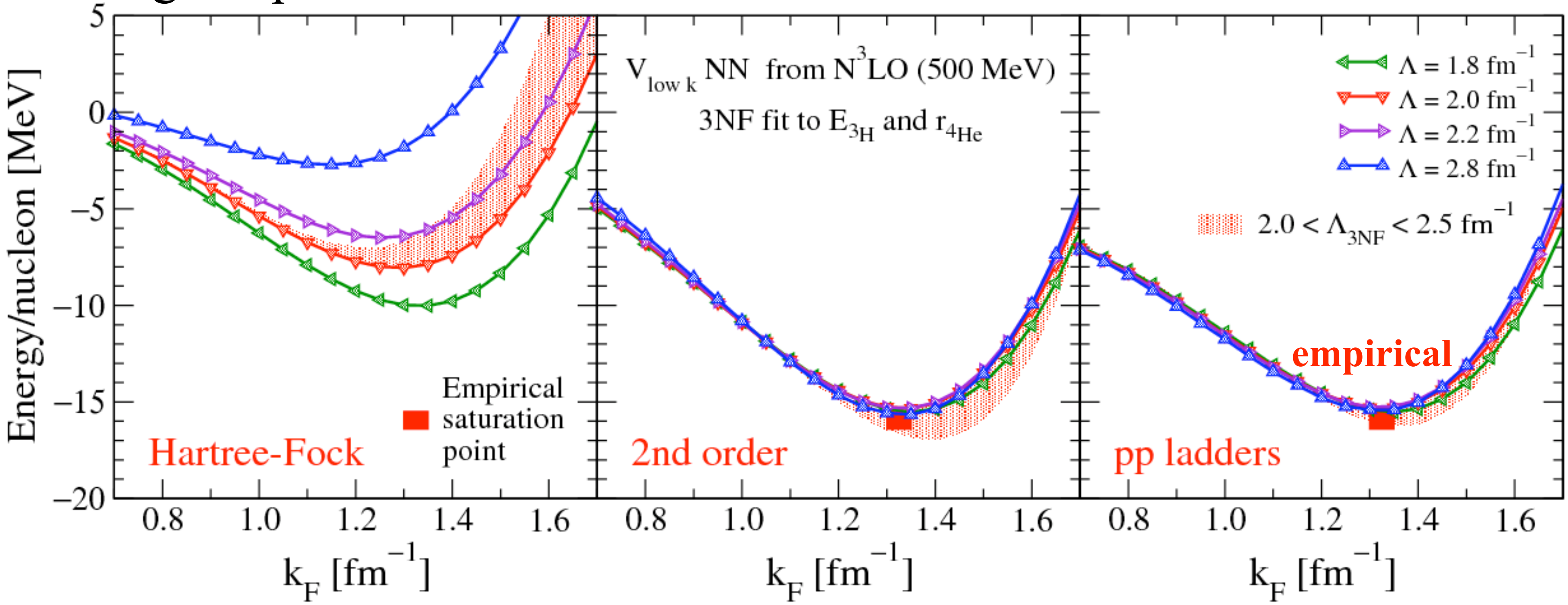
start from chiral EFT and RG evolution:
nuclear matter converged at ≈ 2nd order, 3N drives saturation

weak cutoff dependence, but need to improve 3N treatment

exciting: empirical saturation within theoretical uncertainties



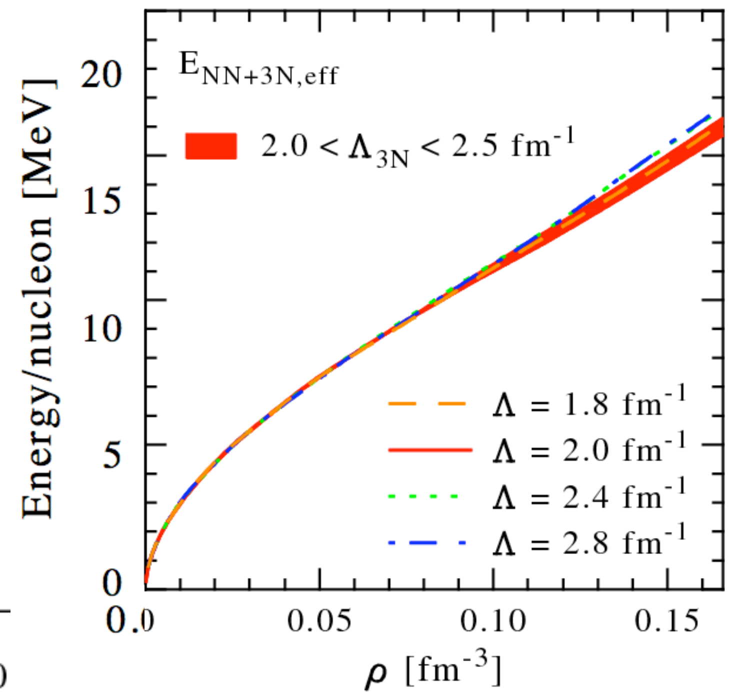
Bogner, AS, Furnstahl, Nogga (2009)

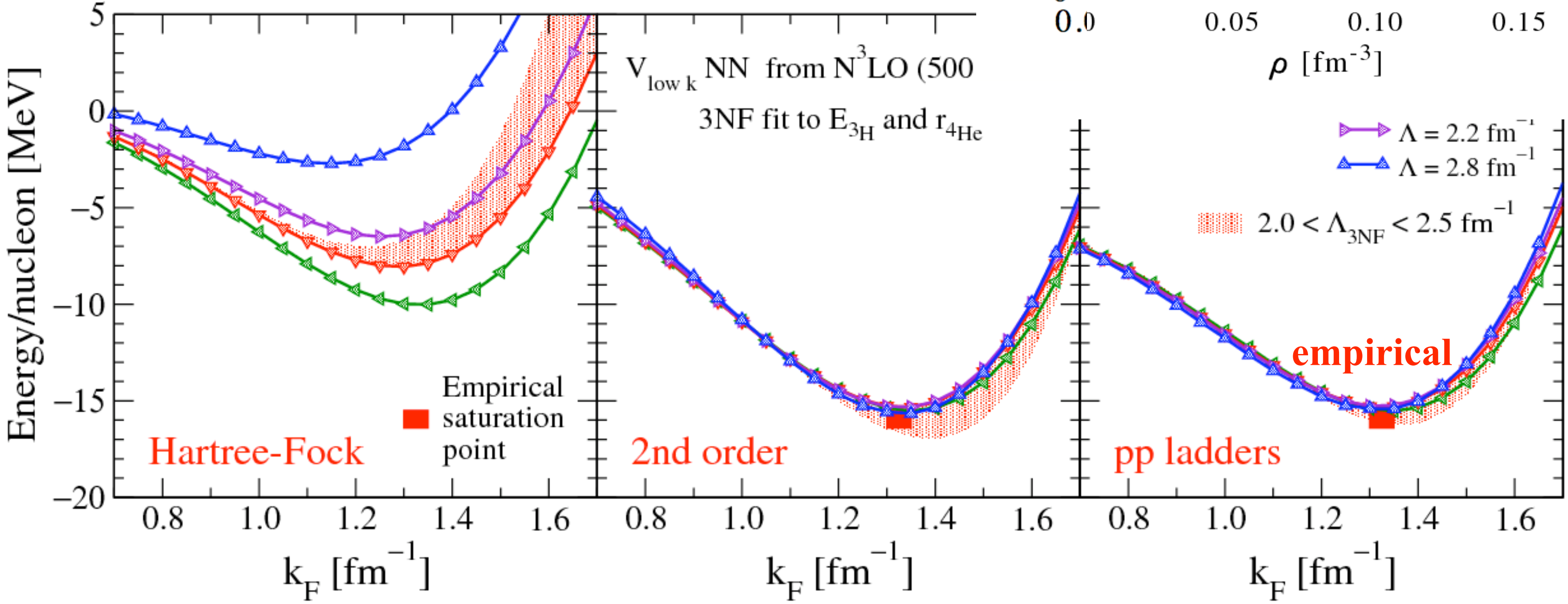
Impact of 3N interactions on neutron matter

Hebeler, AS (2009); Tolos, Friman, AS (2007)

only c_1 and c_3 terms contribute,
c_4, D, E terms vanish in neutron matter

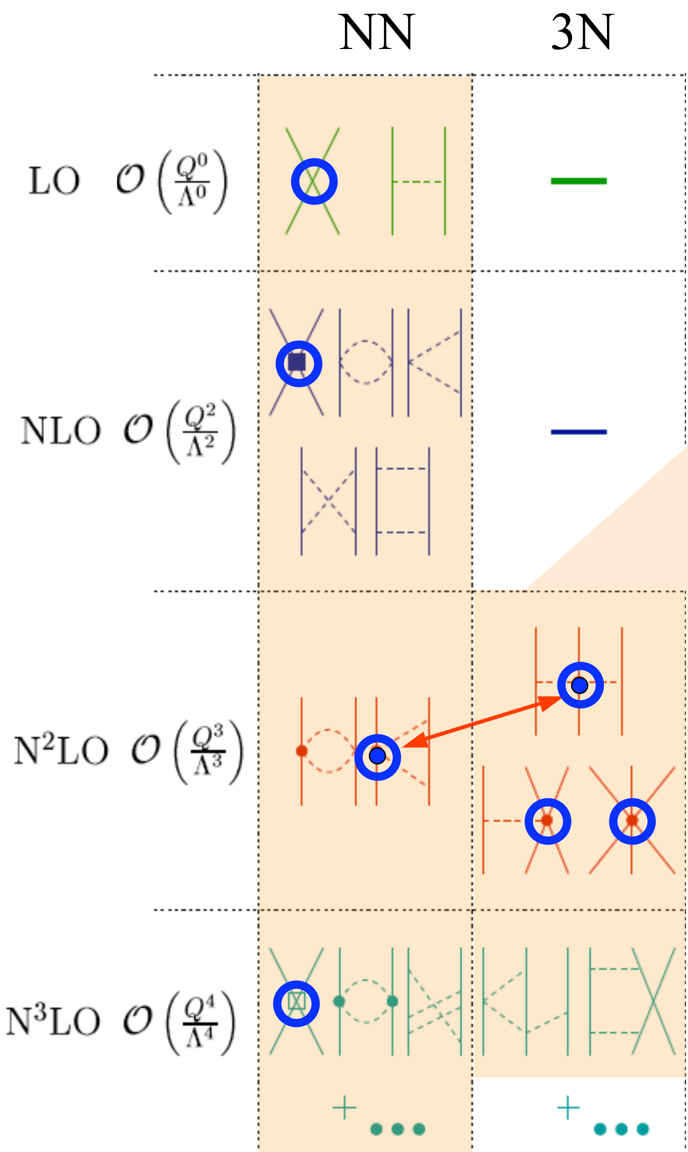
very weak cutoff dependence





Chiral Effective Field Theory for 3N forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



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3N,4N: only 2 new couplings to N^3LO

leading 3N: N^2LO

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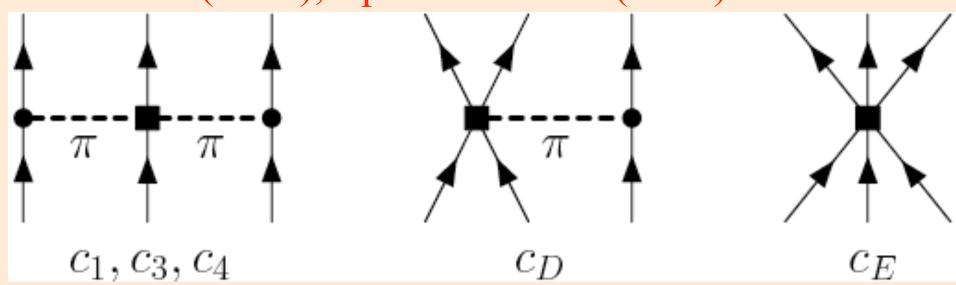

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c_D, c_E fit to ^3H binding energy and ^4He radius (or ^3H beta decay half-life)

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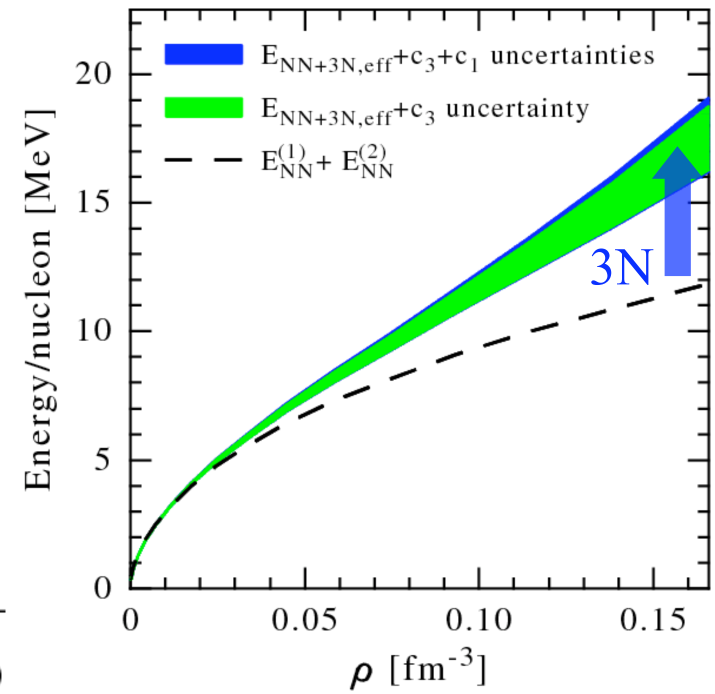
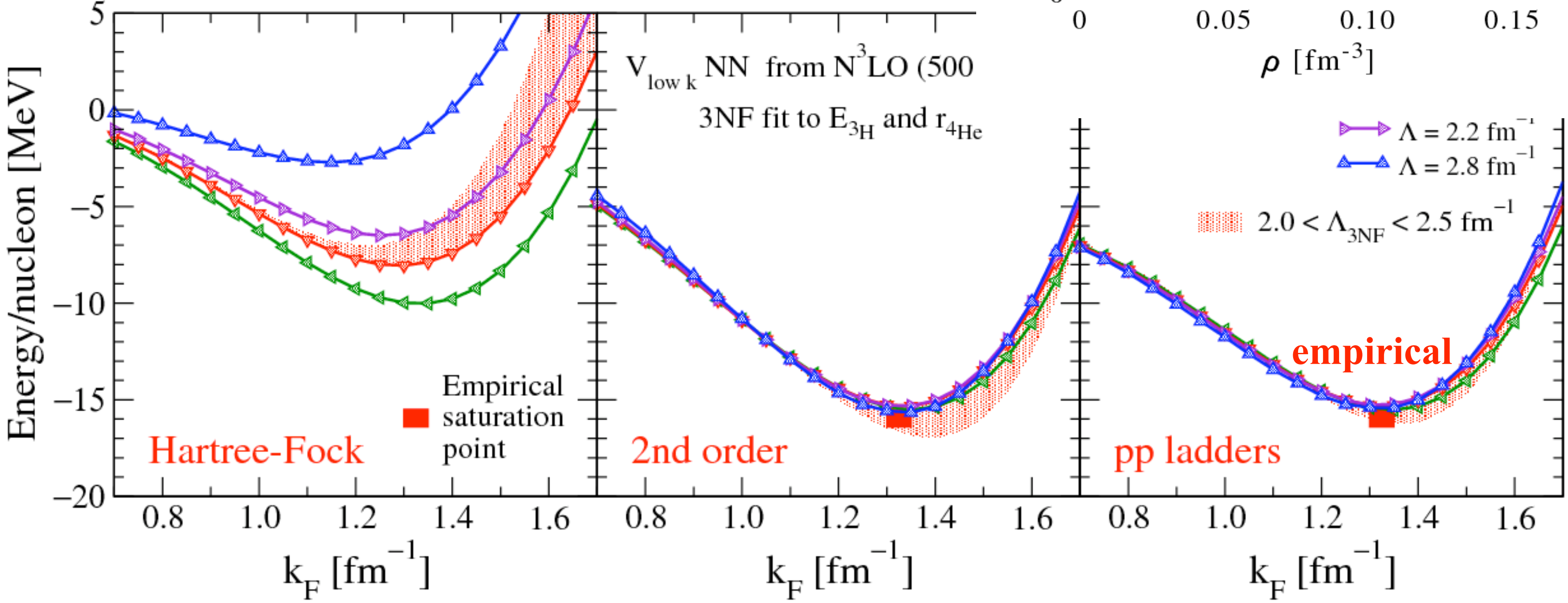
very weak cutoff dependence

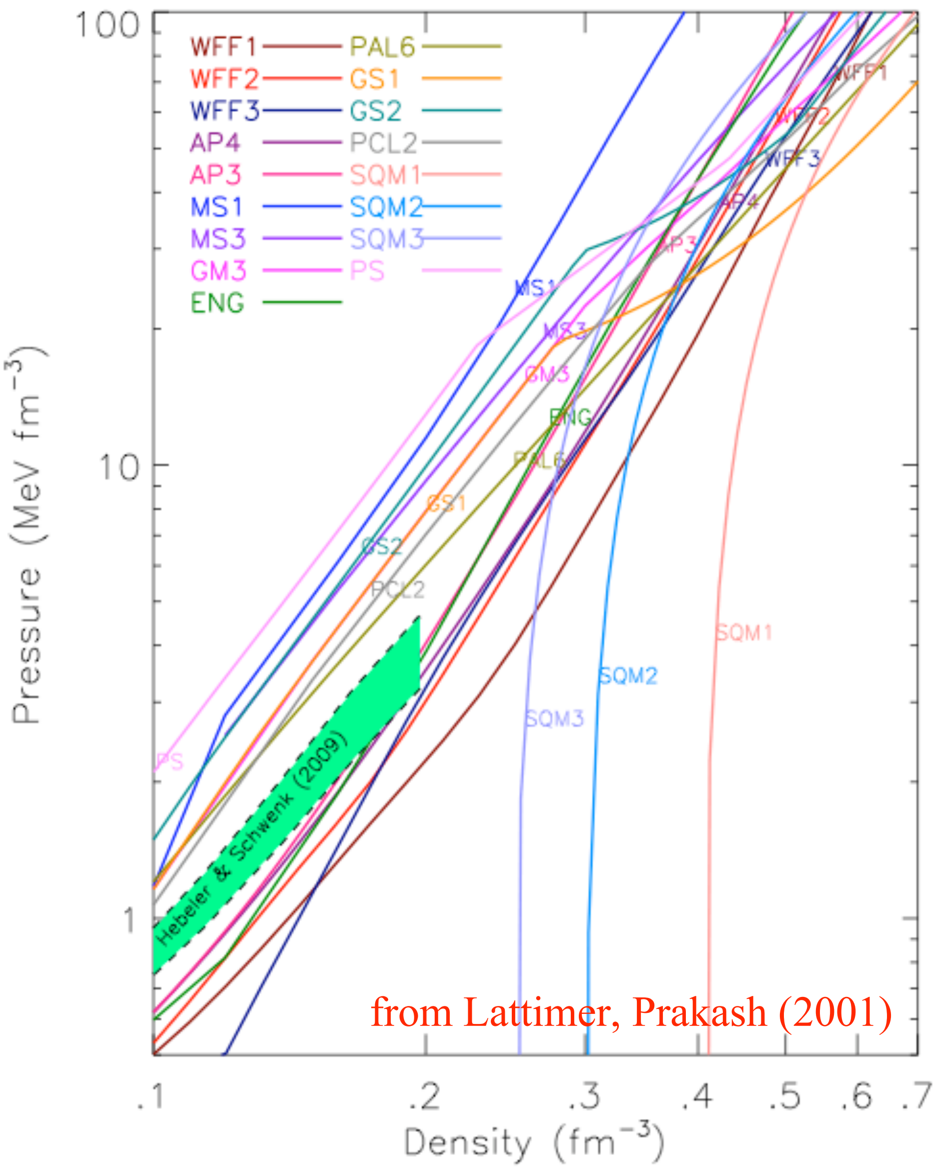
band dominated by c_3 uncertainty

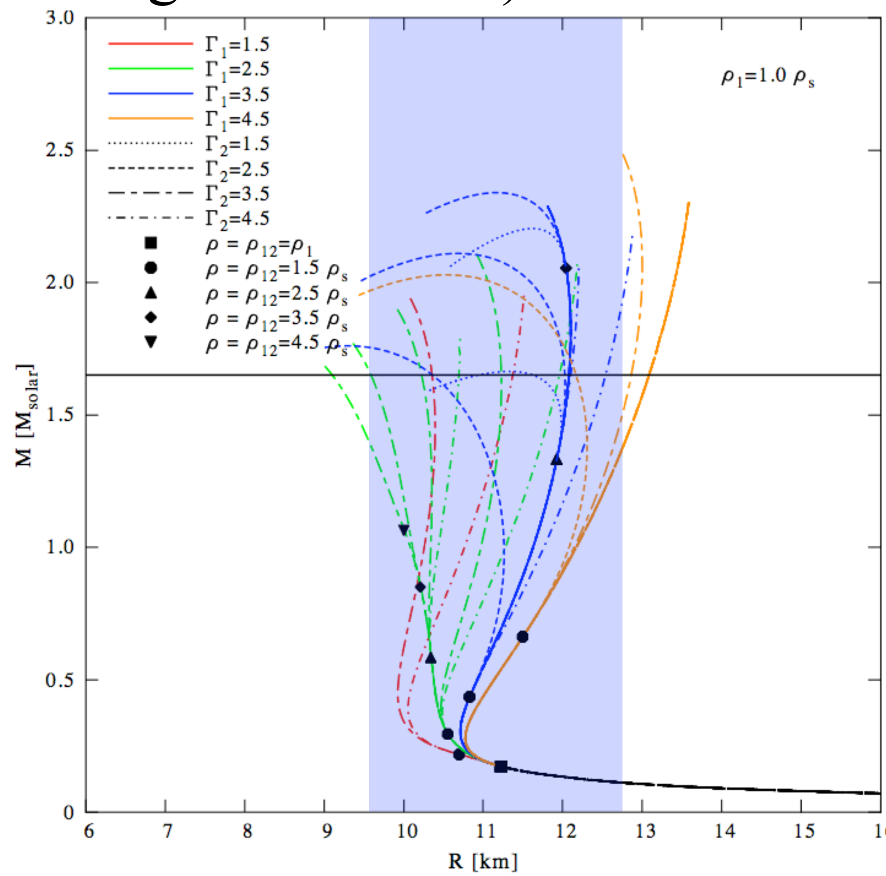




Impact on neutron stars

uncertainty band for pressure, leads to neutron star masses and radii (with general two polytrope extension to higher densities)





low-density pressure sets scale

determines ns radius to ~15%

for typical M=1.4 M_{sun}

Hebeler, Lattimer, Pethick, AS, in prep.

In-medium SRG for nuclei

$$H = \sum_{12} T_{12} a_1^\dagger a_2 + \frac{1}{(2!)^2} \sum_{1234} \langle 12|V|34\rangle a_1^\dagger a_2^\dagger a_4 a_3 + \frac{1}{(3!)^2} \sum_{123456} \langle 123|V^{(3)}|456\rangle a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4$$

normal-order Hamiltonian with respect to reference state
(e.g., Hartree-Fock ground state)

$$H = E_0 + \sum_{12} f_{12} \{a_1^\dagger a_2\} + \frac{1}{(2!)^2} \sum_{1234} \langle 12|\Gamma|34\rangle \{a_1^\dagger a_2^\dagger a_4 a_3\} + \frac{1}{(3!)^2} \sum_{123456} \langle 123|\Gamma^{(3)}|456\rangle \{a_1^\dagger a_2^\dagger a_3^\dagger a_6 a_5 a_4\}$$

with 0-, 1- and 2-body normal-ordered parts

$$E_0 = \langle \Phi|H|\Phi\rangle = \sum_{1} T_{11} n_1 + \frac{1}{2} \sum_{12} \langle 12|V|12\rangle n_1 n_2 + \frac{1}{3!} \sum_{123} \langle 123|V^{(3)}|123\rangle n_1 n_2 n_3$$

$$f_{12} = T_{12} + \sum_{i} \langle 1i|V|2i\rangle n_i + \frac{1}{2} \sum_{ij} \langle 1ij|W|2ij\rangle n_i n_j \,,$$

$$\langle 12|\Gamma|34\rangle = \langle 12|V|34\rangle + \sum_{i} \langle 12i|V^{(3)}|34i\rangle n_i \,,$$

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with 0-, 1- and 2-body normal-ordered parts and in-medium SRG eqns
e.g., for nuclear matter with η=[f,Γ] see Bogner et al., Kehrein (2006)

$$\frac{dE_0}{ds} = \frac{1}{2} \sum_{1234} (f_{12} - f_{34}) |\Gamma_{1234}|^2 \, n_1 n_2 \bar{n}_3 \bar{n}_4 \,,$$

$$\frac{df_1}{ds} = \sum_{234} (f_{41} - f_{23}) |\Gamma_{4123}|^2 (\bar{n}_2 \bar{n}_3 n_4 + n_2 n_3 \bar{n}_4) \,,$$

$$\frac{d\Gamma_{1234}}{ds} = -(f_{12} - f_{34})^2 \, \Gamma_{1234} + \frac{1}{2} \sum_{ab} (f_{12} + f_{34} - 2 f_{ab}) \Gamma_{12ab} \Gamma_{ab34} (1 - n_a - n_b) + \sum_{ab} (n_a - n_b)$$
$$\times \Big\{ \Gamma_{a1b3} \Gamma_{b2a4} \big[(f_{a1} - f_{b3}) - (f_{b2} - f_{a4}) \big] - \Gamma_{a2b3} \Gamma_{b1a4} \big[(f_{a2} - f_{b3}) - (f_{b1} - f_{a4}) \big] \Big\} \,,$$

approx. includes many-body forces and sums pp, hh, ph diagrams

In-medium SRG for nuclei Tsukiyama, Bogner, AS, in prep.

decouple 1p1h, 2p2ph,… ApAh sectors from reference state

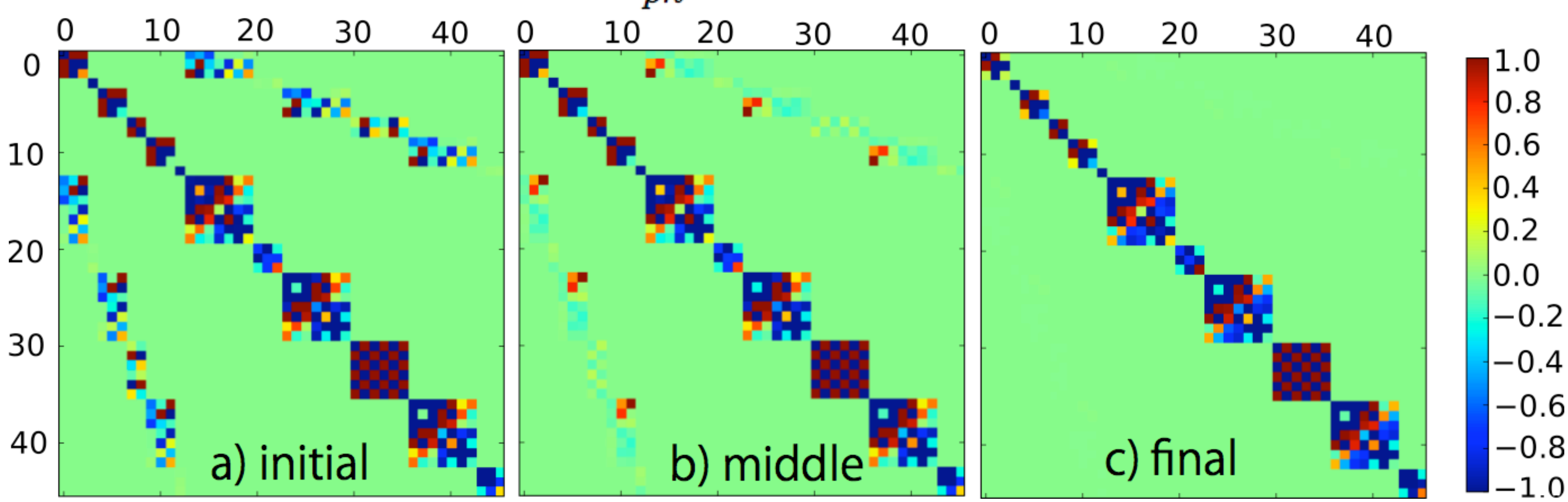
want to suppress pphh and ph couplings,
all other (normal-ordered) couplings annihilate reference state

minimal choice: $\eta(s) = [H^d(s), H(s)] = [H^d(s), H^{od}(s)]$

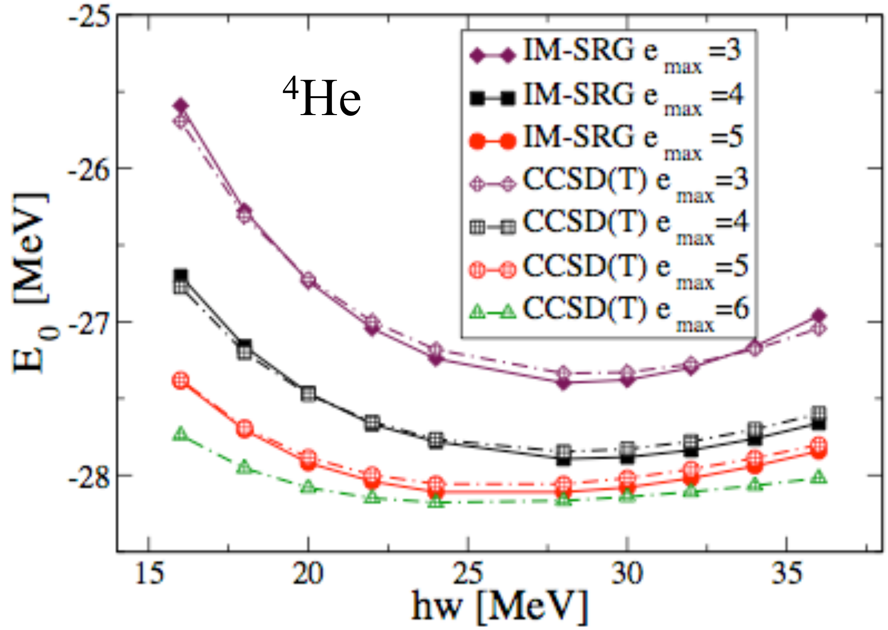
$$H^{od}(s) = g^{od}(s) + \Gamma^{od}(s)$$

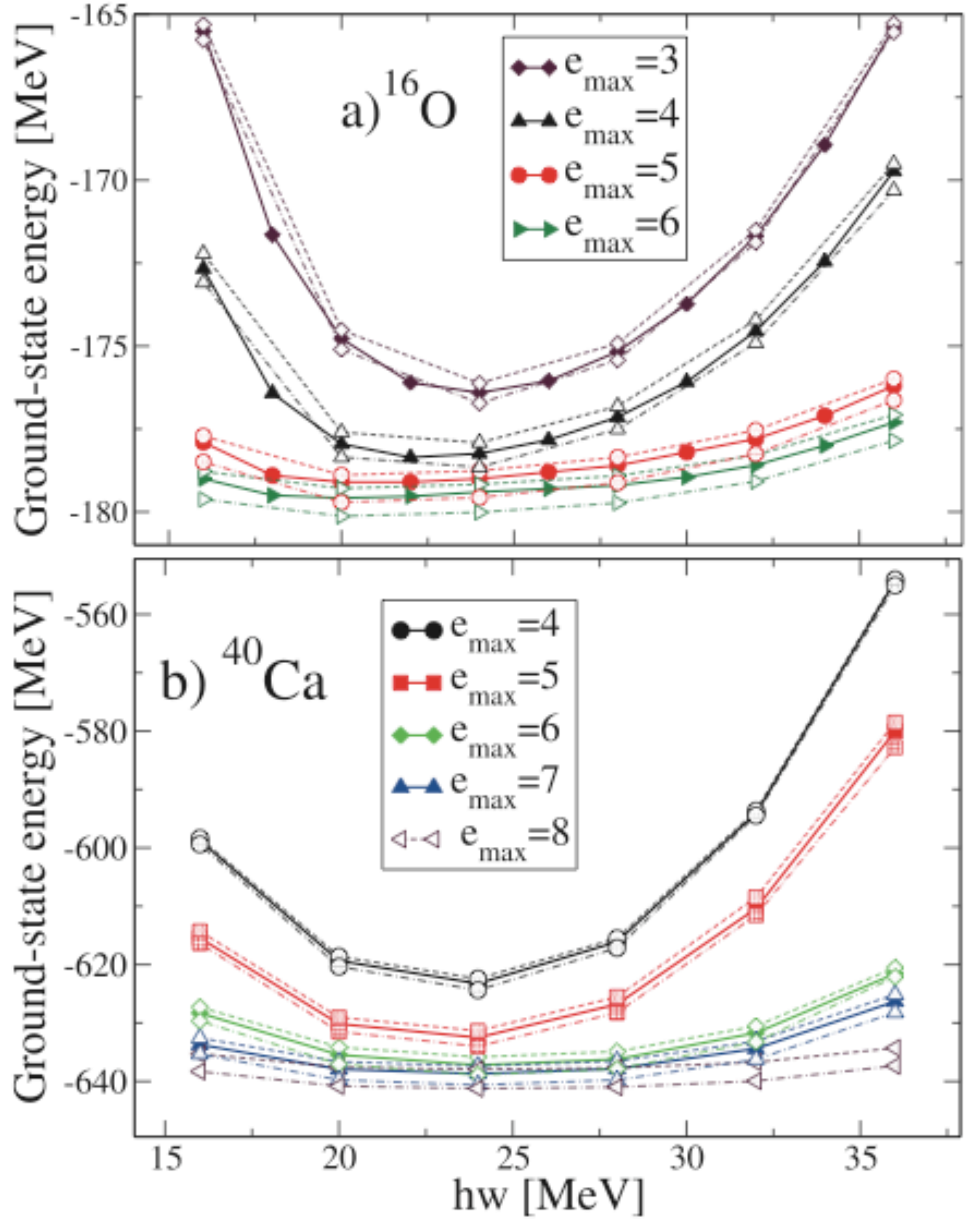
$$\Gamma^{od}(s) = \sum_{pp'hh'} \Gamma_{pp'hh'}(s) a_p^\dagger a_{p'}^\dagger a_h a_{h'} + h.c.$$

$$g^{od}(s) \sum_{ph} g_{ph}(s) a_p^\dagger a_h + h.c.$$



In-medium SRG for nuclei Tsukiyama, Bogner, AS, in prep.





first results for closed-shell nuclei

very promising convergence,
results comparable to CCSD(T)

can be used to derive nonperturbative valence-shell effective interactions

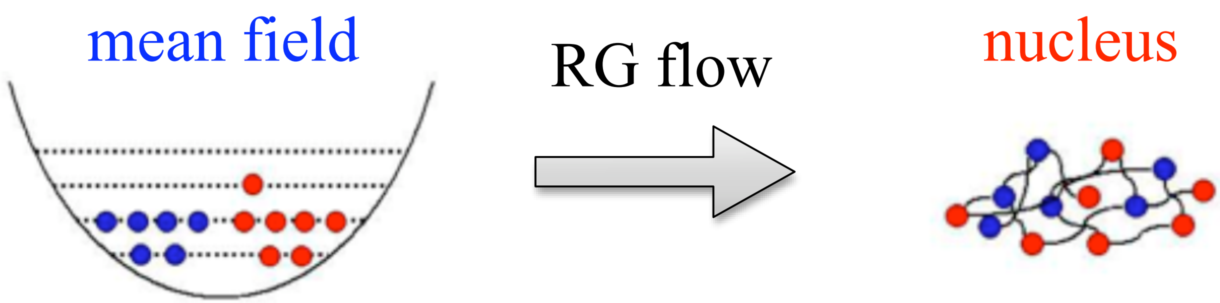
Density Functional RG for Nuclei

Braun, Polonyi, AS, in prep.

density functional: $\Gamma[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S[\psi^\dagger,\psi] + \int J\cdot(\psi^\dagger\psi)}$

with $S[\psi^\dagger, \psi] = \int \psi^\dagger \left[\partial_t - \frac{1}{2m}\Delta\right]\psi + \frac{1}{2}\int \psi^\dagger\psi V_{2\mathrm{b}}\psi^\dagger\psi + V_{3\mathrm{b}}$

main idea:



start from mean-field (background potential) and include interactions

Density Functional RG for Nuclei

introduce background potential U into the path integral

density functional: $\Gamma_\lambda[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S_\lambda[\psi^\dagger,\psi] + \int J\cdot(\psi^\dagger\psi)}$

with $S_\lambda[\psi^\dagger, \psi] = \int \psi^\dagger \Big[\partial_t - \frac{1}{2m}\Delta + (1-\lambda)U_\lambda\Big]\psi + \frac{1}{2}\int \psi^\dagger\psi\lambda V_{2b}\psi^\dagger\psi + \lambda V_{3b}$

(auxiliary) background potential or (physical) trap potential

leads to flow equation

$$\partial_\lambda\Gamma_\lambda[\rho] = [-U_\lambda + (1-\lambda)\,\partial_\lambda U_\lambda]\cdot\rho + \frac{1}{2}\rho\cdot V_{2b}\cdot\rho + \frac{1}{2}\mathrm{Tr}\left[V_{2b}\cdot\left(\frac{\delta^2\Gamma_\lambda[\rho]}{\delta\rho\,\delta\rho}\right)^{-1}\right]$$

introduce kinetic and exchange-correlation part $\tilde{\Gamma}_\lambda$:

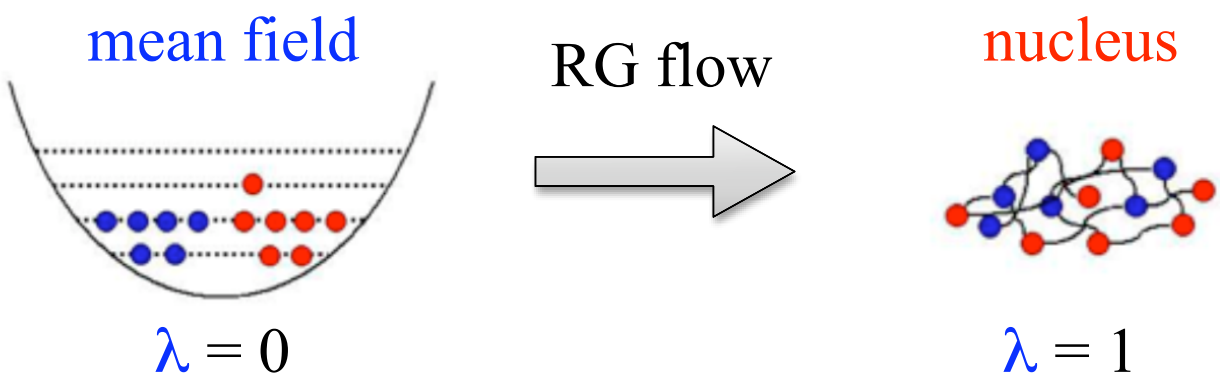
$$\Gamma_\lambda[\rho] = (1-\lambda)U_\lambda\cdot\rho + \frac{\lambda}{2}\rho\cdot V_{2b}\cdot\rho + \tilde{\Gamma}_\lambda[\rho]$$

Density Functional RG for Nuclei

density functional: $\Gamma_\lambda[\rho] = \ln \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \, \mathrm{e}^{-S_\lambda[\psi^\dagger,\psi] + \int J\cdot(\psi^\dagger\psi)}$

with $S_\lambda[\psi^\dagger,\psi] = \int \psi^\dagger \left[\partial_t - \frac{1}{2m}\Delta + (1-\lambda)U_\lambda\right]\psi + \frac{1}{2}\int \psi^\dagger\psi\lambda V_{2b}\psi^\dagger\psi + \lambda V_{3b}$

main idea:



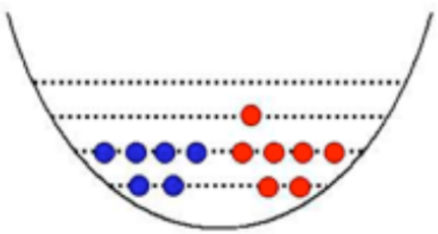
start from mean-field (background potential) and include interactions

$$\partial_\lambda \tilde{\Gamma}_\lambda[\rho] = \frac{1}{2} V_{2b}$$ 

Density Functional RG for Nuclei

main idea:

mean field

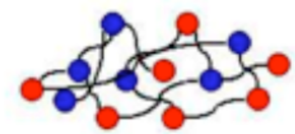


$\lambda = 0$

RG flow

$$\partial_\lambda \tilde{\Gamma}_\lambda[\rho] = \frac{1}{2} V_{2b}$$

nucleus



$\lambda = 1$

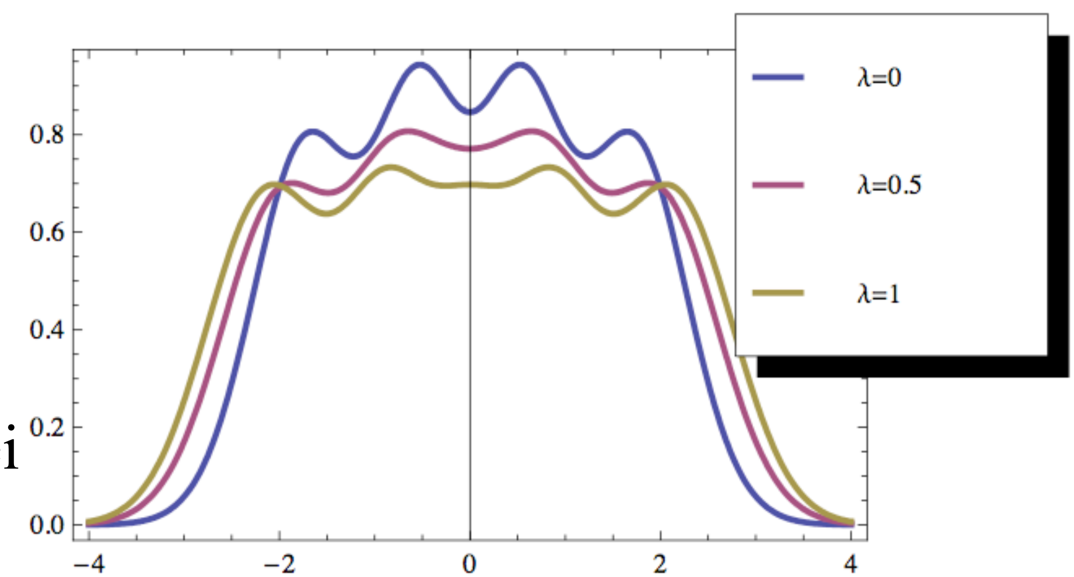
start from mean-field (background potential) and include interactions

currently: comparison to MC results for 1d model Alexandrou et al. (1989)

$$V_{2b}(x) = \sum_{i=1}^{2} \frac{V_i}{\sigma_i \sqrt{\pi}} e^{-\frac{x^2}{\sigma_i^2}}$$

density basis expansion scales favorably to heavy nuclei

benchmark results for real nuclei with coupled-cluster theory



Thanks to collaborators

S. Bacca, S. Baroni, K. Hebeler, J. Holt

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R.J. Furnstahl, R.J. Perry, E. Anderson, E. Jurgenson

A. Nogga

D.J. Dean, G. Hagen, T. Papenbrock

T. Otsuka, K. Tsukiyama, Y. Akaishi

T. Suzuki

C.J. Pethick

B. Friman

J. Lattimer

J. Braun

J. Polonyi

Summary

Exciting era with advances on many fronts: EFT and RG

For the first time, approaches from light to heavy nuclei
and for astrophysics based on the same interactions

Three-nucleon interactions are a frontier:
they impact the structure and existence of neutron-rich
nuclei and neutron-rich matter in astrophysics

Exciting intersections with problems in many related areas