# The Similarity Renormalization Group (SRG)

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On the web: `http://www.physics.ohio-state.edu/~ntg/srg/`

# Motivation for SRG

- Want to use minimal degrees of freedom for precision nonperturbative microscopic low-energy calculations
- As resolution increases degrees of freedom increase without bound
- Need a formalism that rigorously controls resolution $\longrightarrow$ **Exact Renormalization Group**
- The Similarity Renormalization Group is an Exact RG

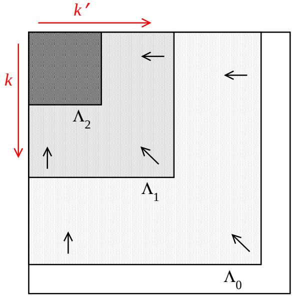
# Motivation for SRG II

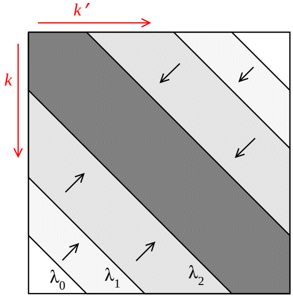
- The SRG produces band-diagonal interactions
  - Unitary transformation designed to decouple low- and high-energy states
  - All observables preserved
  - No relevant changes to low energy observables even when high momenta are removed
  - Natural hierarchy of many-body forces maintained

# Motivation for SRG III

- Designed for light-front QCD
  - Requires Hamiltonian formulation and suitable renormalization
  - $P^- = \frac{P_{perp}^2 + m^2}{P^+}$
  - Wee partons have high energy $\rightarrow$ vacuum decouples
  - Constituent picture emerges naturally
- In QED:
  - $\lambda^2 < m^2 \rightarrow$ electron-positron pairs decouple
  - $\lambda^2 < \alpha m^2 \rightarrow$ photons decouple

# The Similarity Renormalization Group (SRG)



# The Similarity Renormalization Group (SRG)

$$H_s = U(s) H U^\dagger(s) \equiv T + V_s$$

$$\frac{dH_s}{ds} = [\eta(s), H_s] \quad \textit{where} \quad \eta(s) = \frac{dU(s)}{ds} U^\dagger(s) = -\eta^\dagger(s)$$

$$\eta(s) = [G_s, H_s] \quad \Longrightarrow \quad \frac{dH_s}{ds} = [[G_s, H_s], H_s]$$

Projected onto partial-wave momentum representation ($G_s = T$):

$$\frac{dV_s(k,k')}{ds} = -(\epsilon_k - \epsilon_{k'})^2 \, V_s(k,k') + \frac{2}{\pi} \int_0^\infty q^2 \, dq \, (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) \, V_s(k,q) \, V_s(q,k')$$

# "Simple" example: delta function potential

$$H_\Lambda = P^2 - \alpha_\Lambda \Lambda^{2-d} \delta^d_\Lambda(X)$$
$$H_\Lambda|\psi_\Lambda >= E|\psi_\Lambda >$$

$$\begin{aligned} < k|H|\psi > &= k^2\psi(k) - \alpha\Lambda^{2-d} \int d^dq < k|\delta_\Lambda|q > \psi(q) \\ &= E\psi(k) \end{aligned}$$

- Choice of initial regulator affects no qualitative results

$$< k|\delta_\Lambda|q >= (\frac{1}{2\pi})^d \theta(\Lambda - |k|)\theta(\Lambda - |q|)$$

- Focus on s-wave:

$$k^2\psi(k) - \frac{\alpha\Lambda^{2-d}\Omega_d}{(2\pi)^d} \int_0^\Lambda dq\, q^{d-1}\psi(q) = E\psi(q)$$

# Introduce discretization, exponentially spaced grid

$$q = q_0 e^x \rightarrow q_0 e^{an}$$

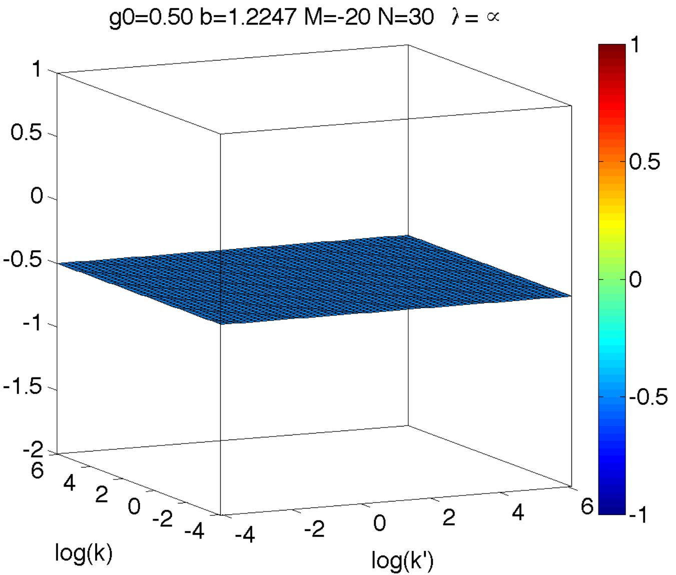
- Discretize x with uniform spacing
- Exponentially small discretization error and cutoff 'error'

$$q_n = b^n$$

$$\frac{dV_{mn}}{ds} = -(b^{2m} - b^{2n})^2 V_{mn} + c_d \sum_k b^{dk} (b^{2m} + b^{2n} - 2b^{2k}) V_{mk} V_{kn}$$

- We want $b \rightarrow 1$ limit, but $b \rightarrow \infty$ limit can be solved analytically and yields main features of numerical results.

# $G = [T, H]$, $g = 0.5$, $E_{bd} = 2.4674$



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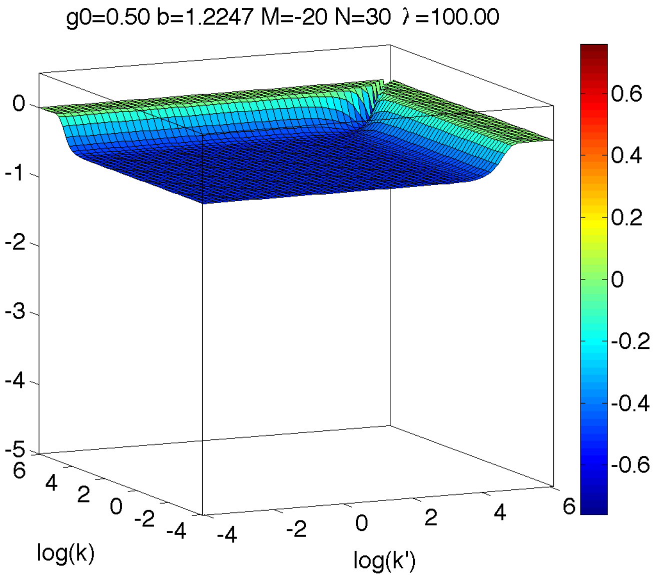

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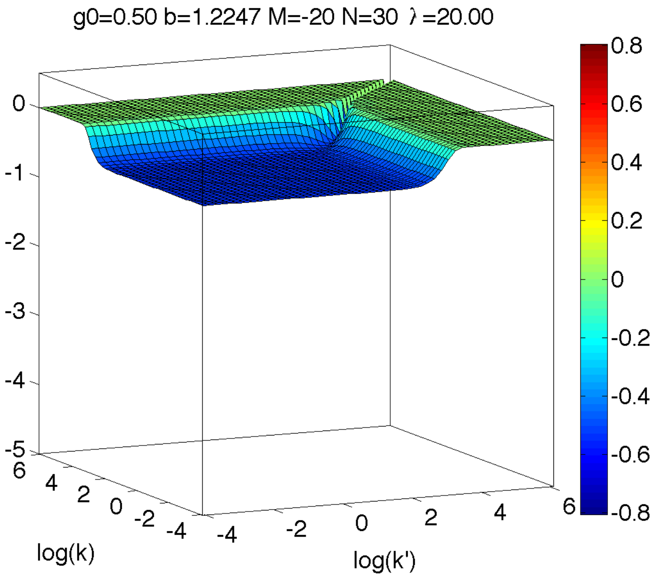

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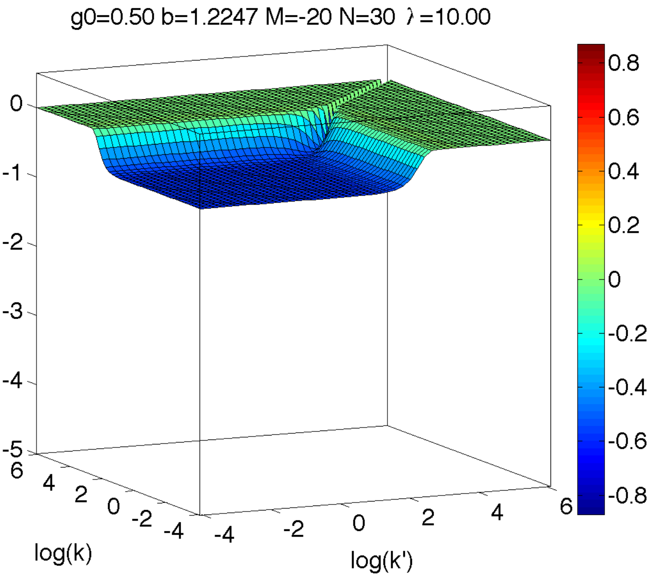

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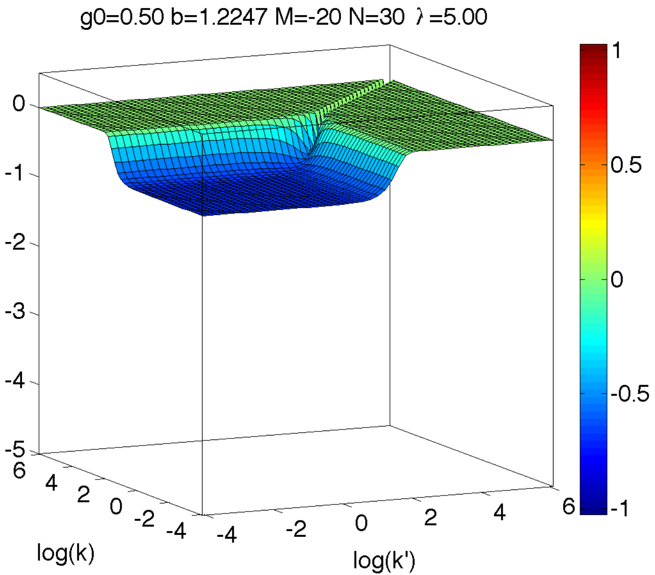

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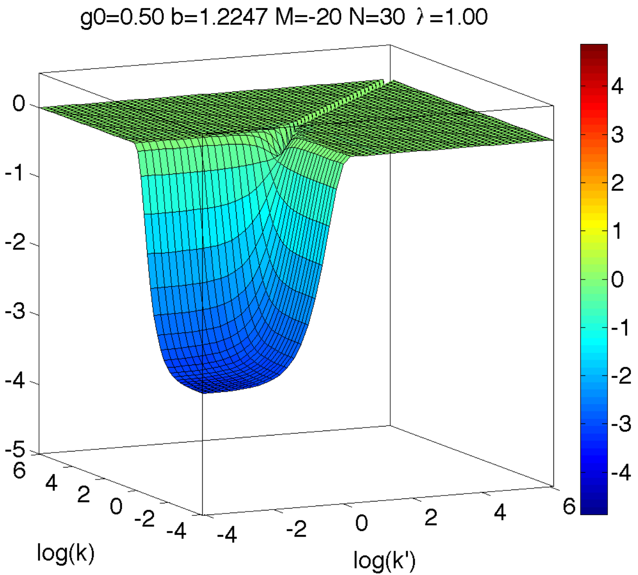

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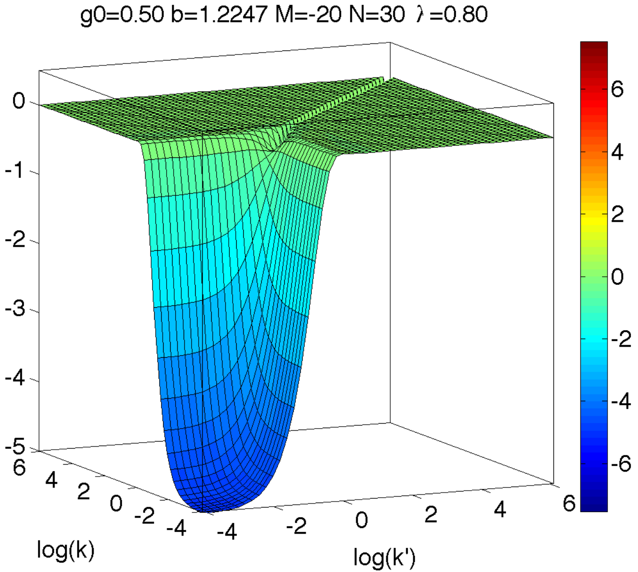

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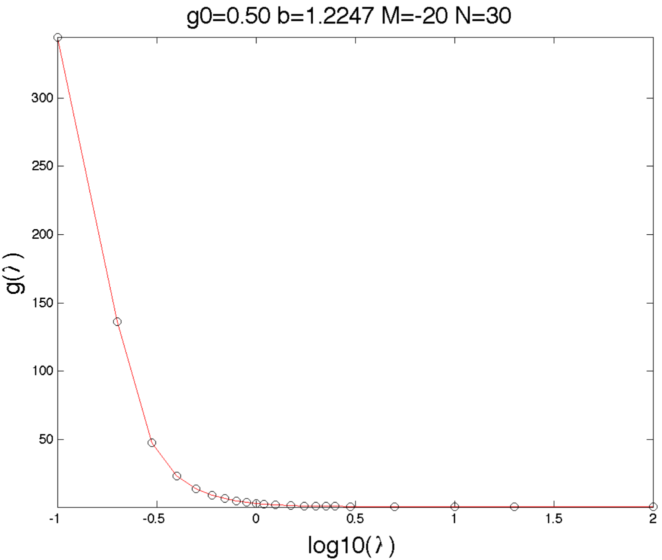

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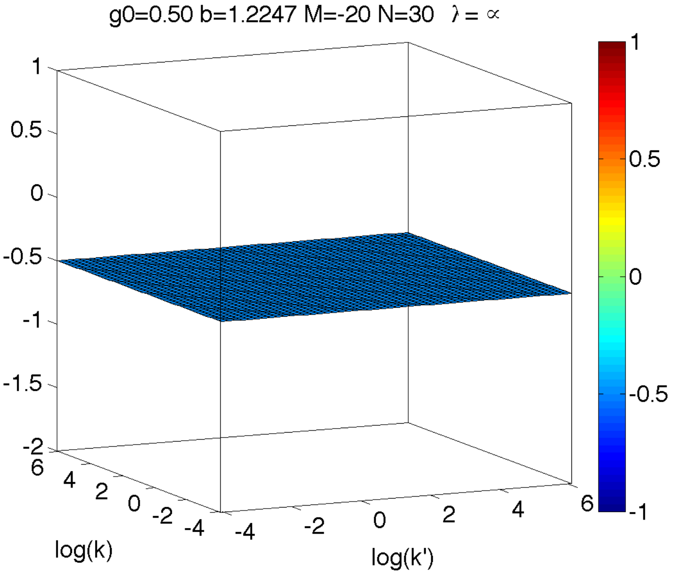

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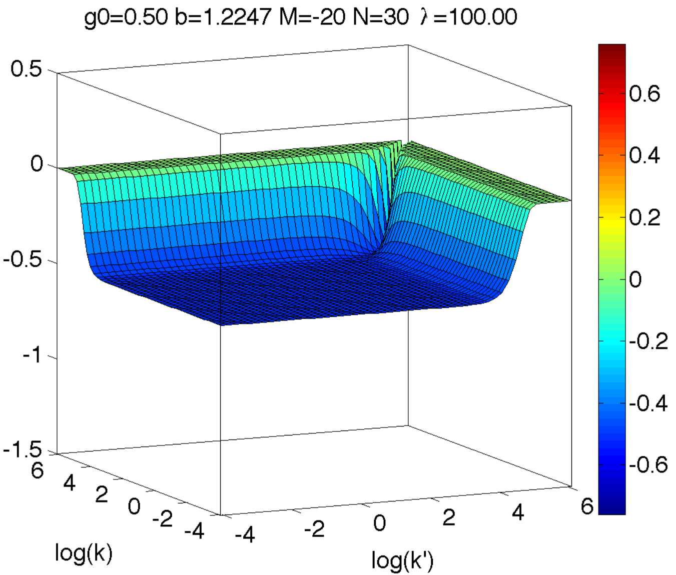

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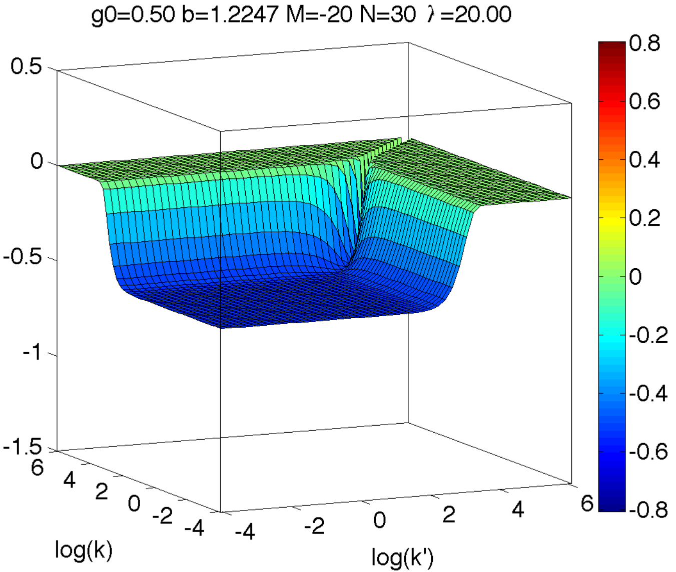

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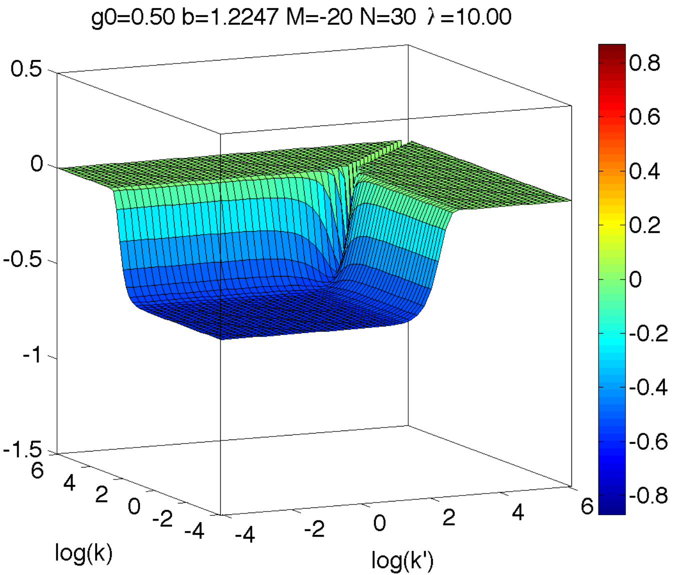

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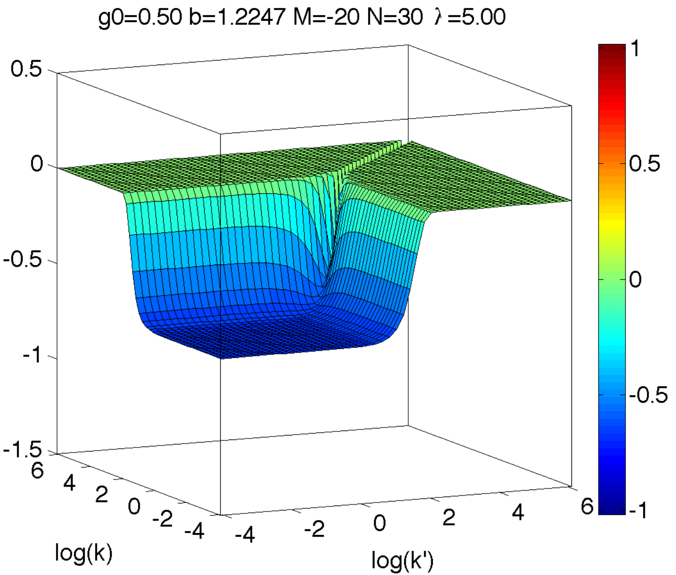

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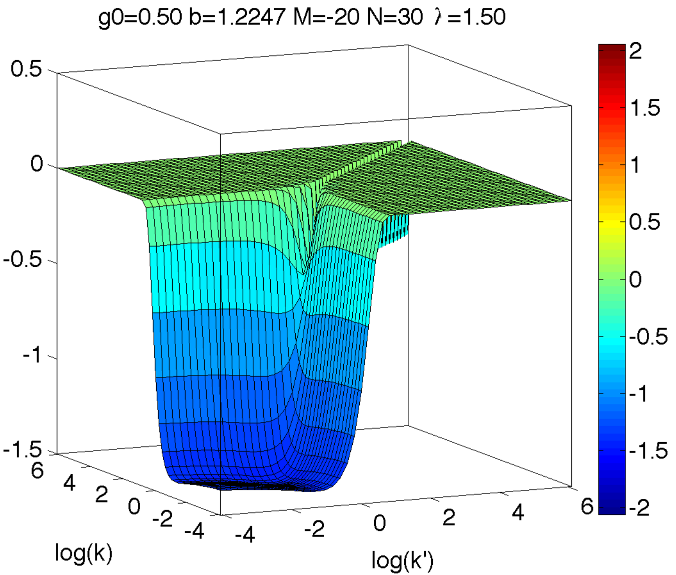

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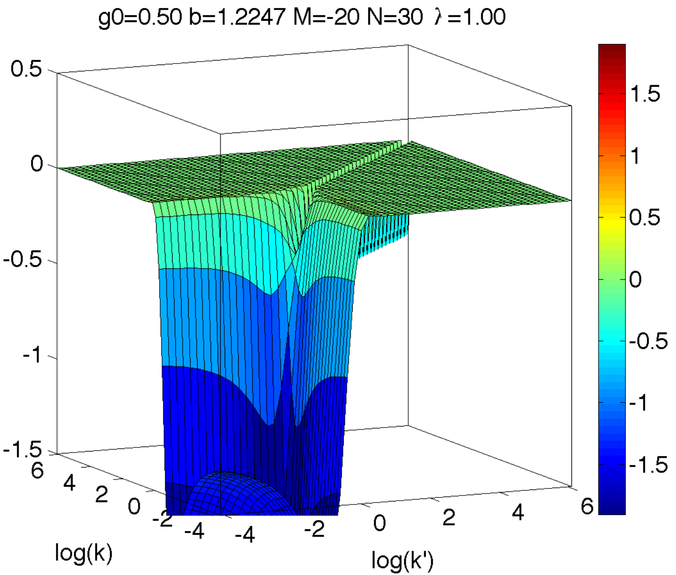

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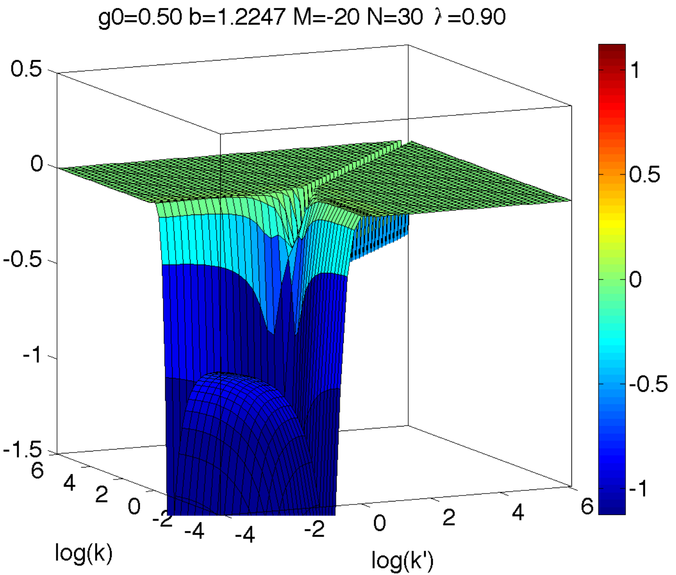

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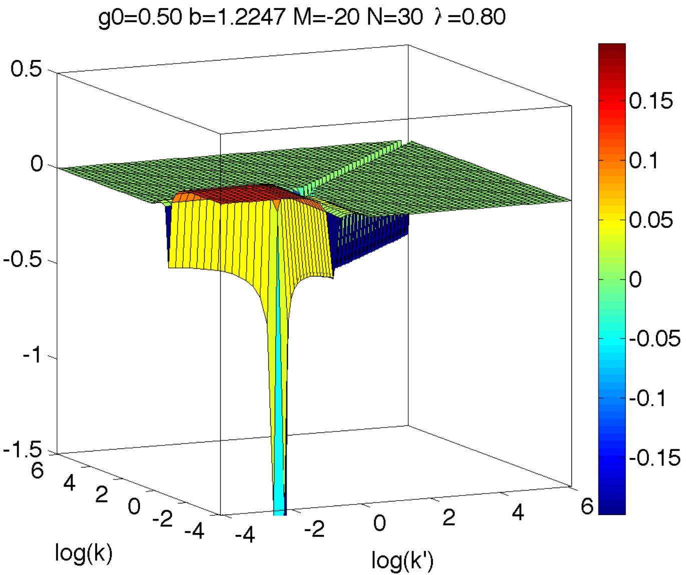

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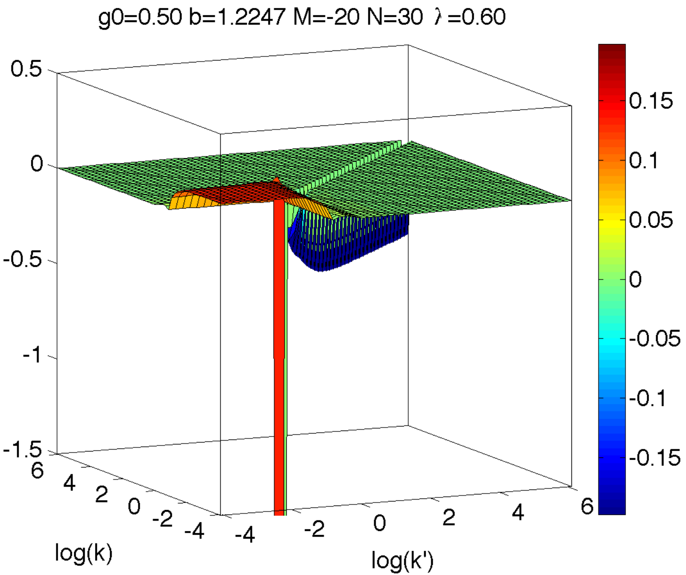

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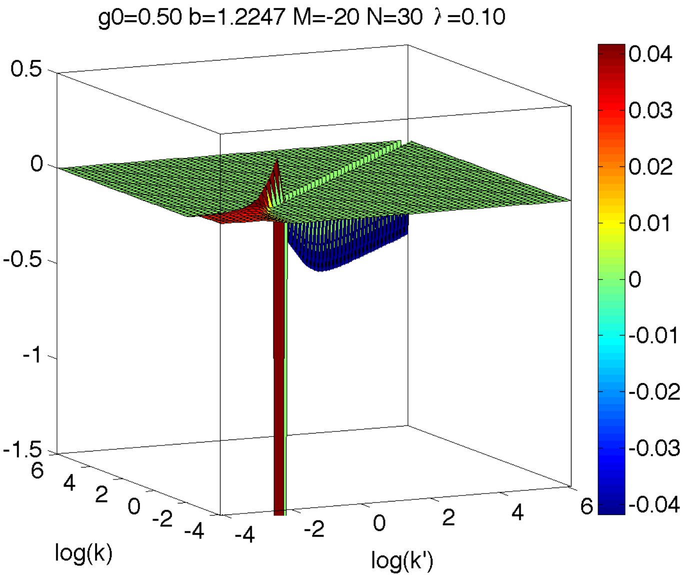

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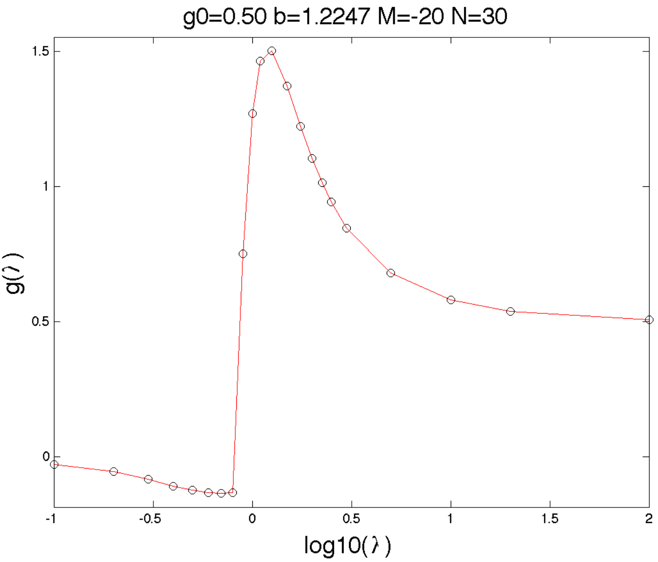
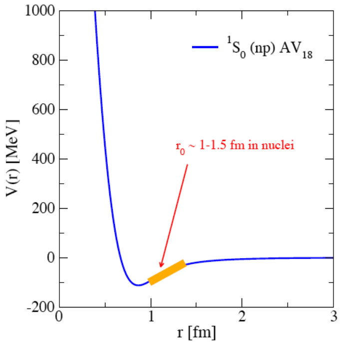


# Sources of Nonperturbative Physics for NN

1. Strong short-range repulsion ("hard core")
2. Iterated tensor interaction
3. Near zero-energy bound states



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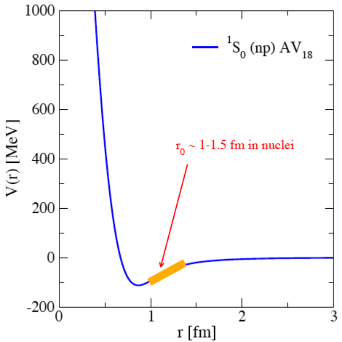


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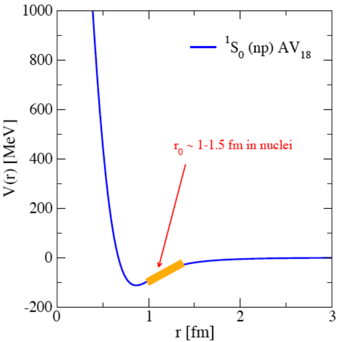
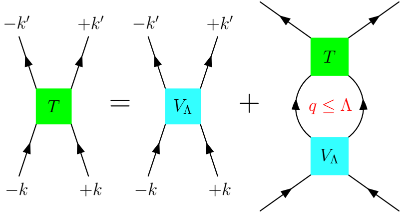


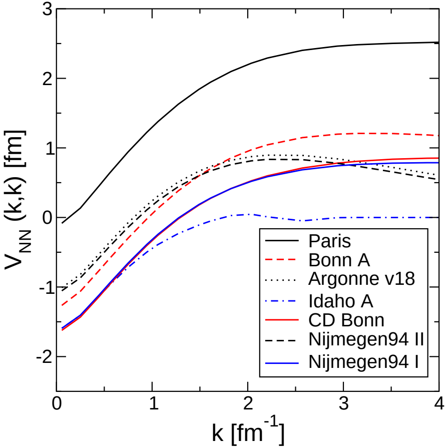

- Consequences:
  - Strong correlations overwhelm Hartree-Fock.
  - Diagrammatic analyses become hopelessly complicated.
- However ...
  - the potential depends on the resolution $\Longrightarrow$ cutoff dependent
  - We can use a RG to control the cutoff.

# Gameplan: Exploit Variable Cutoff Potential

Bogner, Kuo, Schwenk




- Require $\frac{dT}{d\Lambda} = 0$
  $\Longrightarrow$ renormalization group equation for $V_\Lambda$
- Run from $\Lambda_B = 25\,\text{fm}^{-1}$ to $\Lambda = 2\,\text{fm}^{-1} \sim E_{\text{lab}} \doteq 350\,\text{MeV}$



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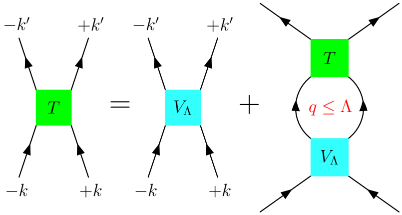


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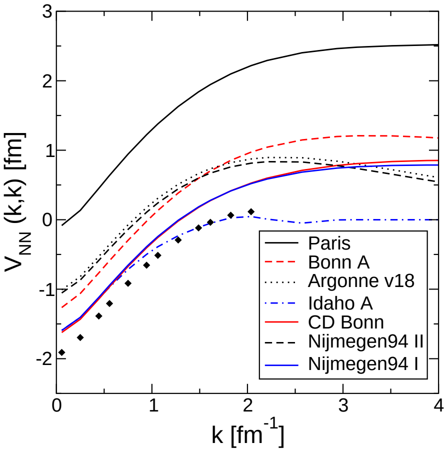

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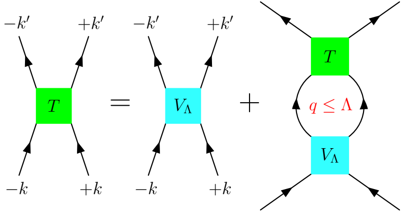


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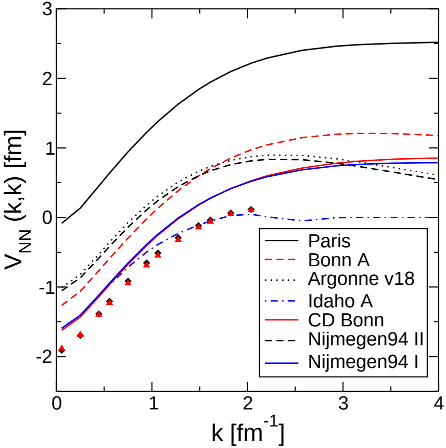

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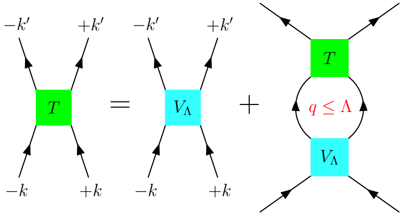


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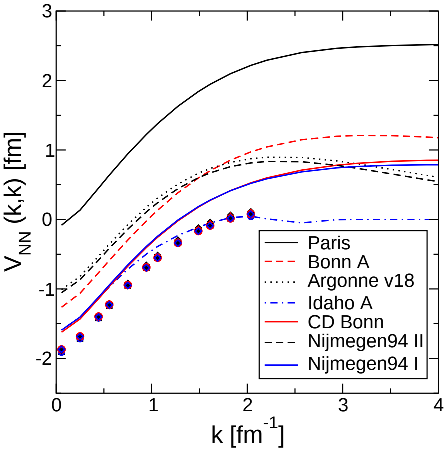
- Same long distance physics $\Longrightarrow$ collapse to "$V_{\text{low }k}$"!



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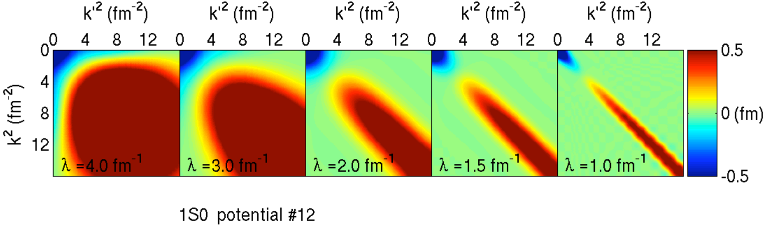
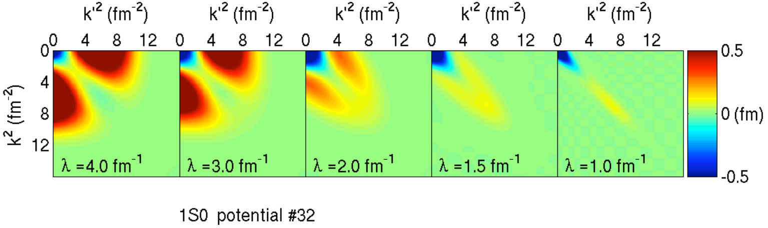
# Typical comparison in $^1S_0$

- N3LO from Entem/Machleidt

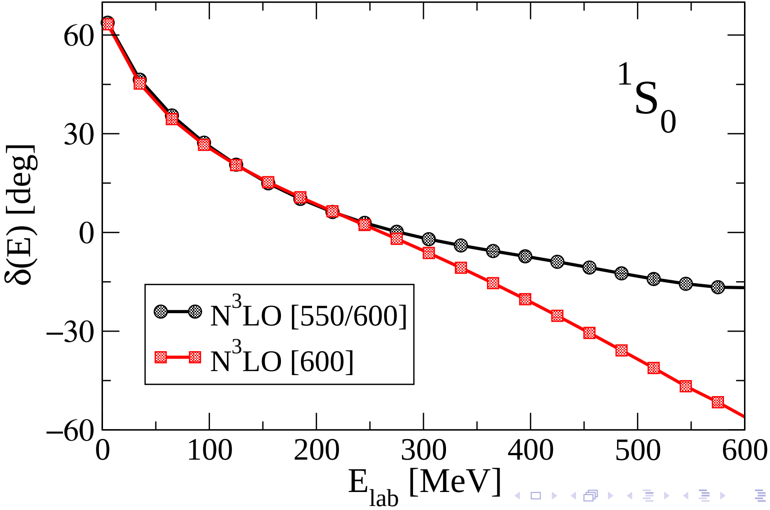



- N3LO from EpelBaum



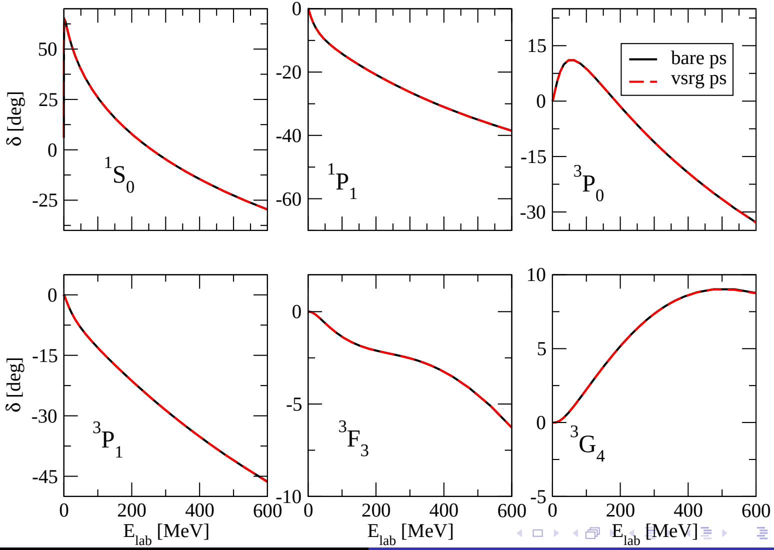
# Phase shifts do not change

- S-wave phase shifts from the two chiral EFT $N^3LO$ potentials.



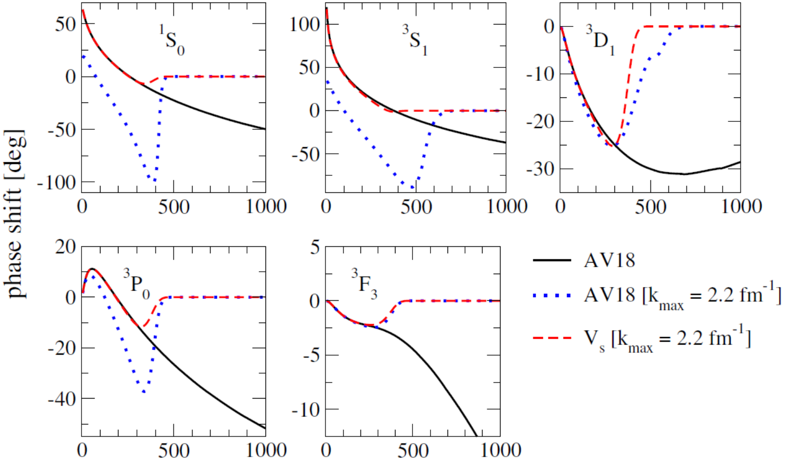
# Bare vs. SRG phase shifts

- Several partial waves



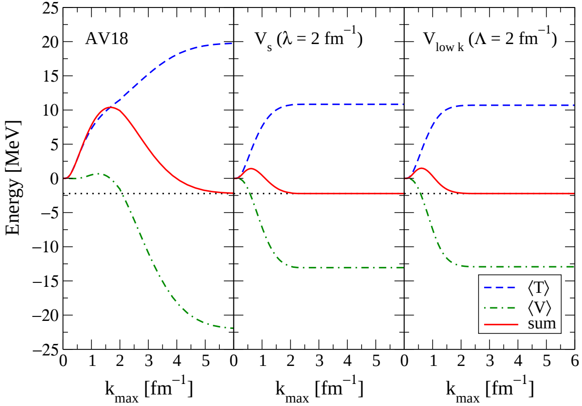
# SRG decouples high momenta from low-energy observables

- Phase shifts with $V_s(k,k') = 0$ for $k, k' > k_{max}$



# The strength of observables shifts considerably

- Distribution of kinetic and potential energies in the deuteron



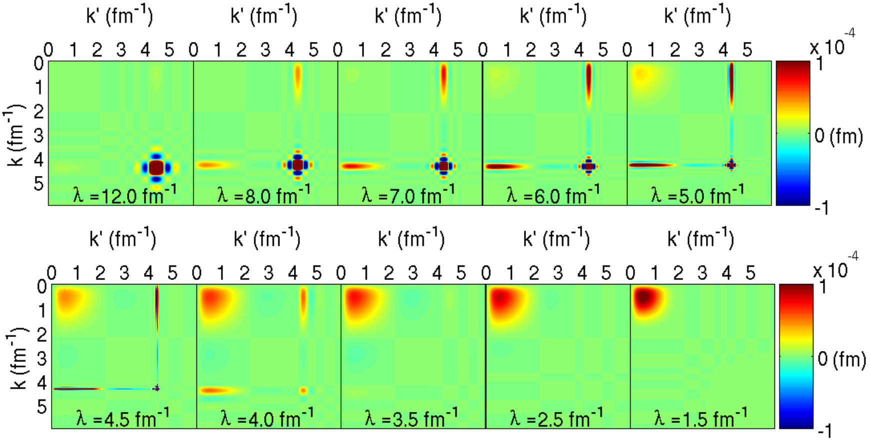
# Every Observable Flows

- The evolution with $s$ of any other operator $O$ is given by the same unitary transformation, $O_s = U(s) O U^\dagger(s)$, which means that $O_s$ evolves according to

$$\frac{dO_s}{ds} = [\eta(s), O_s] = [[T_{\rm rel}, V_s], O_s] \, .$$

- Consider the evolution of $< \psi_d | a^\dagger(q) a(q) | \psi_d >$ .
- The integral does not change, but the strength flows.
- Look at $< \psi_d(s) | k >< k | U_s | q >< q | U^\dagger | k' >< k' | \psi_d >$ .

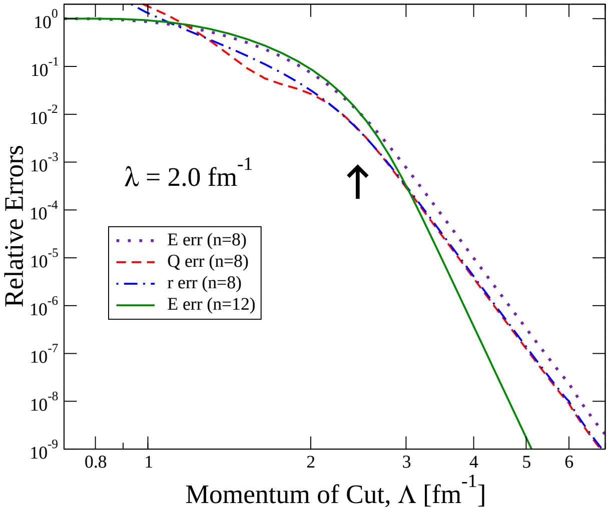
# Flow of strength from $a^\dagger a$



# Deuteron Observables

Deuteron Observables

- Binding Energy
- Quadrupole Moment
- RMS radius



# Soft Potentials in History

- There were active attempts to transform away hard cores and soften the tensor interaction in the late sixties and early seventies.
- But the requiem for soft potentials was given by Bethe:

  > *"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."*

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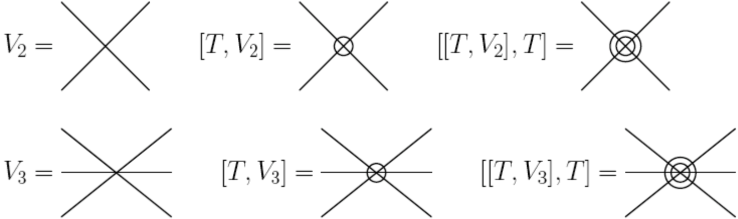
- But that story is not complete: three-nucleon forces (3NF)!

# Three-Body Interactions in the SRG

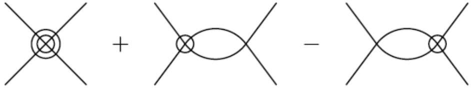
- Same SRG equation: $\frac{dH}{ds} = [[T, H], H]$
- $H = T + V_2 + V_3 + \cdots$
- $V_2 = \frac{1}{2!} a_i^\dagger a_j^\dagger (ij|V_2|kl) a_l a_k$
- $V_3 = \frac{1}{3!} a_i^\dagger a_j^\dagger a_k^\dagger (ijk|V_3|lmn) a_n a_m a_l$



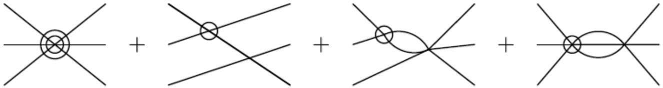
# Fock space SRG equation

- $\frac{dV_2(s)}{ds} + \frac{dV_2(s)}{ds} + \cdots =$
  $[[T, V_2(s) + V_3(s)], T] + [[T, V_2(s) + V_3(s)], V_2(s) + V_3(s)] + \cdots$

$\frac{dV_2}{ds} =$



$\frac{dV_3}{ds} =$



- 2-body and 3-body problems separate

# Summary

- SRG provides a powerful new RG tool
- Produces well-defined RG-improved perturbation theory
- Provides new exact RG calculations
- SRG universality provides well-defined nuclear interactions
- SRG $\rightarrow$ soft NN force & small compensating 3N force

1. Objective: Precise Strong Interaction Calculations at all Scales
2. Similarity Renormalization Group
3. Selecting the right degrees of freedom
4. 3-Body
5. Summary