The QCD phase diagram

with RG methods

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INT Seattle, February 25th 2010

Outline

- QCD phase diagram
- Quark confinement & chiral symmetry breaking
- Chiral phase structure at finite density
- Summary and outlook

QCD phase diagram

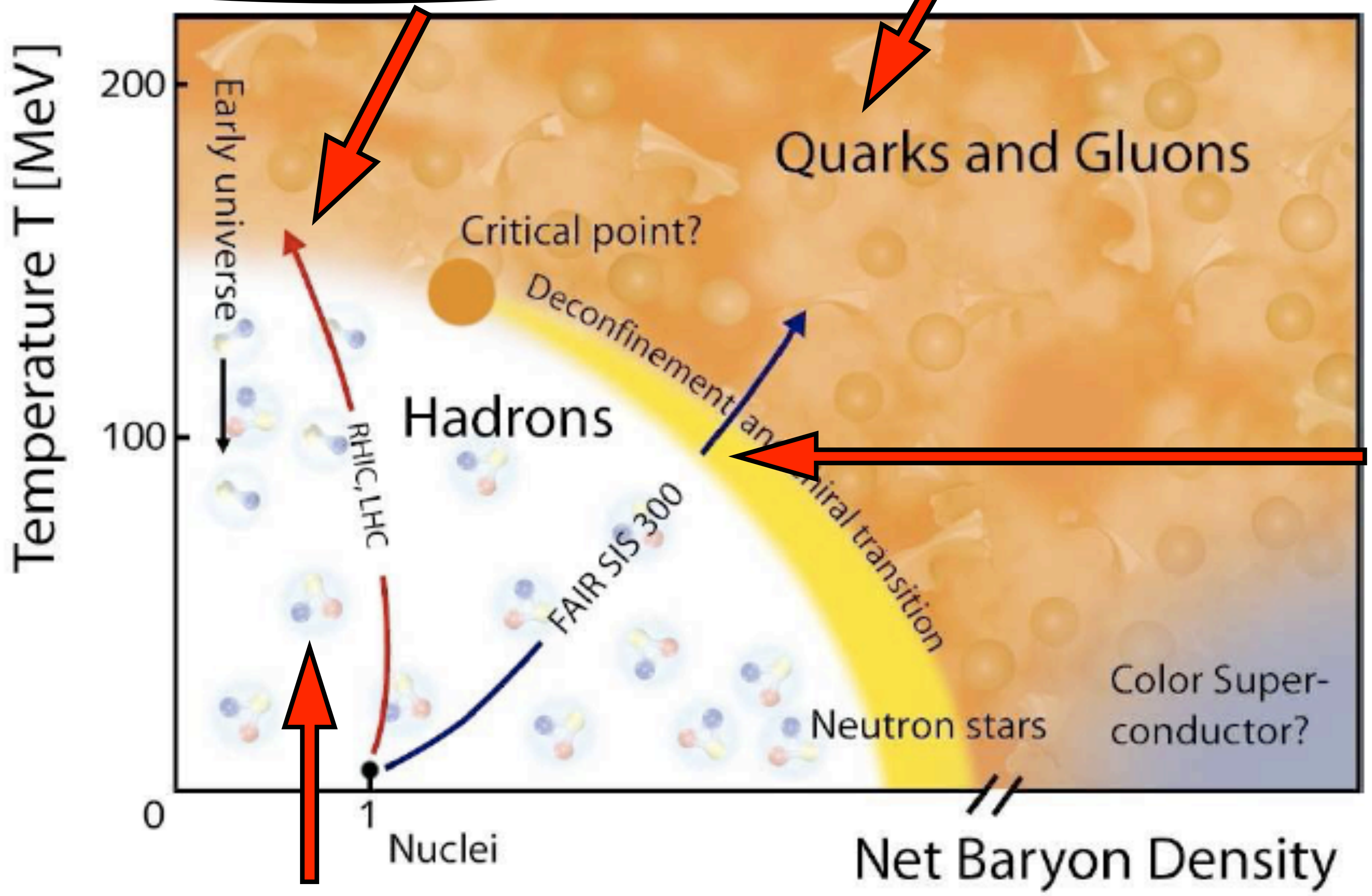
Phase diagram of QCD

Strongly correlated quark-gluon-plasma

'RHIC serves the perfect fluid'

massless quarks (chiral symmetry)

deconfinement



FAIR, www.gsi.de

quarkyonic:

confinement & chiral symmetry?

hadronic phase

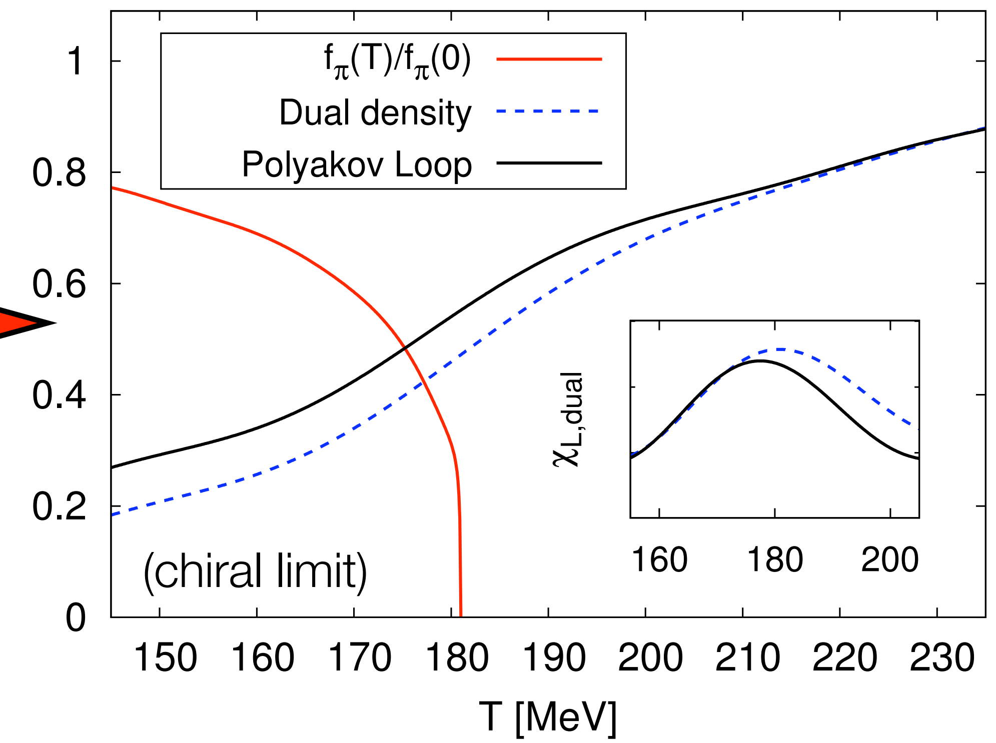
confinement & chiral symmetry breaking

Phase diagram of two flavour QCD

Continuum methods

RG-flows in QCD

Braun, Haas, Marhauser, JMP '09

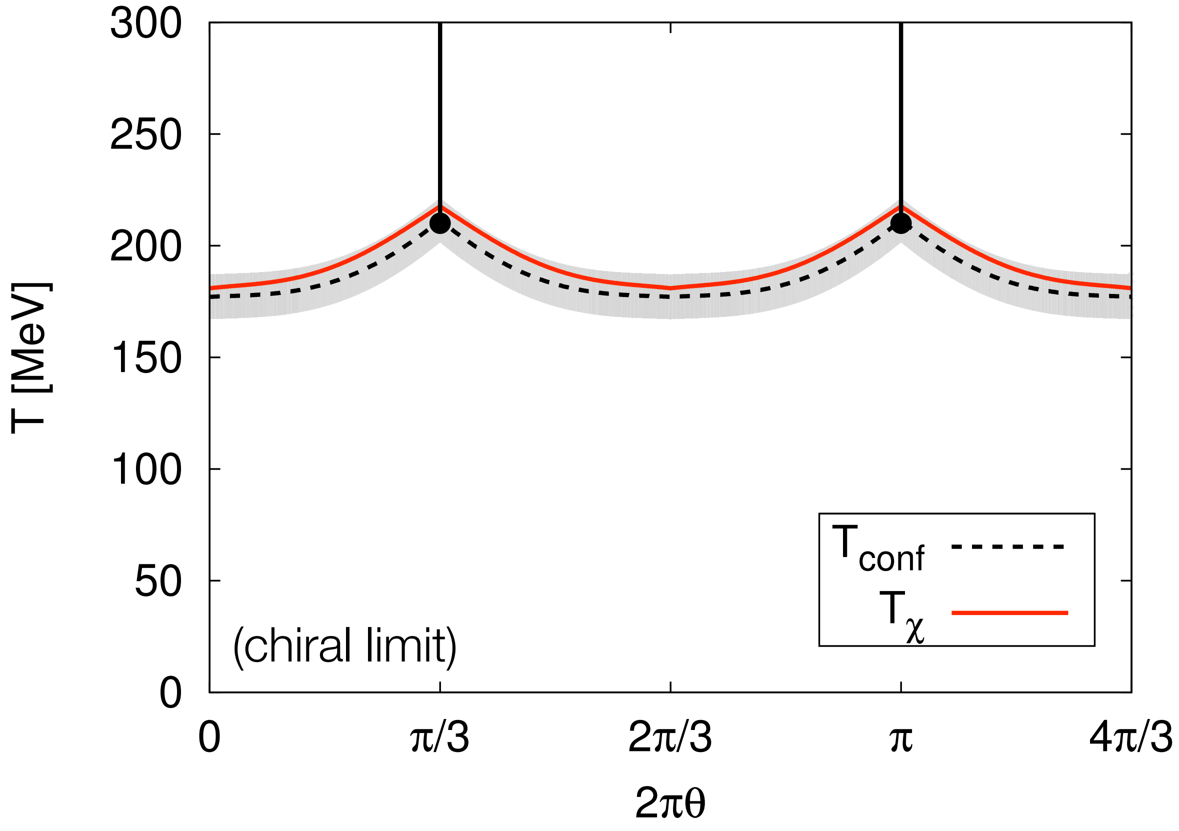


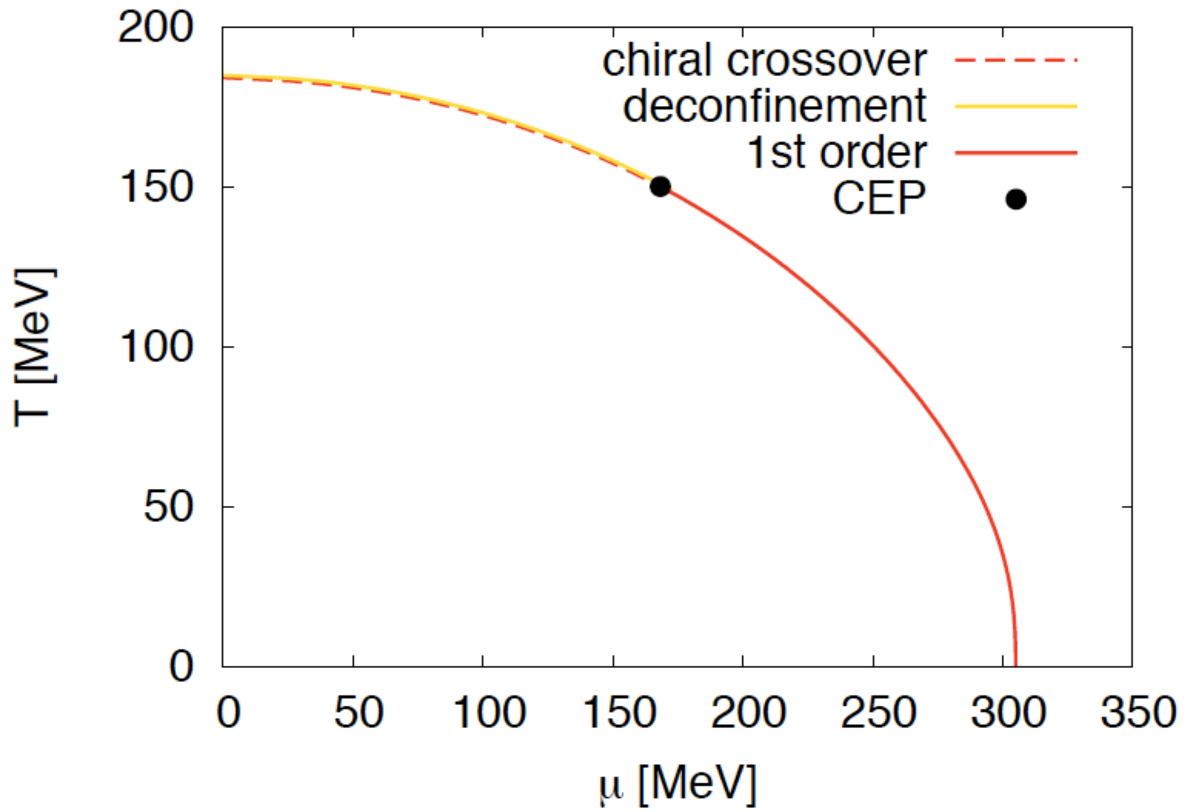
PNJL & PQM model

Fukushima '03
Ratti, Thaler, Weise '06

Back-coupling of matter fluctuations to YM-sector

Schaefer, JMP, Wambach '07





Phase diagram of two flavour QCD

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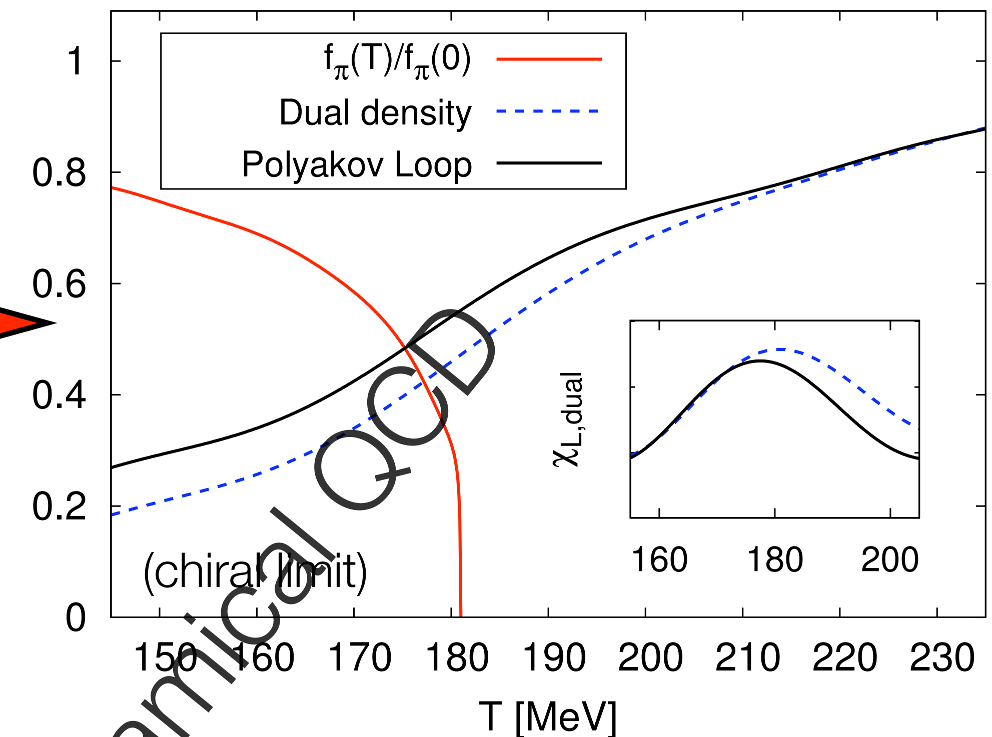

Full dynamical QCD

PNJL & PQM model

Fukushima '03
Ratti, Thaler, Weise '06

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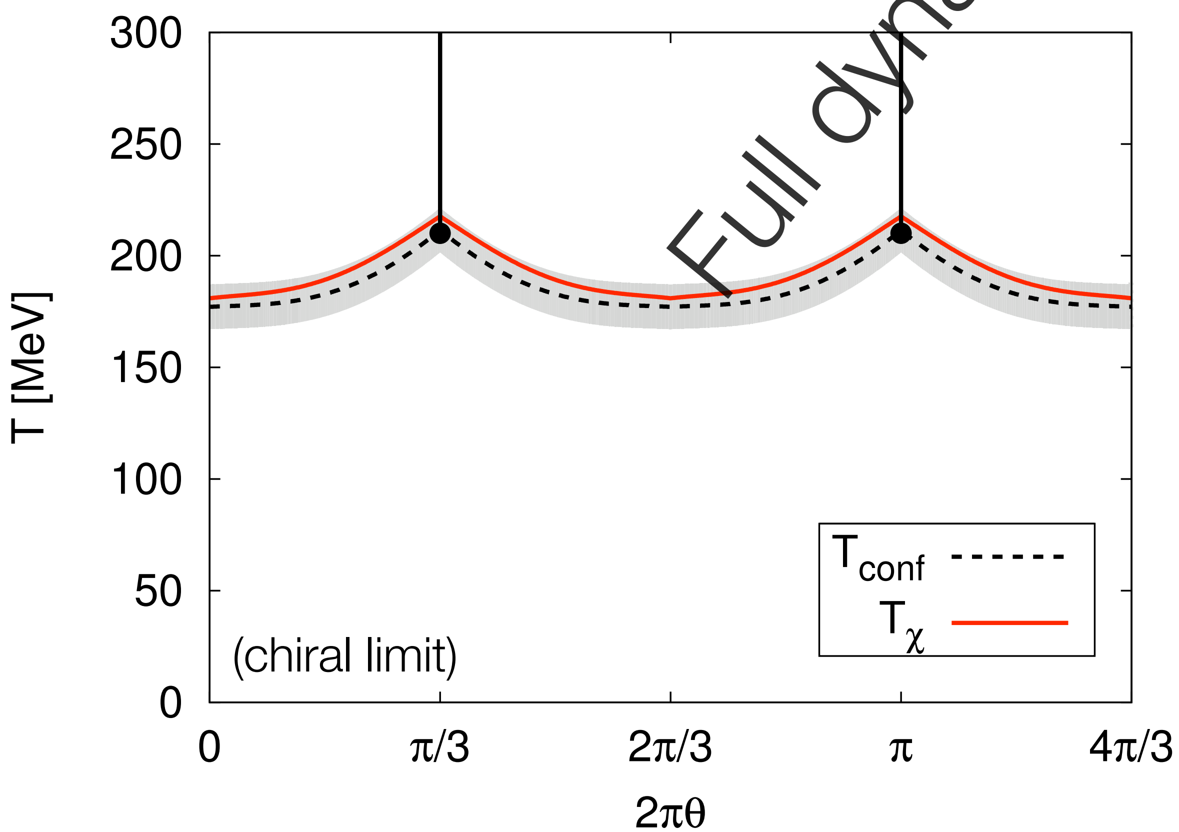
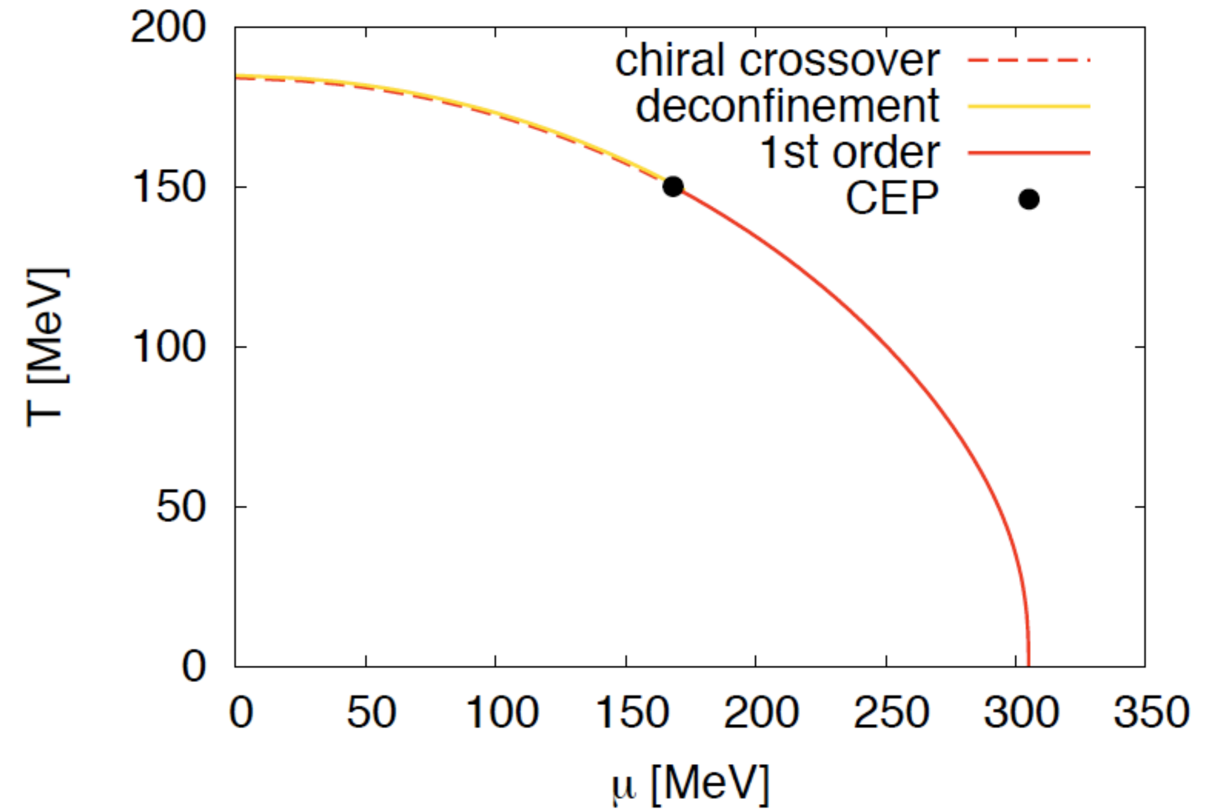
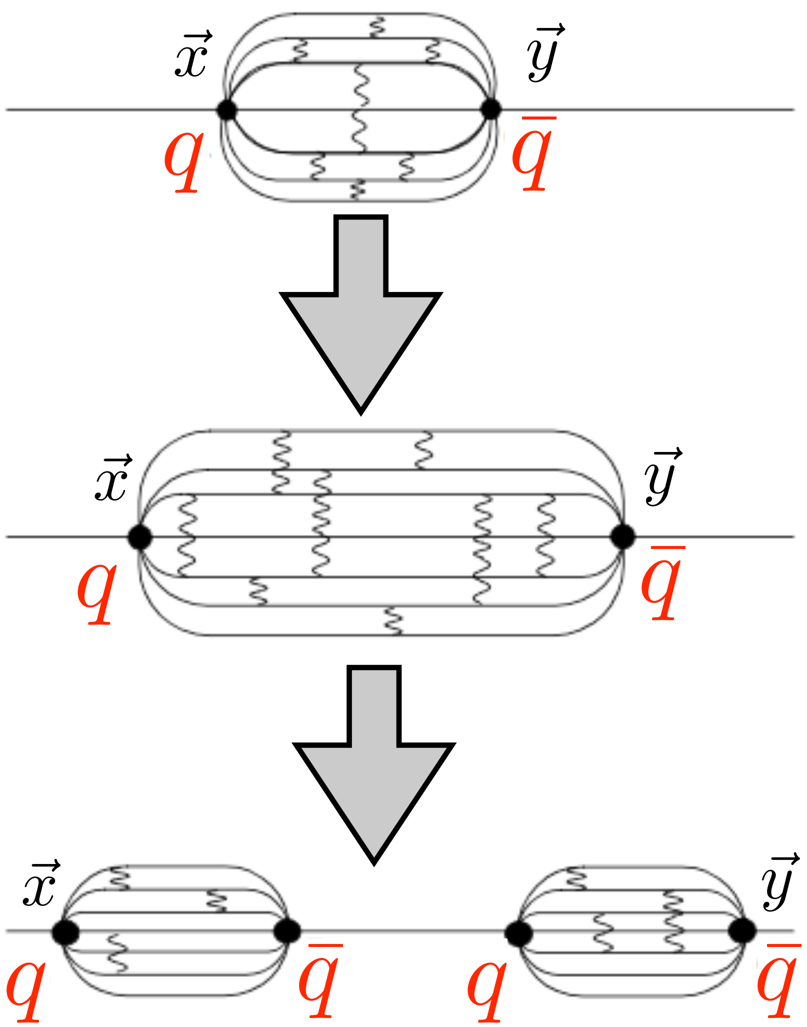




Quark confinement & chiral symmetry breaking

Confinement

$$r = |\vec{x} - \vec{y}|$$



Order parameter $\sim \, '\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

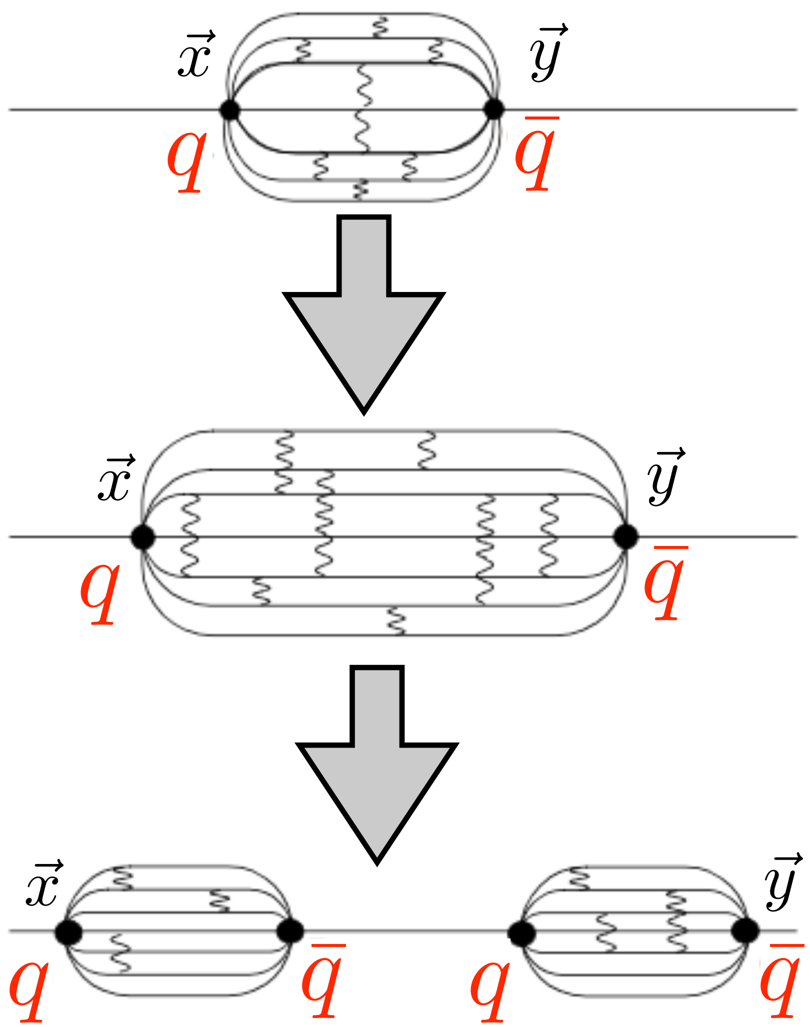
- Confinement: $\Phi = 0$
- Deconfinement: $\Phi \neq 0$

Φ Polyakov loop

$$\Phi = \frac{1}{3}\langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\}\rangle$$

Confinement

$$r = |\vec{x} - \vec{y}|$$



string breaking at $r \approx 1.1 fm$

Order parameter $\sim$ $'\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

- Confinement: $\Phi = 0$
- Deconfinement: $\Phi \neq 0$

Symmetry

- Z_3 - symmetry: $q \rightarrow zq$
- broken by dynamical quarks

Chiral symmetry breaking

chiral symmetry

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4)\times10^3$	

chiral symmetry breaking: $\Delta m \approx 400 MeV$

2 light flavours, one heavy flavour $2+1$

chiral symmetry breaking

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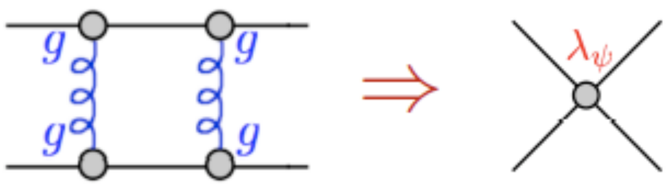

$\propto \alpha_s^2$

$$\int d^4x \, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$

$$\langle \bar{q}q \rangle \neq 0$$

mass term: $\langle \bar{q}q \rangle \, \bar{q}q$

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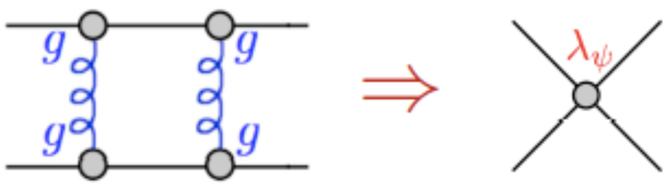

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$\langle \bar{q}q \rangle \neq 0$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$

chiral condensate

- chiral symmetry: $\sigma = 0$
- symmetry breaking: $\sigma \neq 0$

Symmetry

- $SU_L(N_f) \times SU_R(N_f)$
- broken to $SU(N_f)$

Functional RG

Functional RG

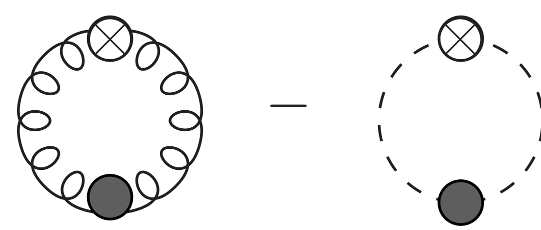
- Introduction to Functional RG flows & some results in QCD (talks & lit)
    - Integrals from differential equations: The FRG-idea in 0+0-dimensions
    - Confinement & chiral symmetry breaking from Functional Methods
    - On the phase diagram of QCD
    - Aspects of the Functional RG

Functional RG

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\;k\partial_k R_k(p)$$

- Yang Mills Theory: $\phi = (A, C, \bar{C})$ RG-scale k: $t = \ln k$

$$\partial_t\Gamma_k[\phi] \;=\; \tfrac{1}{2}$$



- Fermions are straightforward though ‘physically’ complicated

    - no sign problem numerics as in scalar theories
    - chiral fermions reminder: Ginsparg-Wilson fermions from RG arguments
    - bound states via dynamical hadronisation effective field theory techniques applicable

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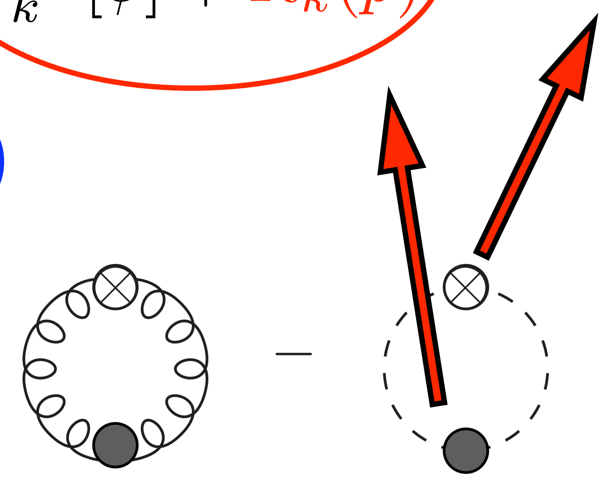

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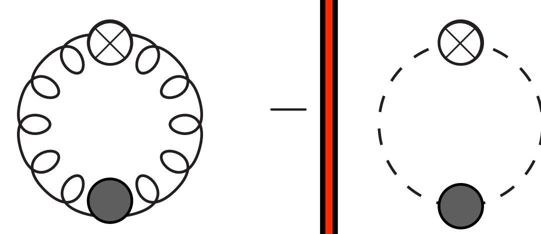
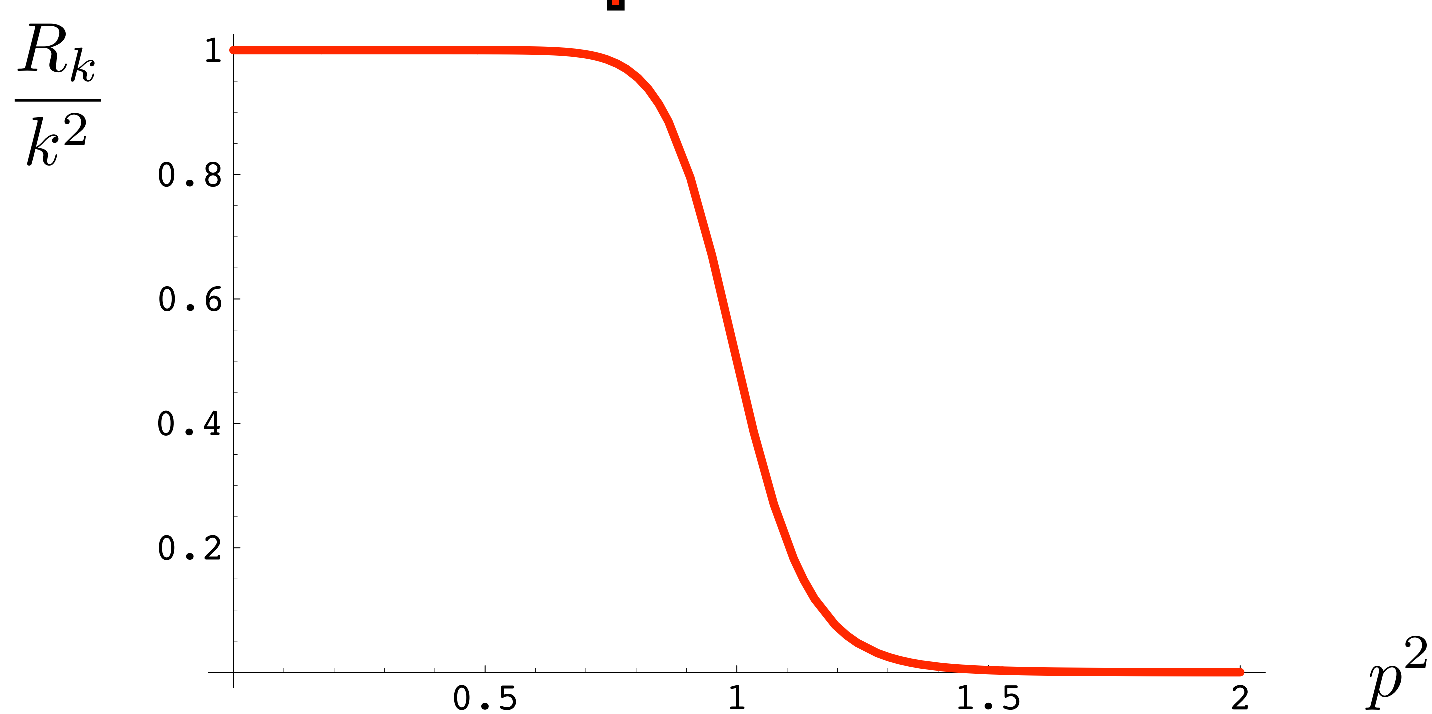
RG-scale k: $t = \ln k$

$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}\;\; [\text{gluon loop}] \;-\; [\text{ghost loop}]$$



- Flow infrared finite



Functional RG

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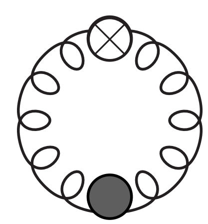
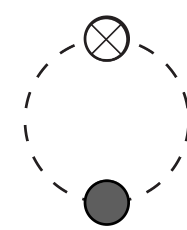
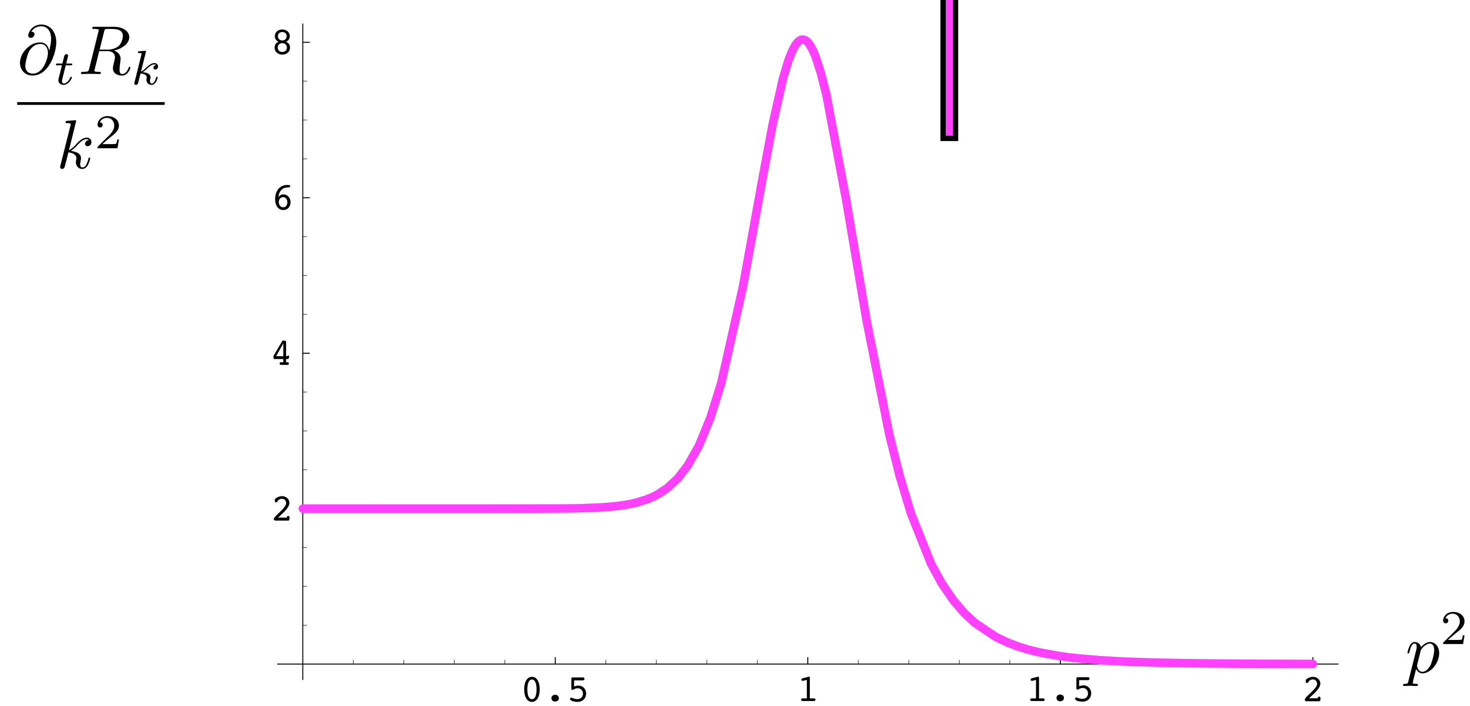
$\partial_t \Gamma_k[\phi] \; = \; \frac{1}{2}$  $-$ 

- Flow ultraviolet finite

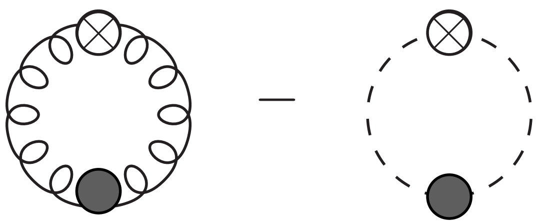


Functional RG

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\,k\partial_k R_k(p)$$

- Yang Mills Theory: $\phi = (A, C, \bar{C})$ RG-scale k: $t = \ln k$

$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}$$



- Perturbation theory

$$\partial_t \Gamma_k[\phi] = \partial_t\,\frac{1}{2}\mathrm{Tr}\ln\left(S_{\mathrm{cl}}^{(2)}[\phi] + R_k(p)\right)$$

Functional RG

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\; k\partial_k R_k(p)$$

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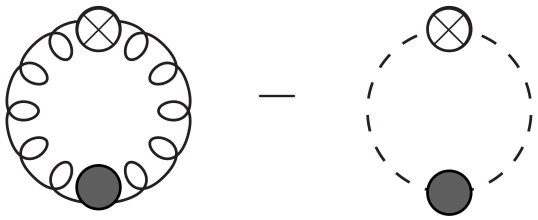
$\partial_t \Gamma_k[\phi] \;=\; \frac{1}{2}$ 

- Full flow

$$\partial_t \Gamma_k[\phi] = \partial_t \frac{1}{2}\mathrm{Tr}\ln\left(\Gamma_k^{(2)}[\phi] + R_k(p)\right) \;+\; \partial_t \Gamma_k^{(2)}[\phi] - \text{terms}$$

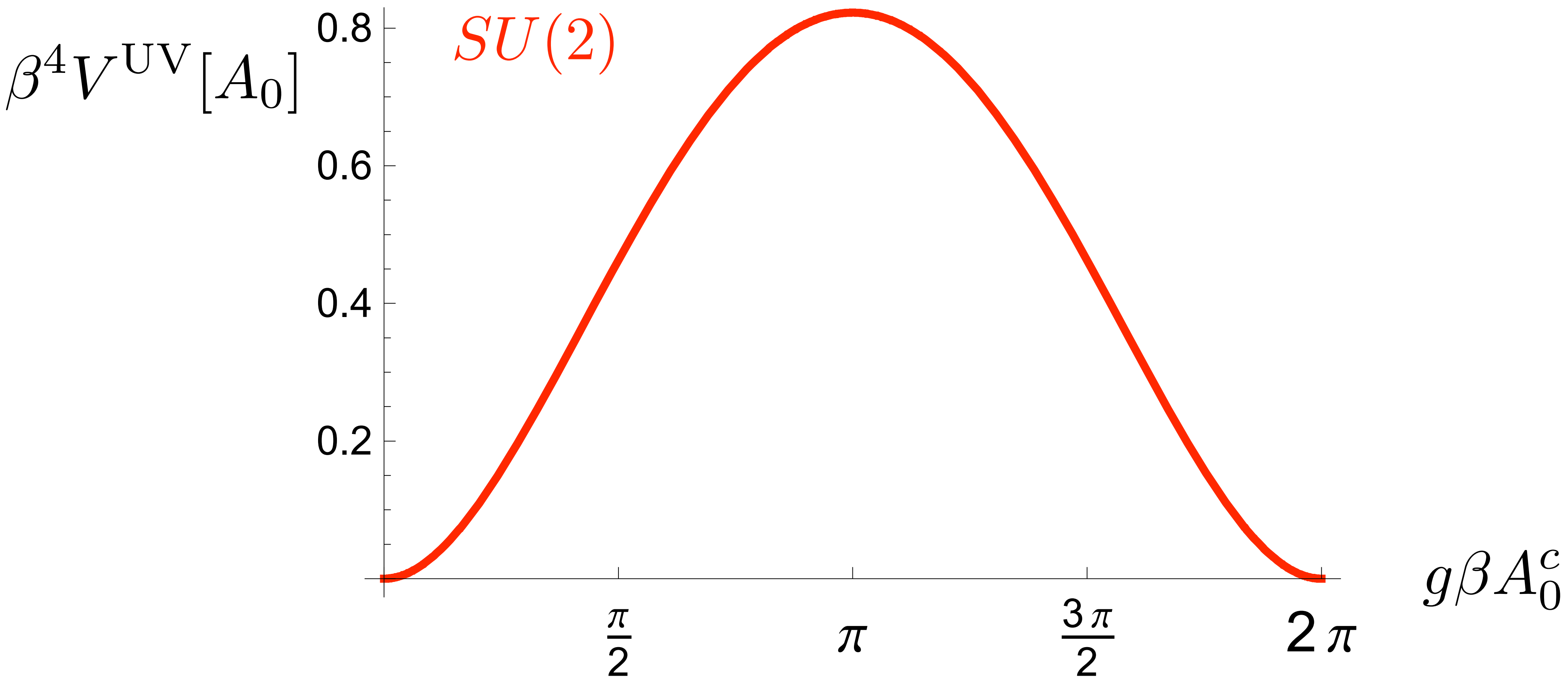
RG-improvement

Confinement

Confinement

Perturbation theory

$$V^{\mathrm{UV}}[A_0] = \frac{1}{2\Omega}\,\mathrm{Tr}\ln S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}\,\mathrm{Tr}\ln S^{(2)}_{C\bar{C}}[A_0]$$



$$SU(2):\ \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

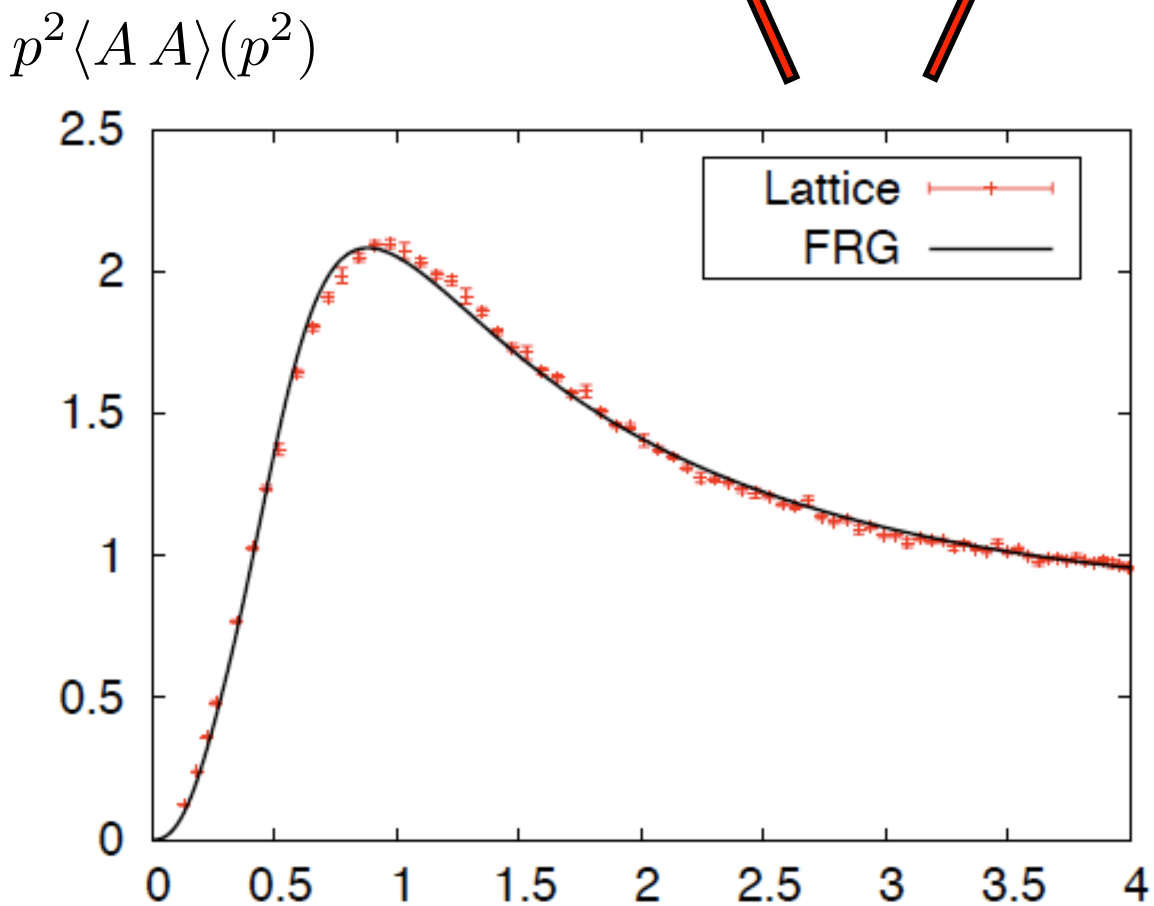
Confinement

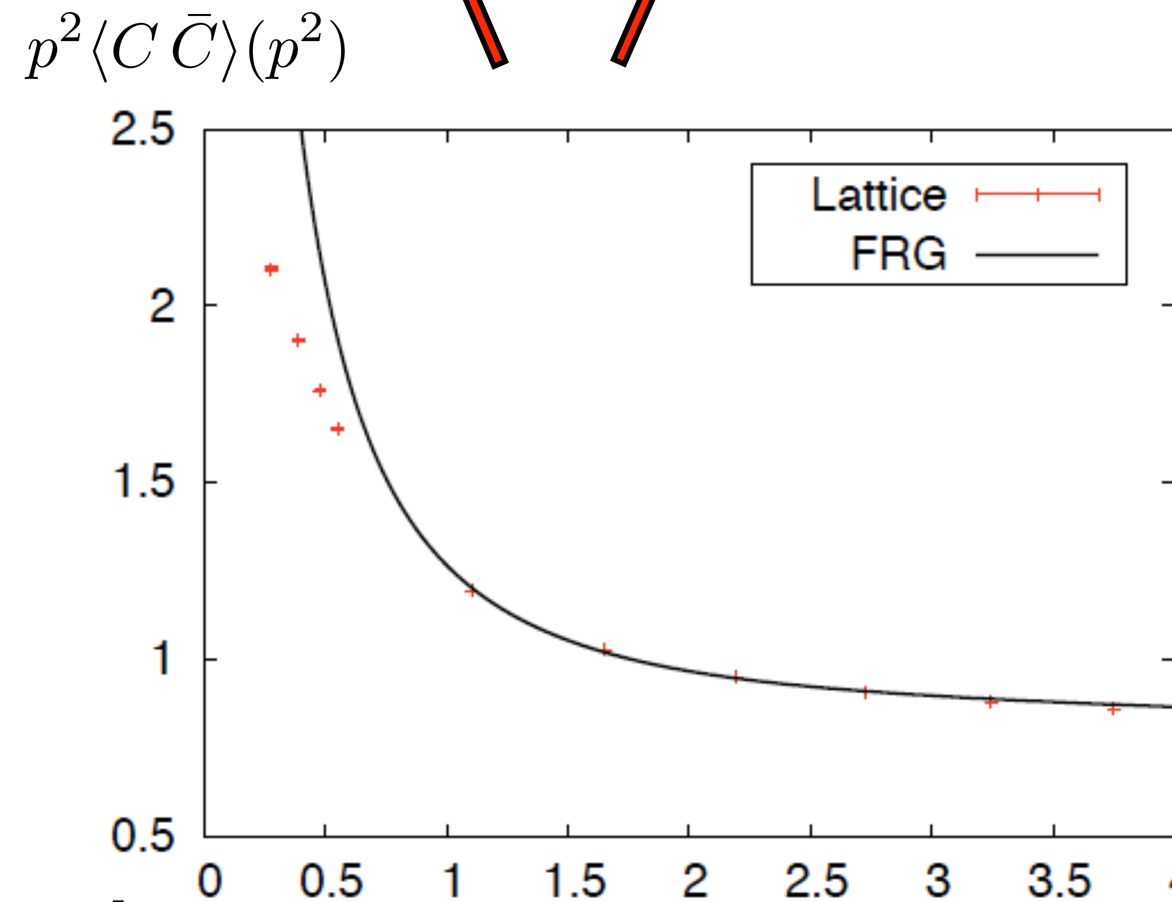
Continuum methods ⟸ (Functional RG-flows)

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) - \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle) + O(V''[A_0])$$

$$p_0 \to 2\pi T n - g A_0$$





p [GeV]

Fischer, Maas, JMP '08

JMP, in preparation

Confinement

Continuum methods

Braun, Gies, JMP '07

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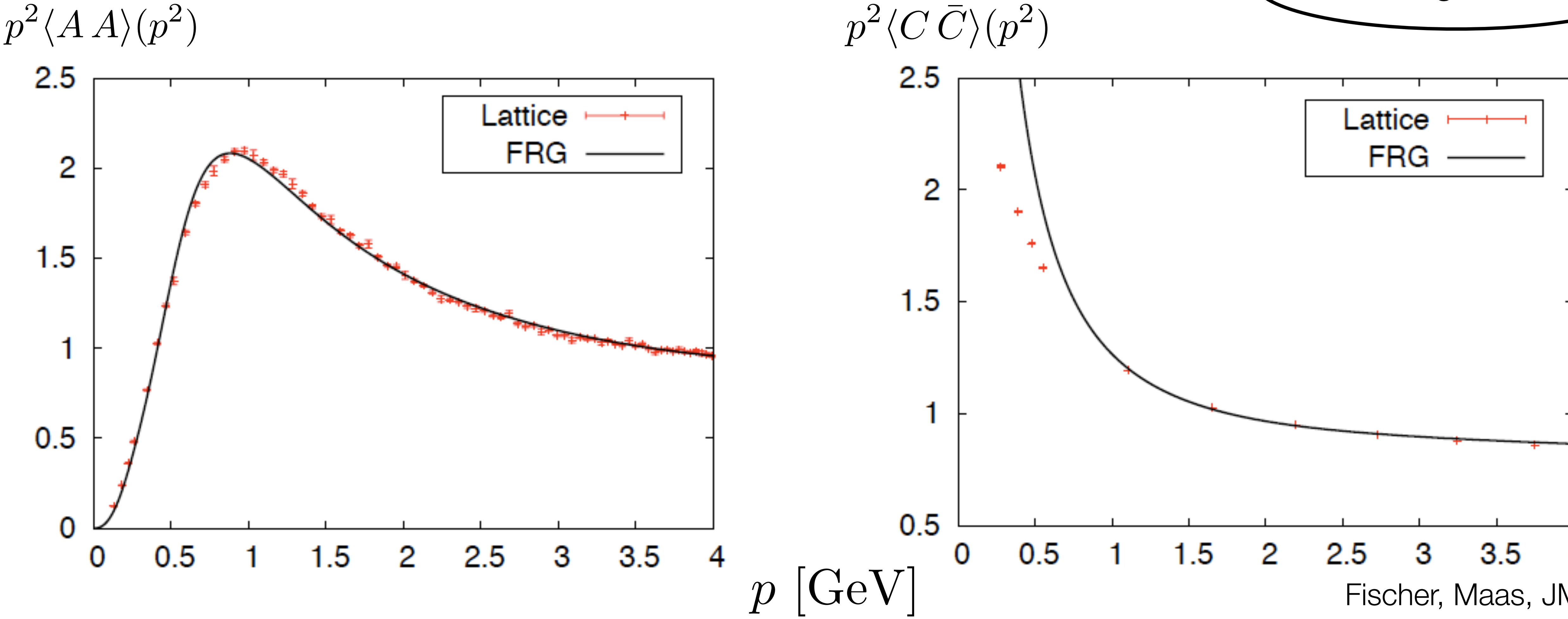
'Polyakov loop potential'

subleading for $T_{c,\mathrm{conf}}$

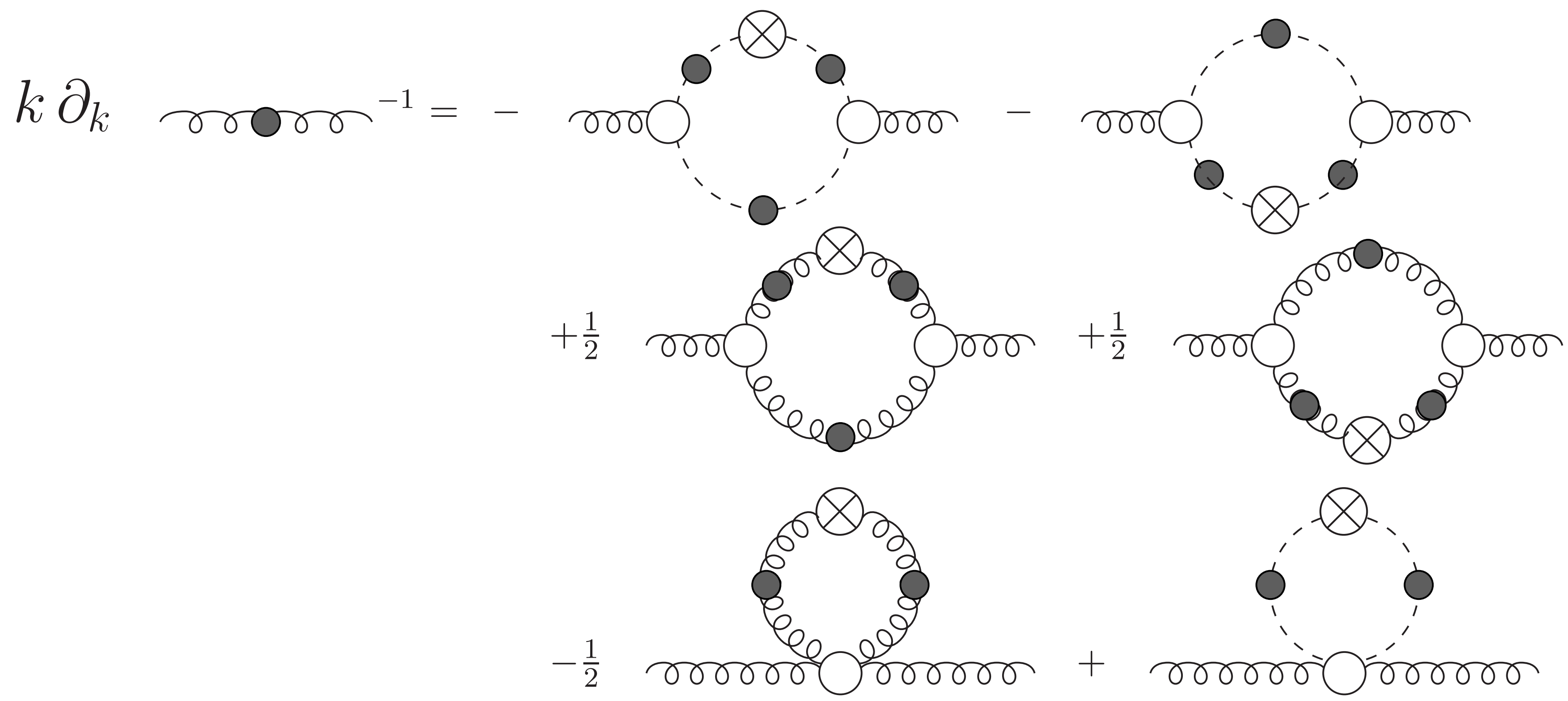


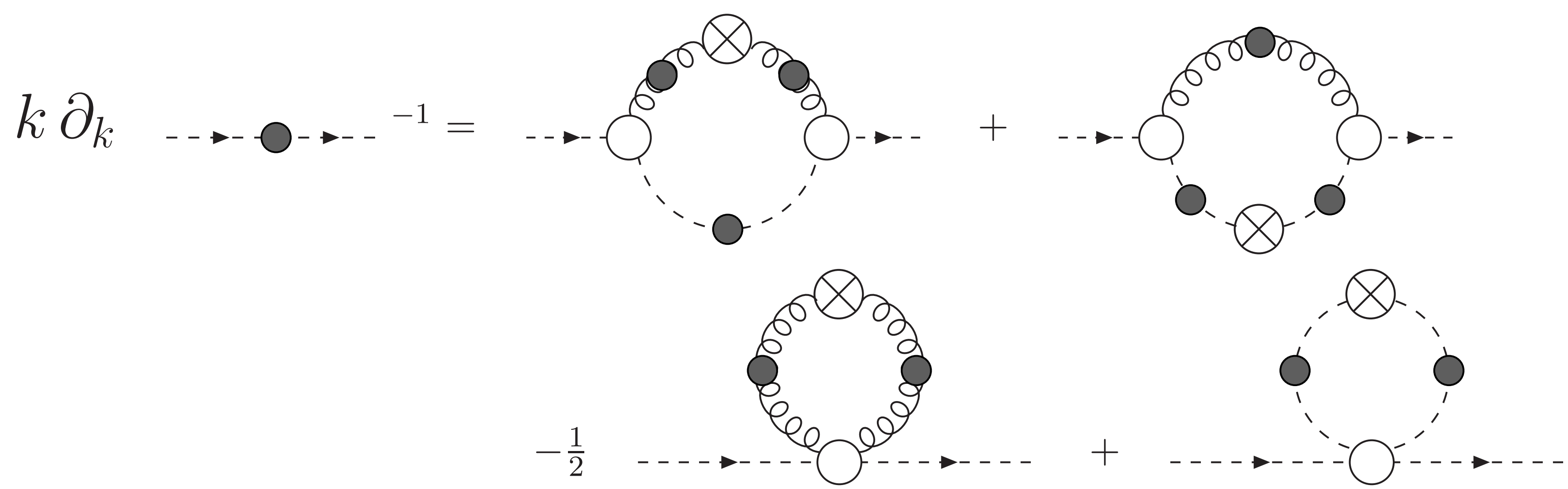
Fischer, Maas, JMP '08

JMP, in preparation

Confinement

Computation of propagators





Confinement

Computation of propagators

- full momentum dependence of propagators
- vertices momentum-dependent RG-dressing
- functional optimisation JMP'05
- functional relations between diagrams: Flow=Flow(DSE)

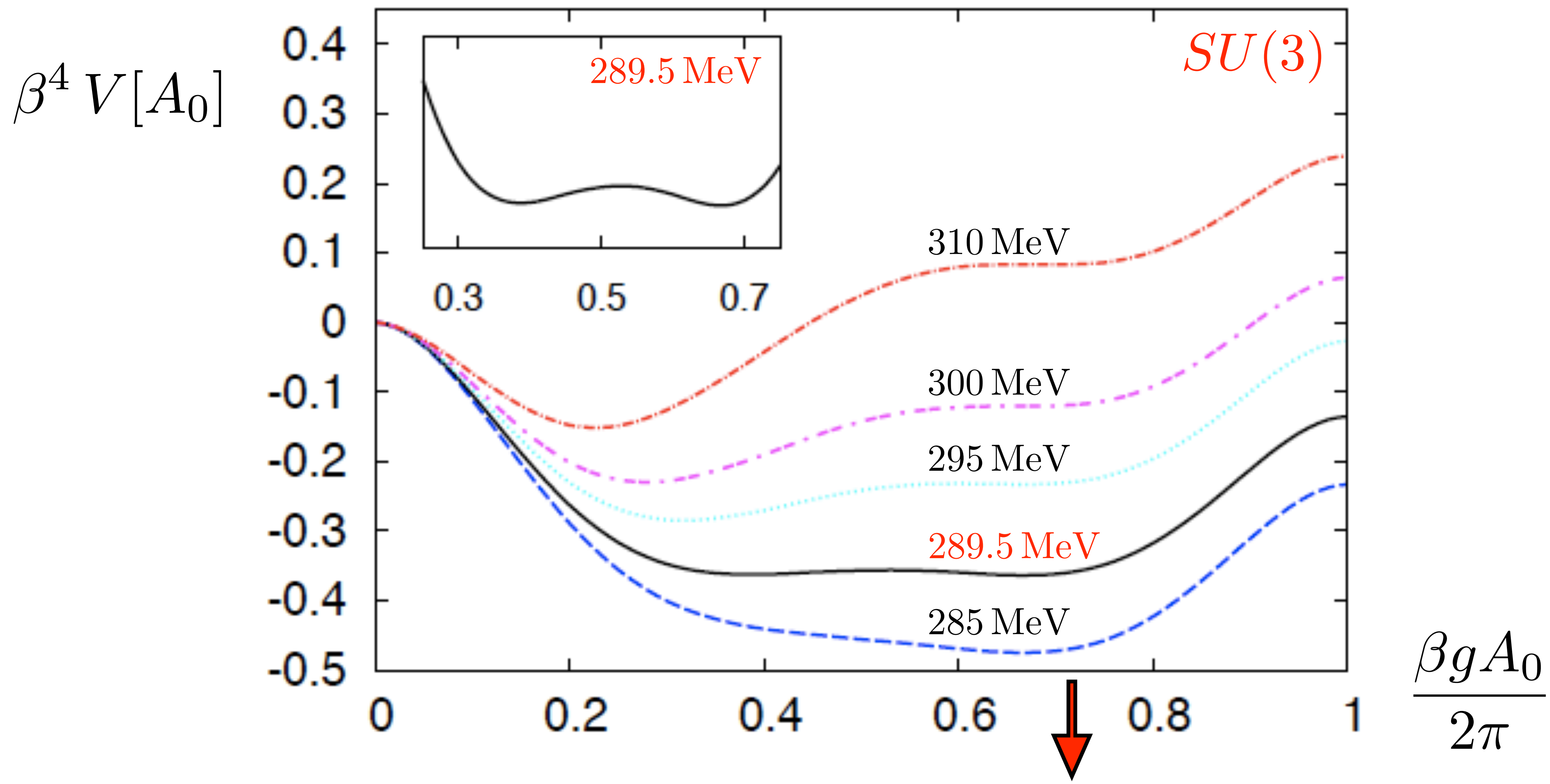
$$\Rightarrow k\partial_k \langle A(p)\, A(-p)\rangle = \mathrm{Flow}_A[\langle A\, A\rangle\,,\,\langle C\bar{C}\rangle]$$

$$k\partial_k \langle C(p)\, \bar{C}(-p)\rangle = \mathrm{Flow}_C[\langle A\, A\rangle\,,\,\langle C\bar{C}\rangle]$$

- scaling/decoupling via boundary conditions at $p^2 = 0$

Confinement

$T_c = 289.5 \pm 10\,\mathrm{MeV}$ $\quad T_c/\sqrt{\sigma} = 0.658 \pm 0.023$ $\quad \mathrm{lattice}: T_c/\sqrt{\sigma} = 0.646$



$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0) \longrightarrow \Phi[\frac{4}{3}\pi\frac{1}{\beta g}] = 0$$

Braun, Gies, JMP '07

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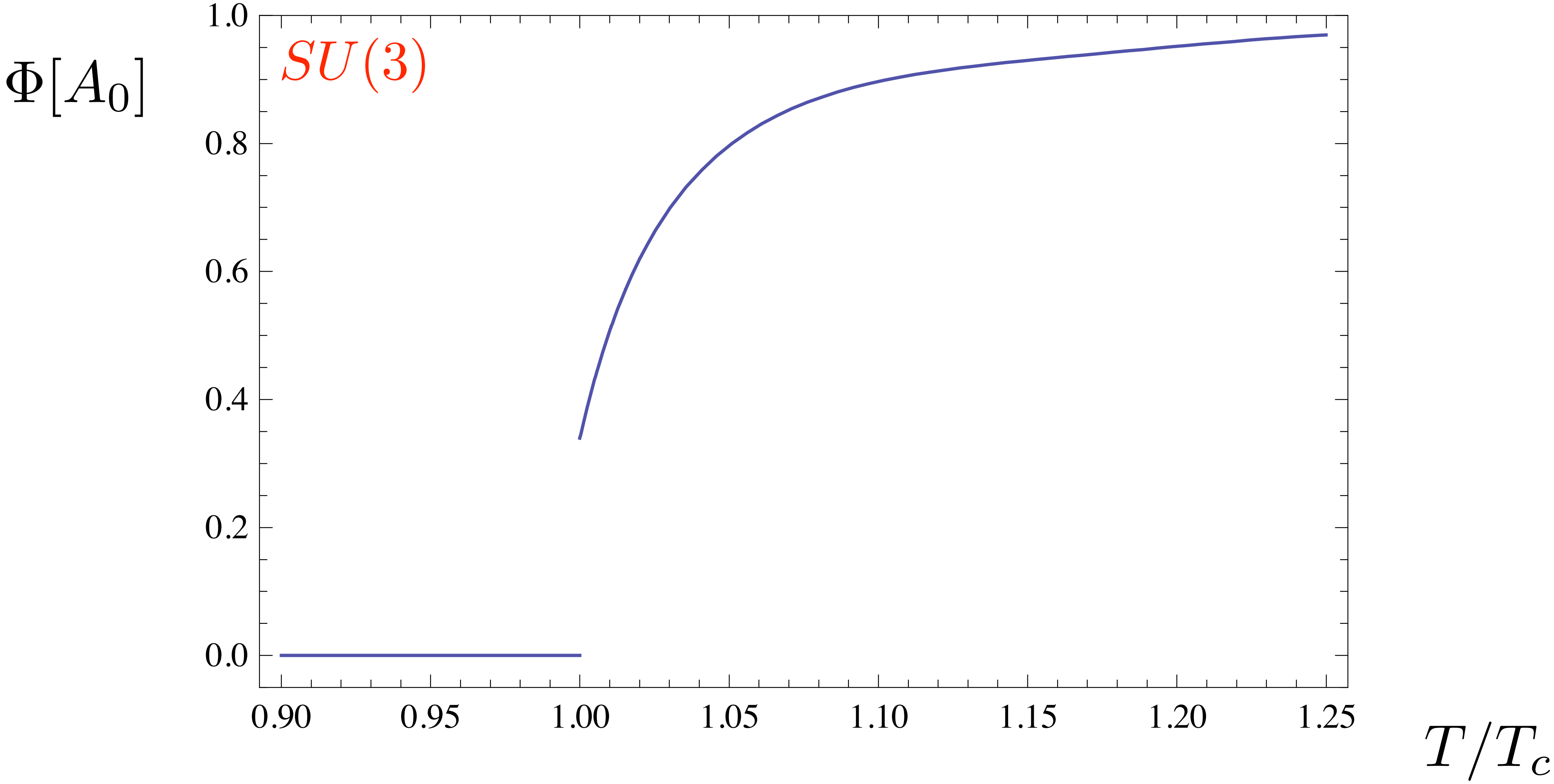


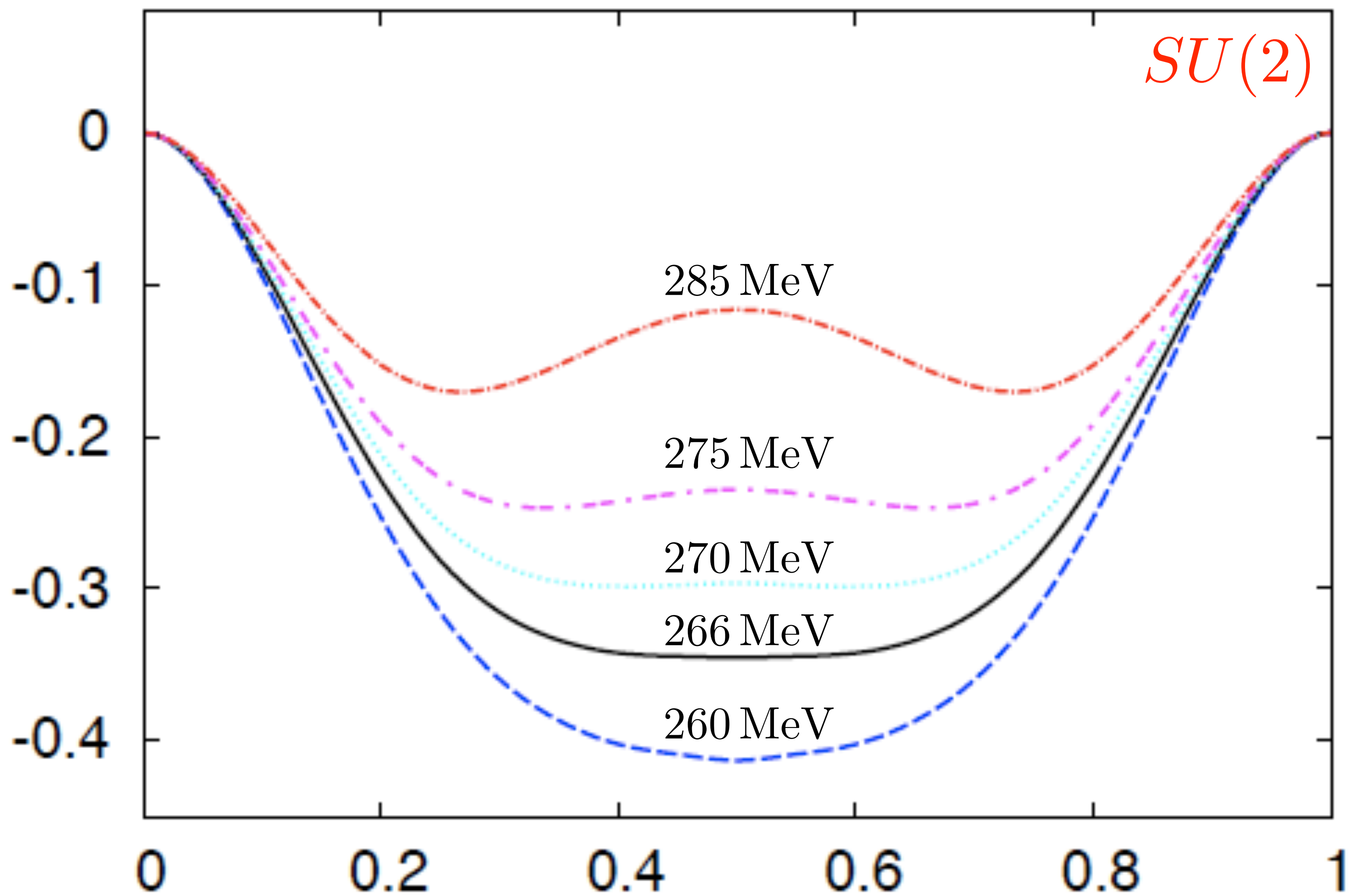
SU(N), G(2), Sp(2): Braun, Eichhorn, Gies, JMP, in preparation

Braun, Gies, JMP '07

Confinement

$T_c = 266 \pm 10\,\mathrm{MeV}$ $T_c/\sqrt{\sigma} = 0.605 \pm 0.023$ lattice : $T_c/\sqrt{\sigma} = 0.709$

$\beta^4\, V[A_0]$



$\frac{\beta g A_0}{2\pi}$

$\Phi[A_0] = \cos\frac{1}{2}\beta g A_0 \longrightarrow \Phi[\pi/(\beta g)] = 0$

Braun, Gies, JMP '07

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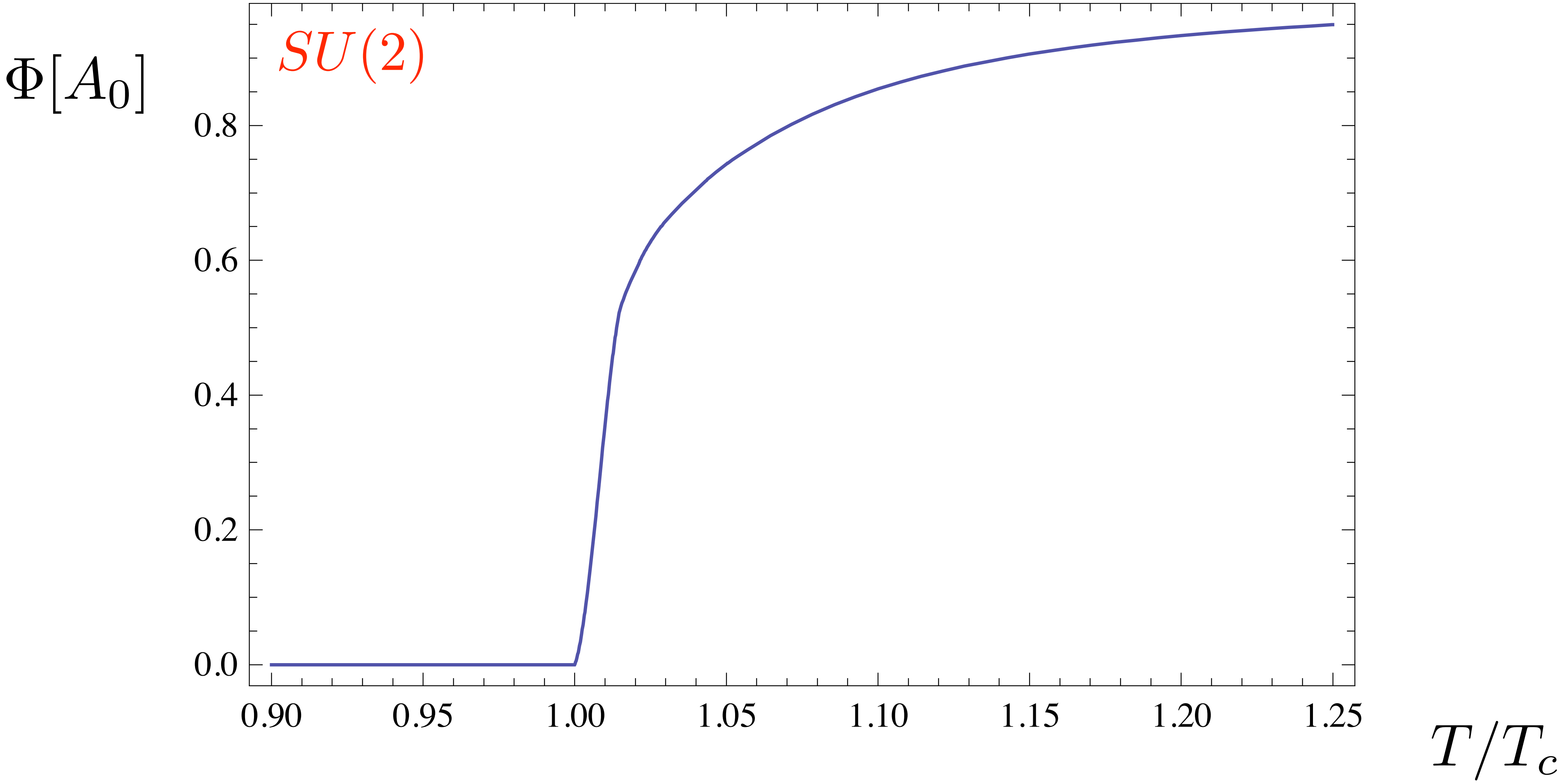

Braun, Gies, JMP ‘07

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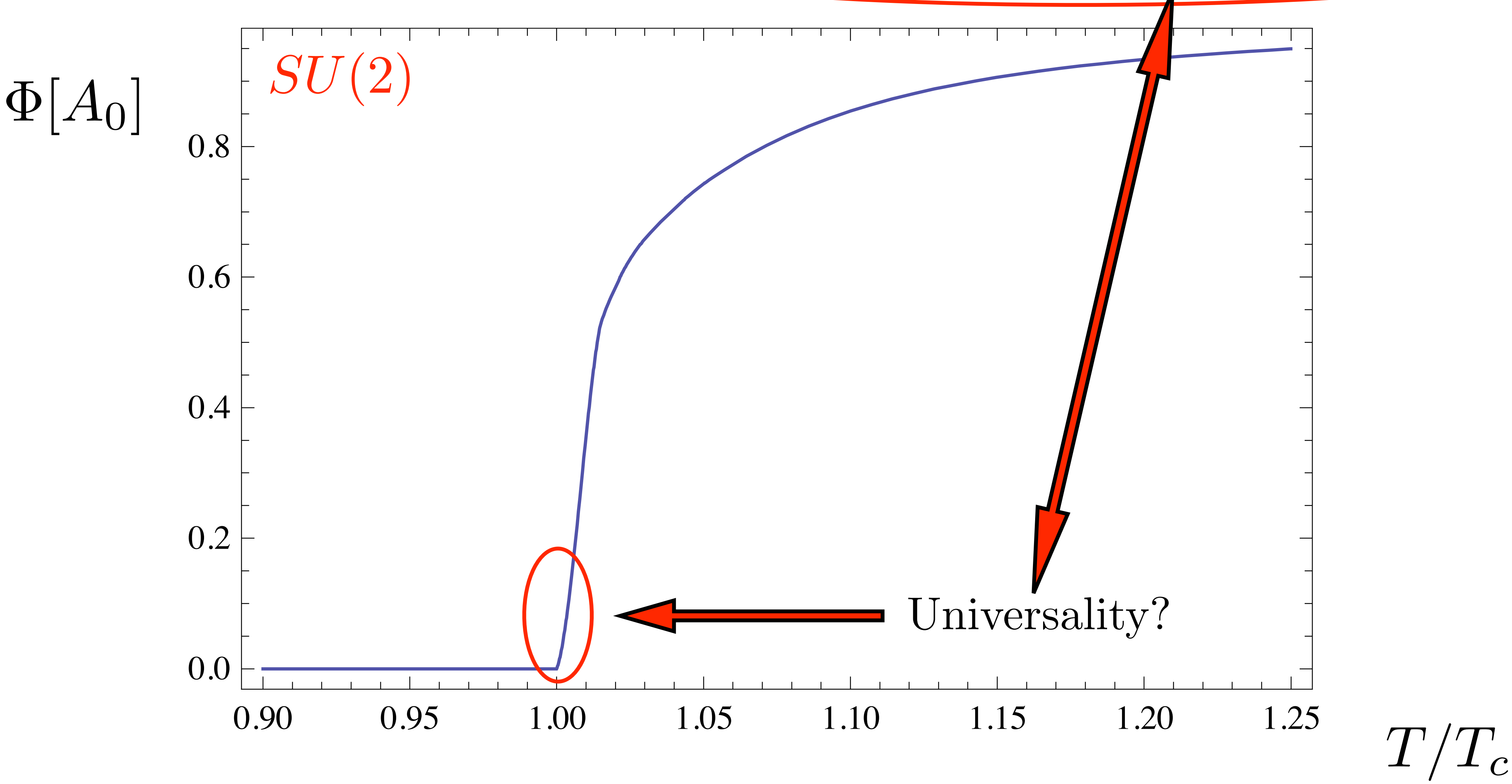
$V'' -$ terms

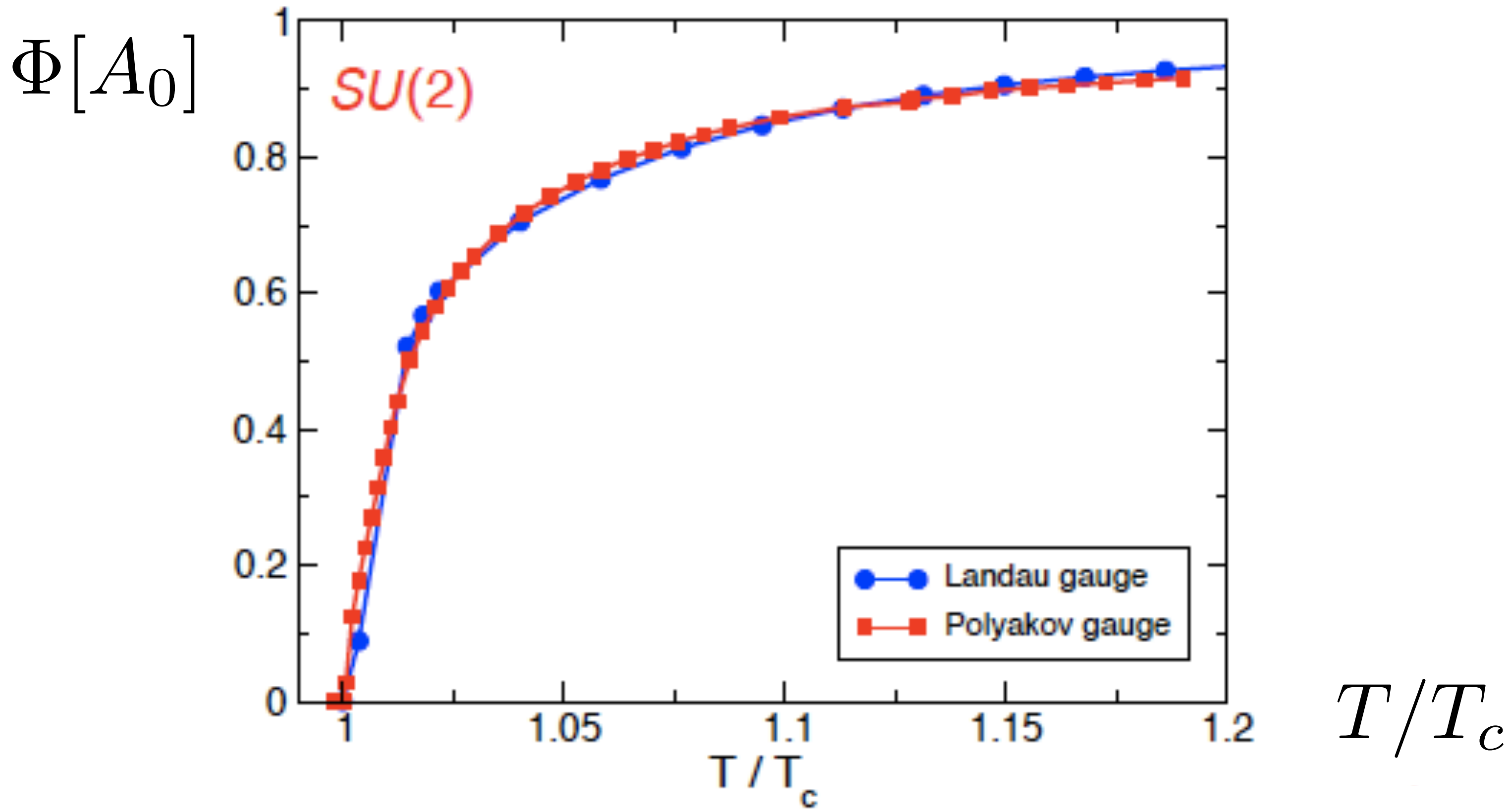


Braun, Gies, JMP '07

Universal properties & gauge independence

Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

$$\text{RG-flow}: \quad V[A_0] = -\int dt\, \text{flow}[V''[A_0]\,,\, \alpha_s]$$



- – –: Polyakov gauge: crit. exp. $\nu = 0.65$ $\qquad \nu_{\text{Ising}} = 0.63$
- – –: Landau gauge propagators

JMP, Marhauser ‘08

Universal properties & gauge independence

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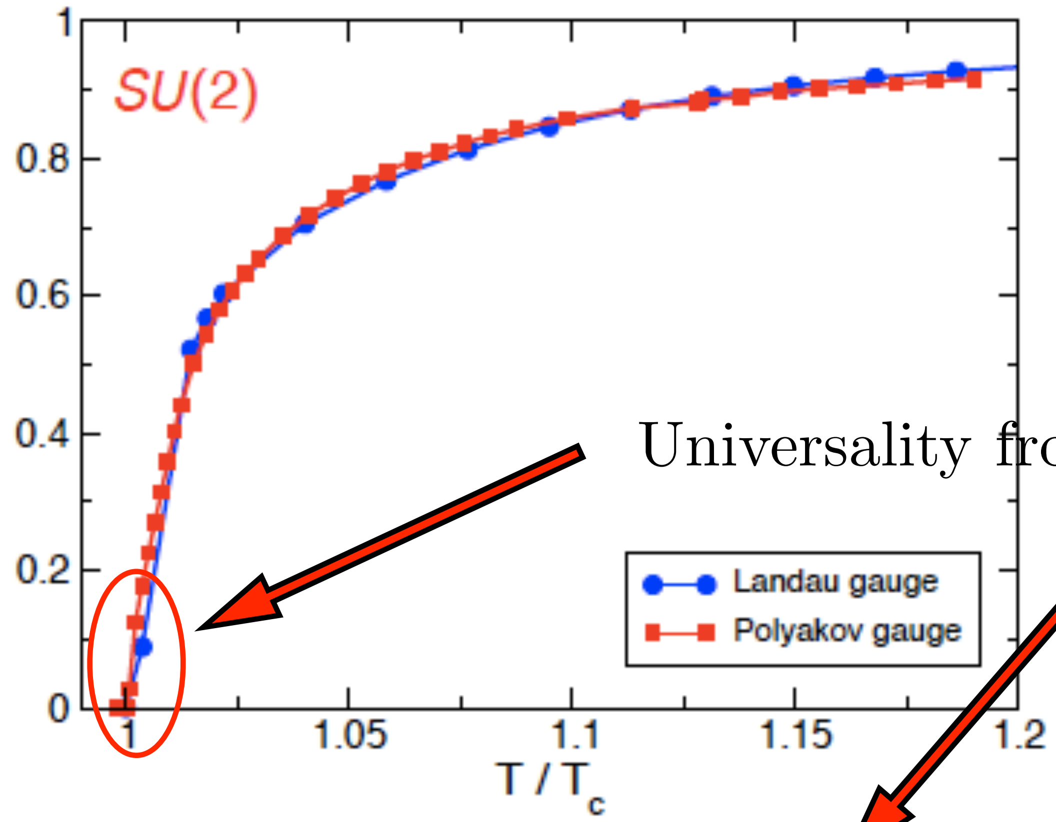
$\Phi[A_0]$



Universality from $V'' -$ terms

T/T_c

- – –: Polyakov gauge: crit. exp. $\nu = 0.65$ $\nu_{\text{Ising}} = 0.63$
- – –: Landau gauge propagators

JMP, Marhauser '08

Phase structure at vanishing density

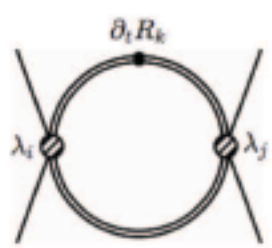
Chiral symmetry breaking

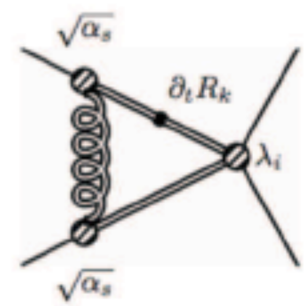
A glimpse at chiral symmetry breaking

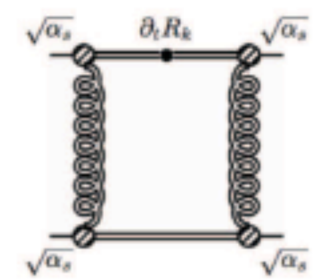
Flow of four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

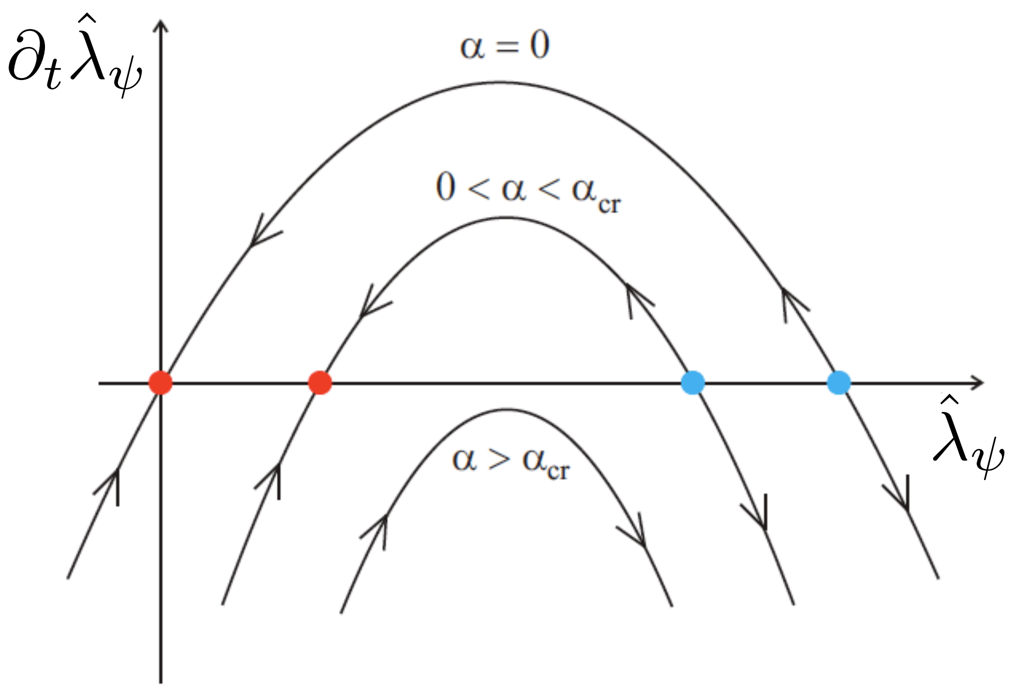
$$k\partial_k \hat{\lambda}_\psi \;=\; 2\hat{\lambda}_\psi \;-\; A(\tfrac{T}{k})\,\hat{\lambda}_\psi^2 \;-\; B(\tfrac{T}{k})\,\hat{\lambda}_\psi\,\alpha_s \;-\; C(\tfrac{T}{k})\,\alpha_s^2$$

$\Uparrow$ $\Uparrow$ $\Uparrow$





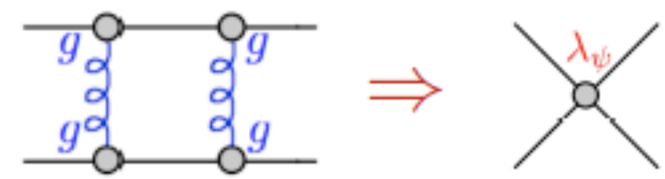




$m^2 \propto \frac{1}{\lambda}$ $\Longrightarrow$ $\alpha_s > \alpha_{s,crit}$: chiral symmetry breaking

Chiral symmetry breaking

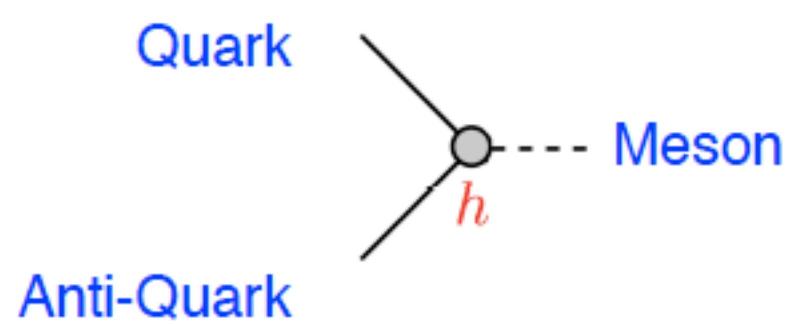
$$g\,\bar{\psi}\not{A}\,\psi \quad \xrightarrow{\int_p}$$



Hubbard-Stratonovitch

$$\lambda_\psi\,(\bar\psi\psi)^2 = h\,\bar\psi\psi\,\sigma - \tfrac{1}{2}m^2\sigma^2$$

with $m^2 = -\dfrac{h^2}{2\lambda_\psi}$ and EoM(σ) $\Longrightarrow$



+Baryons and Glueballs

+Baryonisation

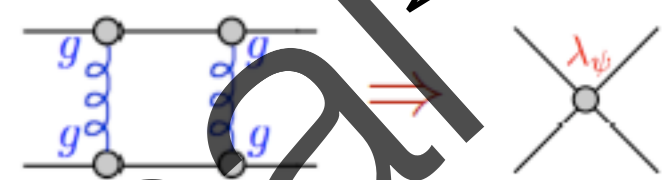
Dynamical degrees of freedom

Quarks, Gluons $\psi,\ A$ $\Longrightarrow$ $\psi,\ A$ + Mesons, Baryons $\phi \sim \bar\psi\psi,\ b \sim \psi^3$

Chiral symmetry breaking

Dynamical hadronisation ⟸ (Functional RG-flows)

$g\,\bar\psi \not{A}\,\psi \quad \xrightarrow{\int_{k-dk}^{k}}$



Hubbard-Stratonovitch

$$\lambda_\psi\,(\bar\psi\psi)^2 = h\,\bar\psi\psi\,\sigma - \tfrac{1}{2}m^2\sigma^2$$

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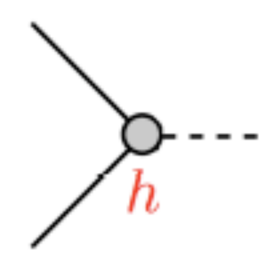

+Baryons and Glueballs

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Dynamical degrees of freedom

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Gies, Wetterich ‘01

JMP ‘05

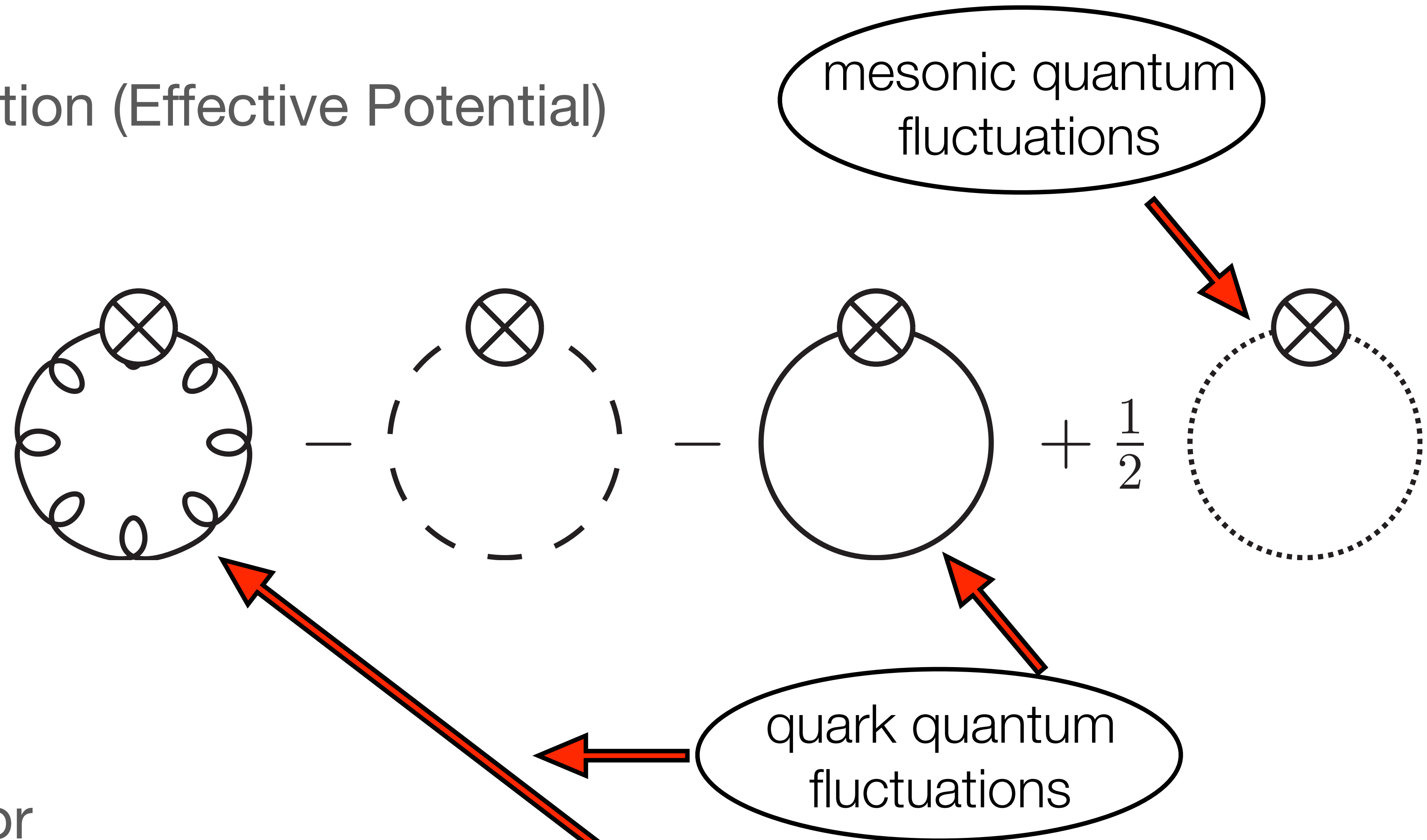
Flörchinger, Wetterich ‘09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods ⟸ (Functional RG-flows)

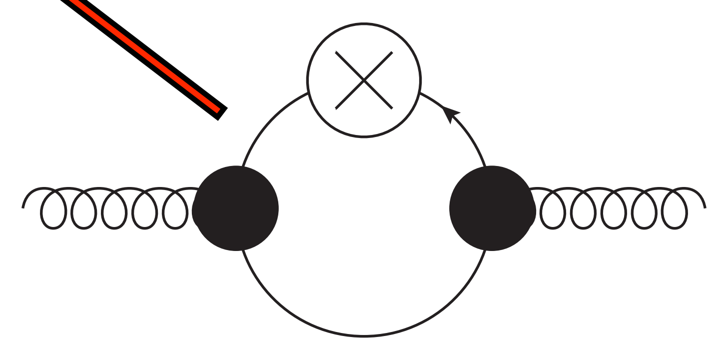
- RG-flow of Effective Action (Effective Potential)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



- flow of gluon propagator

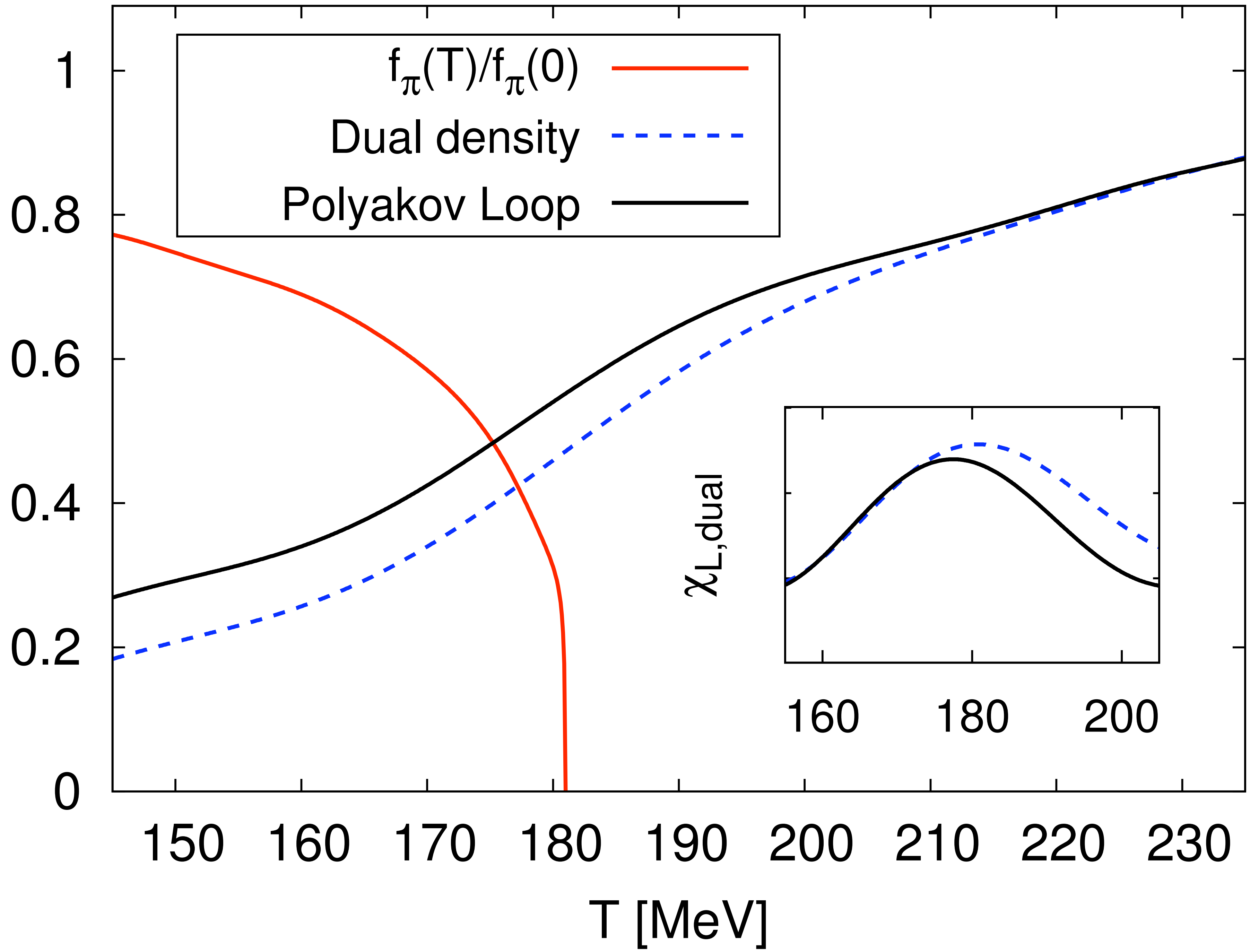
pure gauge theory flow +



+ ...

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods

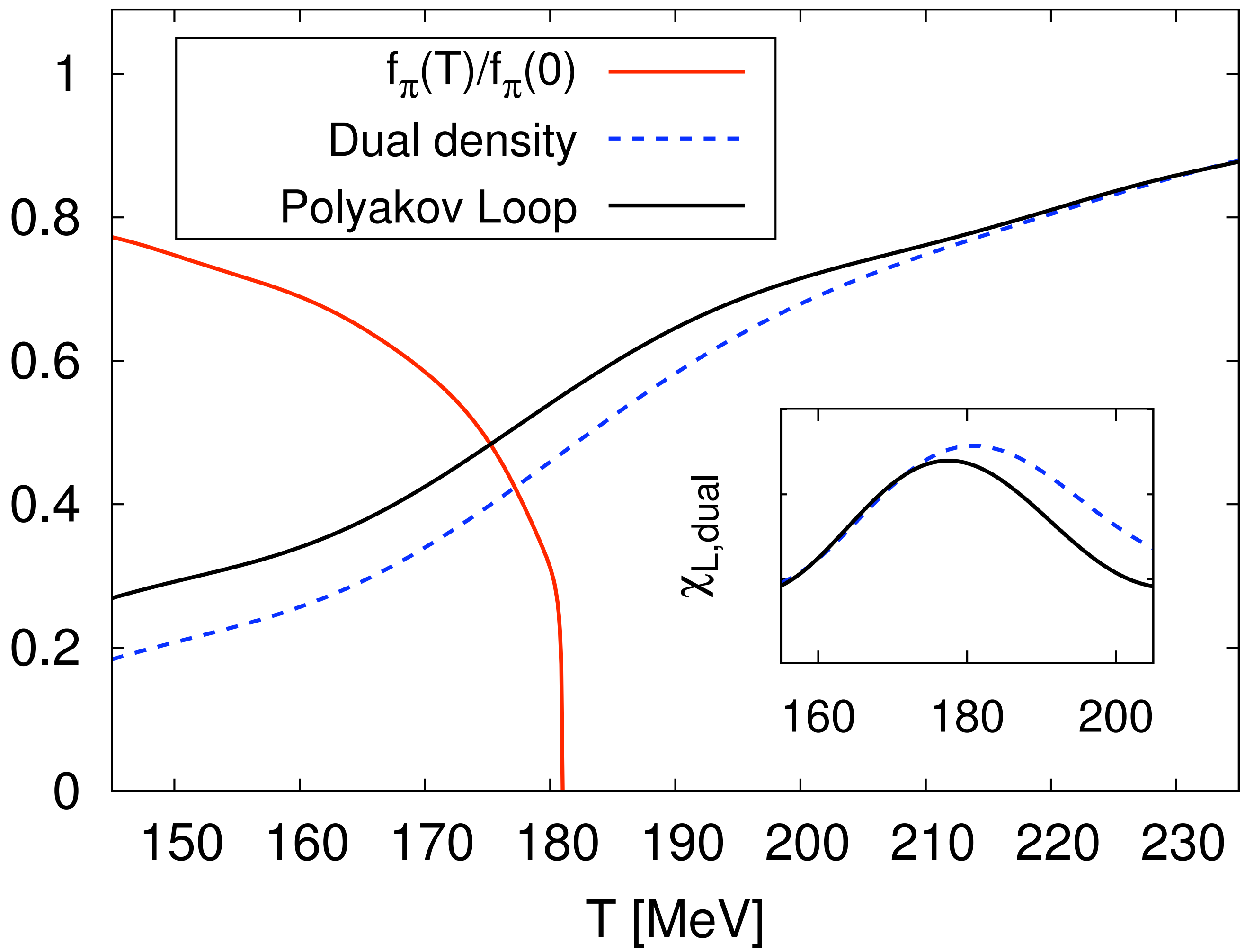


$$T_\chi \simeq T_{\rm conf} \simeq 180\text{MeV}$$

Braun, Haas, Marhauser, JMP ‘09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



compatible with Karsch et al '09

$T_{c,\mathrm{conf}} \simeq T_{c,\chi} \simeq 160\mathrm{MeV}$

$N_f = 2 + 1$

$T_\chi \simeq T_{\mathrm{conf}} \simeq 180\mathrm{MeV}$

$N_f = 2$

compatible with Fodor et al '09

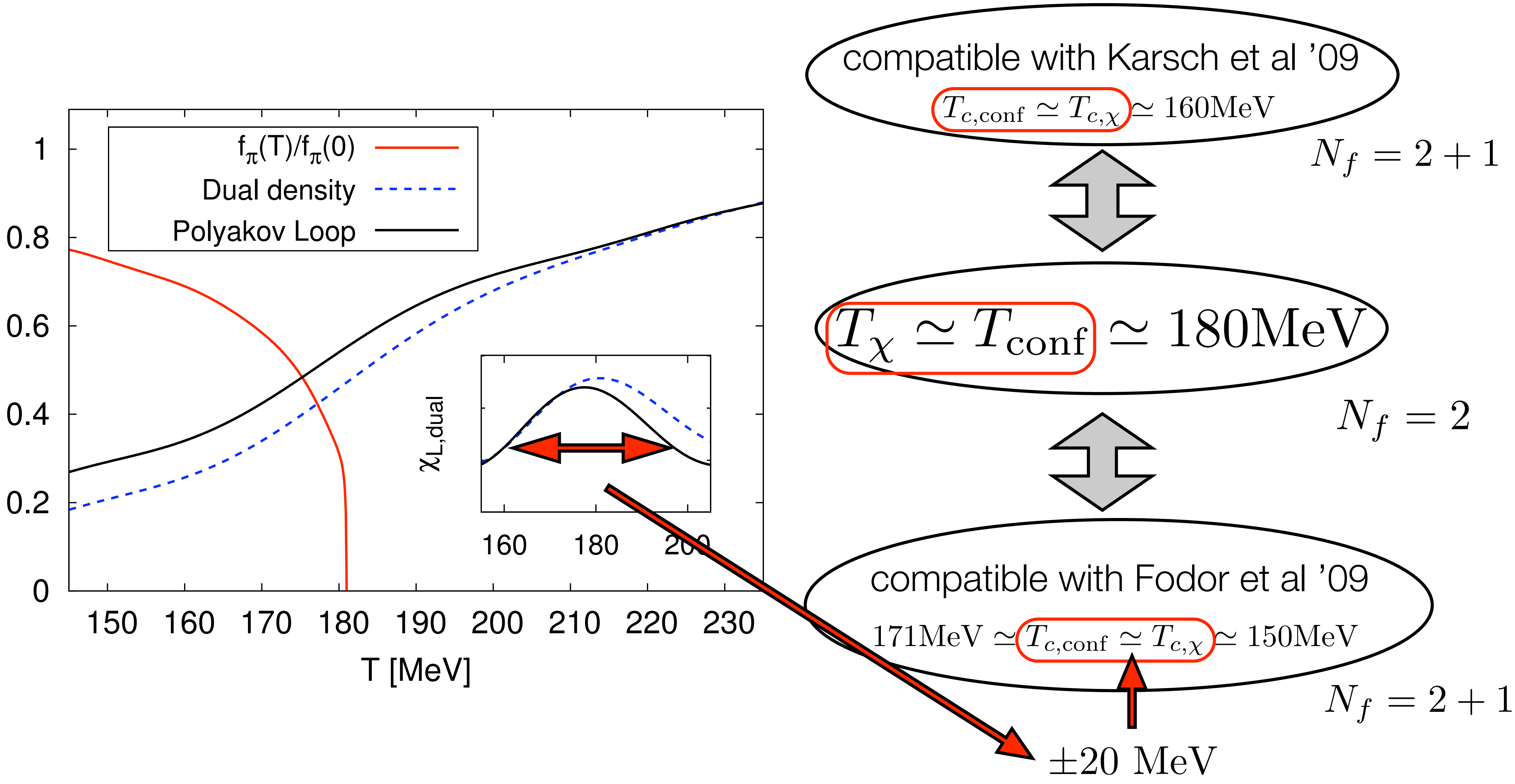
$171\mathrm{MeV} \simeq T_{c,\mathrm{conf}} > T_{c,\chi} \simeq 150\mathrm{MeV}$

$N_f = 2 + 1$

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



Braun, Haas, Marhauser, JMP '09

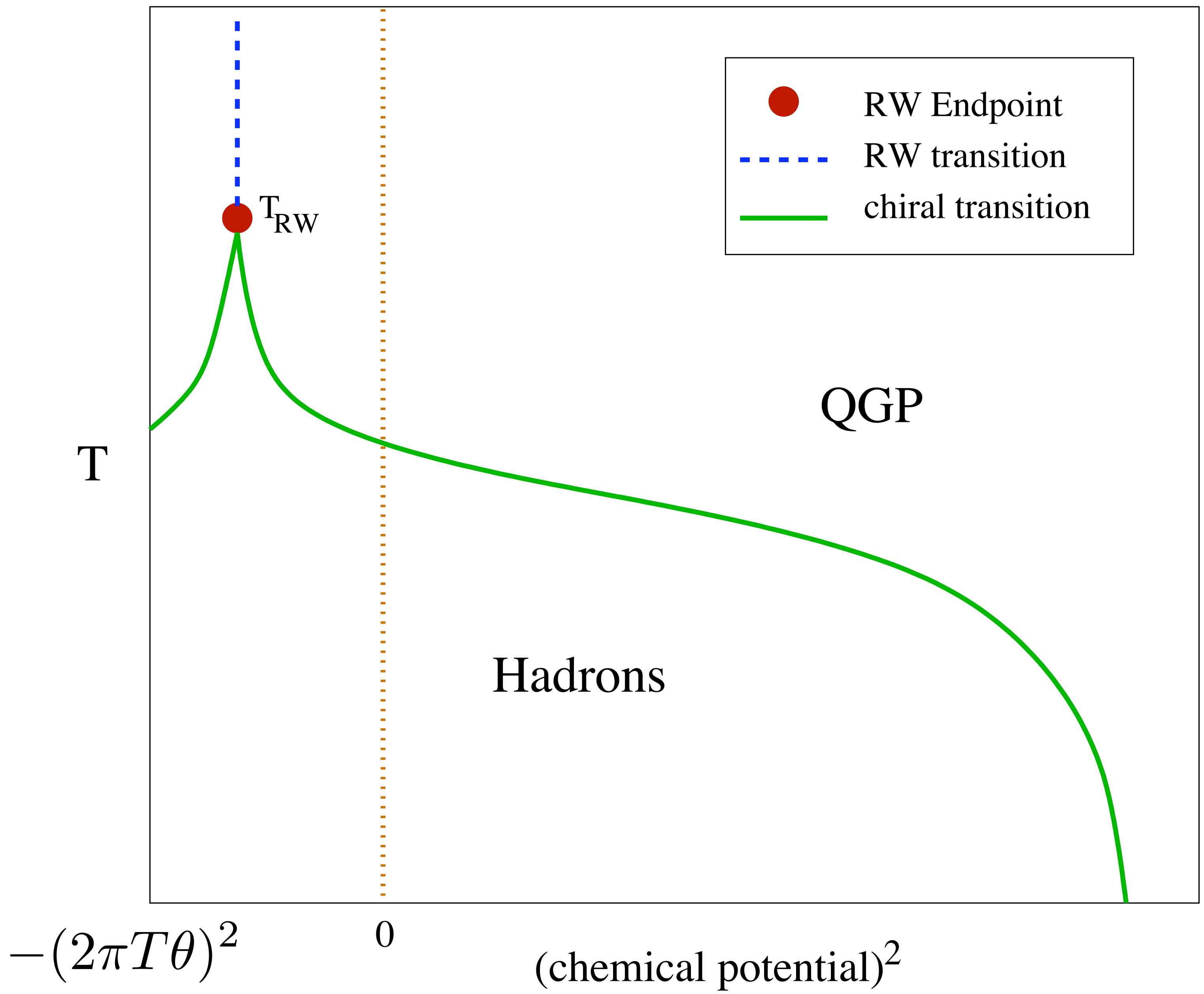
Phase structure at finite density

Imaginary chemical potential

Lattice & Continuum QCD

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$

- Roberge-Weiss symmetry



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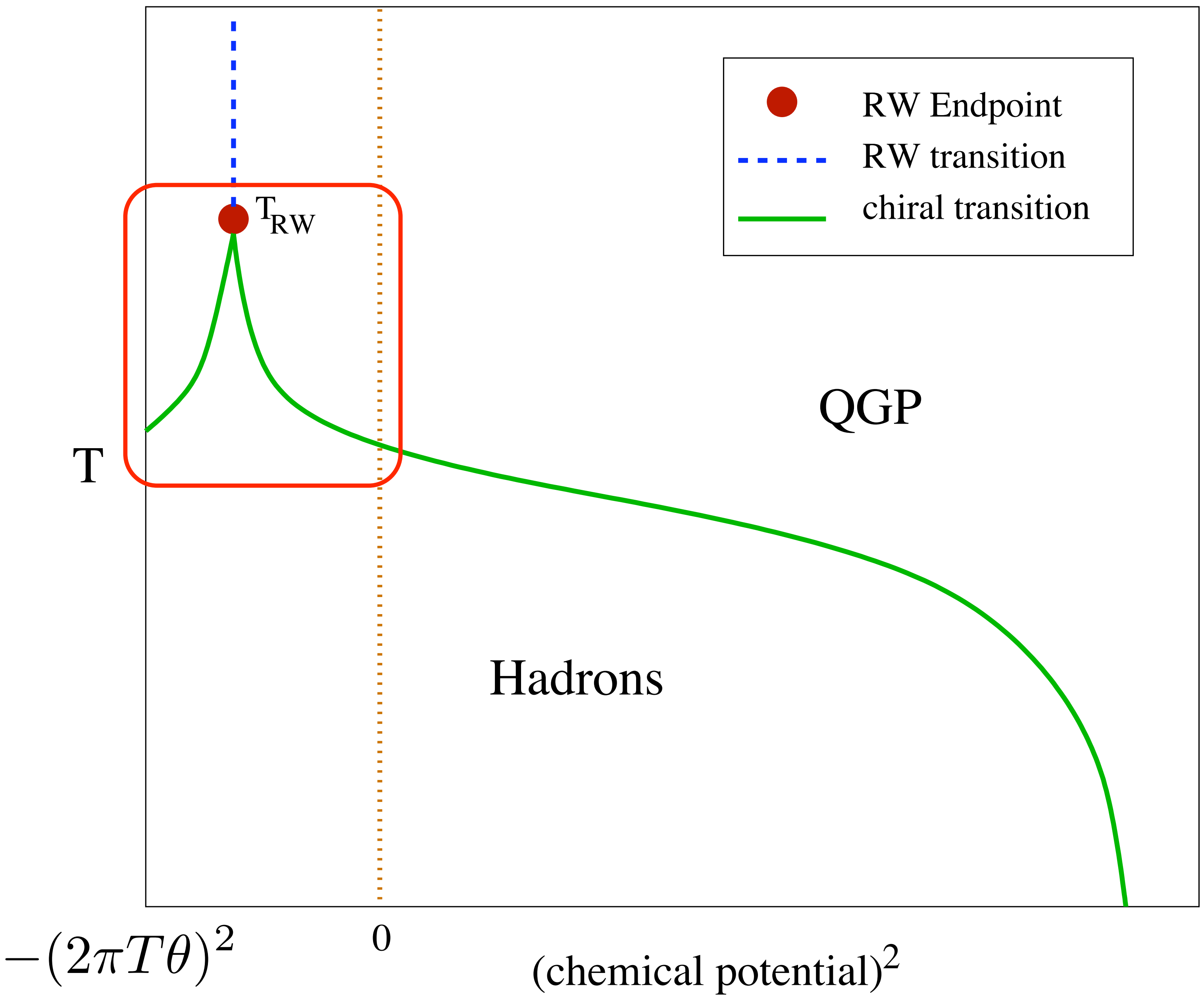
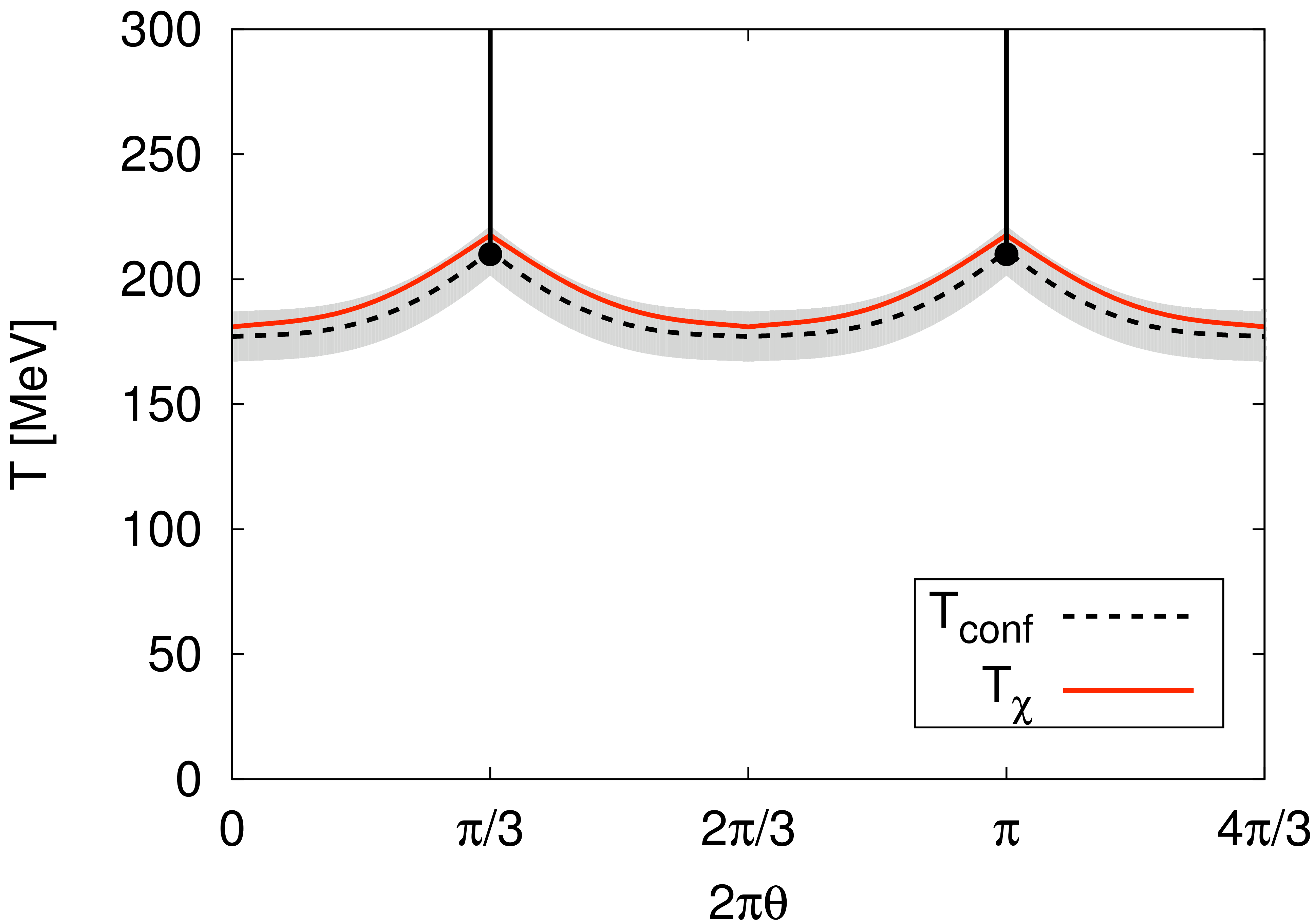


Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods

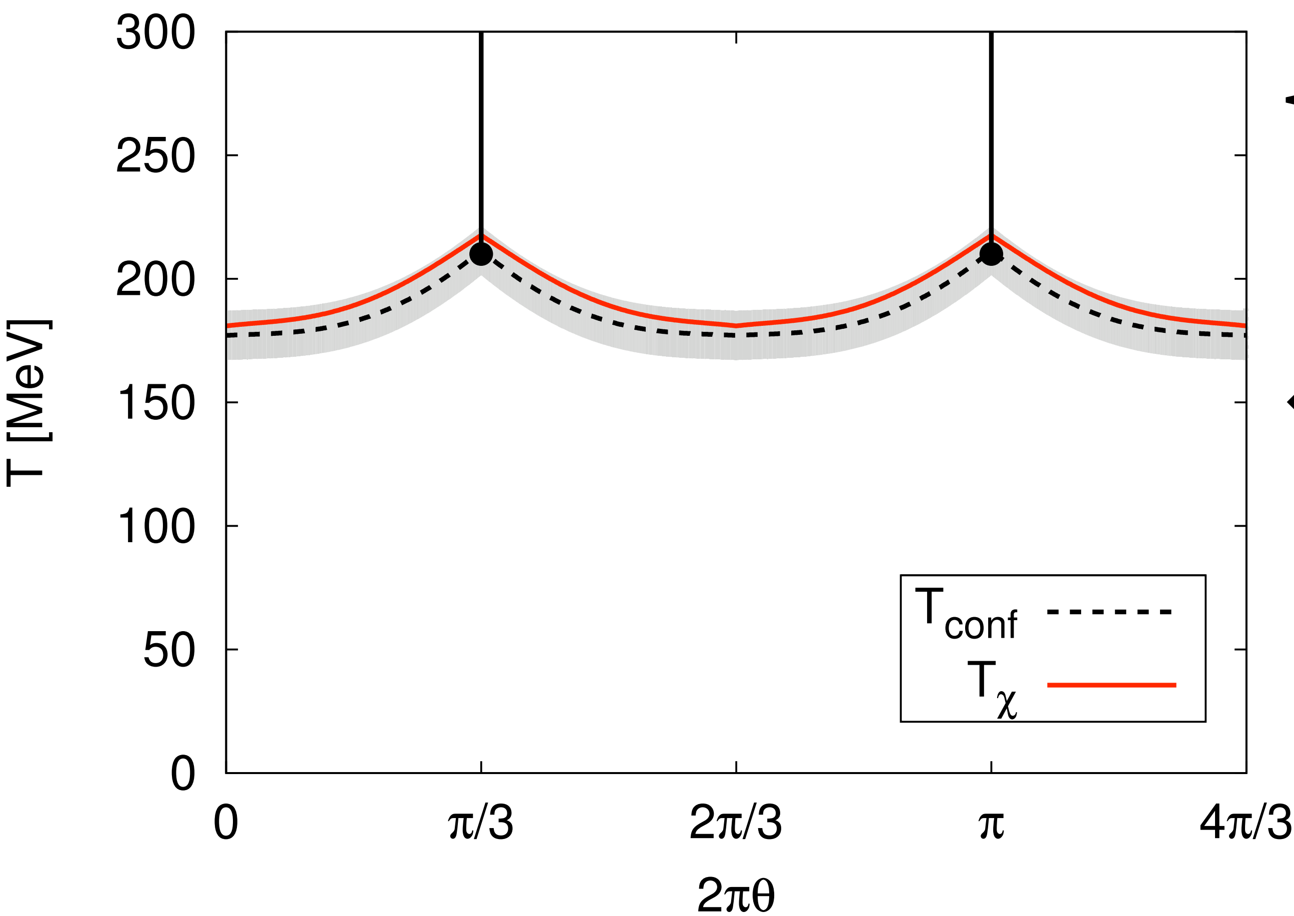


$$\text{chemical potential} : \mu = 2\pi i\, T\, \theta$$

Braun, Haas, Marhauser, JMP ‘09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



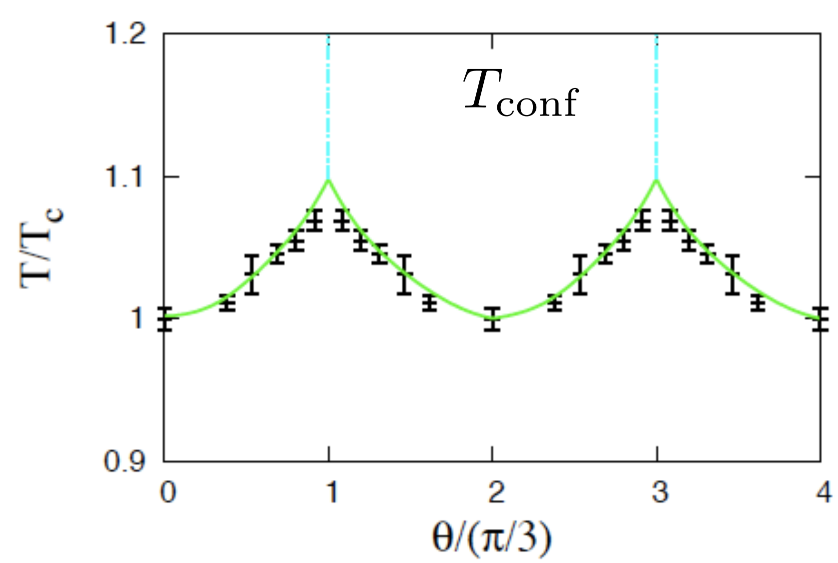
compatibility

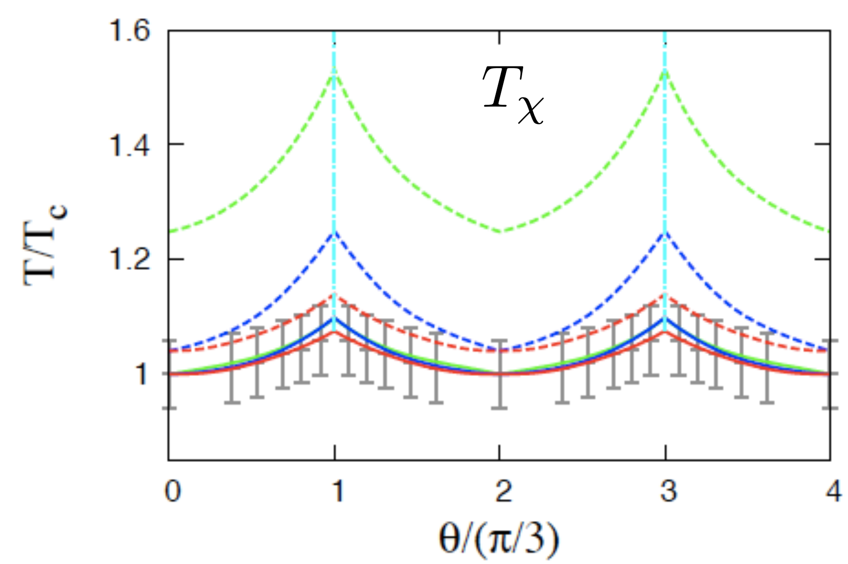
lattice results
Kratochvila et al '06 & Wu et al '06

adjust 8-fermi interaction

Polyakov-NJL model
Sakai et al '09

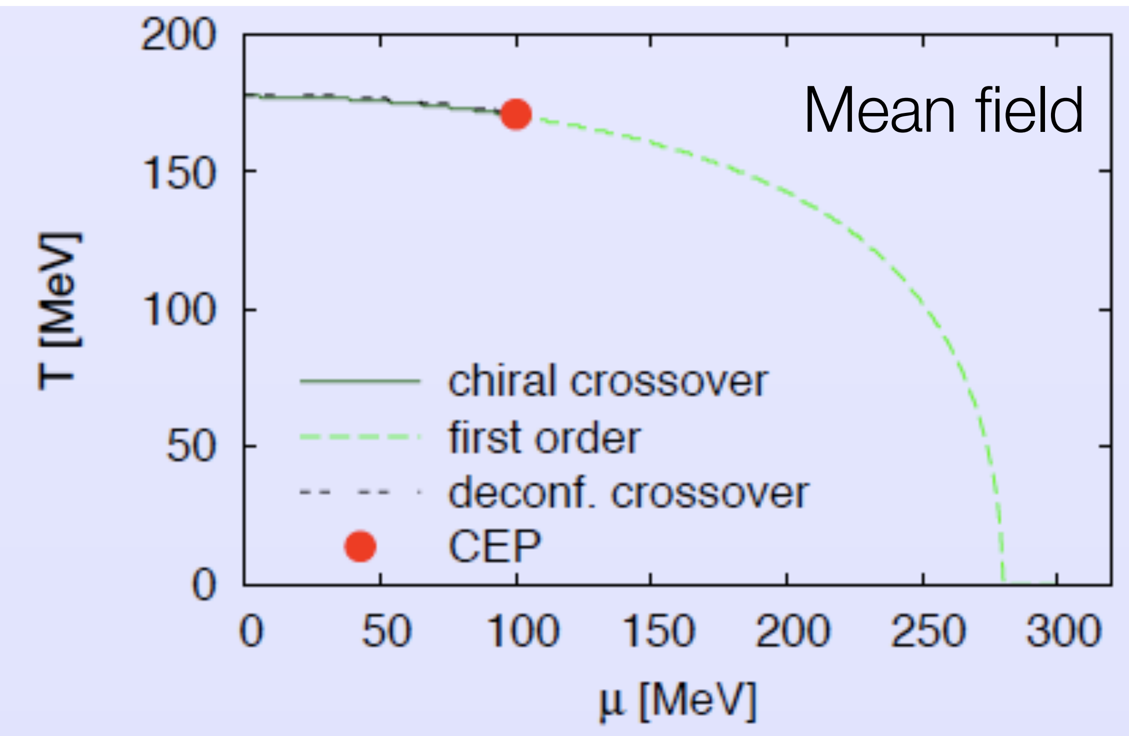
Braun, Haas, Marhauser, JMP '09





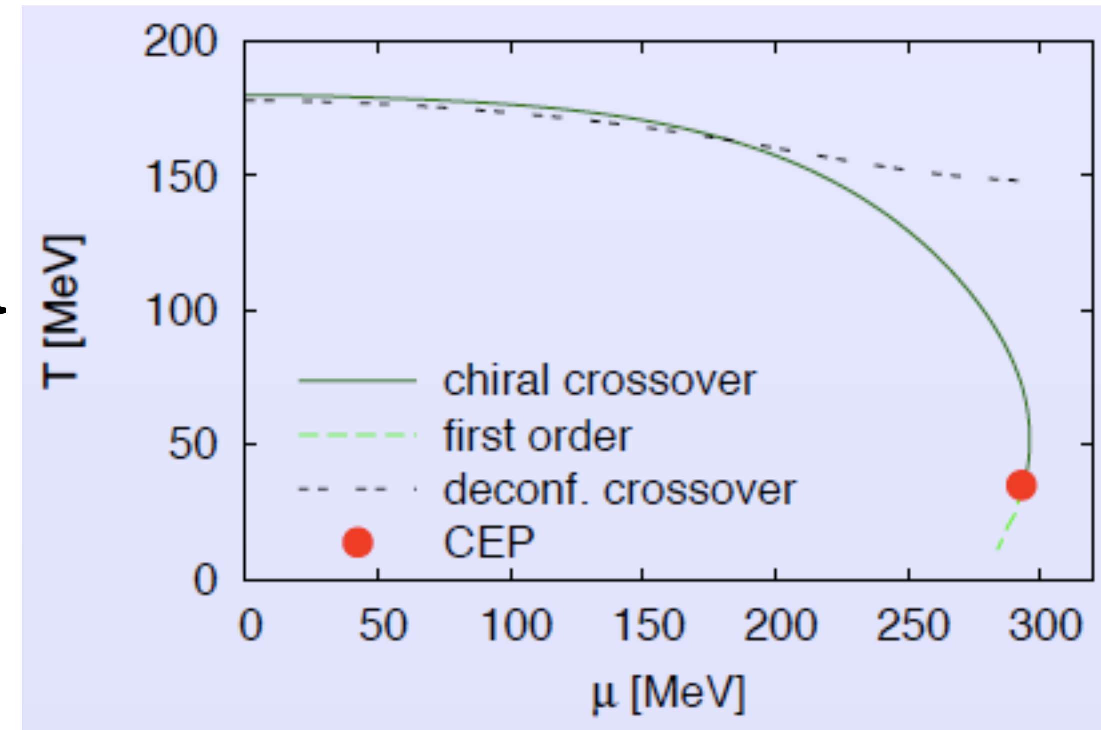
A glimpse at real chemical potential

Polyakov - Quark-Meson model



RG

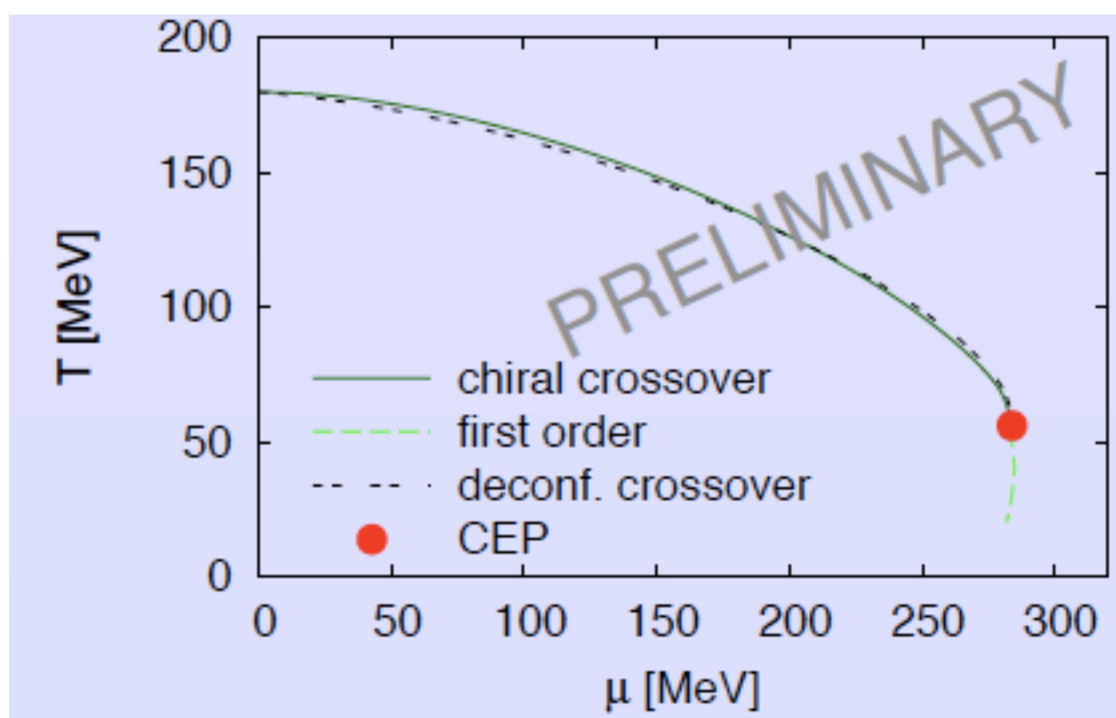
quark-meson fluctuations



HTL/HDL

quark fluctuations in YM sector

Schaefer, JMP, Wambach '07

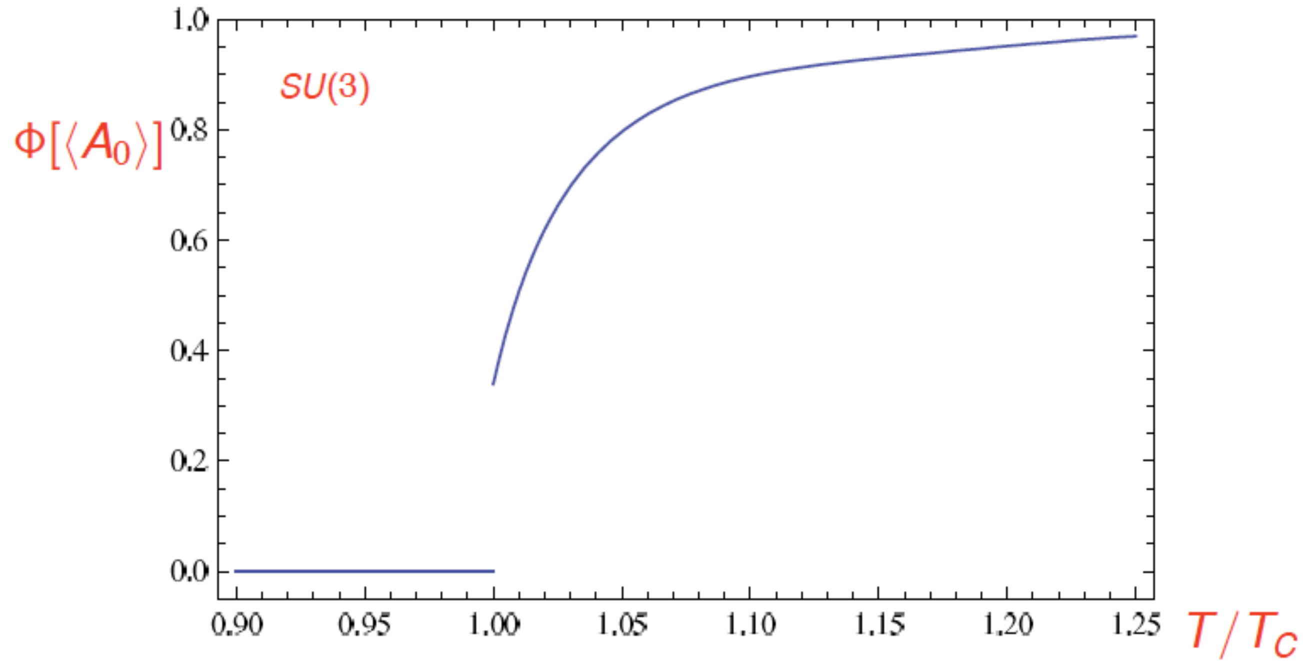


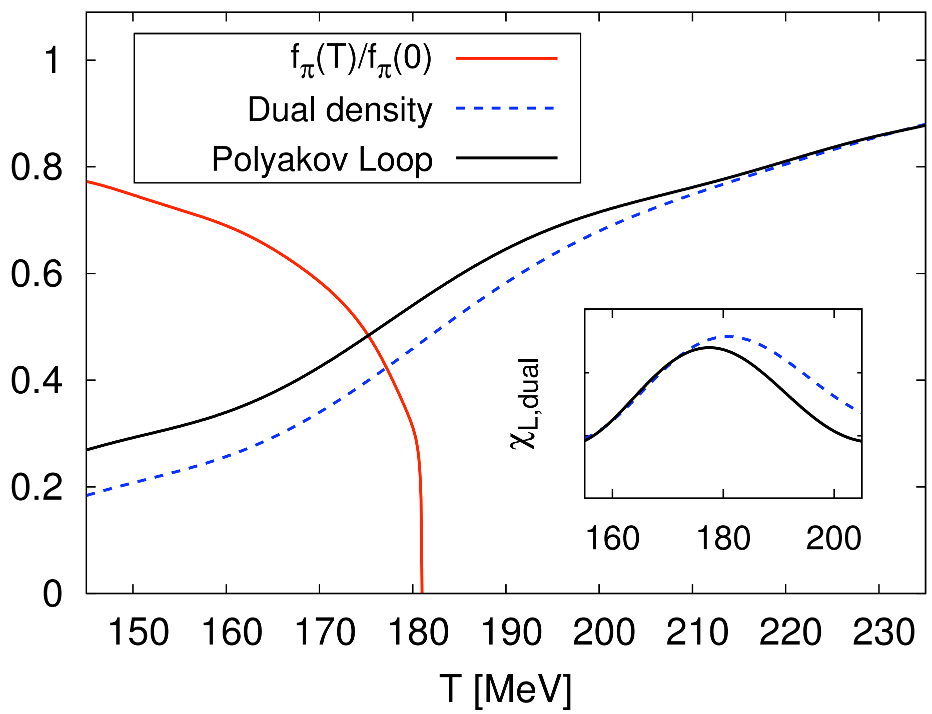
Herbst, JMP, Schaefer, in prep

Summary & Outlook

Summary & outlook

- Phase diagram of QCD
    - Confinement & chiral symmetry breaking at finite temperature





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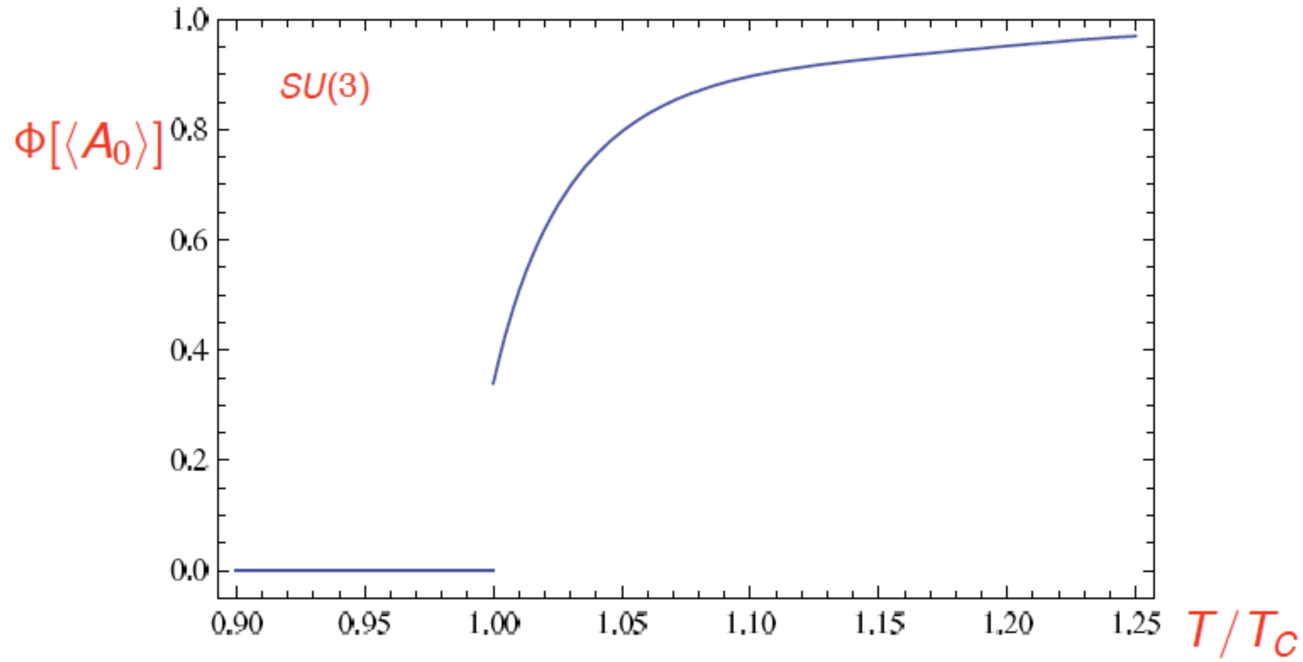
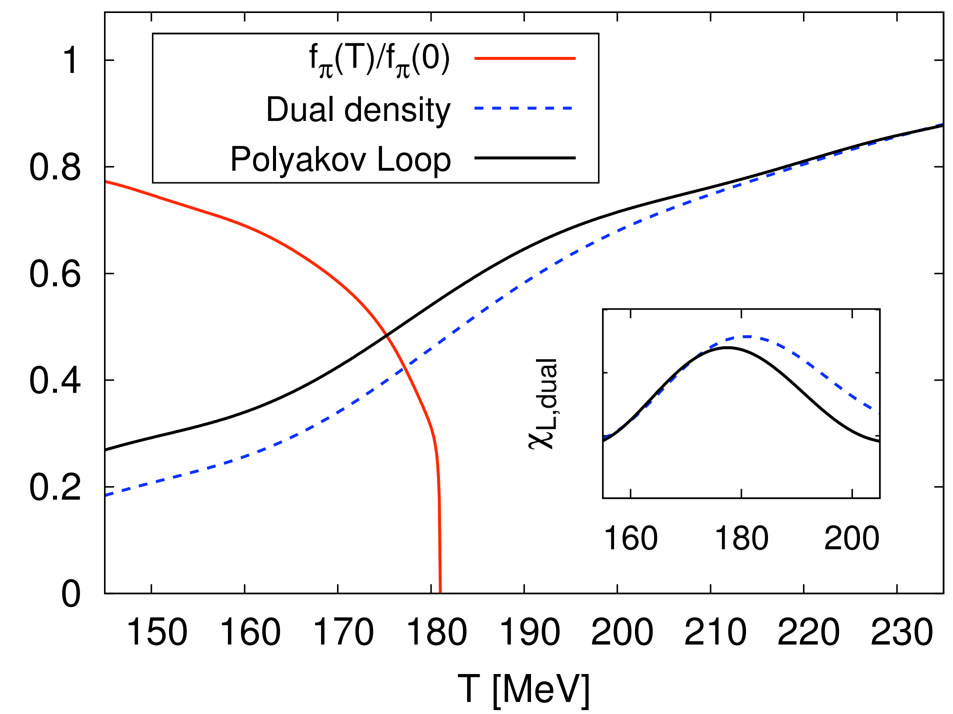




- Dynamical hadronisation

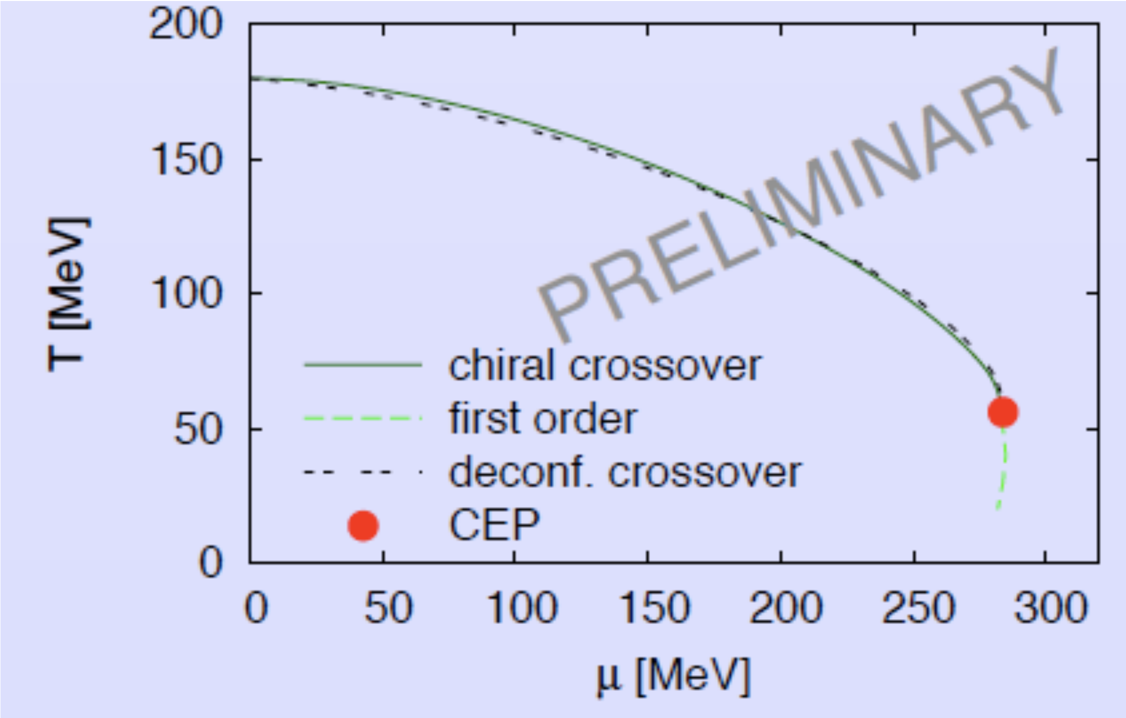
QCD flows dynamically into hadronic effective theories

- Next steps: real chemical potential & 2+1 flavours

work in progress

Summary & outlook

- Phase diagram of QCD
  - Confinement & chiral symmetry breaking at finite temperature
    - Dynamical hadronisation
  - critical point and phase lines in effective theories



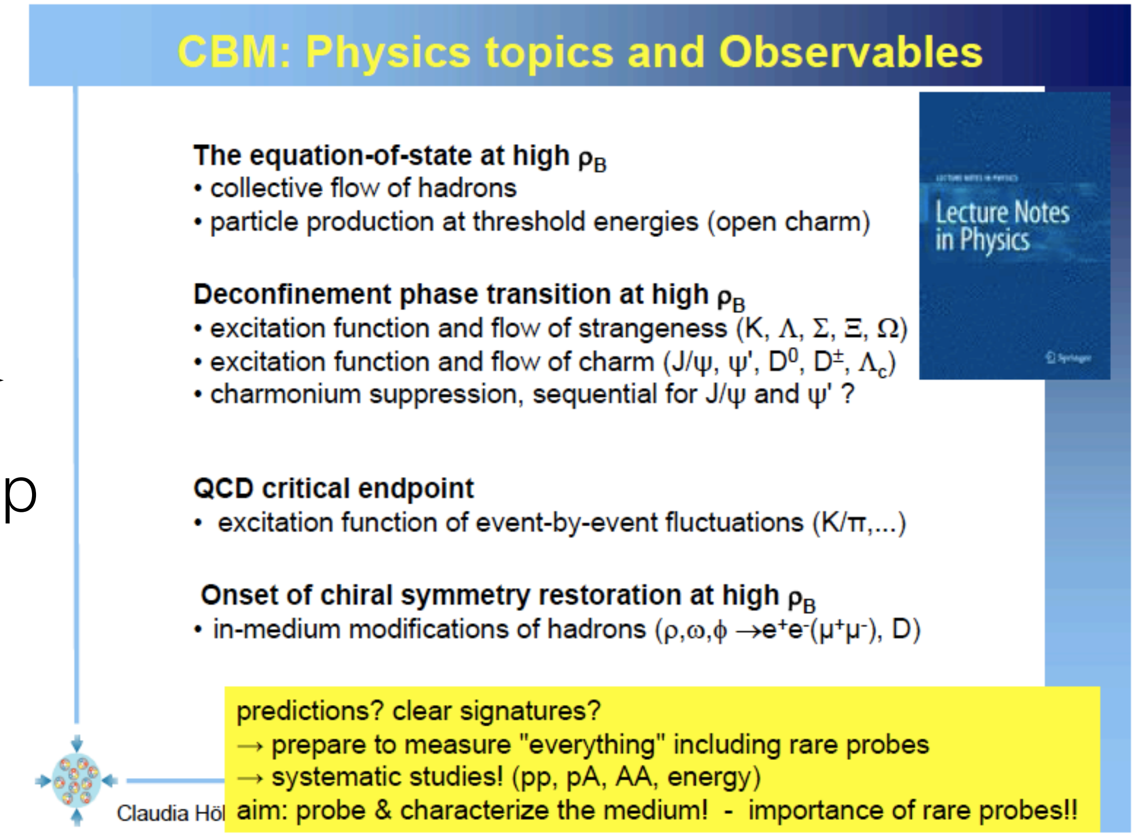
Summary & outlook

- Phase diagram of QCD
  - Confinement & chiral symmetry breaking at finite temperature
    - Dynamical hadronisation
  - critical point and phase lines in effective theories
    - Hadronic properties
  - Next step

e.g.

Top-down meets bottom-up

Refine effective hadronic theories



Summary & outlook

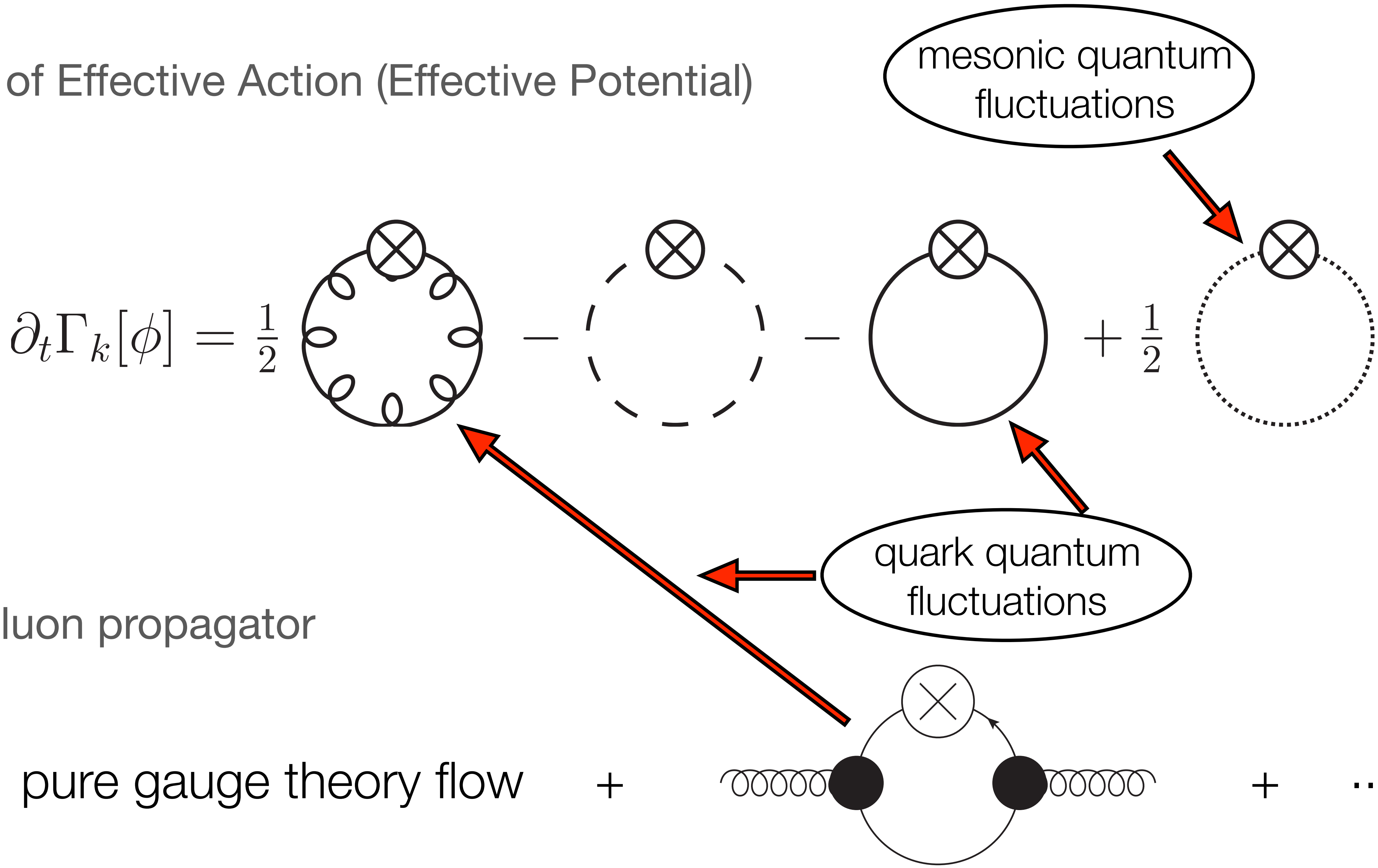
- Phase diagram of QCD
    - Confinement & chiral symmetry breaking at finite temperature
        - Dynamical hadronisation
    - critical point and phase lines in effective theories
        - Hadronic properties
    - non-equilibrium physics

Additional material: dual density

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods ⟸ (Functional RG-flows)

- RG-flow of Effective Action (Effective Potential)
- flow of gluon propagator



Imaginary chemical potential

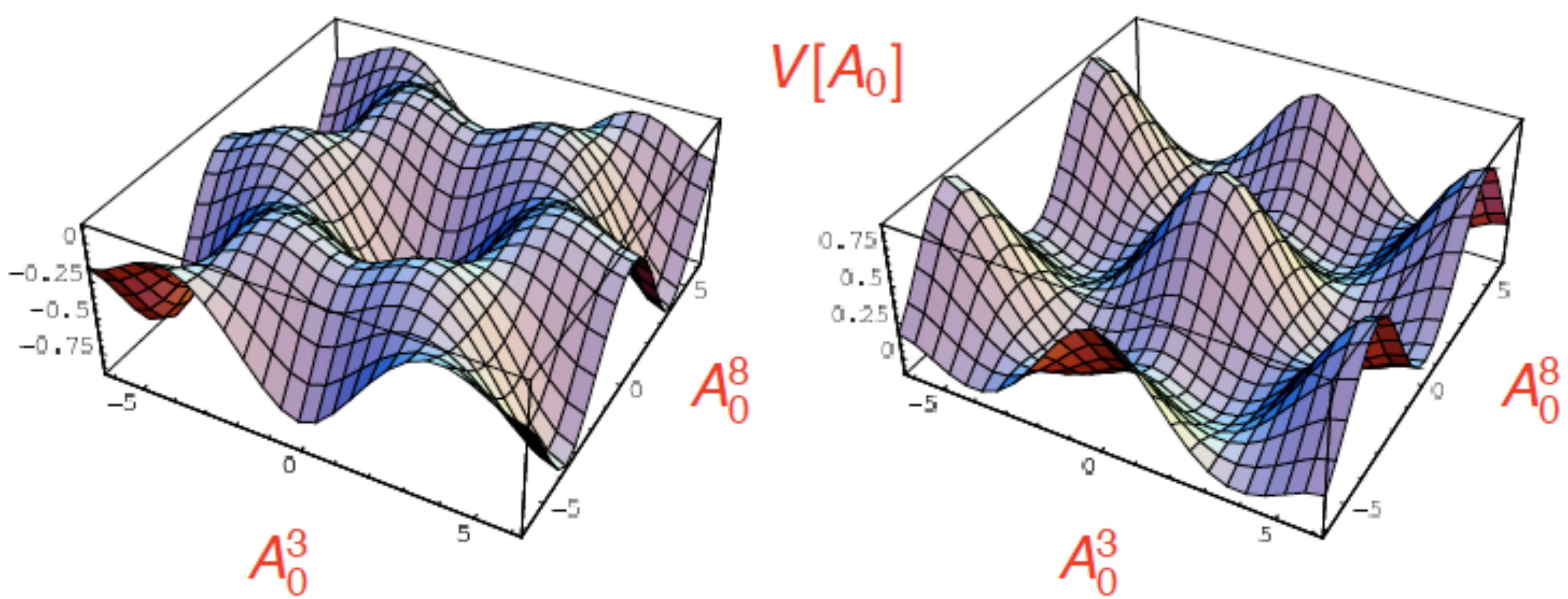
Lattice & Continuum QCD

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$

- Roberge-Weiss symmetry

$$Z_\theta = Z_{\theta+1/3}$$

deconfining confining



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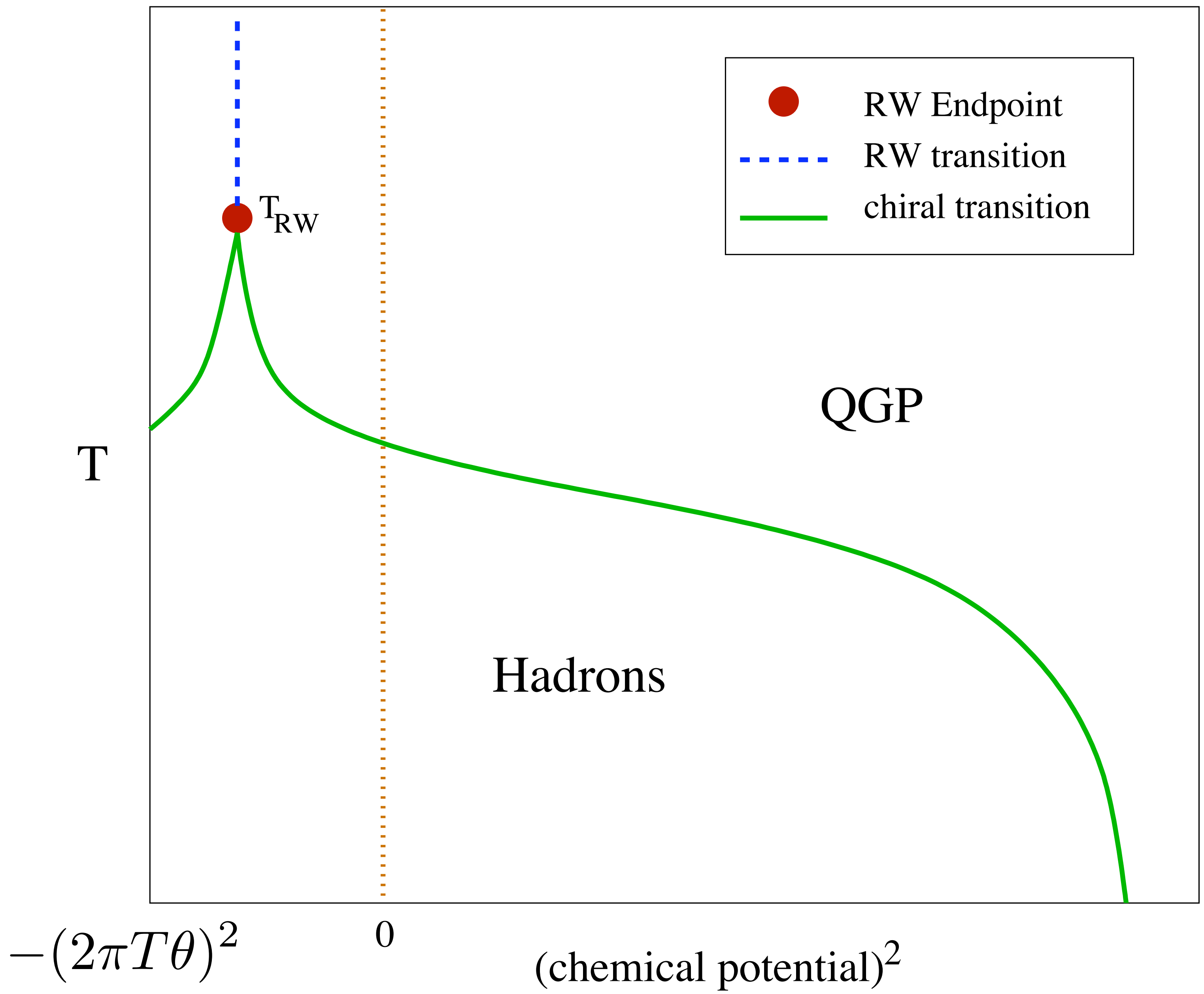

Dual order parameter

Lattice & Continuum QCD

$$\mathcal{O}_\theta = \langle O[e^{2\pi i\theta t/\beta}\psi]\rangle \qquad \text{with} \qquad \psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x)$$

imaginary chemical potential $\mu = 2\pi i\theta/\beta$ for $\psi_\theta = e^{2\pi i\theta t/\beta}\psi$

$$z = e^{2\pi i\theta_z} \longrightarrow \tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

order parameter for confinement

Dual order parameter

- Lattice
  - Gattringer ‘06
  - Synatschke, Wipf, Wozar ‘08
  - Bruckmann, Hagen, Bilgici, Gattringer ‘08
- Continuum
  - Fischer, ’09; Fischer, Mueller ‘09
  - Braun, Haas, Marhauser, JMP ‘09 ← imaginary chemical potential

Dual order parameter

Lattice & Continuum QCD

$$\tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

- no imaginary chemical potential (lattice studies):

DSE: 4 loop and more → $\tilde{\mathcal{O}}$ ← FRG: 3 loop and more

- imaginary chemical potential I: evaluated at equations of motion

$\tilde{\mathcal{O}}[\langle A_0\rangle_\theta] \equiv 0$ ← Roberge-Weiss

- imaginary chemical potential II: evaluated at a fixed background

standard FRG & DSE → $\tilde{\mathcal{O}}[\langle A_0\rangle_\theta] \neq 0$ ← breaking of Roberge-Weiss

Dual order parameter

Lattice & Continuum QCD

$$\tilde{\mathcal{O}} = \int_0^1 d\theta \, \mathcal{O}_\theta e^{-2\pi i \theta}$$

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Dual order parameter

Continuum methods ⟵ (Functional RG-flows)

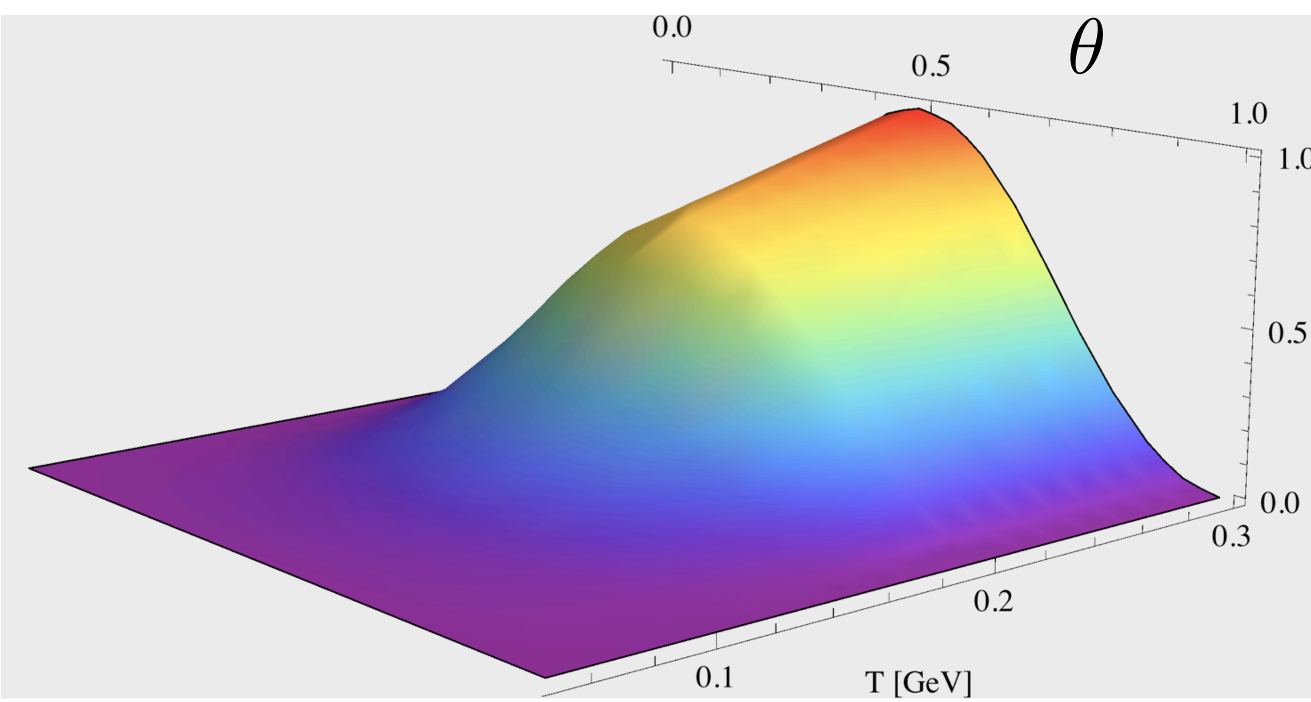
$$\mathcal{O}_\theta = \langle O[e^{2\pi i\theta t/\beta}\psi]\rangle \quad \text{with} \quad \psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x)$$

imaginary chemical potential $\mu = 2\pi i\theta/\beta$ for $\psi_\theta = e^{2\pi i\theta t/\beta}\psi$

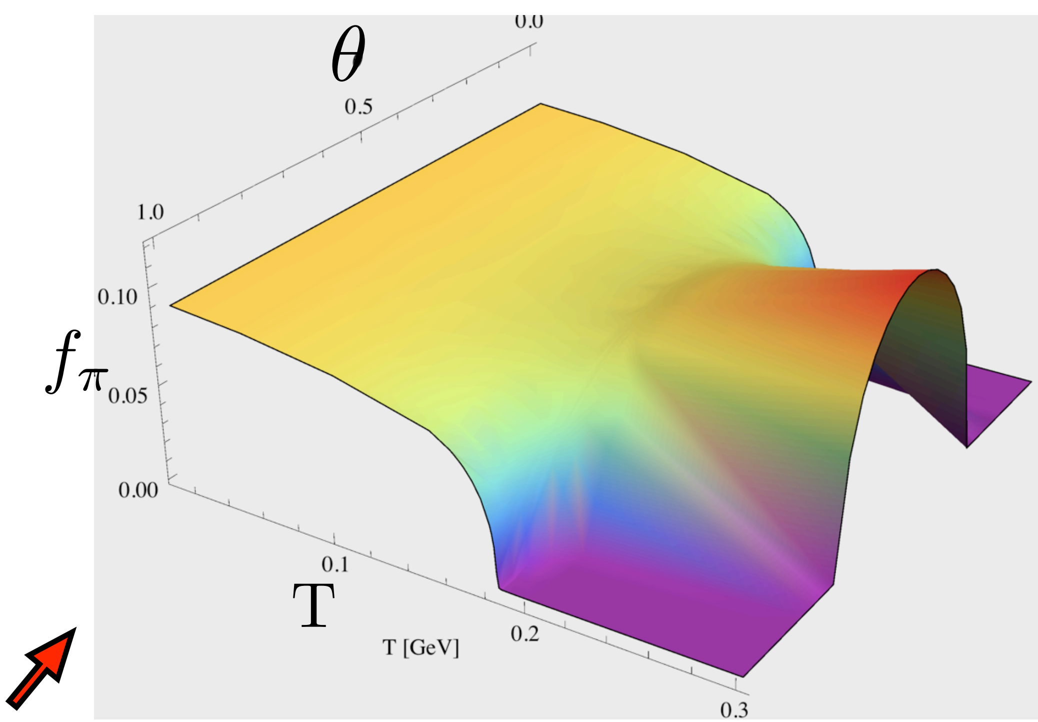
$$z = e^{2\pi i\theta_z} \longrightarrow \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

order parameter for confinement

'fermionic pressure difference' $p(T,\theta) \simeq P(T,\theta) - P(T,0)$



$f_\pi(T,\theta)$

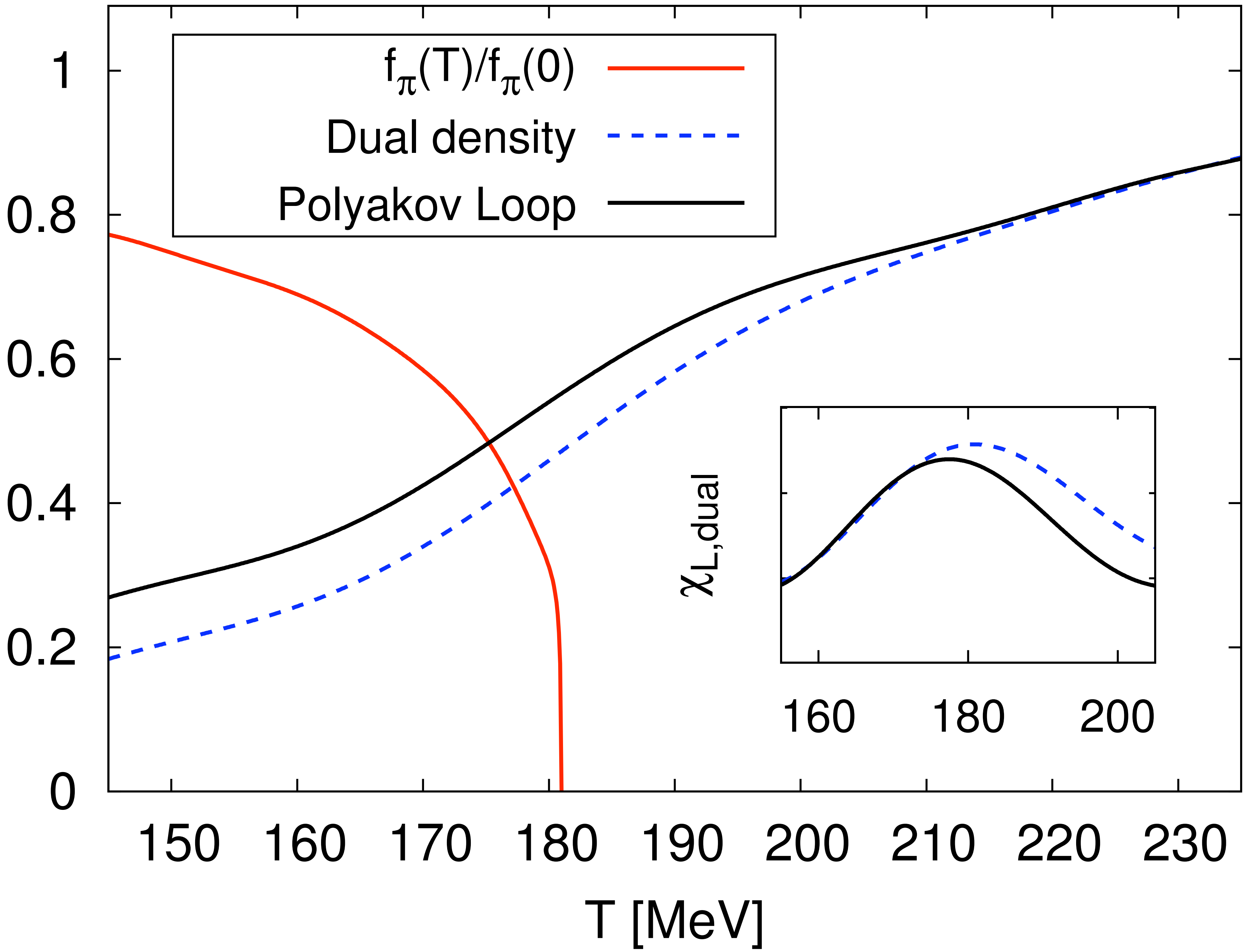


fixed A_0: no Roberge-Weiss periodicity

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods



$$T_\chi \simeq T_{\rm conf} \simeq 180 \rm MeV$$

Braun, Haas, Marhauser, JMP ‘09

Full dynamical QCD: N_f = 2 & chiral limit

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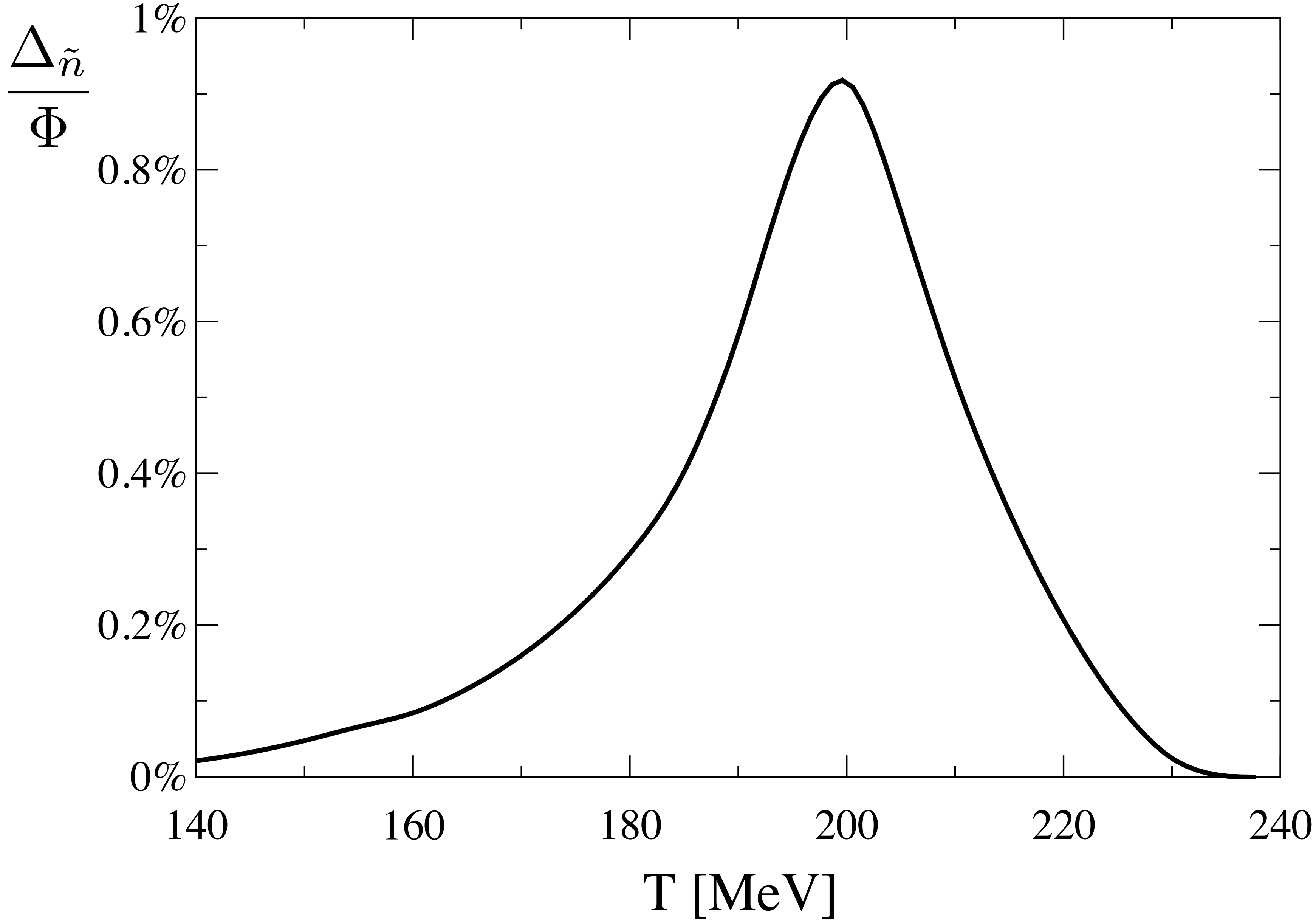
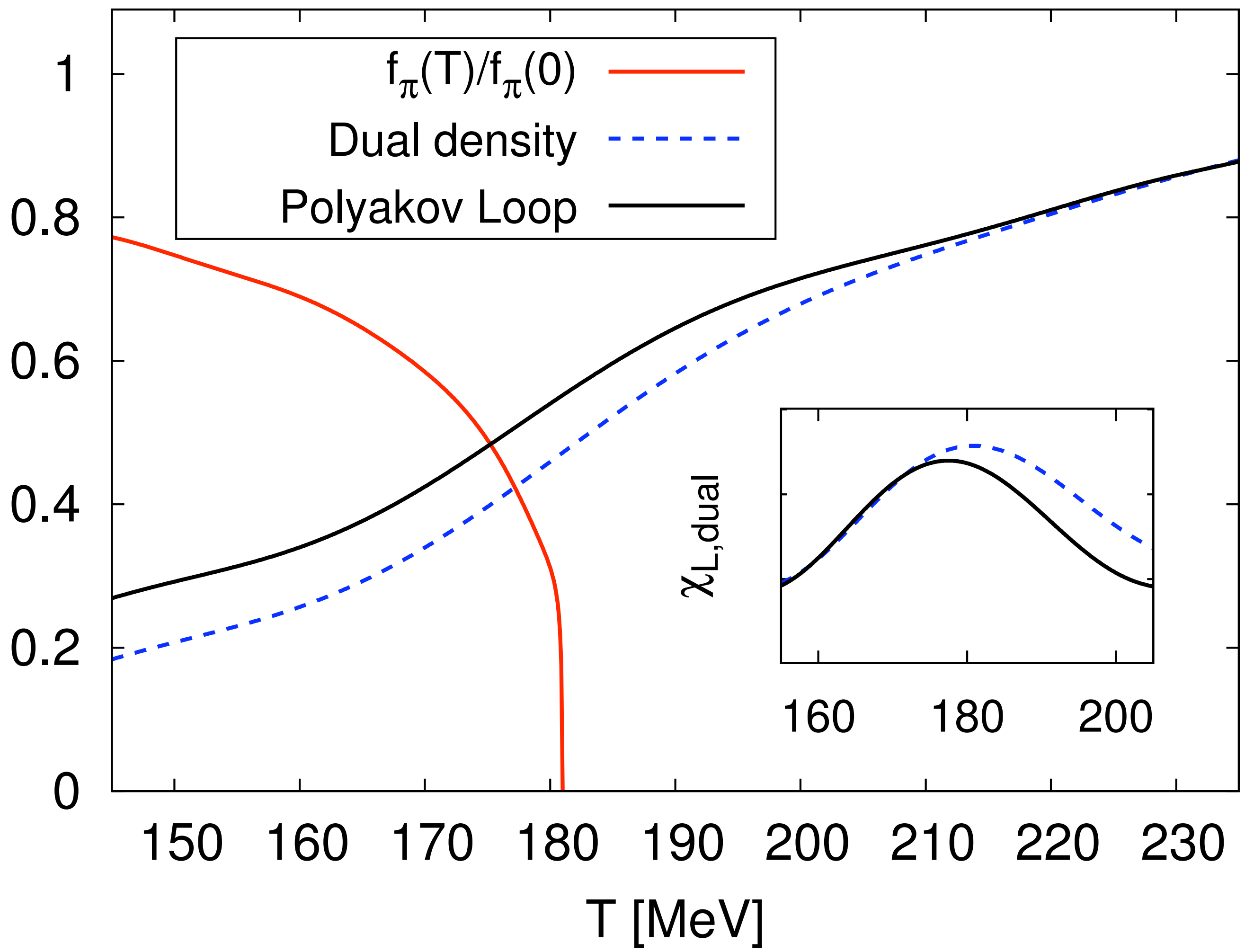


$$\Delta_{\tilde{n}} = \frac{\tilde{n}[\langle A_0 \rangle]}{\tilde{n}[0]} - \Phi[\langle A_0 \rangle] :$$ Deviation of dual density from Polyakov loop

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



compatible with Karsch et al '09

$T_{c,\mathrm{conf}} \simeq T_{c,\chi} \simeq 160\mathrm{MeV}$

$N_f = 2 + 1$

$$T_\chi \simeq T_{\mathrm{conf}} \simeq 180\mathrm{MeV}$$

$N_f = 2$

compatible with Fodor et al '09

$171\mathrm{MeV} \simeq T_{c,\mathrm{conf}} > T_{c,\chi} \simeq 150\mathrm{MeV}$

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Braun, Haas, Marhauser, JMP '09

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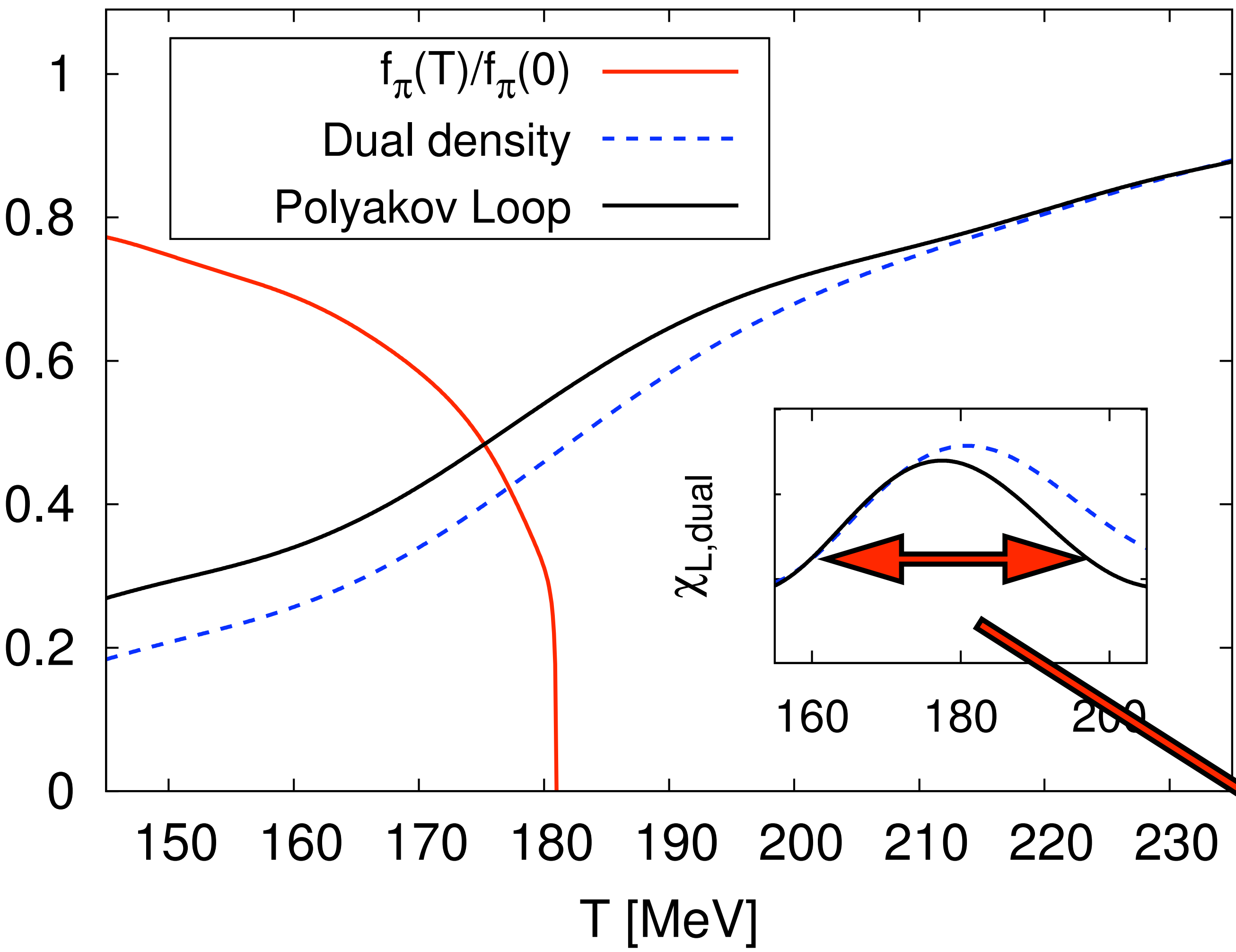

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$171\mathrm{MeV} \simeq T_{c,\mathrm{conf}} \simeq T_{c,\chi} \simeq 150\mathrm{MeV}$

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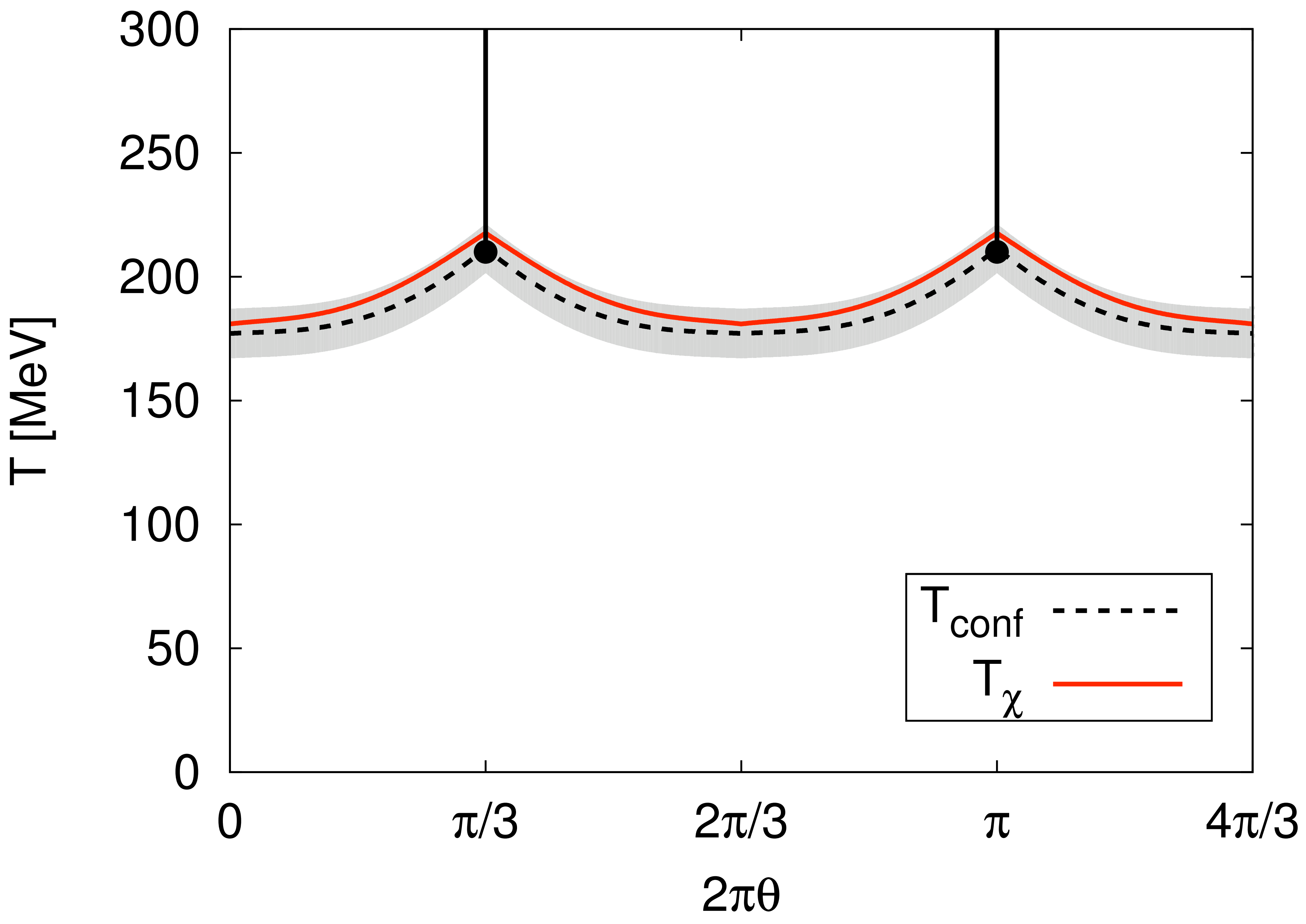
± 20 MeV

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods

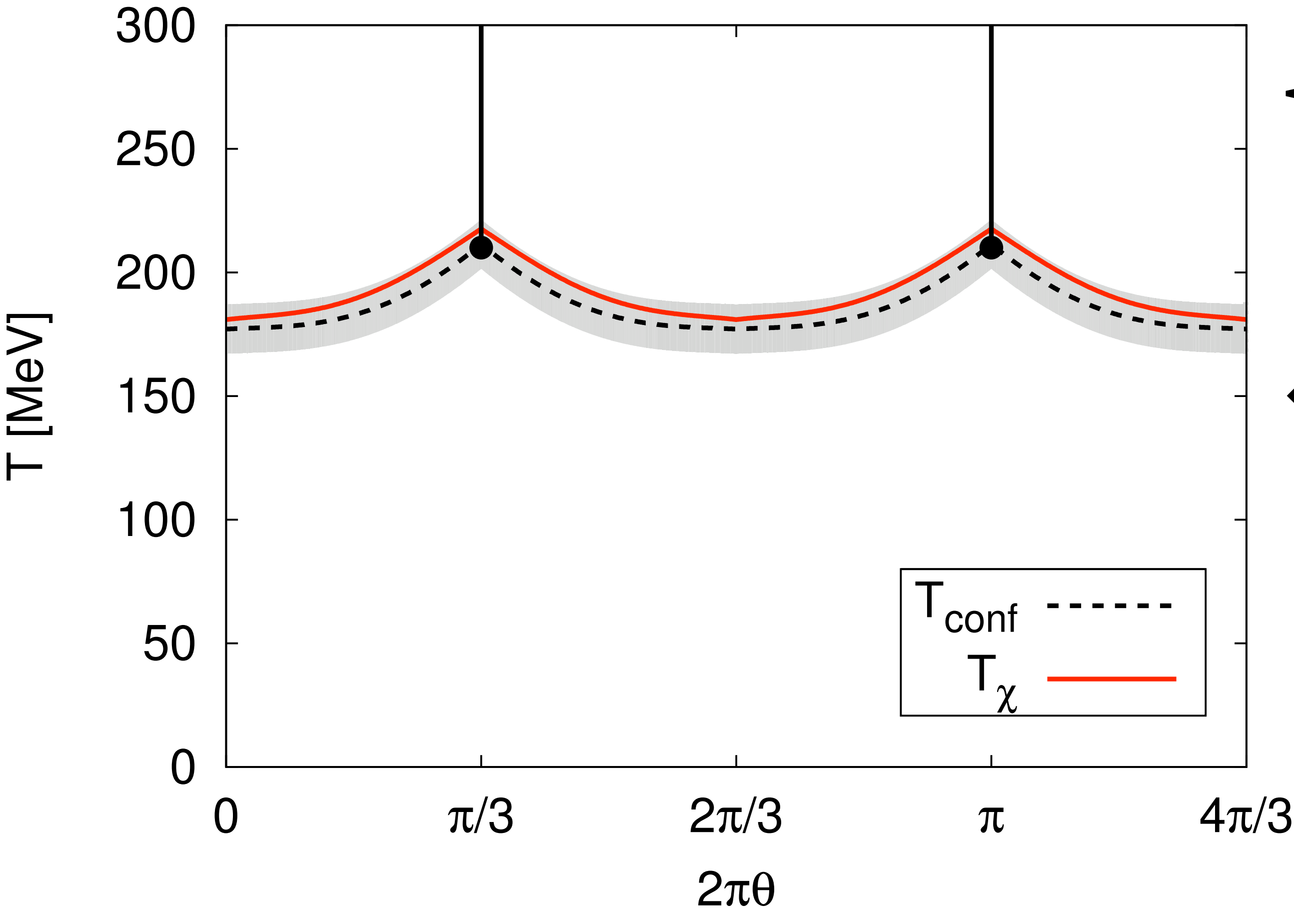


$$\text{chemical potential}: \mu = 2\pi i\, T\, \theta$$

Braun, Haas, Marhauser, JMP '09

Full dynamical QCD: N_f = 2 & chiral limit

Continuum methods & lattice



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Braun, Haas, Marhauser, JMP '09