Confinement Lost, and Regained

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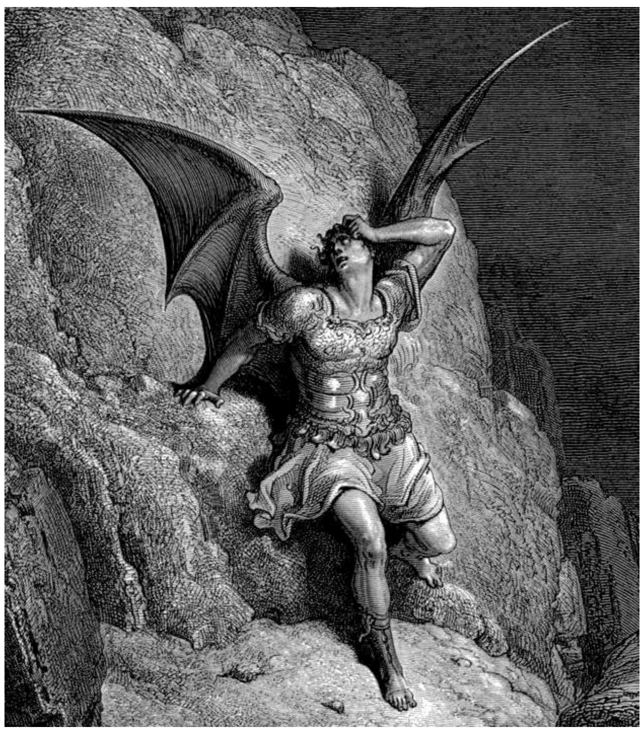
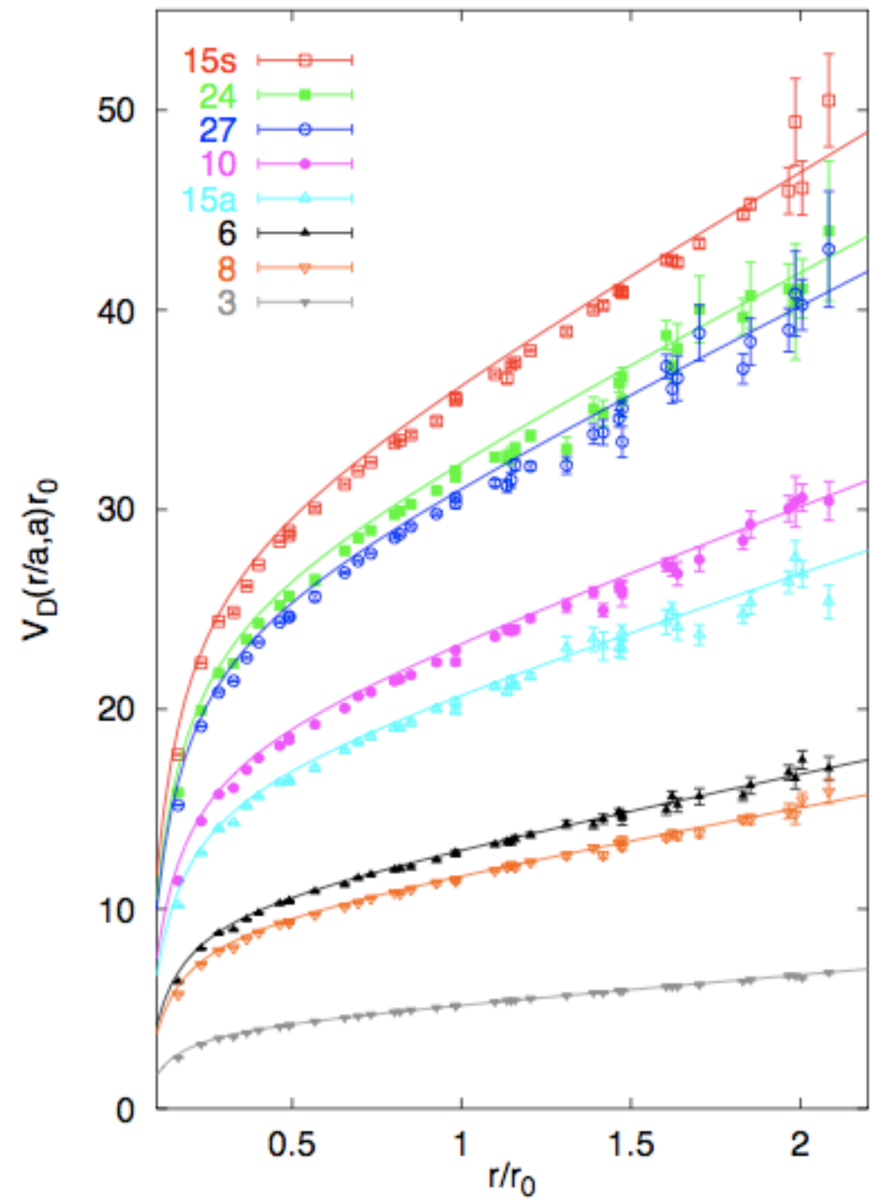


The mind is its own place, and in itself
Can make a heav'n of hell, a hell of heav'n.

Quark Confinement

$$V_{Q\bar{Q}} = \sigma r$$

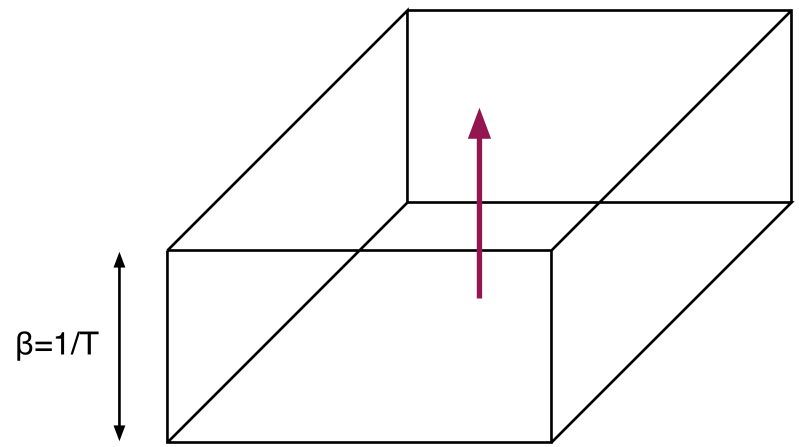
- Large-distance behavior of quark-antiquark potential
- Usually omit dynamical quarks due to string breaking
- Different string tensions for different representations
- Confinement stops at the deconfinement temperature T_D



Bali, 2000

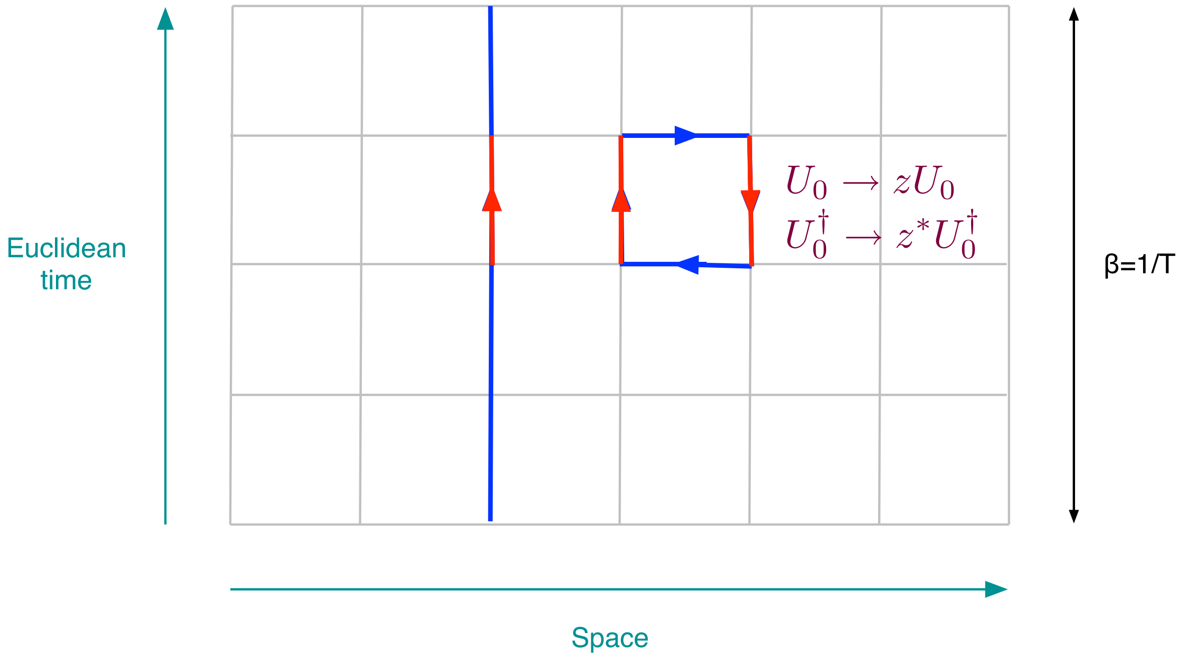
Polyakov loop as order parameter

- P is the phase factor that generalizes the Aharonov-Bohm phase factor to non-Abelian gauge theories
- P is a topologically non-trivial Wilson loop
- The operator $Tr_R P$ represents the insertion of a static color charge in a representation R
- $Tr_R 1$ = # of particle colors in R
- $\langle Tr_F P \rangle = 0$ means confinement



$$P(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{\beta} dt A_4 (\vec{x}, t) \right]$$

Z(N) symmetry



$$P \to zP \qquad z = e^{\frac{2\pi i}{N}}$$

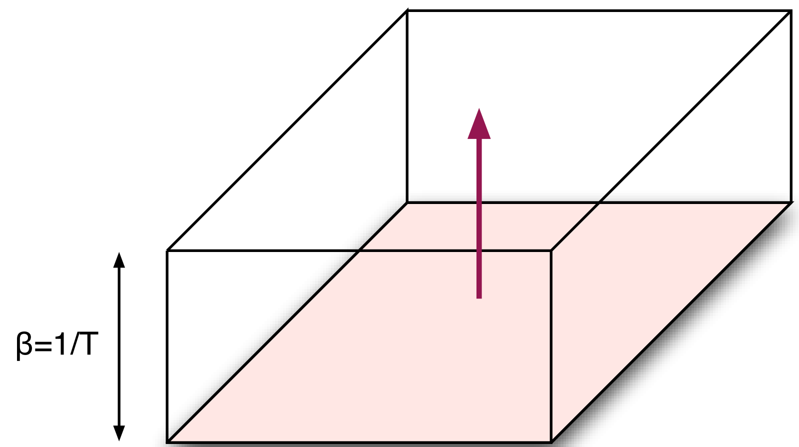
Confined $\langle Tr_F P \rangle = 0$

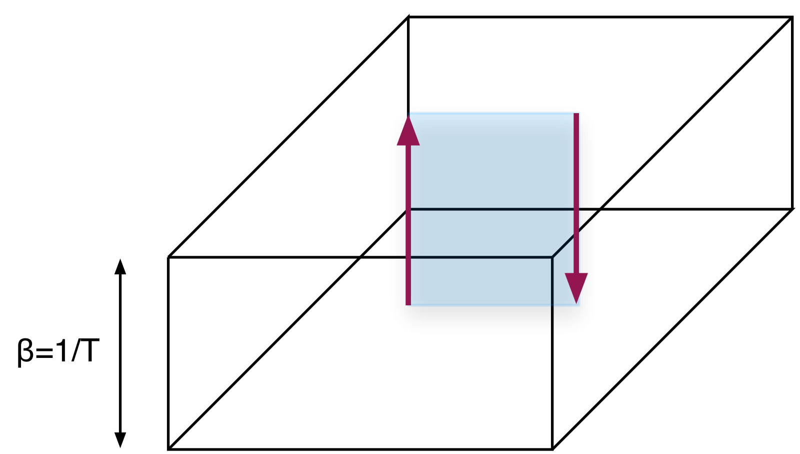
Deconfined $\langle Tr_F P \rangle \neq 0$

- The confinement-deconfinement transition in finite-T SU(N) gauge theories is associated with breaking of a global Z(N) symmetry.
- Confinement = Z(N) symmetry + mass gap

Svetitsky-Yaffe Universality

- Dimensional reduction: 4d gauge theory to effective 3d model
- SU(N) gauge theories are in the universality class of Z(N) spin systems
- Fundamental representation quarks explicitly break Z(N) symmetry, behaving like an external magnetic field.
- High T = broken symmetry makes sense via spin system analogy

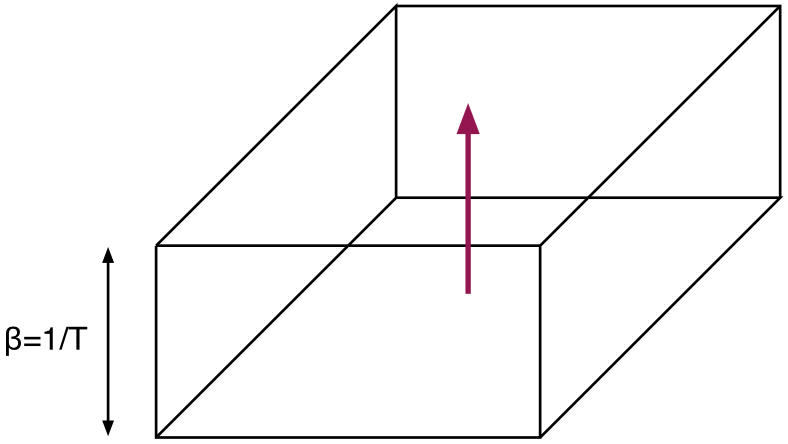




$$\langle Tr_R P(\vec{x}) \, Tr_R P^{+}(\vec{y}) \rangle \simeq \exp\left[-\frac{\sigma_R^{(t)}}{T} |\vec{x} - \vec{y}| \right]$$

Deconfinement at high T

Free energy density of a boson in a representation R with spin degeneracy s moving in a Polyakov loop background P at non-zero temperature and density. The fermion expression is similar.



$$V_b = sT \int \frac{d^d k}{(2\pi)^d} Tr_R \left[\ln\left(1 - P e^{\beta\mu - \beta\omega_k}\right) + \ln\left(1 - P^+ e^{-\beta\mu - \beta\omega_k}\right) \right]$$

With standard boundary conditions (periodic for bosons, antiperiodic for fermions), perturbative effects always favor the deconfined phase.

$$V_b = -sT \int \frac{d^d k}{(2\pi)^d} \sum_{n=1}^{\infty} \frac{1}{n} \left[e^{n\beta\mu - n\beta\omega_k} Tr_R P^n + e^{-n\beta\mu - n\beta\omega_k} Tr_R P^{+n} \right]$$

m=0, μ=0, P=I gives black body formula!

μ≠0 shows sign problem at finite density

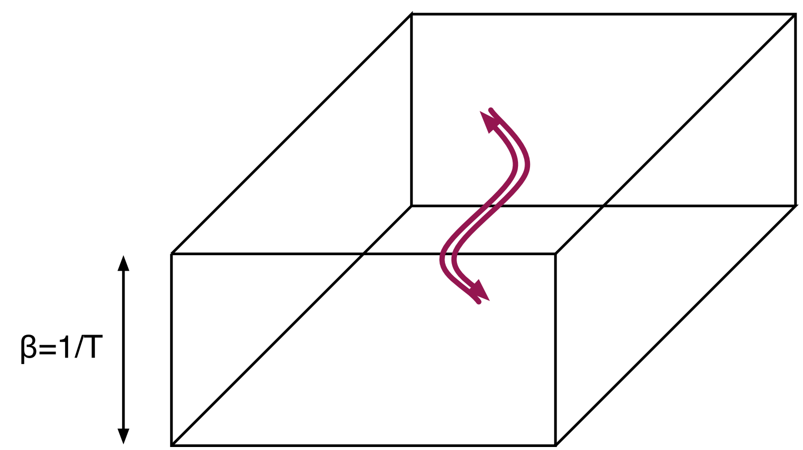
Restoring Confinement via Adjoint Fermions

$$\int d^4x \, \frac{1}{4g^2} \left(F^a_{\mu\nu}\right)^2 \to \int d^4x \, \frac{1}{4g^2} \left(F^a_{\mu\nu}\right)^2 + \int d^4x \, \bar{\psi} \left(\gamma \cdot D + m\right) \psi$$

- Adjoint fermions preserve Z(N) symmetry
- Periodic boundary conditions required: no longer a thermal ensemble
- Advantages: local, renormalizable field theory
- Disadvantages: dynamical fermions expensive to simulate; chiral symmetry breaking an issue; complicated phase diagram

$$Z = Tr\left[(-1)^F e^{-\beta H}\right]$$

$$\frac{1}{2} \leq N_f \leq \frac{11}{4}$$

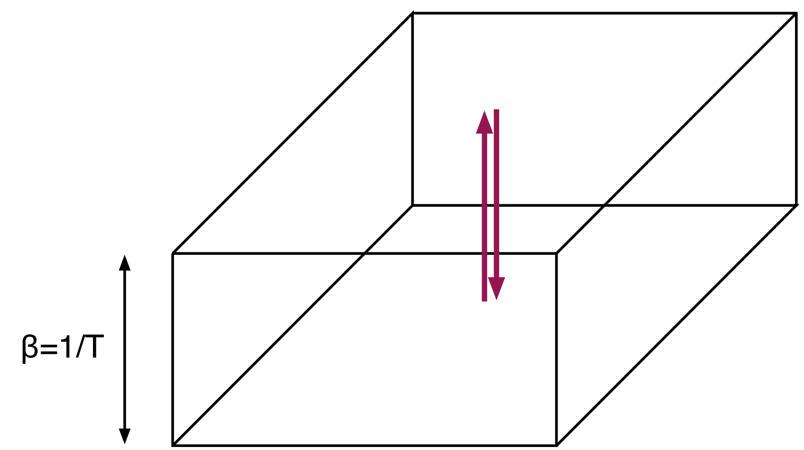


Unsal 2007; 2009

Restoring Confinement via Deformation

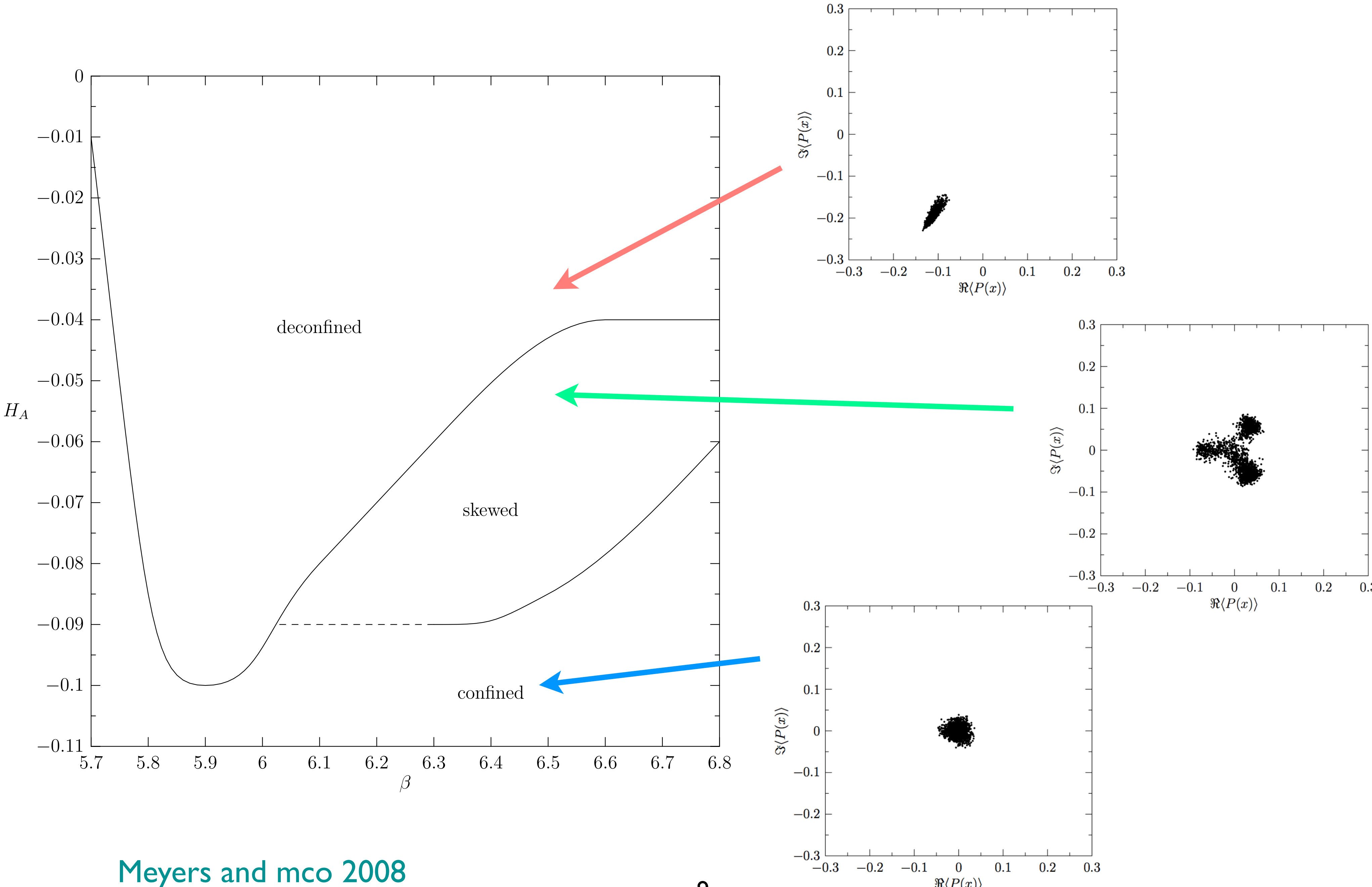
$$\int d^4x \, \frac{1}{4g^2} \left(F^a_{\mu\nu}\right)^2 \to \int d^4x \, \frac{1}{4g^2} \left(F^a_{\mu\nu}\right)^2 - \int d^3x \, h_A \left|Tr_F P\right|^2$$

- Like adding $|\Phi|^2$ term to Landau-Ginsburg free energy to change phase
- Mimics effect of heavy adjoint quark (static limit)
- Advantages: conceptually simple; easy to simulate for small N
- Disadvantages: non-local in time; not a renormalizable theory; harder to simulate as N increases



Meyers and mco 2008

SU(3) via deformation

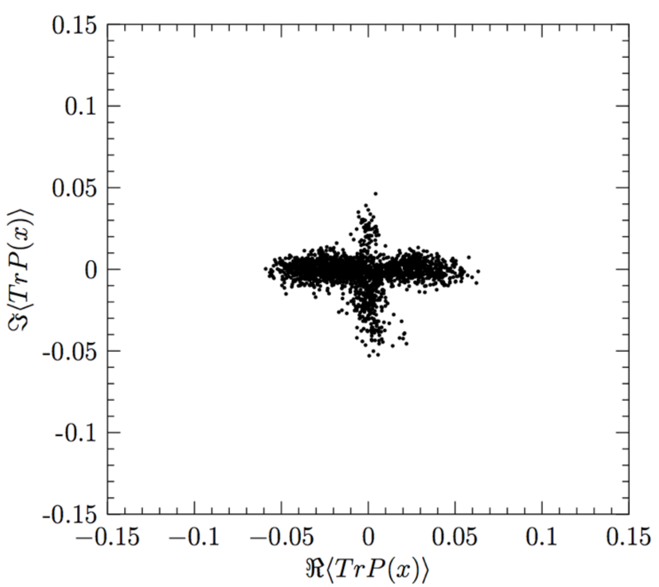


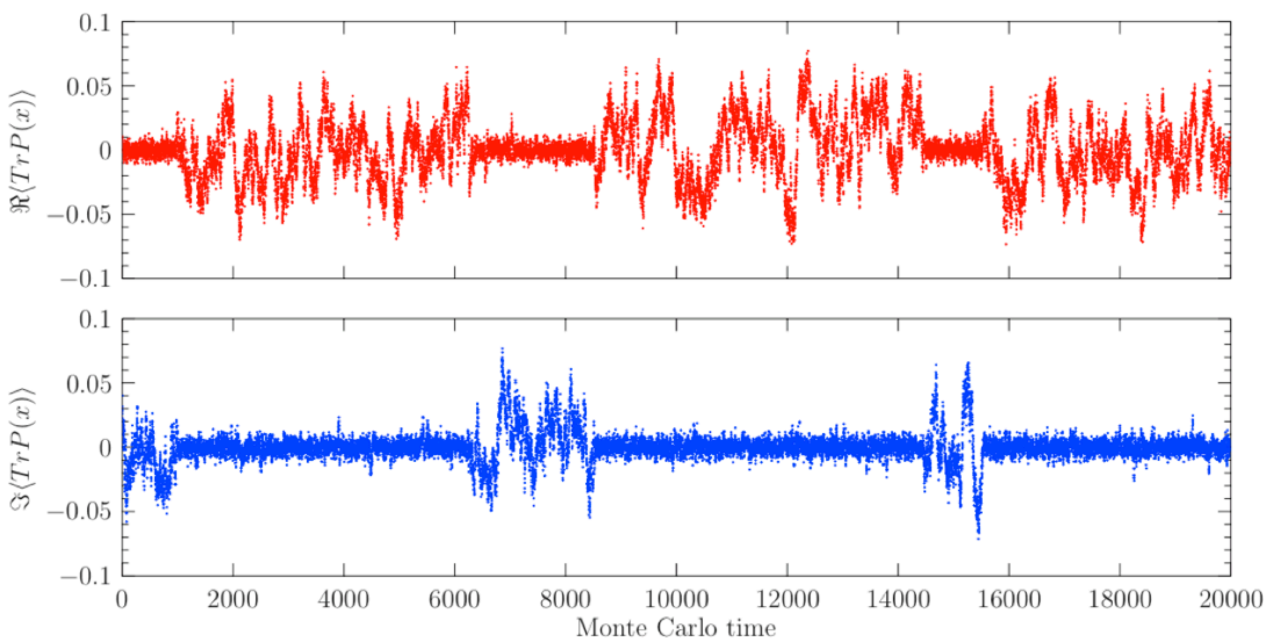
Meyers and mco 2008

Partial Confinement in SU(4)

- Double trace deformation in SU(4) leads to partially confined phase: quarks are confined, but pairs of quarks are not

$$\langle Tr_F P \rangle = 0 \qquad \langle Tr_F P^2 \rangle \neq 0$$





Meyers and mco 2008

What is confinement?

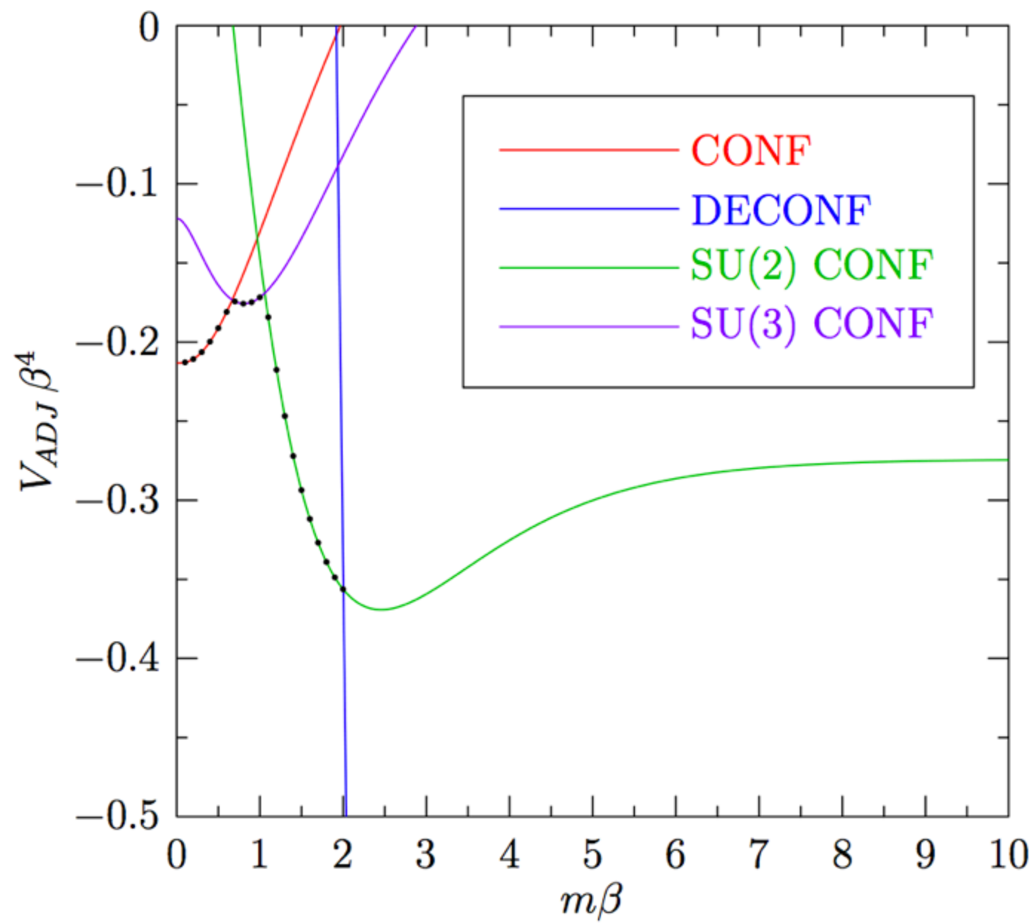
- Confined phase confines all states with non-zero N-ality
- Actually [N/2] order parameters; all are zero in the confined phase
- Eigenvalues of P in confined phase are uniformly spaced on the unit circle

$$Tr_F\left[P_0^k\right] = 0 \rightarrow \det\left(\lambda - P_0\right) = 0 \rightarrow \lambda^N + (-1)^N = 0$$

Meisinger, Miller and mco 2002

Many phases as N increases

- With adjoint fermions, hierarchy of phases between deconfined phase and confined phase
- Not all phases partially confined
- Electric string tensions are calculable from 1-loop effective potential, and are of order $(gT)^2$



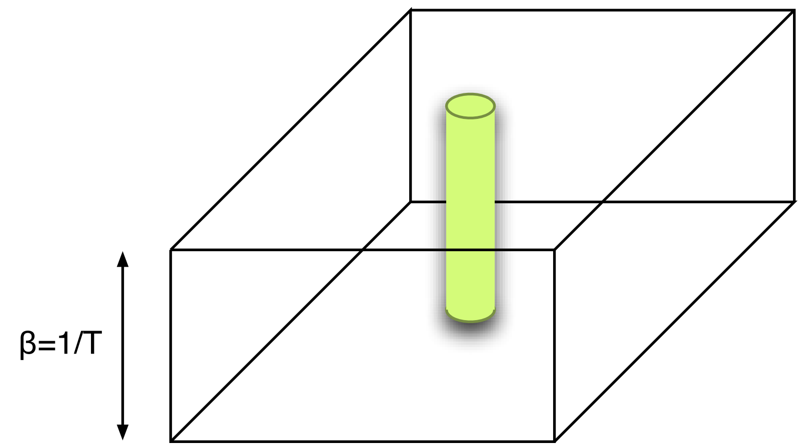
Meyers and mco 2009

$$V_{1-loop}\left(P,\beta,m,N_f\right)=\frac{1}{\pi^2\beta^4}\sum_{n=1}^{\infty}\left[2N_f\beta^2m^2K_2\left(n\beta m\right)-\frac{2}{n^2}\right]\frac{\left|Tr_FP^n\right|^2-1}{n^2}$$

Meisinger and mco 2010

Color Magnetic Monopoles

- A_0 acts as an adjoint Higgs field in 3d.
- $\mathrm{Tr}_F\, P = 0$ implies A_0 breaks SU(N) to $U(1)^{N-1}$
- N different monopoles
- Connects to finite temperature instantons (calorons): each caloron contains N monopoles
  Lee & Lu 1998; Kraan & van Baal 1998
- Monopoles are the complete answer for $N_f=1/2$ ($\mathcal{N}=1$ SuSy) Davies et al. 1999, 2000



Monopoles make a non-perturbative contribution to the partition function

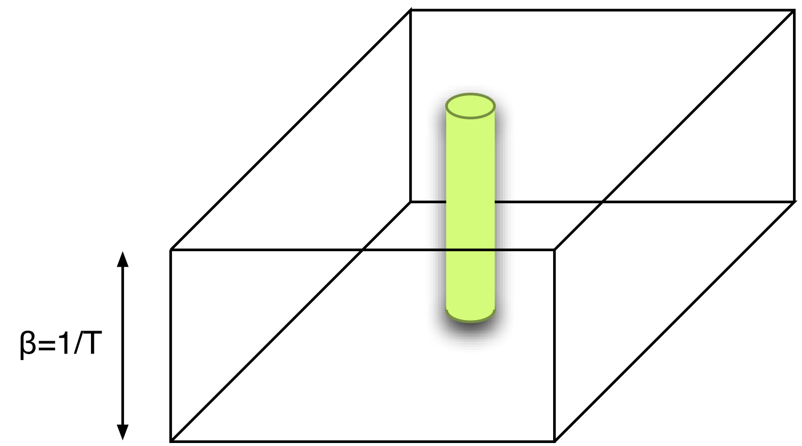
$$\xi \propto \exp\left[-\beta M_{mono}\right] = \exp\left[-c/g^2\right]$$

$$S_{mag} = \int d^3x \left[\frac{T}{2}\left(\partial\rho\right)^2 + 2\xi \sum_{j=1}^{N}\left[1 - \cos\left(\frac{2\pi}{g}\alpha_j \cdot \rho\right)\right]\right]$$

Unsal and Yaffe 2008

Monopoles and Spatial String Tensions

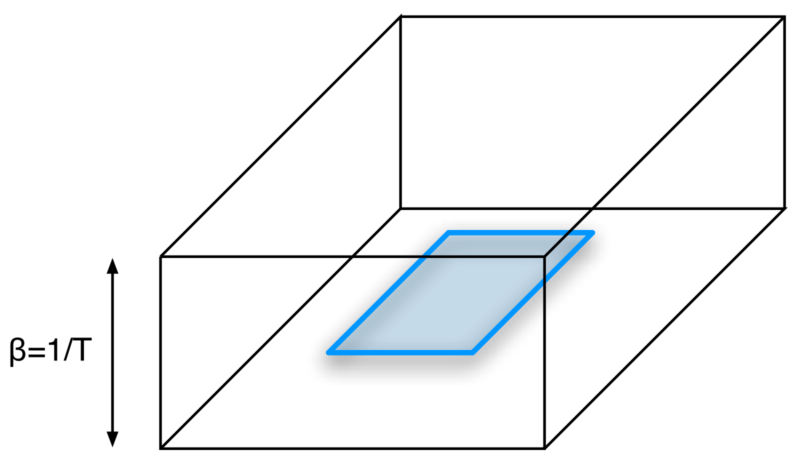
- Monopole gas implies generalized sine-Gordon model
- Realization of dual superconductor picture of confinement
- Narrow region of validity for semiclassical physics; shrinks as N increases
- Physics has correct RG behavior
- Spatial Wilson loops show area law



$$S_{mag} = \int d^3x \left[\frac{T}{2} (\partial \rho)^2 + 2\xi \sum_{j=1}^{N} \left[1 - \cos\left(\frac{2\pi}{g} \alpha_j \cdot \rho \right) \right] \right]$$

$$\xi \propto \exp\left[-\beta M_{mono}\right] = \exp\left[-c/g^2\right]$$

$$\beta N \Lambda \ll 1$$

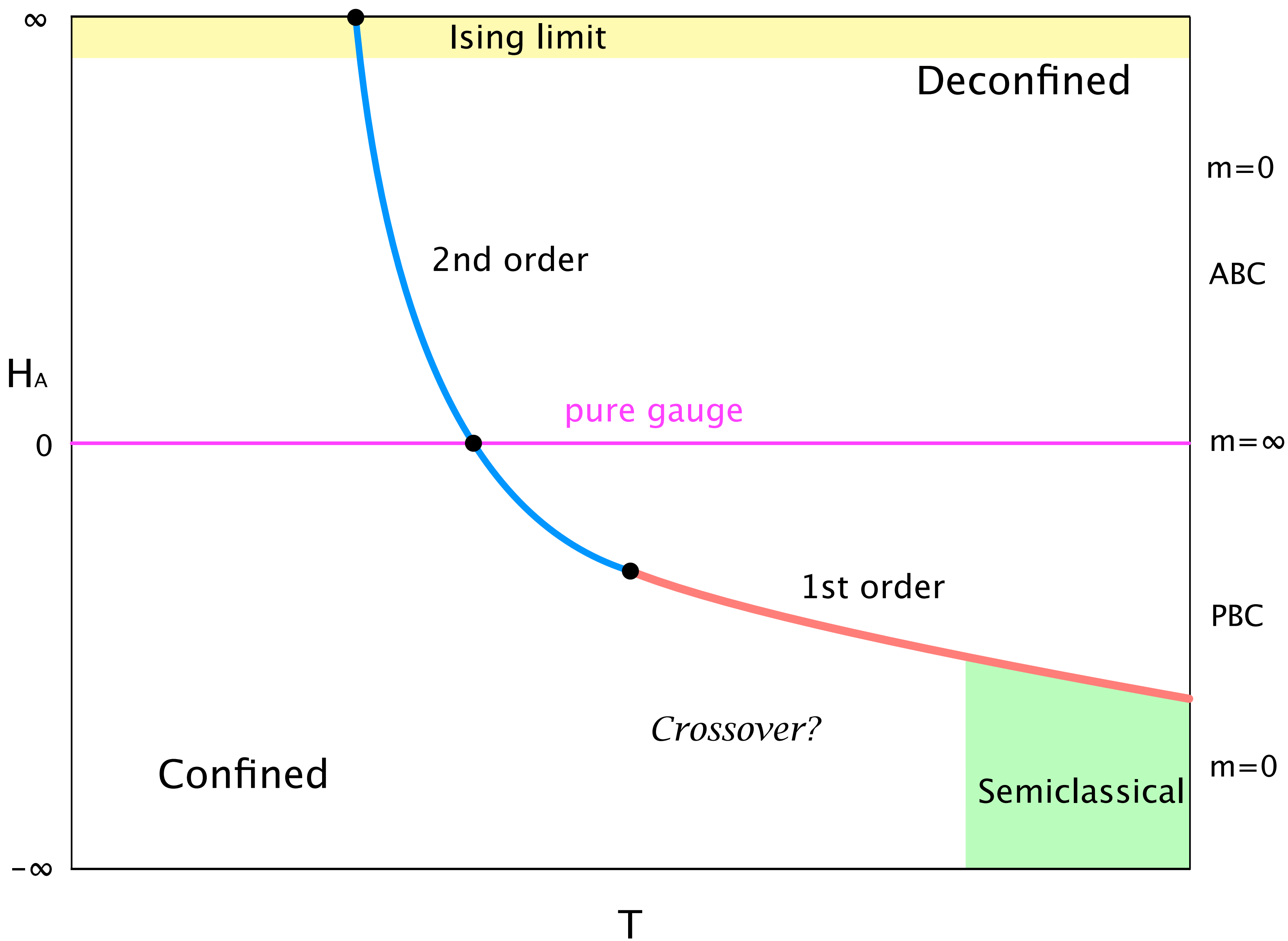


$$\sigma_k^{(s)} \leq \frac{8}{\pi} \left[\frac{g^2 T \xi}{N} k \left(N - k\right) \right]^{1/2}$$

Exact for N=2 & 3!

Meisinger and mco 2010

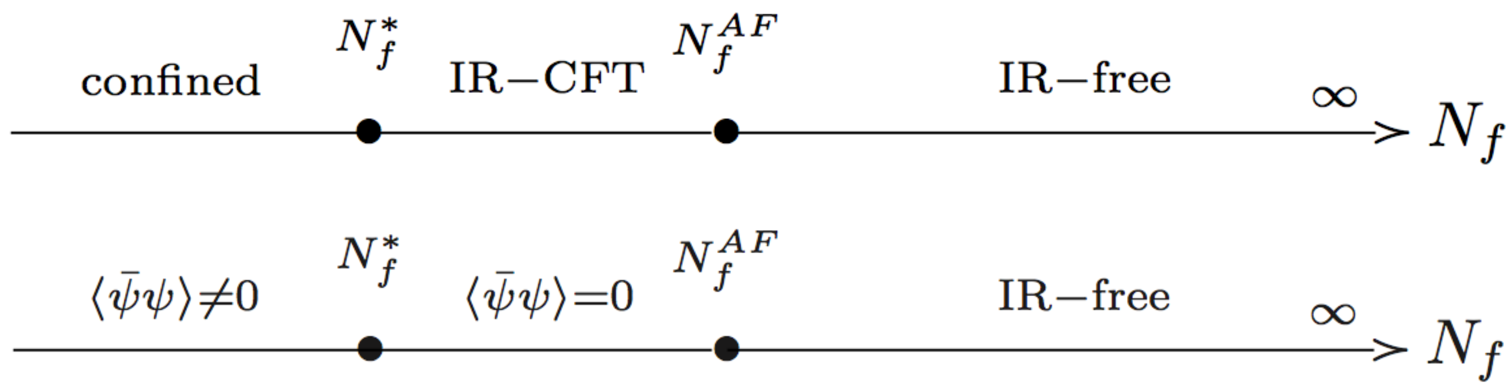
Conjectured SU(2) Phase Diagram



Confinement vs Conformality

Poppitz and Unsal 2009

- Relevance of topological operators as an indicator of confinement

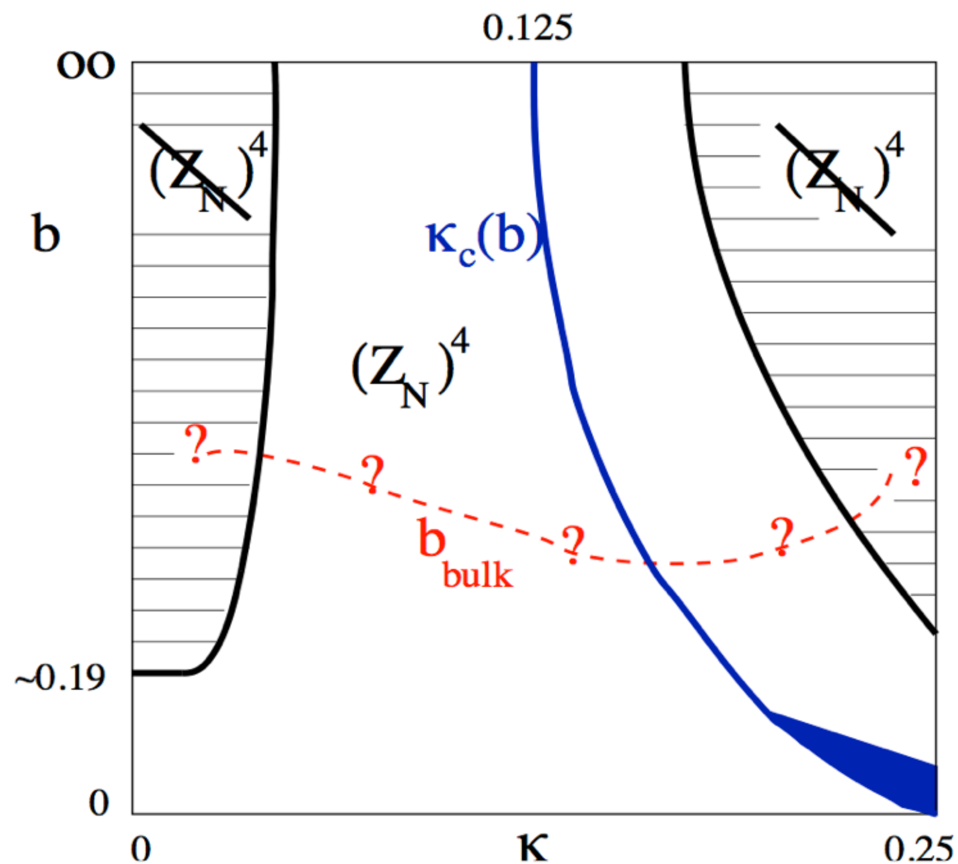


$$\frac{5}{2}N < N_f^D < \frac{11}{2}N, \qquad \text{QCD(F)}$$

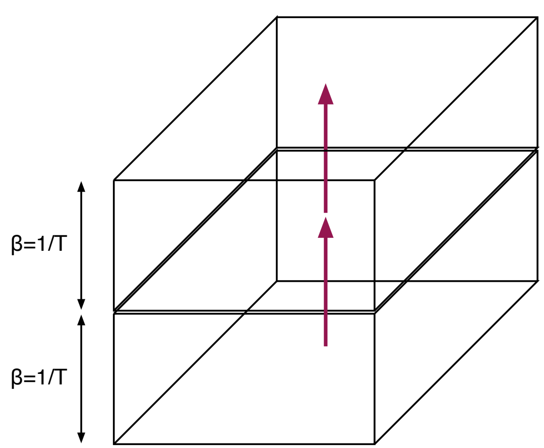
$$4\left(1-\frac{2}{N+2}\right) < N_f^D < \frac{11}{2}\left(1-\frac{2}{N+2}\right), \qquad \text{QCD(S)} \quad N \geq 3$$

Large N

- Large N limit implies existence of "master field"
- Eguchi-Kawai: reduces infinite-volume large-N limit to 1^4 lattice
- Eguchi-Kawai fails if Z(N) breaks; problem for over 20 years
- Eguchi-Kawai may work now!



Bringoltz and Sharpe, 2009



Conclusions

- We have found a region connected to the low-T phase of SU(N) gauge theories where confinement can be understood semiclassically
- Continuum theory with correct RG behavior
- Accessible to lattice simulations
- Lots of questions remain
- Lots of room for further research
- Stay tuned!