PNJL Model and QCD Phase Transitions

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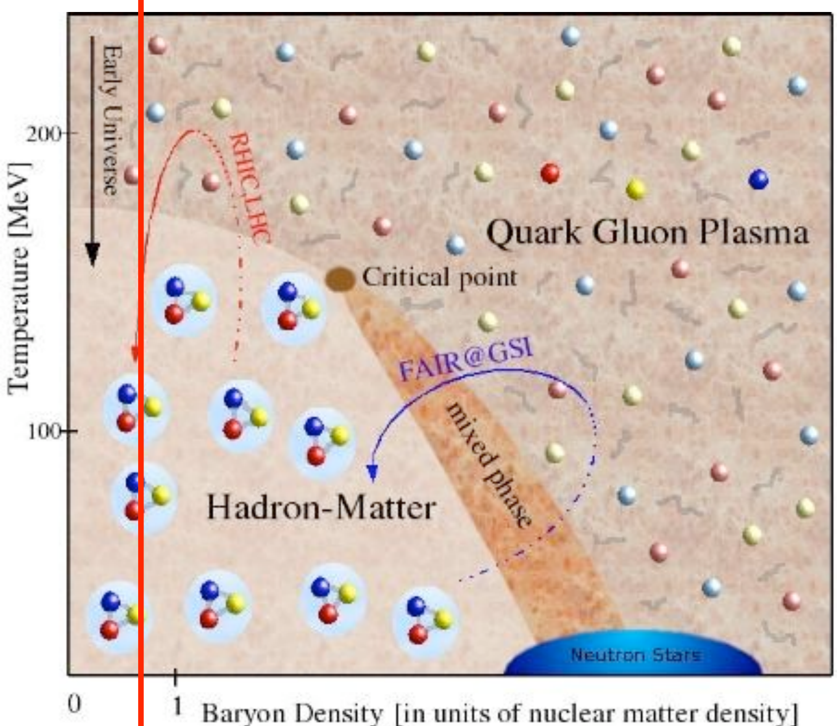
Phase Transitions in Quantum Chromodynamics

This Talk



- Low Temperature

  Lattice and Phenomenology say it is confined.

- High Temperature

  Perturbation theory is reliable and shows deconfinement.

Want to make a phenomenological model which describe both low and high temperature QCD behavior in a single picture.

$\longrightarrow$ PNJL

Outline

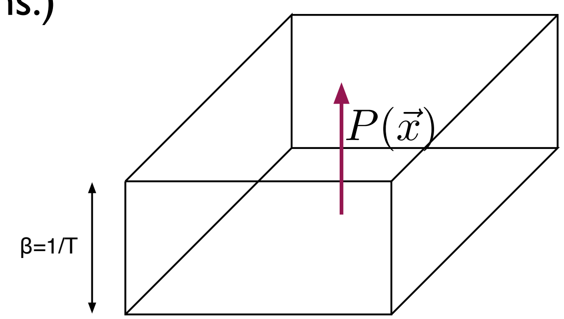
- Confinement and Chiral Symmetry Breaking
- Nambu-Jona Lasinio Model (NJL)
- Polyakov-Nambu-Jona Lasinio Model (PNJL)
  - Fundamental Fermions (Finite-Temperature QCD)
  - Adjoint Fermions
    - Antiperiodic Boundary Conditions (ABC)
    - Periodic Boundary Conditions (PBC)

Confinement

Consider pure gauge theory (i.e. only gauge bosons and no fermions.)

Order Parameter: Polyakov Loop

$P(\vec{x}) = \mathcal{P}e^{i\int_0^\beta dt A_0(x)}$ → The world line of massive quark at $\vec{x}$.

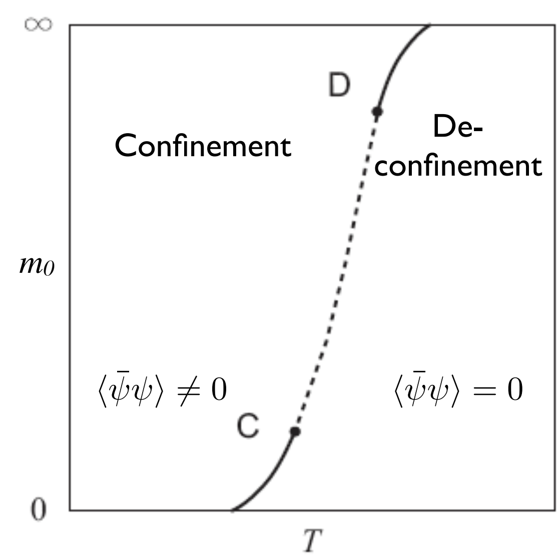


$$\boxed{\langle TrP(\vec{x})\rangle = e^{-\beta F_q}}$$

$g \in SU(N_c)$ $\quad z \in Z(N_c)$: $P(\vec{x}) \to zP(\vec{x})$

$\langle TrP(\vec{x})\rangle = 0 \longrightarrow F_q = \infty$ Confinement

$\langle TrP(\vec{x})\rangle \neq 0 \longrightarrow F_q = Finite$ Deconfinement



<K. Fukushima, PRD69, 2004>

χSB

Condensate of bound quark-antiquark pairs,
almost like Cooper pairs in BCS.

Order Parameter: $\langle\bar{\psi}\psi\rangle$

Dynamically generated quark mass (constituent mass)

NJL Model (1/2): Effective Lagrangian

How does low energy QCD lagrangian look like?

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_0)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + g\bar{\psi}\gamma^\mu A_\mu \psi$$

Guess

1. Effective Quark Theory.
2. "Integrate out" high energy degrees of freedom.
3. Chiral Symmetry.

<Y. Nambu, and G. Jona-Lasinio, PR122 & PR124, 1961>

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma \cdot \partial - m_0\right)\psi + \underbrace{\frac{g_S}{2}\left[\left(\bar{\psi}\lambda^a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda^a\psi\right)^2\right]}_{\boxed{\text{4 Fermion Interaction}}} + \underbrace{g_D\left[\det\bar{\psi}\left(1-\gamma_5\right)\psi + h.c.\right]}_{\boxed{U_A(1)\text{ Breaking}}}$$

- Similar to the microscopic theory of superconductivity.
- Non-renormalization theory in three dimension.

2 Flavors: Set $g_D = 0$.

$$\mathcal{L}_{NJL} \rightarrow \bar{\psi}\left(i\gamma \cdot \partial - m_0\right)\psi + \frac{g_S}{2}\left[\left(\bar{\psi}\lambda^a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\lambda^a\psi\right)^2\right]$$

NJL Model (2/2): Gap Equation

$$\langle\bar{\psi}\psi\rangle = \sigma$$

$$S_{eff} = -i\sum_{j=1}^{N_f} \ln\left[\det\left(i\gamma\cdot\partial - m_{0j} + 2g_S\sigma_j\right)\right] + \int d^4x \left[-\sum_{j=1}^{N_f} g_S\sigma_j^2\right]$$

Effective mass: $m = m_0 - 2g_s\sigma$

$$V_{eff} = 2g_S\sigma^2 - 4N_c\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\sqrt{p^2+m^2} \xrightarrow[\frac{dV_{eff}}{dm}=0]{} m = m_0 + 4g_SN_c\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{m}{\sqrt{p^2+m^2}}$$

"Gap Equation"

If $g_S\Lambda^2 > \dfrac{\pi^2}{8}$, then $\sigma \neq 0$ and chiral symmetry is broken.

- There is a great deal of work on the model.
- A downside is NJL model lacks confinement.

PNJL Model (1/3): Setup

$$S_{eff} = -i \sum_{j=1}^{N_f} \ln \left[\det \left(i\gamma \cdot (\partial - \underline{iA}) - m_{0j} + 2g_S \sigma_j \right) \right] + \int d^4x \left[-\sum_{j=1}^{N_f} g_S \sigma_j^2 \right]$$

- The covariant derivative couples fermions to a background Polyakov loop.
- A_0 is constant and diagonal. ("Polyakov Gauge")
- Interplay between Polyakov loop and σ is now easy to see.

$$\boxed{V_{PNJL} = V_{cond} + V_{zero} + V_{quark,T} + V_{gluon,T}}$$

$$V_{cond} + V_{zero} = 2g_S \sigma^2 - 4N_c \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m^2}$$

$$V_{quark,T} = -2N_f Tr_R [T \int \frac{d^3p}{(2\pi)^3} ln(1 \pm Pe^{-\omega_p/T}) + h.c.] = \frac{4m^2T^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(\mp 1)^n Tr_R P^n}{n^2} K_2\left(nm/T\right)$$

+ Antiperiodic B.C.
- Periodic B.C.

<P. Meisinger, and M. Ogilvie, PRD65, 2002>

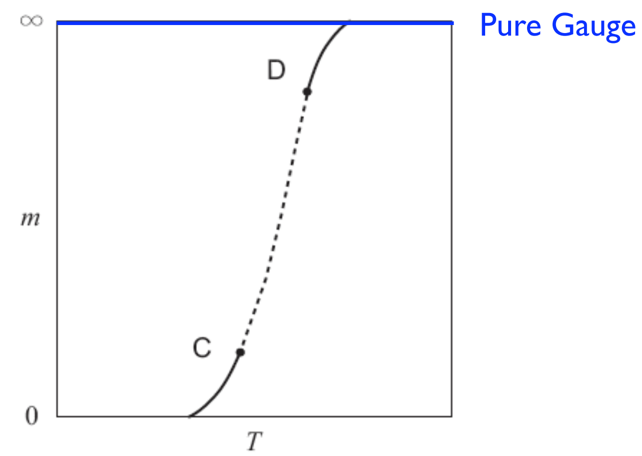
$V_{gluon,T} \cdots$ Need a phenomenological model.

PNJL Model (2/3): Gluonic Part

$$V_{pert} = -2Tr_A[\int \frac{d^3p}{(2\pi)^3} ln(1 - Pe^{-\omega_p/T})] \propto T^4$$

$$\omega_p = |p|$$

$\boxed{\text{Valid at high T}}$



Add subleading term (T^2) by hand to get correct behavior at low T. Introduce "a mass term", M:

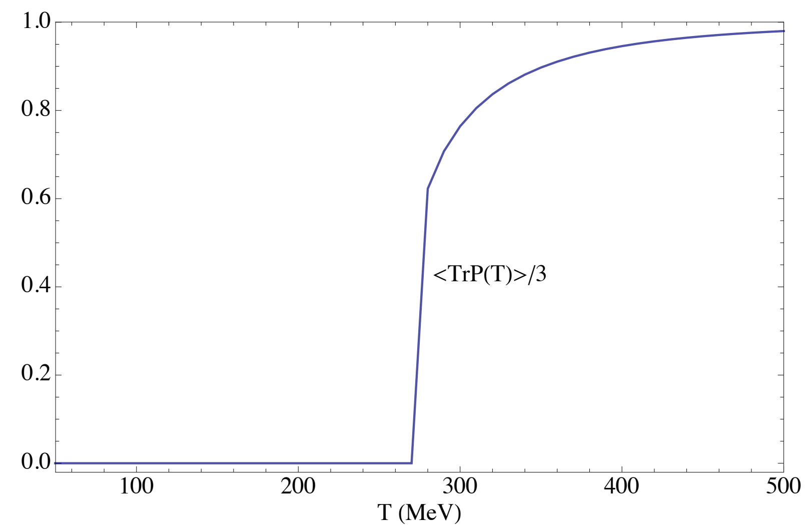
$$\omega_p = \sqrt{p^2 + M^2}$$

M is just a parameter, not mass of gluon.

<P. Meisinger, T. Miller, and M. Ogilvie, PRD65, 2002>

$$V_{gluon,T} = f(P)T^4 + g(P)M^2T^2$$

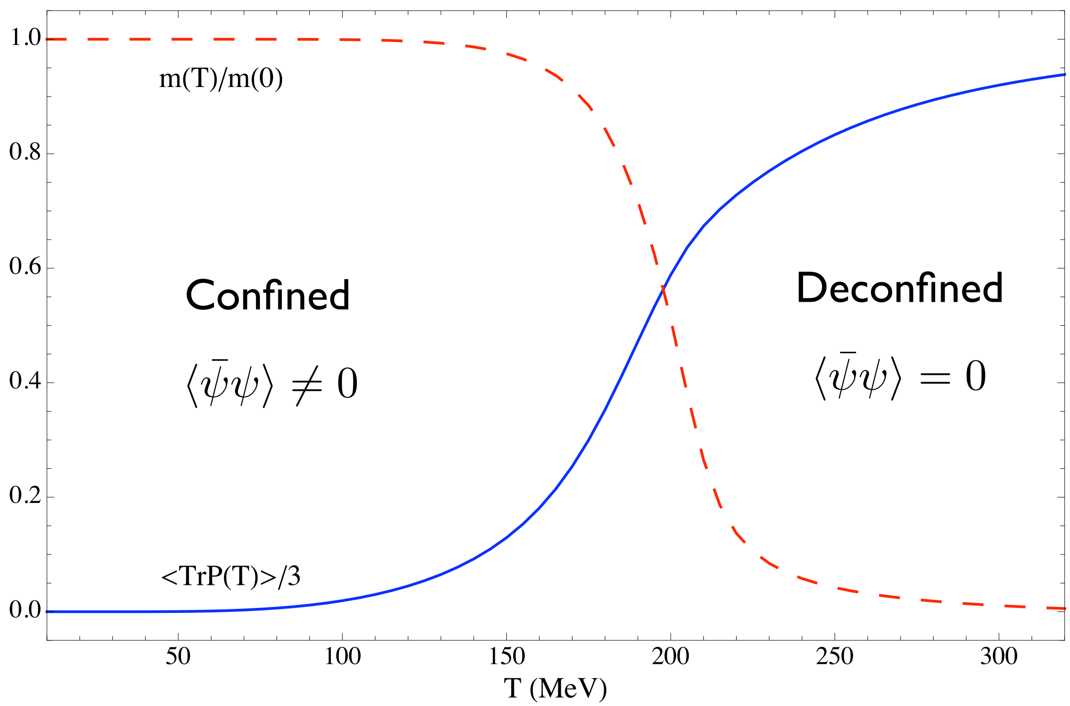
$$M = 596\, MeV \rightarrow T_d = 270\, MeV$$



Pure Gauge

PNJL Model (3/3): Results

Now we have a whole potential: $V_{PNJL} = V_{cond} + V_{zero} + V_{quark,T} + V_{gluon,T}$



Fundamental Fermions (N_f =2)

$m_0 = 5.5\, MeV$

$\Lambda = 631\, MeV$

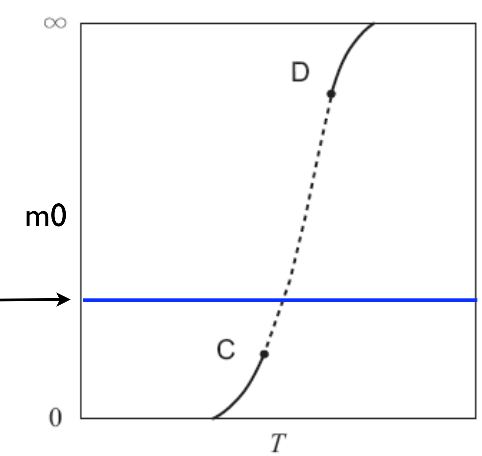
$g_S = 2 \times 5.496\, GeV^{-2}$

<K. Fukushima,PLB59, 2004>

<HN and M. Ogilvie, PRD81, 2010>

PNJL model works!

$m_0 = 5.5\, MeV$ →



Why Adjoint QCD?

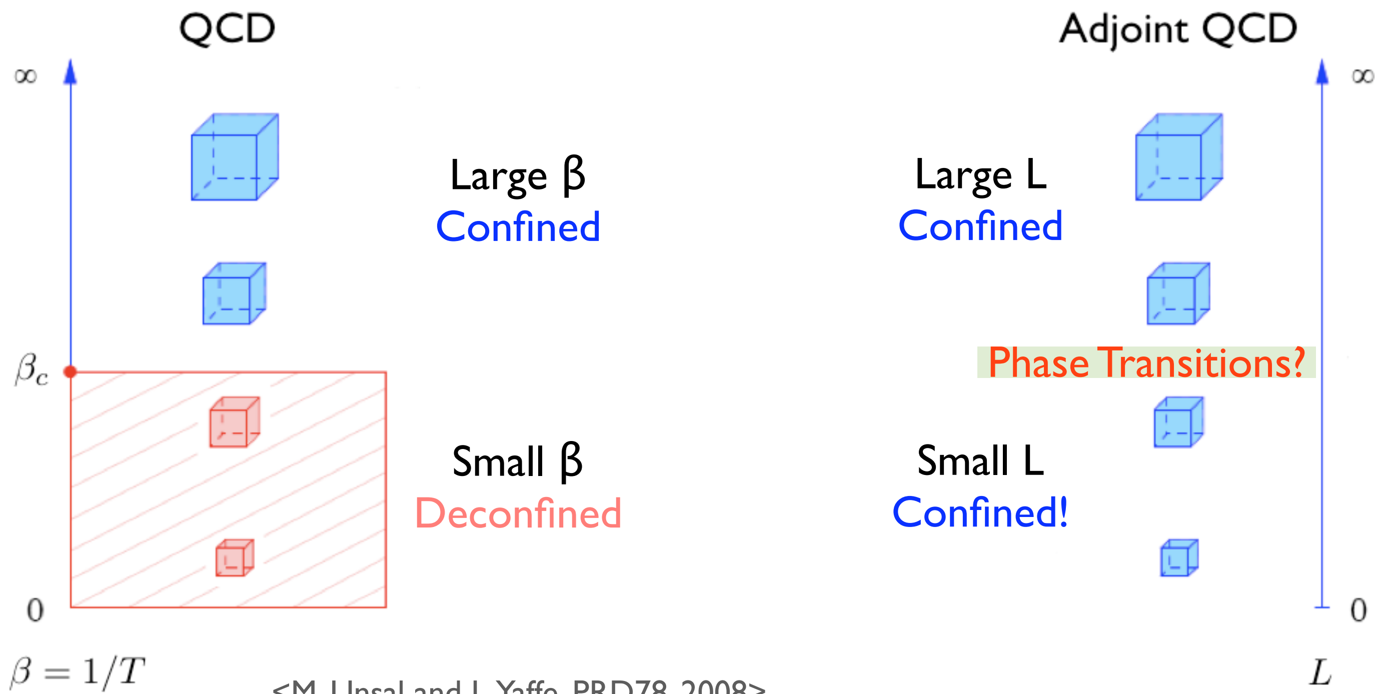
- SUSY
  - $N_f = 1/2$

- Conformal Theory <E. Poppitz and M. Unsal, JHEP0909, 2009>
  - $2 < N_f < N^*$ is in the conformal window.

- Eguchi Kawai Model
  - Volume independence of SU(N) gauge theories in the large N limit.

- Confinement
  - Confinement at "high temperature" or small L.

Confinement with Small Radius



<M. Unsal and L.Yaffe, PRD78, 2008>

$$\mathcal{Z}_E = \int \mathcal{D}A\,\mathcal{D}\Psi\,\mathcal{D}\bar{\Psi}\, e^{-\int_0^\beta d\tau \int d^3x \mathcal{L}_E} = Tr\, e^{-\beta H}$$

ABC: Temporal Component → $\beta = 1/T$
PBC: Spatial Component → $\beta = L$

Adjoint Fermions with PBC

- Confinement at "high T" or small radius.

  <P. Kovtun, M. Unsal, and L. Yaffe, JHEP, 2007>

- New confinement is analytically tractable.

  <M. Unsal, PRD, 2009>

Adjoint fermions with ABC (finite T) in PNJL

Lattice shows that scale of deconfinement and chiral symmetry restoration are different!

$$T_\chi/T_d \approx 7.8$$

<J. Engels, S. Holtmann, and T. Schulzel, NPB724, 2005>

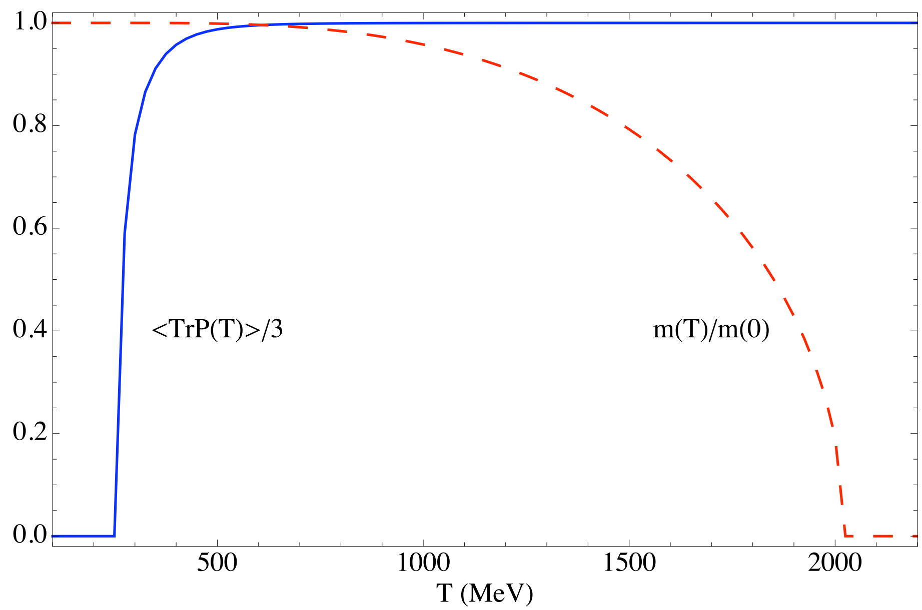
Choose the parameters;

$$m_0 = 0$$

$$\Lambda = 23.22\,GeV$$

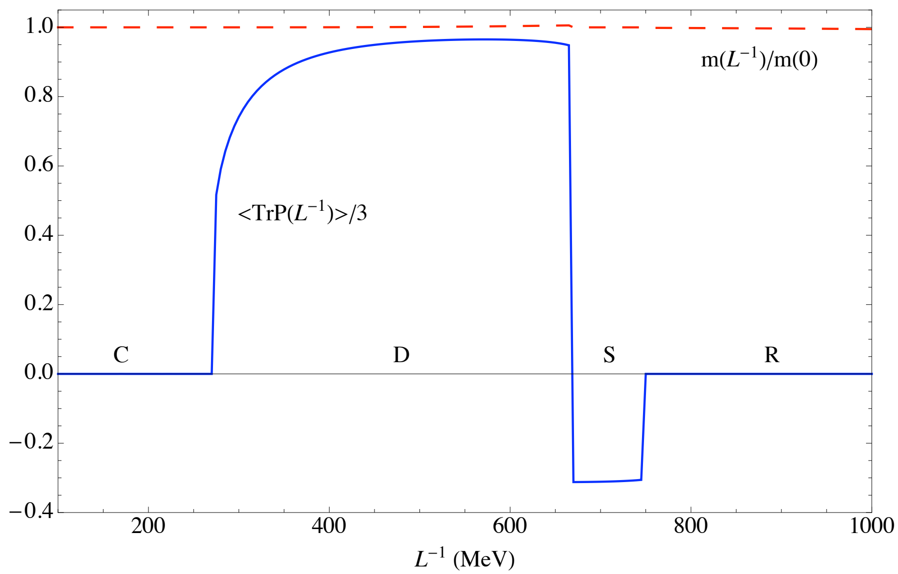
$$g_S\Lambda^2 = 1.2653$$

such that $T_\chi/T_d \approx 7.8.$



<HN and M. Ogilvie, PRD81, 2010>

Adjoint fermions with PBC in PNJL (1/2)



<HN and M. Ogilvie, PRD81, 2010>

- No chiral symmetry restoration up to 10 GeV.
- 3 distinct 1st order phase transitions for TrP.

$m_0 = 0$

$\Lambda = 23.22\, GeV$

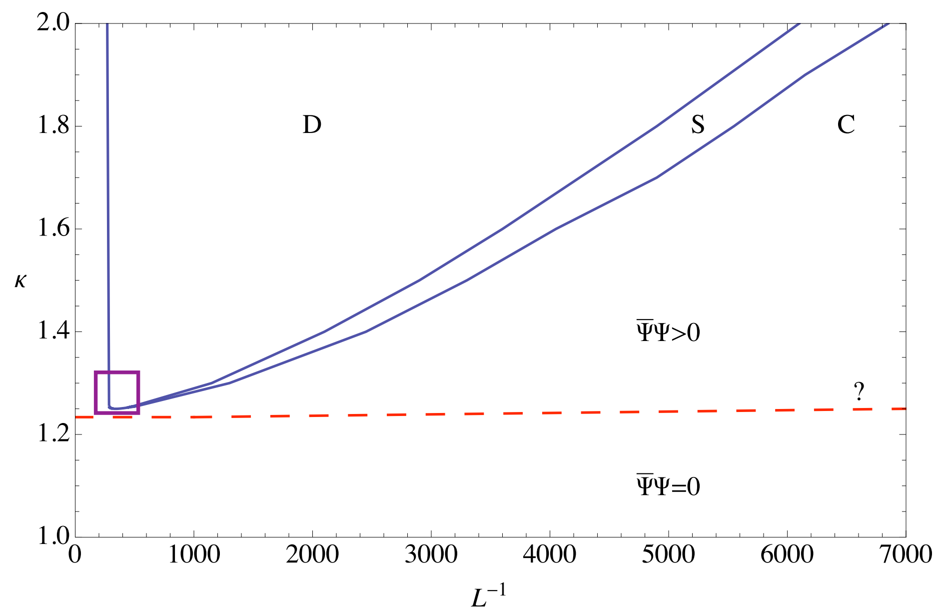
$\kappa = g_S \Lambda^2 = 1.2653$

C = Confined

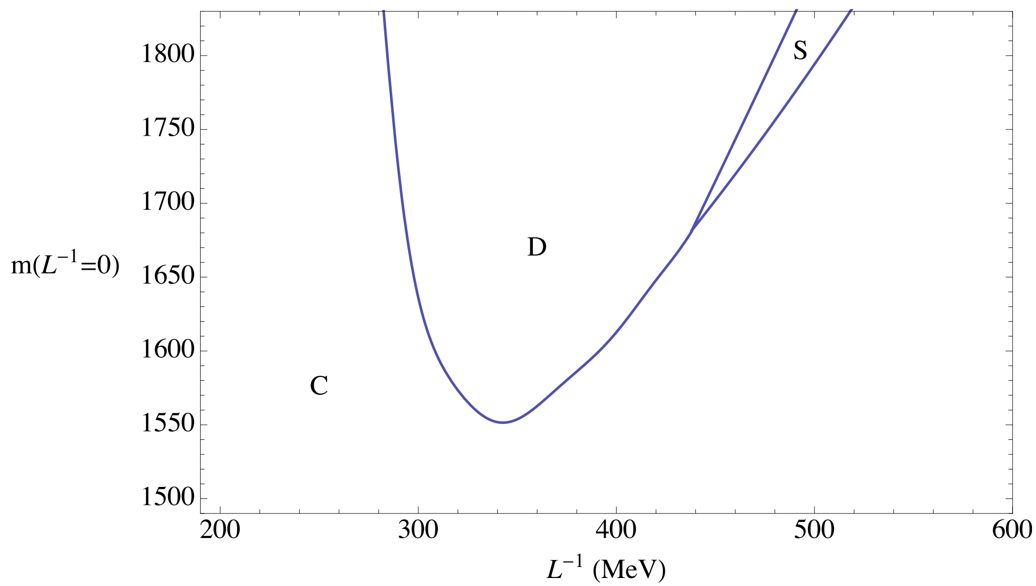
D = Deconfined

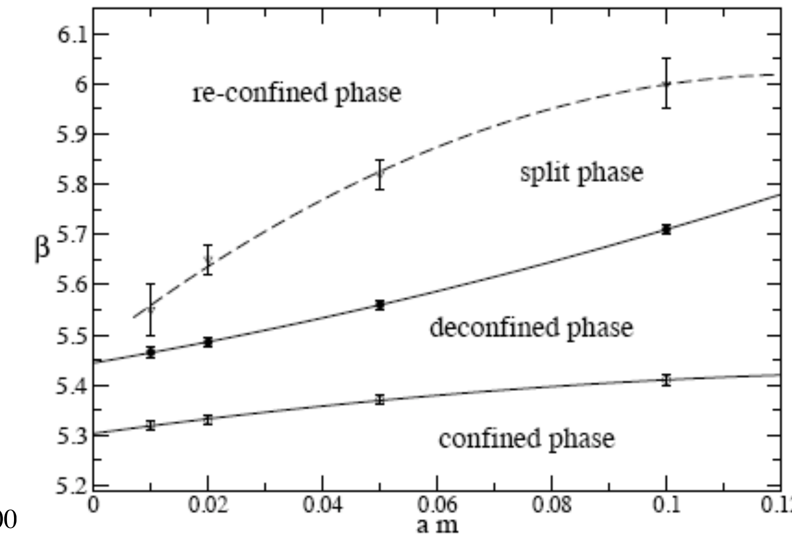
S = Skewed Phase

R = Reconfined



Adjoint fermions with PBC in PNJL (2/2)



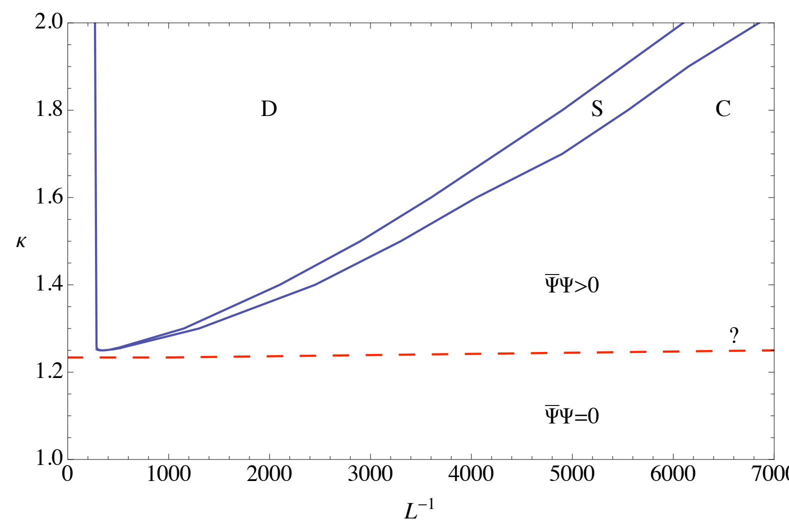


<G. Cossu and M. D'Elia, JHEP0907, 2009>

Remaining Puzzle: Casher's Argument

- Unsal's scenario predicts confinement without chiral symmetry breaking at small L.

- We need more lattice results!



Conclusions

- Two fundamental fermions with ABC.

  The PNJL model can reproduce two crossover transitions at T≈170 MeV.

- Two adjoint fermions with ABC.

  The PNJL model can reproduce $T_\chi/T_d \approx 7.8.$
  PNJL works for finite-temperature cases very well.

- Two adjoint fermions with PBC.

  The PNJL model shows that two confining regions are connected.
  We need more lattice results.