Complex Zeros of the β function, Confinement and Discrete Scaling

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New applications of the renormalization group method
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work done in part with Alexei Bazavov, Alan Denbleyker, Daping Du, Yuzhi "Louis" Liu, Bugrahan Oktay, Alex Velytsky and Haiyuan Zou

Theme of the talk: improvement of approximations

- Physical quantities should not depend on the energy scale (usually denoted μ) or the distance of reference (often denoted r_0) used to specify renormalized couplings.

- Physical quantities should not depend on the RG scale parameter (usually denoted b or s) which expresses the lowering of the UV cutoff ($\Lambda \longrightarrow \frac{\Lambda}{b}$) or equivalently the increase of the lattice spacing ($a \longrightarrow ba$).

- Unfortunately, due to approximations such as perturbative expansions or local potential approximations, physical quantities become artificially dependent on unphysical scale parameters.

Approximations in need of improvement

The talk is divided into two parts:

1. Continuous scaling (with r_0/a or μ). How to improve weak coupling expansions in QCD? Expect digressions about large field contributions to the partition function.

2. Discrete scaling (with b). Numerical calculations using the hierarchical model (b^D integer). Numerical instabilities encountered while attempting to extend b^D to non-integer values will be discussed by Yuzhi "Louis" Liu. An important question not addressed in the talk: how to improve the hierarchical and local potential approximations?

Continuous scaling

- How does the mass gap depend on the bare coupling in 4D lattice gauge theory and 2D $O(N)$ sigma models ?

- Conjecture: in the infinite volume limit, the complex zeros of the nonperturbative β_{CS} function delimit the boundary of a region without (Fisher's) zeros of the partition function in the complex $\beta = 2N_c/g^2$.

- Haiyuan Zou will discuss finite volume aspects.

- For confining models, these zeros stay away from the real axis (in the complex coupling plane).

Discrete scaling

- Numerical block-spinning in configuration space can only be done for an integer number of sites b^D (hierarchical model, Migdal-Kadanoff).

- Attempts of continuations at non-integer b^D lead to numerical instabilities (Yuzhi “Louis” Liu’s talk).

- The discrete scaling allows (small, order $10^{-11} - 10^{-16}$ in 3D examples) log-periodic corrections to the scaling laws which in principle conflict with the continuum limit and are amplified in series expansions.

- Critical exponents have a (small, order 10^{-4} in 3D examples) b-dependence connecting smoothly with the $b \rightarrow 1$ limit (Wilson-Polchinski equation in the local potential approximation (Litim, Bervilliers , ...)).

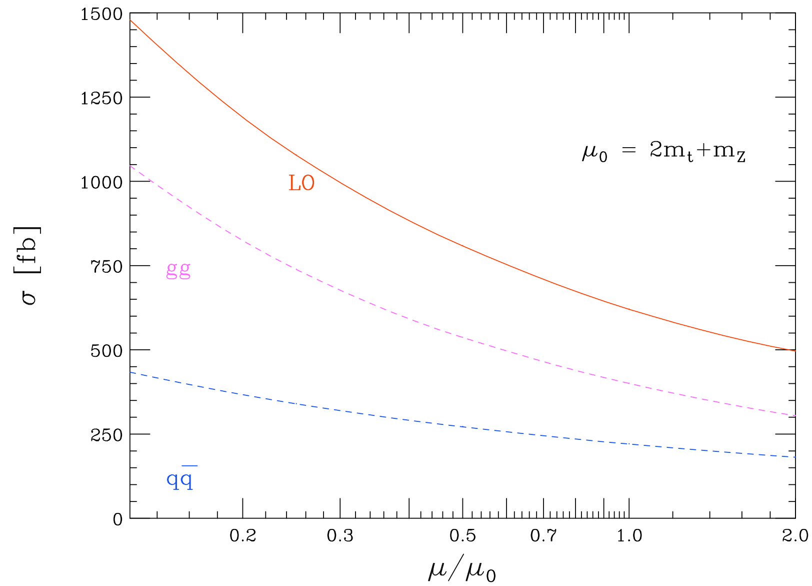
Continuous scaling: μ-independence

Physical quantities such as renormalized n-point functions $\Gamma^{(n)}$ should not depend on the scale μ used to define the renormalized coupling g. This generates Callan-Symanzik equations

$$(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - n\gamma(g))\Gamma^{(n)}(p, g, \mu) = 0$$

Unfortunately, due to the truncation of perturbative series, significant μ dependence is often observed in lowest order perturbative QCD calculations.

$pp \longrightarrow t\bar{t}Z$: LO



- Significant scale dependence contributes to $\Delta\mathcal{N}$.
- NLO QCD calculation is necessary!

Figure 1: From, "NLO QCD Corrections to $t\bar{t}Z$ Production" Thomas McElmurry et al., Loopfest VII and arXiv:0804.2220[hep-ph]

Need for NLO, NNLO, NNNLO, ...

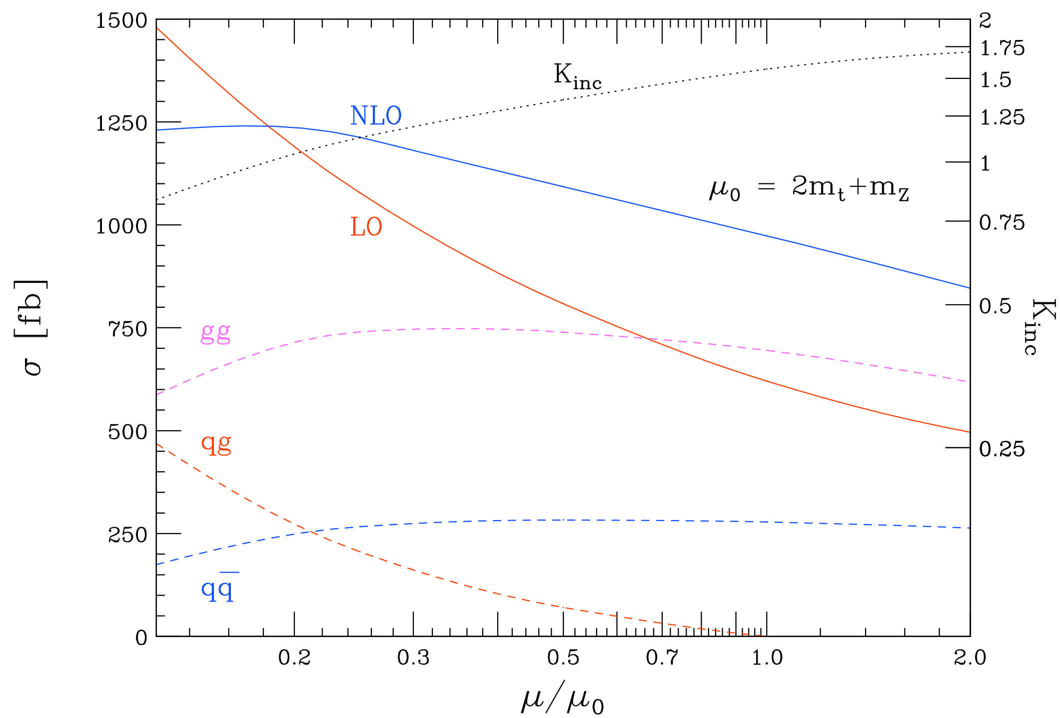
- Higher order corrections reduce the μ dependence.

- NNLO calculations are very time consuming.

- If Dyson's argument for QED (loss of vacuum under $e^2 \to -e^2$) applies to perturbative QCD, the series are not supposed to converge, so the long term prospective of the reduction of the μ-dependence is unclear and requires some optimistic attitude.

- But with LHC data coming up, the short term needs are tremendous

Results



- Vary scale from $\mu_0/4$ to μ_0: uncertainty is $\pm 11\%$.
- $K_{inc} = 1.35$ for $\mu = \mu_0/2$.

Figure 2: From, "NLO QCD Corrections to $t\bar{t}Z$ Production" Thomas McElmurry et al., Loopfest VII and arXiv:0804.2220[hep-ph]

The force scale on the lattice

A slightly easier problem: how does the lattice spacing a expressed in units of, for instance, $r_0 = 0.5fm$ depend on $\beta \equiv 2N_c/g^2$? (Dimensional transmutation)

Note: this β is not the β_{CS} function nor $1/kT_{phys.}$.

For the interval $5.7 < \beta < 6.92$, in pure gauge $SU(3)$, the following empirical power series (Sommers and Necco) is obtained from Wilson loops of various sizes

$$\begin{aligned}\ln(a/r_0) &= -1.6804 - 1.7331\ (\beta - 6) \\ &+ 0.7849\ (\beta - 6)^2 - 0.4428\ (\beta - 6)^3\ .\end{aligned}$$

Parametrization of the non-perturbative part

We proposed the following parametrization (PRD 74, 2006)

$$d\ln(a/r_0)/d\beta = -(4\pi^2/33) + (51/121)\beta^{-1} + A_1 \mathrm{e}^{-A_2\beta}\ .$$

with $A_1 = -1.35\ 10^7$ and $A_2 = 2.82$ (the first two coefficients are universal)

The assumption of $a^2_{pert.}$ corrections (Allton) fixes $A_2 = 8\pi^2/33 \simeq 2.4$ which is close to the value 2.82 obtained above.

Can we extract A_1 and A_2 from (factorially diverging?) weak coupling series for Wilson loops, or semi-classical arguments?

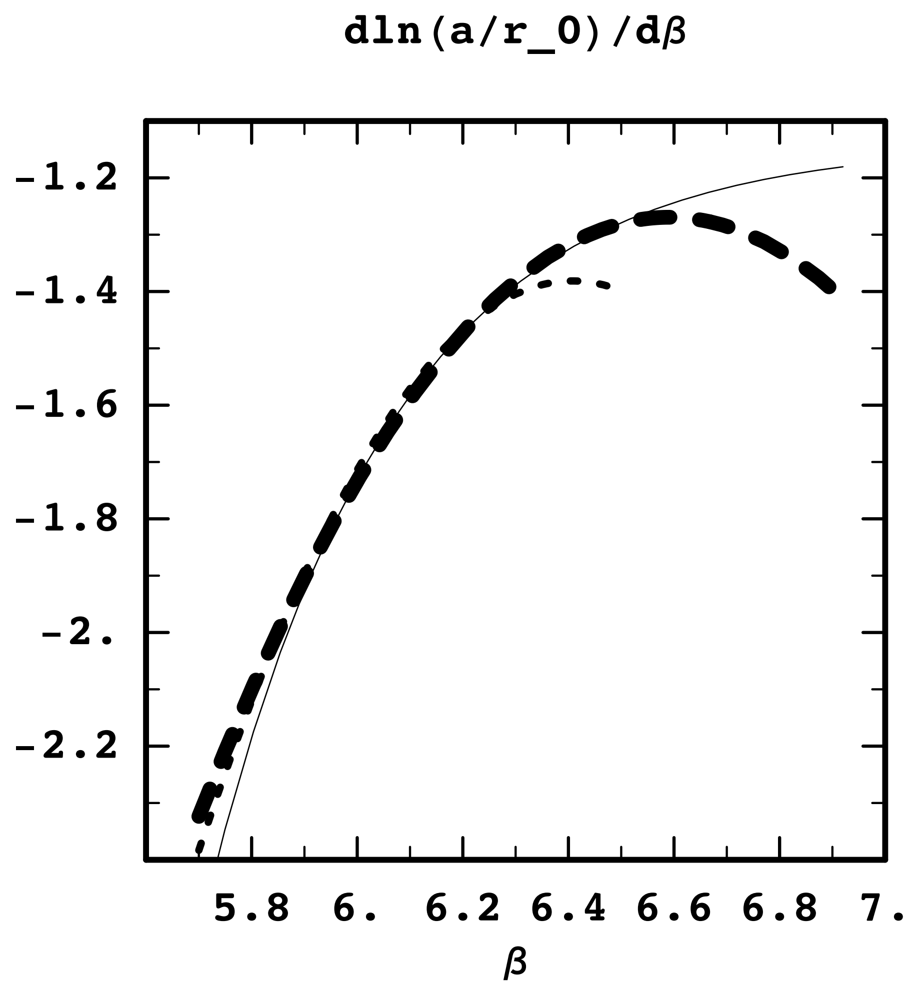


Figure 3: $d\ln(a/r_0)/d\beta$ using Necco et al. (thick dashes), and Guagnelli et al. (small dashes) and our parametrization (solid line).

The rule of thumb for divergent series

Drop the order with the smallest contribution (and all higher orders) in an asymptotic series: $A \sim \sum_k a_k \lambda^k$.

Error at order $k \equiv \Delta_k(\lambda) = A_{numerical}(\lambda) - \sum_{l=0}^{k} a_l \lambda^l$. We assume that $\Delta_k \simeq \lambda^{k+1} a_{k+1}$ (for λ small enough).

Large order behavior: $|a_k| \sim |C_1||C_2|^k \Gamma(k + C_3)$.

The error is minimized for $k^\star \simeq (\lambda|C_2|)^{-1} - C_3 - (1/2) + \mathcal{O}(1/k^\star)$.

$Min_k \quad |\Delta_k| \quad \simeq \quad \sqrt{2\pi}|C_1|(\lambda|C_2|)^{1/2-C_3} \mathrm{e}^{-\frac{1}{|C_2|\lambda}}$ (order independent) . Sometimes C_2 is related to a classical action (instantons ....).

Lattice series show no sign of factorial growth

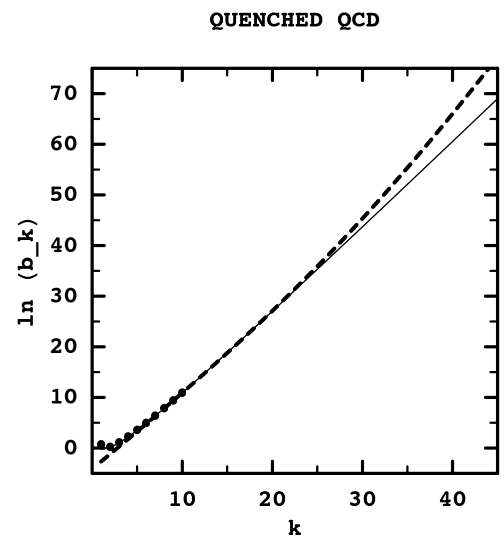


Figure 4: ln(b_k) for a dilogarithm model (power growth, solid line) and an integral model (factorial growth, dashes). The dots up to order 10 are the known values (di Renzo et al.) for the 1x1 Wilson loop. More recent calculations up to order 30 (Rakow, Perlt et al. arXiv:0910.2795) are consistent with a power growth.

Lattice series reflect the zeros of the partition function

The 1x1 Wilson loop (plaquette) $P = -\frac{1}{VZ}\frac{d}{d\beta}Z$.

Singularities can only come from (Fisher's) zeros of Z.

Infinities on the real axis for the second derivatives requires long range correlations (massless particles).

The existence of a mass gap (confinement) keeps the zeros away from the real axis.

Confirmed by MC reweighting on a 4^4 lattice. Higher volume and $SU(2)$ require more systematic methods (density of states).

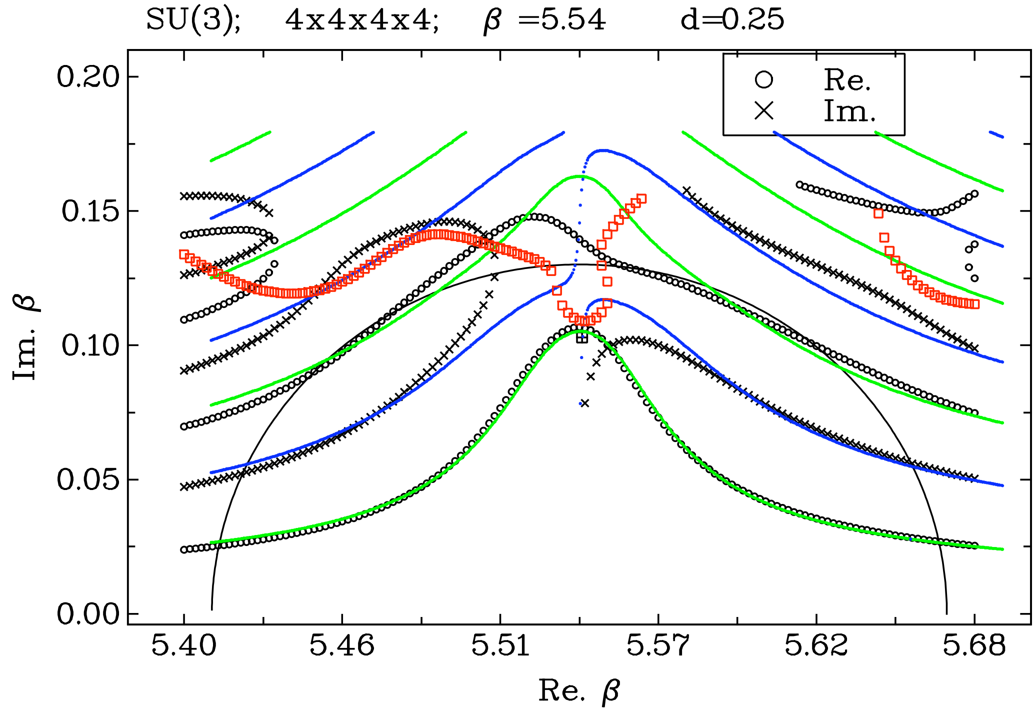


Figure 5: Zeros of the real (crosses) and imaginary (circles) using MC for $SU(3)$ on a 4^4 lattice at $\beta = 5.54$. (see PRD 76 and Lattice 2007 for details).

Perturbation theory for a simple integral

$$\int_{-\infty}^{+\infty} d\phi e^{-\frac{1}{2}\phi^2-\lambda\phi^4} \neq \sum_{0}^{\infty} \frac{(-\lambda)^l}{l!} \int_{-\infty}^{+\infty} d\phi e^{-\frac{1}{2}\phi^2} \phi^{4l}$$

The peak of the integrand of the r.h.s. moves too fast when the order increases. On the other hand, if we introduce a field cutoff, the peak moves outside of the integration range and

$$\int_{-\phi_{max}}^{+\phi_{max}} d\phi e^{-\frac{1}{2}\phi^2-\lambda\phi^4} = \sum_{0}^{\infty} \frac{(-\lambda)^l}{l!} \int_{-\phi_{max}}^{+\phi_{max}} d\phi e^{-\frac{1}{2}\phi^2} \phi^{4l}$$

General expectations: for a finite lattice, the partition function Z calculated with a field cutoff is convergent and $\ln(Z)$ has a finite radius of convergence.

Compact groups means no large field contribtions

Lattice gauge theories with a compact group (e.g., Wilson's lattice QCD) have a build-in large field cutoff: the group elements associated with the links are integrated with dU_l the compact Haar measure. The partition function $Z(\beta)$ is the Laplace transform of $n(S)$, the density of states

$$Z(\beta) = \int_0^{S_{max}} dS\, n(S)\, \mathrm{e}^{-\beta S} \ ,$$

$$n(S) = \prod_{links} \int dU_l \delta(S - \sum_p (1 - (1/N) ReTr(U_p)))$$

ln($n(S)$) is a "color entropy" ($\propto \mathcal{N}_p$, extensive); $n(S) = \mathrm{e}^{\mathcal{N}_p f(S/\mathcal{N}_p)}$

$S_{max} = 2\mathcal{N}_p$ for $SU(2N)$, $\frac{3}{2}\mathcal{N}_p$ for $SU(3)$; ($\mathcal{N}_p$: number of plaquettes)

One plaquette ($SU(2)$)

$Z(\beta) = \int_0^2 dS n(S) \mathrm{e}^{-\beta S} = 2\mathrm{e}^{-\beta} I_1(\beta)/\beta$ (analytical in the entire β plane)

$n(S) = \frac{2}{\pi}\sqrt{S(2-S)}$ (invariant under $S \to 2-S$)

The large order of the weak coupling expansion $\beta \to \infty$ is determined by the behavior of $n(S)$ near $S = 2$, itself probed when $\beta \to -\infty$ in agreement with the common wisdom that the large order behavior of weak coupling series can be understood in terms of the behavior at small negative coupling.

$\sqrt{2-S}$ is easy to approximate near $S = 0$ (radius of convergence $= 2$)

$Z(\beta) = (\beta\pi)^{-3/2} 2^{1/2} \sum_{l=0}^{\infty} (2\beta)^{-l} \frac{\Gamma(l+1/2)}{l!(1/2-l)} \int_0^{2\beta} dt \mathrm{e}^{-t} t^{l+1/2}$ is convergent

The crucial step

$\int_0^{2\beta} dt\mathrm{e}^{-t}t^{l+1/2} \simeq \int_0^{\infty} dt\mathrm{e}^{-t}t^{l+1/2} + O(\mathrm{e}^{-2\beta})$ is responsible for the factorial behavior

The peak of the integrand crosses the boundary near order 2β

Dropping higher order terms (than order $\simeq 2\beta$) agrees with the rule of thumb (minimizing the first contribution dropped)

The non-perturbative part can be fully reconstructed (higher orders + "tails", PRD 74 096005)

For L^4 lattices, the crossing is expected near order $2\beta\mathcal{N}_p$. Non-perturbative effects should be explainable by the contributions near S_{max}.

More large field considerations

The RG has been designed to integrate progressively over large momenta modes. Could we design a procedure to integrate progressively over large field configurations?

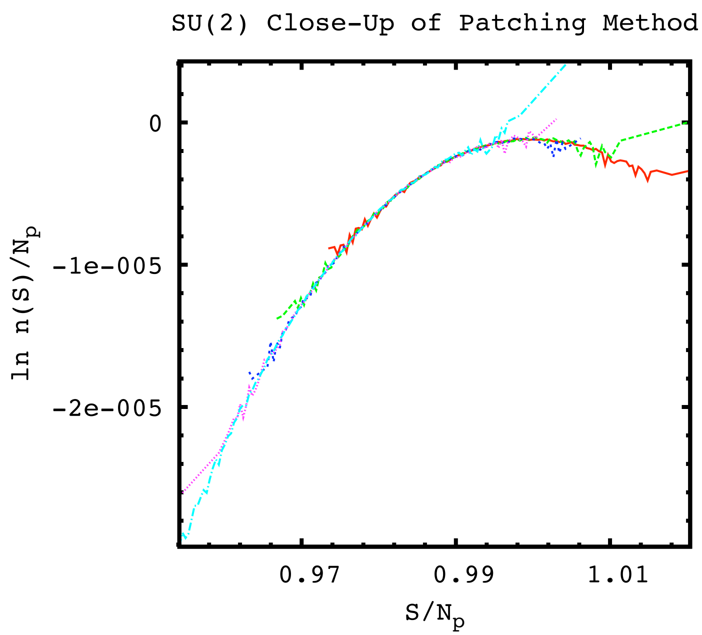
MCRG for $O(N)$ non-linear sigma models:

$$\vec{n}'_{block} = \frac{\sum_{x \in block} \vec{n}_x}{|| \sum_{x \in block} \vec{n}_x ||}$$

By design, $||\vec{n}_{block}|| = ||\vec{n}_x|| = 1$

Naively it looks like we can't build large fields over large blocks (is there some Jacobian compensating?)

Numerical calculation of $n(S)$ for $SU(2)$



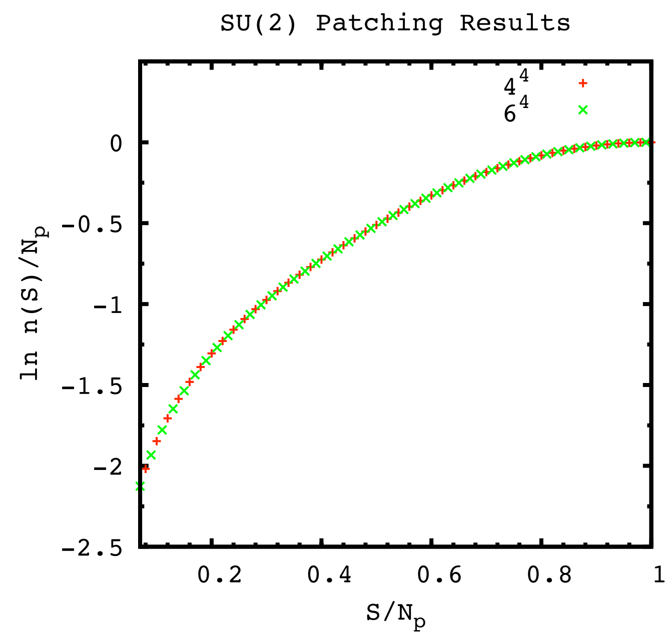


Figure 6: Results of patching $P_\beta(S)e^{\beta S}$ for 4^4 and 6^4 (PRD78 054503).

$U(1)$ lattice gauge theory (A. Bazavov)

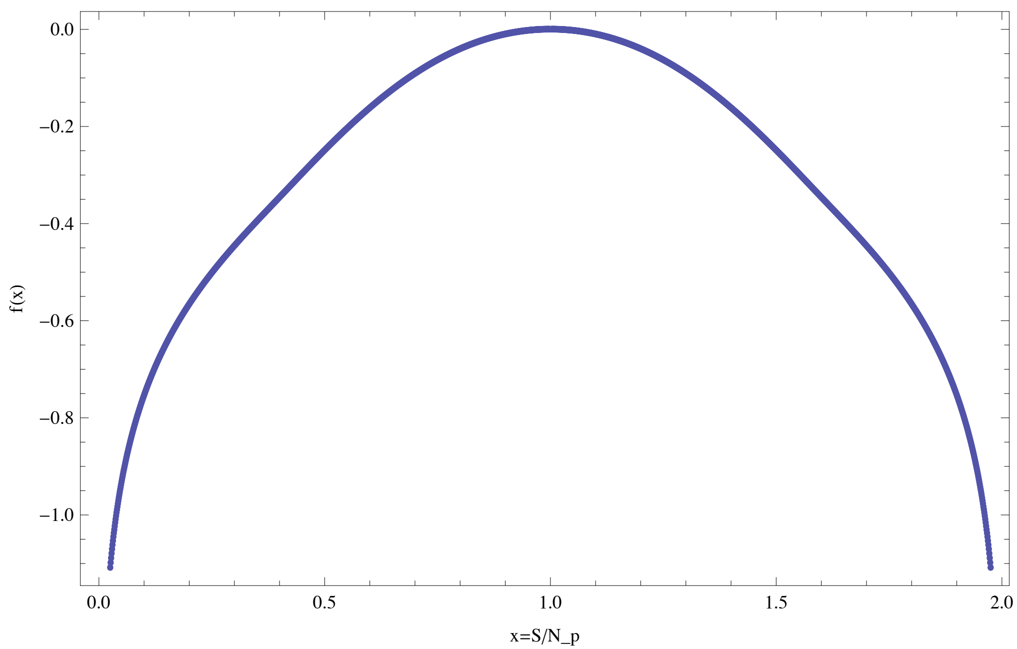


Figure 7: Density of states for $U(1)$ on a 4^4 lattice by multicanonical methods.

Semi-classical calculations

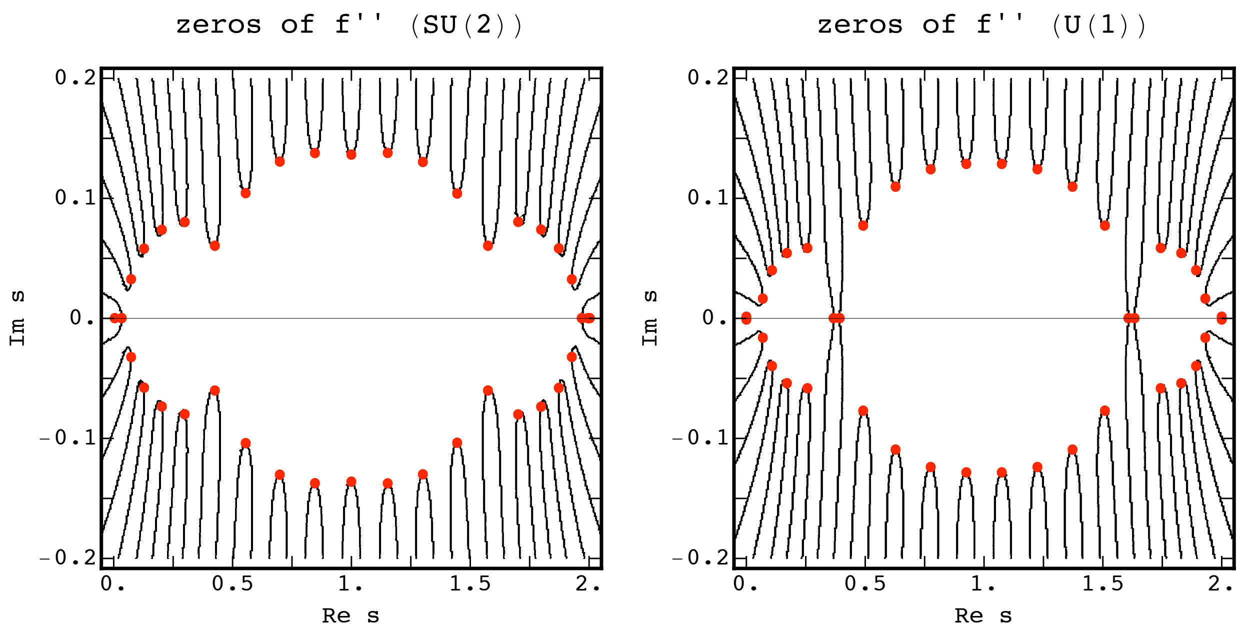


Figure 8: Complex zeros and zeros of the real part of $f''(s)$ in the complex s plane with a Chebyshev (40) on 4^4 for $SU(2)$ (left) and $U(1)$ (right). $f"(s) = 0$ means that Gaussianity breaks down.

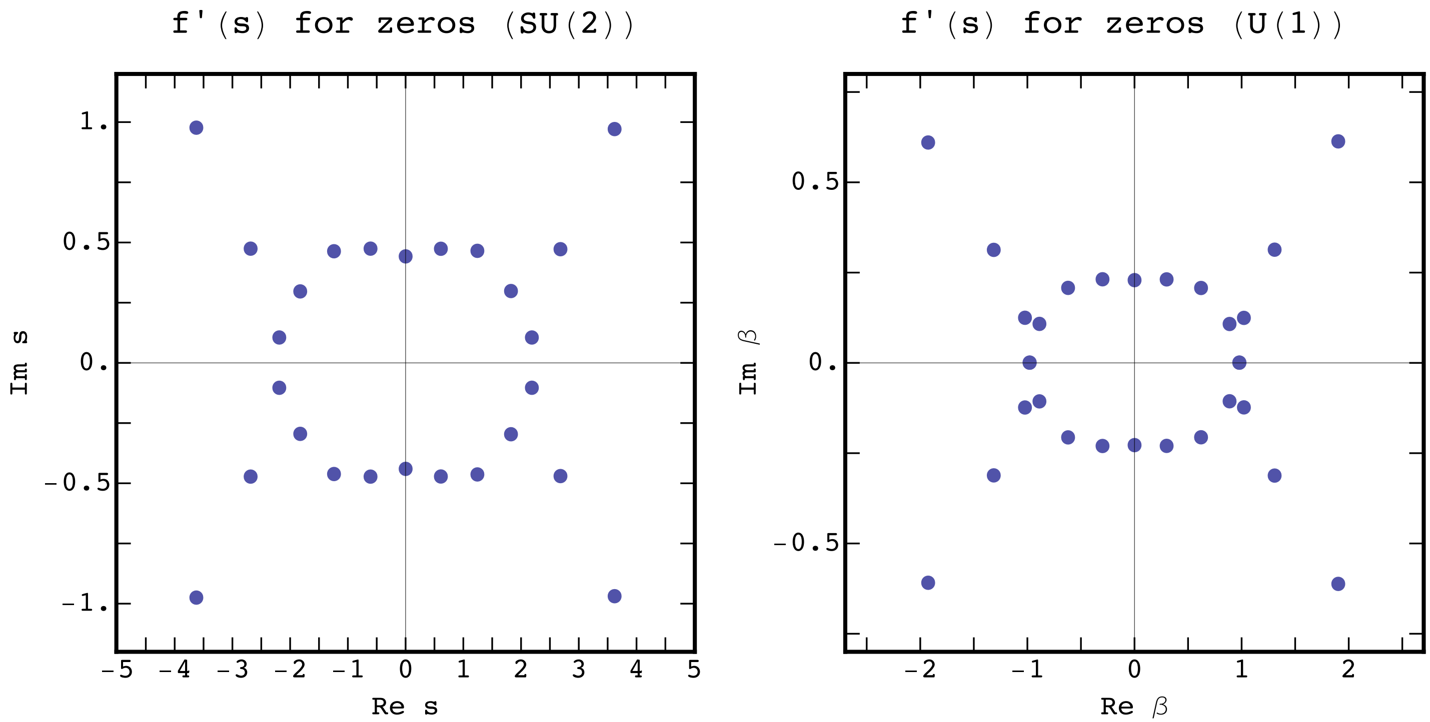


Figure 9: Candidate Fisher's zeros: $f'(s)$ evaluated at the complex zeros of $f''(s)$ shown on the previous figure for $SU(2)$ (left) and $U(1)$ (right). $n(S) = \mathrm{e}^{\mathcal{N}_p f(S/\mathcal{N}_p)}$.

Poles of the average Plaquette = Fisher's zeros

$Z = \sum_{n=0}^{\infty} z_n \beta^n$ with $|z_n| < S_{max}^n / n!$, so at finite volume, Z is an analytical function, not only on the negative real axis, but over the entire β plane.

$P = -(dZ/d\beta)/Z$, and the worse thing that can happen to P is that Z has a zero of order k, say at β_0. Then $(dZ/d\beta)/Z \simeq k/(\beta - \beta_0)$ for $\beta \simeq \beta_0$. If we now integrate over a closed contour C,

$$(i2\pi)^{-1} \oint_C d\beta (dZ/d\beta)/Z = \sum_k n_k(C) \; ,$$

where $n_k(C)$ is the number of zeros of order k inside C . The fact that the loop integral is an integer (no imaginary part) allows to monitor the accuracy of the calculation.

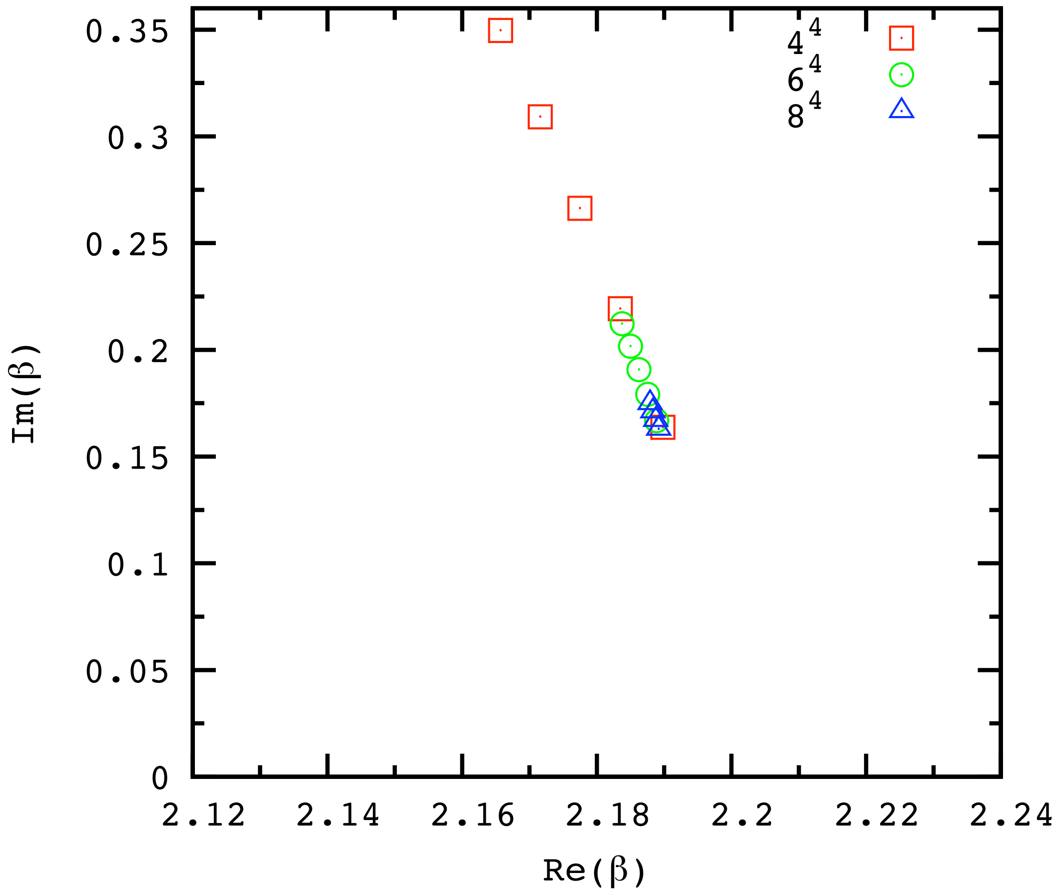


Figure 10: Fisher's zeros for $SU(2)$ on 4^4 , 6^4 and 8^4 (left). The distance between scales approximately $(L)^{-3.7}$. Work done by Daping Du

$$(i2\pi)^{-1}\oint_C d\beta(dZ/d\beta)/Z = \sum_k n_k(C) \ ,$$

where $n_k(C)$ is the number of zeros of order k inside C.

Near $\beta = 0$ everything is regular for $(dZ/d\beta)/Z$ (strong coupling).

At large β we have a regular perturbative series in $1/\beta$ and a non-perturbative part which we assume to be an integer power of the mass gap M^2. As long as we don't cross singular points where $d\beta/dM^2 = 0$ we can change variable from β to M^2 without running into a cut and the loop integral is zero.

$\beta = 2N_c/g^2$ and $d\beta/dM^2 = (2N_c/g^3)(1/M^2)\beta_{CS}(g)$ and so singular points coincide with complex zeros of $\beta_{CS}(g)$ where $M^2 \neq 0$.

This sounds complicated but it works well for the 2D $O(N)$ sigma models.

Nonlinear $O(N)$ sigma model on a square lattice (hep-lat 0907.2980; PRD80 054020)

$Z = \int \prod_x d^N \phi_x \delta(\vec{\phi}_x \vec{\phi}_x - 1)\ \mathrm{e}^{-(1/g_0^2)E[\{\phi\}]}$

with $E[\{\phi\}] = \sum_{x,e}(1 - \vec{\phi}_x \vec{\phi}_{x+e})$

We assume a cubic lattice with an even number of sites in each directions and periodic boundary conditions. Under these conditions (as for $SU(2N)$ LGT) $Z[-g_0^2] = \mathrm{e}^{4DL^D/g_0^2} Z[g_0^2]$

Gap equation: $\int_{-\pi}^{\pi} \frac{dk^D}{(2\pi)^D} \frac{1}{2(\sum_{j=1}^{D}(1-\cos(k_j))+M^2} = 1/\lambda^t \ \equiv b$

$\lambda^t = g_0^2 N$ kept constant as N becomes large.

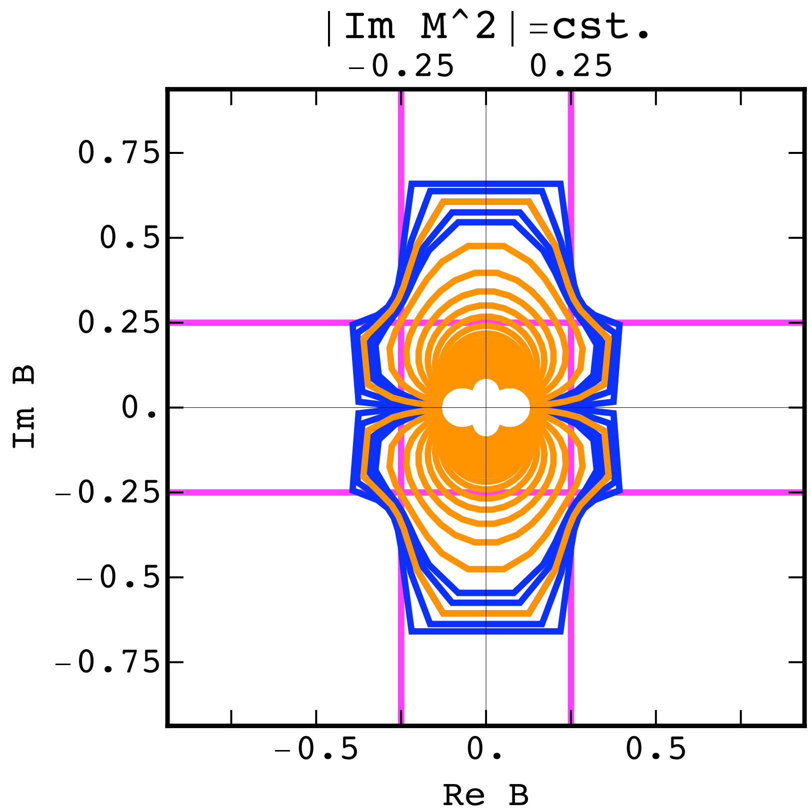


Figure 11: Complex values taken by $b = 1/\lambda^t$ when M^2 varies over the complex plane (here on horizontal lines in the M^2 plane; spacing 0.1 (blue) and 0.5 (brown)). Asymptotic limits are ± 0.25 and represent the logarithmic singularities at $M^2 = 0,\ 4$ and 8 (magenta).

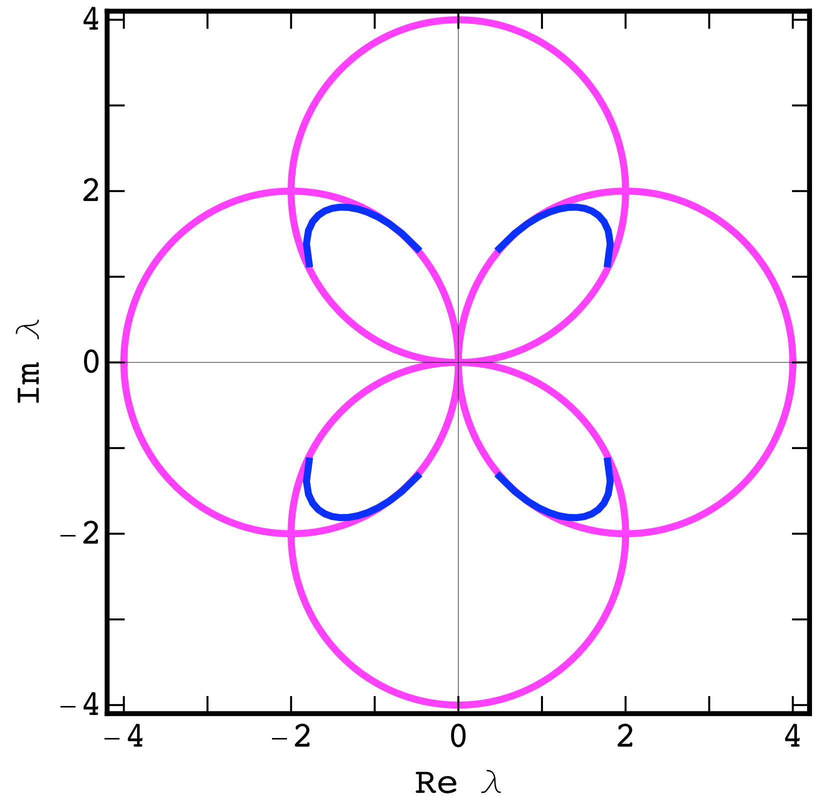


Figure 12: Complex values taken by λ^t when M^2 varies over lines above and below the cut with $ImM^2 = \pm 0.01$ (blue); the circles are the inverses of the asymptotic lines in the b plane.

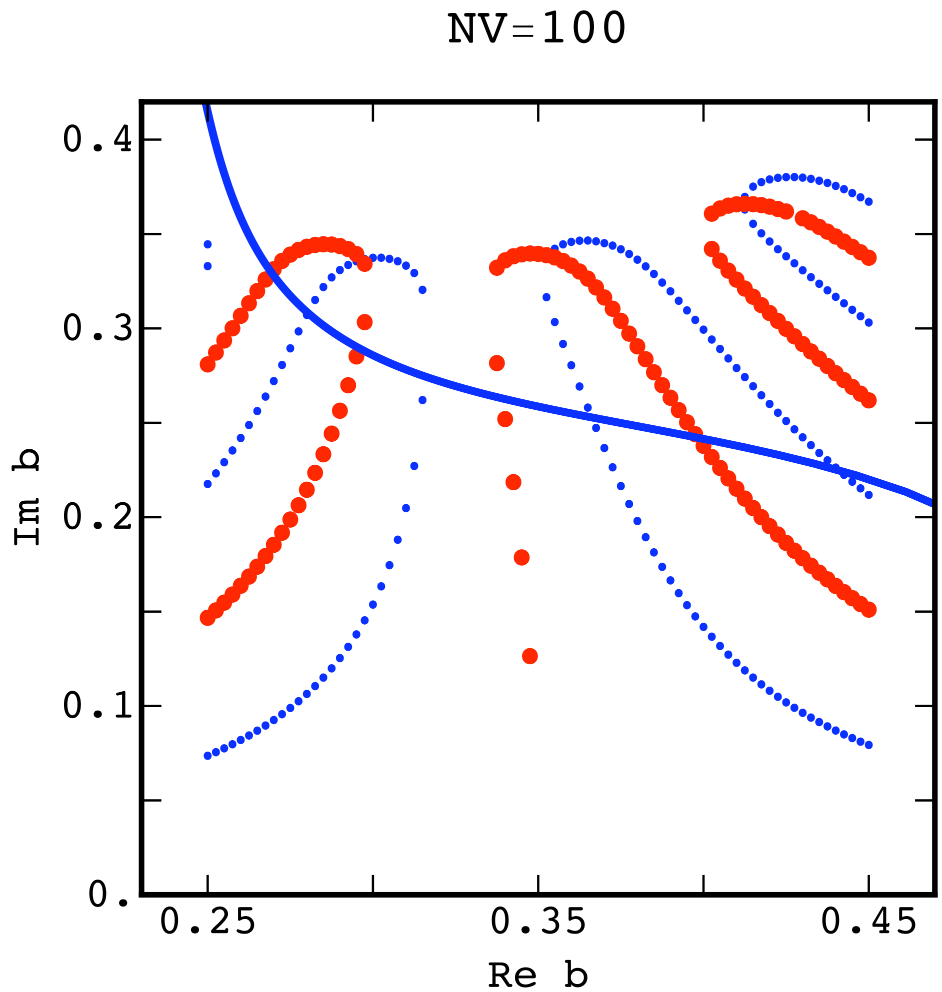


Figure 13: Preliminary search for zeros for $NV = 100$ zeros of Re (blue), zeros of Im (red); the solid blue line is the b- image of the $[-8, 0]$ cut.

Finite Volume Zeros (with Haiyuan Zou)

At finite volume, there are only L^2 momenta for periodic boundary conditions. To fix the ideas, for a square 4×4 lattice the gap equation is

$$b = (1/16)\Big(1/M^2 + 4/(2+M^2) + 6/(4+M^2) + 4/(6+M^2) + 1/(8+M^2)\Big)$$

In general, after reducing to a common denominator, we obtain a rational form $b = Q(M^2)/P(M^2)$, where Q and P are polynomials of degrees q and $q+1$ respectively. The value of q depend on accidental degeneracies. In the 4×4 example, $q =$ 4. For 8×8, $q =$ 12. The inversion $M^2(b)$ requires a Riemann surface with $q+1$ sheet.

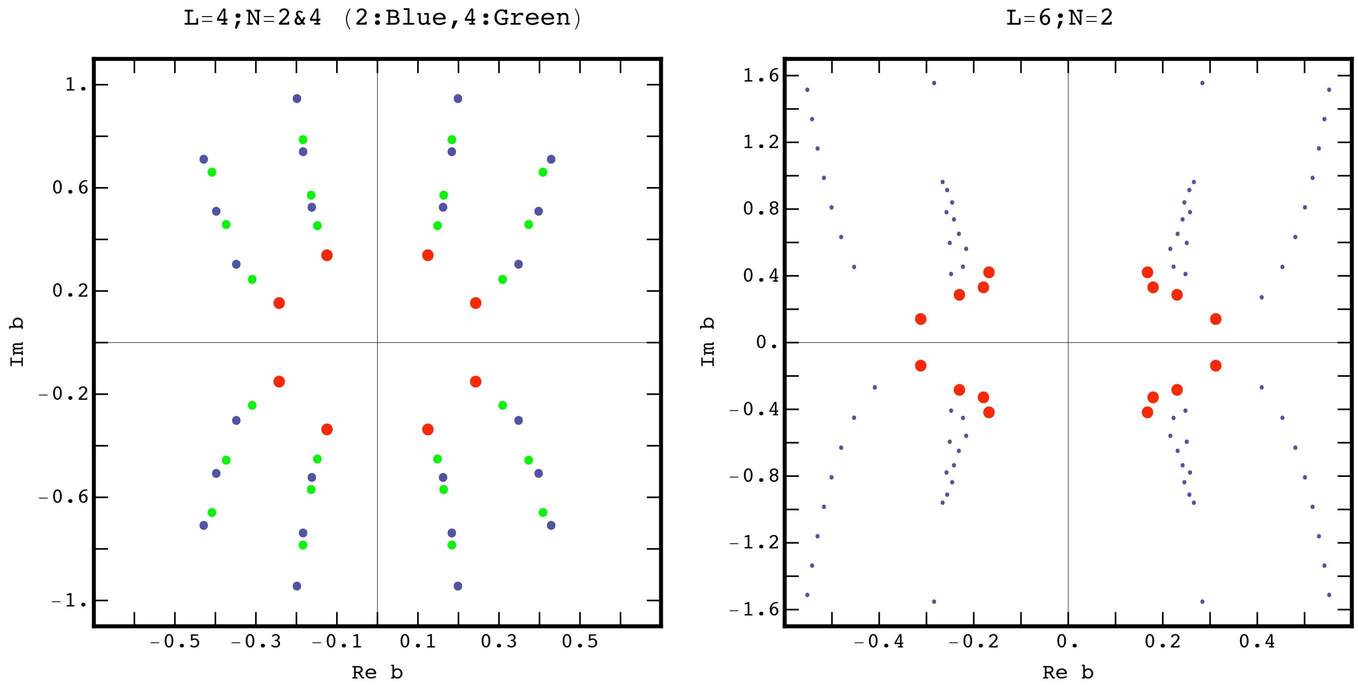


Figure 14: Finite volume zeros and images of singular points (red) (Haiyuan Zou).

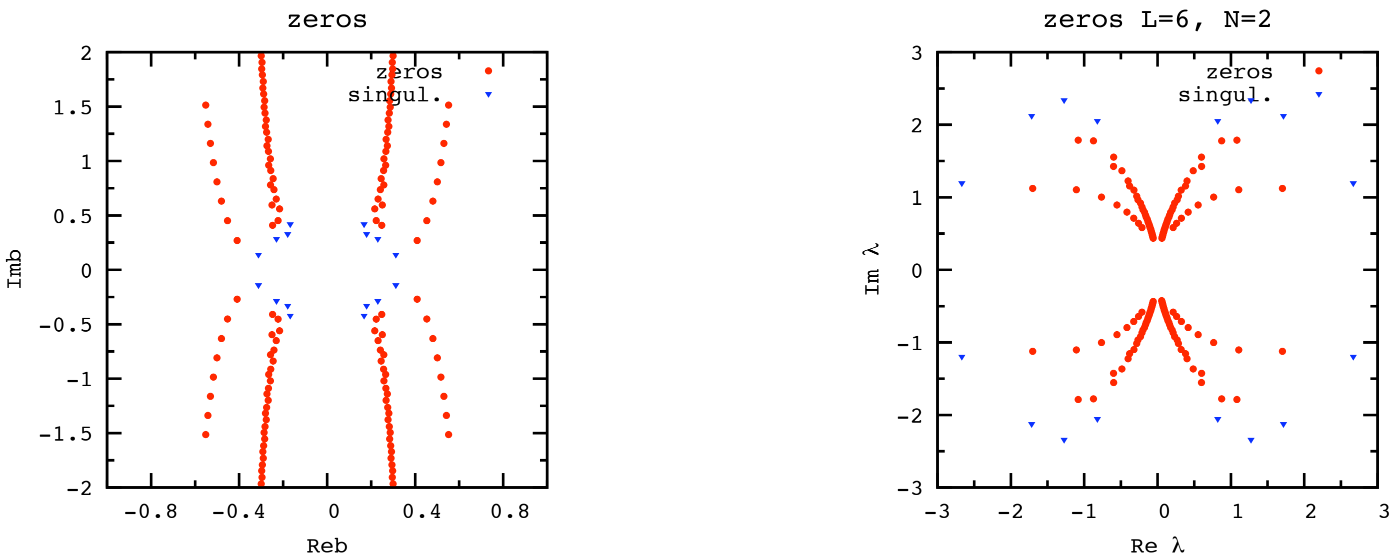


Figure 15: Fisher's zeros (red) and singular points of the $b(M^2)$ map for $N = 2$ and $L = 6$ in the b plane (left) and the λ^t plane (right). The distance between the zeros scales like $1/N$ and $1/L^2$. Work done by Haiyuan Zou.

Fisher's zeros stay away from the real axis

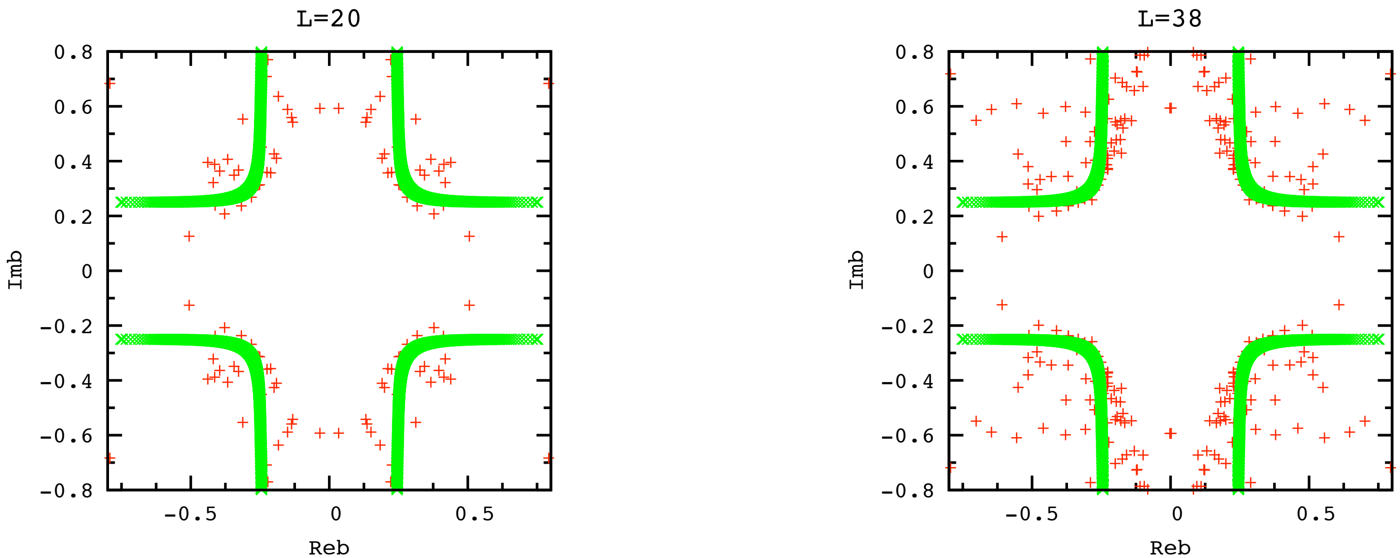


Figure 16: Singular points of $b(M^2)$ for $L = 20$ and 38 (red) compared to the image of the cut in the M^2 plane at infinite volume (green). Work done by Haiyuan Zou.

Discrete Scaling: Dyson Hierarchical Model

2^n sites Labeled with n indices $x_n, ...x_1$, each index being 0 or 1 (think about a tree with n branching levels).

Kinetic term (sum over blocks of all 2^l sizes; not renormalized):

$$S = -\frac{1}{2}\sum_{l=1}^{n}(\frac{c}{4})^l \sum_{x_n,\ldots,x_{l+1}} (\sum_{x_l,\ldots,x_1} \phi_{(x_n,\ldots,x_1)})^2$$

If $c = 2^{(D-2)/D}$, Gaussian fields scale like in D-dimensions

$2^{\frac{1}{D}}$: "linear" scale factor (block spin: $2 \rightarrow 1$). $D = 3$ hereafter

Exact RG transformation affects only the local potential

Important facts about Dyson Hierarchical Model

- The LPA is exact
- It is a lattice model
- Its recursion formula is related to Wilson's approximate recursion formula (that allowed the first numerical RG calculations) but the exponents are different. (JPA 29 L635, 1996)
- It is a model on the 2-adic line. The classification of the multiplicative characters provides in principle a systematic method of improvement of the hierarchical approximation (YM, Europhysics 93, hep-th/9307128). This has a wavelet translation (Haar system). Analogous to the derivative expansion. Never tried beyond one dimension.

Recursion Formula

Initial local measure: $W_0(\phi)=\delta(\phi^2-1)$ (Ising) or $W_0(\phi)=e^{-A\phi^2-B\phi^4}$

Block spin transformation:

$$W_{n+1}(\phi)=C_{n+1}e^{\frac{\beta}{2}(\frac{c}{4})^{n+1}\phi^2}\int d\phi' W_n(\frac{(\phi-\phi')}{2})W_n(\frac{(\phi+\phi')}{2})\ ,$$

Fourier Representation of the RG transformation $(c=2^{1-1/D})$

$$R_{n+1}(k)=C_{n+1}exp(-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2})(R_n(\frac{\sqrt{c}k}{2}))^2$$

M_n: the total field $\sum\phi_x$ inside blocks of side 2^n ; notice $1/(2q)!$.

$R_n(k)=\sum_{q=0}^{\infty}\frac{(-ik)^{2q}}{(2q)!}\frac{<(M_n)^{2q}>_n}{(4/c)^{qn}}$ Polynomial truncations of $R_n(k)$: very accurate in the symmetric phase

Generalization to $O(N)$ straightforward (PRD 73 047701 2006)

We can calculate very accurately the critical exponents and amplitudes

N	γ	Δ	β_c/N
1	1.29914073	0.425946859	1.179030170
2	1.41644996	0.475380831	1.236763288
3	1.52227970	0.532691965	1.275794011

Using $b = 1 + \epsilon$ one recovers the Wilson-Polchinski equation in the LPA approximation (See Felder CMP 111 101 1987):

$$\dot{u} = \frac{2y}{N}u'' + \left(1 + \frac{2}{N} + (2-d)y - 2yu\right)u' + (2-u)u,$$

For $N = 1$, γ_{WP}= 1.29923547 (Litim, Bervillier,Juttner)

Recursion formula with $b^D \neq 2$

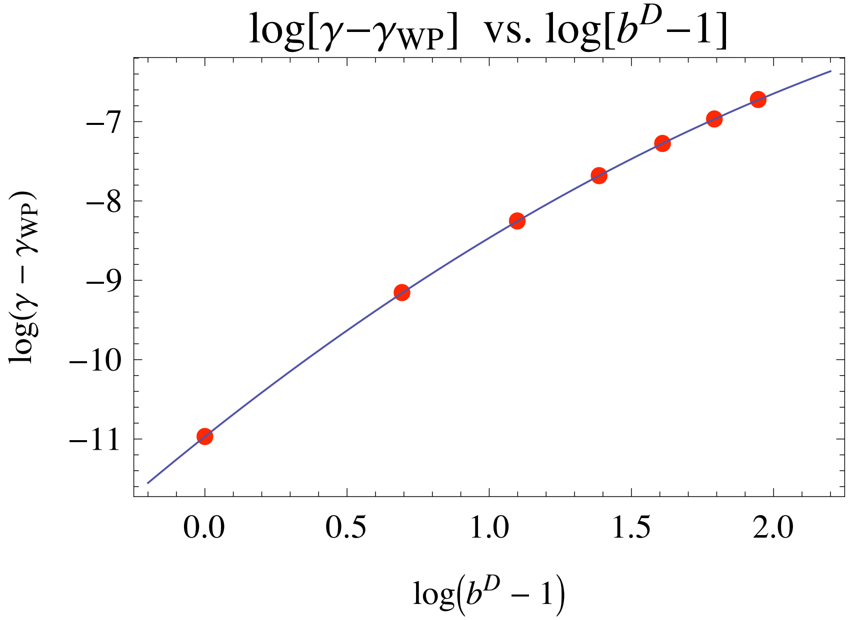


Figure 17: γ for integer values of b^D, γ_{WP} was first calculated by Litim . Work done with B. Oktay and Y. Liu.

Linear analysis of instabilities for non-integer b^D

$b^D = 2 + \zeta$ with ζ small but continuous.

$R_\zeta(k)$ the Fourier transform of the total field distribution.

FP: $R^\star_\zeta \simeq R^\star_0 + \zeta R^\star_1$ and at first order in ζ: $L[R^\star_1 + G] = R^\star_1$

L is the linear operator for the $\zeta = 0$ problem: $L[\delta R_n] = \lambda_n \delta R_n$

$G = -(5/6)k^2 \frac{\partial}{\partial k^2} R^\star_0 + (1/2) R^\star_0 * \mathrm{Log}(R^\star_0)$

$\mathrm{Log}(R^\star_0)$ is not analytical ($R^\star_0$ has zeros)

Expansions in eigenvectors of L do not converge (Y. Liu' talk)

Need for an "extension" as in the case of $1/r^2$ potential.

Consequences of discrete scaling

The magnetic susceptibility near criticality has the form:

$$\chi = (\beta_c - \beta)^{-\gamma}(A_0 + A_1(\beta_c - \beta)^{\Delta} + ...) \ , \qquad (1)$$

With discrete scaling, the constants A_0 and A_1 can be replaced by functions $A(\beta_c - \beta) = A(\lambda(\beta_c - \beta))$ that can be expanded in integer powers of $(\beta_c - \beta)^{\frac{i2\pi}{ln(\lambda)}}$ with λ the relevant eigenvalue. These amplitude "prevent" continuous scaling(?)

$\nu = \ln(b)/\ln(\lambda)$ but $\omega = 2\pi/\ln(\lambda)$ is obviously b-dependent

$\chi(\beta) = 1 \ + \ b_1\beta \ + \ b_2\beta^2 \ + \ldots$

$r_m = b_m/b_{m-1}$, the ratio of two successive coefficients.

$$S_m = -m(m-1)(r_m - r_{m-1})/(mr_m - (m-1)r_{m-1})$$

$$\widehat{S}_m = mS_m - (m-1)S_{m-1} \simeq \gamma - 1$$

$$(\beta_c - \beta)^z = \beta_c^z \sum_{m=0}^{\infty} \binom{z}{m}(-1)^m\left(\frac{\beta}{\beta_c}\right)^m$$

$$\binom{z}{m}(-1)^m = \frac{m^{-z-1}}{\Gamma(-z)} \times \left(1 + \frac{z+z^2}{2\,m} + \ldots\ \right)$$

$|\Gamma(\gamma + i\omega)| \simeq \sqrt{2\pi}\omega^{\gamma-1/2}\mathrm{e}^{-\omega\pi/2}$; $\omega = 17.66$ for the HM but about 6 for $b = 2$ (could this explain some oscillations observed in some MCRG calculations?).

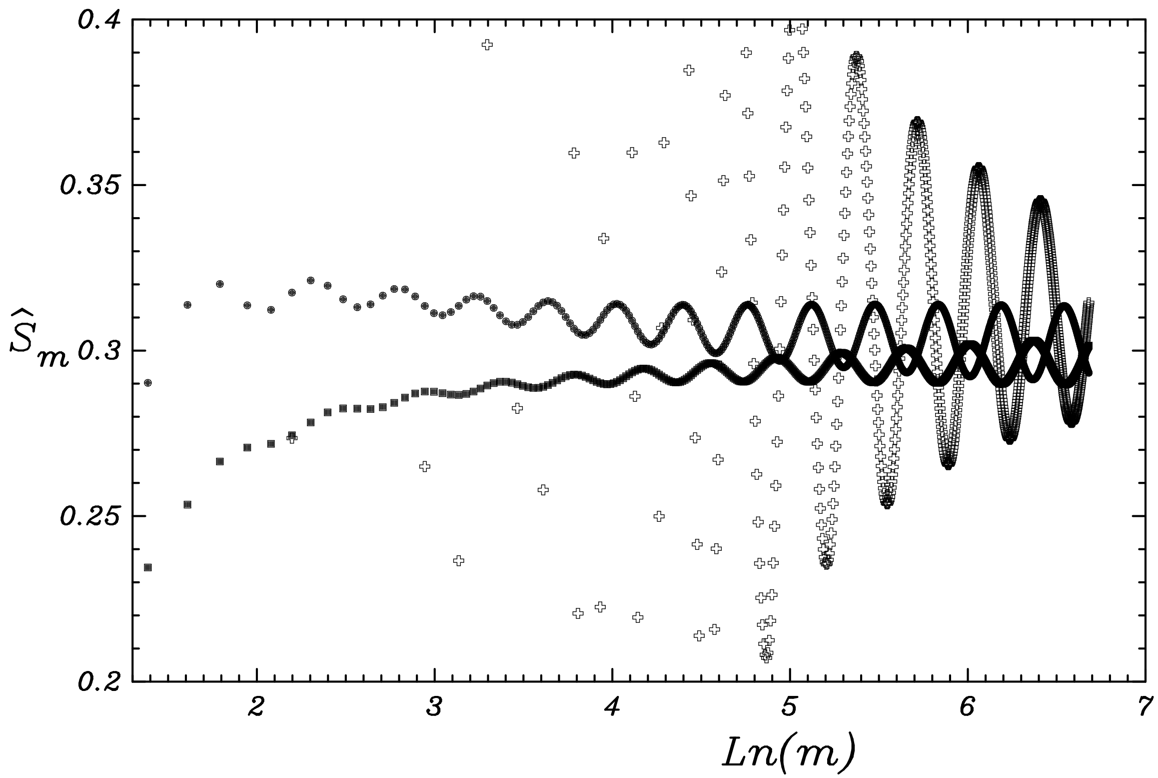


Figure 18: $\widehat{S}_m$ for the Ising model (crosses) and the Landau-Ginzburg model with $B = 1$ (circles) and $B = 0.1$. (squares) (PRL 75, JSP 87)

Conclusions

- There is support to the idea that the complex zeros of the β function delimit the boundary of a region without Fisher's zeros. (Universality? complex RG flows?)

- The existence of log-periodic corrections in models with discrete scaling is an obstruction to continuous blocking. $b-$independence may be a guide for improving the hierarchical approximation.

- $\Delta\beta$ calculations in progress in $O(N)$ models (INT preprints to come up!)

- Thanks!