Numerical Instabilities Associated with Block Spinning Non-Integer Numbers of Sites

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Outline

- Introduction and Motivation
- Linear Analysis
- Window of Stability
- Improvement and Results
- Further Plan

Dyson's Hierarchical Model

$$H = -\frac{1}{2}\sum_{p=1}^{N}(\frac{c}{4})^m \sum_{p_N,\ldots,p_{m+1}} (\sum_{p_m,\ldots,p_1} \phi_{(p_N,\ldots p_m)})^2 \ .$$

D : Dimension of the system.

ϕ : Scalar field in the configuration space.

$\frac{c}{4} = b^{-2-D}$: Control the decay of the interaction.

b^D : Number of sites blocked in each RG iteration.

a : Lattice Spacing.

Λ : UV cutoff.

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- Block Spin:

  Lattice Spacing $a \to ba$

  UV cutoff $\Lambda \to \frac{\Lambda}{b}$

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The blocking parameter b can be continuous in the momentum space.

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Dyson's Hierarchical Model

- Choices of initial measure:

Ising Measure: $W_0(\phi) = \delta(\phi^2 - 1)$

Landau-Ginsburg measure: $W_0(\phi) = exp(-\frac{1}{2}m^2\phi^2 - \lambda\phi^4)$.

- Block Spin transformation:

$$W_{n+1}(\phi) = C_{n+1}e^{\frac{\beta}{2}(\frac{c}{4})^{n+1}\phi^2}\int d\phi' W_n(\frac{(\phi-\phi')}{2})W_n(\frac{(\phi+\phi')}{2})$$

- Fourier transform of the measure:

$$W_n(\phi) = \int \frac{dk}{2\pi}e^{ik\phi}R_n(k)$$

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Dyson's Hierarchical Model

- Recursion formula of the measure: $R(k)$

$$R_{n+1}(k) = C_{n+1}\, \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(\sqrt{\frac{c}{4}}\, k)\right)^2$$

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- Ponchiski's equation:

$$\frac{\partial V}{\partial t} = DV + (1 - \frac{D}{2})\phi\frac{\partial V}{\partial \phi} - (\frac{\partial V}{\partial \phi})^2 + \frac{\partial^2 V}{\partial \phi^2}$$

$HM \underset{b \to 1}{\Longrightarrow}$ Polchinski's Equation $\Longrightarrow \gamma_{WP}$

Dyson's Hierarchical Model

HM with Ising measure($b^D = 2$):

$$\gamma = 1.29914073015 9$$

WP:

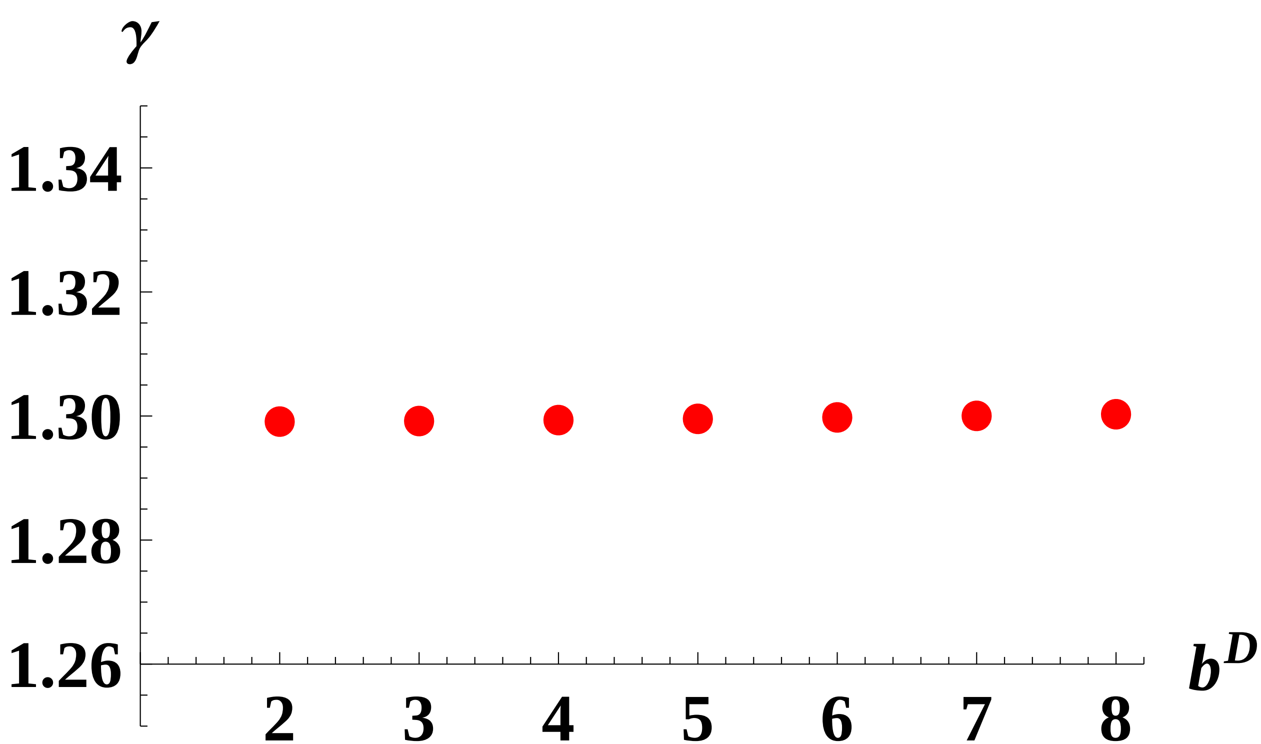
$$\gamma = 1.29912354776 1$$

$\gamma_{WP} \neq \gamma_{b^D=2}$

The difference is in the 5th digit.

Dyson's Hierarchical Model

- Critical exponent γ for integer b^D:



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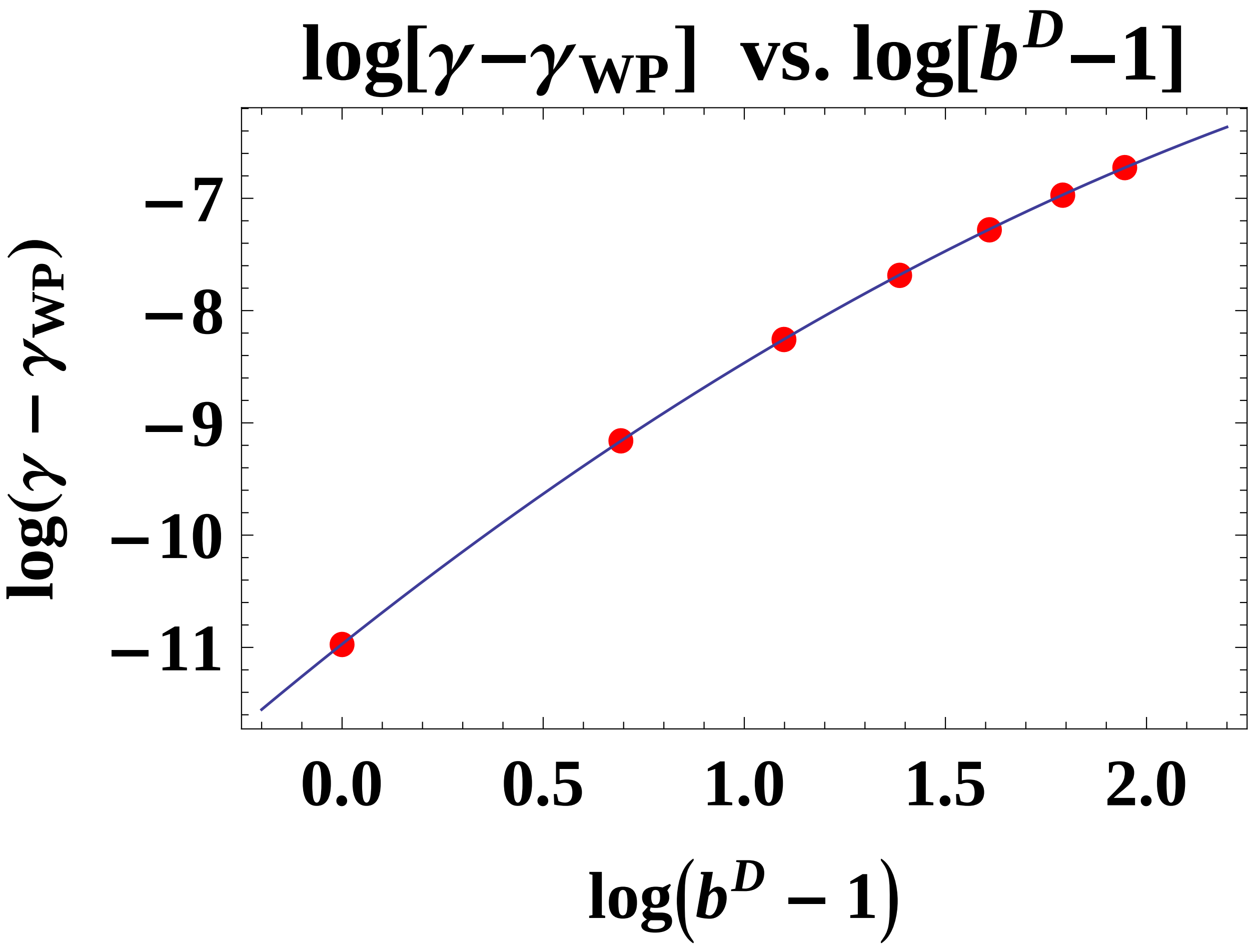
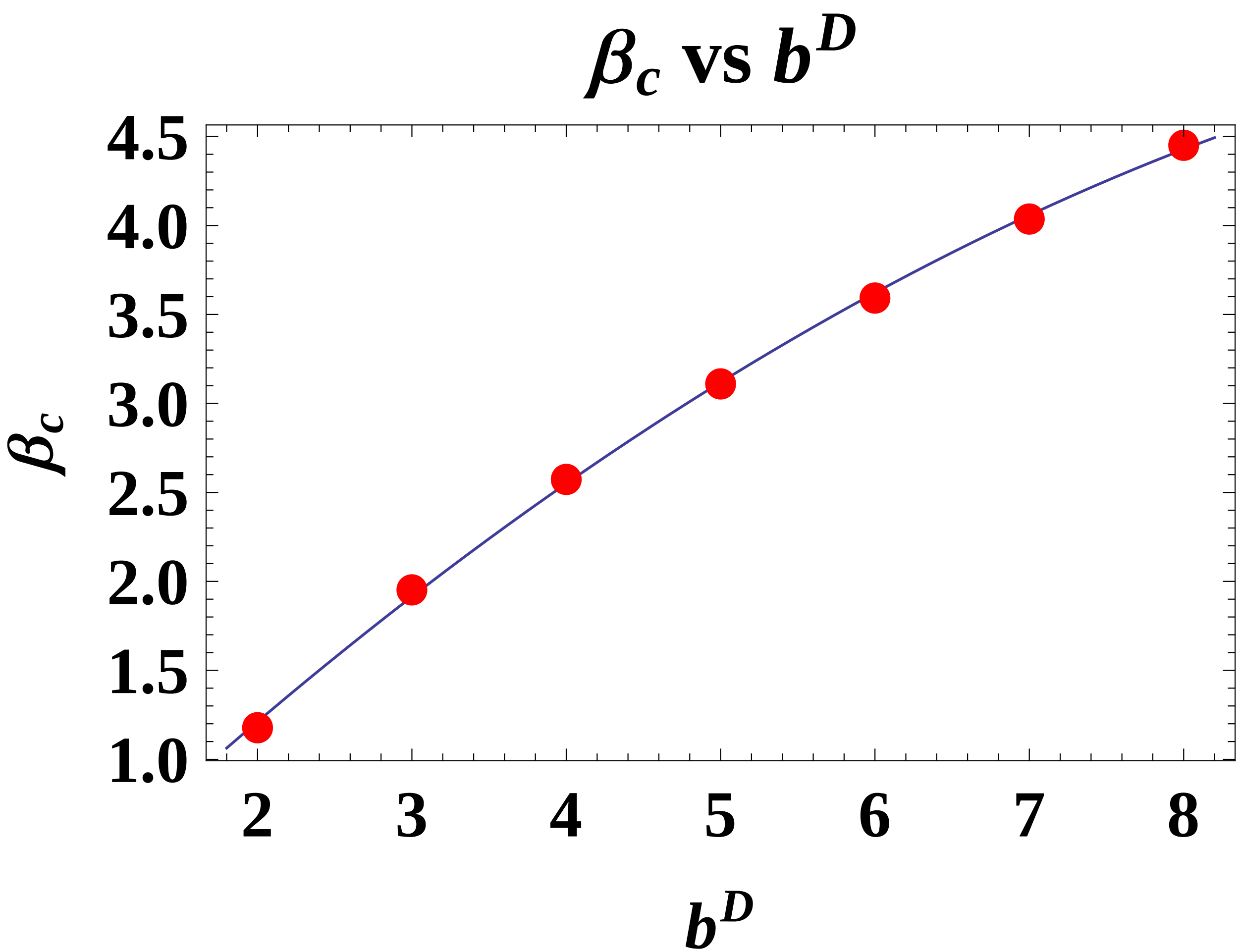


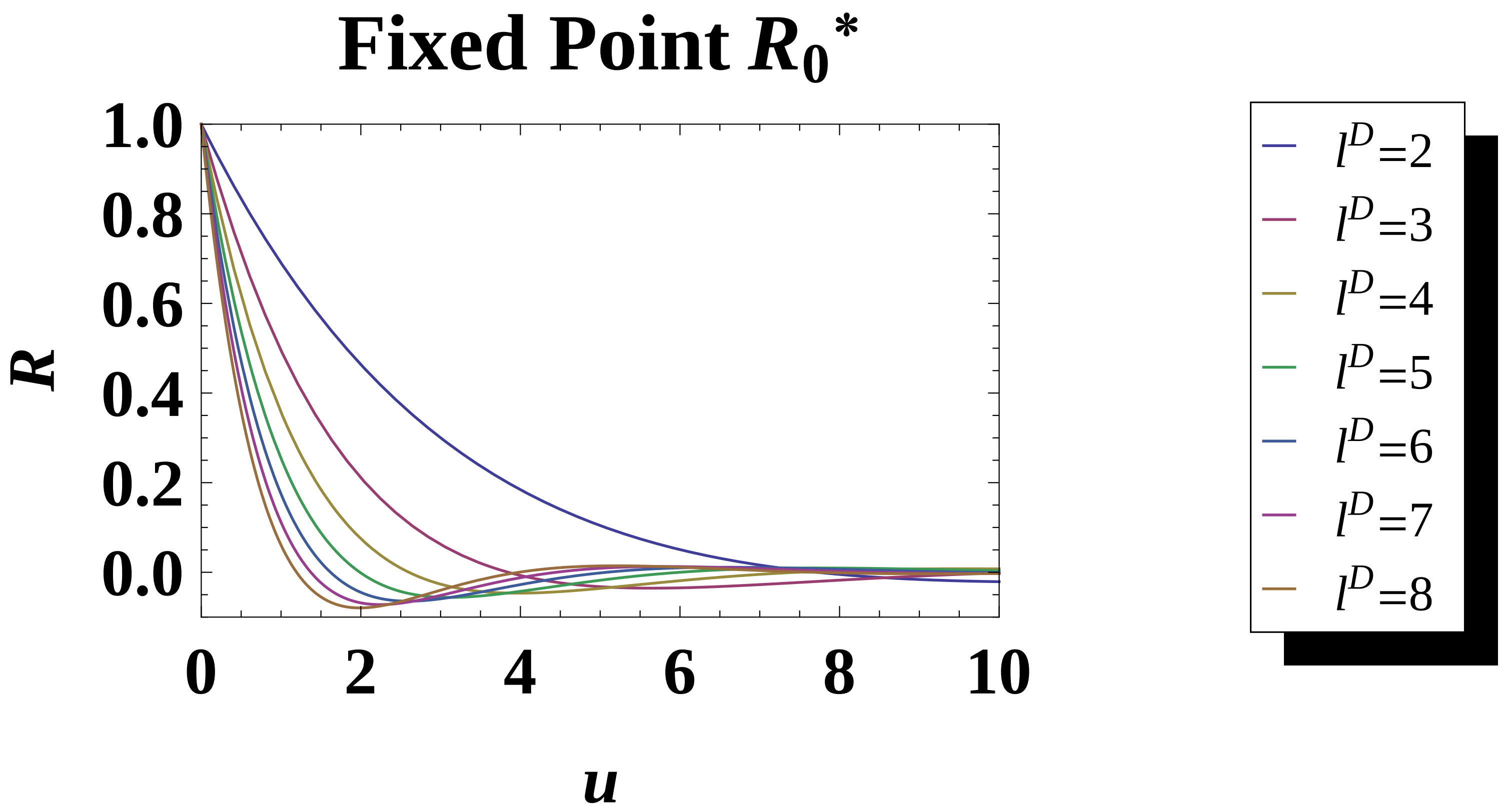
Dyson's Hierarchical Model

- Critical temperature for integer b^D:



Dyson's Hierarchical Model

- Fixed point Polynomial for integer b^D:



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Linear Analysis

- Unnormalized recursion:

$$R_{n+1}(k) = \mathrm{e}^{-\frac{1}{2}\beta\frac{\partial^2}{\partial k^2}} \left(R_n(\sqrt{\frac{c}{4}}\,k)\right)^{b^D}$$

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- Truncation:

$$R_n(k) = a_{n,0} + a_{n,1}k^2 + a_{n,2}k^4 + ... + a_{n,l_{max}}k^{2l_{max}}$$

or equivalently,

$$R_n(u) = a_{n,0} + a_{n,1}u + a_{n,2}u^2 + ... + a_{n,l_{max}}u^{l_{max}}$$

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Linear Analysis

- Small ζ expansion:

$$b^D = 2 + \zeta$$

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$$RG_{\zeta=0}[R_0^*] = \mathrm{e}^{-\frac{1}{2}\beta_c \frac{\partial^2}{\partial k^2}} \left(R_0^*(\sqrt{\frac{c_0}{4}}\, k)\right)^2 = R_0^*$$

$$RG_\zeta[R_0^* + \zeta R_1^*] = R_0^* + \zeta R_1^*$$

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- Linearization:

$$R_0^* + \delta R' = RG_{\zeta=0}[R_0^* + \delta R] = R_0^* + L[\delta R]$$

$$\frac{RG_\zeta[R_0^*] - R_0^*}{\zeta} \underset{\zeta\to 0}{\longrightarrow} L[G]$$

where $L \approx \mathrm{e}^{-\frac{1}{2}\beta_c \frac{\partial^2}{\partial k^2}}[2R^*]$ is the transfer matrix.

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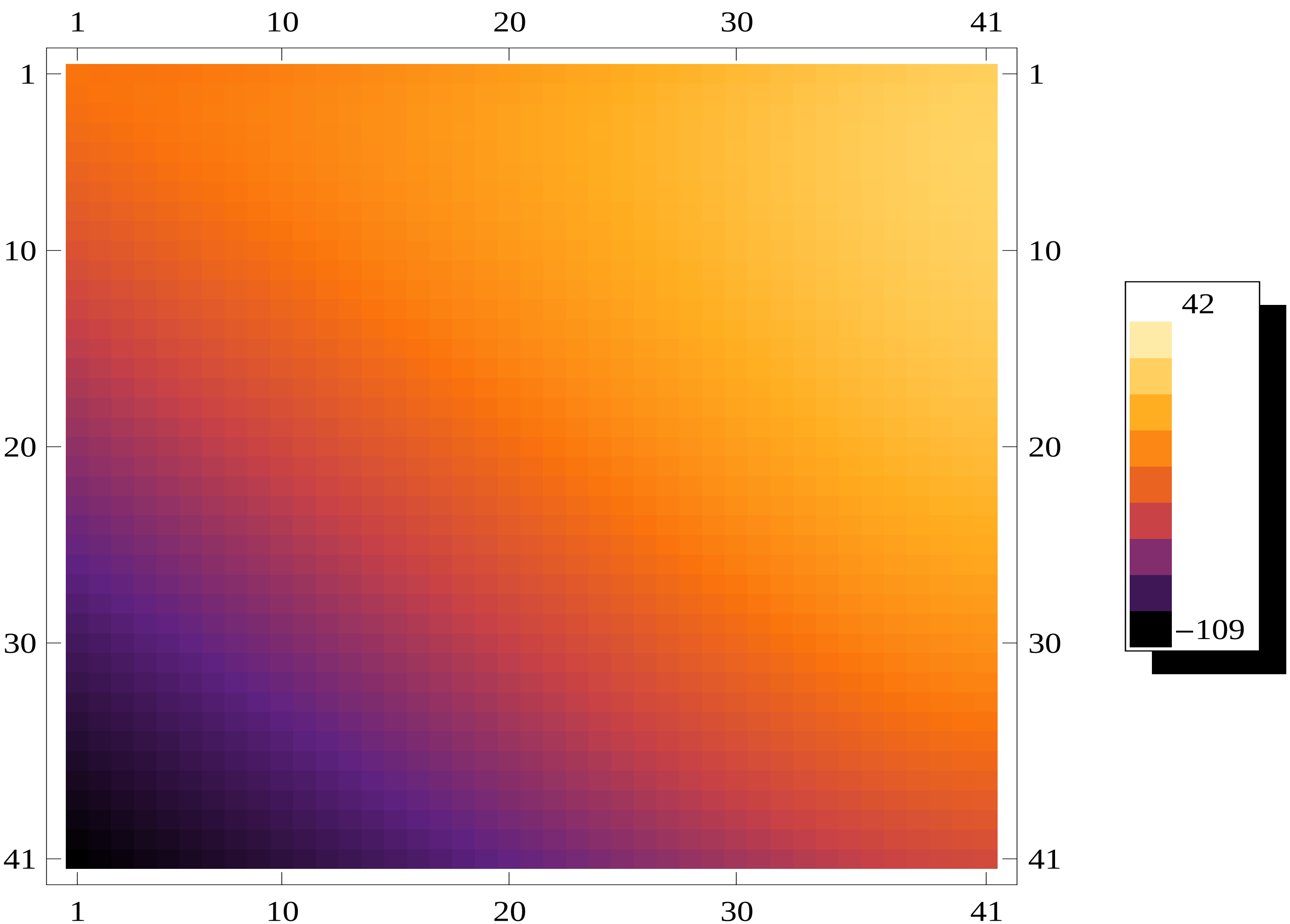
Linear Analysis

- Transfer Matrix:



Linear Analysis

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v_n : Right eigenvectors of the stability matrix L.

$$\lambda_n \alpha_n + \lambda_n \beta_n = \alpha_n$$

$$\Rightarrow \alpha_n = \frac{\beta_n \lambda_n}{1-\lambda_n}$$

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Window of Stability

$$G = -\frac{5}{6}k^2\frac{\partial R_0^*}{\partial k^2} + \frac{1}{2}R_0^* \ln R_0^* = \sum \beta_n v_n$$

We need β_n to be stable when we increase the polynomial order. However, it is only stable for some finite lmax.

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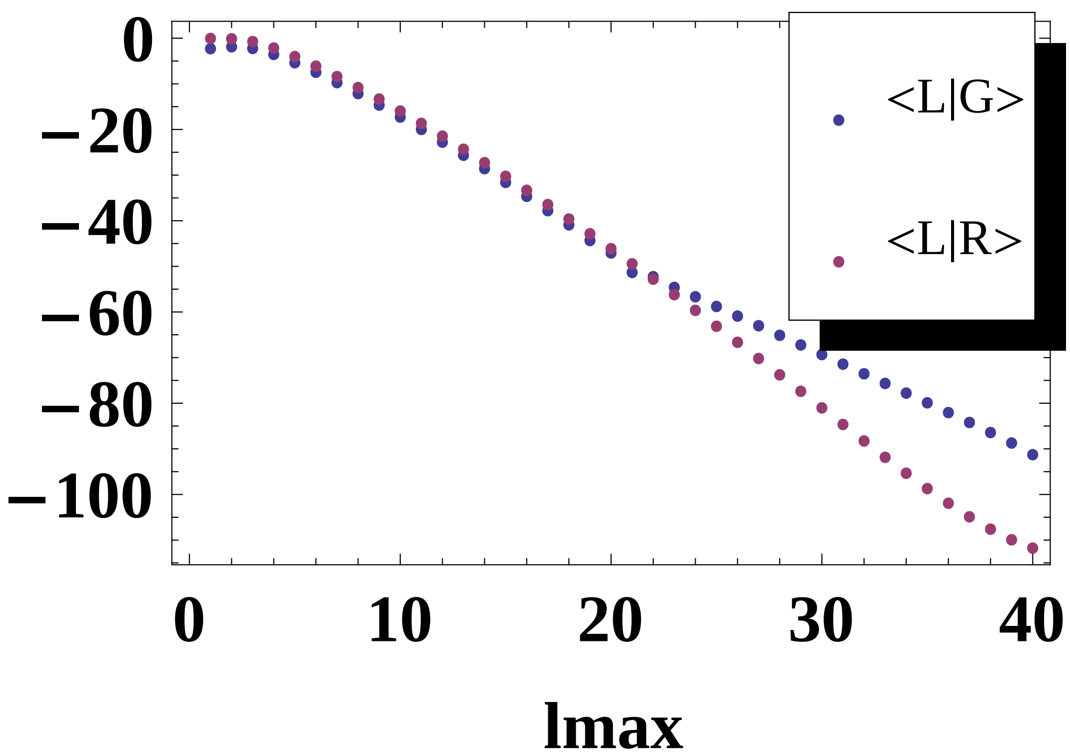
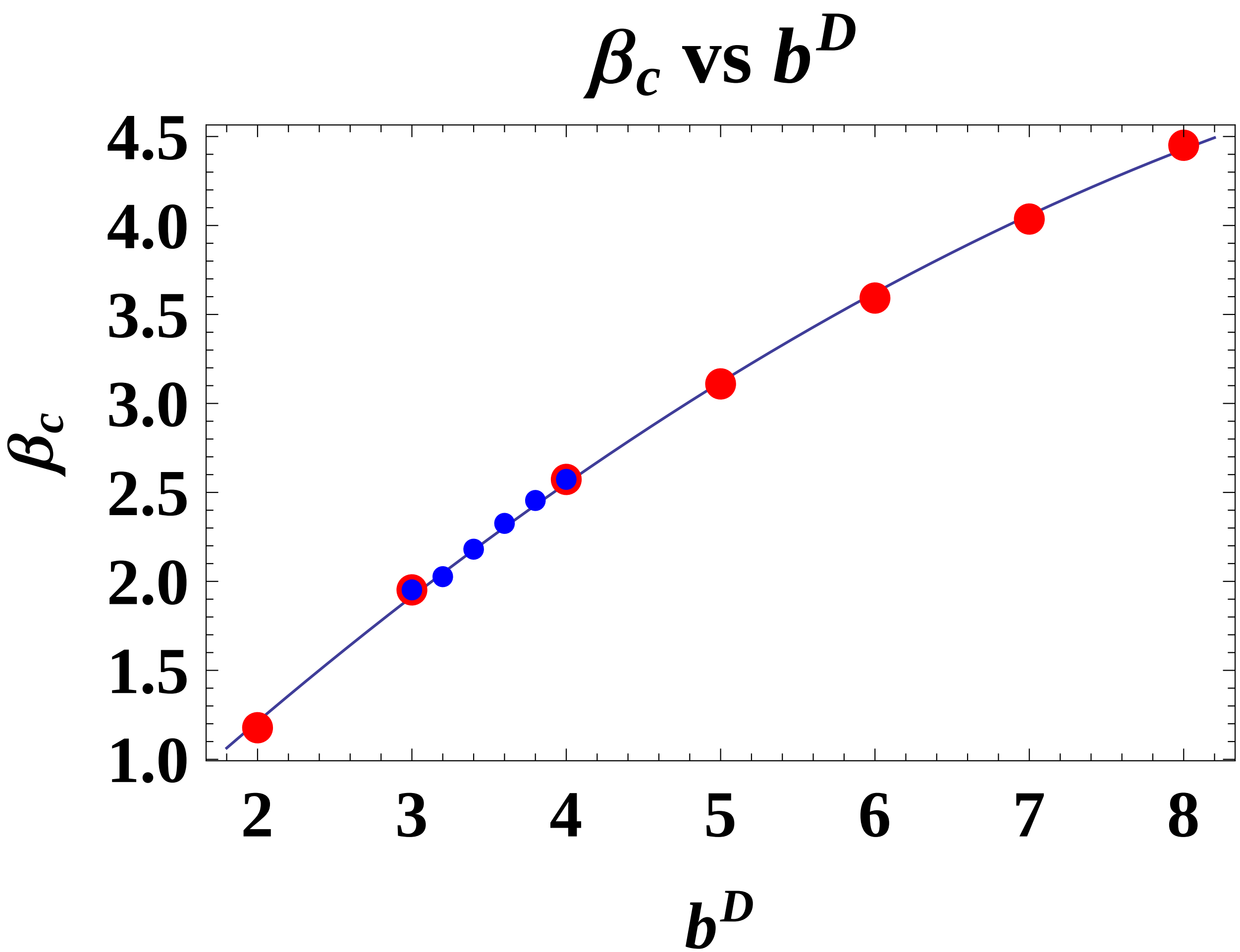


Improvement and Results

- Results for noninteger b^D:



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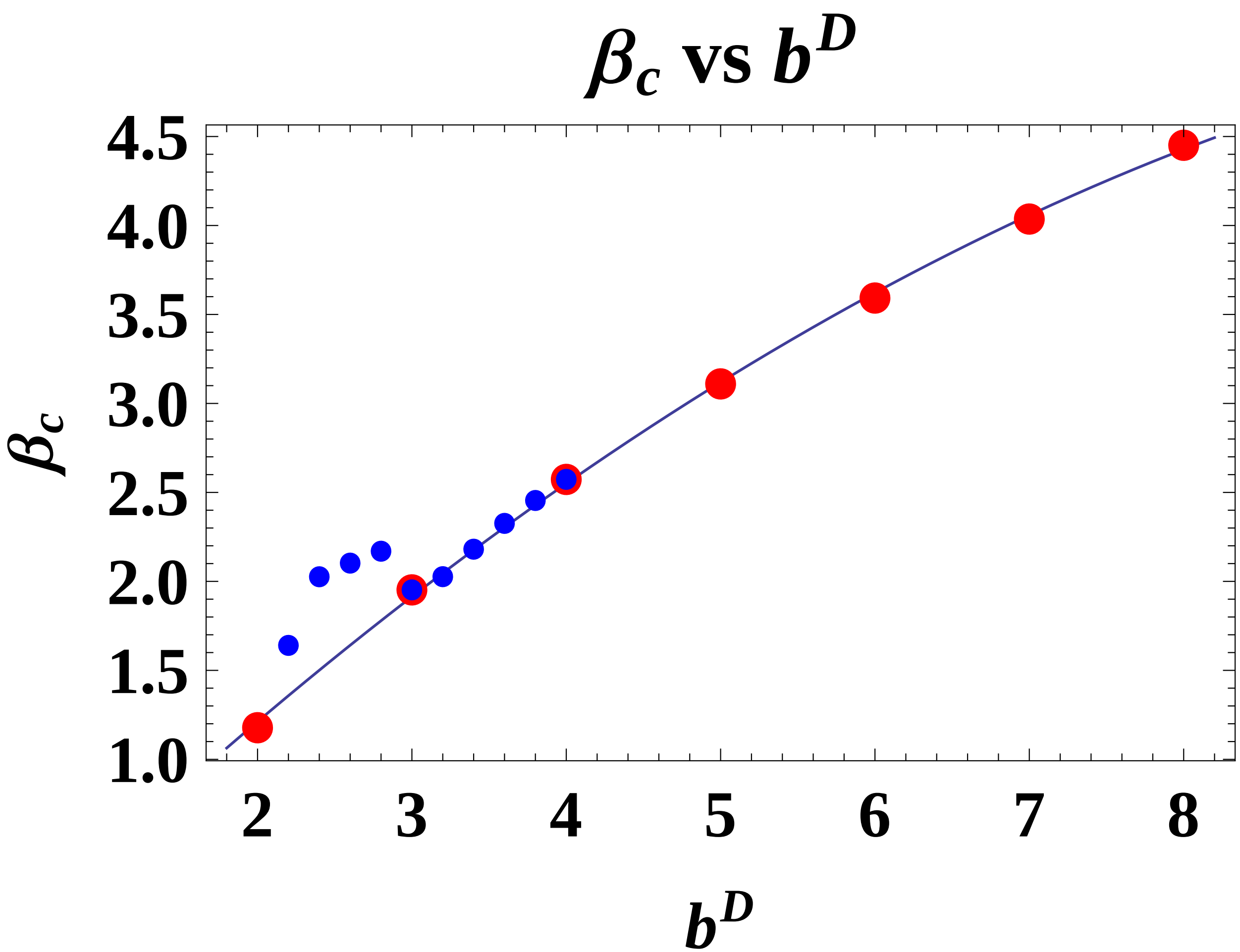

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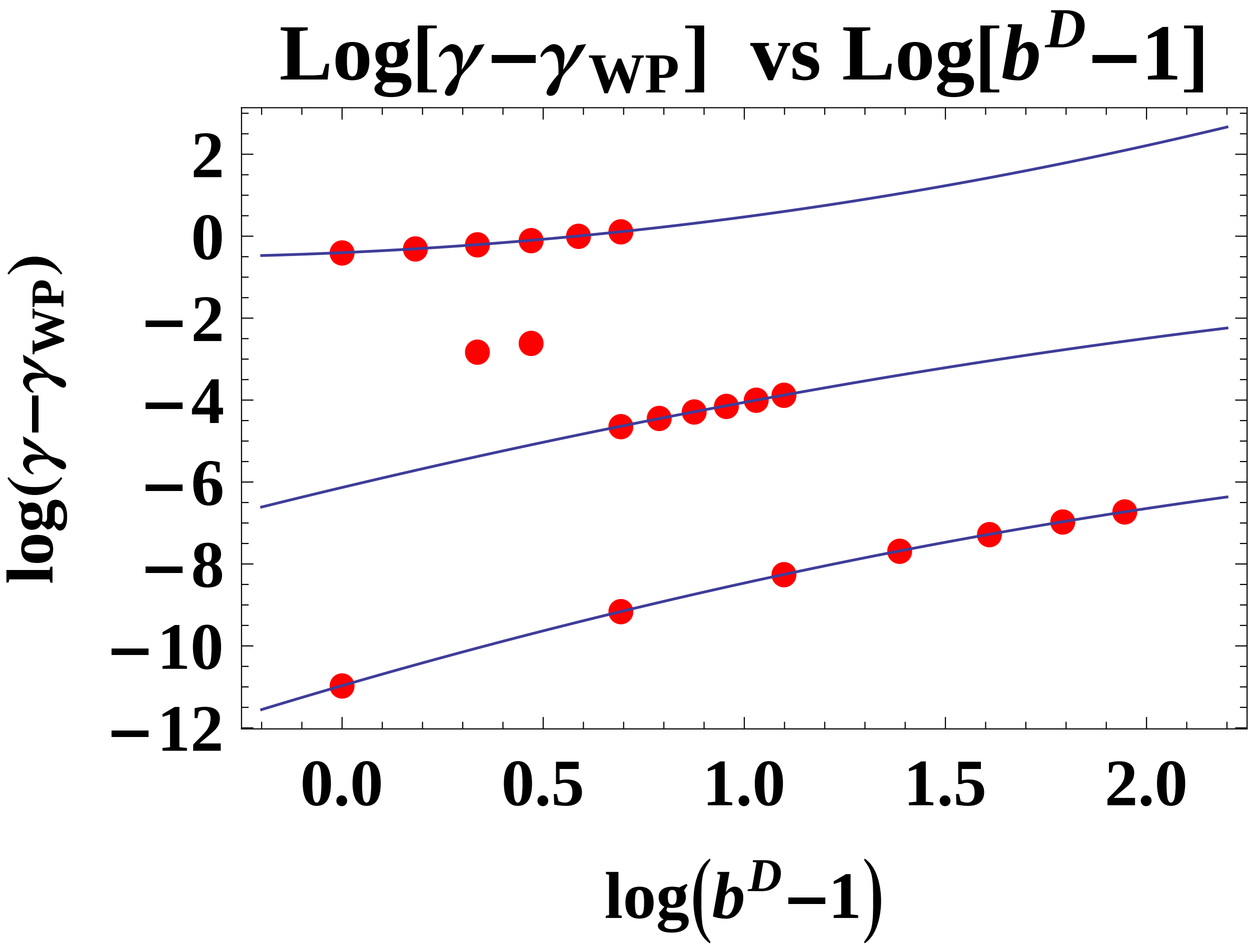

Further Plan

- Develop technique to do large b^D calculation.
- Try to observe the log-periodic corrections.
- Apply two lattice matching to the O(N) models.

Thank You!

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