Conformality Lost

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Motivation: QCD at LARGE N_c and N_f

Colors

Flavors

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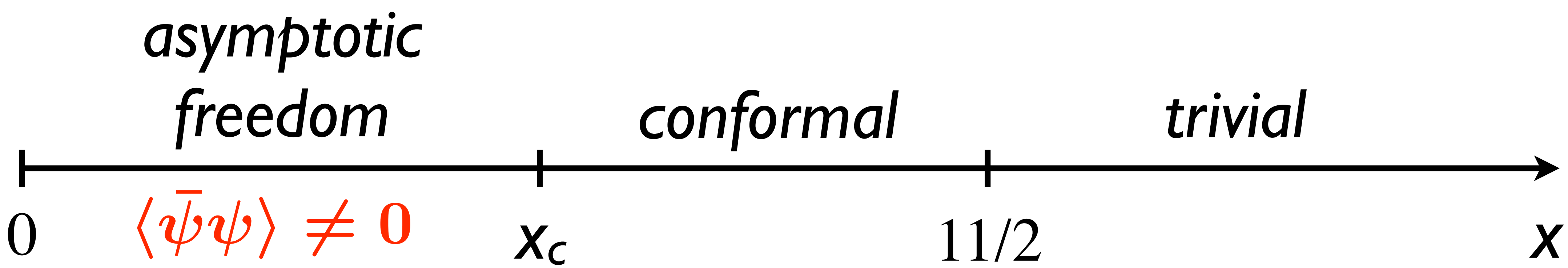
Define $\mathbf{x = N_f/N_c}$, treat as a continuous variable



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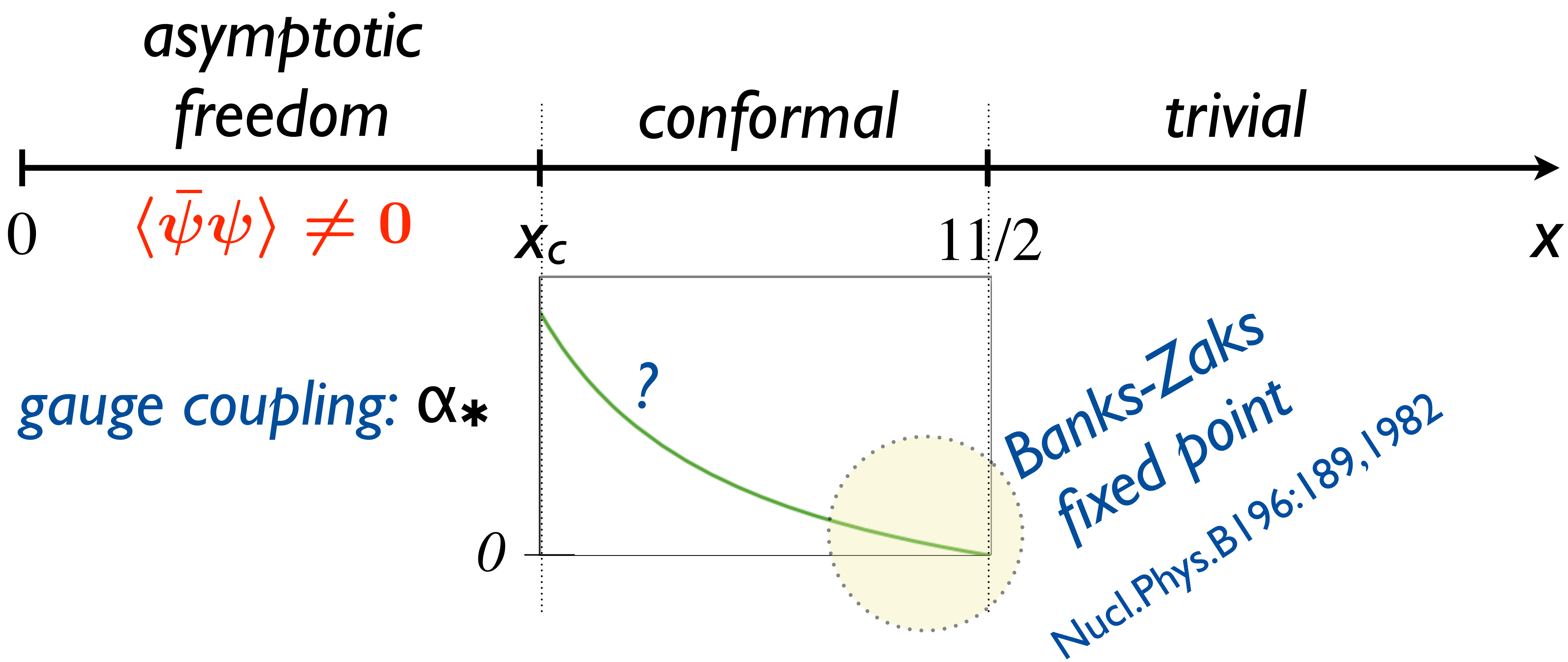

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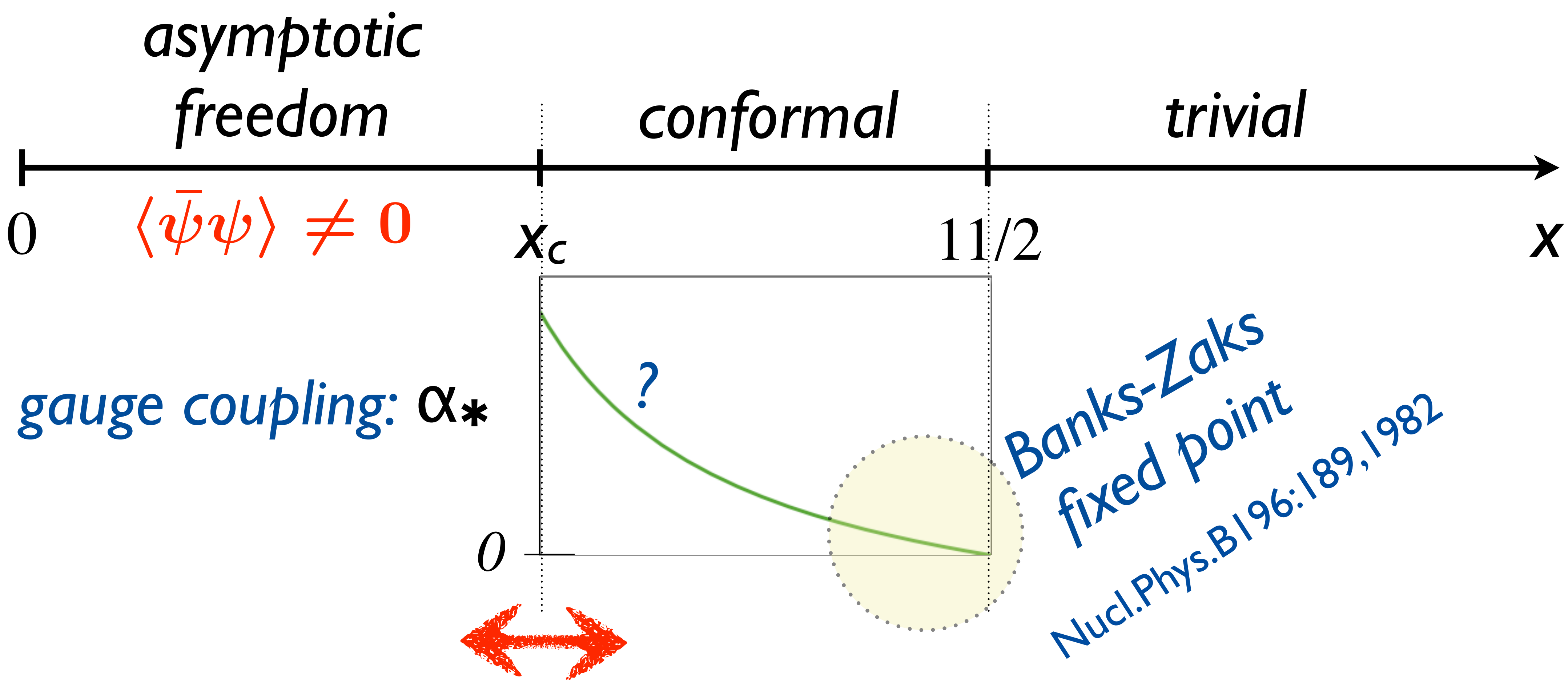


What is the nature of this transition?

How does the IR scale appear as conformality is lost?

Outline:

I. A mechanism for vanishing conformal invariance

II. The Berezinskii-Kosterlitz-Thouless (BKT) transition

III. A quantum mechanics model: the $1/r^2$ potential

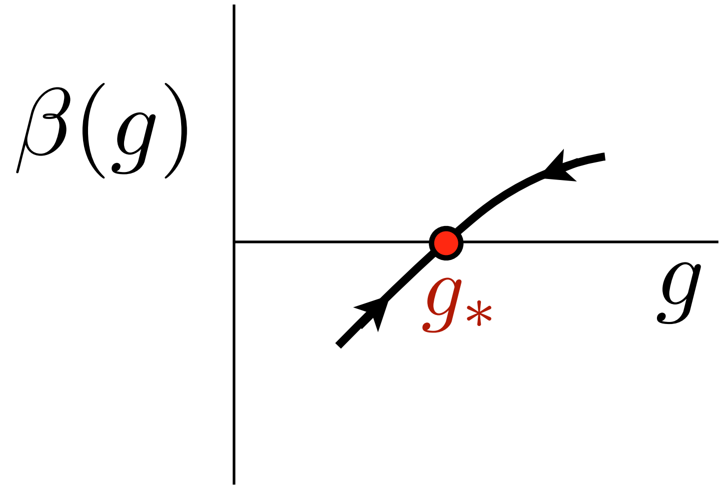
IV. AdS/CFT

V. Relativistic model: defect Yang-Mills

VI. QCD with many flavors? A partner theory QCD* with a nontrivial UV fixed point?

A theory with an infrared conformal fixed point at $g=g_*$ has a zero in the beta function:

$$\beta(g) = \mu\frac{\partial g}{\partial \mu} = \frac{\partial g}{\partial t}$$



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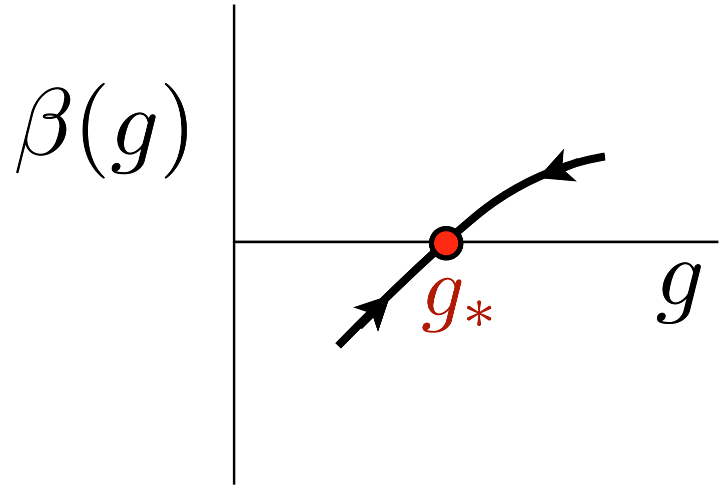

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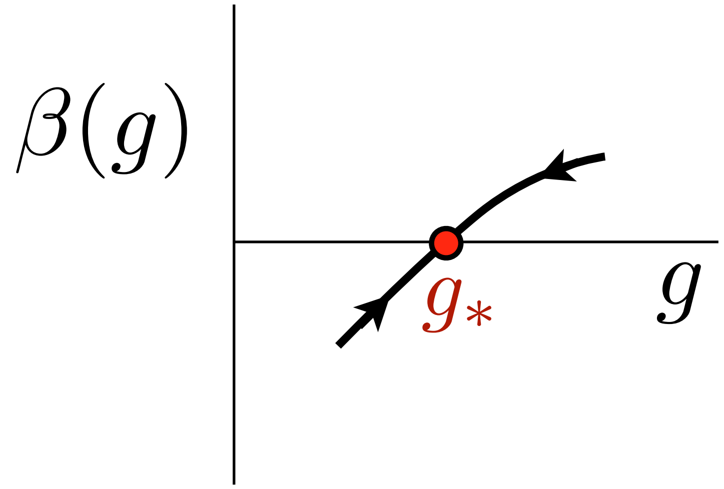

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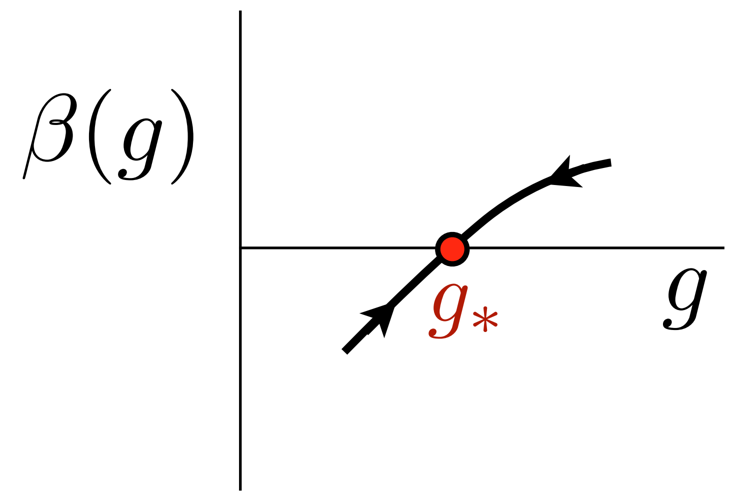

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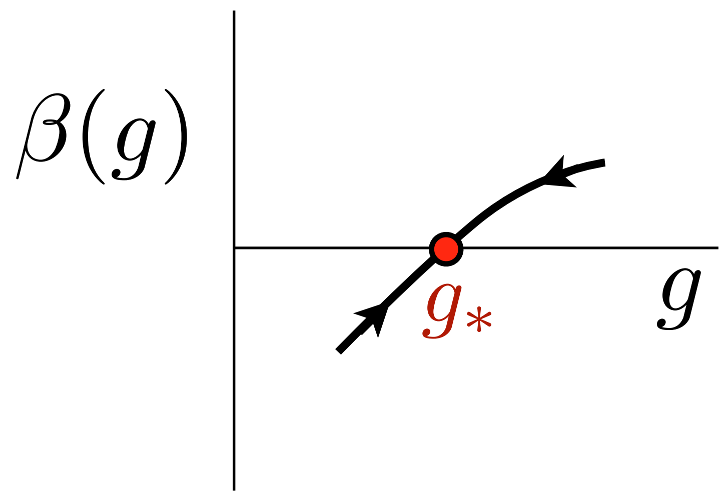

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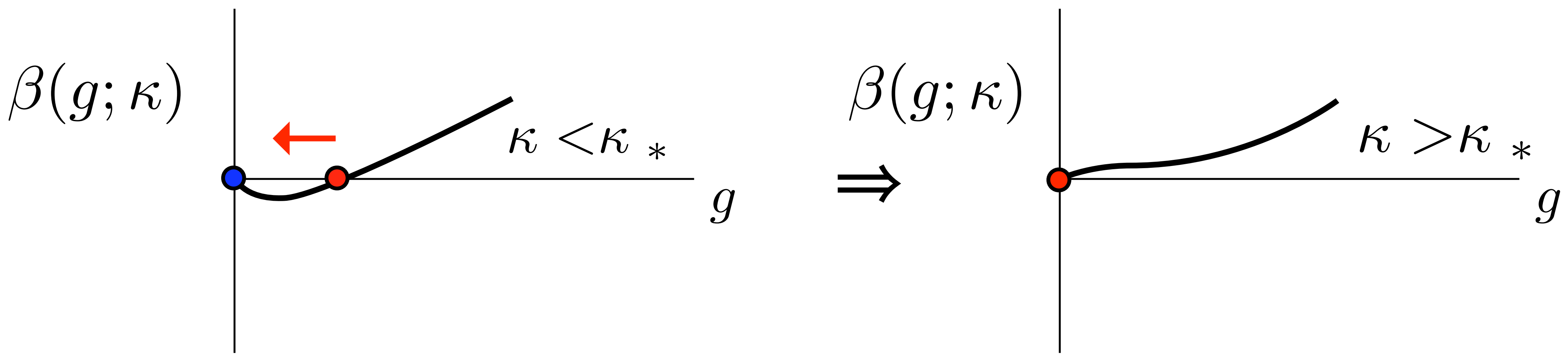
How is conformality lost?

Three ways to lose an infrared fixed point:

#1: Fixed point runs to zero:



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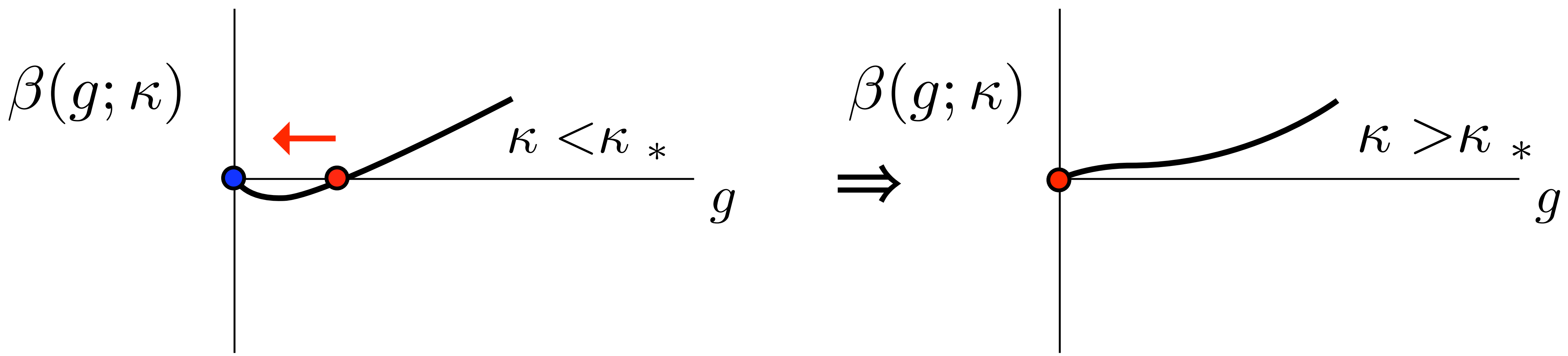

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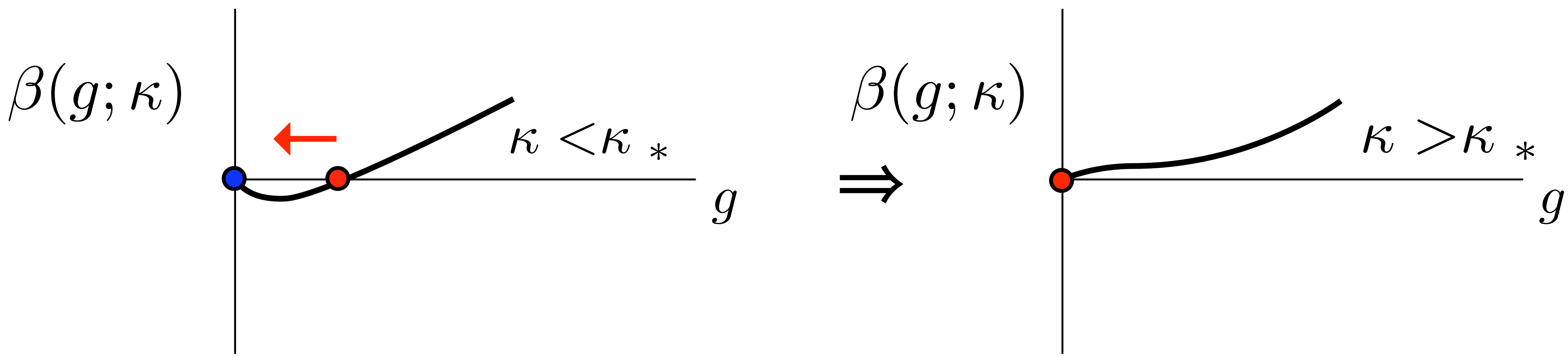

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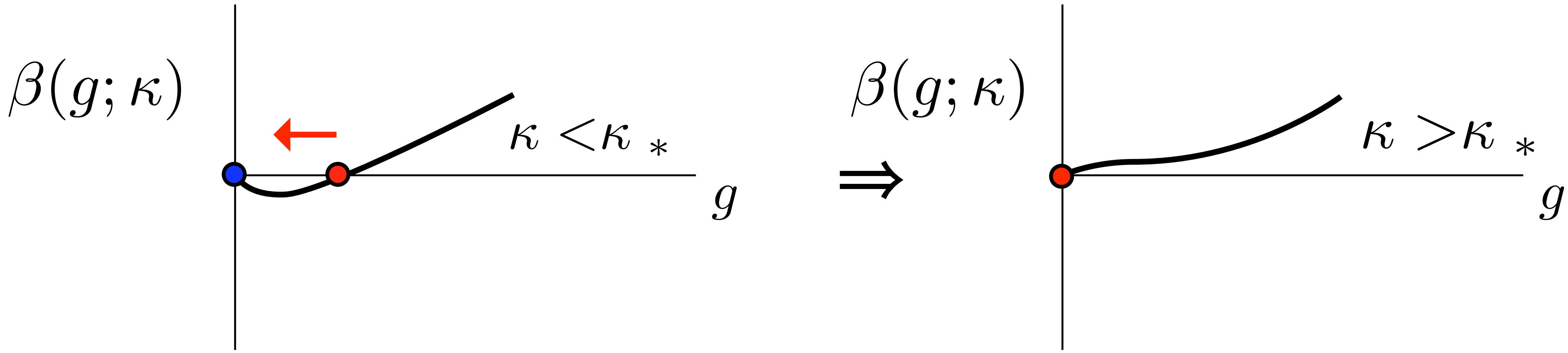

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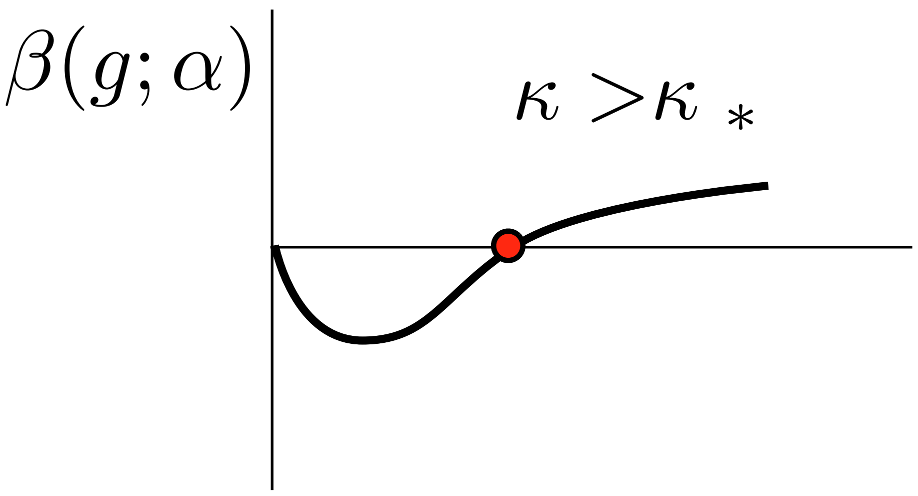

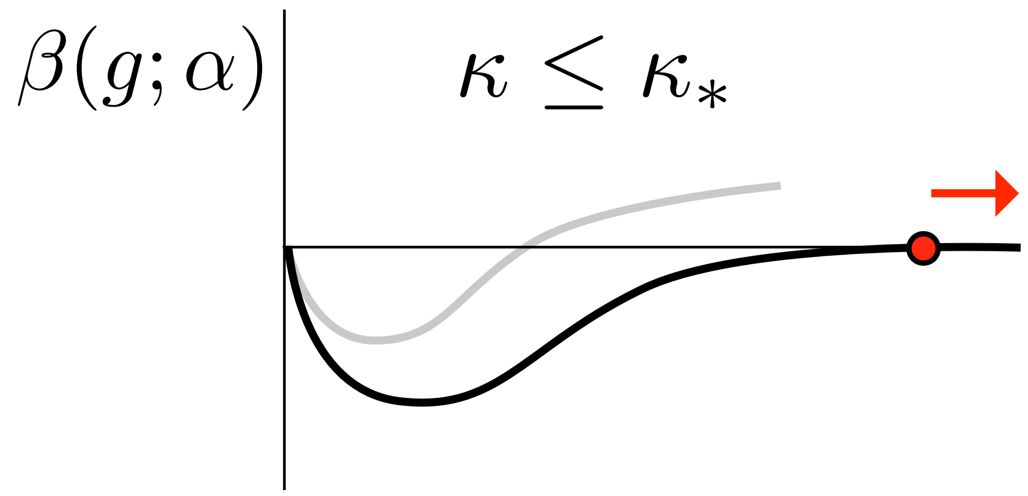

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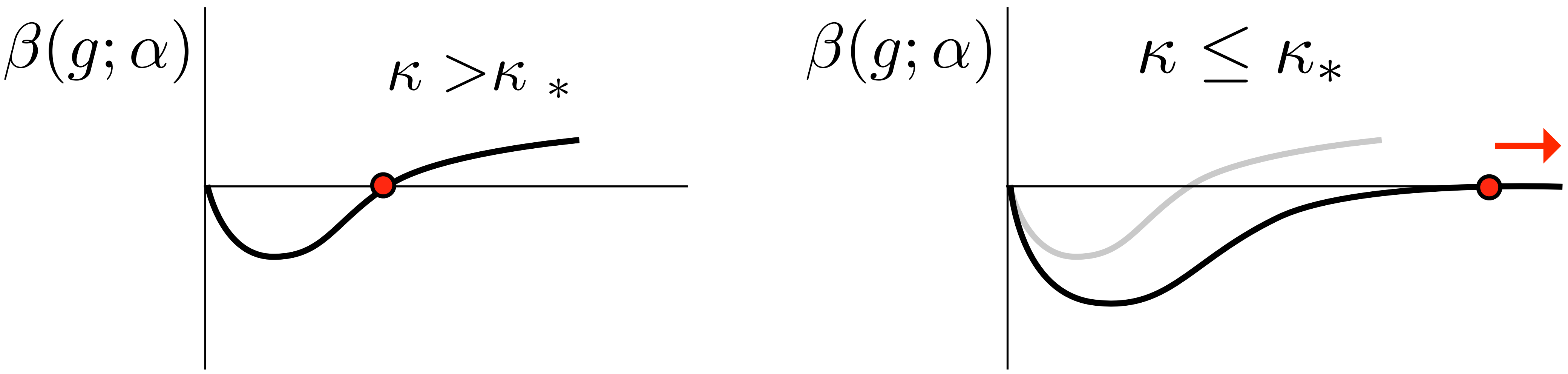

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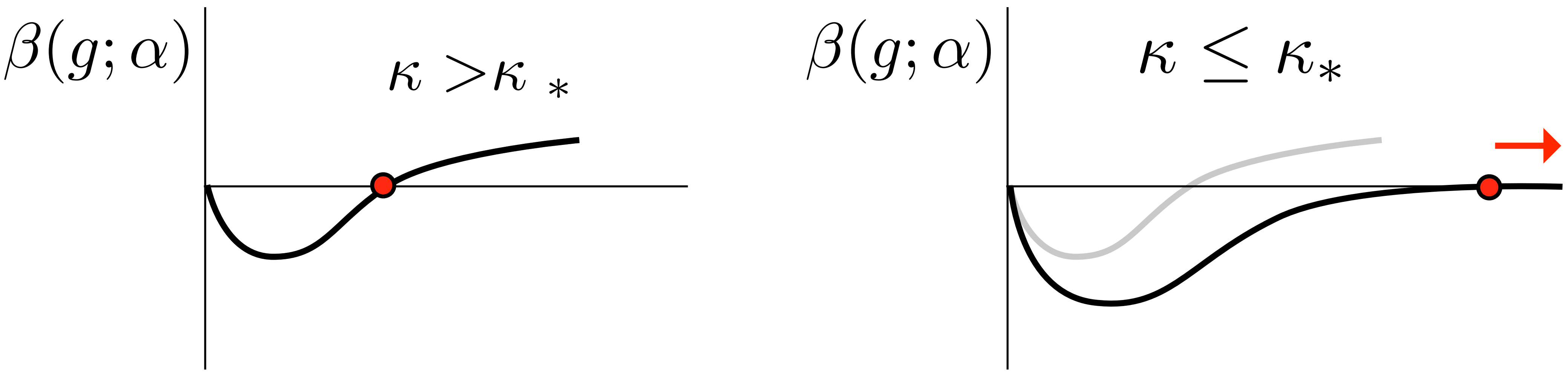

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➤ electric theory dual to a QED-like magnetic theory:

$$F_E \sim \frac{g^2 \ln(r\,\Lambda_{\rm UV})}{r^2} \qquad F_M \sim \frac{g_M^2}{r^2 \ln(r\,\Lambda_{\rm UV})} \qquad g_M \sim 1/g$$

#3: UV and IR fixed points annihilate:

A toy model:

$$\beta(g;\kappa) = (\kappa - \kappa_*) - (g - g_*)^2$$

$\kappa \geq \kappa_*$: $\quad g_\pm = g_* \pm \sqrt{\kappa - \kappa_*}$

UV, IR fixed points

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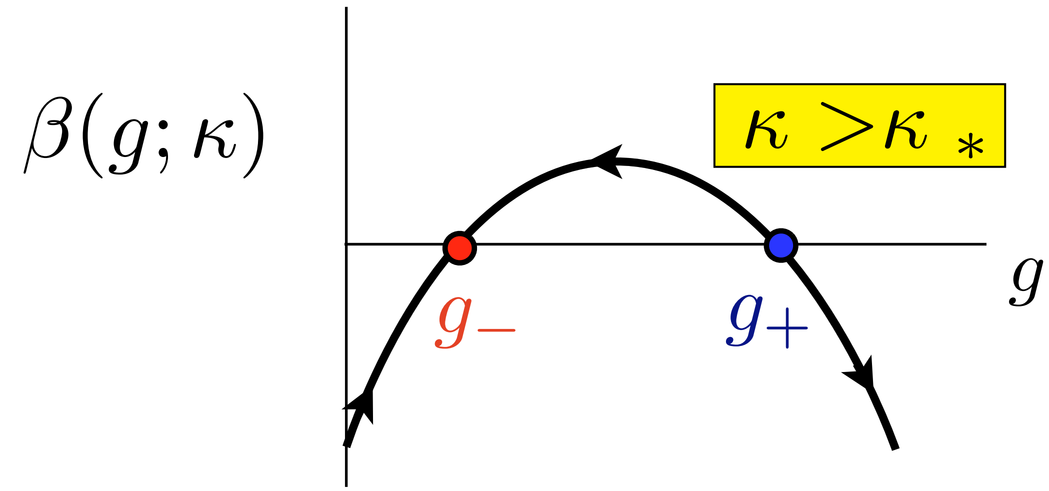

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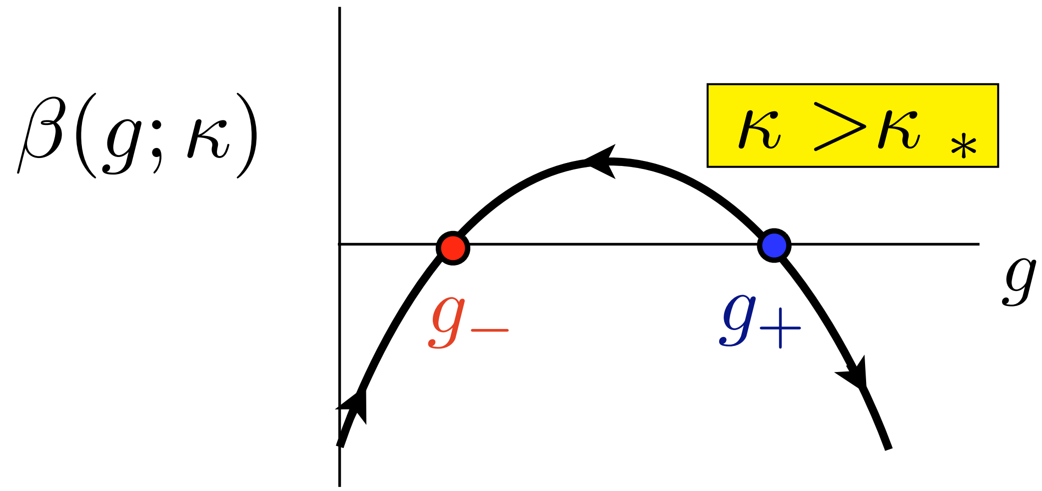

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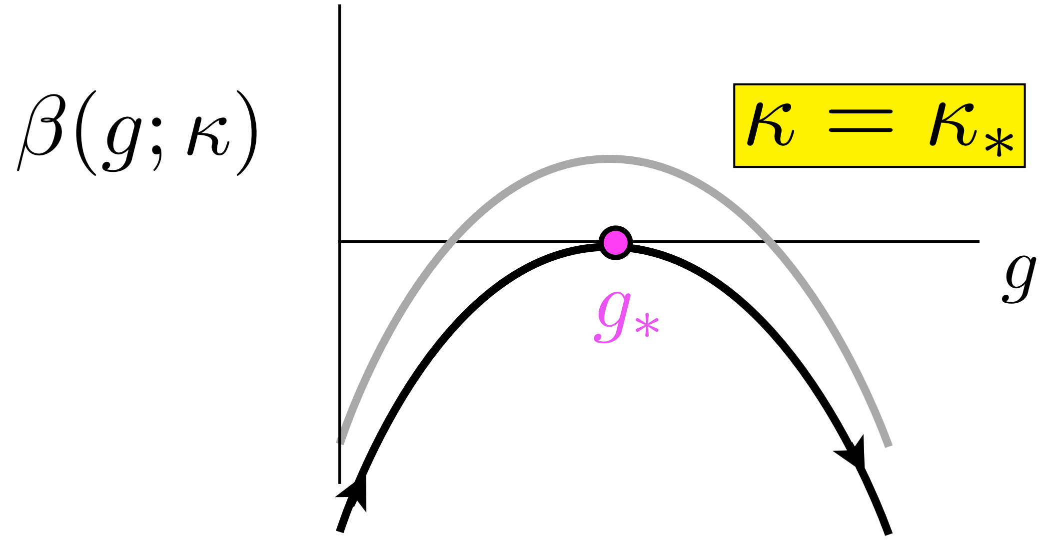

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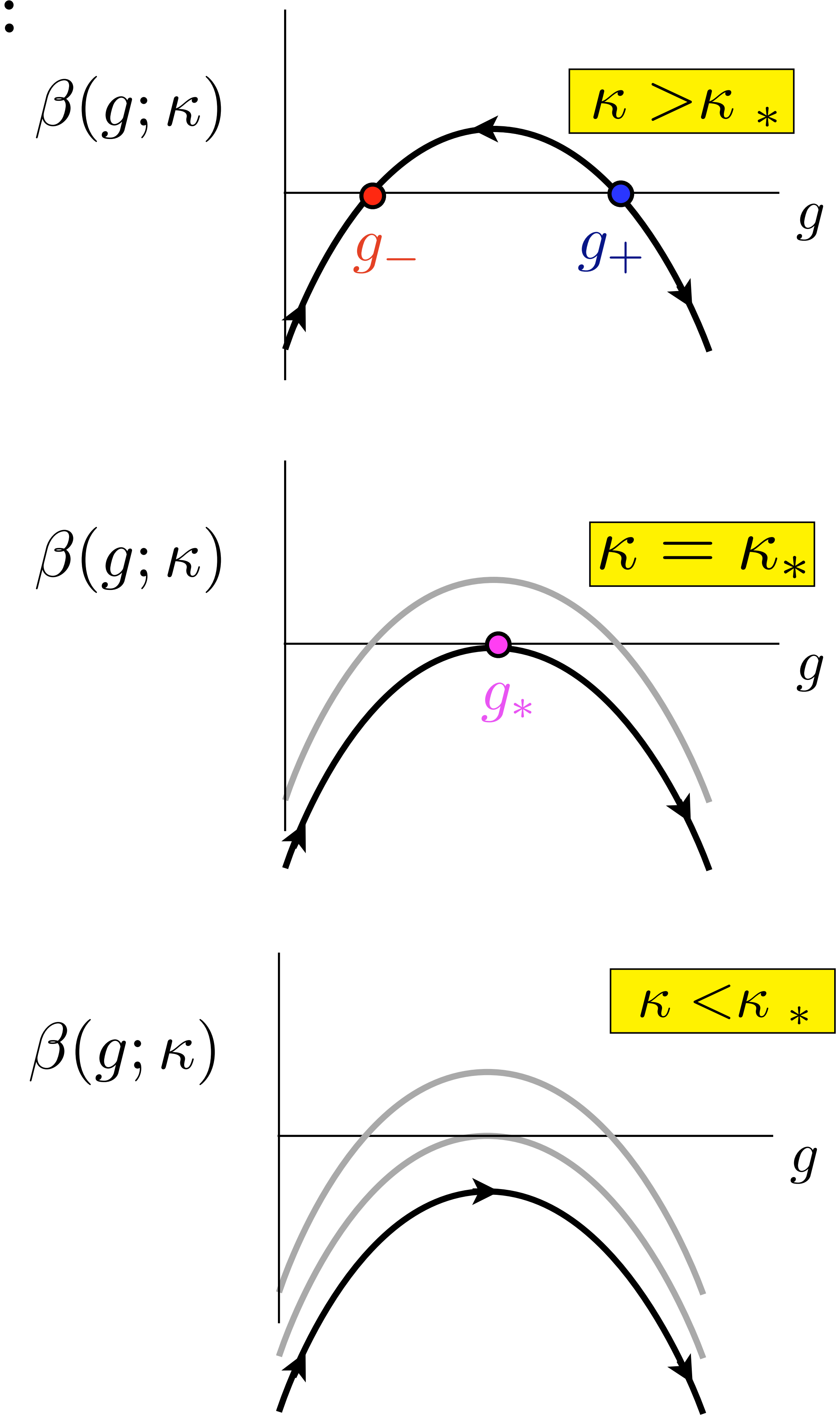

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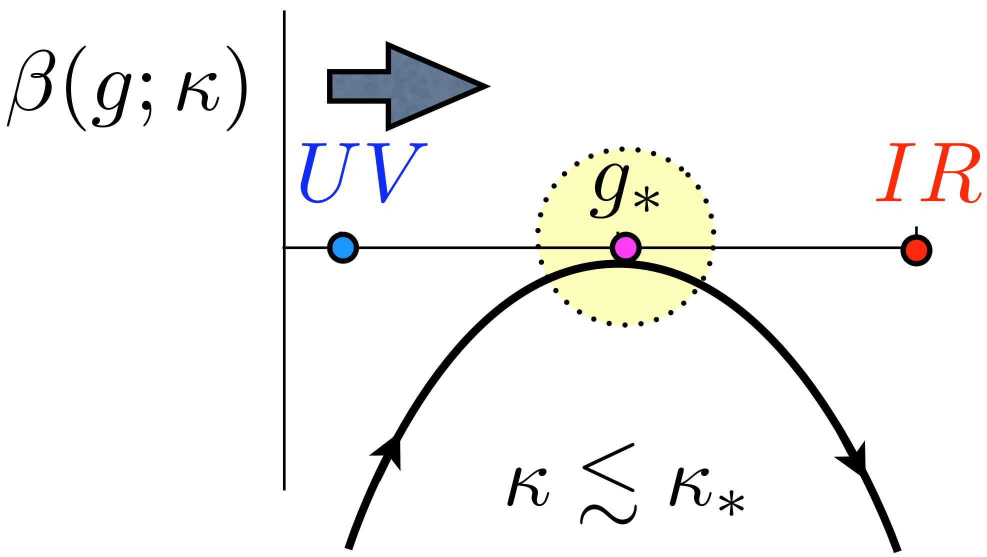

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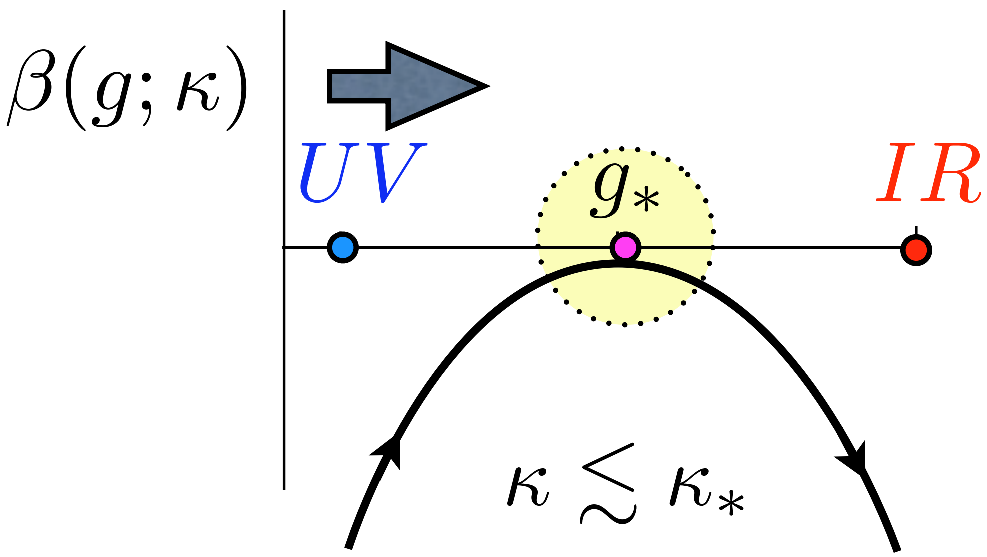

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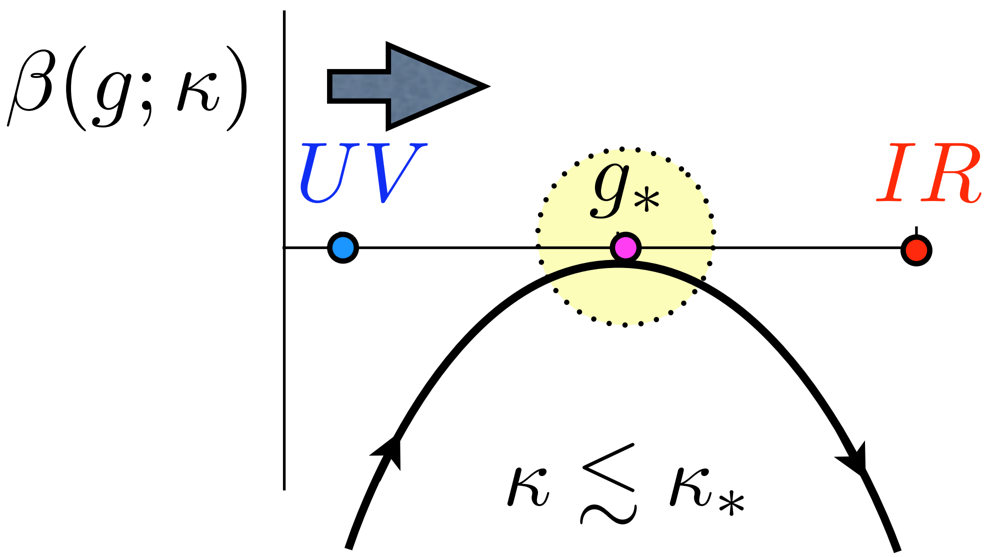

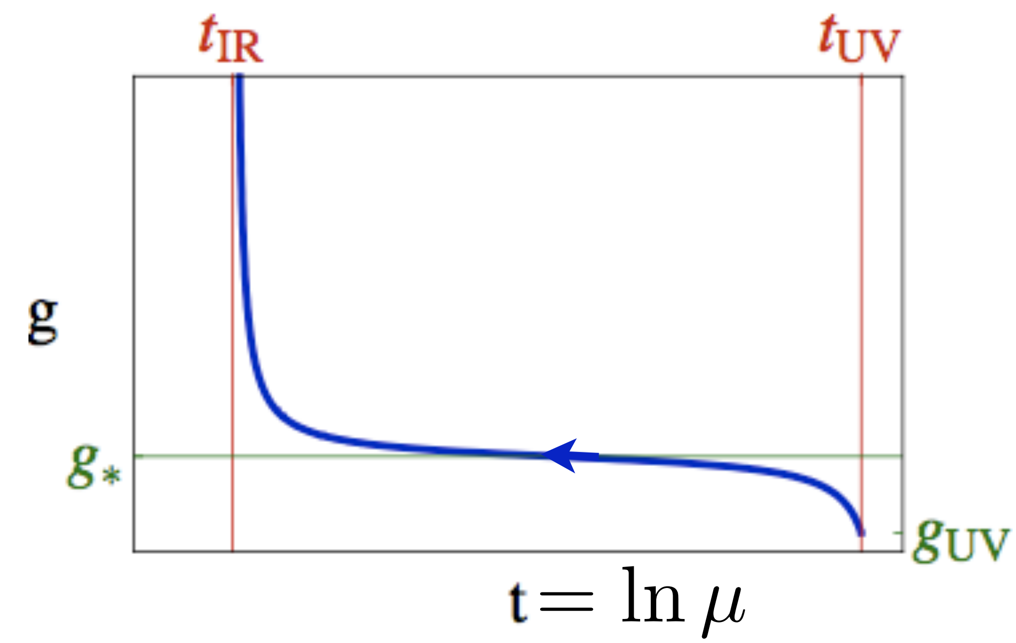

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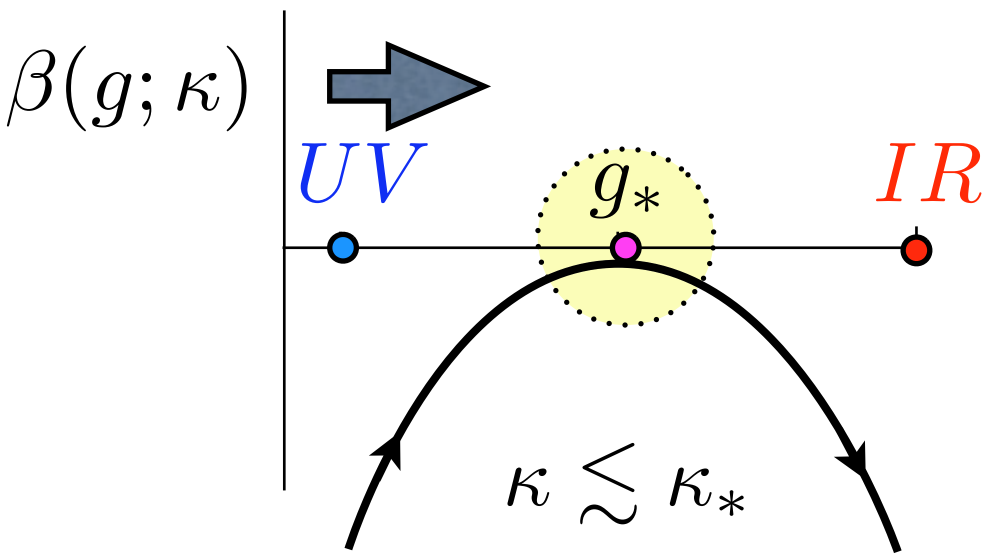

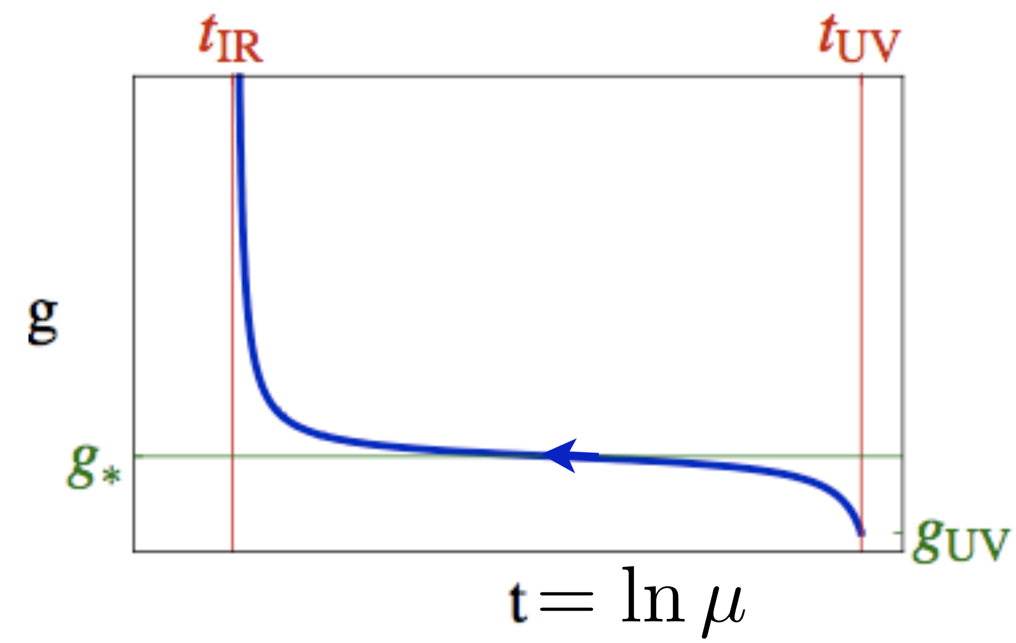

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(Not like 2nd order phase transition: $\Lambda_{IR} \simeq \Lambda_{UV}\sqrt{|\kappa-\kappa_*|}$)

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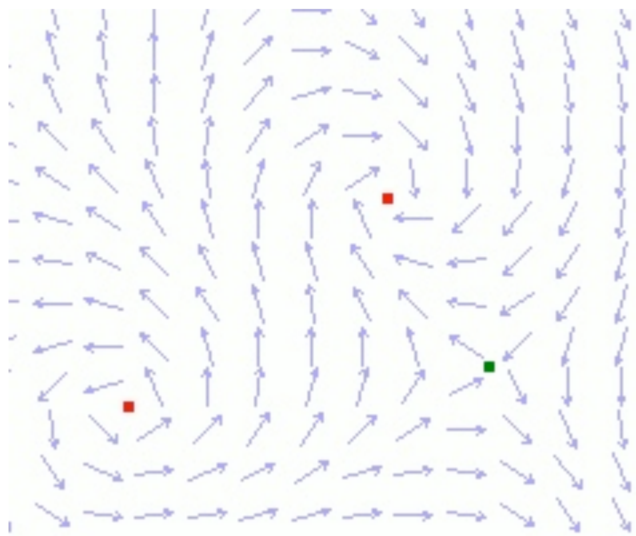

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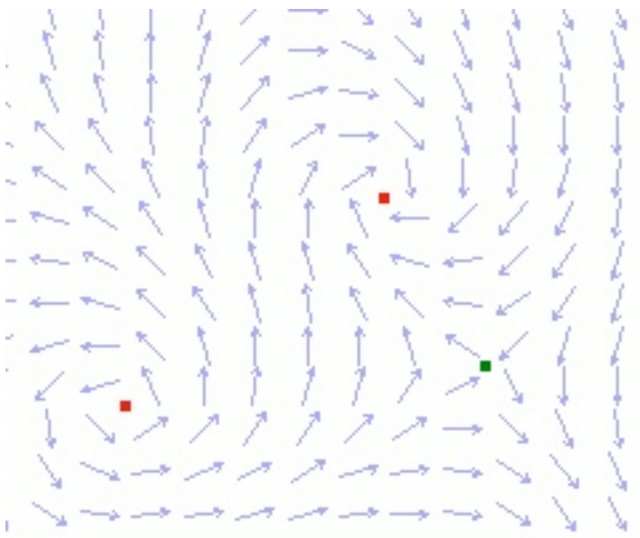

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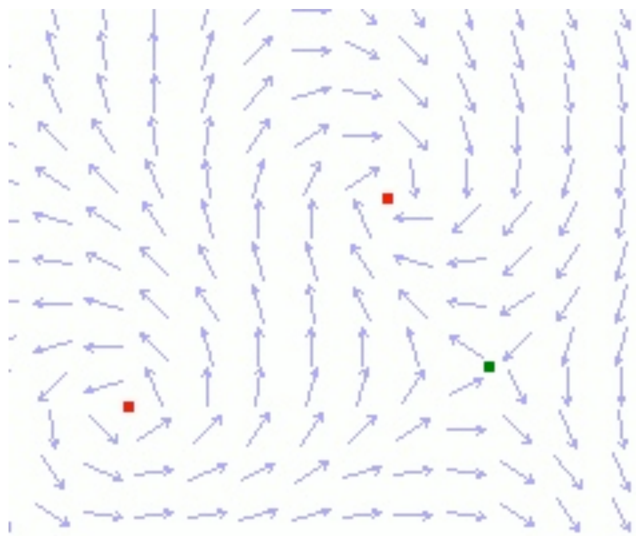

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Vortices condense for $T > T_c = E_0/2$;
can show correlation length forms:

$$\xi \simeq a\, e^{b/\sqrt{T-T_c}}$$

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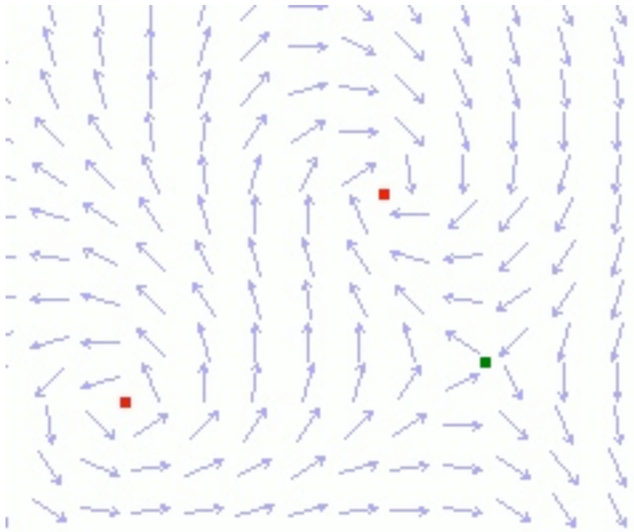

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Coulomb field

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Coulomb field vortices anti-vortices

$$= \mathcal{N} \int D\phi\, e^{-\int d^2x \left[\frac{T}{2}(\nabla\phi)^2 - 2z\cos\phi\right]}$$

temp. fugacity

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The XY model is equivalent to the Sine-Gordon model

Classical XY model BKT transition = zero temperature quantum transition in Sine-Gordon model:

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Perturbative β-functions:

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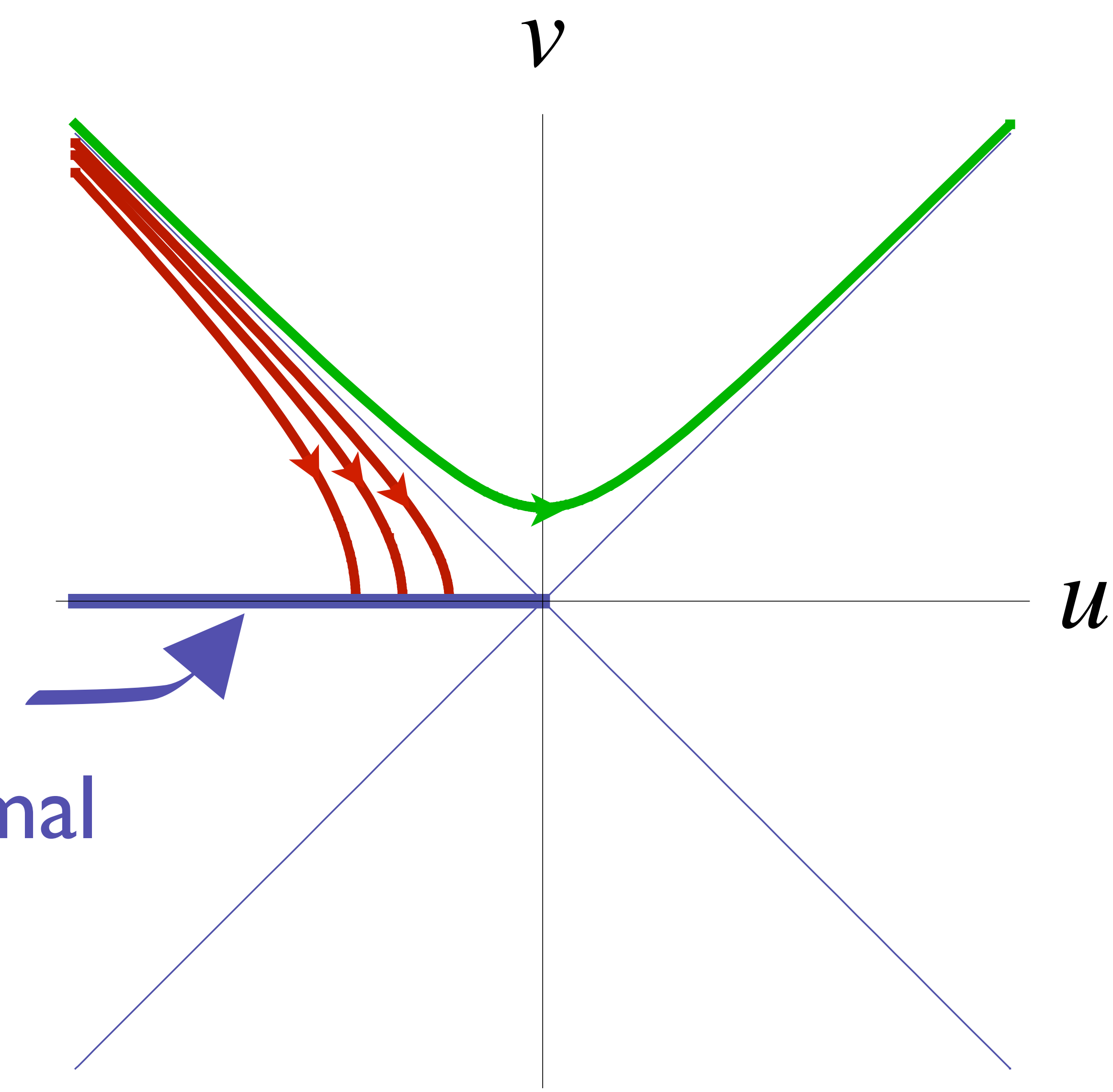

- $T<T_c$
- bound vortices
- trivially conformal

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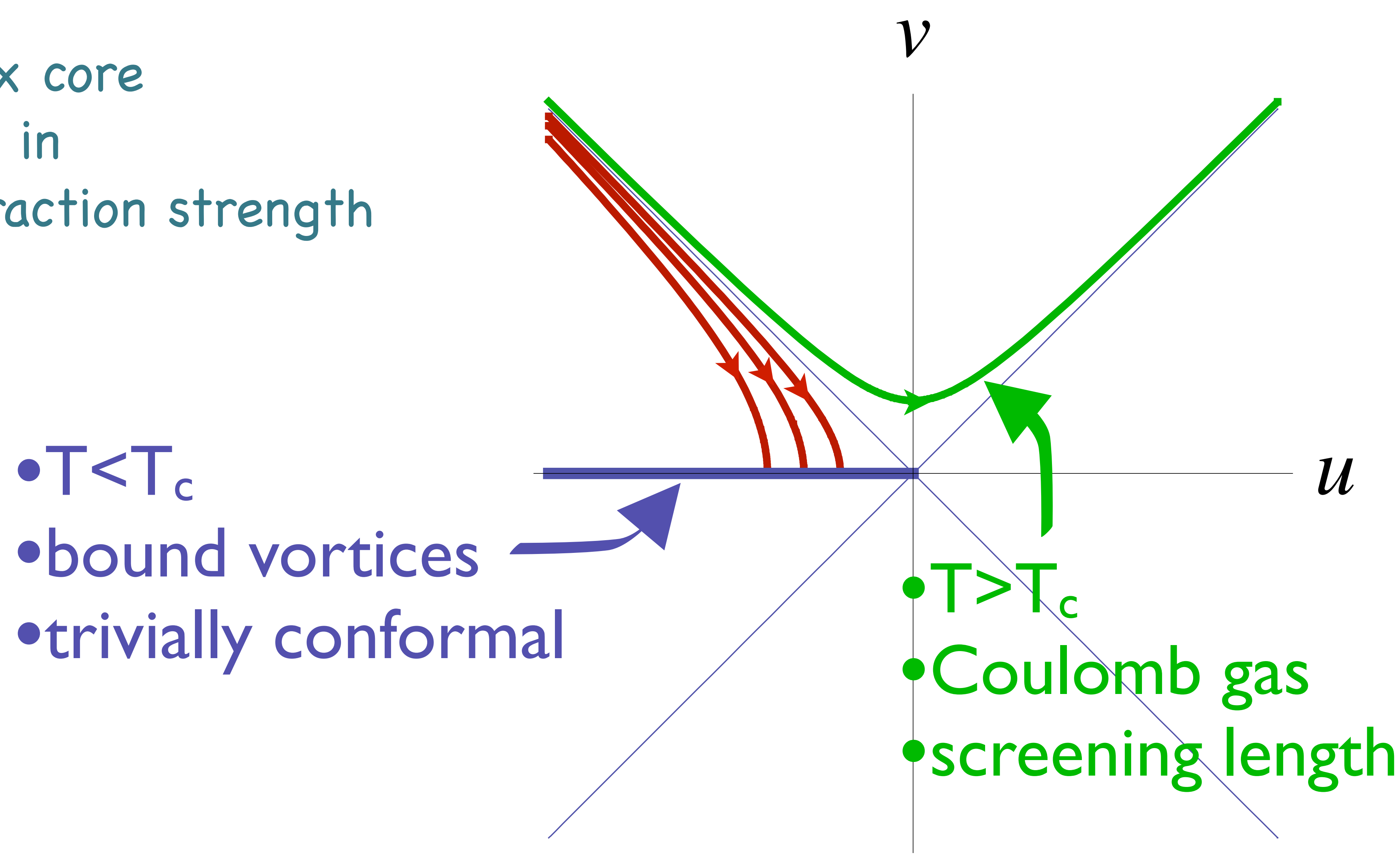

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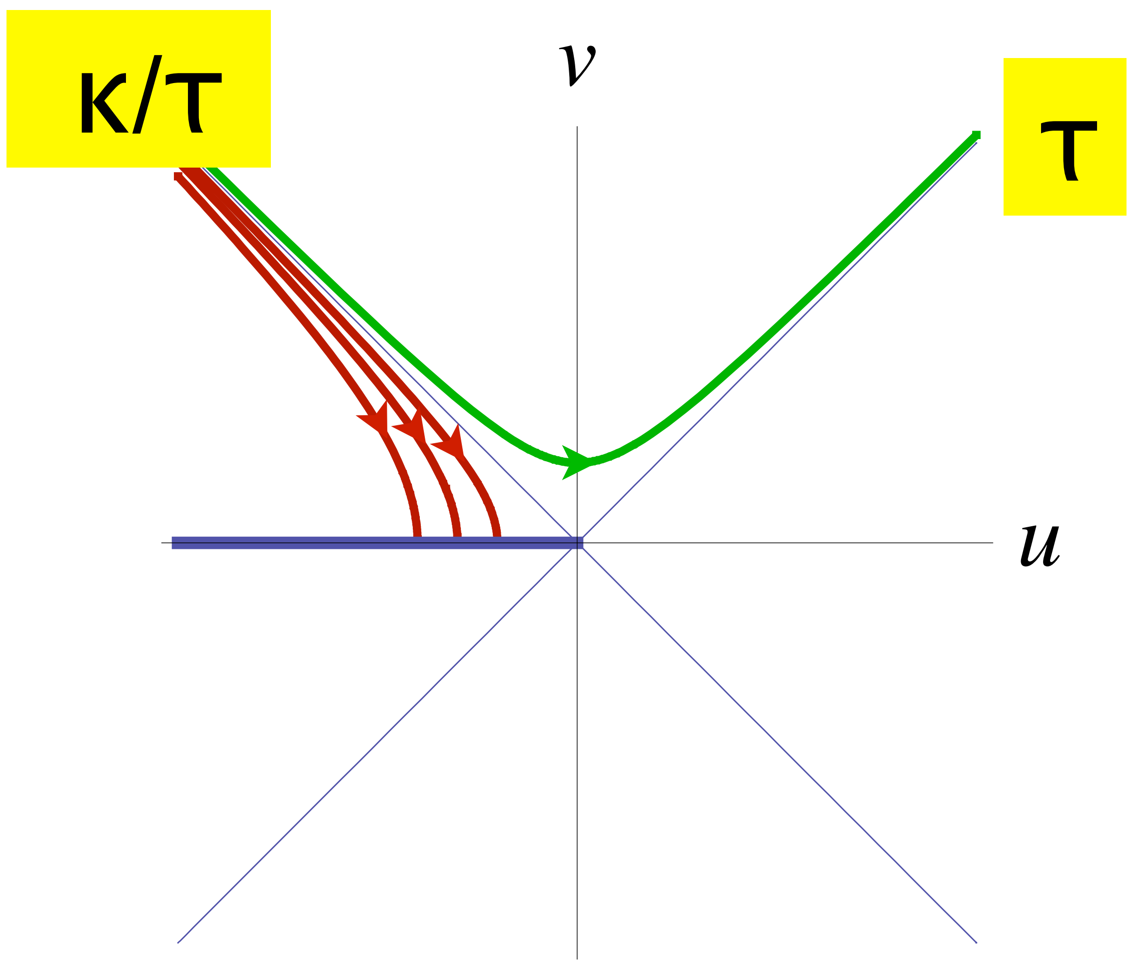
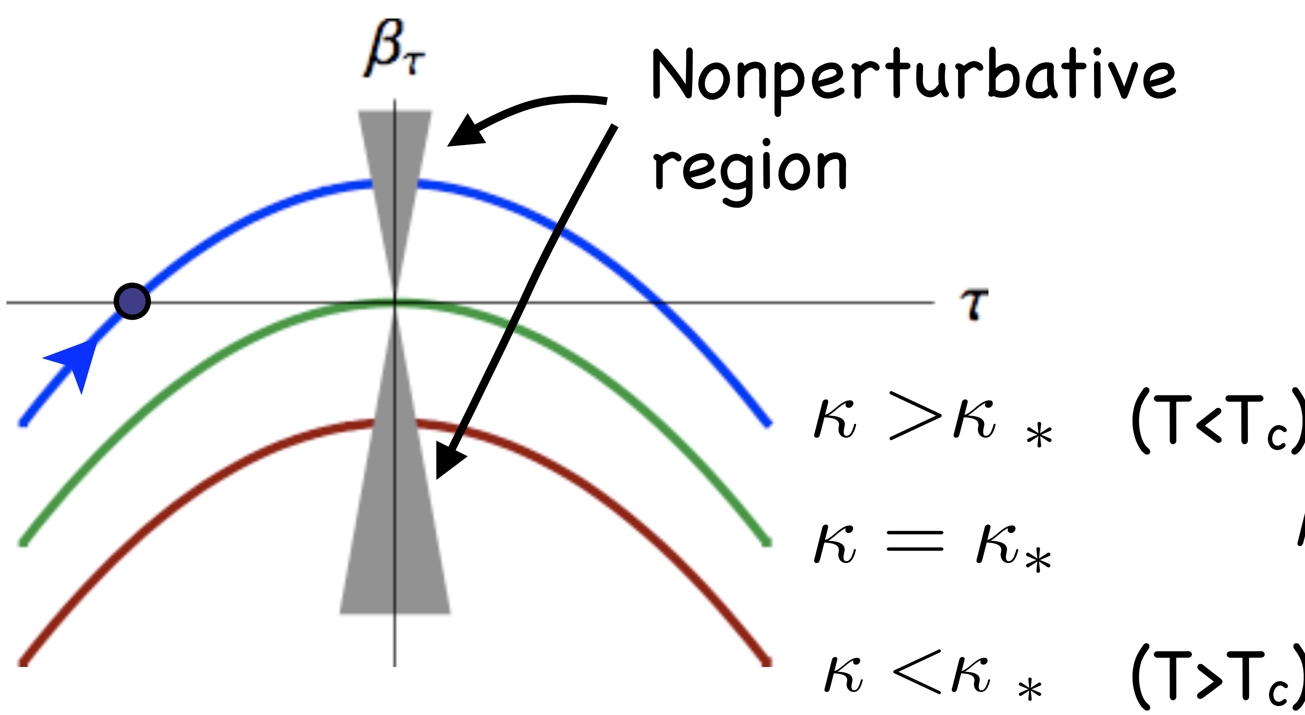


Newer variables:

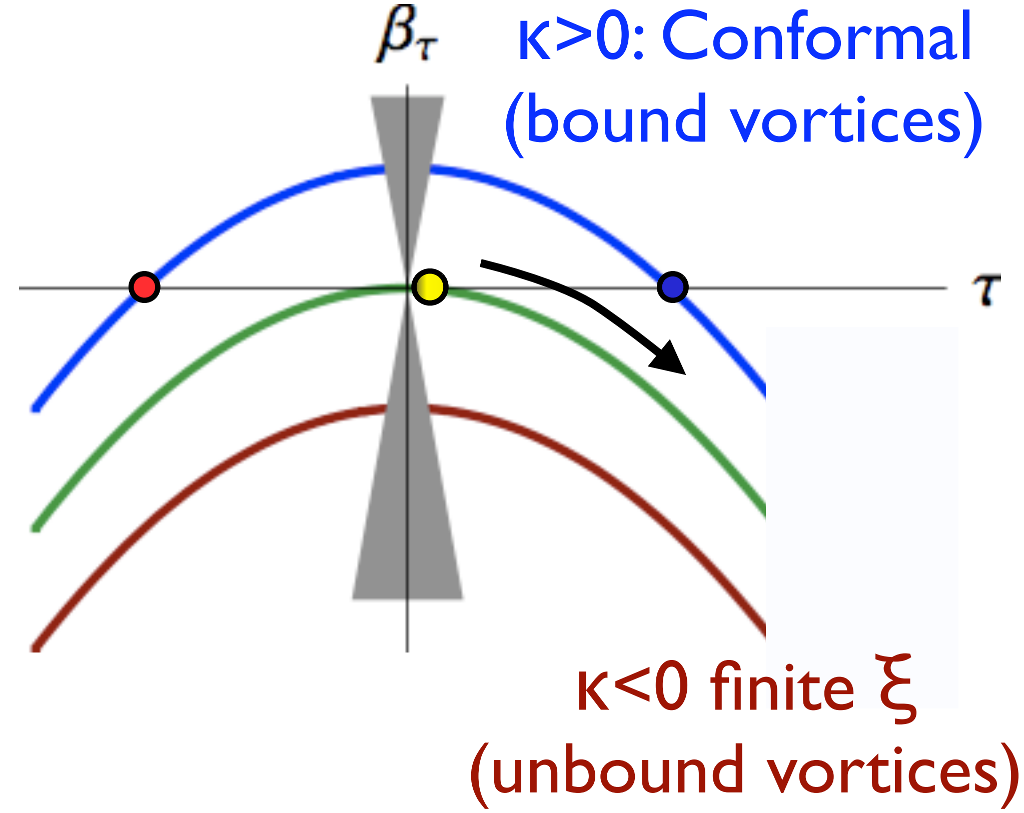
$$\tau = (u+v)\ , \qquad \kappa = (u^2 - v^2)$$

$$\beta_\tau = \kappa - \tau^2\ , \qquad \beta_\kappa = 0$$



$$\kappa_* = 0$$

Correlation length in BKT transition:



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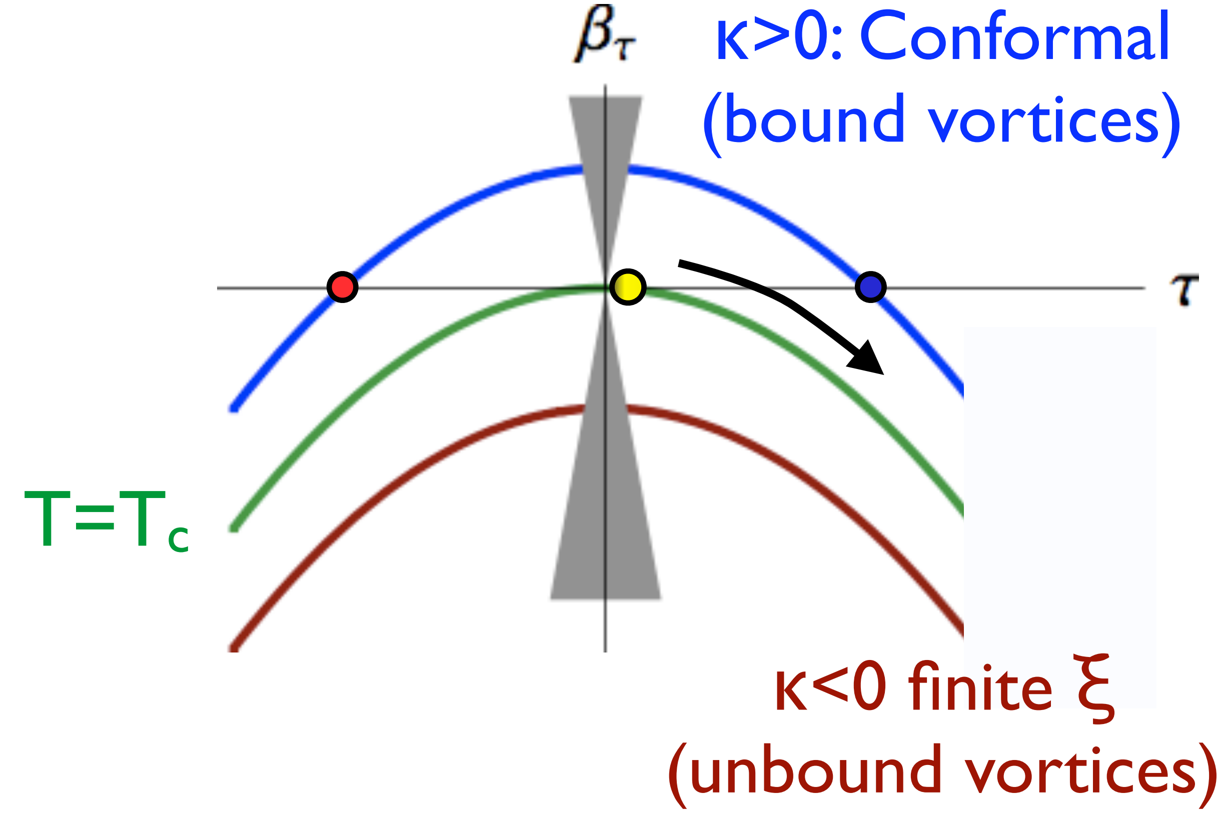

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...giving rise to an IR scale (like Λ_{QCD}) which sets the scale for the finite correlation length for α<0:

$$\xi_{\rm BKT} \sim \frac{1}{\Lambda} e^{\frac{\pi}{2\sqrt{-\alpha}}}$$

So far:

- BKT transition = loss of conformality via fixed point merger
- Mechanism of fixed point merger in general gives rise to "BKT scaling":

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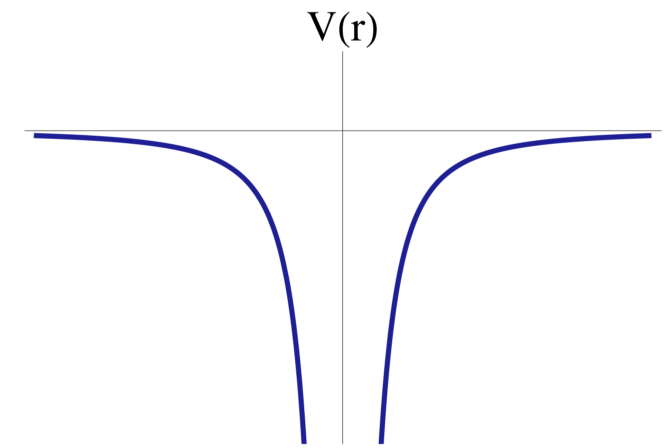
Next: other examples:

- QM with $1/r^2$ potential
- AdS/CFT
- Defect Yang-Mills
- QCD with many flavors

Example: QM in d-dimensions with $1/r^2$ potential

$$\left[-\nabla^2 + V(r) - k^2\right]\psi = 0\ , \qquad V(r) = \frac{\kappa}{r^2}$$



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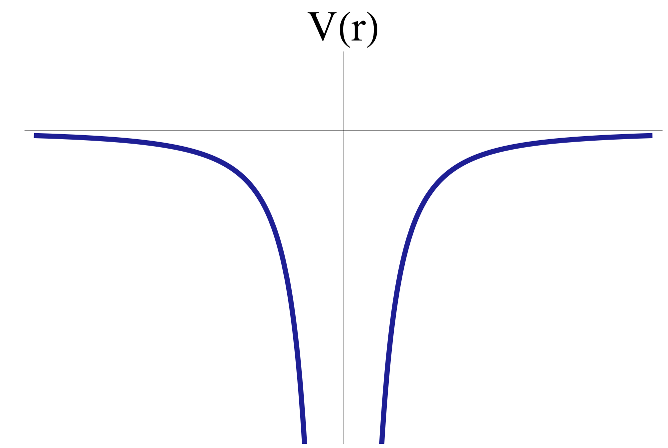

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k=0 solutions: $\psi = c_- r^{\nu_-} + c_+ r^{\nu_+}$

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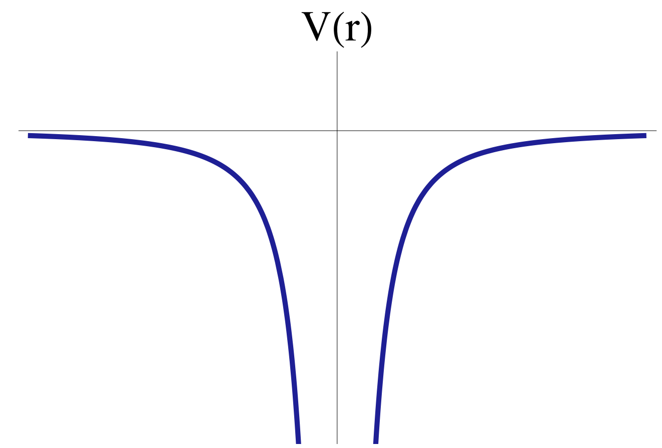

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- valid for $\kappa_* < \kappa < (\kappa_*+1)$
  - $\kappa < \kappa_*$: $\nu_\pm$ complex, no ground state
  - $\kappa = \kappa_*$: $\nu_+ = \nu_-$
  - $\kappa > (\kappa_*+1)$: r^{ν_-} too singular to normalize

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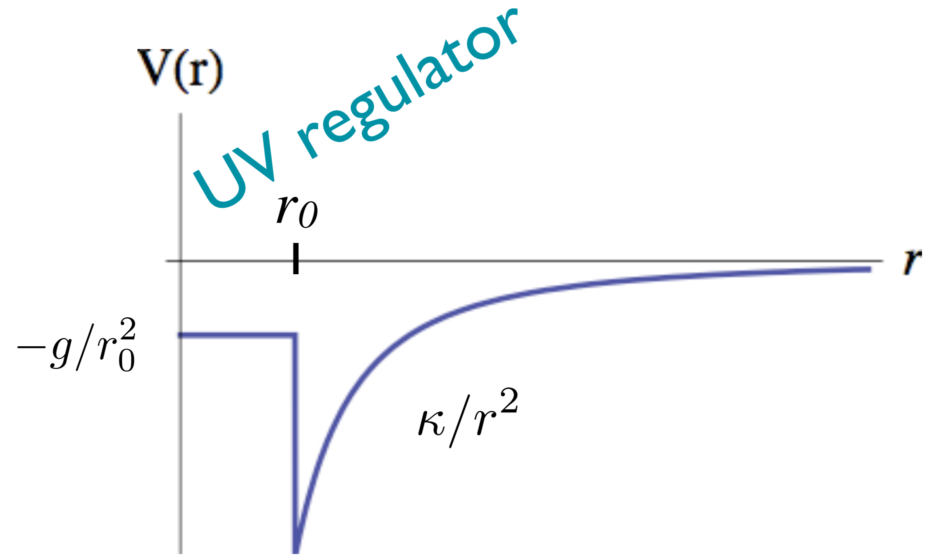
- r^{ν_+} dominates at large r -- corresponds to IR fixed point of g
- r^{ν_-} dominates at small r -- corresponds to UV fixed point of g

1. *Non-perturbative* RG treatment of $1/r^2$ potential:

regulate with square well:

$$V(r) = \begin{cases} \kappa/r^2 & r > r_0 \\ -g/r_0^2 & r > r_0 \end{cases}$$



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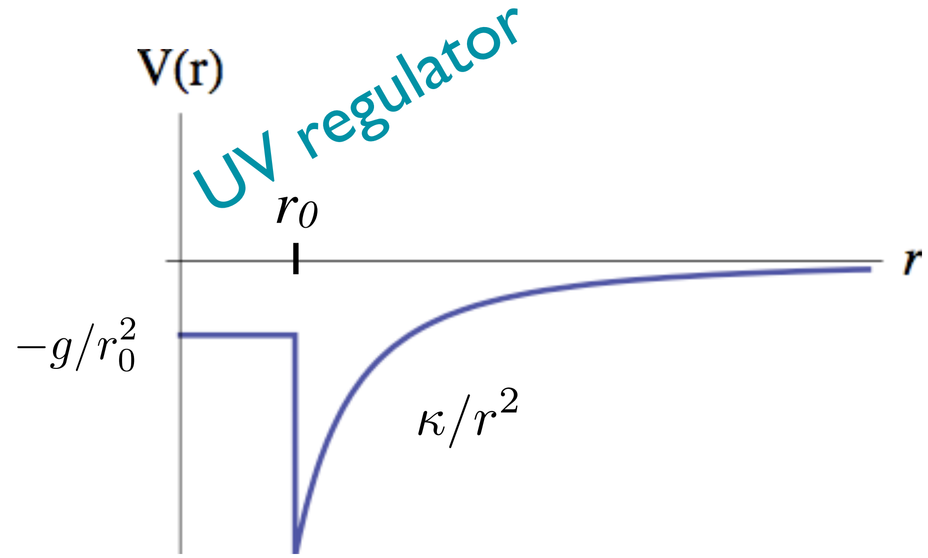

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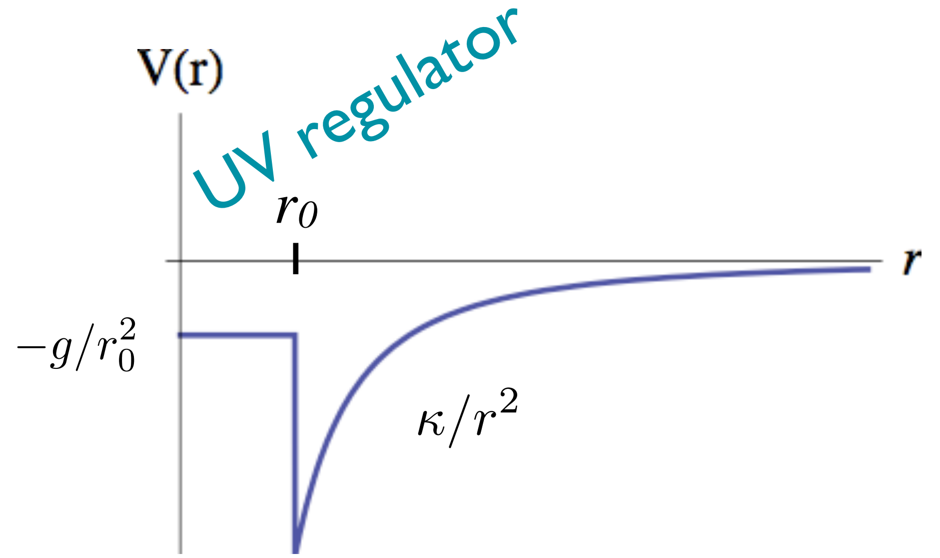

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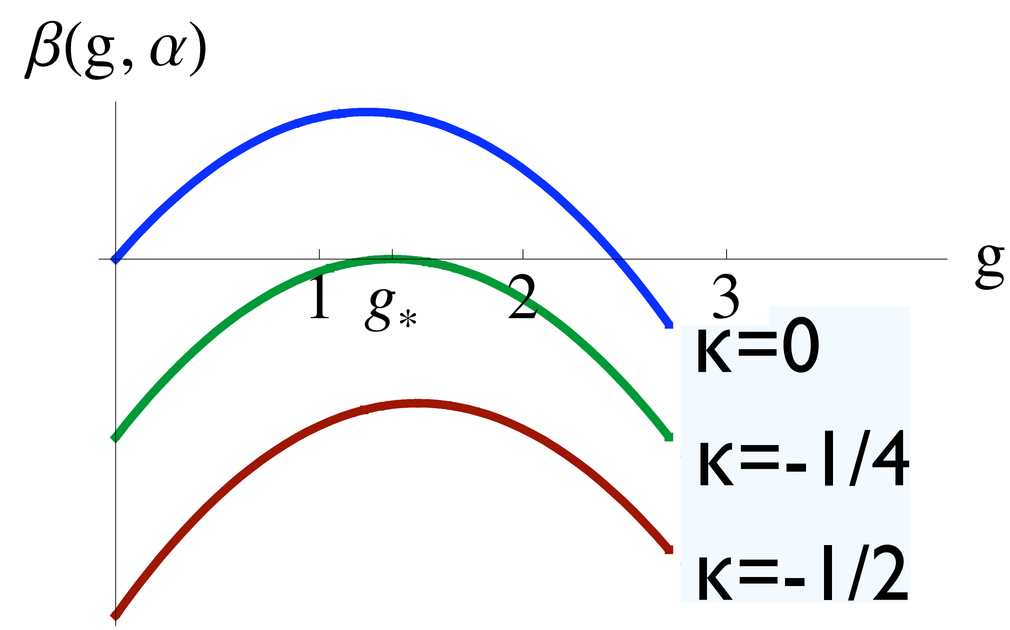
Solve for c_+/c_- (a physical dimensionful quantity)
and require invariance: $d(c_+/c_-)/dr_0 = 0$:

Find <u>exact</u> β-function for g. Eg, for d=3:

$$\beta = \frac{2\sqrt{g}\left(\kappa + \sqrt{g}\cot\sqrt{g} - g\cot^2\sqrt{g}\right)}{-\cot\sqrt{g} + \sqrt{g}\csc^2\sqrt{g}}$$

$\kappa_* = -1/4$, $g_* \approx 1.36$



Aside: Even better to define a modified coupling constant

$$\gamma = \left(\frac{\sqrt{g}\, J_{d/2}(\sqrt{g})}{J_{d/2-1}(\sqrt{g})} \right)$$

Condition $d(c_+/c_-)/dr_0$ yields exact β-function in d-dimensions:

$$\beta_\gamma = \frac{\partial \gamma}{\partial t} = (\kappa - \kappa_*) - (\gamma - \gamma_*)^2, \quad \gamma_* = \frac{d-2}{2}$$

- Toy model is exact!
- γ is a periodic function of g, γ=±∞ equivalent
- Aside: Limit cycle behavior for $\kappa<\kappa_*$: describes "Efimov states"

II. Perturbative RG treatment of κ/r^2 potential:

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$$S = \int dt\, d^d\mathbf{x} \left(i\psi^\dagger \partial_t \psi - \frac{|\nabla\psi|^2}{2m} + \frac{g\pi}{4}\psi^\dagger\psi^\dagger\psi\psi \right)$$
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propagator: $\dfrac{i}{\omega - \mathbf{p}^2/2m}$

contact vertex: $i\pi g\mu^{-\epsilon}$

“meson exchange”: $\dfrac{2\pi i\kappa}{\epsilon}\dfrac{1}{|\mathbf{q}|^\epsilon}$

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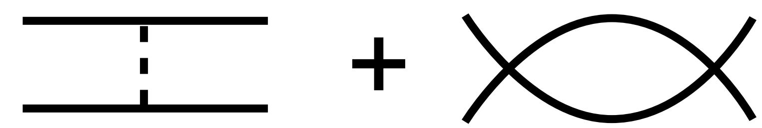
Find g runs: 

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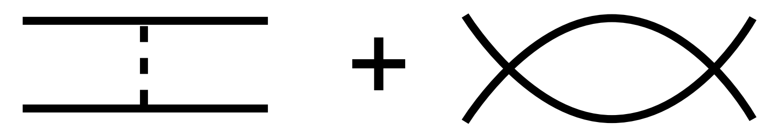

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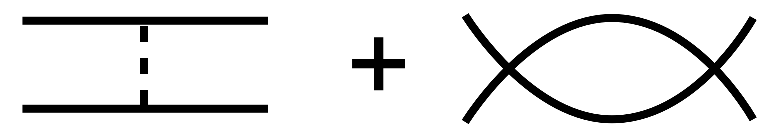
Find g runs: 

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$\kappa > \kappa_*$: conformal

$\kappa = \kappa_*$: critical

$\kappa < \kappa_*$: g blows up in IR

$$B \sim \left(\frac{\Lambda_{\rm IR}^2}{m}\right) \sim \left(\frac{\Lambda_{\rm UV}^2}{m}\right) e^{-2\pi/\sqrt{\kappa_* - \kappa}}$$

BKT scaling

bound state energy

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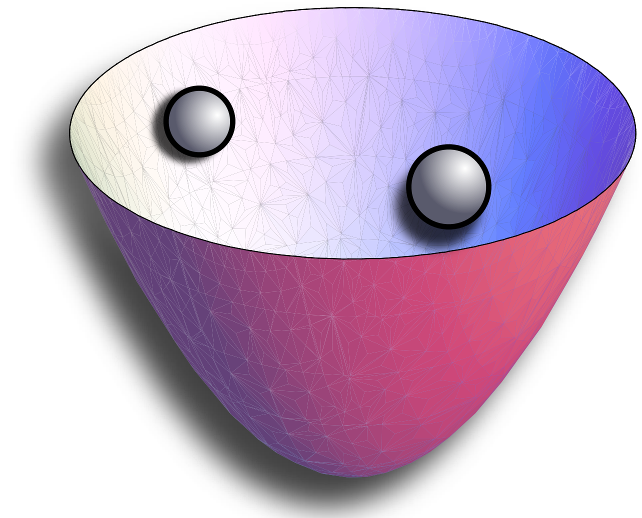

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- Replace $V(r_1-r_2) \to V(r_1-r_2) + \frac{1}{2}\,\omega^2|r_1{}^2+r_2{}^2|$
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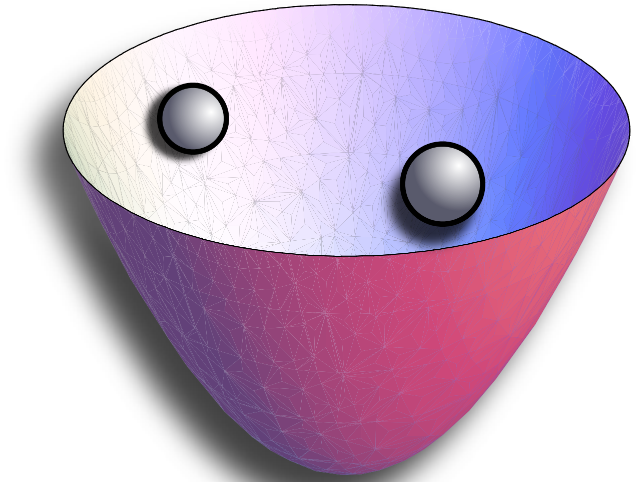

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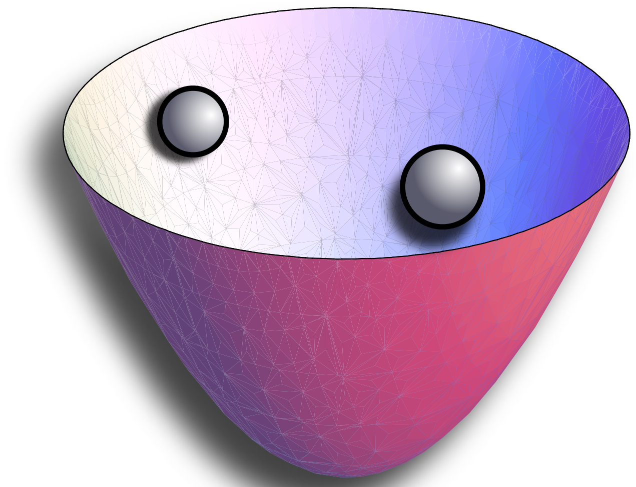

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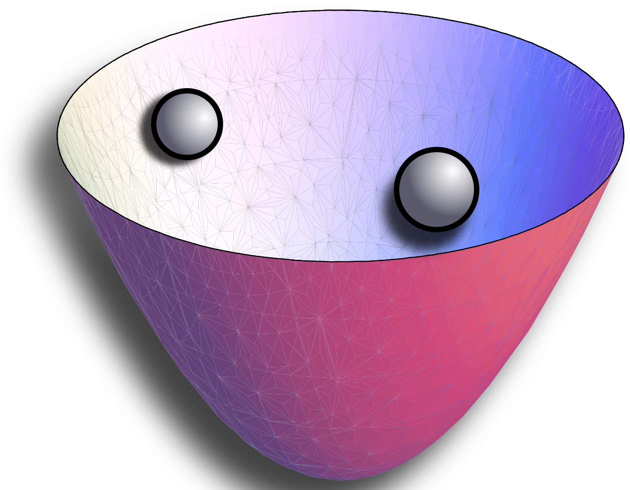

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For $1/r^2$ potential -- find for the two conformal theories:

$[\psi\psi]$: $$\Delta_\pm = (d+\nu_\pm) = \left(\frac{d+2}{2}\right) \pm \sqrt{\kappa-\kappa_*}$$

"+" = UV fixed point
"−" = IR fixed point

Note: $(\Delta_+ + \Delta_-) = (d+2)$: scaling dimension of nonrelativistic spacetime.

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UV fine-tuning: m²φ²...adds OO operator. Eg: O=Ψ̄Ψ, OO =Ψ̄ΨΨ̄Ψ

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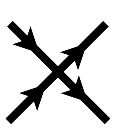
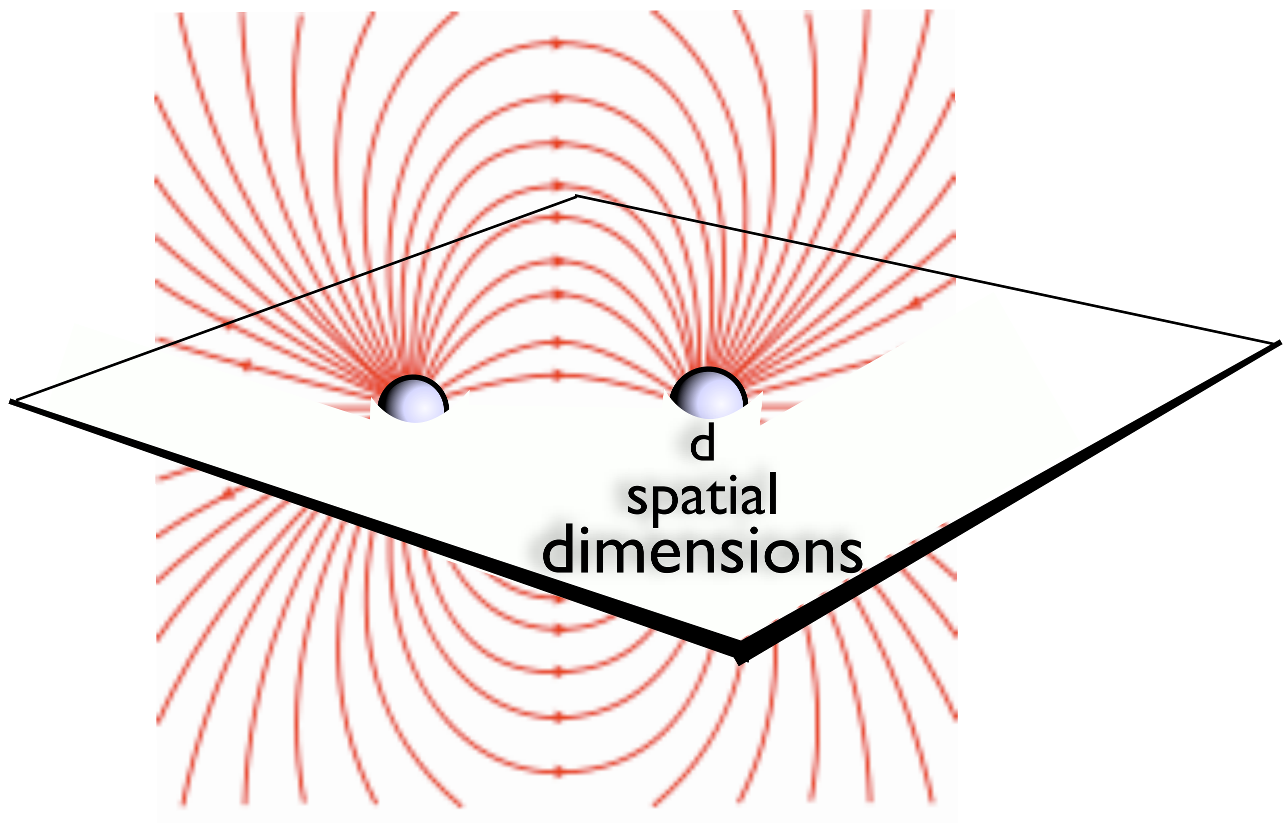
UV fine-tuning: $m^2\varphi^2$...adds OO operator. Eg: O=$\bar{\Psi}\Psi$, OO =$\bar{\Psi}\Psi\bar{\Psi}\Psi$

 ⇒ analog of δ(r) in QM example tuned to unstable UV fixed pt.

A relativistic example: defect Yang-Mills theory



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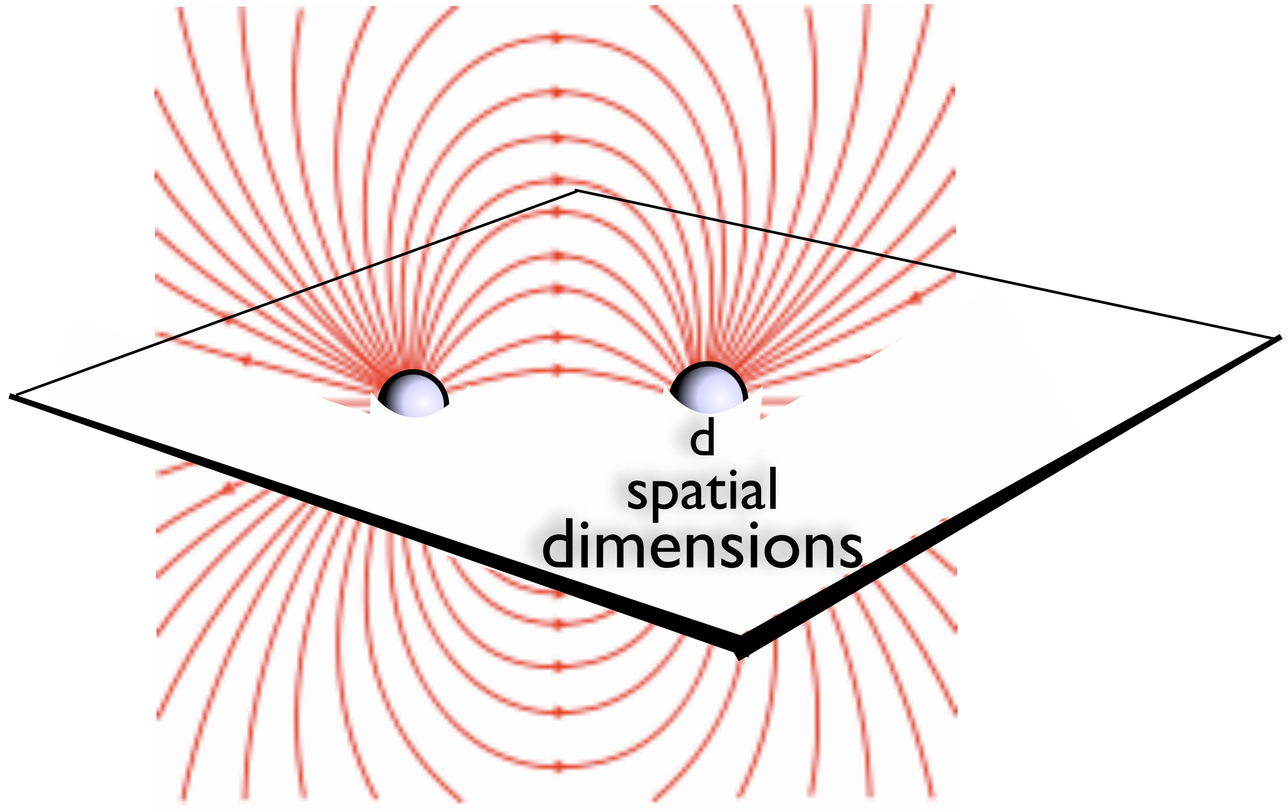

Charged relativistic fermions on a d-dimensional defect
+ 4D conformal gauge theory (eg, N=4 SYM)

$$S = \int d^{d+1}x\, i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - \frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{a,\mu\nu}$$

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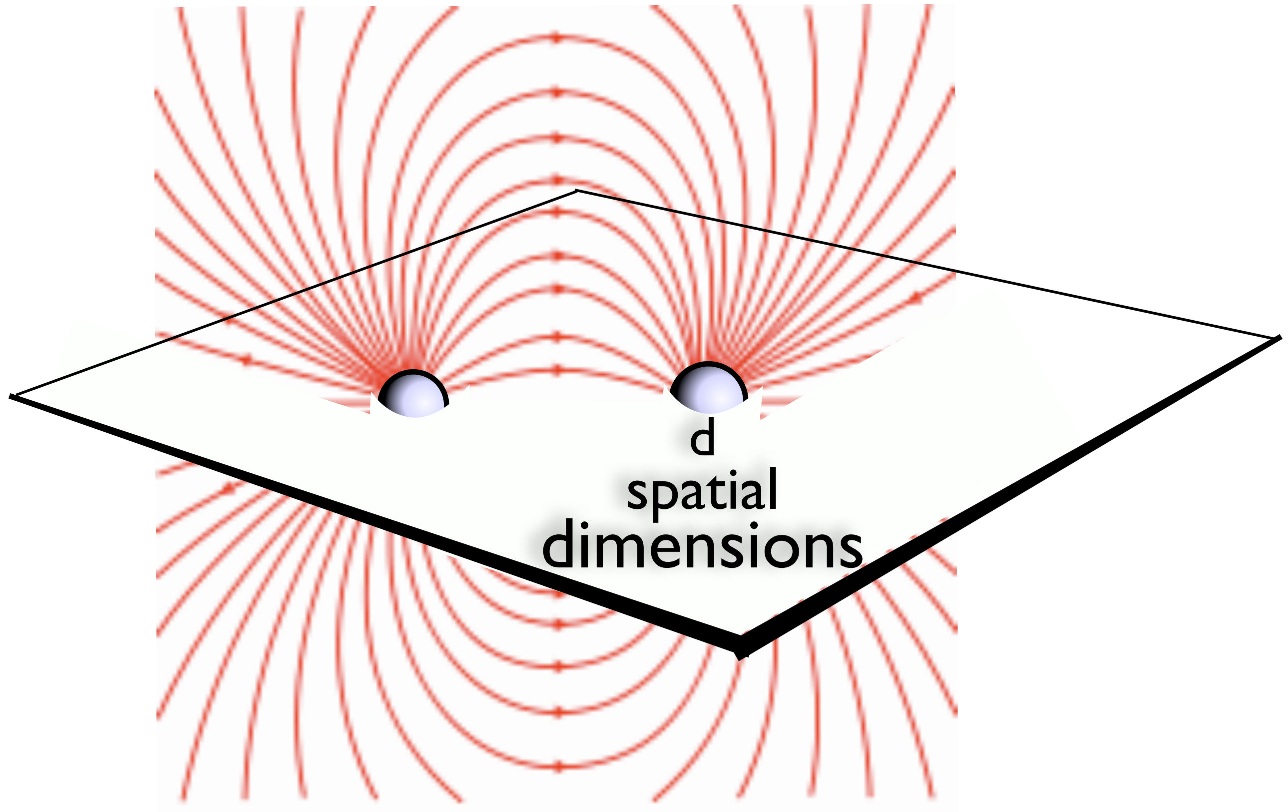

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g doesn't run

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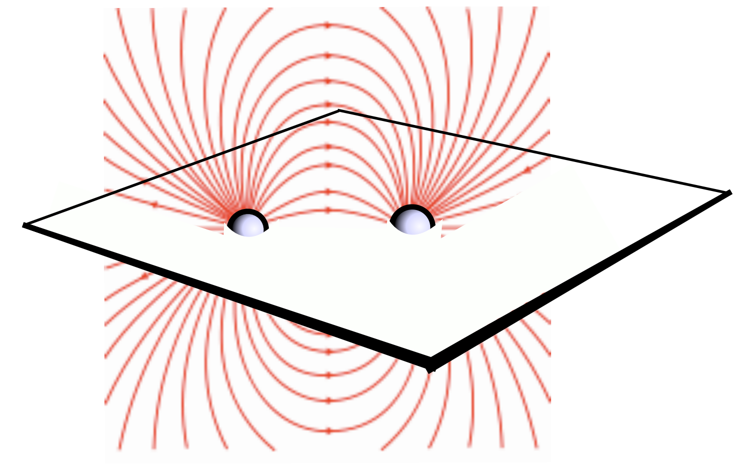

Expect a phase transition as a function of g:

$$\langle\bar{\psi}\psi\rangle = \begin{cases} 0 & g < g_* \\ \Lambda_{\rm IR}^d & g > g_* \end{cases}$$

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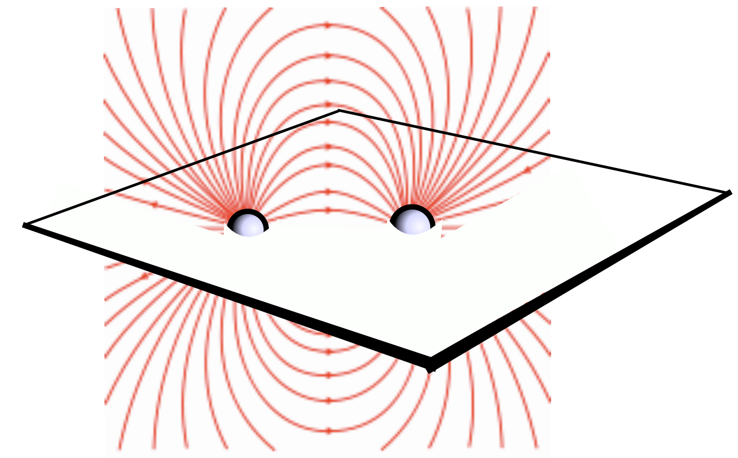

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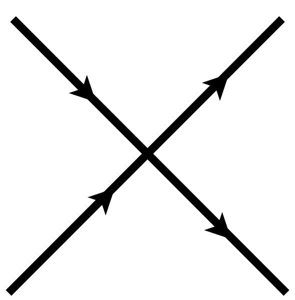

$$\Delta S = \int d^{d+1}x \ \left(-\frac{c}{2}(\bar{\psi}\gamma_\mu T_a \psi)^2\right)$$

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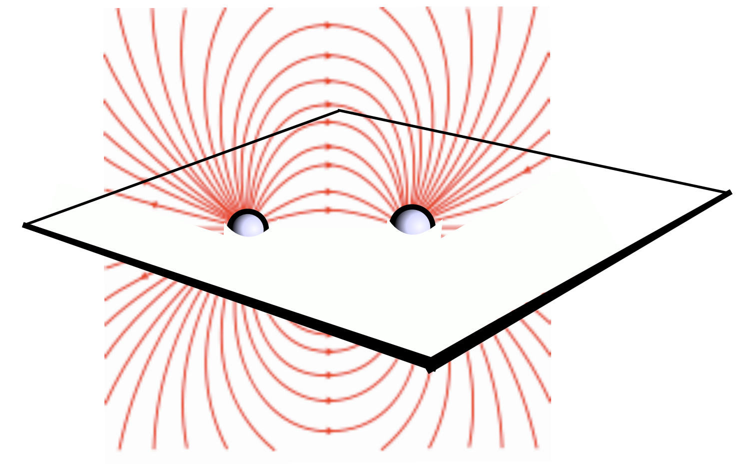

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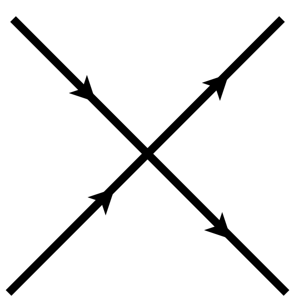

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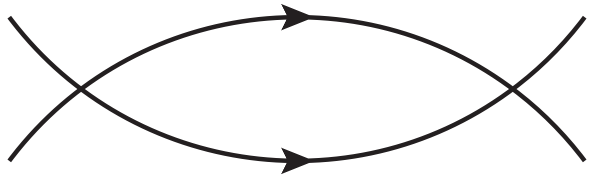
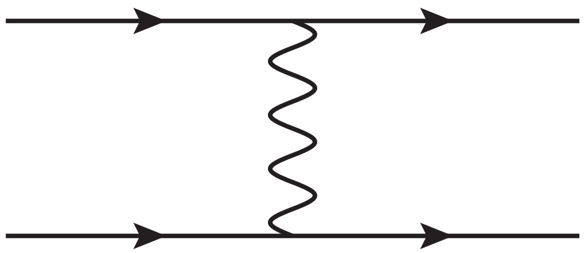
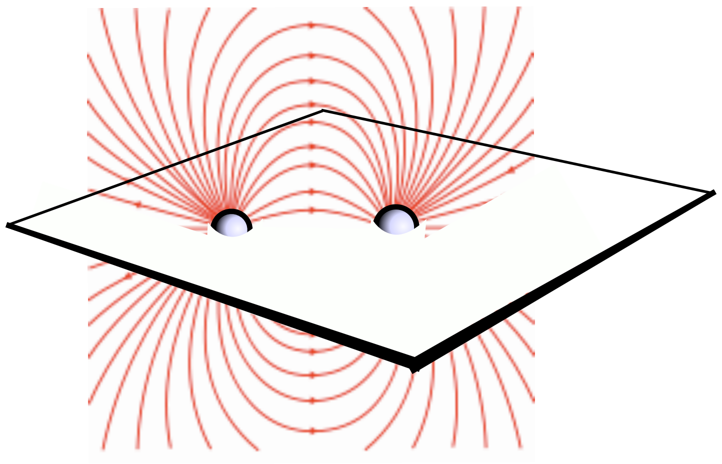
Phase transition is in perturbative regime for d=1+ε (spatial dimensions of "defect"): compute β-function

$\beta(c)$:

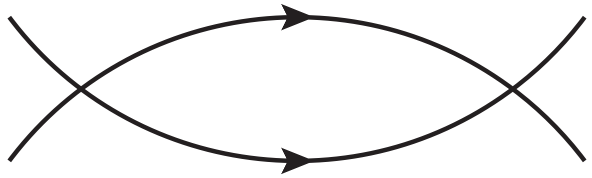


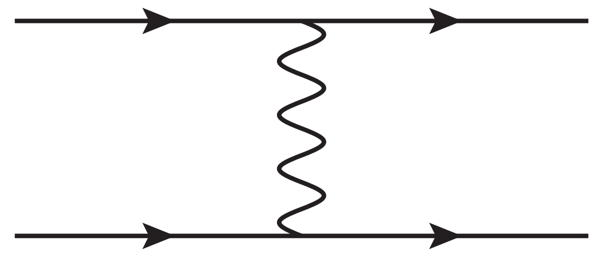


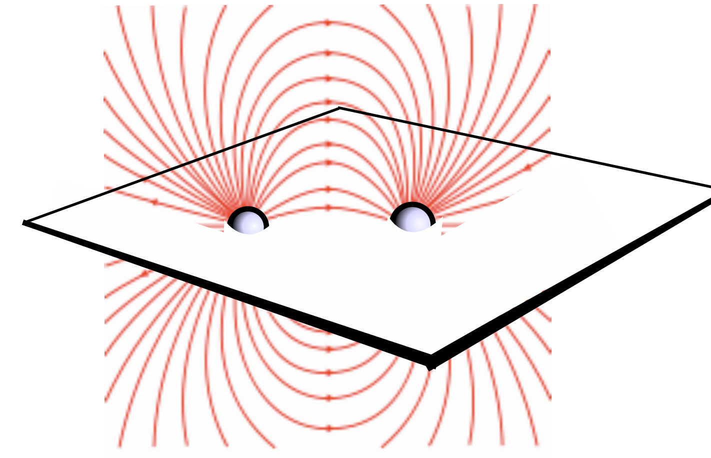


$1/\varepsilon$ pole for $d=(1+\varepsilon)$

β(c):







1/ε pole for d=(1+ε)

$$
\begin{aligned}
\beta(c) &= -\frac{g^2}{2\pi} - \epsilon c - \frac{N_c}{2\pi} c^2 \\
&= \frac{1}{2\pi}\left(\frac{\pi^2\epsilon^2}{N_c} - g^2\right) - \frac{N_c}{2\pi}\left(c - \frac{\epsilon\pi}{N_c}\right)^2
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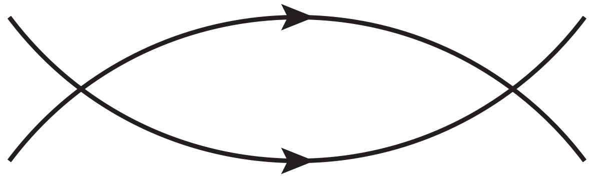

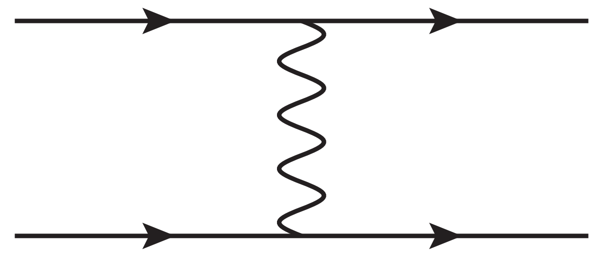

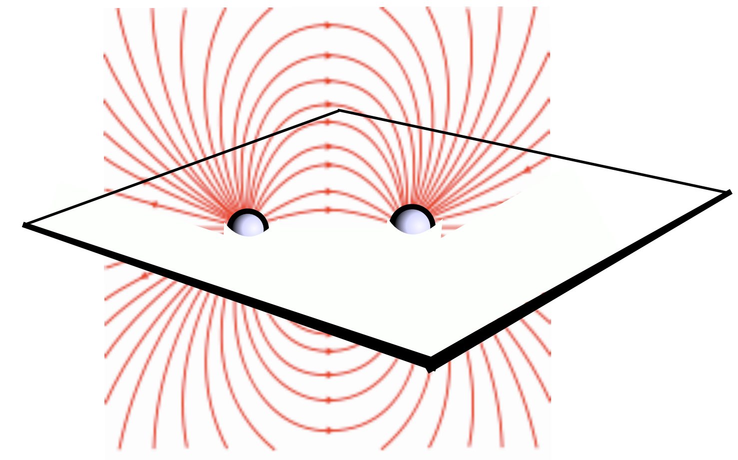

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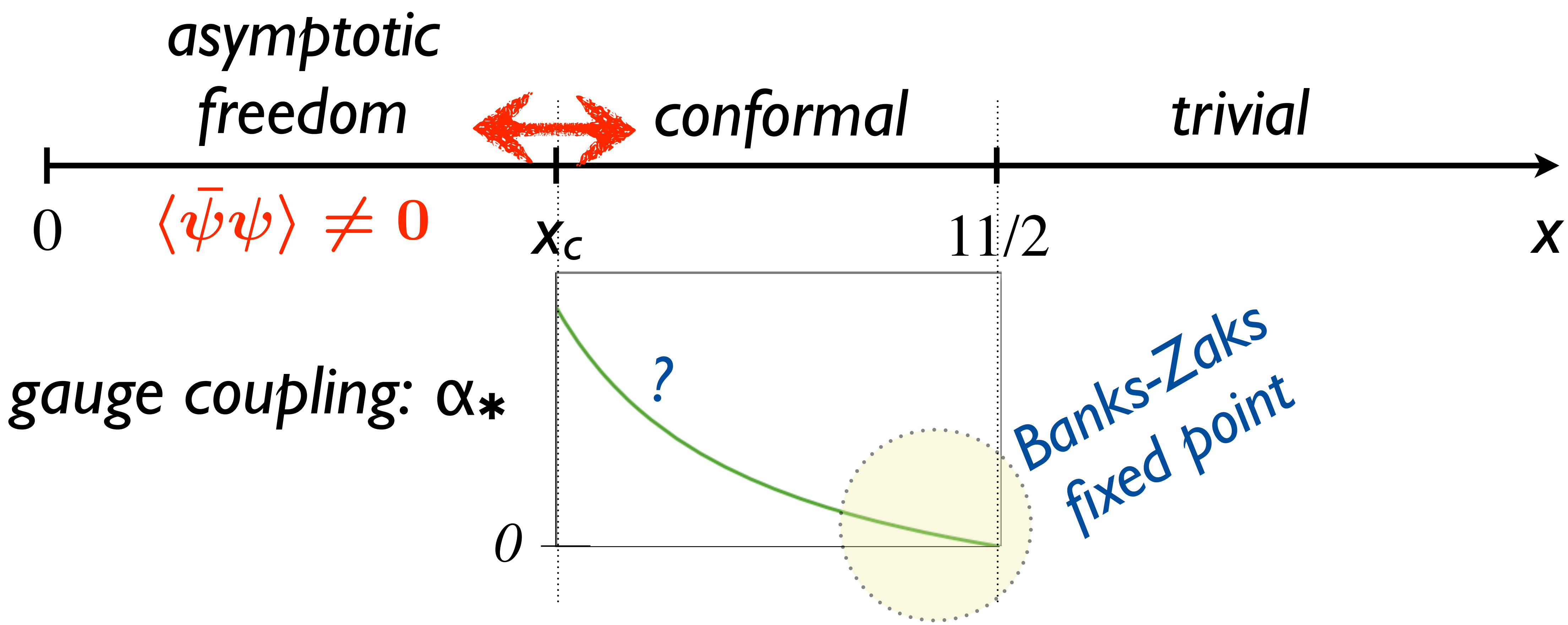
- Find BKT transition at $g^2 = g_*^2 = (\varepsilon\pi)^2/N_c$
  $\Lambda_{IR} \sim \Lambda_{UV} \exp[-\pi/\sqrt{(g^2-g_*^2)}]$
- Schwinger-Dyson gap eq (rainbow approx) gives qualitatively same results

Back to QCD at LARGE N_c and N_f:



Transition at $x=x_c$?

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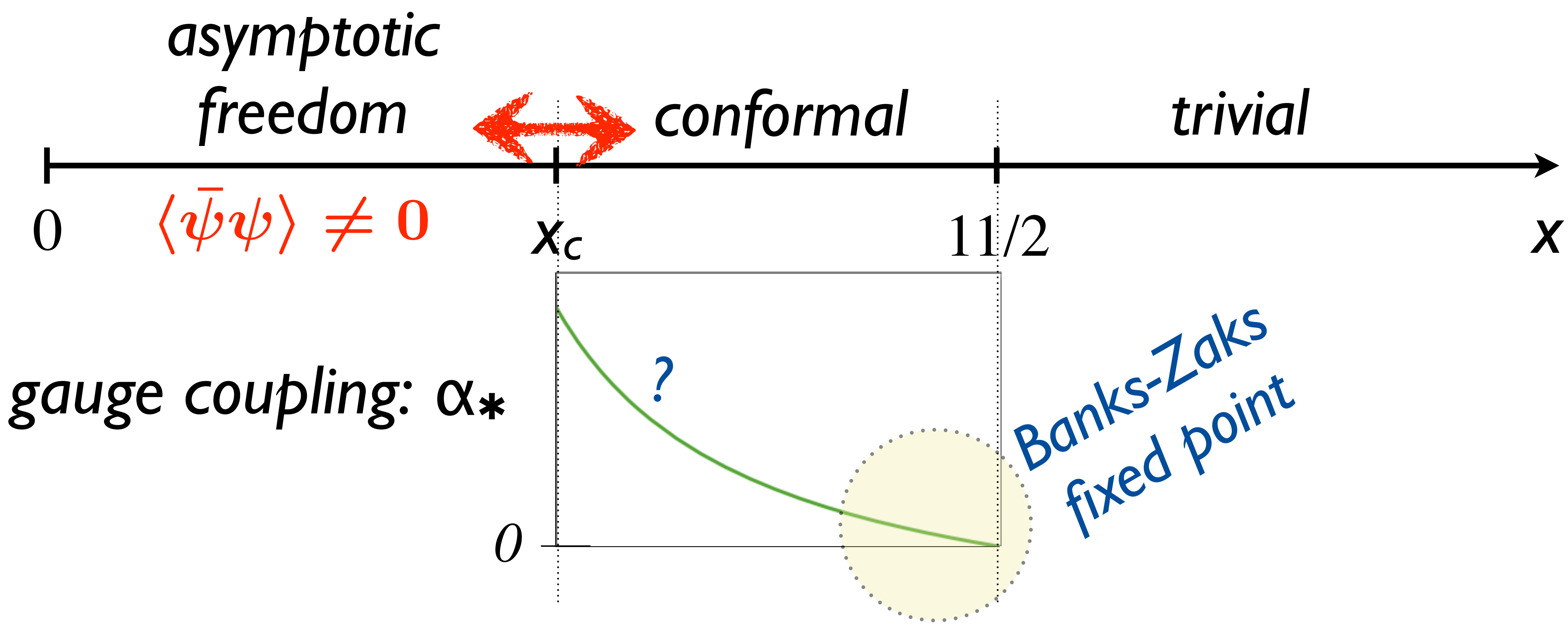

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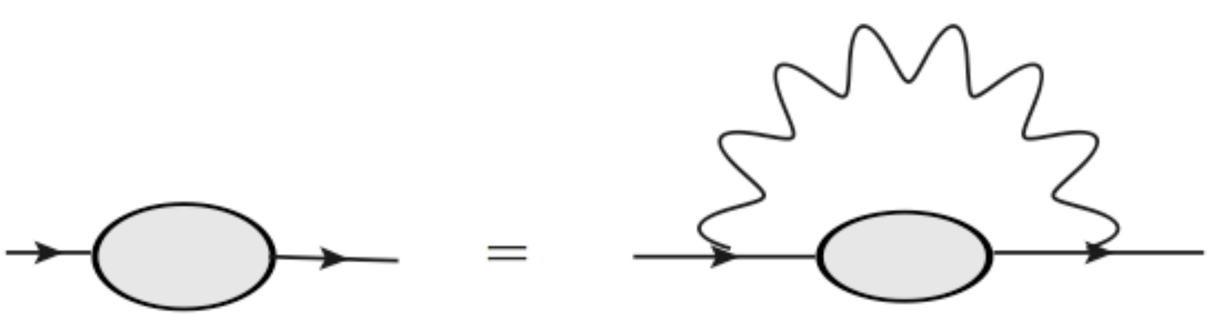

Miransky 1985

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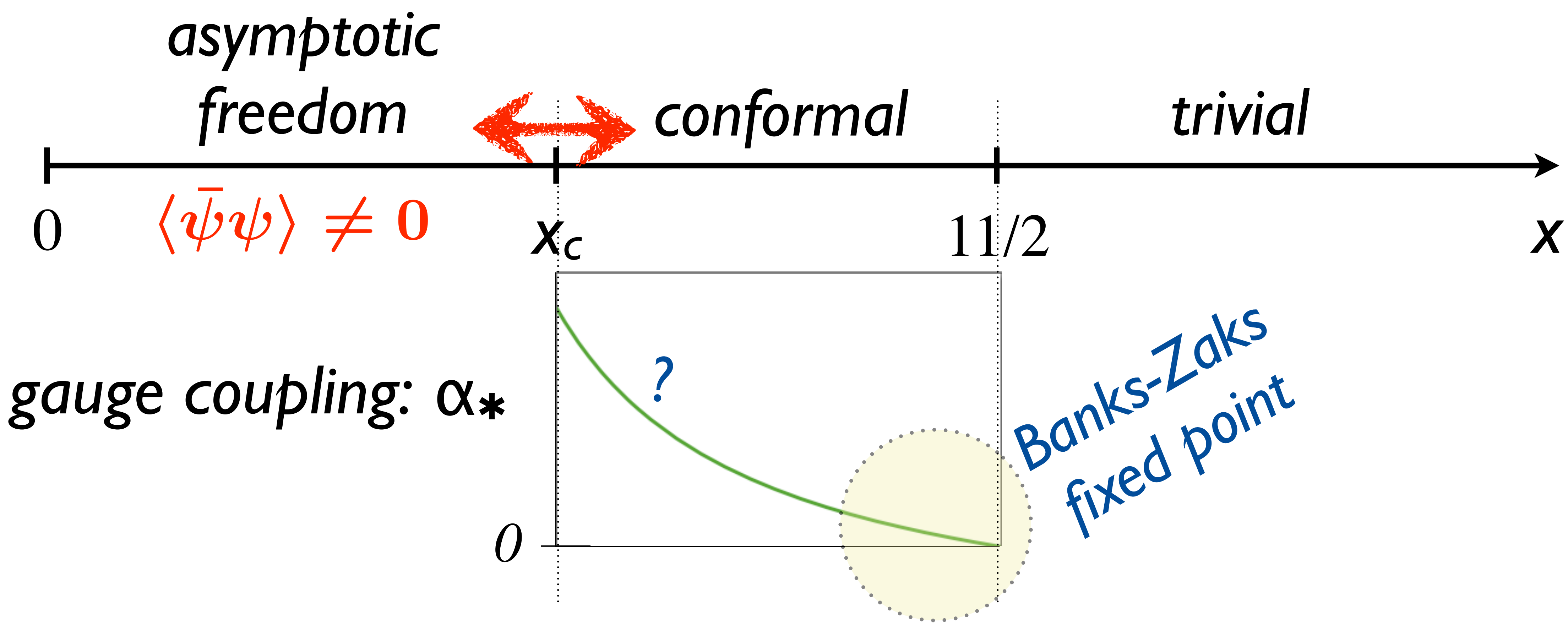

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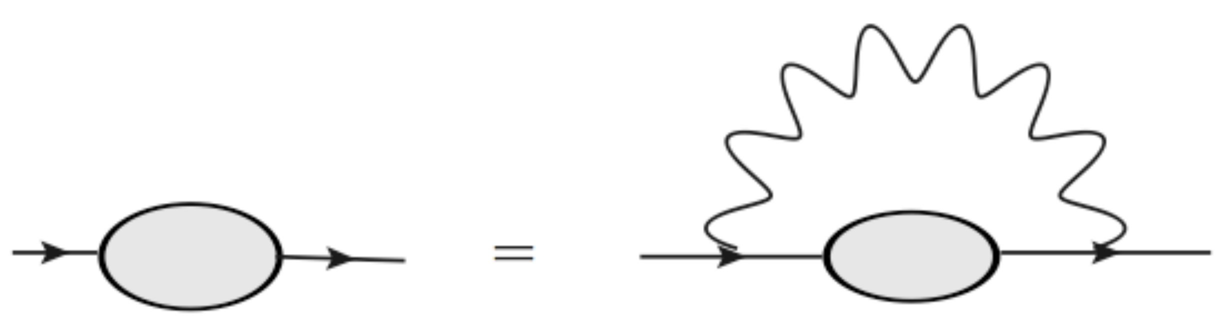

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Appelquist, Terning, Wijerwardhana 1996

Found: BKT scaling for $\langle\bar{\psi}\psi\rangle$...not rigorous, but qualitatively correct?

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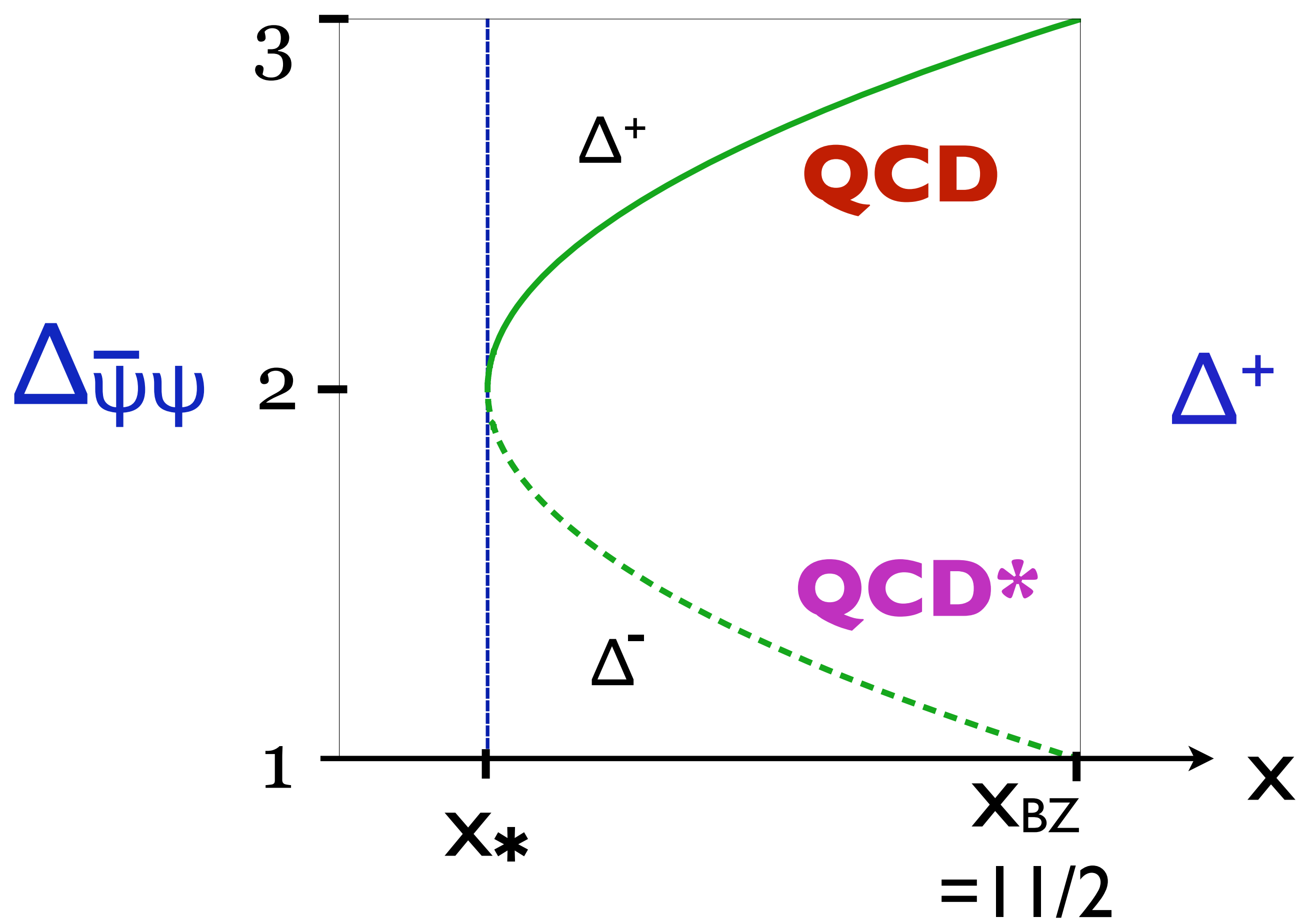

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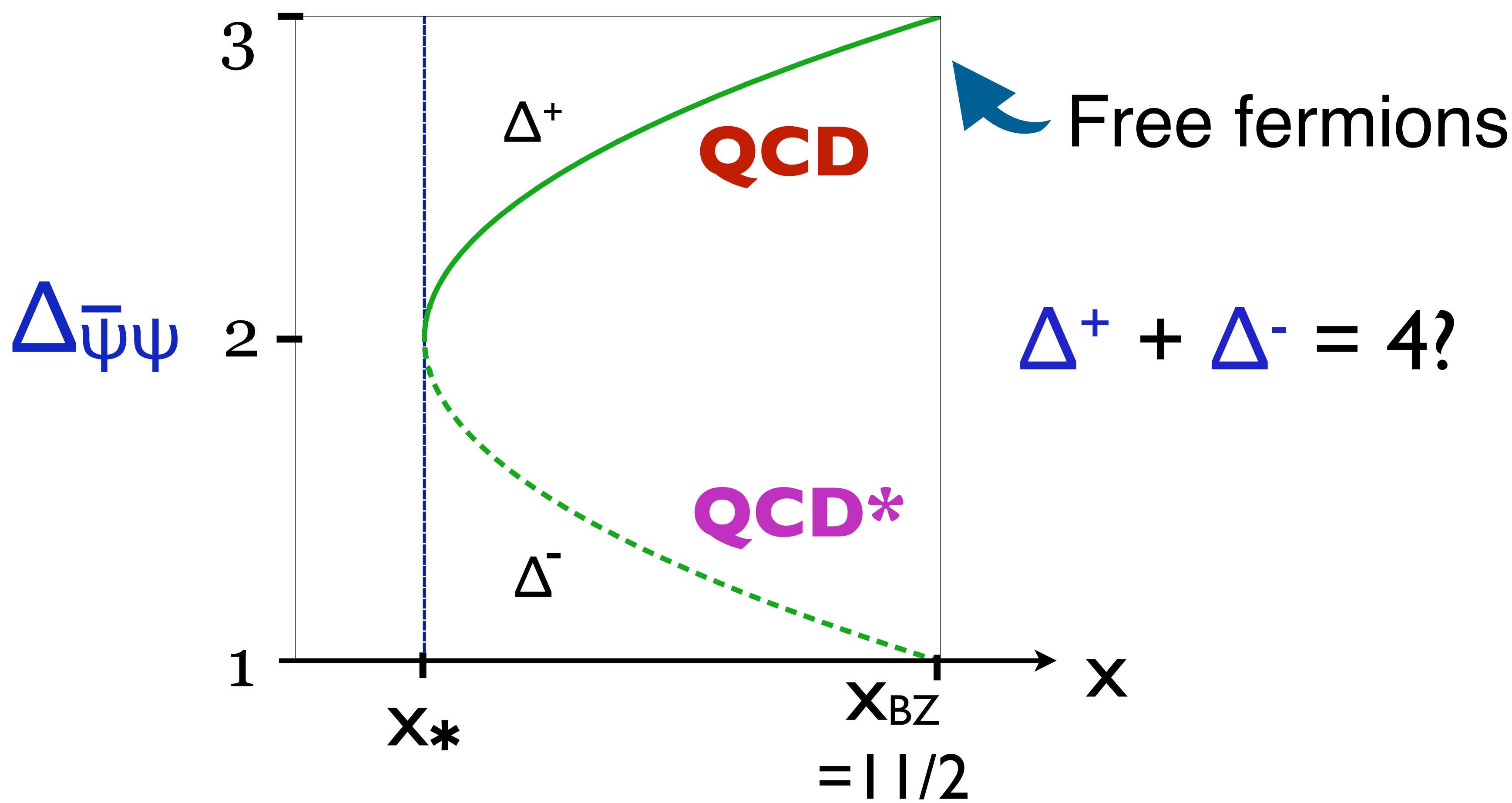

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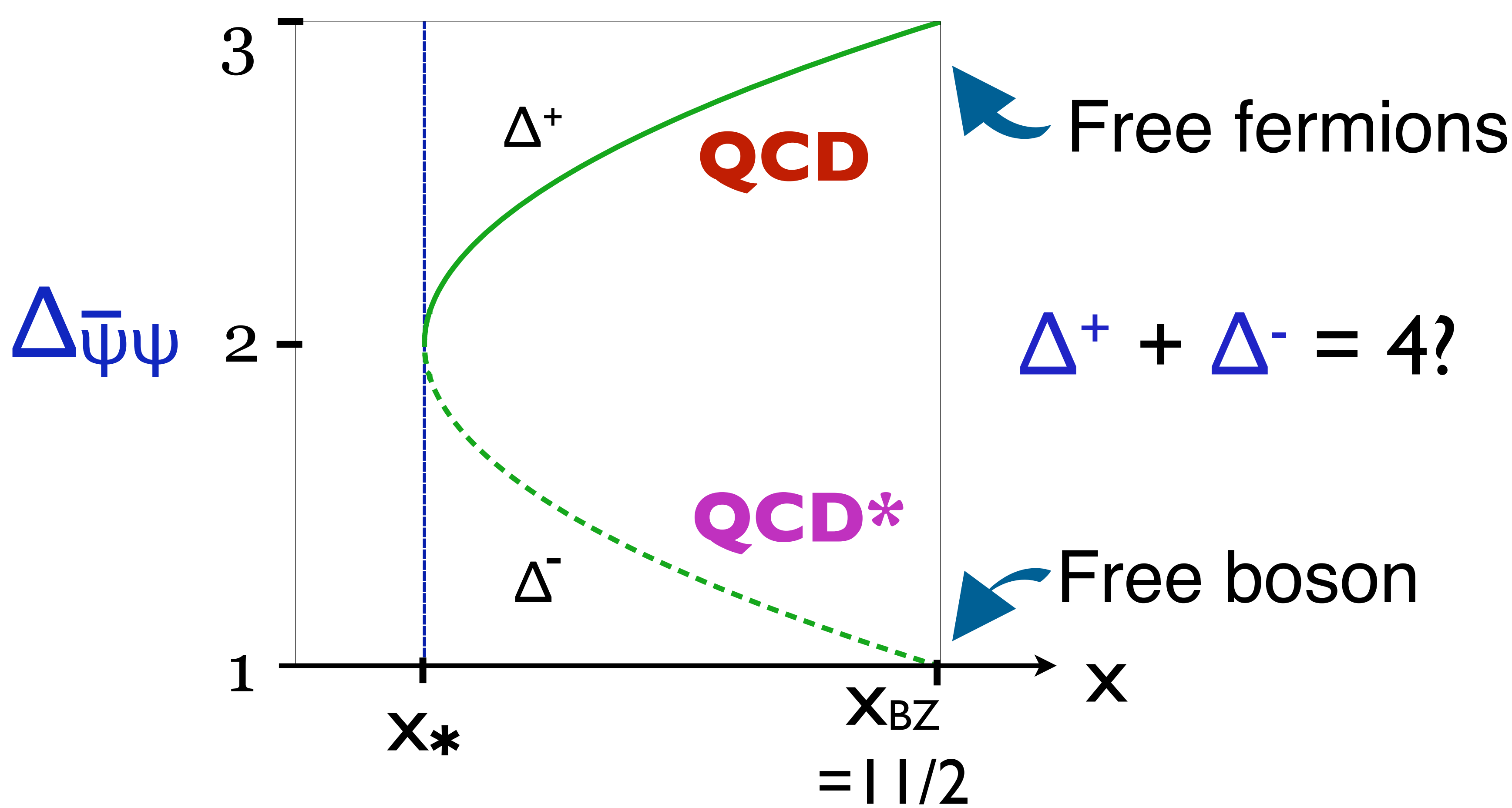

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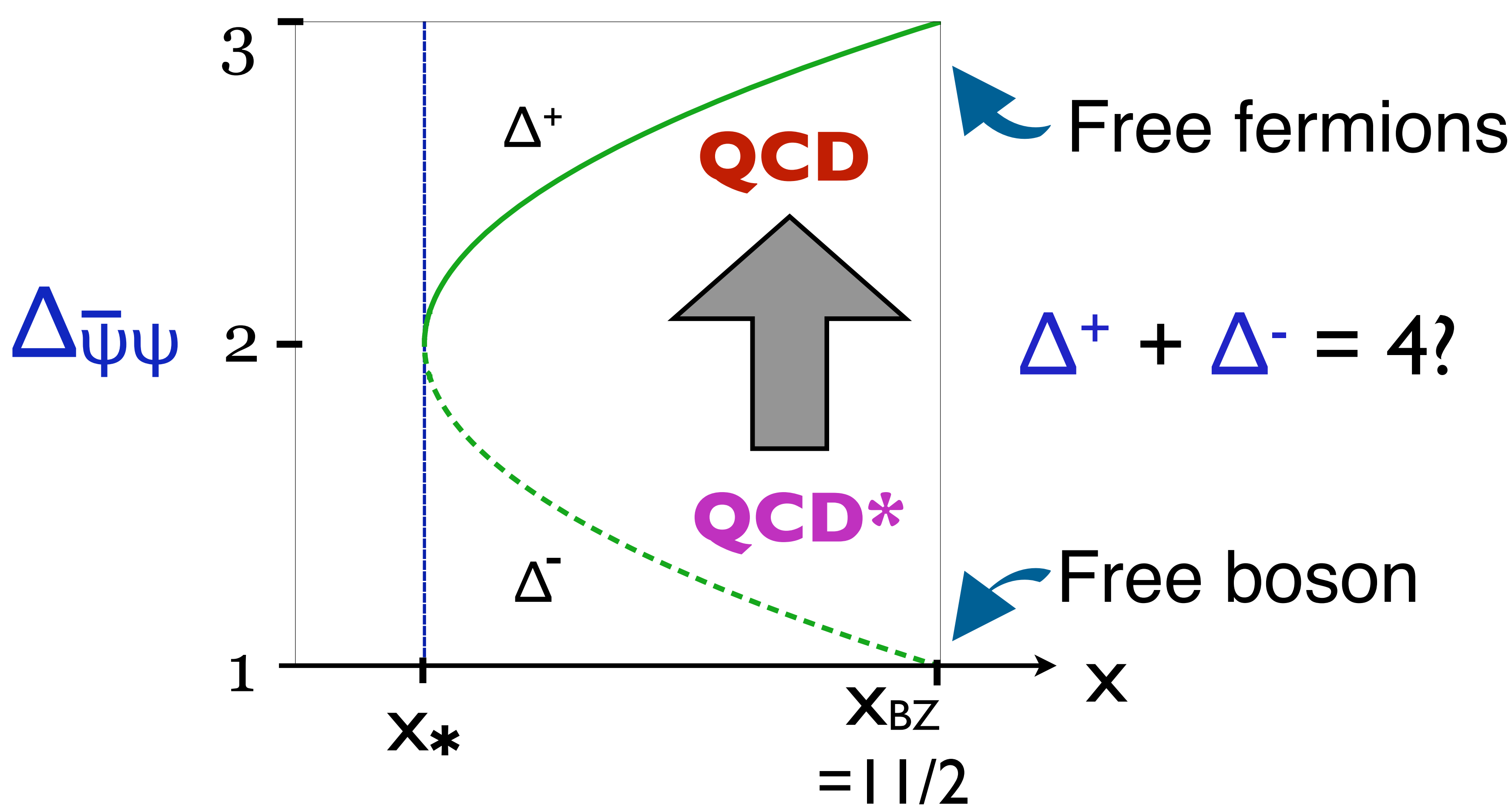

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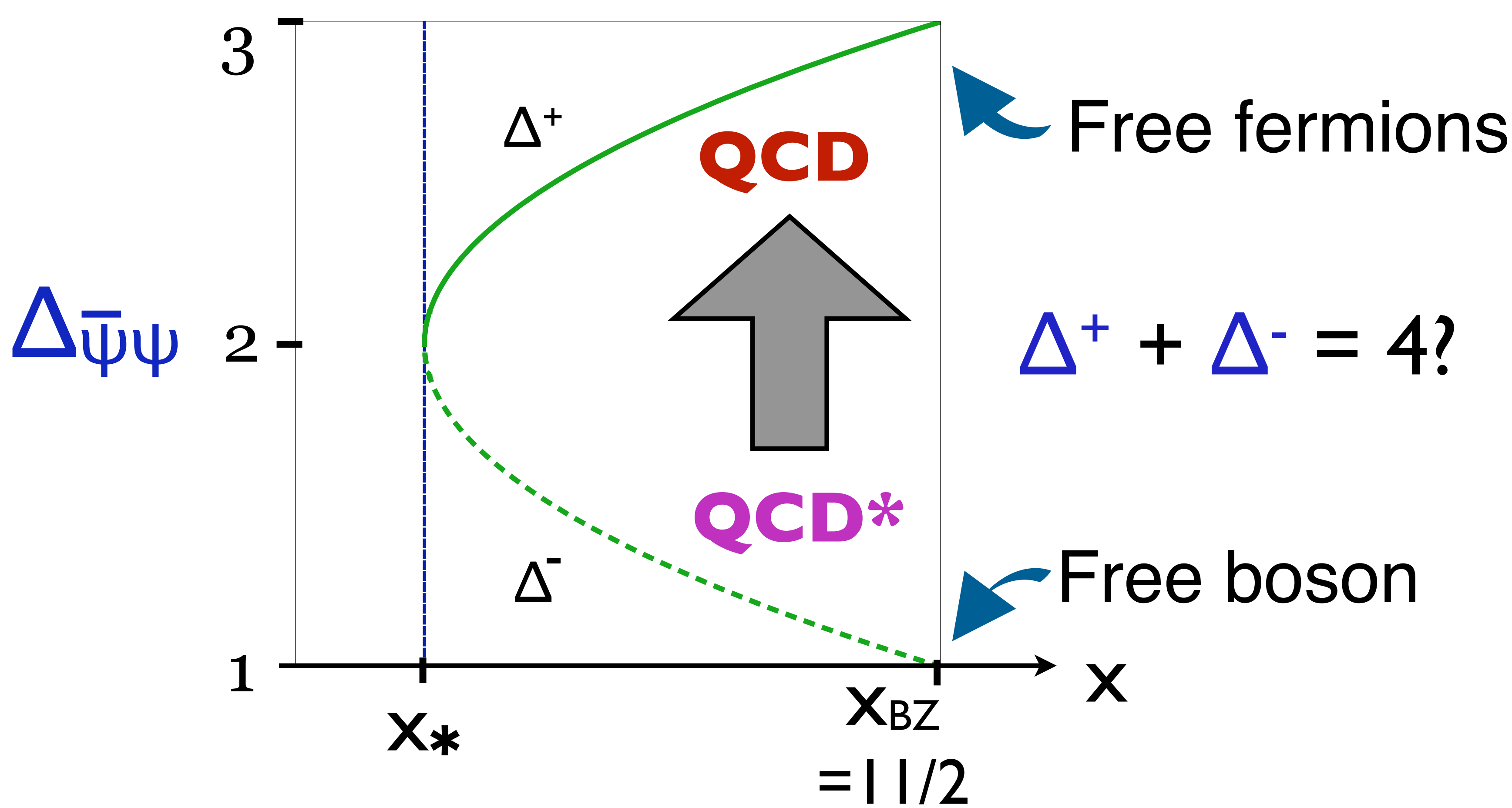

Near Banks-Zaks (IR) fixed point:

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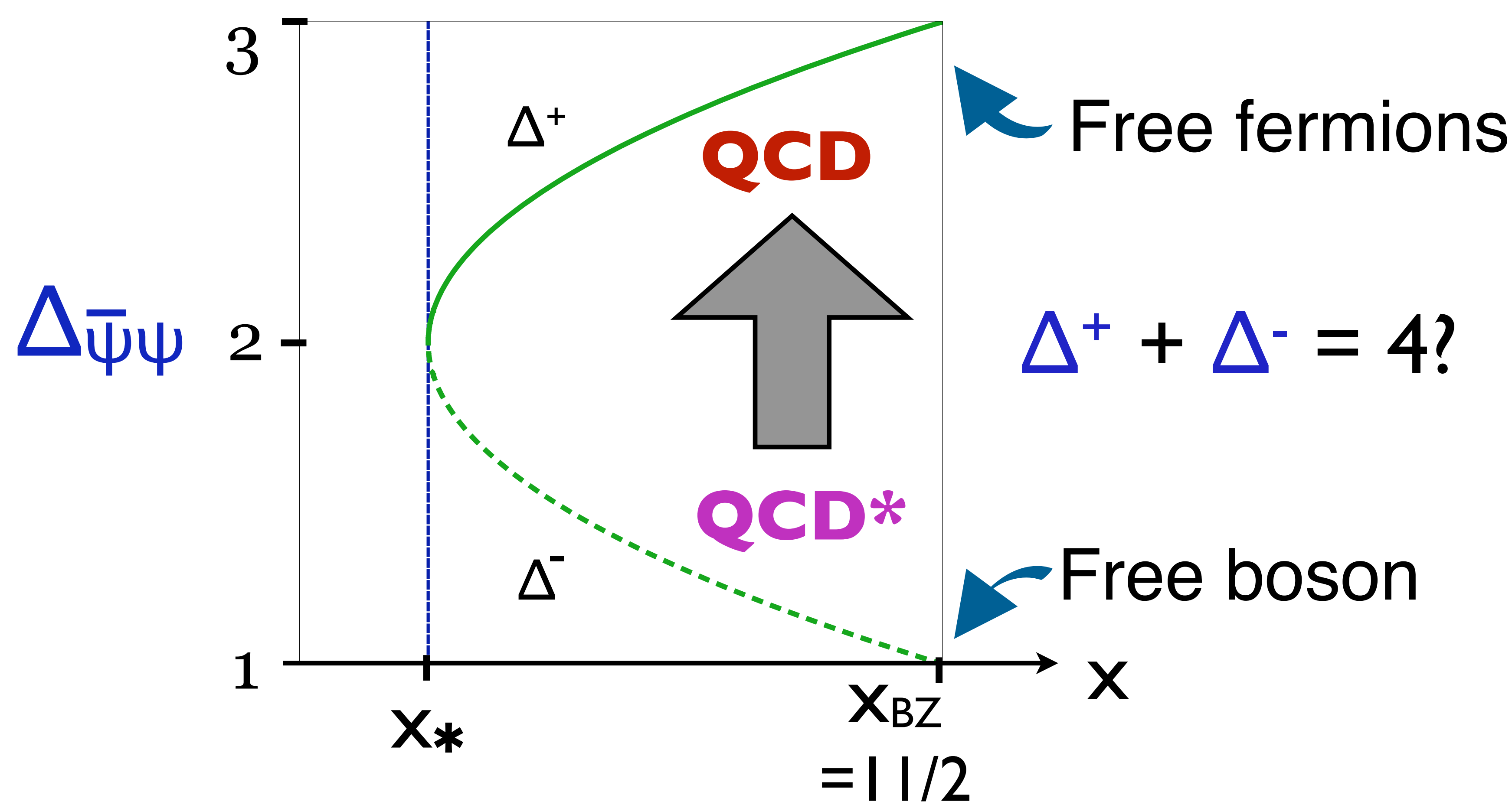

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$$\Delta^+_{\psi\bar\psi} = 3 - \#\, g^2 N_c$$

(almost free quarks)

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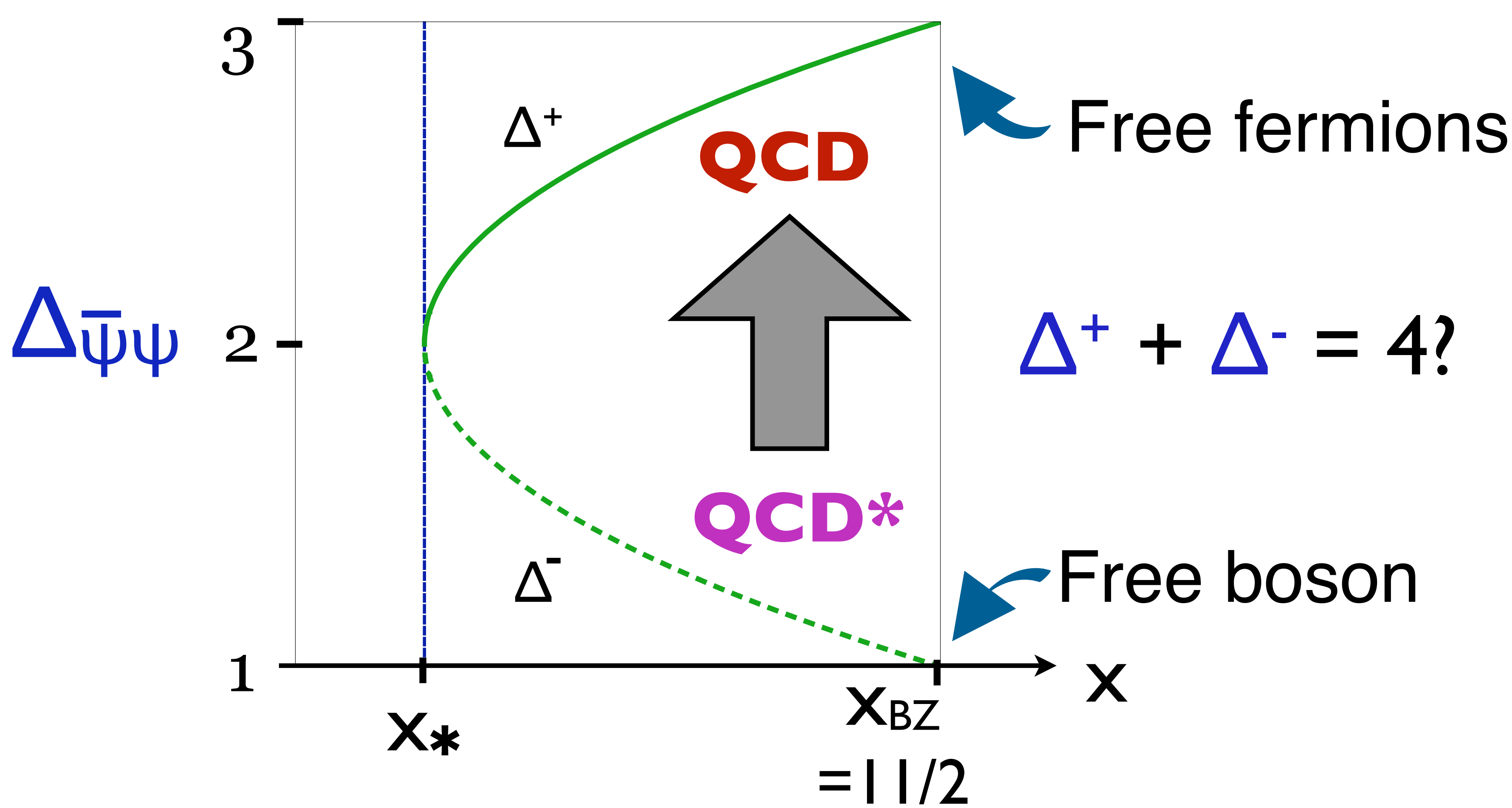

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(almost free quarks)

Partner theory QCD*:

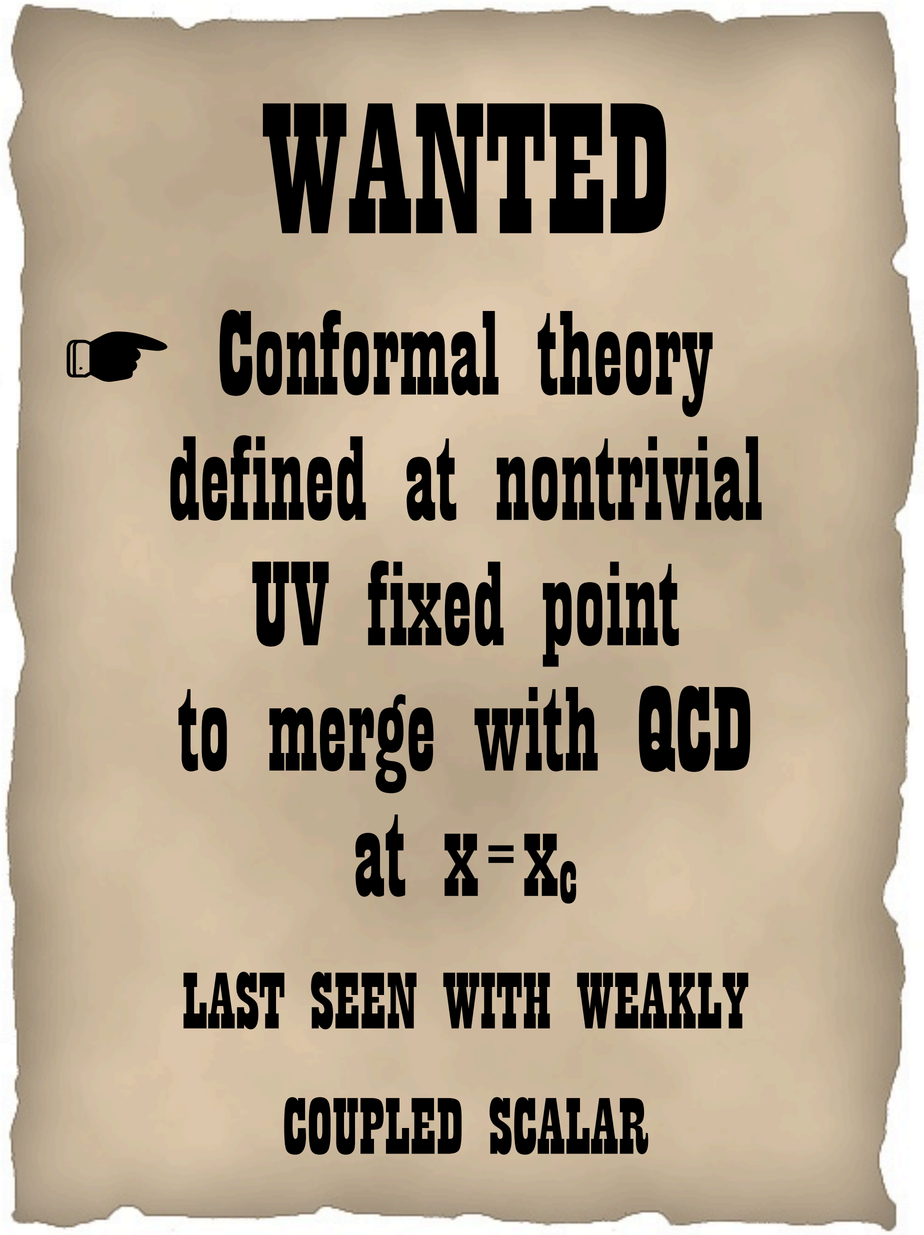
$$\Delta^-_{\psi\bar{\psi}} = d - \Delta^+_{\psi\bar{\psi}} = 1 + \#\, g^2 N_c$$

(almost free scalar?)

WANTED

☛ Conformal theory defined at nontrivial UV fixed point to merge with QCD at $x=x_c$

LAST SEEN WITH WEAKLY COUPLED SCALAR



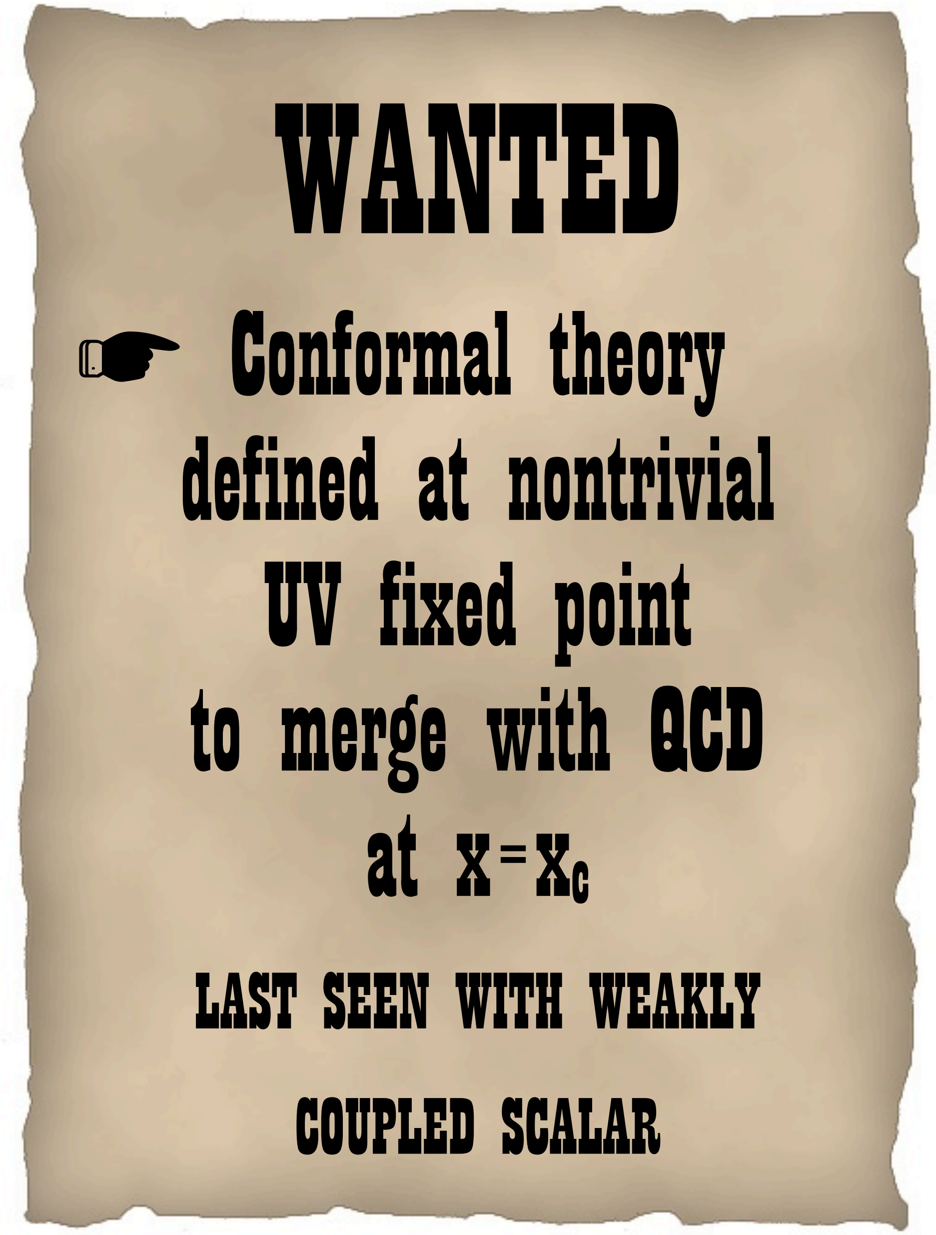
Haven't found a Lorentz invariant <u>perturbative</u> example

with:

(i) weakly coupled scalar;

(ii) full $SU(N_f) \times SU(N_f)$ chiral symmetry

(iii) Matching anomalies



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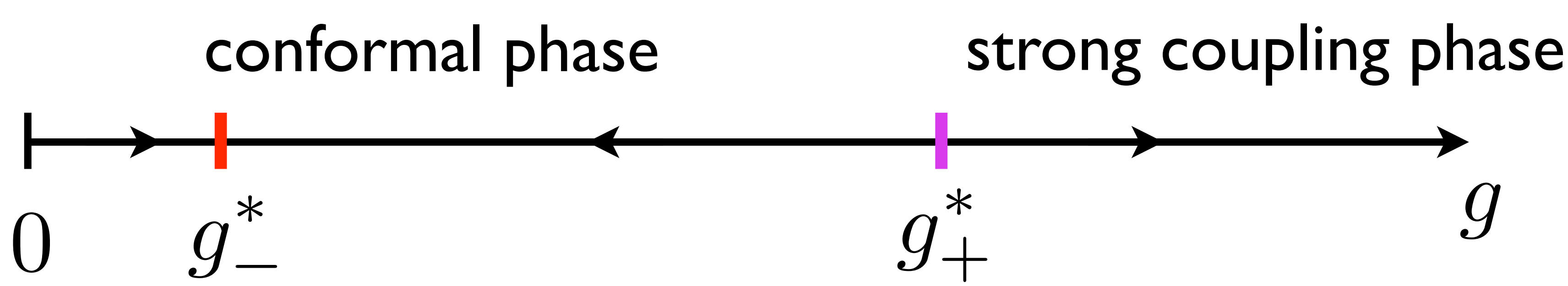
Look for nonperturbative QCD* on the lattice?

One place to start: strong/weak transition for QCD with N_f in conformal window?

(A. Hasenfratz)



QCD* possibly at $g_+{}^*$?

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VI. Finding QCD* should be on field theory / lattice QCD "to-do" list.


E paradiso ab Angelo pelluntur. Genes. III.