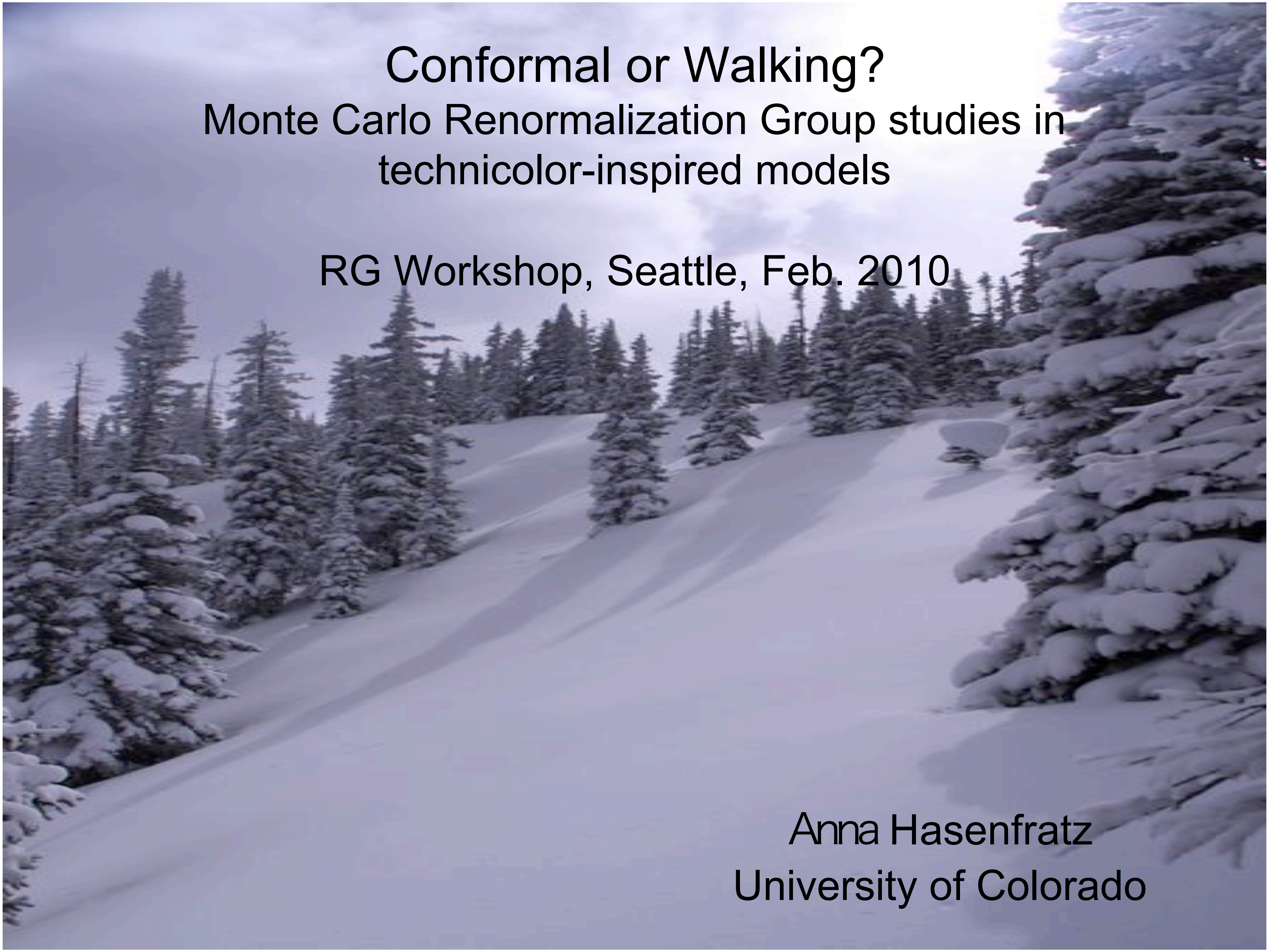


Conformal or Walking?

Monte Carlo Renormalization Group studies in technicolor-inspired models

RG Workshop, Seattle, Feb. 2010

Anna Hasenfratz
University of Colorado

Beyond Standard Model physics on the lattice

Many of interesting candidates are fermion-gauge systems

In Euclidean space they can be discretized

→ they turn into 4d statistical systems

→ identifying phases, critical points, measuring critical exponents can be done just like in a statistical physics

Dictionary:

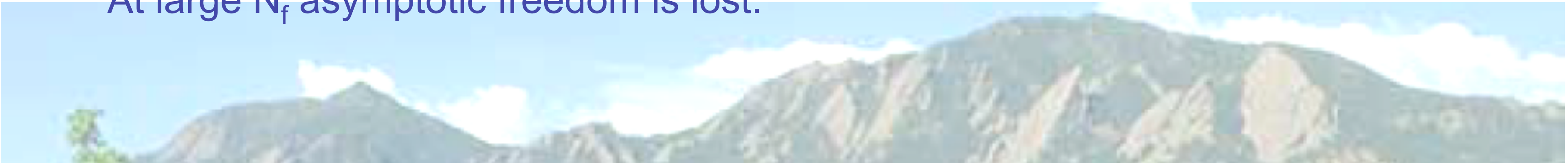
m or m_q	:	fermion mass of N_f degenerate fermions
g	:	gauge coupling of $SU(N_c)$ gauge system
β	:	either $\beta=2N/g^2$ or
		$\beta = \mu\, dg/d\mu = -b_0\, g^3 - b_1\, g^5 + \ldots.$
		RG β function
γ_m	:	mass anomalous dimension, $\gamma_m = y_m - 1$

If the system is asymptotically free it can be

–confining and chirally broken (small N_f) or

–conformal in the $m_q=0$ limit (increasing N_f)

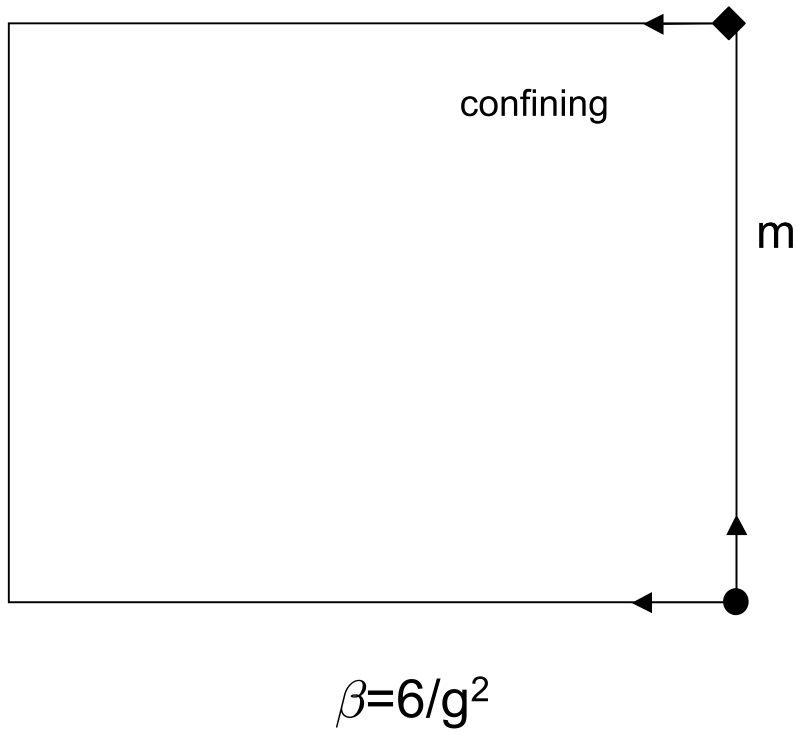
At large N_f asymptotic freedom is lost.



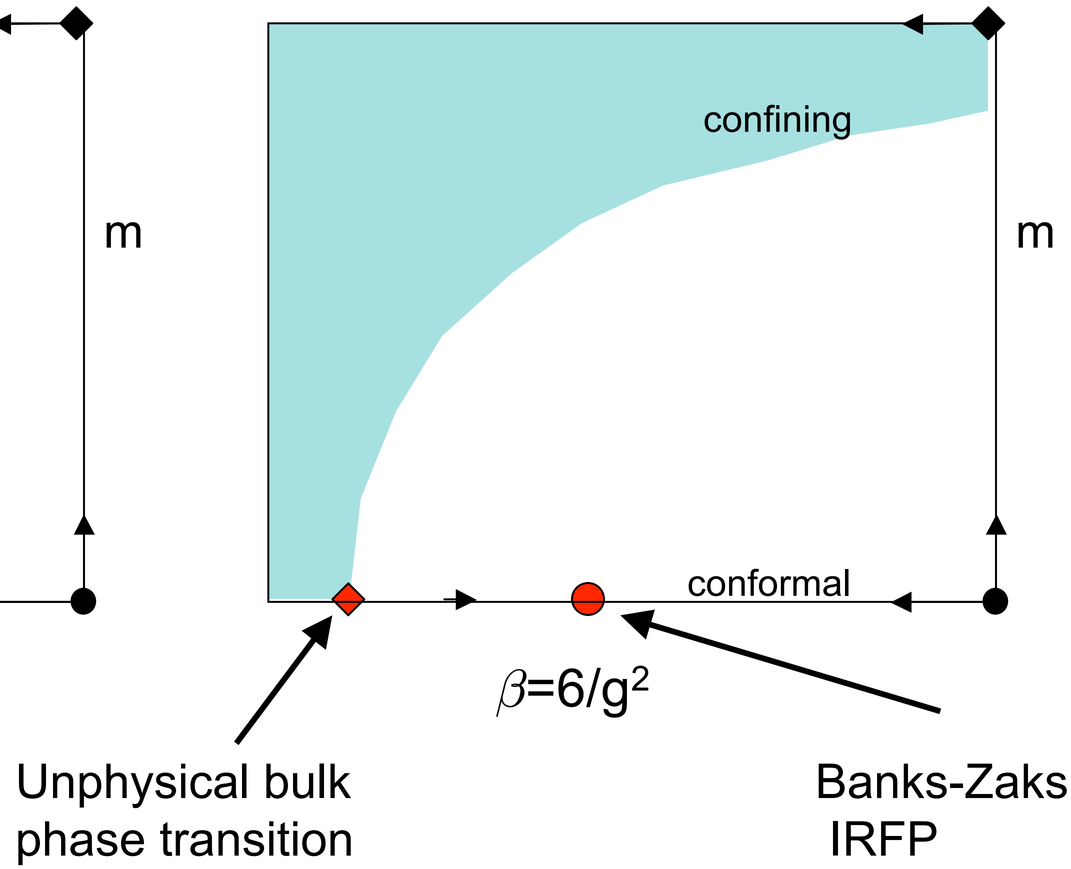
The lattice phase diagram

(arrows: UV to IR)

QCD like



Conformal



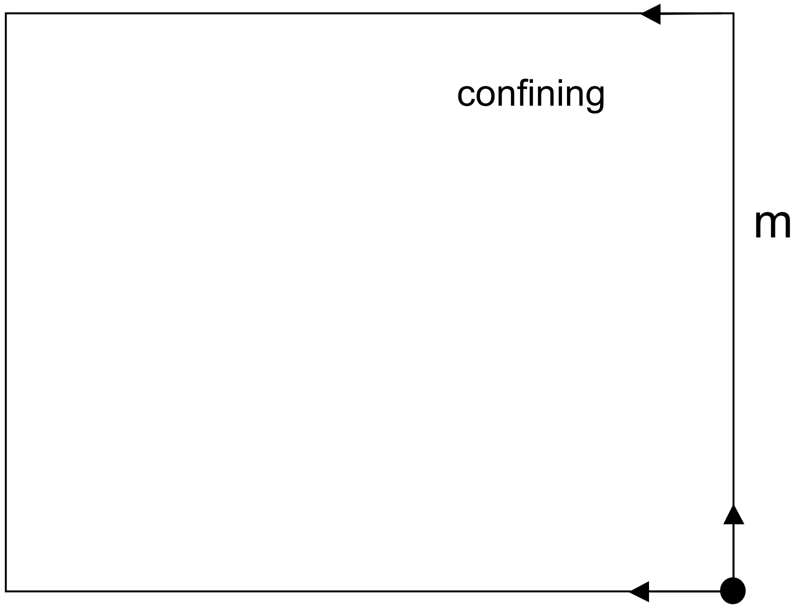
At ● g=0, m=0, both couplings are relevant

At ◆ g=0, m=∞, g is still relevant

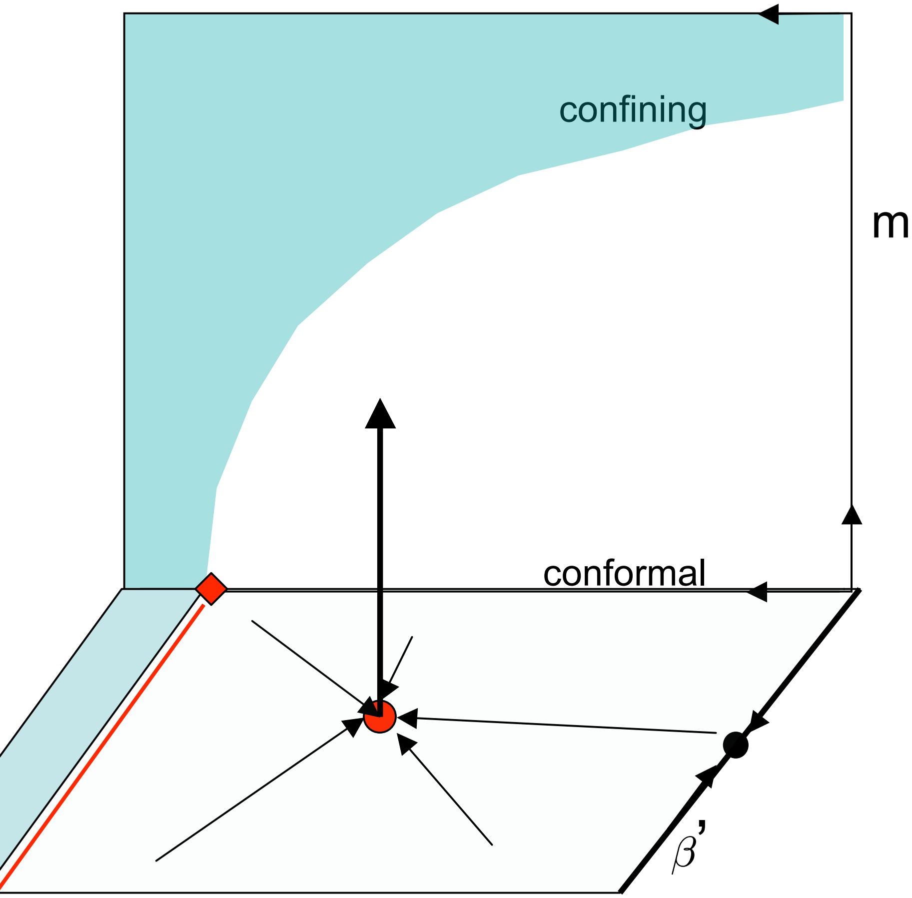
The lattice phase diagram

(arrows: UV to IR)

QCD like



Conformal



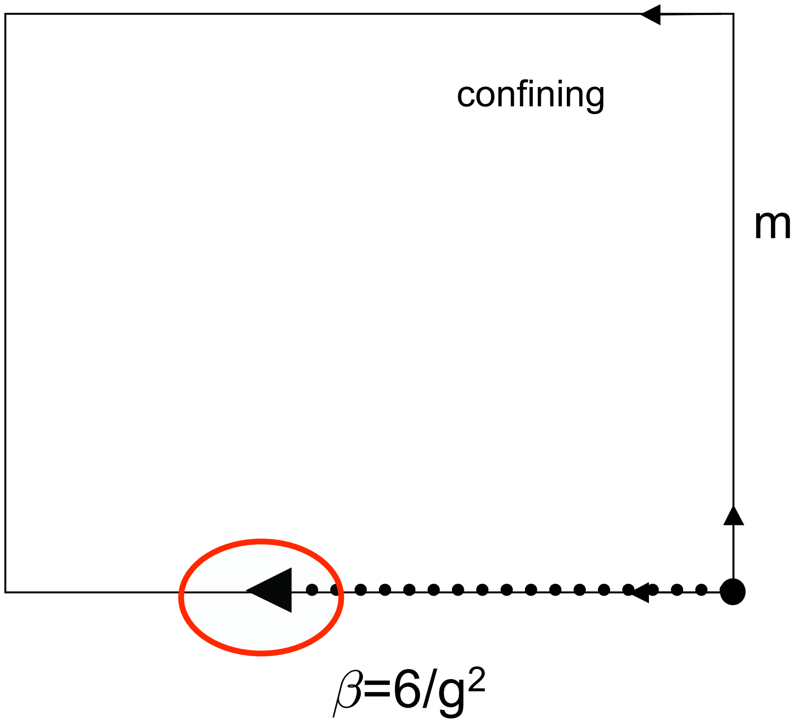
Existence of IRFP is universal;
location is RG dependent

At ● IRFP only m is relevant

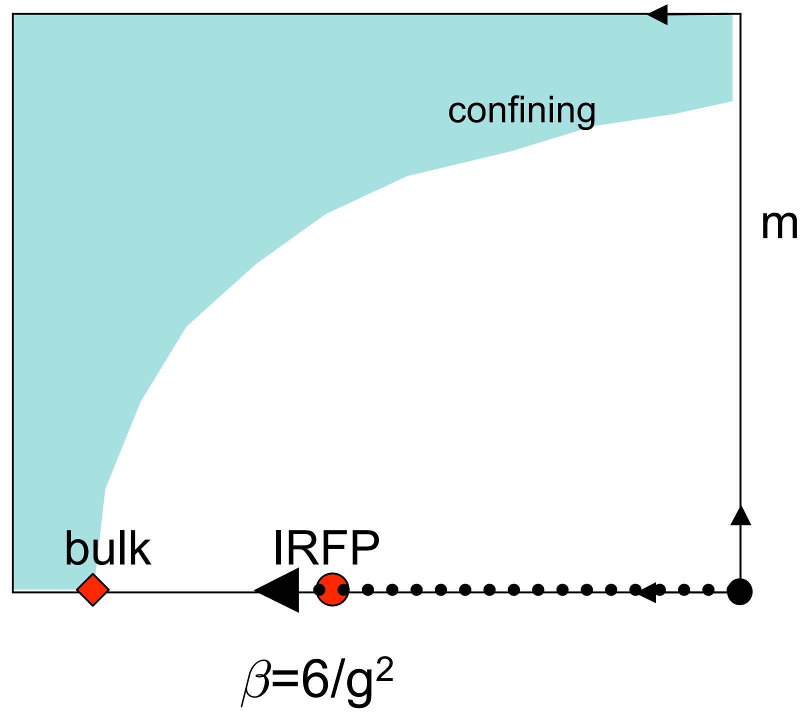
We want to distinguish QCD-like and conformal systems:

(arrows: UV to IR)

QCD like



Conformal



Lattice simulations can connect the perturbative FP and strong coupling

- Found IRFP ? Done ✔
- No IRFP? Show that it is confining before a bulk transition is reached
- Strong lattice artifacts can interfere

Outline

- The bare step scaling function as an alternative to the RG β function
- Monte Carlo Renormalization Group method
  - The 2-lattice matching approach
  - Optimization
  - The role of different RG transformations
- Some results:
  - Pure gauge SU(3) – testing case for MCRG
  - N_f=8,16,12 fundamental fermions, SU(3) gauge
    - The steps scaling function
    - The anomalous dimension of the mass

The step scaling function around a UVFP

I measure the bare differential step scaling function s_b
(instead of the RG β function)

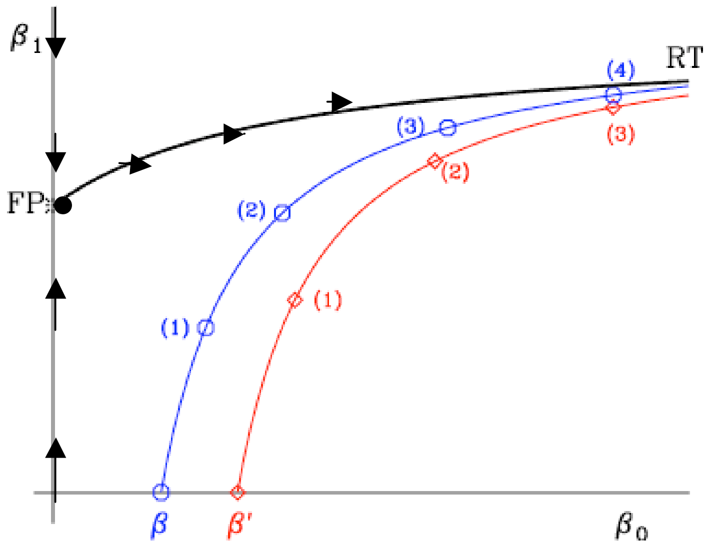
$$s_b(\beta) = \beta - \beta' \quad \text{where} \quad \xi(\beta) = \xi(\beta')/2 \qquad (\beta=2N_c/g_0^2)$$

ξ is the correlation length defined by some physical mass

- Can be measured directly or
- Use RG flow

RG flow lines along a relevant direction (UVFP):

$\xi=\infty$



– do simulations at β and β'(m=0)

– do RG blocking and compare the blocked actions

– if $S(\beta^{(n)}) = S(\beta'^{(n-1)})$ --> $a(\beta)=a(\beta')/2$

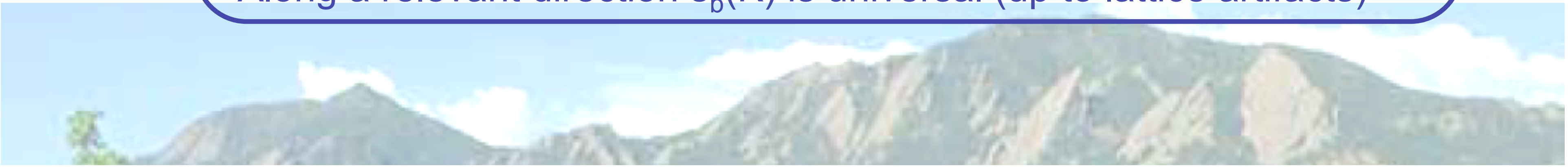
– the step scaling function is

$$s_b(\beta) = \lim_{n_b \to \infty} (\beta - \beta')$$

• The location of the FP on the critical surface depends on the RG transformation

• Tuning free parameters in the RG transformation can pull the FP and its RT close, reducing systematical errors

Along a relevant direction $s_b(K)$ is universal (up to lattice artifacts)



The step scaling function in a conformal system

In the chiral limit $\xi = \infty$ everywhere !

$s_b(\beta)$ can be defined through the RG flow or the running coupling

$$s_b(\beta) = \beta - \beta' \quad \text{where} \quad S^{(n)}(\beta) = S^{(n-1)}(\beta')$$

or

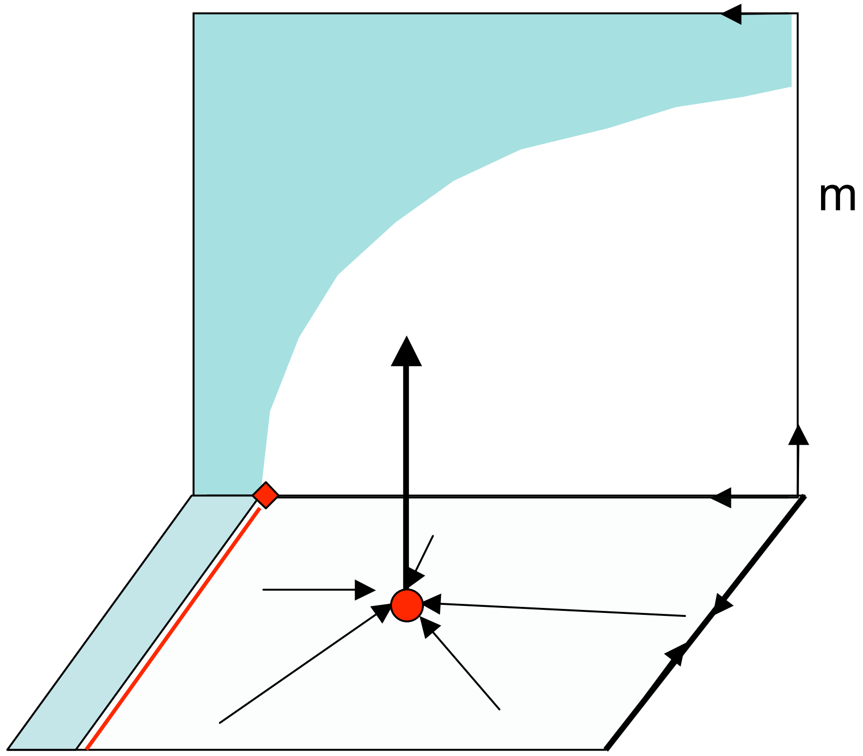
$$g^2(\beta;L) = g^2(\beta';L/2) \qquad (\beta = 2N_c/g_0^2)$$

($g^2(\beta;L)$ can be defined via Schroedinger functional or other RG transformation)

RG flow lines around an IRFP

On the critical surface (m=0) around an IRFP the flows converge to the FP when $n_b \to \infty$

With finite n_b the flow picks up the slowest flowing operator



The location of the IRFP depends on the RG transformation

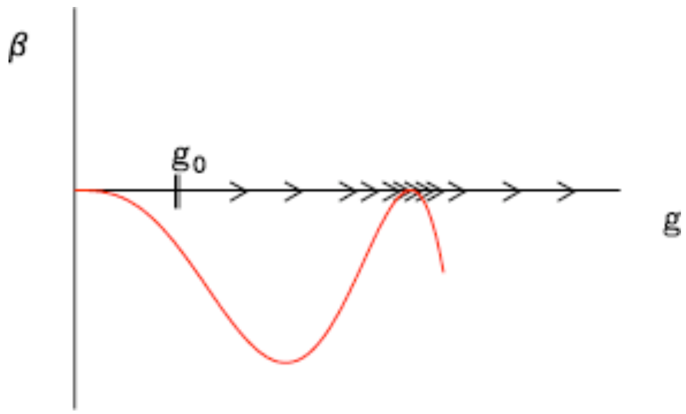
$s_b(\beta)$ along an irrelevant direction depends on the blocking (scheme dependence)

This is a signal for non-QCD-like behavior

The step scaling function of a walking theory

What happens with a walking theory?

- It is QCD-like, ξ is defined, in the scaling region of the UVFP $s_b(\beta)$ is universal (up to $O(a^2)$ corrections).
- The near-zero of the RG β function depends on the RG transformation. At that region $s_b(\beta)$ is RG dependent.



Can confinement and χ SB be established before lattice artifacts overwhelm the system?

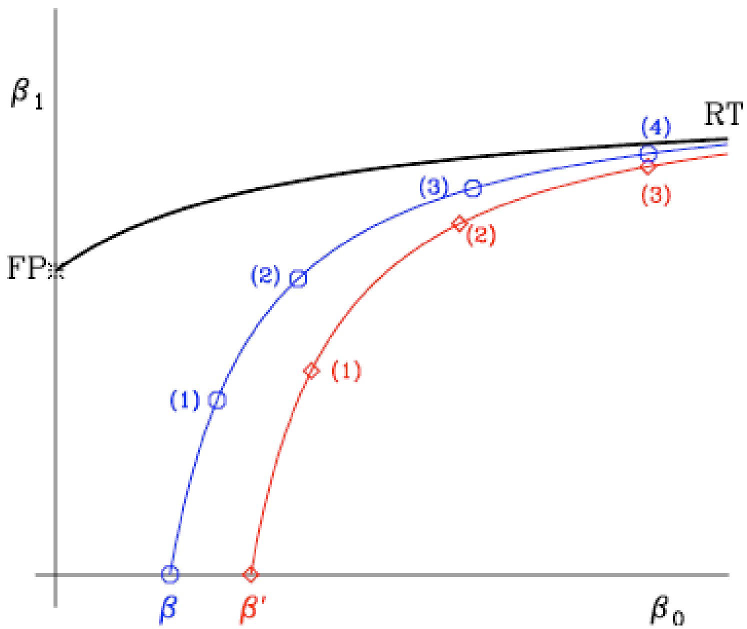
Properties of the step scaling function

This is the bare differential step scaling function

- at a fixed point $s_b(\beta^*)=0$
- the value of s_b is related to the scaling dimension of the coupling
  - for AF models $s_b = 3\ \ln(2)/(4\pi^2)\ b_0\ + O(g^2)$
  - $s_b > 0$ where the RG β-function is $\beta(g) < 0$ (sorry)
  - s_b in the mass predicts the anomalous dimension of the mass

$$m = m'\ 2^{1/y}$$

Calculating $s_b(\beta)$ with MCRG



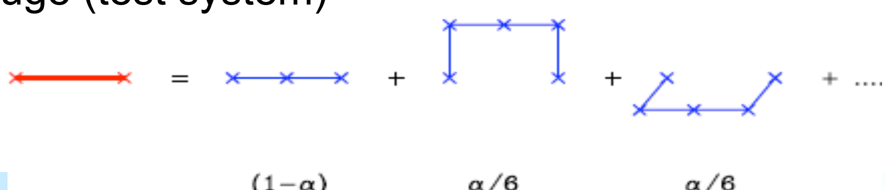
Two actions are identical if all operator expectations values agree

⇓

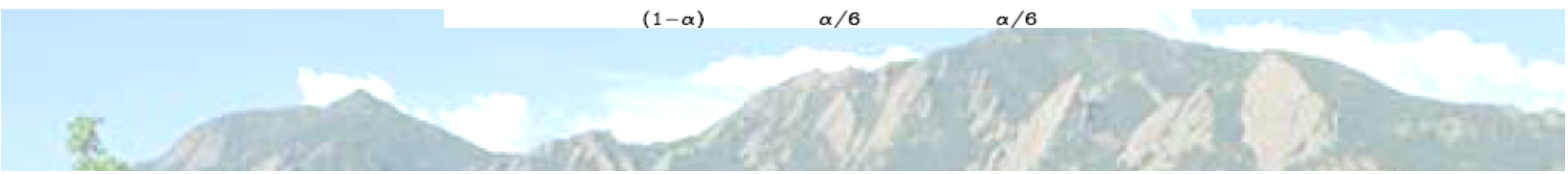
Match operators after several blocking steps

Tests: SU(3) pure gauge (test system)

blocking: "Original"



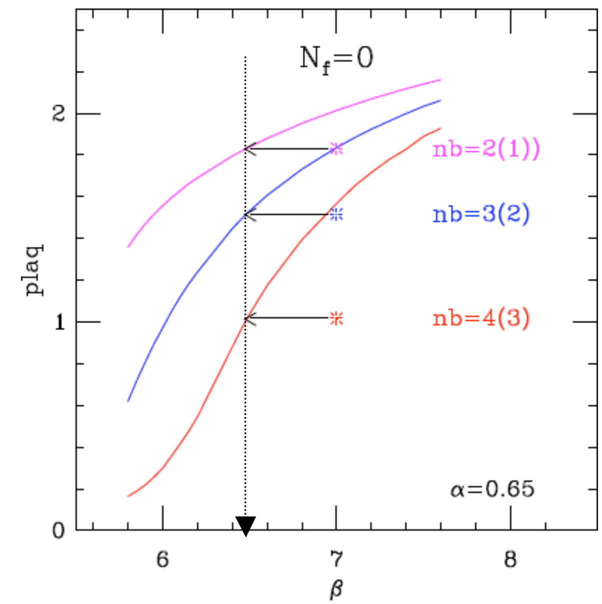
α :optimization parameter



The plaquette* after 1-4 levels of blocking

$32^4 \to 16^4 \to 8^4 \to 4^4 \to 2^4$ (symbols) compare to

$16^4 \to 8^4 \to 4^4 \to 2^4$ (lines)



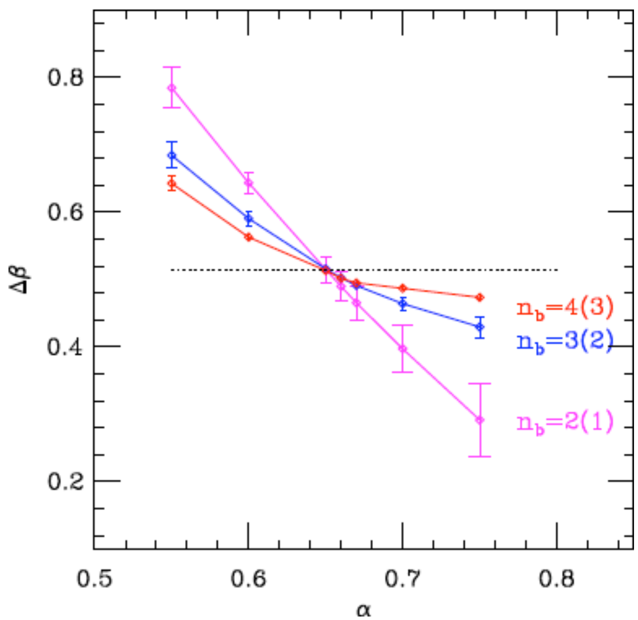
Repeat with many different operators. If they all give the same result, we found matching

$s_b(\beta=7.0) = \Delta\beta = \beta\text{-}\beta' = 7.0\text{-}6.49 = 0.51$

*plaquette: $\mathrm{Tr}(U_\square)$

Optimization of the RG transformations

$\Delta\beta = \beta - \beta'$ at $\beta = 7.0$ as the function of the RG parameter



- Optimizing the RG transformation is essential

  α_{opt}=0.65
- Optimized RG gives the same matching value at each level, for each operator

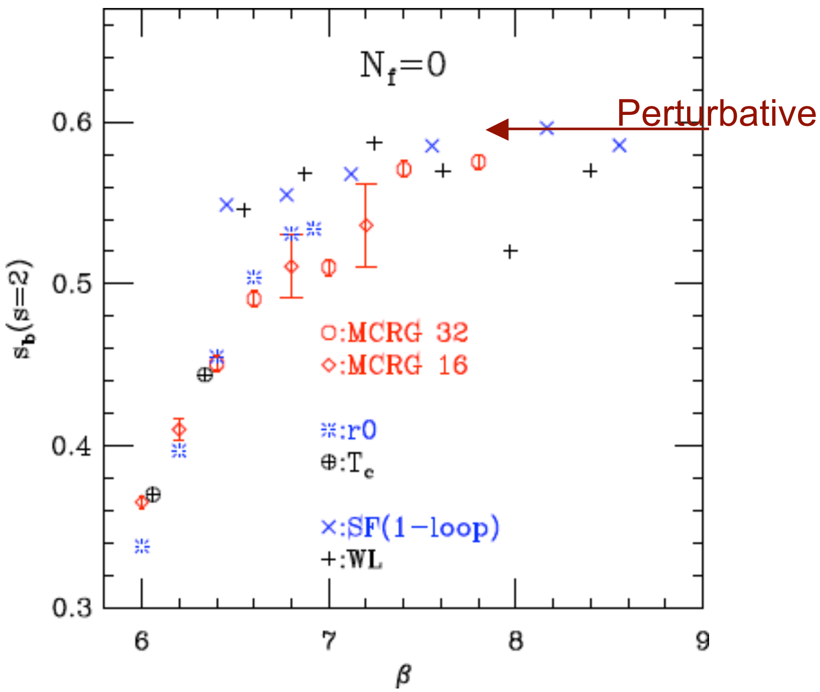
$a(\beta=7.0) = a(\beta'=6.49)/2$

$s_b = \beta - \beta' = 0.51$

SU(3) pure gauge

The bare step scaling function can be calculated in many ways

- Schrodinger fn; Wilson loop ratios,
- physical observables r_0, T_c
- RG matching: $32^4 \rightarrow 16^4$ and $16^4 \rightarrow 8^4$

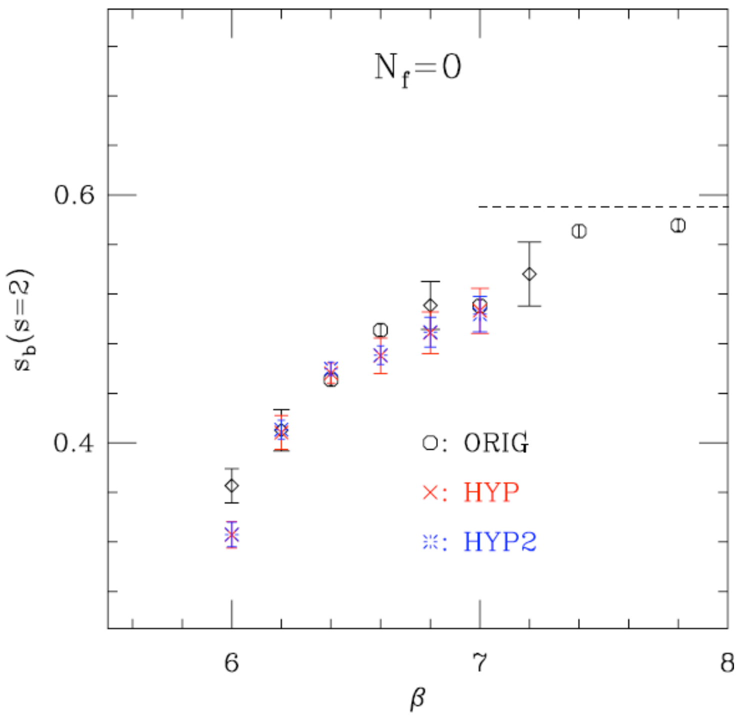


- Excellent agreement between r_0, T_c and MCRG
- Both SF and MCRG approach the perturbative value
- Since at β=6 we can test confinement, we know there is no physical IRFP

Compare different RG transformations:

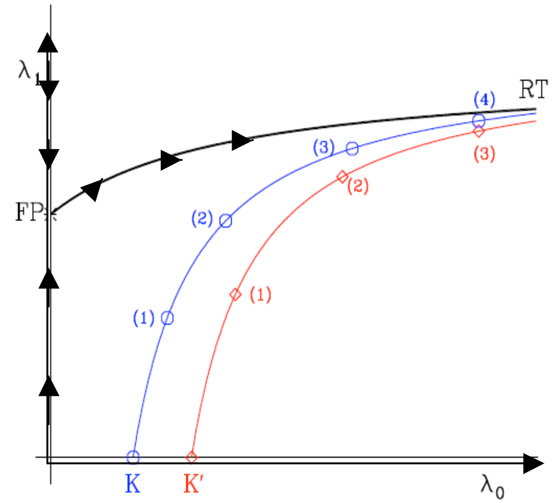
When the flow is governed by a UVFP, $s_b(\beta)$ is universal (up to lattice corrections).

Compare 3 different RG transformations:



Excellent agreement between the 3 RG blockings
→ attractive region of a UVFP

Why do we need different RG's ?



For matching the RG flow has to approach the renormalized trajectory

- The RT describes "perfect actions", i.e. no lattice artifacts

A "good" RG should remove most of the UV modes

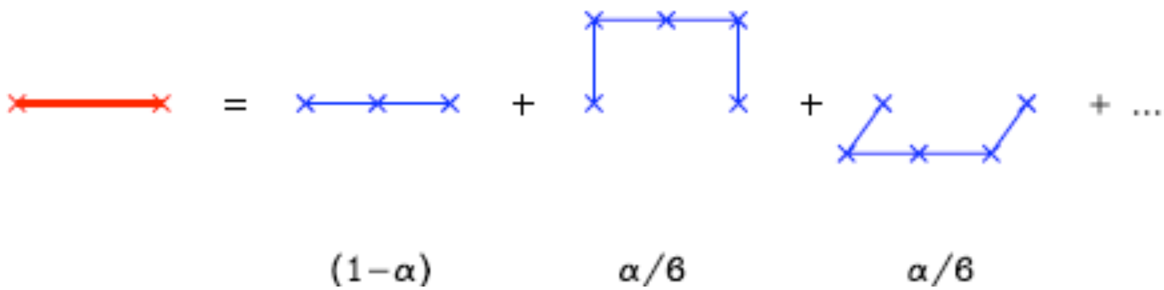
- A "good" RG will approach the RT faster

This is especially important towards strong coupling where UV fluctuations are large

The 3 Renormalization Group transformations

A real space block transformation averages out the short distance modes
Many possibilities - I tried 3 types:

Original



optimize with α

HYP

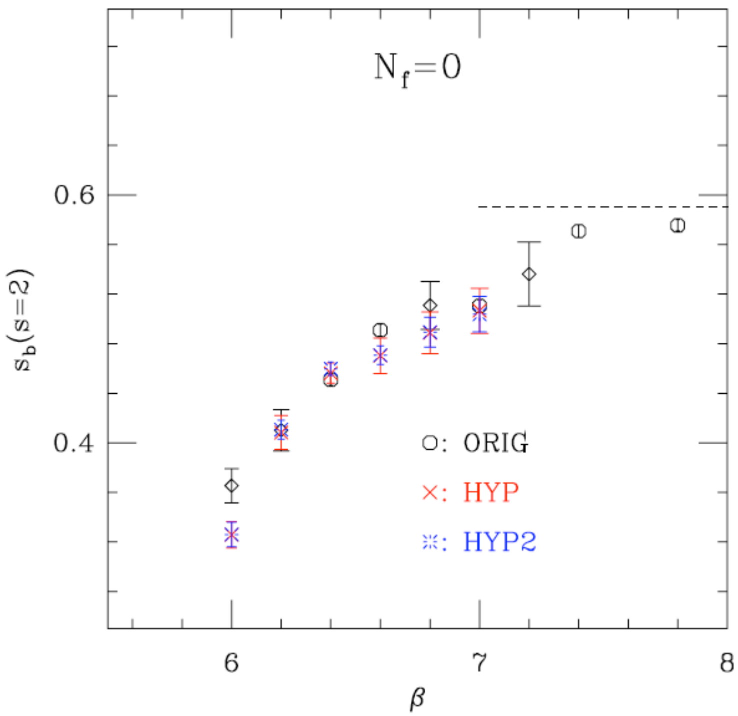
optimize with α_1
(play with α_2, α_3)

HYP2 like HYP, but with twice blocked links

Compare different RG transformations:

When the flow is governed by a UVFP, $s_b(\beta)$ is universal (up to lattice corrections).

Compare 3 different RG transformations:



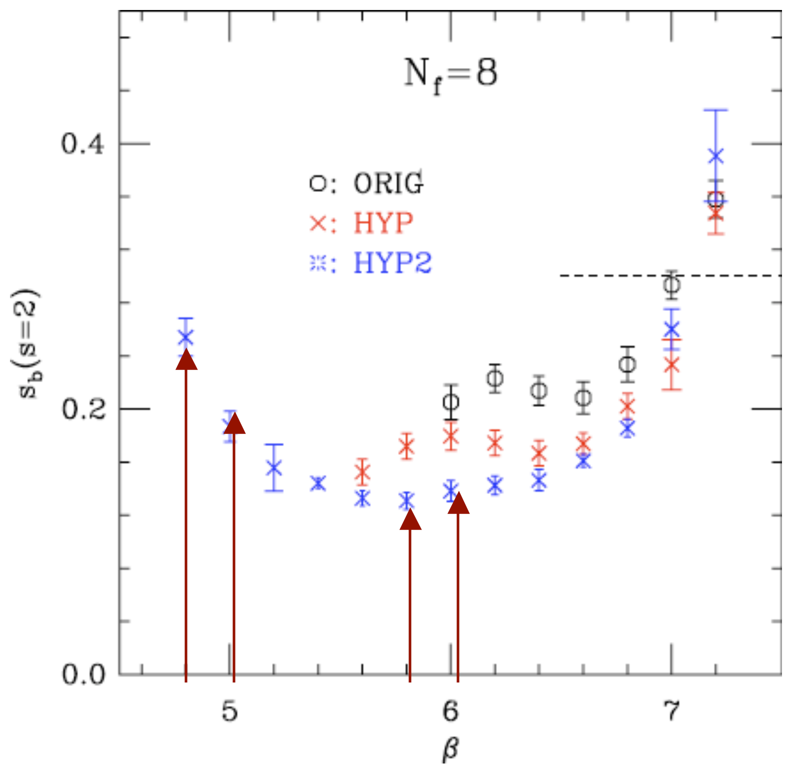
Excellent agreement between the 3 RG blockings

→ attractive region of a UVFP

N_f=8 flavors

Expected to be QCD-like: analytical & numerical results

Compare the different RG transformations:



s_b>0 everywhere - no IRFP

At β~5.0 RG matching gets difficult, but by then confinement develops (string tension is $a\sqrt{\sigma}$ ~ 0.2)

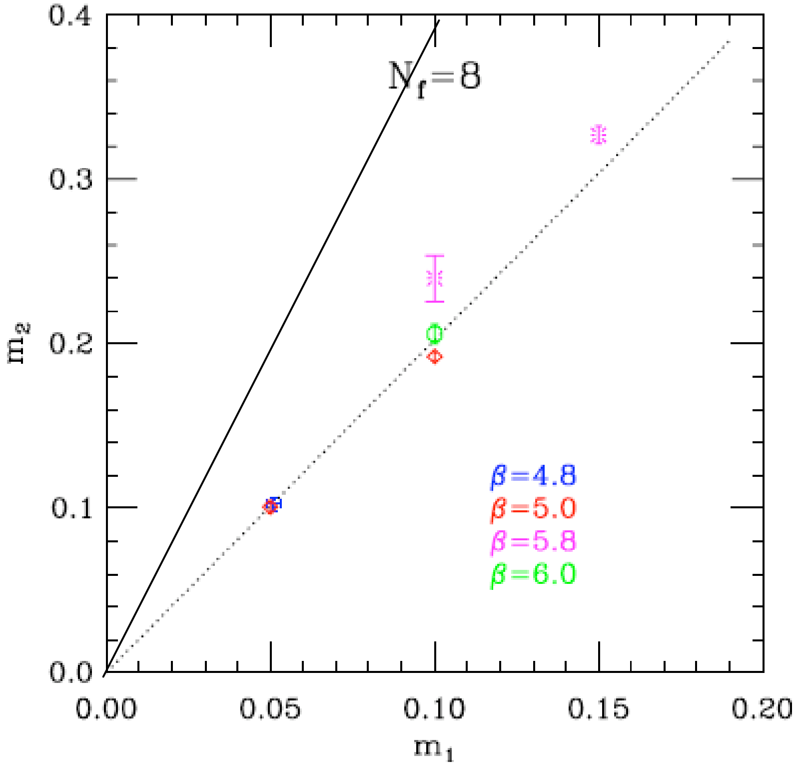
Considerably larger difference between the 3 RG blockings
non-QCD like behavior?

Look at the anomalous mass dimension

N_f=8 flavors, anomalous mass

4 different couplings (β=4.8,5.0,5.8,6.0), optimal RG from m=0 data

y_m=2



$m_2 = m_1\, 2^{-1/y_m}$
$\gamma_m = y_m - 1$

All 3 couplings predict the same value
y_m= 1.02(5)
close to free field exponent

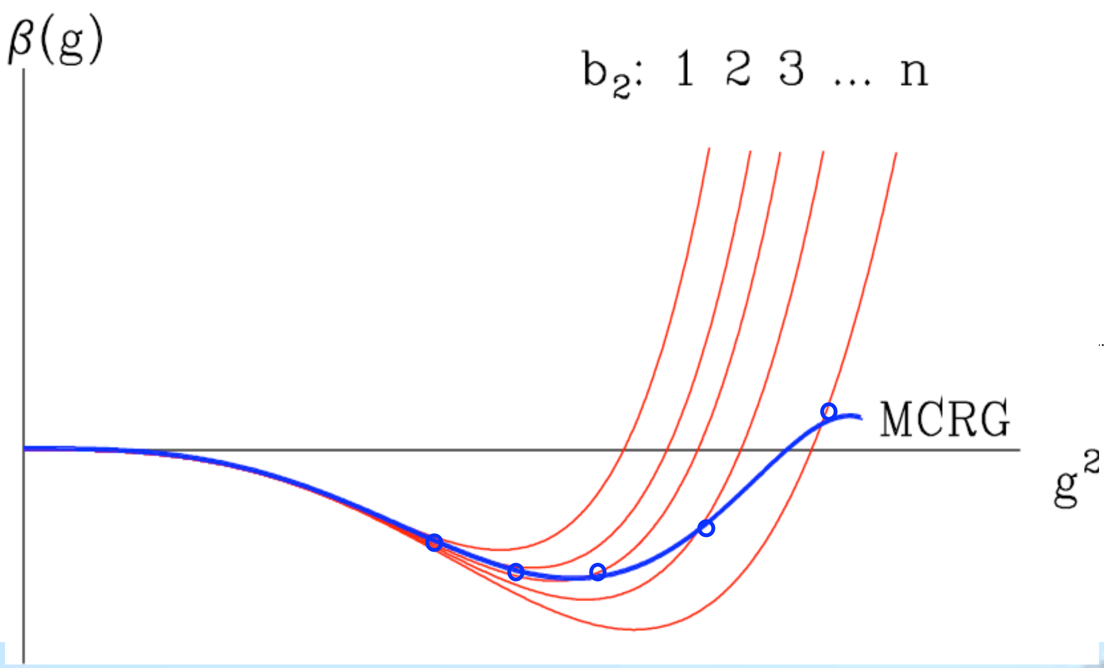
N_f=16 flavors

MCRG optimizes the RG as the function of the bare coupling . Along an irrelevant direction

- -it can pick a different RG at each coupling
- -the β function it determines does not correspond to any “real” β function

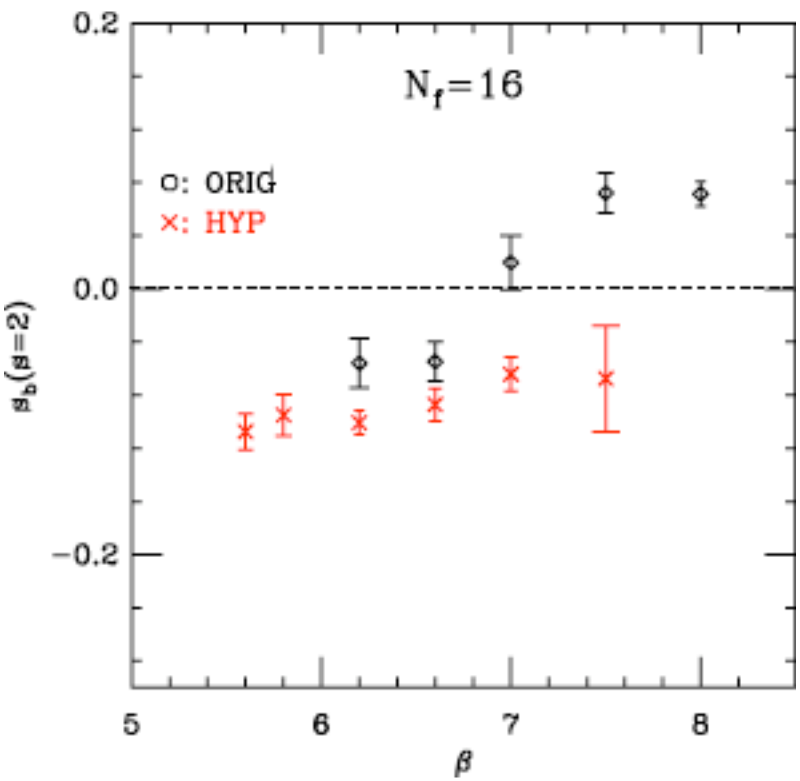
Warning!

But: a zero is a zero



N_f=16 flavors

$16^4 \rightarrow 8^4$ MCRG



ORIG blocking shows $s_b(\beta)$=0 around β=7.0

HYP blocking has an IRFP around β=8.0

Different block transformations predict different $s_b(\beta)$=0 but they both show a positive RG β function

N_f=12 flavors

Some history:

- The analytic work of Appelquiest et al predicts N_f=12 is just above the conformal window
- Yale group found an IRFP at fairly strong coupling, using Schrodinger functional method
- Groningen group identified a bulk phase transition characteristic to a conformal system and claim chiral symmetry at weaker coupling
- Two groups (San Diego and Columbia) have studied the spectrum of the model with improved and unimproved actions. Both see QCD-like behavior, though at strong gauge couplings.

If N_f=12 is conformal,

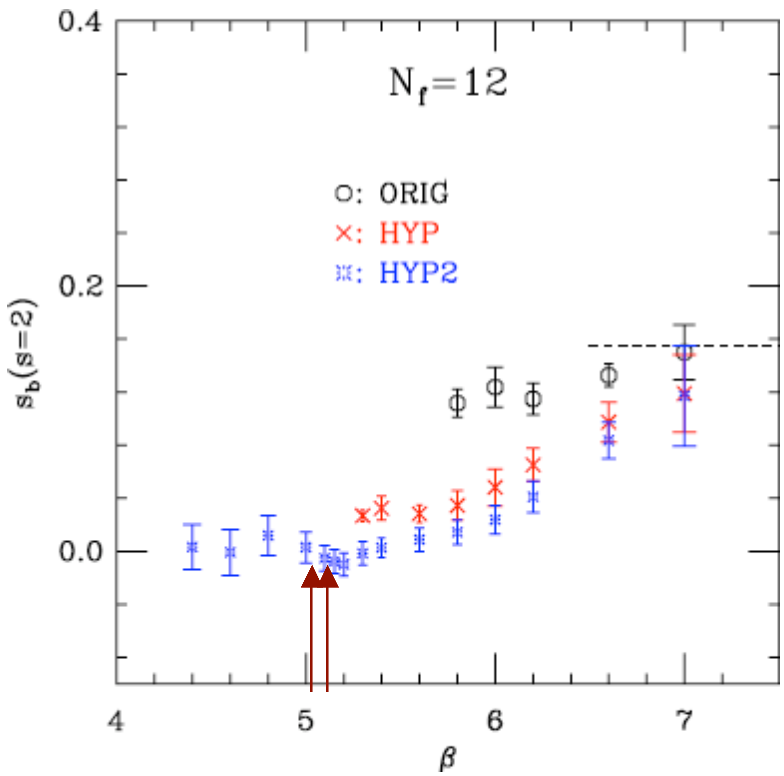
the spectral measurements are in the strong coupling lattice artifact phase

If N_f=12 is QCD-like,

the unimproved actions used with Schrodinger functional are unreliable

N_f=12 flavors with MCRG

Use the same techniques as before; $16^4 \to 8^4$

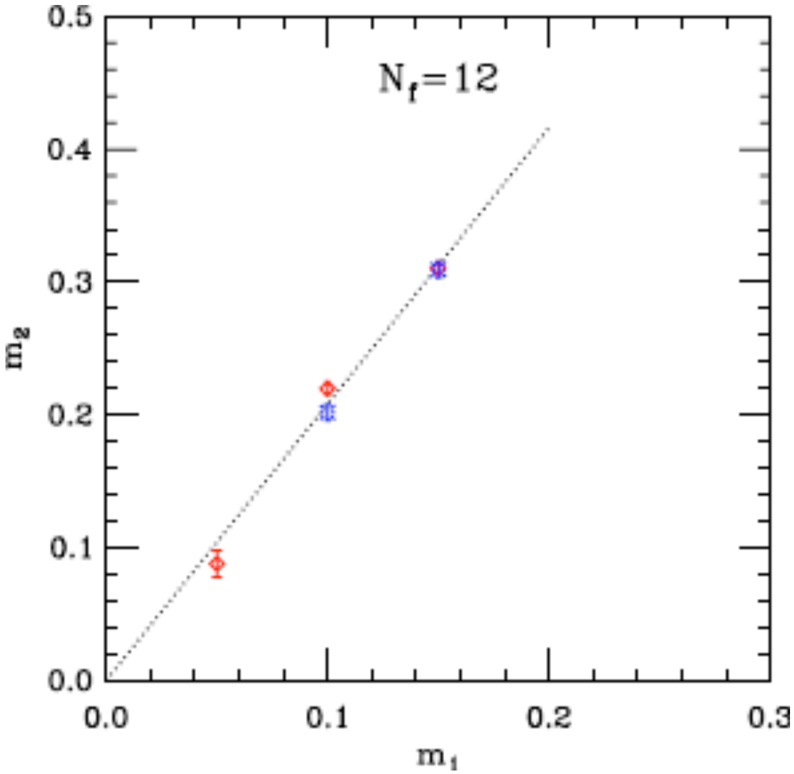


- Orig/HYP blockings predicts different $s_b(\beta)$ functions
- HYP2 hovers around 0 -- likely IRFP
- String tension remains zero at β=4.4 on 16^4 volumes, but lattice artifacts are large

N_f=12 could be walking between β=4 and 6 - but that would be strange, the least

N_f=12 - anomalous mass

- β=5.0,5.1 : both where s_b ~0;
  using optimal RG from m=0 data



Again, consistently

y_m=1.06(3)

for both couplings, masses

At a strongly coupled IRFP one expects a large y_m

Maybe it is not strongly coupled

Note:
SU(2) with adjoint fermions (DelDebbio et al) looks the same ; SU(3) with sextet (T. DeGrand) has y_m~1.5

Summary: N_f=12 flavors

- It would be nice to firmly establish a back flow or confinement
  - Larger volume simulations
  - Different action
  - Different RG transformation
- For now, N_f=12 looks conformal, but with a trivial exponent

Summary: 2- lattice matching MCRG

- Can be optimized by tuning the free parameter(s) of the RG transformation
- Finite volume effects are largely controlled
- Requires relatively small statistics
- Has a lot of built-in consistency checks
  - compare several blocking levels
  - compare several operators
  - compare different RG transformations

Conclusion

MCRG is an effective alternative method to study the phase structure and scaling properties of lattice QFT's

- The method is very universal, straightforward to implement for any other system

N_f=0-8,16 as expected. N_f=12 is difficult:

- y_m~ 1.0 for 12 flavors; far above the opening of the conformal window?

What is next?

- Could the different groups come up with a consistent picture for N_f=12?
- N_f =10,(9,11?) would be very interesting
- SU(2) gauge, other fermion representations can be studied the same way

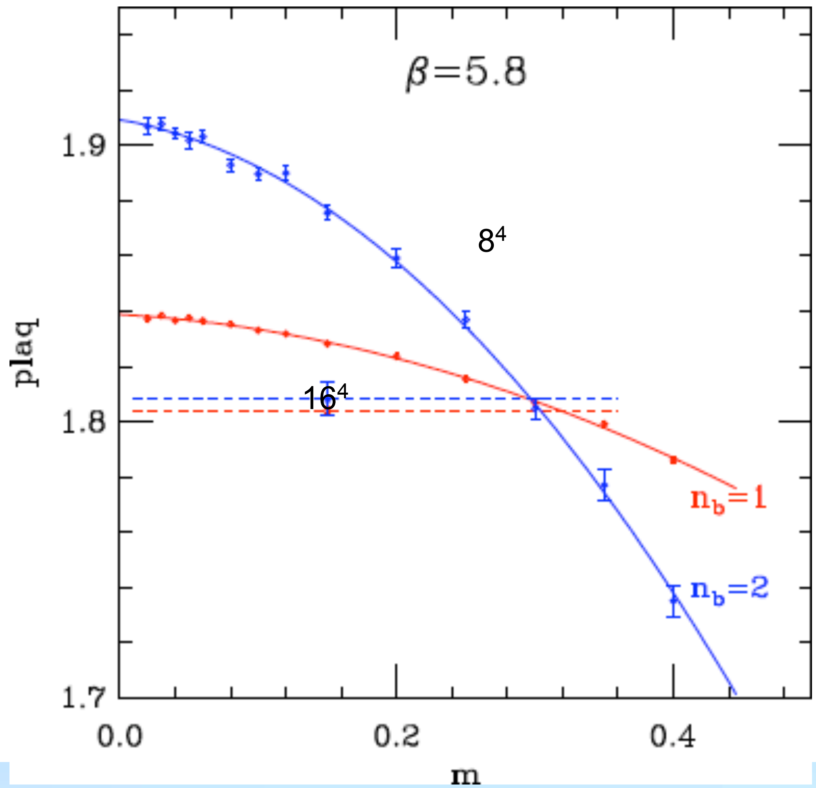
EXTRA SLIDES

MCRG to find the mass anomalous dimension

N_f=16 flavor SU(3) model

Matching in the mass at fixed $\beta = 5.8$

$m_2 = m_1\, 2^{1/\nu}$

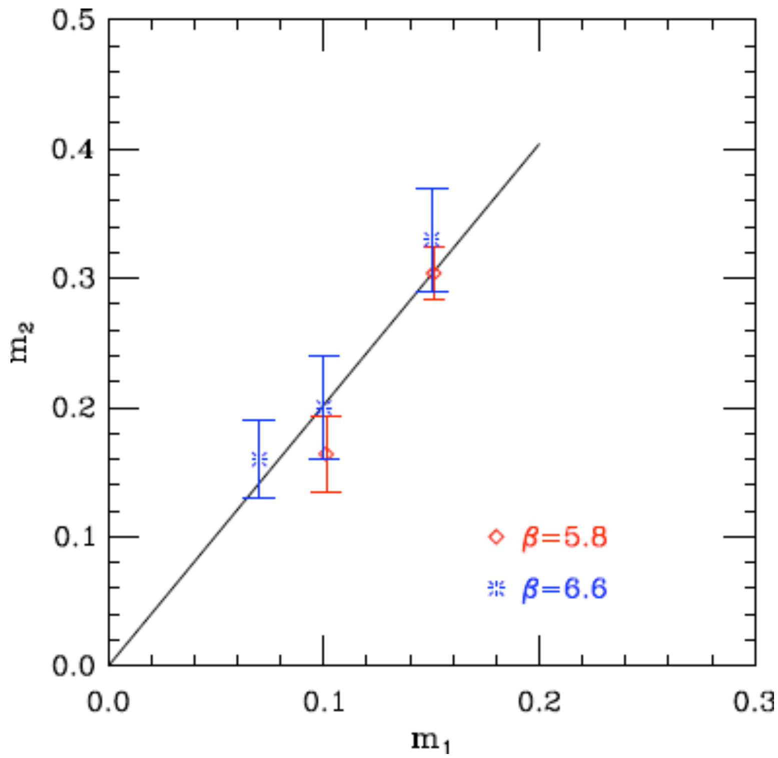


- use the same gauge observables (probably not the best choice)

-at α_{opt} both n_b=2(1) and 3(2) predicts the same matching pair

The critical exponent for the mass

At several couplings, mass values

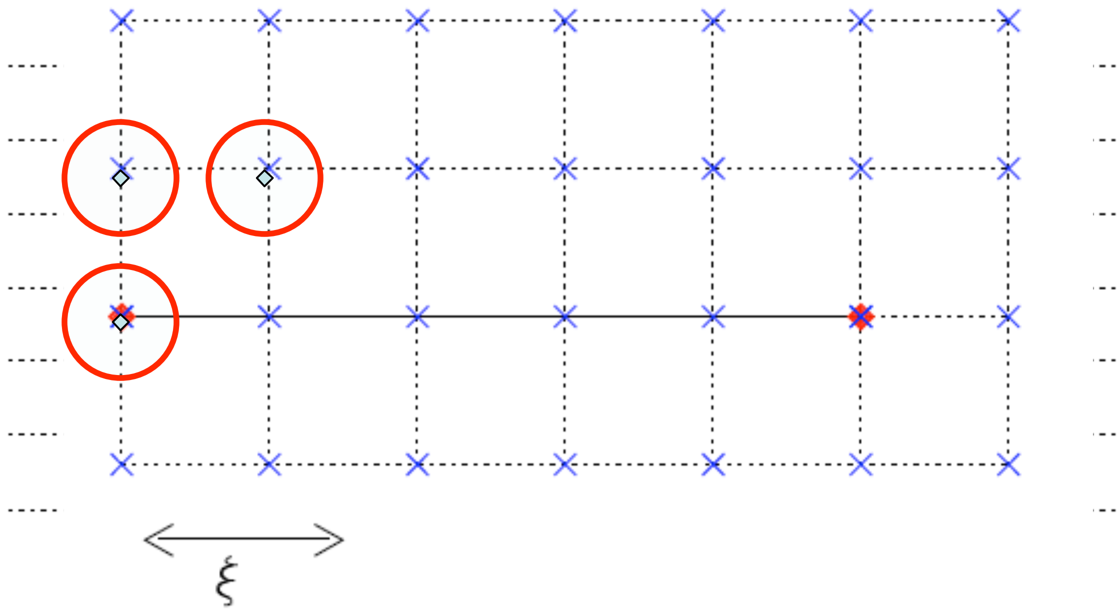


$m_2 = m_1\, 2^{1/\nu}$

ν=1.0(1)

Free field exponent (close to GFP)

Real space block transformation



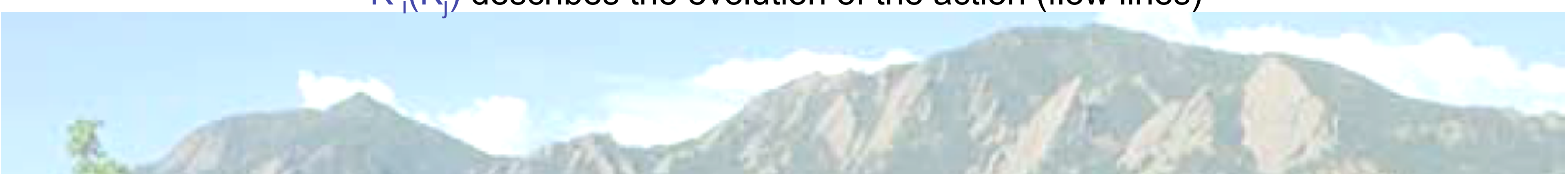
$\langle s(0)s(x)\rangle \sim e^{-x/\xi}$

if $x \gg \xi$

Original :		Blocked in $(b=2)^d$
variables :	$s(x)$	$s'(x)$ = (average of $s(x)$ in block)
lattice spacing:	a	$a'=2a$
correlation length:	$\xi=\xi_{lat}\, a$	$\xi'_{lat}=\xi_{lat}/2$, (but $\xi' = \xi$!)
Action:	$\{K\}$	$\{K'\}$

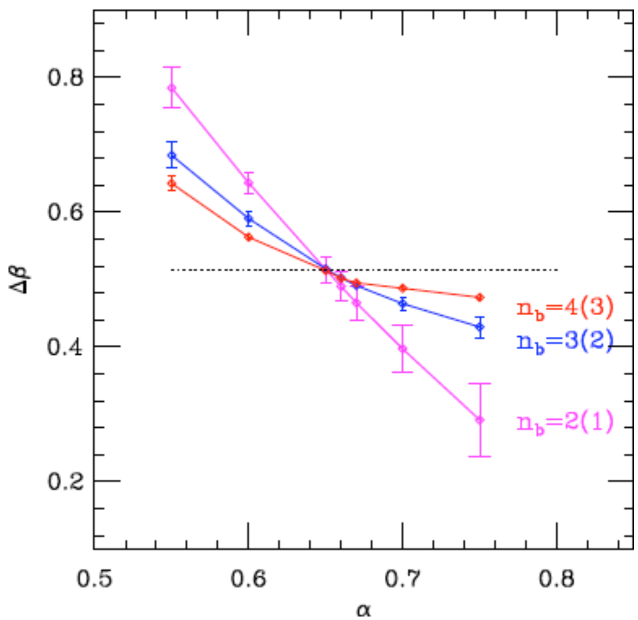
The two systems have the same IR (as long as $\xi_l > 1$)

$K'_i(K_j)$ describes the evolution of the action (flow lines)



Optimization of the RG transformations

$\Delta\beta=\beta-\beta'$ at $\beta=7.0$ with different RG paremeter, blocking levels, operators



- Optimizing the RG transformation is essential
  α_{opt}=0.65
- Optimized RG gives the same matching value at each level, for each operator

$a(\beta=7.0) = a(\beta'=6.49)/2$

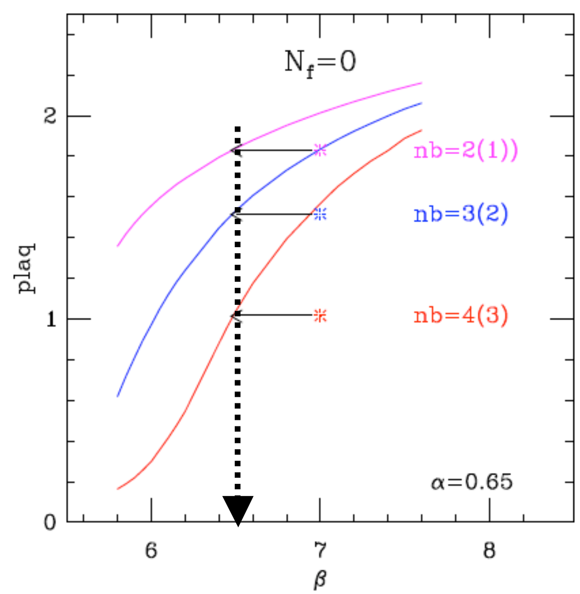
$s_b = \beta-\beta'=0.51$

2- lattice matching MCRG - in practice:

Two actions are identical if every expectation value measured with the 2 actions are identical

MCRG identifies matched couplings (β,β') by comparing expectation values after n_b (n_b -1) RG blocking steps.

Example: pure gauge SU(3)



The plaquette after 1-4 levels of blocking

$32^4 \rightarrow 16^4 \rightarrow 8^4 \rightarrow 4^4 \rightarrow 2^4$ (symbols)

(n_b 2 3 4)

compared to

$16^4 \rightarrow 8^4 \rightarrow 4^4 \rightarrow 2^4$ (lines)

(n_b 1 2 3)

$\Delta\beta = \beta - \beta' = 0.51$

Walking : not quite that simple….

The RG β function is scheme dependent:

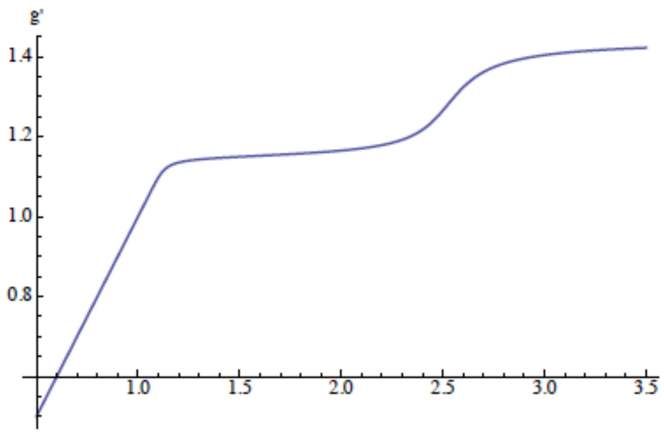
$\beta(g) = \mu\,(d\,g\,/d\,\mu\,) = -\beta_0\,g^3 - \beta_1\,g^5 + O(g^7)$

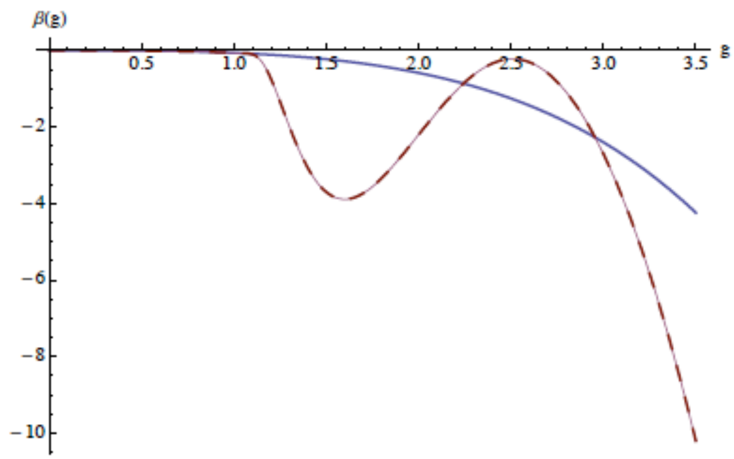
only β_0, β_1 are universal

Change $g \rightarrow g' = \Phi(g)$

$\beta(g) \rightarrow \beta'(g') = (d\,\Phi(g)/d\,g)\beta(g)$

Example from DelDebbio (Leiden workshop)





(this is N_f=0 SU(3) !)

One really needs to look at physical quantities

Why walking?

2 energy scales:

- weak scale v=250GeV : techni-pion decay constant F_π ~ 250 GeV
- cut-off scale (extended technicolor symm. breaking) : Λ_{ETC}

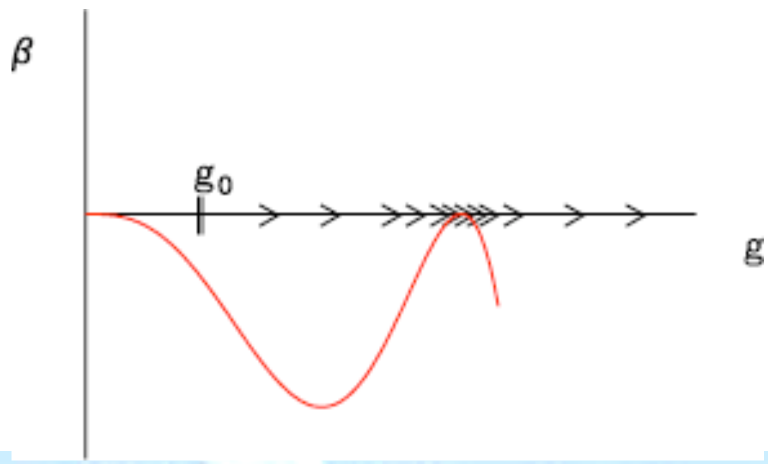
Light fermion masses : $m_q(\Lambda_{ETC}) \sim \frac{<\bar{\psi}\psi>_{ETC}}{\Lambda^2_{ETC}}$

Flavor changing neutral current limits require Λ_{ETC} > 10^3 TeV

$\rightarrow$ large $<\bar{\psi}\psi>_{ETC}$

while QCD like theories have $<\bar{\psi}\psi>_{TC} \approx 4\pi F_\pi^3$

$\rightarrow$ small $<\bar{\psi}\psi>_{TC}$

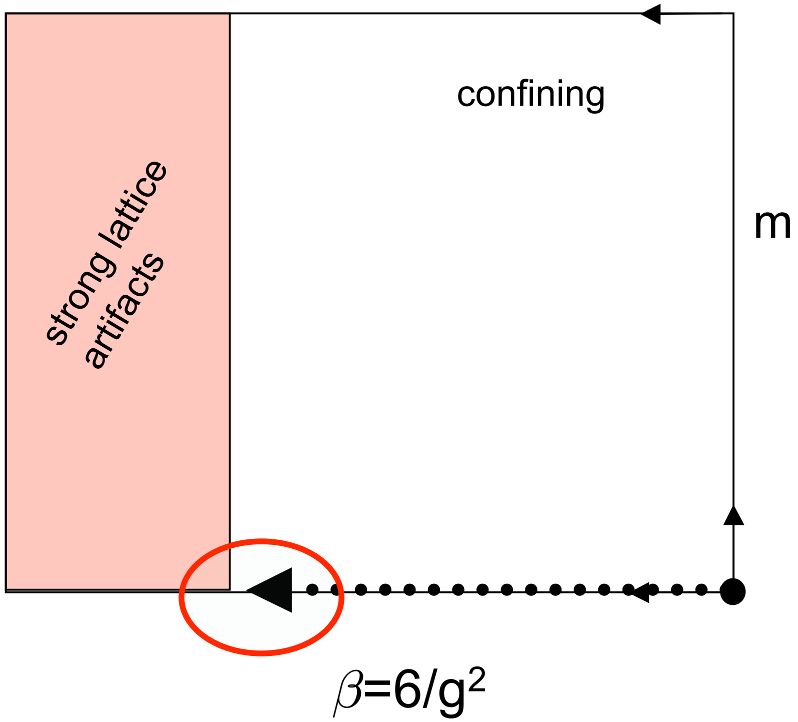


The problem can be solved if the running coupling and the anomalous mass remain strong across a large scale : the coupling walks

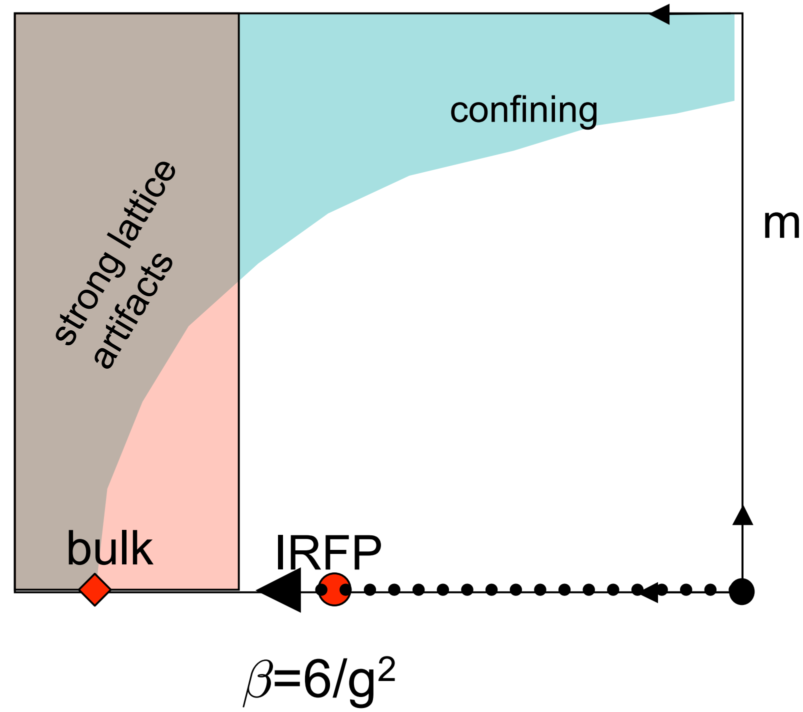
The lattice phase diagram

(arrows: UV to IR)

QCD like



Conformal



Lattice simulations can connect the perturbative FP and strong coupling

- Found IRFP ? Done ✔
- No IRFP? Show that it is confining before a bulk transition is reached
- Strong lattice artifacts can interfere

Calculating the step scaling function

- Schroedinger functional method:
  - generalization of the approach used to calculate the renormalized coupling in QCD
- 2-lattice matching Monte Carlo Renormalization Group method:
  - based on Wilson RG description to study critical behavior
  - has been used in QCD and statistical systems extensively
  - Has a lot of built-in checks and controls
  - works with bare quantities
  - could be used to calculate the renormalized coupling in QCD