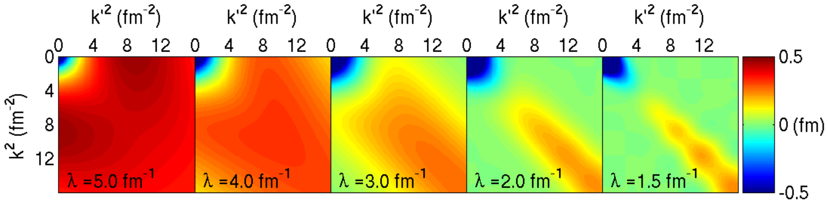
Similarity RG and Many-Body Operators

Dick Furnstahl

Department of Physics
Ohio State University

February, 2010



Collaborators: E. Anderson, S. Bogner, S. Glazek, E. Jurgenson, P. Navratil, R. Perry, L. Platter, A. Schwenk, K. Wendt

Outline

Prelude: Many-body operators from SRG

Context: Nuclear physics and three-body forces

Explicit running of three-body (and higher) interactions

Open questions and issues

Outline

Prelude: Many-body operators from SRG

Context: Nuclear physics and three-body forces

Explicit running of three-body (and higher) interactions

Open questions and issues

Recap: SRG flow equations [arXiv:0912.3688]

- Transform an initial hamiltonian, $H = T + V$:

$$H_s = U_s H U_s^\dagger \equiv T + V_s \ ,$$

where s is the *flow parameter*. Differentiating wrt s:

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \text{with} \qquad \eta_s \equiv \frac{dU_s}{ds} U_s^\dagger = -\eta_s^\dagger \ .$$

- η_s is specified by the commutator with “generator” G_s:

$$\eta_s = [G_s, H_s] \ ,$$

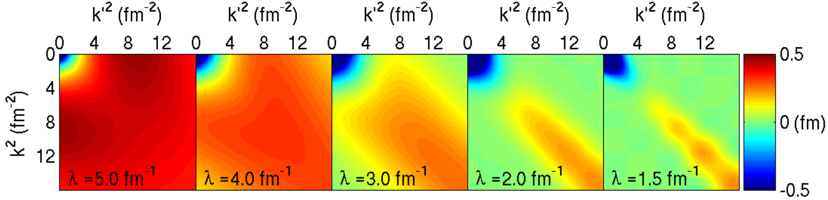
which yields the flow equation (T held fixed),

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s] \ .$$

- G_s determines flow $\Longrightarrow$ many choices (T, H_D, H_{BD} ...)

Recap: SRG flow equations [arXiv:0912.3688]

- Implementation issues
  - choosing G_s
  - choosing a basis
- Features evident from two-body system
  - decoupling of low-energy from high-energy
  - evolves toward universal low-k interactions in free space
  - consistent evolution of operators



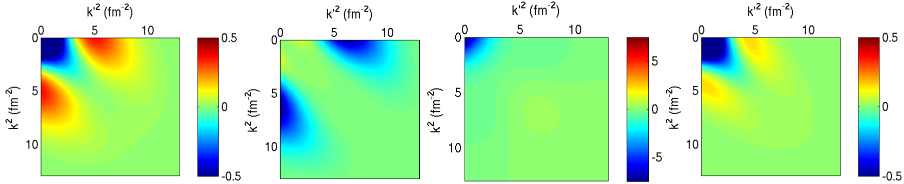
- What about applications to few- or many-body systems?

Flow in momentum basis with $\eta(s) = [T, H_s]$

- For NN only, project onto relative momentum states $|k\rangle$

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \quad \text{with} \quad T_{\rm rel}|k\rangle = |\epsilon_k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



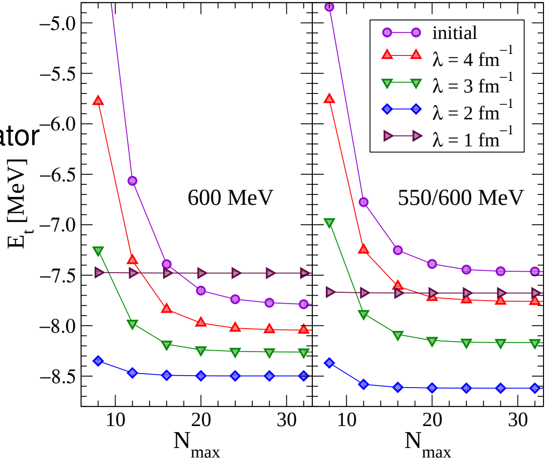
$V_{\lambda=2.5}(k,k')$ | 1st term | 2nd term | $V_{\lambda=2.0}(k,k')$

- First term drives V_λ toward diagonal:

$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \cdots$$

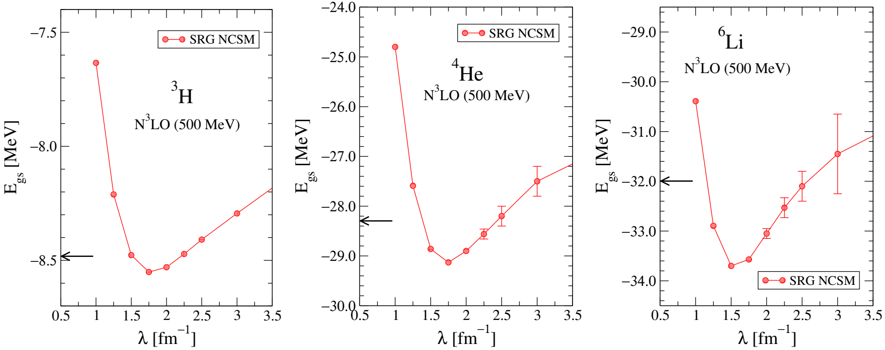
Variational Calculations in Three-Nucleon Systems

- Triton ground-state energy vs. size of harmonic oscillator basis ($N_{\max}\hbar\omega$ excitations)
- Rapid convergence as λ decreases
- Different binding energies $\Longrightarrow$ 3-body contribution



Running of E_{gs} with two-body interaction (NN) only

- Not unitary for $A \geq 3 \Longrightarrow$ ground-state energy depends on λ



- Same qualitative behavior (from NN repulsion/attraction)
- Gives running of net three–body contribution
- Error bars are from extrapolated results (not converged)

Flow equations lead to many-body forces

- Schematically:

$$\frac{dV_s}{ds} = \Big[\Big[\sum a^\dagger a, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big] = \cdots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body!}} + \cdots$$

  so there will be A-body forces generated
- Is this a problem?
  - Normal ordering and truncation at NN may be sufficient [Achim]
  - Ok if “induced” many-body forces are same size as natural ones
- Nuclear 3-body forces already needed in unevolved potential
  - In fact, there are A-body forces initially
  - Natural hierarchy from chiral EFT
    $\Longrightarrow$ stop flow equations before unnatural 3-body size
  - Many-body methods must deal with them!
- SRG is a tractable method to evolve many-body operators

Outline

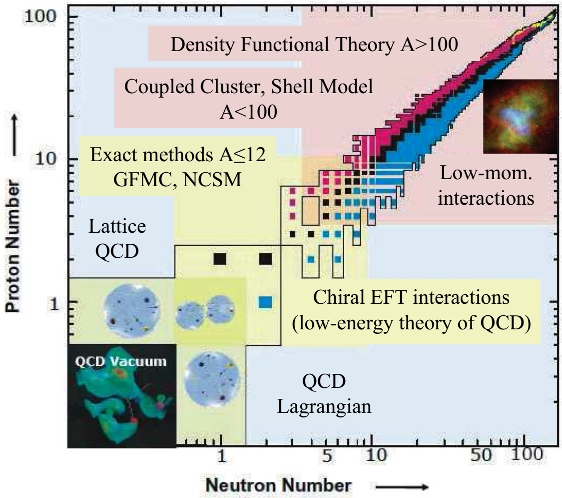
Prelude: Many-body operators from SRG

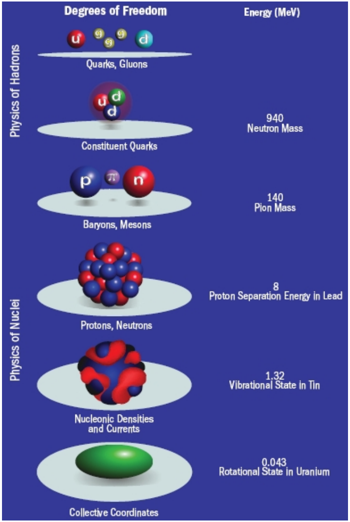
Context: Nuclear physics and three-body forces

Explicit running of three-body (and higher) interactions

Open questions and issues

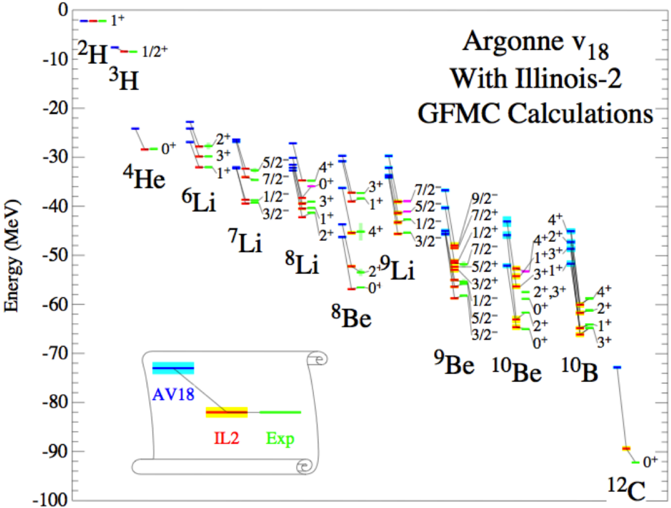
Degrees of Freedom: From QCD to Nuclei





- RG $\Longrightarrow$ make nuclear structure look more like quantum chemistry

Light nuclei: Pieper/Wiringa (Bonner Prize!)

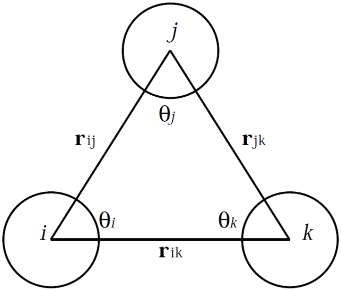


- Three-body forces needed for energies, splittings, . . .

Atomic 3-body forces: Axilrod-Teller term (1943)

- Three-body potential for atoms/molecules from triple-dipole mutual polarization (3rd-order perturbation correction)

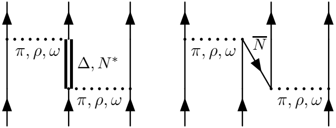
$$V(i,j,k) = \frac{\nu(1 + 3\cos\theta_i \cos\theta_j \cos\theta_k)}{(r_{ij} r_{ik} r_{jk})^3}$$

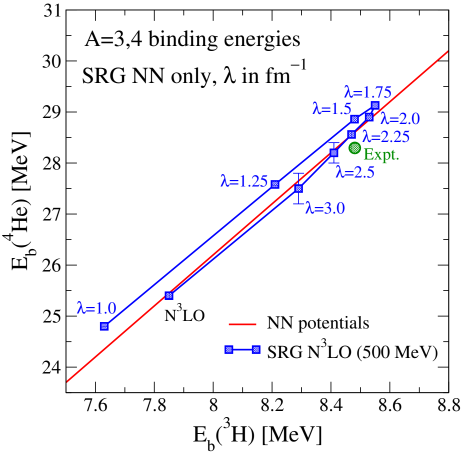


- Usually negligible in metals and semiconductors
- Can be important for ground-state energy of solids bound by van der Waals potentials
- Bell and Zuker (1976): 10% of energy in solid xenon

Observations on three-body forces

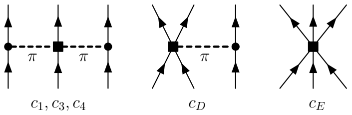
- Three-body forces arise from eliminating dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - e.g., Tjon line

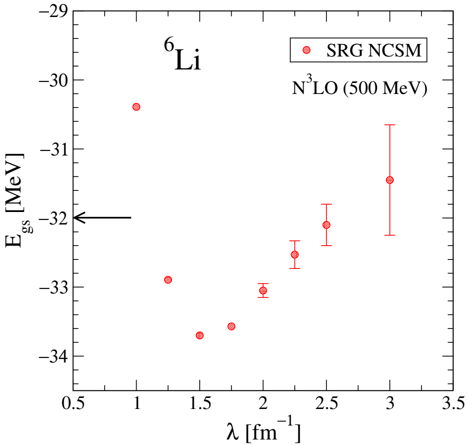




Observations on three-body forces

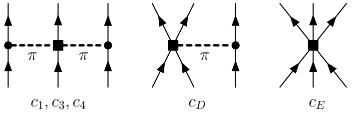
- Three-body forces arise from eliminating dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - e.g., Tjon line
- NNN at different Λ/λ can be fit to χEFT or evolved
  - how large is 4-body?

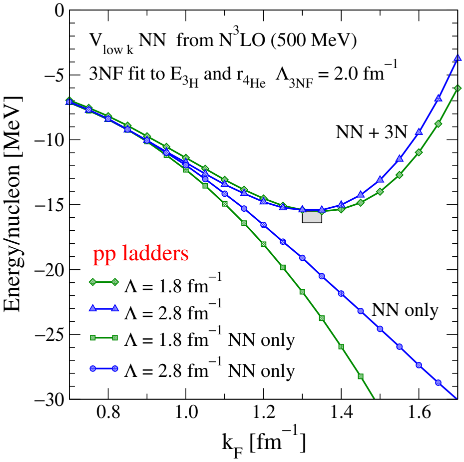




Observations on three-body forces

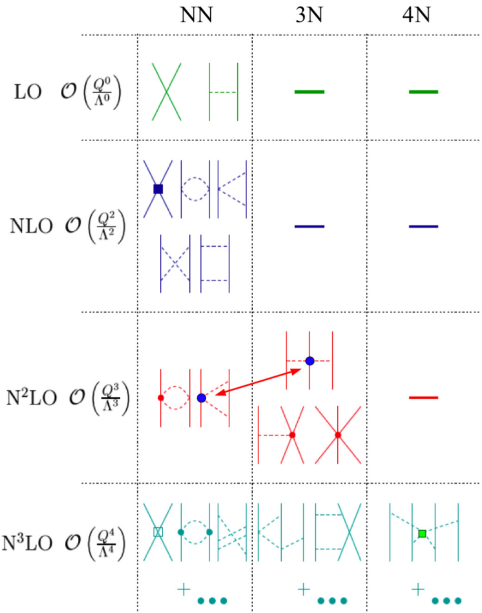
- Three-body forces arise from eliminating dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - e.g., Tjon line
- NNN at different Λ/λ can be fit to χEFT or evolved
  - how large is 4-body?
  - saturation of nuclear matter
  - cutoff dependence as tool



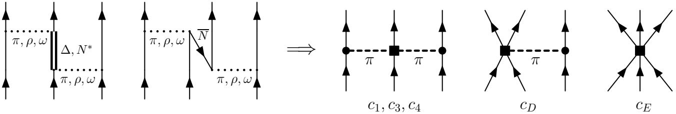


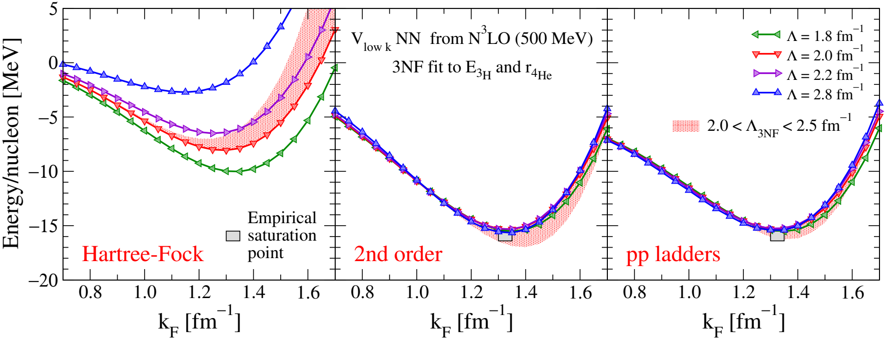
Chiral EFT hierarchy

- Power counting still unsettled
- But many-body hierarchy consistent with calculations in few-body systems
- 3-body at N^{2}LO without Δ and NLO with Δ
- 4-body at N^{3}LO



Nuclear matter with RG-evolved NN plus fit NNN





- At low resolution, nuclear saturation driven by NNN
- Can we validate use of the chiral EFT operator basis?

Outline

Prelude: Many-body operators from SRG

Context: Nuclear physics and three-body forces

Explicit running of three-body (and higher) interactions

Open questions and issues

SRG with normal-ordering in the vacuum

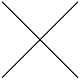
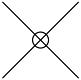
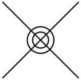
- SRG flow equation $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$, e.g., $G_s = T_{\rm rel}$
  - Right side evaluated w/o solving bound-state or scattering eqs.
  - Can be applied directly in three-particle space
- *A*-body operators completely fixed in *A*-particle subspace
- What about spectator nucleons?
  - Decoupling of 3N part in momentum space

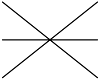
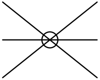
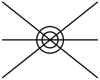
$$\frac{dV_s}{ds} = \frac{dV_{12}}{ds} + \frac{dV_{13}}{ds} + \frac{dV_{23}}{ds} + \frac{dV_{123}}{ds} = [[T_{\rm rel}, V_s], H_s] \, ,$$

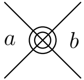
$$\Longrightarrow \quad \frac{dV_{123}}{ds} = [[T_{12}, V_{12}], (T_3 + V_{13} + V_{23} + V_{123})] + \{123 \to 132\} + \{123 \to 231\} + [[T_{\rm rel}, V_{123}], H_s]$$

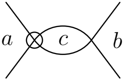
  - No “multi-valued” two-body interactions (dependence on excitation energy of unlinked spectators)
  - Or, direct solution in discrete harmonic oscillator basis

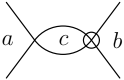
Diagrams for SRG $\Longrightarrow$ Disconnected cancels

$V_s^{(2)} =$  $[T, V_s^{(2)}] =$  $[[T, V_s^{(2)}], T] =$ 

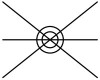
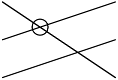
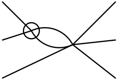
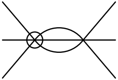
$V_s^{(3)} =$  $[T, V_s^{(3)}] =$  $[[T, V_s^{(3)}], T] =$ 

$\dfrac{dV_s^{(2)}(a,b)}{ds} =$ 

$+$ 

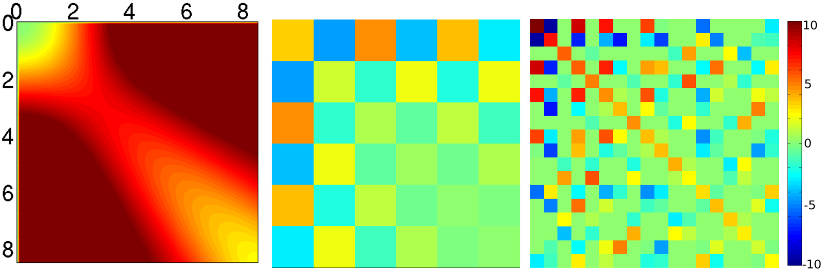
$-$ 

$-(\epsilon_a - \epsilon_b)^2\, V_s^{(2)}(a,b)$ $\qquad$ $\sum_c \left[(\epsilon_a - \epsilon_c) - (\epsilon_c - \epsilon_b)\right] V_s^{(2)}(a,c)\, V_s^{(2)}(c,b)$

$\dfrac{dV_s^{(3)}}{ds} =$  $+$  $+$  $+$  $+ \cdots$

Embedding: Initial potential

- Symmetrized Jacobi oscillator basis (here: 1D bosons)

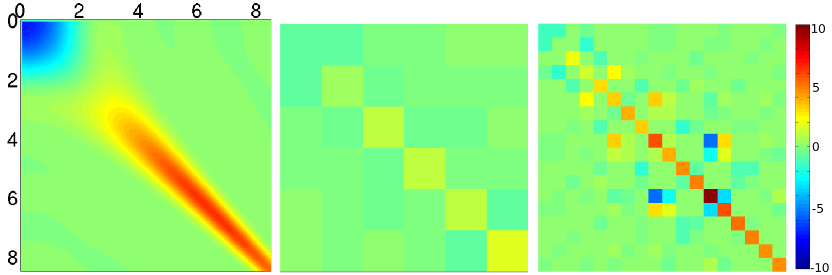


$V(p,p') \longrightarrow V(N_2,N_2') \longrightarrow V(N_3,N_3')$

- Diagonalize symmetrizer $\Rightarrow \langle N_A || N_{A-1}; n_{A-1}\rangle$; use recursively
- Embedding is everything, SRG coding is trivial
- 3D: Use Navratil et al. technology for NCSM

Embedding: SRG evolved potential at $\lambda = 2$

- Symmetrized Jacobi oscillator basis (here: bosons)



$V(p, p') \longrightarrow V(N_2, N_2') \longrightarrow V(N_3, N_3')$

- Diagonalize symmetrizer $\Rightarrow \langle N_A || N_{A-1}; n_{A-1} \rangle$; use recursively
- Embedding is everything, SRG coding is trivial
- 3D: Use Navratil et al. technology for NCSM

Few-Body Embedding

Legend: Embedding, Evolving, BE calculation, Initial 3NF

- A=3 (2N only):

$$V^{(2)}_{osc} \overset{SRG}{\Longrightarrow} V^{(2)}_{\lambda,osc} \overset{embed}{\Longrightarrow} V^{(2)}_{\lambda,3Nosc} \overset{diag}{\Longrightarrow} BE_3^{(2Nonly)}$$

- A=4 (2N only):

$$V^{(2)}_{osc} \overset{SRG}{\Longrightarrow} V^{(2)}_{\lambda,osc} \overset{embed}{\Longrightarrow} V^{(2)}_{\lambda,3Nosc} \overset{embed}{\Longrightarrow} V^{(2)}_{\lambda,4Nosc} \overset{diag}{\Longrightarrow} BE_4^{(2Nonly)}$$

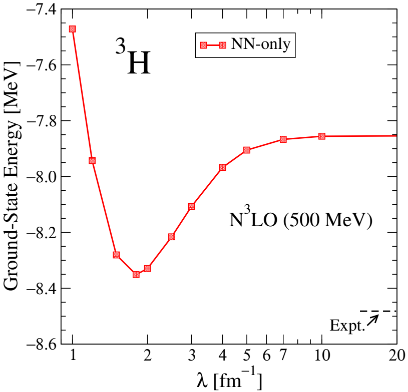
- A=4 (2N+3N only):

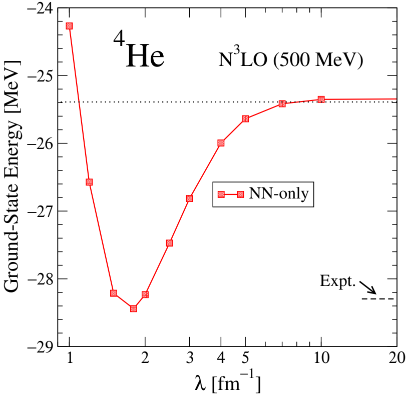
$$V^{(2)}_{osc} \overset{embed}{\Longrightarrow} V^{(2)}_{3Nosc} \overset{SRG}{\Longrightarrow} V^{(2+3)}_{\lambda,3Nosc} \overset{embed}{\Longrightarrow} V^{(2+3)}_{\lambda,4Nosc} \overset{diag}{\Longrightarrow} BE_4^{(2N+3Nonly)}$$

$$\overset{3NF}{\Longrightarrow} \quad + V^{(3init)}_{3Nosc} \ldots$$

3D SRG Evolution with $T_{\rm rel}$ in a Jacobi HO Basis

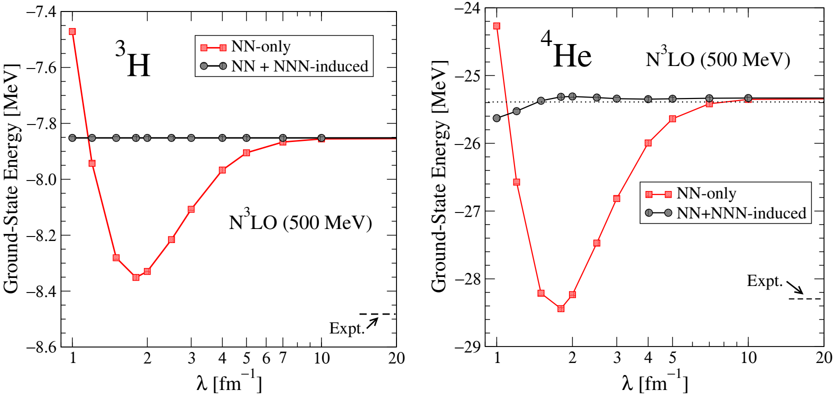
- Evolve in *any* basis [momentum space in progress by L. Platter]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - directly obtain SRG matrix elements in HO basis
  - separate 3-body evolution not needed
- Compare 2-body only to full $2+3$-body evolution:





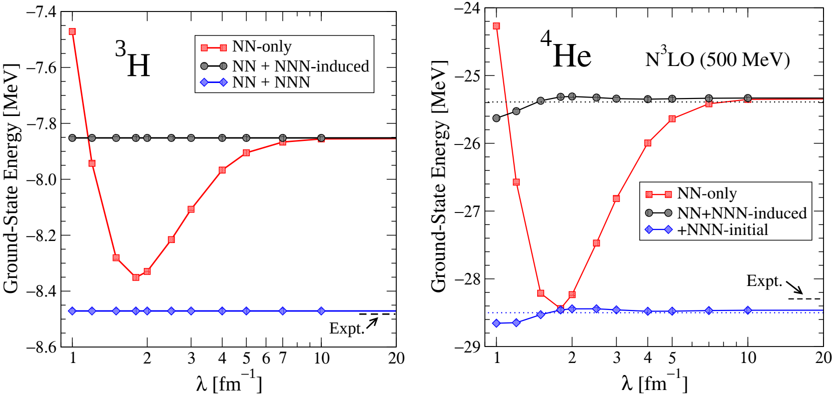
3D SRG Evolution with T_{rel} in a Jacobi HO Basis

- Evolve in *any* basis [momentum space in progress by L. Platter]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - directly obtain SRG matrix elements in HO basis
  - separate 3-body evolution not needed
- Compare 2-body only to full $2 + 3$-body evolution:



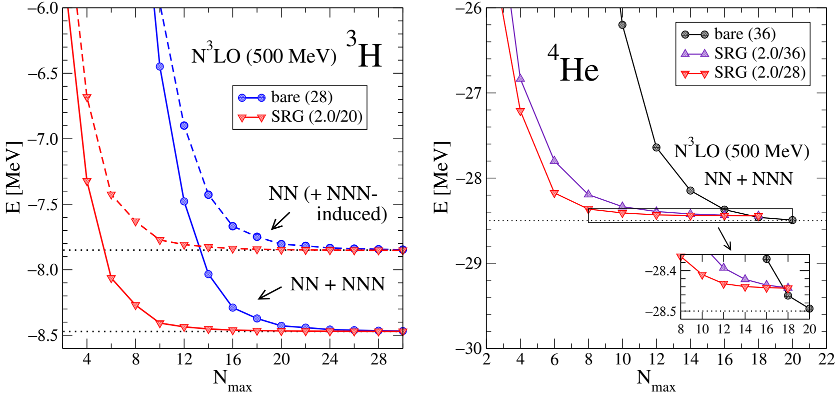
3D SRG Evolution with $T_{\rm rel}$ in a Jacobi HO Basis

- Evolve in *any* basis [momentum space in progress by L. Platter]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - directly obtain SRG matrix elements in HO basis
  - separate 3-body evolution not needed
- Compare 2-body only to full $2 + 3$-body evolution:



3D SRG Evolution with T_{rel} in a Jacobi HO Basis

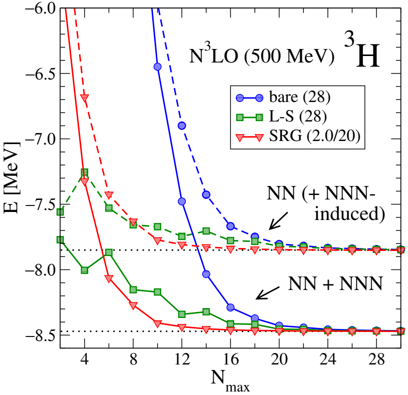
- Good convergence properties independent of 3-body:

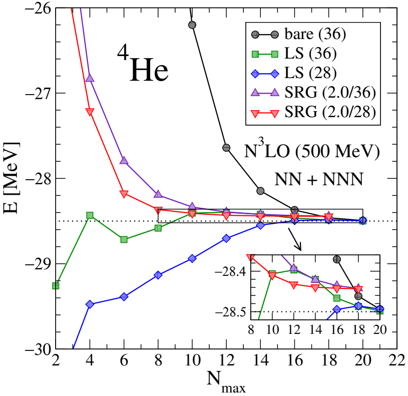


- HO matrix elements (to be) available for NCFC, CC, …
- Challenge: efficient (on-the-fly) conversion to m-scheme

3D SRG Evolution with T_{rel} in a Jacobi HO Basis

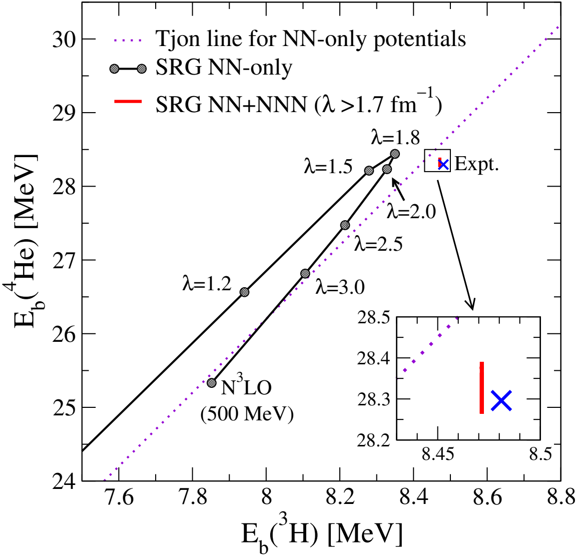
- Good convergence properties independent of 3-body:





- HO matrix elements (to be) available for NCFC, CC, …
- Challenge: efficient (on-the-fly) conversion to m-scheme

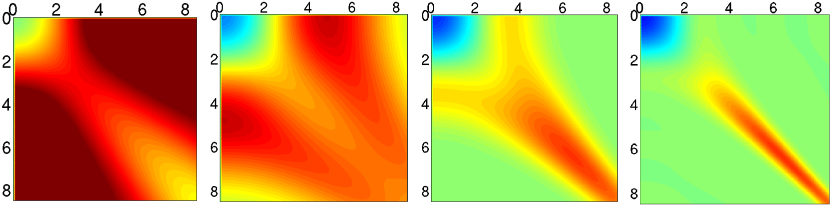
Tjon line revisited



Explore Using a One-Dimensional Model

- 1-D model [Negele et al.: Phys.Rev.C **39** 1076 (1989)]:

$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$

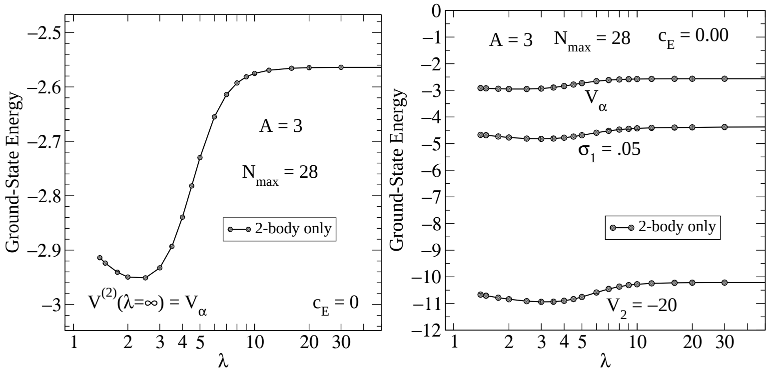


$\lambda = \infty$ $\quad$ $\lambda = 5$ $\quad$ $\lambda = 3$ $\quad$ $\lambda = 2$

- Same features as in 3D, but much easier!
- See E. Jurgenson, rjf, arXiv:0809.4199 for details

Induced Many-Body Forces: $A = 3$

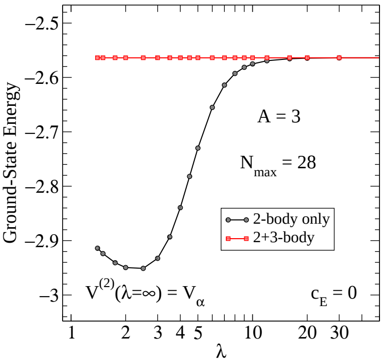
$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$

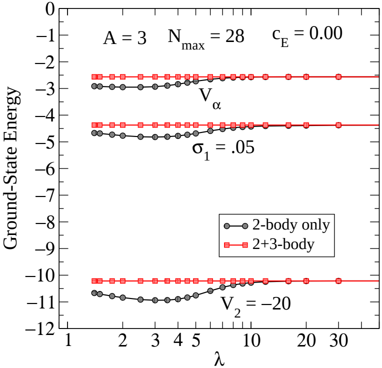


- Basis independent: same evolution in k or HO basis
- Black: Same evolution pattern for 2-body-only as 3D NN-only

Induced Many-Body Forces: $A = 3$

$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} + \frac{V_2}{\sigma_2\sqrt{\pi}} e^{-x^2/\sigma_2^2}$$

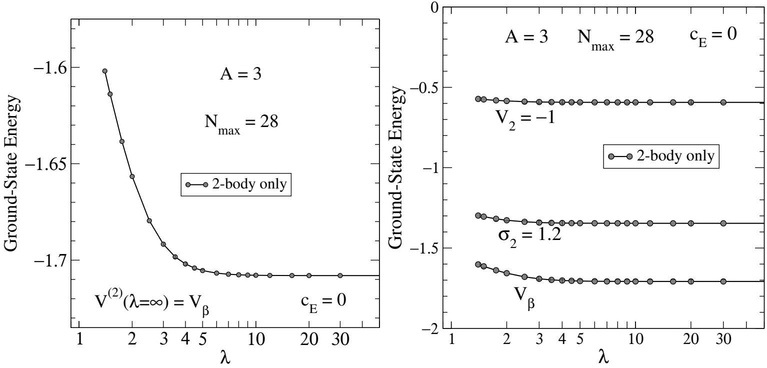




- Basis independent: same evolution in k or HO basis
- Black: Same evolution pattern for 2-body-only as 3D NN-only
- Red: Includes induced 3NF - Unitary!

Induced Many-Body Forces: $A = 3$

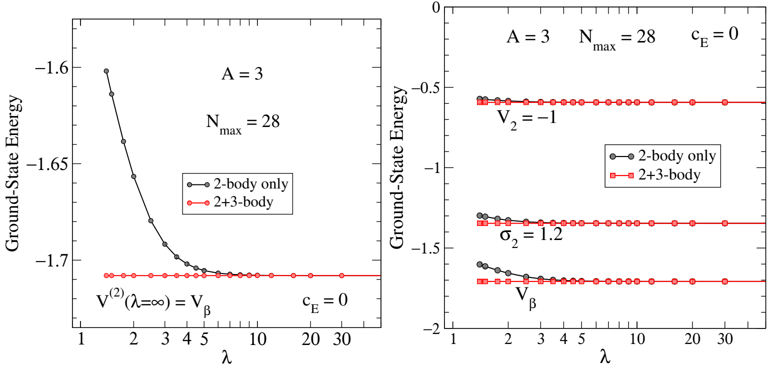
$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2}$$ attractive only



- Basis independent: same evolution in k or HO basis
- Black: Evolution pattern for attractive 2-body-only

Induced Many-Body Forces: $A = 3$

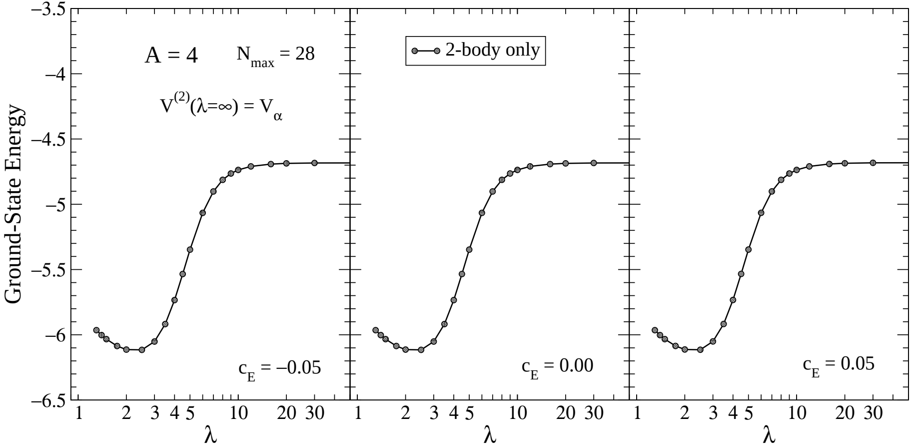
$$V^{(2)}(x) = \frac{V_1}{\sigma_1\sqrt{\pi}} e^{-x^2/\sigma_1^2} \qquad \text{attractive only}$$



- Basis independent: same evolution in k or HO basis
- Black: Evolution pattern for attractive 2-body-only
- Red: Includes induced 3NF - Unitary!

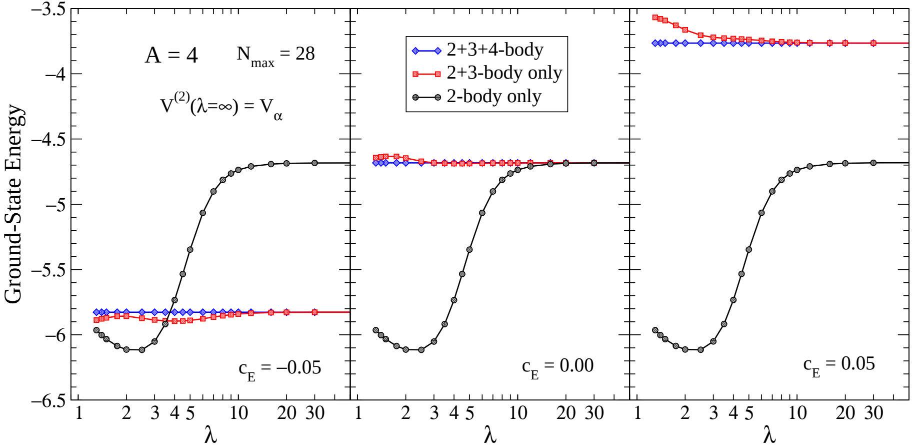
Induced Many-Body Forces: $A = 4$

$$V^{(3)}(p,q,p',q') = c_E e^{-((p'^2+q'^2)/\Lambda^2)^n} e^{-((p^2+q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4)$$

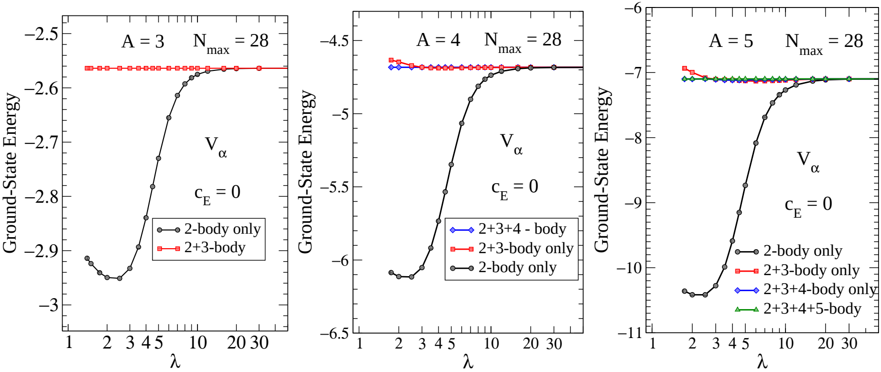


Induced Many-Body Forces: $A = 4$

$$V^{(3)}(p,q,p',q') = c_E e^{-((p'^2+q'^2)/\Lambda^2)^n} e^{-((p^2+q^2)/\Lambda^2)^n} \quad (\Lambda = 2 \quad n = 4)$$



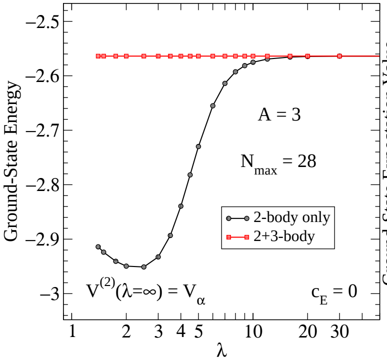
Induced Many-Body Forces: $A = 5$

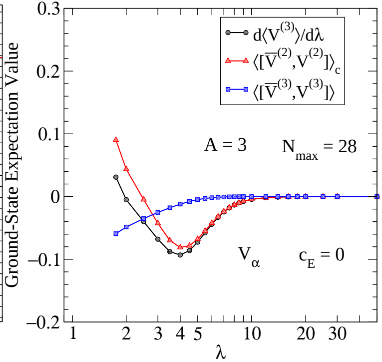


- Five-body force is negligible
- Hierarchy of induced many-body forces

$V^{(3)}$ analysis

$$\frac{d}{d\lambda}\langle\psi_\lambda^{(3)}|V_\lambda^{(3)}|\psi_\lambda^{(3)}\rangle = \langle\psi_\lambda^{(3)}|[\overline{V}_\lambda^{(2)}, V_\lambda^{(2)}]_c - [\overline{V}_\lambda^{(3)}, V_\lambda^{(3)}]|\psi_\lambda^{(3)}\rangle$$

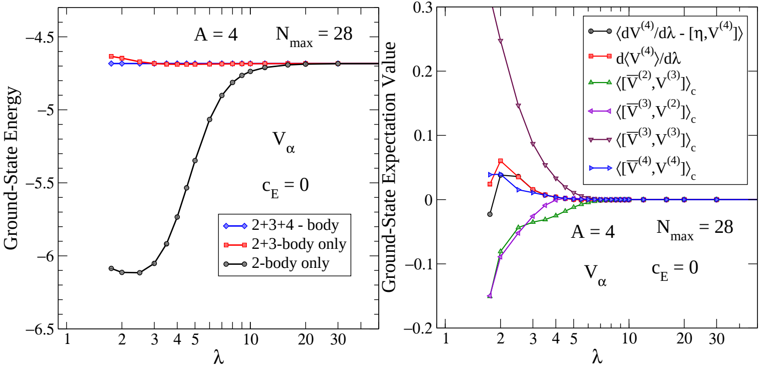




- Majority evolution dominated by $[\overline{V}^{(2)}, V^{(2)}]$, $(\overline{V} \equiv [T, V])$
- Hierarchy of contributions

$V^{(4)}$ analysis in $A = 4$

$$\frac{d}{d\lambda}\langle\psi_\lambda^{(4)}|V_\lambda^{(4)}|\psi_\lambda^{(4)}\rangle = \langle\psi_\lambda^{(4)}|[\overline{V}_\lambda^{(2)}, V_\lambda^{(3)}]_c + [\overline{V}_\lambda^{(3)}, V_\lambda^{(2)}]_c + [\overline{V}_\lambda^{(3)}, V_\lambda^{(3)}]_c - [\overline{V}_\lambda^{(4)}, V_\lambda^{(4)}]|\psi_\lambda^{(4)}\rangle$$



- No $[\overline{V}^{(2)}, V^{(2)}] \Longrightarrow$ Induced 4-body is small!
- Initial hierarchy of few-body forces is maintained

Outline

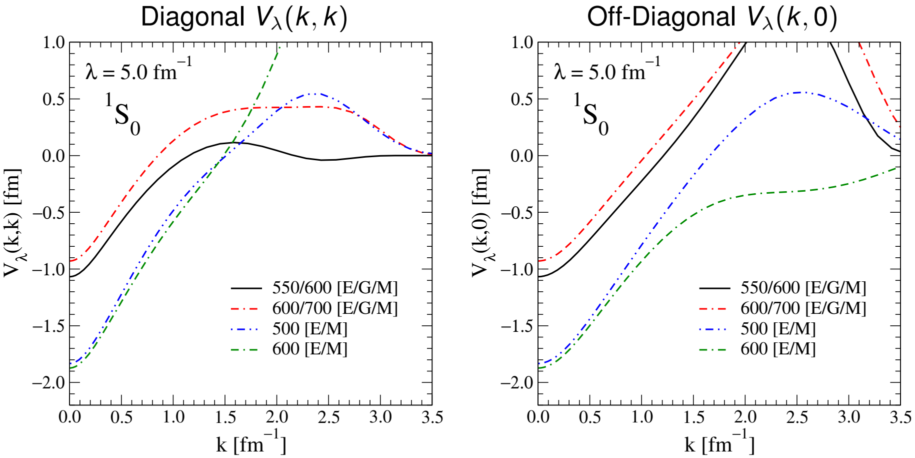
Prelude: Many-body operators from SRG

Context: Nuclear physics and three-body forces

Explicit running of three-body (and higher) interactions

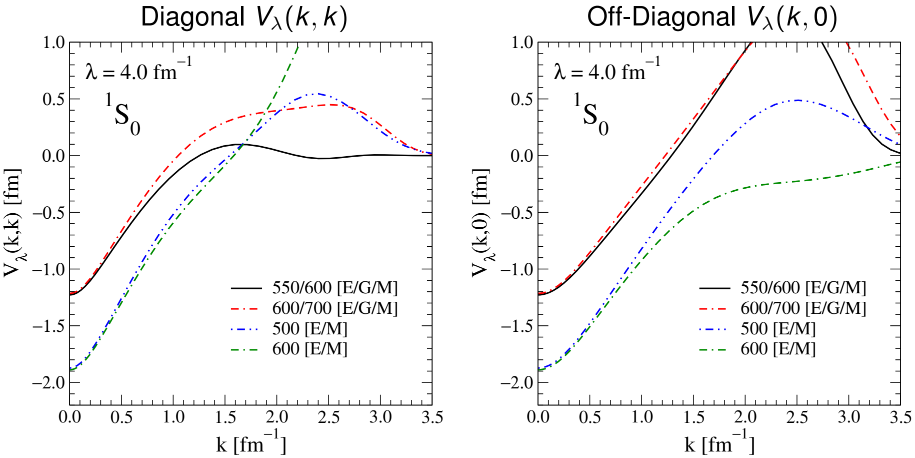
Open questions and issues

Run to Lower λ via SRG $\Longrightarrow$ ≈Universal



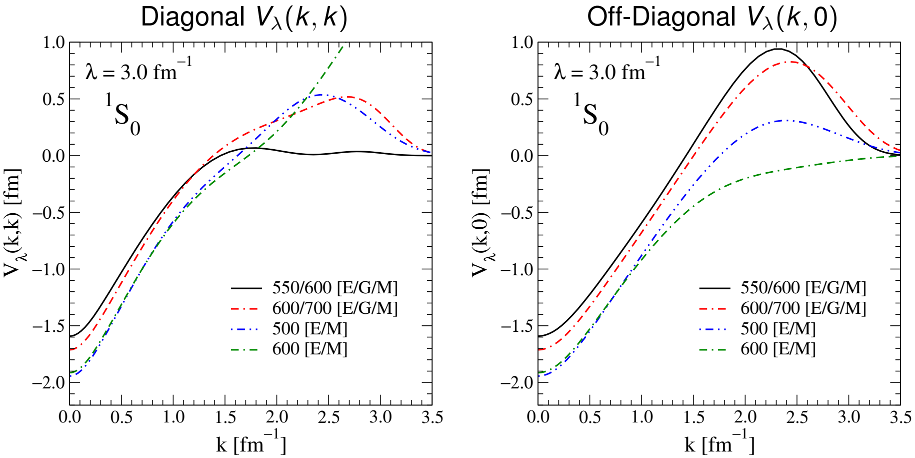
- Will evolved NNN interactions be universal?

Run to Lower λ via SRG $\Longrightarrow$ $\approx$Universal



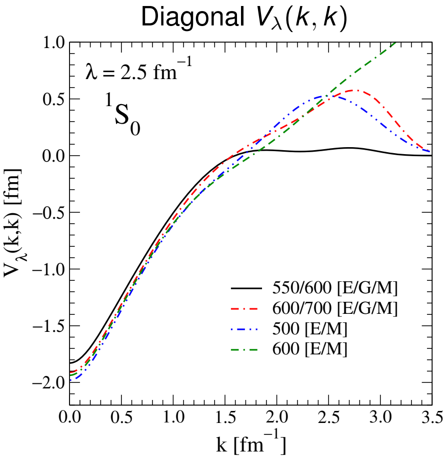
- Will evolved NNN interactions be universal?

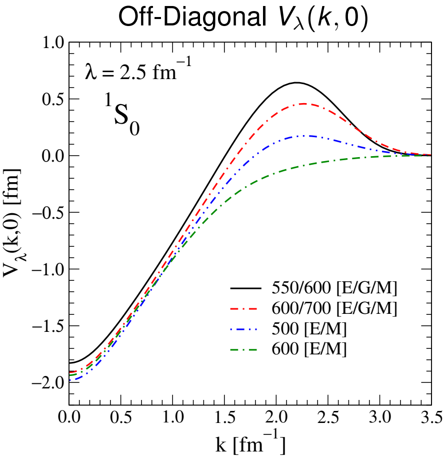
Run to Lower λ via SRG $\Longrightarrow$ ≈Universal



- Will evolved NNN interactions be universal?

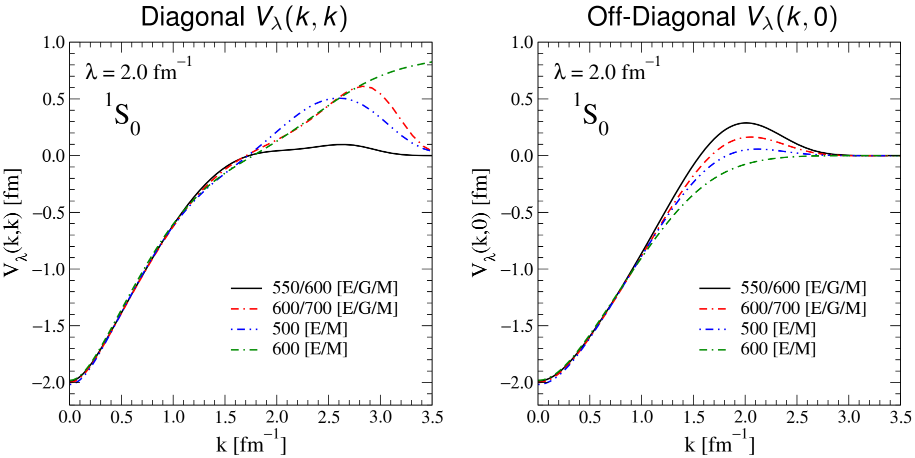
Run to Lower λ via SRG $\Longrightarrow$ ≈Universal





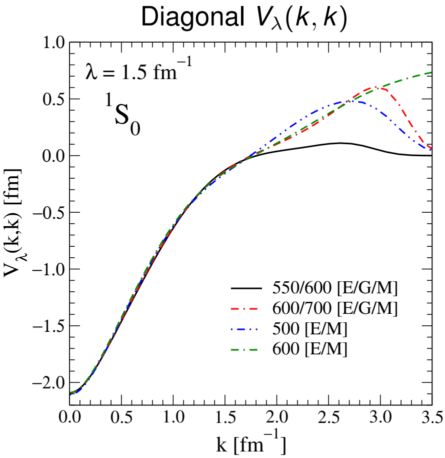
- Will evolved NNN interactions be universal?

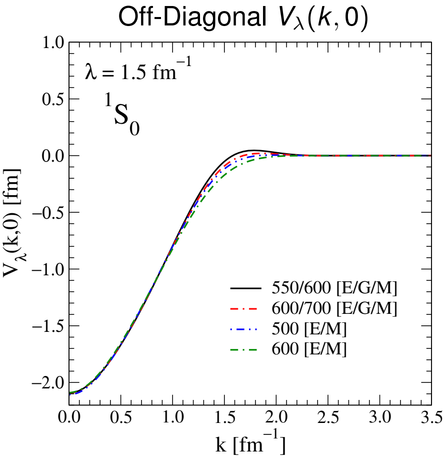
Run to Lower λ via SRG $\Longrightarrow$ $\approx$Universal



- Will evolved NNN interactions be universal?

Run to Lower λ via SRG $\Longrightarrow$ ≈Universal





- Will evolved NNN interactions be universal?

Every operator flows

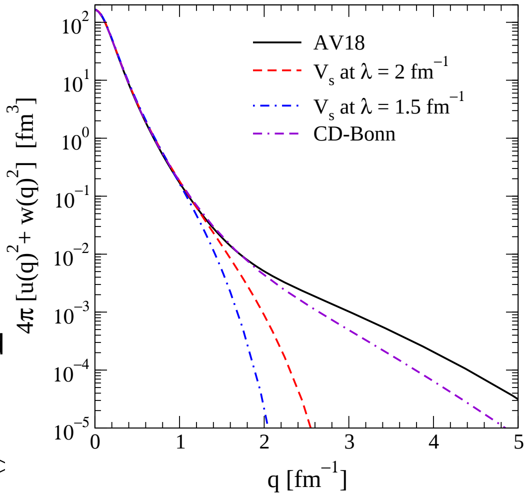
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

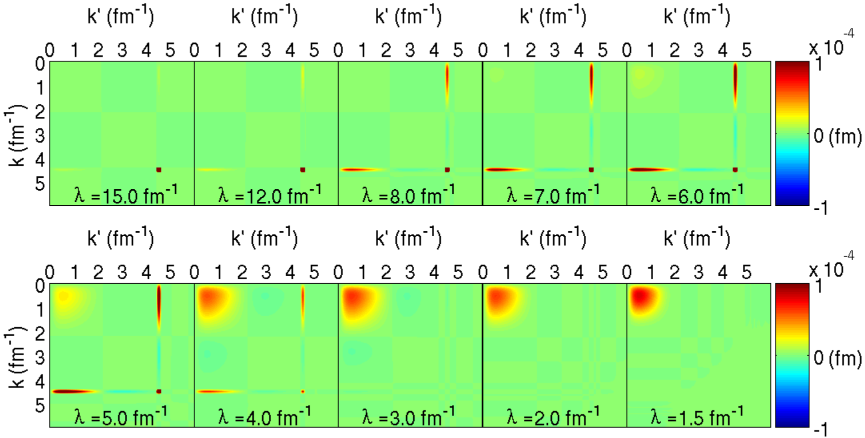
so O_s evolves via

$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi(0)_i\rangle\langle\psi(s)_i|$
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 4.5\,\mathrm{fm}^{-1}$



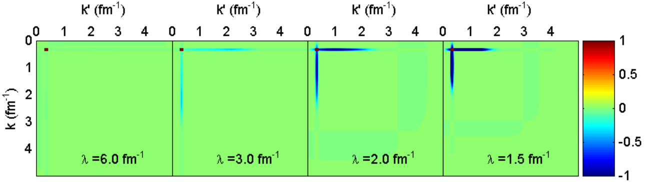
Integrand of $< \psi_d | U a_q^\dagger a_q U^\dagger | \psi_d >$ at $q = 4.5\,\text{fm}^{-1}$



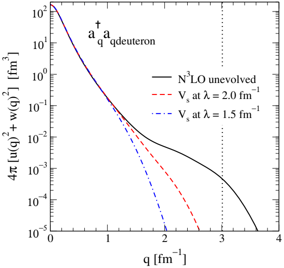
- Flow of deuteron matrix element integrand is toward low k
- Simple variational ansatz works well $\Longrightarrow$ No fine-tuning
- Factorization: $U(k,q) \longrightarrow K(k)Q(q)$ for $k \leq \lambda$, $q \gg \lambda$

High and low momentum operators in deuteron

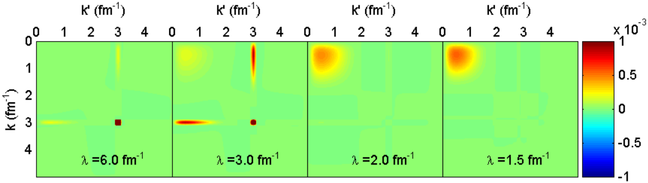
- **Integrand** of $\langle\psi_d|\,(Ua^\dagger_q a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



- Momentum distribution



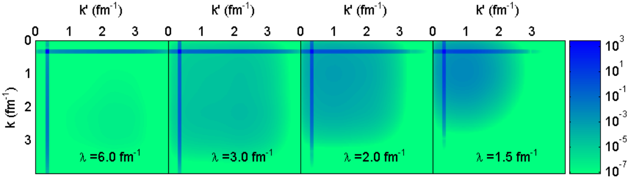
- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



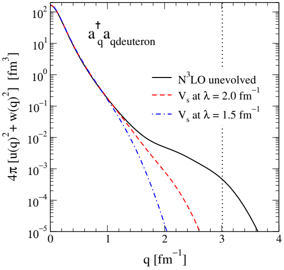
- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$

High and low momentum operators in deuteron

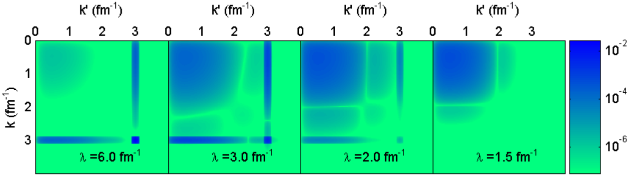
- **Integrand** of $\langle\psi_d|\,(U a^\dagger_q a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



- Momentum distribution



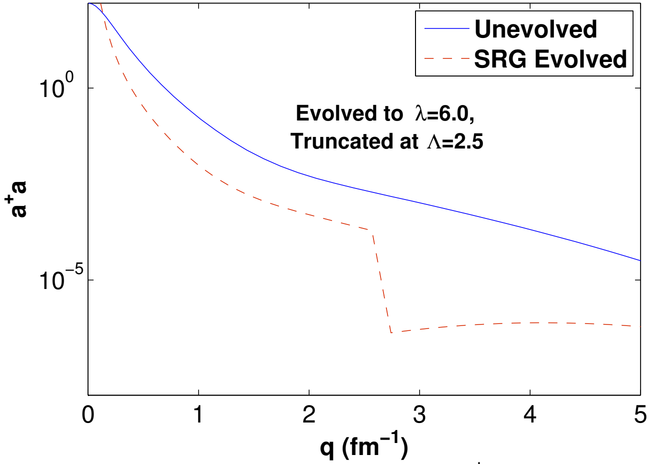
- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$

Decoupling in operator expectation values

- Evolve to λ in full space $\rightarrow$ **TRUNCATE at** $\Lambda = 2.5\,\mathbf{fm}^{-1}$:

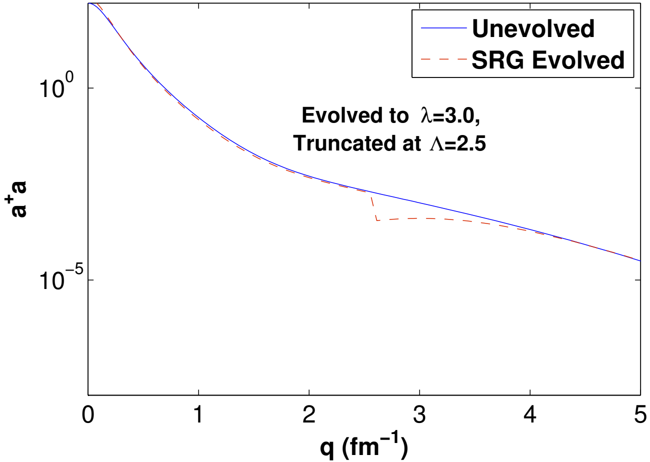


$\lambda = 6\,\text{fm}^{-1}$

- Here: momentum distribution $a_q^\dagger a_q$
- Decoupling for all q works when $\lambda < \Lambda$

Decoupling in operator expectation values

- Evolve to λ in full space $\rightarrow$ **TRUNCATE at** $\Lambda = 2.5\,\textbf{fm}^{-1}$:

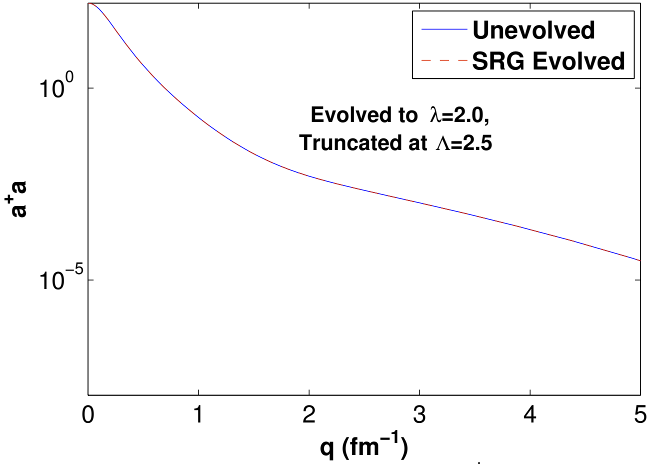


$\lambda = 3\,\text{fm}^{-1}$

- Here: momentum distribution $a_q^\dagger a_q$
- Decoupling for all q works when $\lambda < \Lambda$

Decoupling in operator expectation values

- Evolve to λ in full space $\rightarrow$ **TRUNCATE at** $\Lambda = 2.5\,\mathbf{fm}^{-1}$:



$\lambda = 2\,\text{fm}^{-1}$

- Here: momentum distribution $a^\dagger_q a_q$
- Decoupling for all q works when $\lambda < \Lambda$

Factorization

- If $k < \lambda$ and $q \gg \lambda \Longrightarrow$ factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$?
- Operator product expansion for nonrelativistic wf's (Lepage)

$$\Psi_{true}(r) = \overline{\gamma}(r) \int d\mathbf{r}'\, \Psi_{eff} \delta_a(\mathbf{r}') + \overline{\eta}(r) a^2 \int d\mathbf{r}'\, \Psi_{eff} \nabla^2 \delta_a(\mathbf{r}') + \mathcal{O}(a^4)$$

- Similarly, in momentum space

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi_\alpha^\lambda(p) + \cdots$$

- By projecting potential in momentum subspace, recover OPE via:

$$\gamma^\lambda(q) \equiv -\int_\lambda^\infty q'^2 dq'\, \langle q| \frac{1}{\widehat{Q}_\lambda H^\infty \widehat{Q}_\lambda} |q'\rangle V^\infty(q', 0)$$

$$\eta^\lambda(q) \equiv -\int_\lambda^\infty q'^2 dq'\, \langle q| \frac{1}{\widehat{Q}_\lambda H^\infty \widehat{Q}_\lambda} |q'\rangle \frac{\partial^2}{\partial p^2} V^\infty(q', p)|_{p^2=0}$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle \langle \psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p)\Big] \gamma^\lambda(q) + \cdots$$

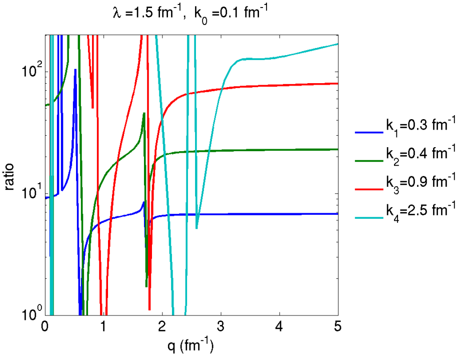
Numerical Factorization

- Test of factorization of U:

$$\frac{U_\lambda(k_i, q)}{U_\lambda(k_0, q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)}$.

- Look for plateaus for $q \gtrsim 2\text{fm}^{-1}$



- Singular value decomposition
  - quantitatively analyze the extent to which U factorizes
  - outer product expansion

$$G = \sum_i^r d_i \mathbf{u}_i \mathbf{v}_i^t$$

where r is the rank and the d_i are decreasing singular values

- Example: results for $\lambda = 2\,\text{fm}^{-1}$, for $q > \lambda$ and $k < \lambda$

	$^1S_0$		
Potential	d_1	d_2	d_3
AV18	0.763	0.033	0.007
N3LO 500 MeV	1.423	0.221	0.015
N3LO 550/600 MeV	3.074	0.380	0.061
	$^3S_1$–$^3S_1$		
AV18	0.671	0.015	0.008
N3LO 500 MeV	1.873	0.225	0.044
N3LO 550/600 MeV	4.195	0.587	0.089

Practical use of factorization

- Decoupling implies

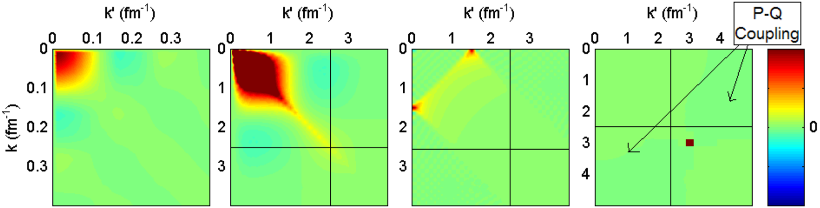
$$\langle\psi_\lambda|\, U_\lambda \widehat{O}\, U_\lambda^\dagger\, |\psi_\lambda\rangle \cong \int_0^\lambda dk' \int_0^\infty dq' \int_0^\infty dq \int_0^\lambda dk\; \psi_\lambda^\dagger(k') U_\lambda(k',q') \widehat{O}(q',q) U_\lambda^\dagger(q,k) \psi_\lambda(k)$$

- Factorization: set $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$, where $k < \lambda$ and $q \gg \lambda$

$$\Longrightarrow \int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \left[\int_0^\lambda \int_0^\lambda \underbrace{U_\lambda(k',q')\widehat{O}(q',q)U_\lambda^\dagger(q,k) + I_{QOQ}\, K_\lambda(k')K_\lambda(k)}_{\textit{Low Momentum Structure}} \right] \psi_\lambda(k)$$

where $I_{QOQ} \equiv \int_\lambda^\infty dq' \int_\lambda^\infty dq \left[Q_\lambda(q') \widehat{O}(q',q) Q_\lambda(q) \right]$ ← *Universal*

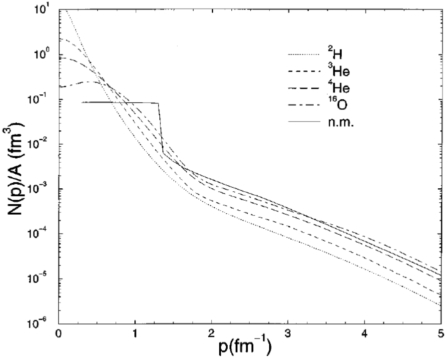
– Valid when initial operators weakly couple high and low momentum:



r^2 $1/r$ $G_C(q = 3.02\,\mathrm{fm}^{-1})$ $a_q^\dagger a_q(q = 3.02\,\mathrm{fm}^{-1})$

Factorization in few-body nuclei: $n(k)$ at large k

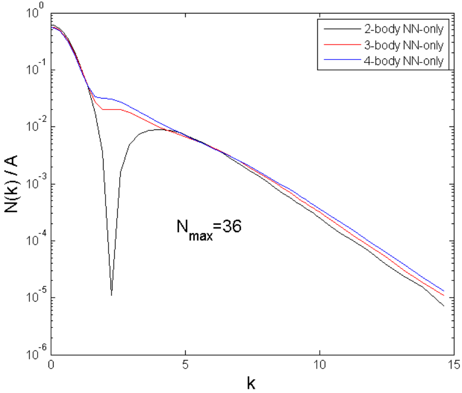
- AV14 NN with VMC



From Pieper, Wiringa, and Pandharipande (1992).

- Conventional explanation: Dominance of NN potential and short-range correlations (Frankfurt et al.)

- *A* bosons in 1D model

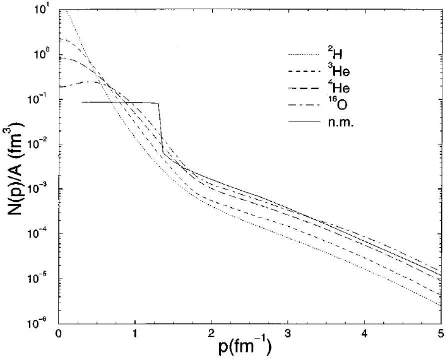


- Alternative: *factorization*

$$\int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \left[I_{QOQ} K_\lambda(k') K_\lambda(k) \right] \psi_\lambda(k)$$

Factorization in few-body nuclei: $n(k)$ at large k

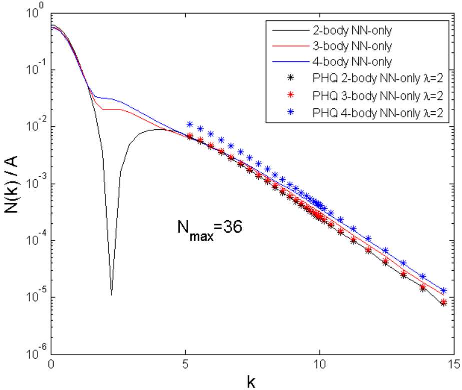
- AV14 NN with VMC



From Pieper, Wiringa, and Pandharipande (1992).

- Conventional explanation: Dominance of NN potential and short-range correlations (Frankfurt et al.)

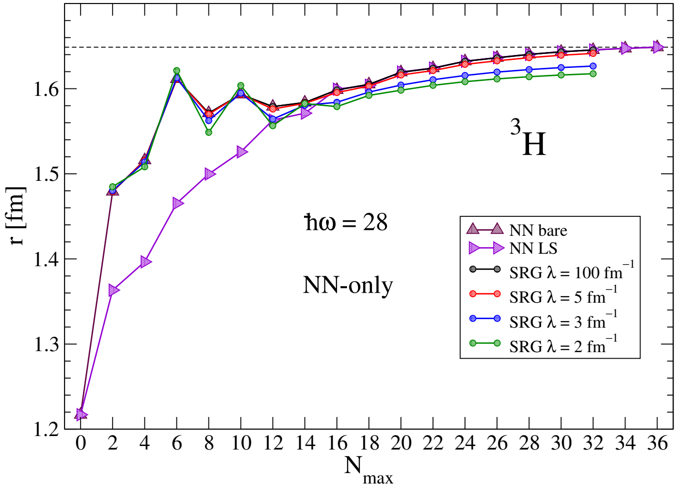
- *A* bosons in 1D model



- Alternative: *factorization*

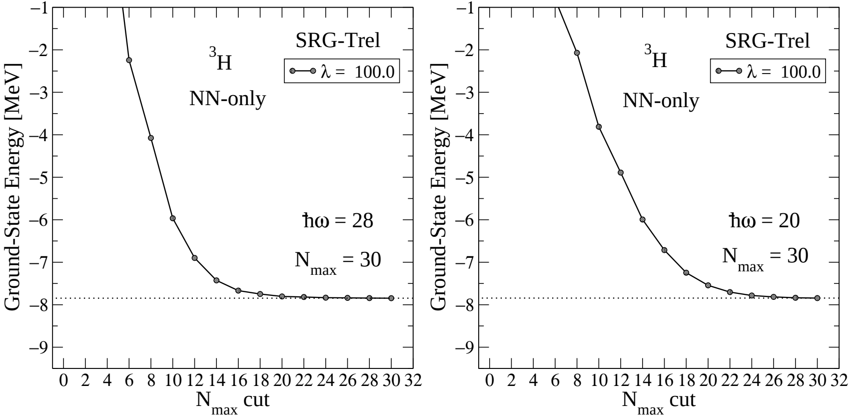
$$\int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \left[I_{QOQ} K_\lambda(k') K_\lambda(k) \right] \psi_\lambda(k)$$

Long-distance observables: radius



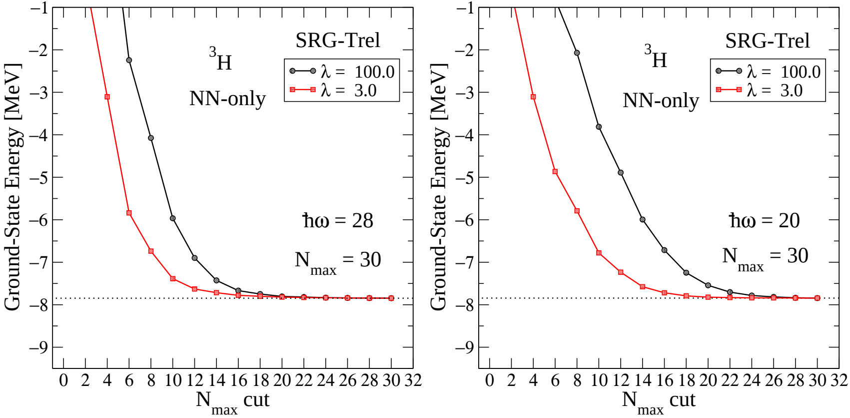
- Unevolved operator
- Harmonic oscillator basis is problematic!

Evolving NN forces in NCSM A=3 space



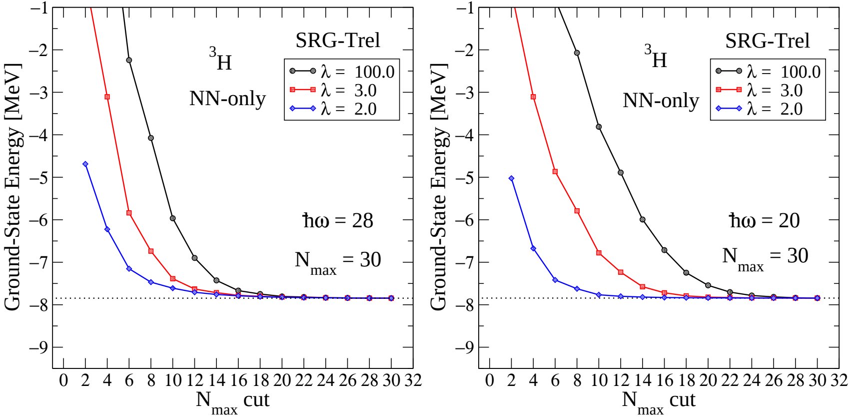
- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

Evolving NN forces in NCSM A=3 space



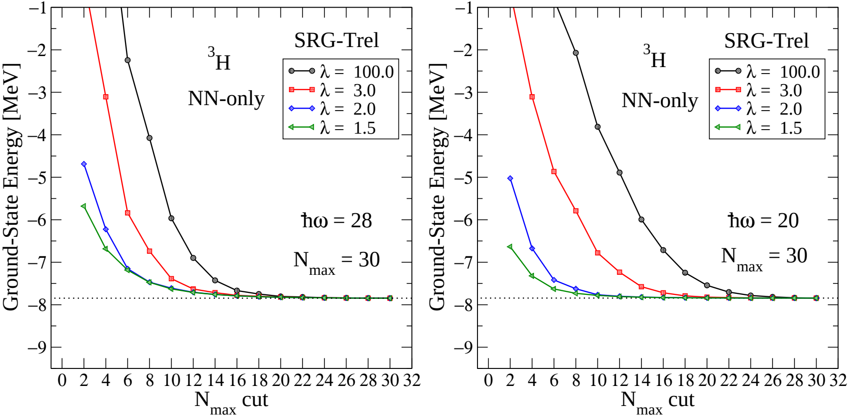
- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

Evolving NN forces in NCSM A=3 space



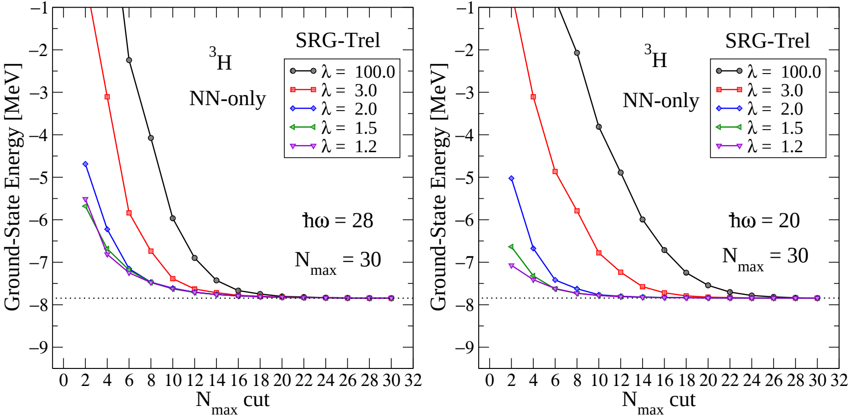
- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

Evolving NN forces in NCSM A=3 space



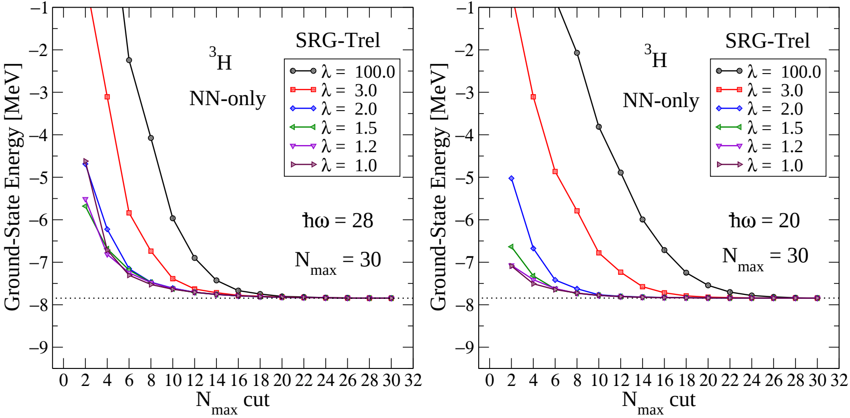
- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

Evolving NN forces in NCSM A=3 space



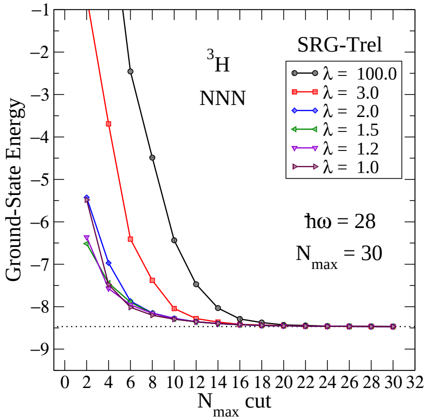
- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

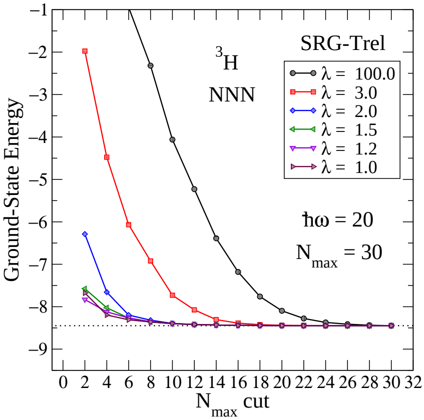
Evolving NN forces in NCSM A=3 space



- $\hbar\omega = 28$ is optimal for the bare interaction
- $\hbar\omega = 20$ is optimal for $\lambda = 2$ evolution
- No improvement in convergence for small λ

Evolving NN + NNN in NCSM A=3 space

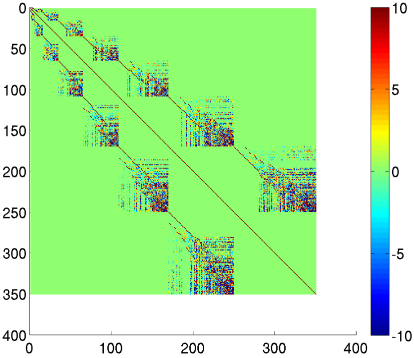


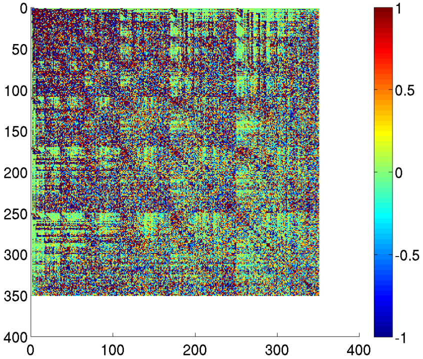


- Same plots but now including an initial 3NF from N2LO
- No improvement in convergence for small λ

Using other SRG Generators

- 1D matrices $T_{\rm rel}$ and V in NCSM basis:

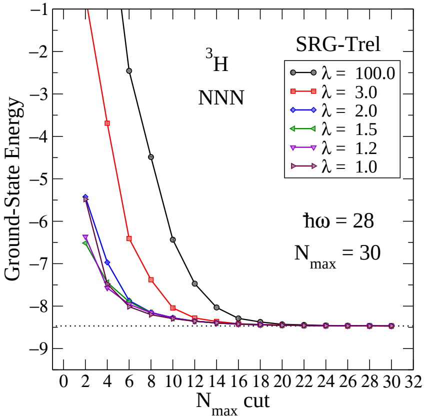


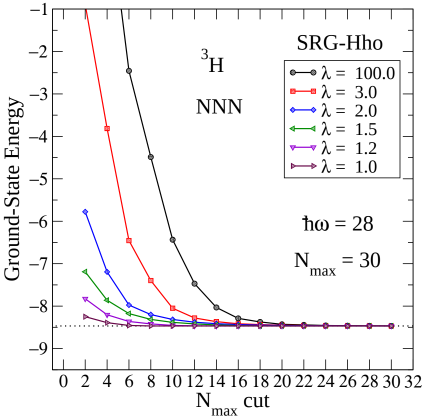


- In this basis T_{rel} will not drive H to diagonal form
- But harmonic oscillator Hamiltonian will!
  - $H_{\rm ho} = T_{\rm rel} + V_{\rm ho}$ is diagonal in this basis

Evolving with H_{ho} in HO Basis

- Using $G = H_{\mathrm{ho}}$ improves convergence dramatically





- Compare T_{rel} on the left with H_{ho} on the right
- But: 1D study indicates spurious bound states contaminate evolution with H_{ho} with many-body truncation

Summary of open questions and issues

- Power counting for evolved many-body interactions
- Do many-body interactions flow to universal form?
- Operator issues
  - Scaling of many-body operators
  - Factorization for few-body systems
- Can choices for G_s ...
  - reduce the many-body forces?
  - improve convergence in HO basis?