Introduction to recent RG studies for QCD – and beyond

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Outline

- Introduction to lattice QCD and its RG roots
- The Schrödinger functional – running coupling from volume dependence
- Beyond QCD – lattice models of technicolor

QCD and Lattice QCD

QCD Lagrangian: quarks and gluons

$$\mathcal{L}_{QCD} = \sum_j [\bar{\psi}_j(\gamma^\mu(i\partial_\mu - gA_\mu^a) + m_j]\psi_j - \frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} \quad (1)$$

Modern era began in 1973 with perturbative calculation of beta function

$$\beta(g^2) = \frac{dg^2}{d\log(\mu^2)} = \frac{b_1}{16\pi^2}g^4 + \ldots \quad (2)$$

$$b_1 = -\frac{11}{3}N_c + \frac{2}{3}N_f < 0$$

- $1/\alpha(q) = -\frac{b_1}{4\pi}\log q^2/\Lambda^2$ – Effective coupling is weak at short distance, stronger at long distance
- Explains "scaling" in deep-inelastic scattering
- "Color" ($N_c = 3$) explains regularities in spectroscopy
- Discovery of heavy flavors (1974, 1977) gave NR confining systems
- Where does confinement fit into this story?

Lattice QCD

- Formulated by Wilson, 1974
- Introduce lattice spacing a (=UV cutoff)
  - Quark fields defined on sites of lattice
  - Gauge fields defined on links of lattice, $U_\mu(x)$ = group element of $SU(N)$
  - Lattice action a discretized version of $\mathcal{L}$, ex. $S = \beta \mathrm{Tr} U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu(x+\hat{\nu}) U_\nu(x)$
    - Preserving local gauge invariance (sum of traces of loops of U's)
    - Not respecting continuum space-time symmetries
- Large-a strong coupling limit is confining (confinement = disorder)

$$Z = \int [d\phi] \exp(-S(\phi)) \tag{3}$$

$$\langle O \rangle = \frac{1}{Z} \int [d\phi] O(\phi) \exp(-S(\phi)) \tag{4}$$

- In finite volume, Z becomes a multidimensional integral
- Monte Carlo (Creutz ++, 1979): Generate $\phi_1 \rightarrow \phi_2 \rightarrow \ldots$ by Markov chain

$$\langle O \rangle = \frac{1}{N} \sum_{j=1}^{N} O(\phi_j) \tag{5}$$

Lattice QCD – how it's done today

- Begin with some discretization of $\mathcal{L}_{QCD}$
- Input dimensionless coupling constant g^2 plus am_q
- "a" is an output parameter – you may not know where you are
- Figure out "a", measure $m_H a \to m_H$

In early days, use pert RG to relate a to $g(a)$, compute m/Λ via

$$a\Lambda = \left(\frac{16\pi^2}{b_1 g^2(a)}\right)^{b_2/(2b_1^2)} \exp\left(-\frac{8\pi^2}{b_1 g^2(a)}\right). \tag{6}$$

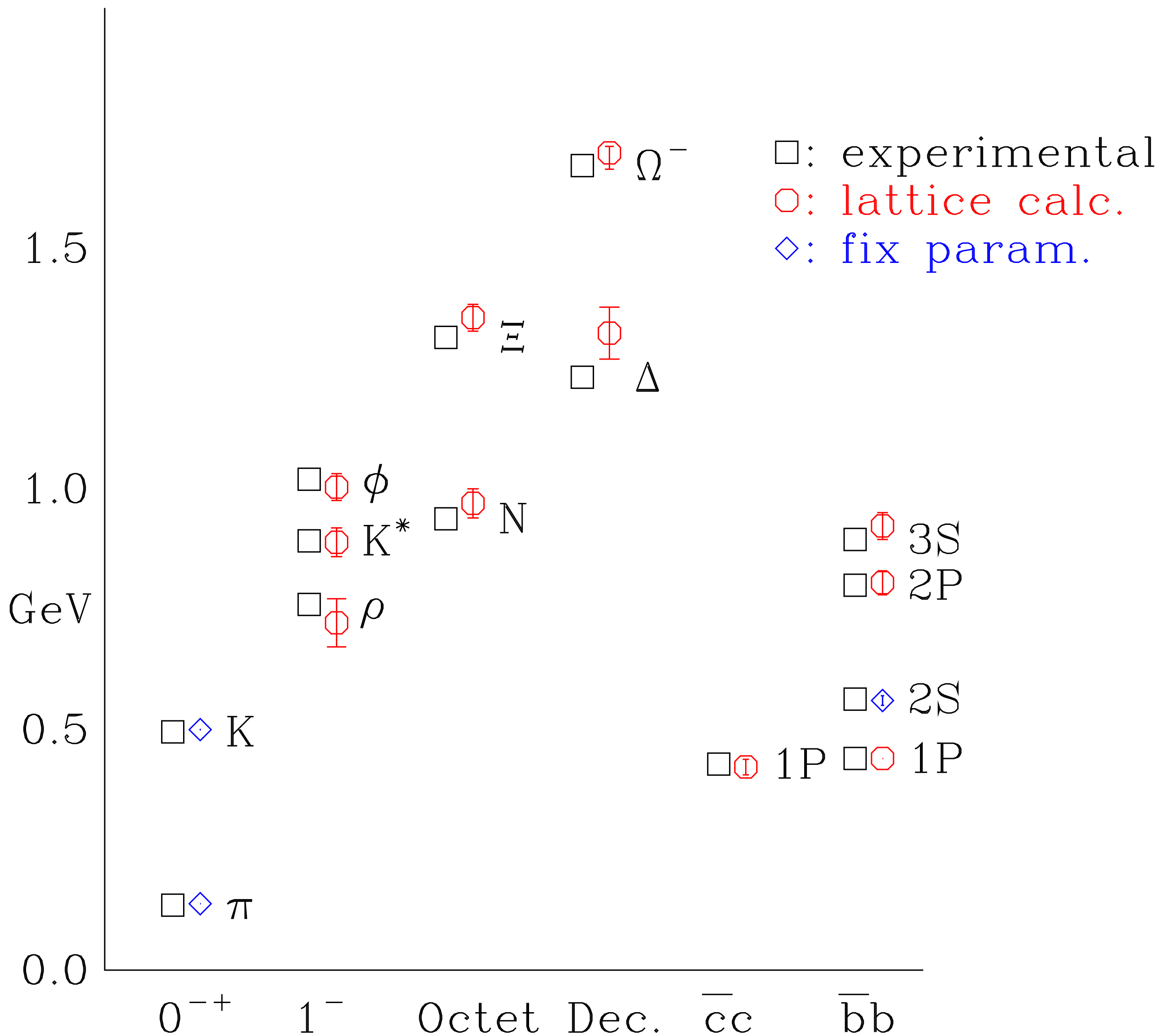
Not so useful/unambiguous. Instead, just look for "scaling"

$$[am_1(a)]/[am_2(a)] = m_1(0)/m_2(0) + \mathcal{O}(m_1 a) + \mathcal{O}[(m_1 a)^2] + \ldots \tag{7}$$

- Universality as $a \to 0$; $g(a) \to 0$
- No proof of confinement, but strong and weak coupling regions connected

Recent development: faster computers, improved algorithms

Simulations with light dynamical fermions have made lattice QCD a precision undertaking



Milc collab 2009

Physics/simulation issues

1) QCD has a scale associated with confinement, call it R_H

- Need $L >> R_H >> a$ and NEVER get $>>$

Need to find a useful window of bare couplings and L for confinement physics

2) QCD has massless particles, too

- Pions are Goldstone bosons of spontaneously broken chiral symmetry, $m_\pi^2 \propto m_q$
- Lots of chiral PT
- $m_q \rightarrow 0$ is expensive
- Issues with $m_\pi L \sim 1$

Need to find a useful window in m_q and L for chiral physics

3) Universality (Do answers depend on discretization?)

Lattice QCD summary

- RG ideas, more than specific RG methodology, underlie lattice calcs
- Effective field theory story also heavily exploited
  - S_{latt} = action at cutoff scale $p \sim 1/a$
  - $S_{latt} = S_{continuum} + O(a^2)$ irrelevant operators
  - Predictions at $p << 1/a$ should be QCD predictions
- Trust but verify: lots of work to demonstrate this
- RG/EFT-based "lattice action design"

The Schrödinger Functional

- Goal: Nonperturbative def'n of α, which heals to PT – used to predict Λ
- Designed for (and used mostly for) asymptotically free theories
  - $d = 2$ O(N) $\sigma-$ model
  - $d = 4$ pure YM, QCD
- Basically background field method for lattice in box of size L^4
- Boundary conditions for fields depend on parameter η

$$Z = \int_{\eta-boundaries} [d\phi] \exp(-\frac{1}{g^2} S(\phi)) \qquad (8)$$

- Classical action depends on η
- $\Gamma_{cl} = -\log Z_{cl} = g^{-2} S^{cl}$
- Promote this to $\Gamma = -\log Z = g(L)^{-2} S^{cl}$
- Classically, $\frac{\partial \Gamma}{\partial \eta}|_{\eta=0} = \frac{K}{g^2}$
- $\langle \frac{\partial \Gamma}{\partial \eta}|_{\eta=0} \rangle =$messy lattice operator on edge of box, measured in a simulation $\equiv \frac{K}{g^2(L)}$

The Schrödinger Functional – Running

Simulate at same bare parameters on volumes L_0 and sL_0, compute the change in the coupling

Interpret as integrated beta function

$$\beta(g) = -L\frac{dg^2}{dL}, \tag{9}$$

$$-\int_{L_0}^{sL_0} \frac{dL}{L} = \int_{g^2(L_0)}^{g^2(sL_0)} \frac{dg^2}{\beta(g^2)} \equiv \int_{u}^{\sigma(s,u)} \frac{dv}{\beta(v)}, \tag{10}$$

Issues:

- Artifacts in sims go as $O(a^2/L^2)$ so vary a/L, tune bare params to fix $g^2(L)$, check $g^2(sL)$
- "Daisy chain" $L \rightarrow sL \rightarrow s^2L \rightarrow \ldots$ for running over large range of scales
- Fix overall scale from energy observable at one bare coupling
- Match to $\overline{MS}$ deep in weak coupling
- Predict $\alpha_s(M_Z)$ or $\Lambda = 245$ MeV in terms of a low energy observable

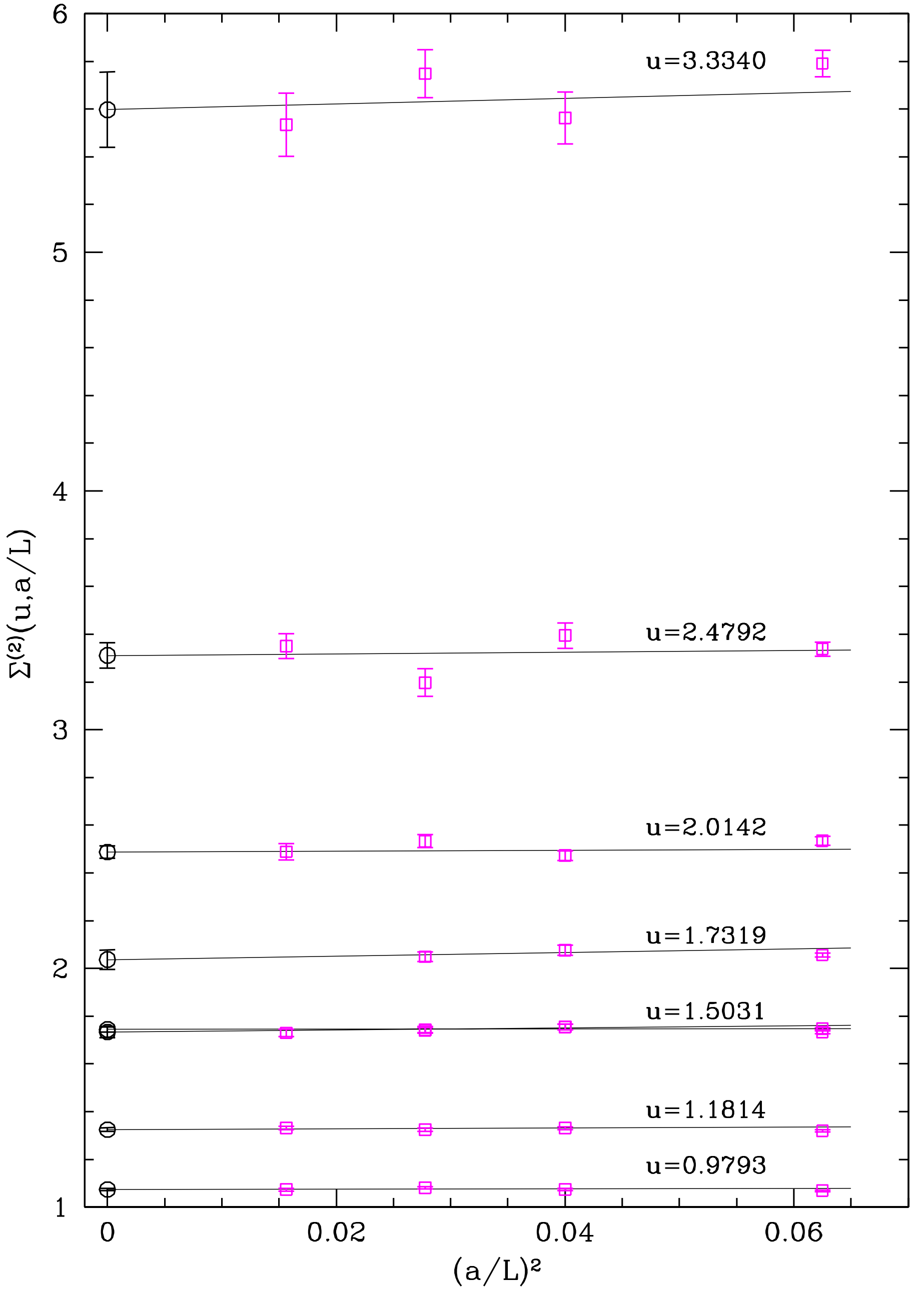
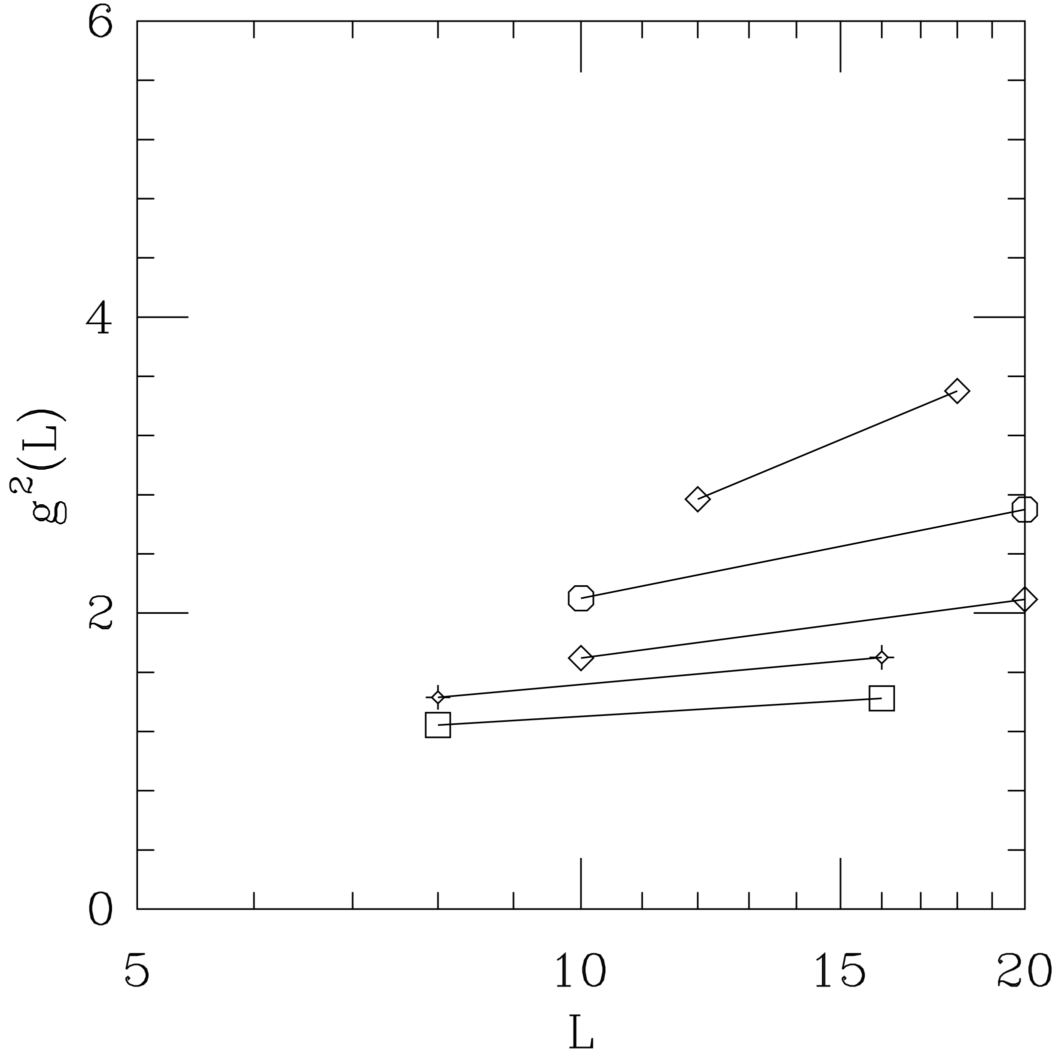
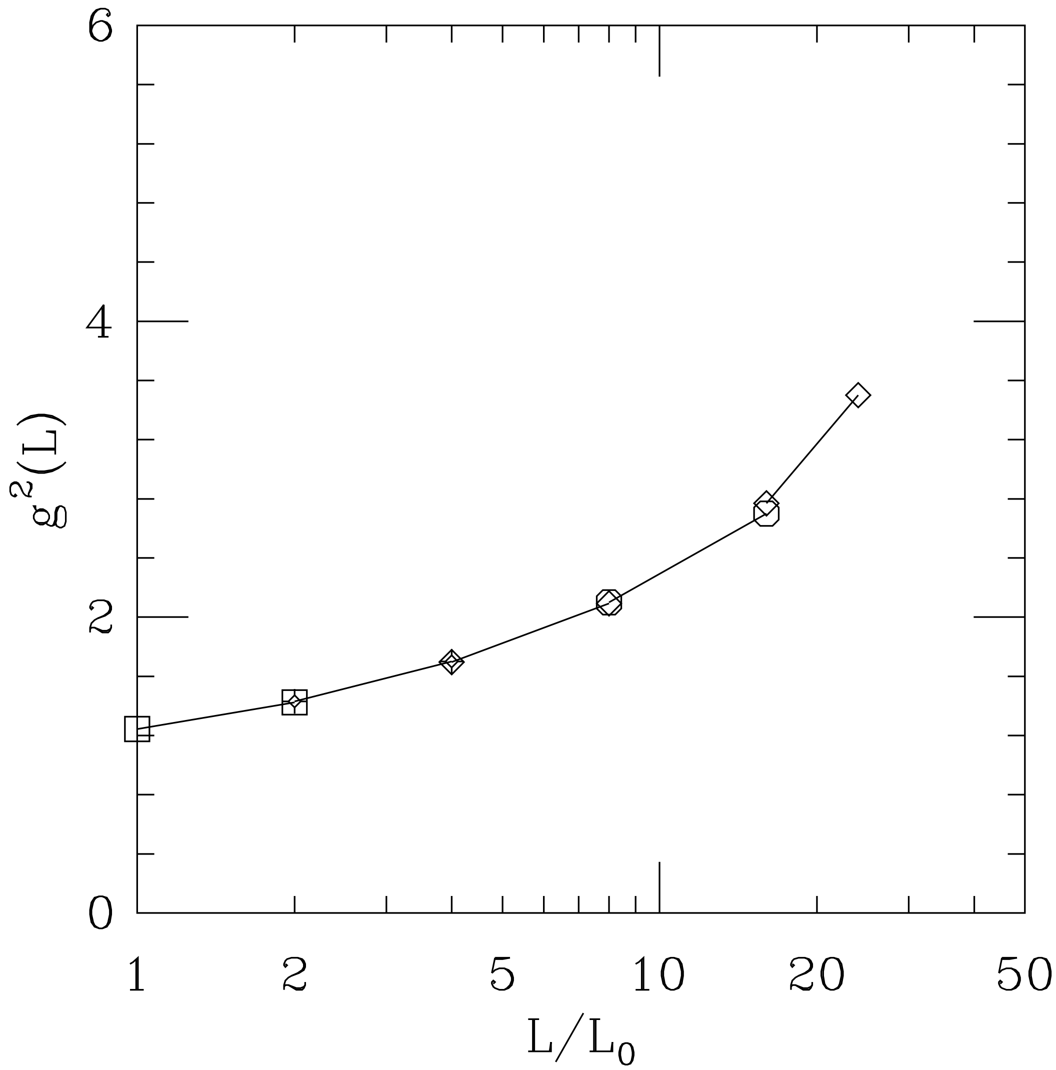


Figure 1: a/L test for $N_f = 2$ QCD running coupling (della Morte et al, NP B713(2005) 378)



Quenched QCD: “connect the lines” to see the coupling constant run (note slope!)



Connecting the lines to see running over a wide range of scales

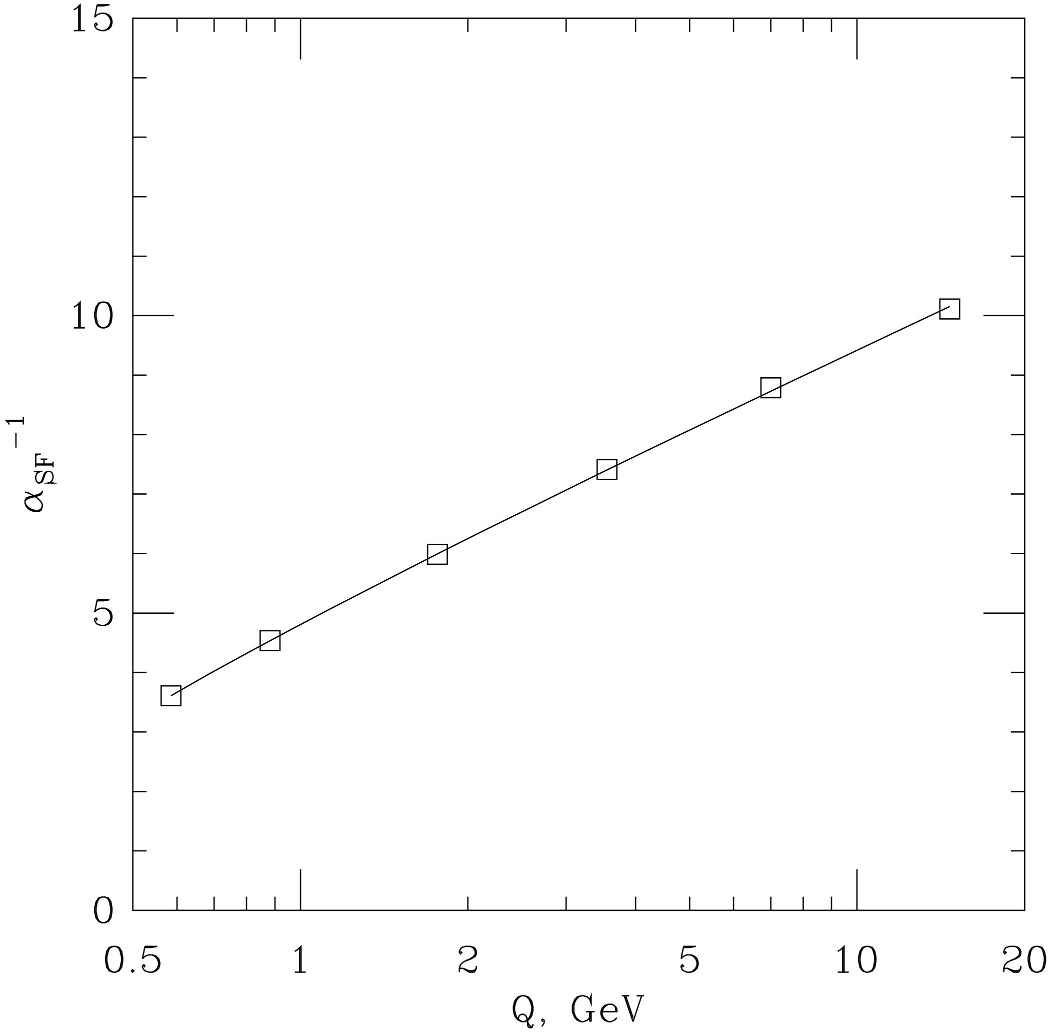


Figure 2: The pure gauge $SU(3)$ coupling constant from the Schrödinger functional method (Lüscher et al, 1993), with superimposed three-loop prediction.

Positives and Negatives

Positives:

- Boundary conditions permit simulations at $m_q = 0$
- Straightforward (perturbative) scheme matching to continuum regularization
- Allows precision calculation of Λ parameter in continuum regularization scheme
- Cumbersome but straightforward extension to other anomalous dimensions

Negatives

- Simulations turn out to be noisy
- Choice of RG is choice of boundary conditions – before the simulation begins
- Even coupling itself is not unique – only 2 loop β function is scheme independent

Several other recent related methods (objects of size pL in boxes of size L) under development

Beyond QCD

In Standard Model, Higgs boson is a fundamental field

Long standing desire to replace fundamental Higgs field by something more "natural"

- Analogy with superconductivity
- Hierarchy or Naturalness problem – quadratic fine tuning of Higgs mass $m_H^2 = \mu^2 + \Lambda^2$
- Triviality problem: Higgs coupling grows without bound into UV

One possible scenario – "technicolor" –

- New strongly interacting sector, with new fermons and gauge fields
- Higgs $v \rightarrow \langle \bar{Q} Q \rangle$
- $W - \pi - W$ coupling $\rightarrow M_W^2 = (\frac{g_2}{2})^2 f_\pi^2$
- A tower of new excitations (techni-particles)...
- A long history (since 1977) based on semi-analytic methods
- Not favored by precision electroweak measurements – maybe!

Perturbative analysis

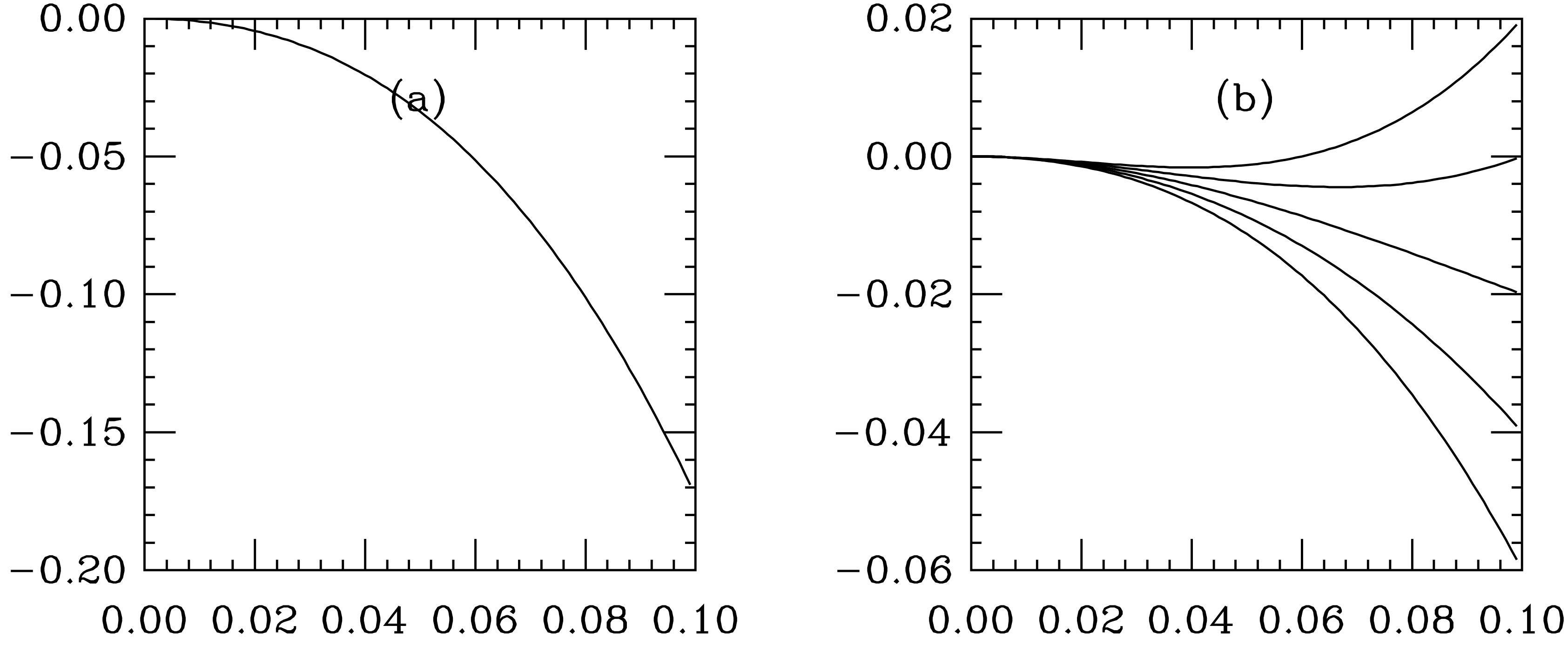
$$\beta(g^2) = \frac{dg^2}{d\log(\mu^2)} = \frac{b_1}{16\pi^2} g^4 + \frac{b_2}{(16\pi^2)^2} g^6 + \ldots \tag{11}$$

$$b_1 = -\frac{11}{3} N_c + \frac{4}{3} N_f T(R)$$

$$b_2 = -\frac{34}{3} N_c^2 + N_f T(R)(\frac{20}{3} N_c + 4C_2(R))$$

- For large enough $N_f T(R)$, $b_1 > 0$: trivial theory
- For small enough $N_f T(R)$, b_1 and $b_2 < 0$: QCD-like theory? "Classical" TC
  - This is "technicolor:" techni-pions eaten by W's, techni-particles as new physics
  - Phenomenology wants slow running or "walking" (while PT makes sense)
- In between, $b_1 < 0$, $b_2 > 0$: possibility of an IR attractive fixed point (IRFP) – $\beta(g^{*2}) = 0$
  - No confinement
  - No chiral symmetry breaking
  - No particles
  - If $g \longrightarrow 0$ at cutoff, Λ parameter governs short distances – but not long distances
- Solvable model here: large N_c, fixed N_f/N_c can put $g^{*2} \sim O(\epsilon)$

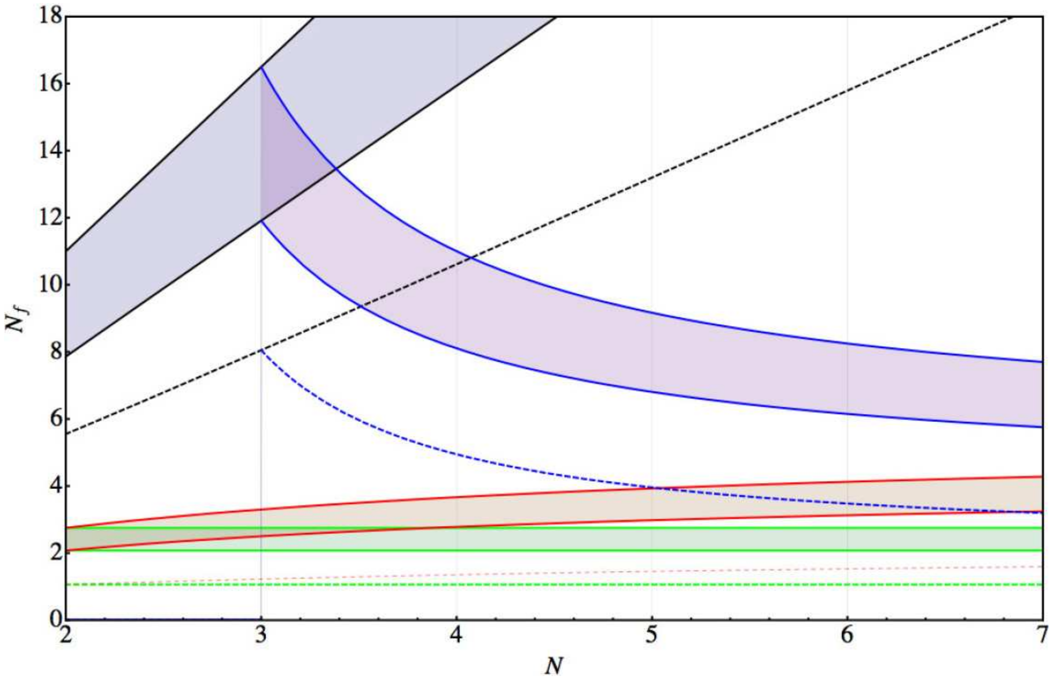
Many weak points in this old (Caswell, Banks-Zaks, . . .) story – so lattice people move in



Artist's conception: Asymptotic freedom, IRFP, walking technicolor...

(a) $N_c = 3$ $N_f = 2$

(b) $N_c = 3$, $N_f = 12$ with $b_3 x^4$ term dialed up



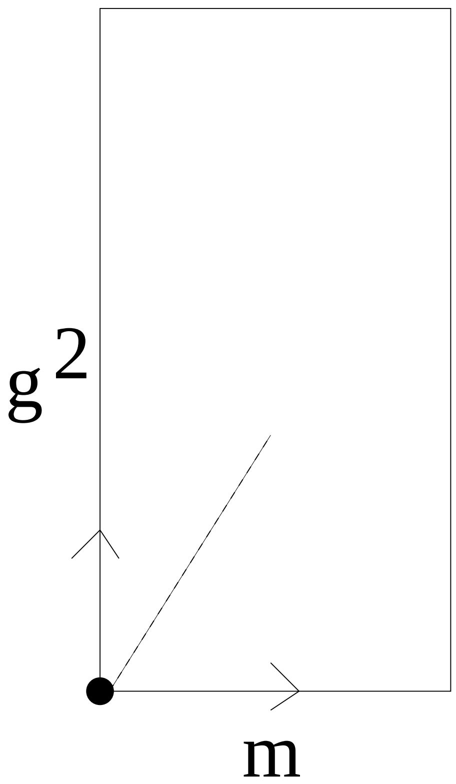
An expectation from the continuum literature (Dietrich & Sannino, PRD 2007)

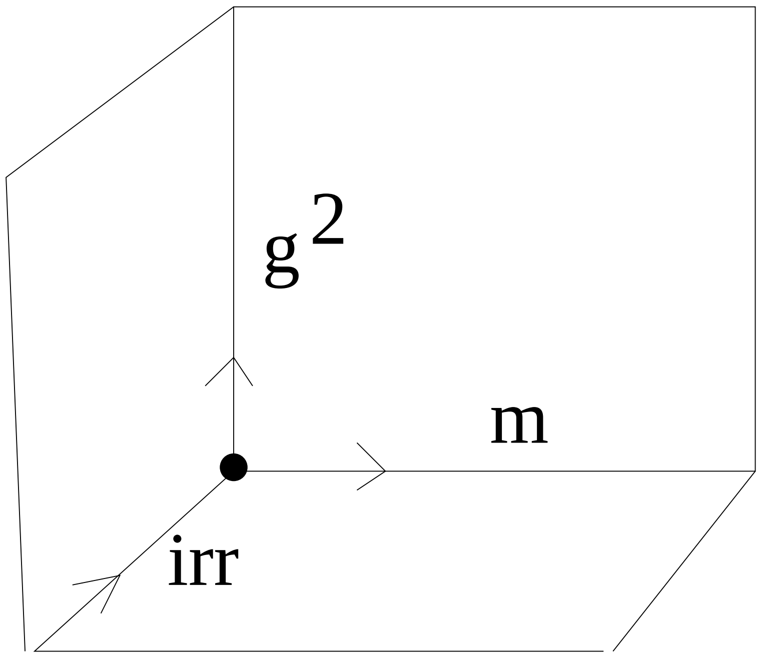
- bands show model predictions for conformal window vs (N_f, N_c, and fermion rep)
- Colors for different fermion representations, shading for different vacua

Strategies for studying candidate theories

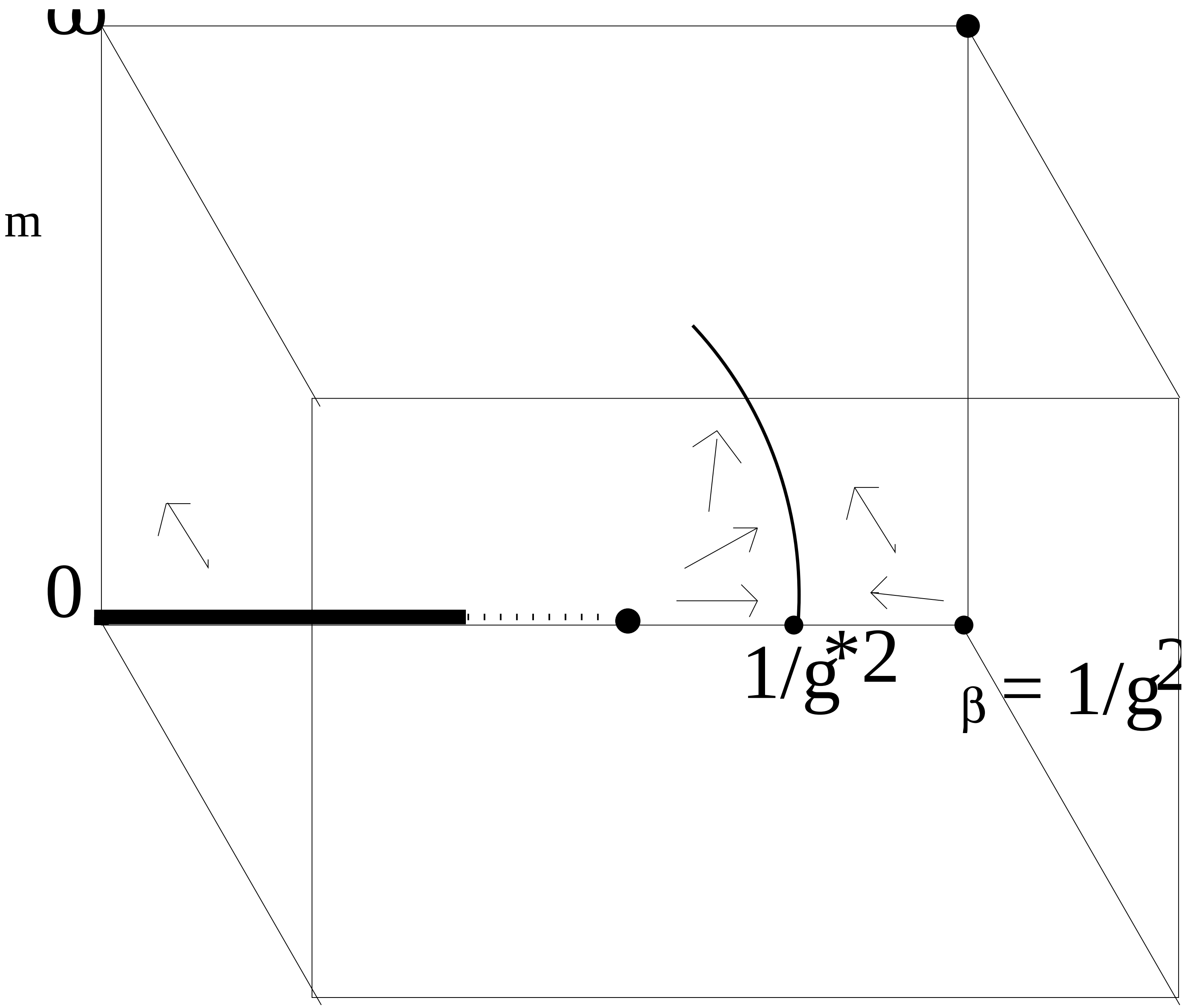
- Compute running coupling constant (typically via Schrodinger functional)
  - Want to see slow running, or zero in beta function (= no running = IRFP)
  - In fact, quite easy to see slow running, the zero is hard
- Attempt to do "usual" lattice calculations (spectroscopic observables)
  - Remember, if a coupling is strong, its definition becomes ambiguous
  - For "classical TC" expect to see chiral symmetry breaking
  - Care about value of m_H, f_π, $\langle\bar{\psi}\psi\rangle/f_\pi^3$
  - In conventional TC, gauge coupling AND mass are relevant
  - If in the conformal window, quark mass is relevant perturbation – $\xi \sim m_q^{-1/ym}$

Running in QCD-like parameter space





Running in conformal window of lattice BSM theory



My Overview of Lattice Work

Many groups studying (mostly) a few kinds of models

- $SU(2)$ with $N_f = 2$ adjoint rep fermions
  - Perhaps the most "minimal" model
  - Simulations use Wilson type fermions (dangerous chiral symmetry issues)
  - Coupling certainly walks, claim of IRFP in beta fn and in observables
  - Lattice theory has confining strong coupling phase, curious weak coupling phase
- $SU(3)$ with $N_f = 2$ sextet rep fermions
  - Similar results as above
- $SU(3)$ with large-N_f fundamentals
  - Mostly done with staggered fermions (dangerous flavor symmetry issues)
  - Lattice simulations generally show confining phase at strong coupling
  - $N_f \leq 8$ seem QCD-like from beta fn and spectra
  - (Disputed) claim $N_f = 12$ has IRFP
  - Lower N_f's with (more chiral lattice) fermions look technicolor-like
  - Anna H. will talk about these systems

Physics/simulation issues

If the theory has a (confinement) scale, need $L >> R_H >> a$

Need to find a useful window of bare couplings and L for confinement physics

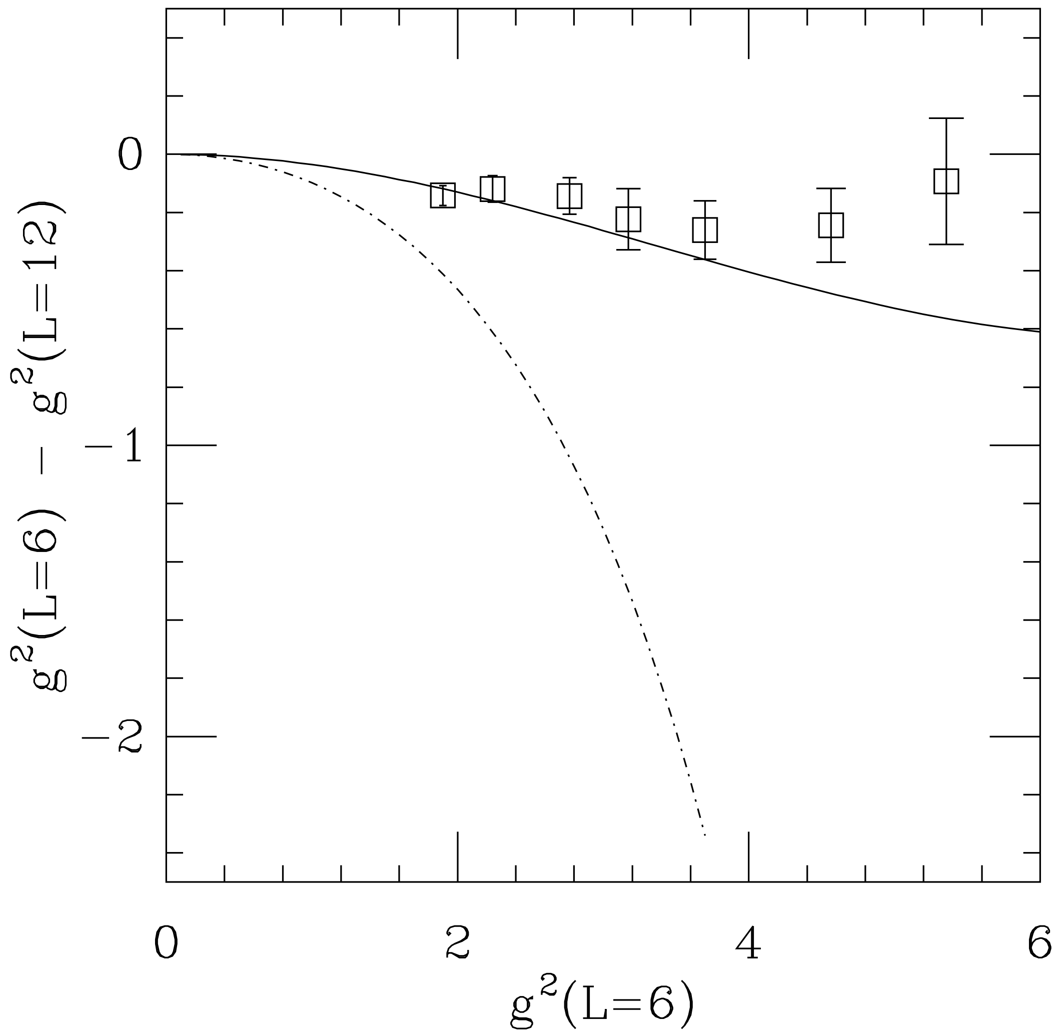
If the theory has Goldstone particles

- Then you have to see them, $m_\pi <$ other mass scales
- Issues with $m_\pi L \sim 1$

Need to find a useful window in m_q and L for chiral physics

In QCD, these scales are not so different; here, they might be

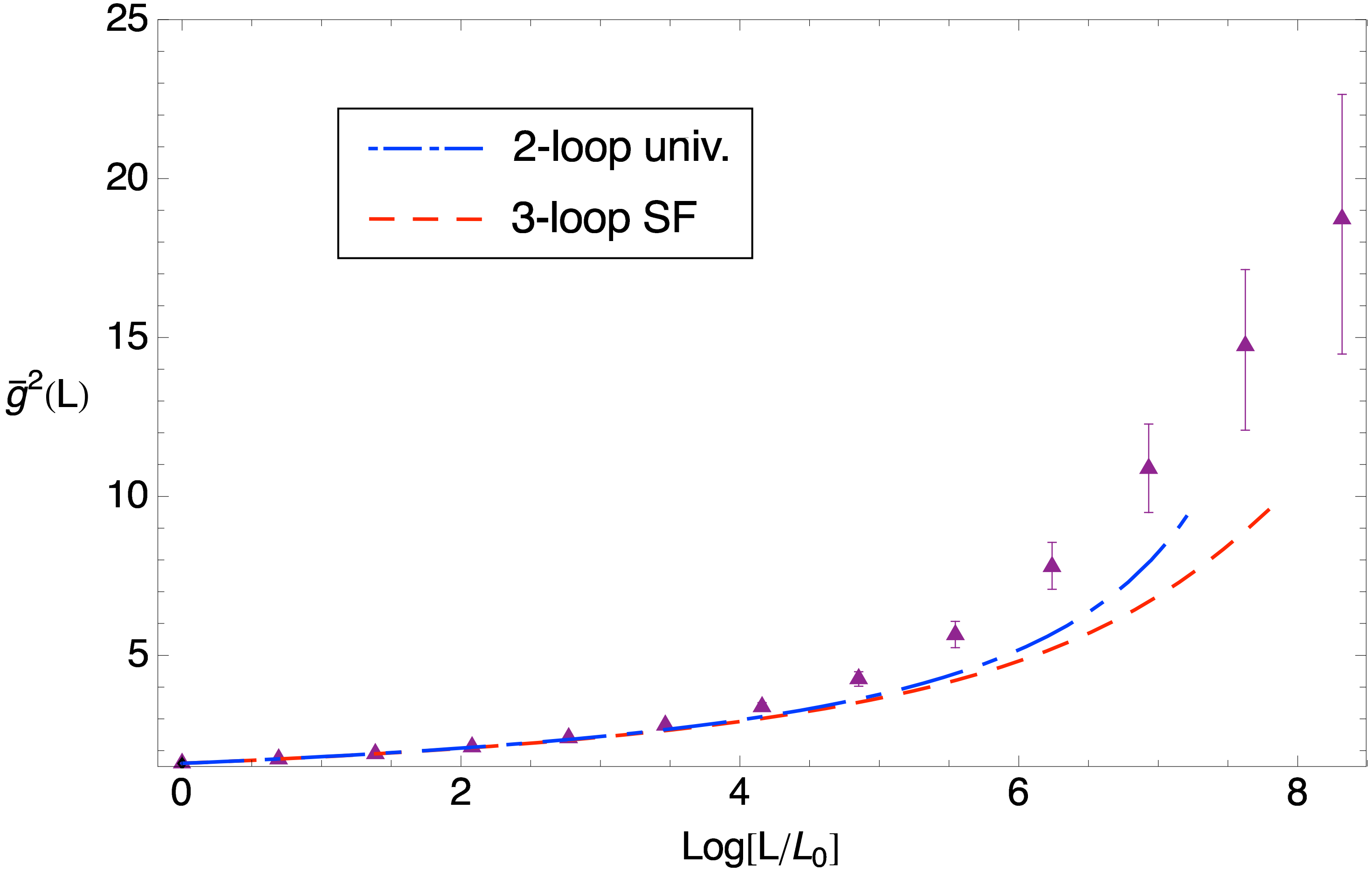
Universality (Do answers depend on discretization?)



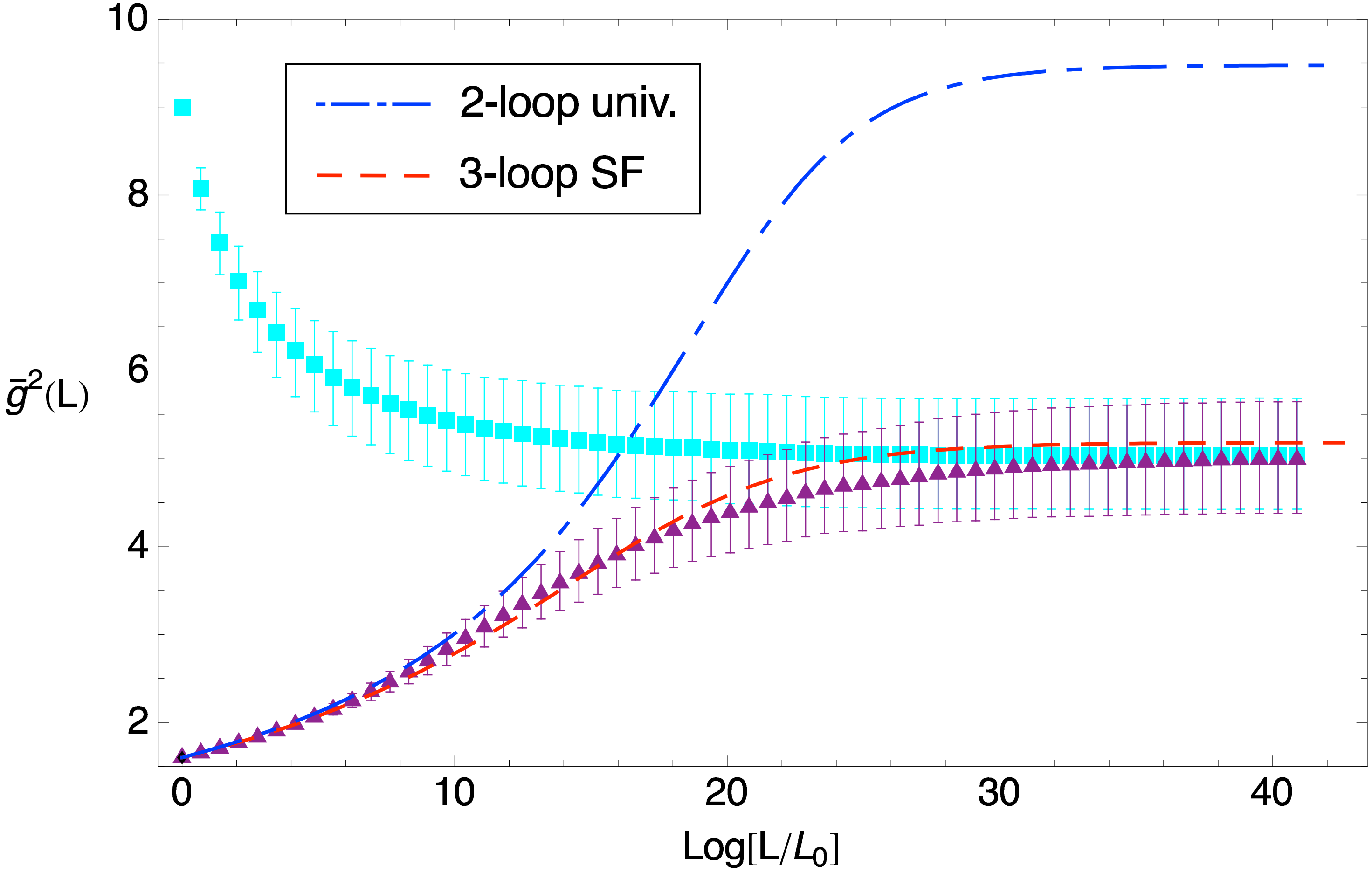
Walking vs running – Data and line –$N_c = 3$, $N_f = 2$ sextet rep fermions

Dashed line is (integrated) 2-loop beta function for $N_c = 3$, $N_f = 2$ fundamentals

$N_f = 8$, $N_c = 3$ running coupling (Yale)



$N_f = 12,\ N_c = 3$ running coupling (Yale)



Why it's hard to see a zero

It's because

$$s\frac{\partial g^2}{\partial s} \tag{12}$$

is always small and the small slope hides the zero

Suppose

$$s\frac{\partial g^2}{\partial s} = y_g(g^2 - g^{*2}) \tag{13}$$

Then

$$\begin{aligned} \Delta g^2 = g^2(1) - g^2(s) &= (g^2(1) - g^{*2})(1 - s^{y_g}) \\ &\sim (y_g \ln \frac{1}{s})(g^2(1) - g^{*2}) \end{aligned} \tag{14}$$

if y_g is small. It multiplies everything –

Expected behavior at top of conformal window; expect y_g grows near the bottom

Slow running is almost no running

- IRFP theory has one relevant coupling, m_q, criticality at $m_q \rightarrow 0$
- g^2 is irrelevant, even location of g^{*2} is RGT dependent

This implies correlation length diverges as

$$\xi \sim m_q^{-1/ym} \tag{15}$$

or

$$M^{ym} \propto m_q \tag{16}$$

- This could be absolutely true (in a real IRFP theory, $g \rightarrow g^*$ so it's irrelevant)
- This could only be approximately true but suppose g runs slowly

If you only look over scales where g doesn't change much, you get power laws

$$\begin{aligned} \Gamma(sp) &= s^{dn}\Gamma(p)\exp\int_1^s \frac{dt}{t}\gamma(g(t)) \\ &\simeq s^{dn}\Gamma(p)s^{\gamma(g(s))} \end{aligned} \tag{17}$$

This is power law, $\Gamma(k) \sim k^{dn+\gamma}$

y_m is the ingredient phenomenologists want (related to scaling dimension of $\bar{\psi}\psi$)

$$\langle \bar{\psi}\psi \rangle_{TC} = \langle \bar{\psi}\psi \rangle_{ETC} \exp \int_{TC}^{ETC} \frac{d\mu}{\mu} \gamma(\mu) \tag{18}$$

with $y_m = 1 - \gamma = 4 - d$, $d =$ scaling dimension of condensate

Theorems say $3 > d > 1$ or $1 < y_m < 3$

- $y_m = 1$ is free field fermions ($d = 1$ is a free boson)
- Large $|\gamma|$ often desired by phenomenology
- y_m is expected to grow near the bottom of the conformal window, perhaps big y_m marks its end

Summary

- QCD
  - RG calculation started QCD
  - RG notions underly precision lattice QCD simulations
- Schrödinger functional
  - coupling defined through system size plus b.c.'s
  - minireview of results
- Beyond-SM systems
  - We don't already know the answer
  - If the theory has a scale, you have to find a useful window of bare couplings where $L > R_H > a$
  - If the theory has massless particles at $m_q = 0$, are they Goldstone bosons, or something else
  - And what scale L, a captures their physics?
  - We don't know yet, what is physics and what is lattice artifact