Nuclear Physics from lattice QCD at strong coupling

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with Michael Fromm (ETH)

arXiv:0811.1931, 0907.1915 $\longrightarrow$ PRL, 0912.2524
and in progress

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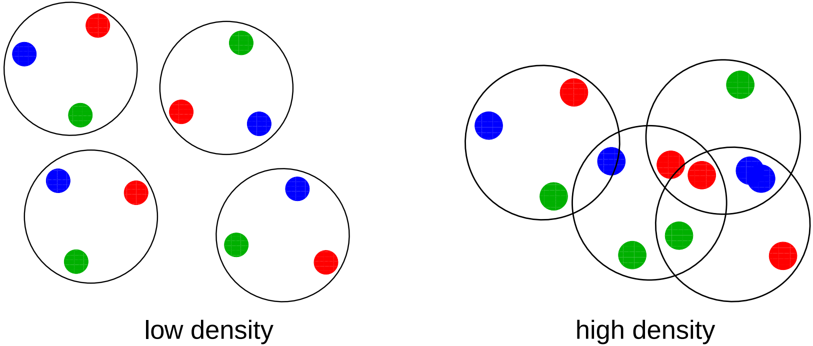
Matter under extreme conditions: simple ideas

- Confinement loses meaning when hadrons overlap

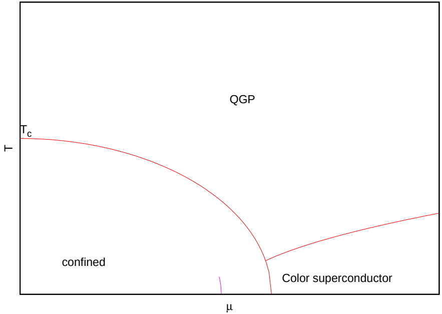


- Same at high temperature (overlapping with pions)
- Bag picture: percolation $\rightarrow$ chiral symmetry restoration

More formal

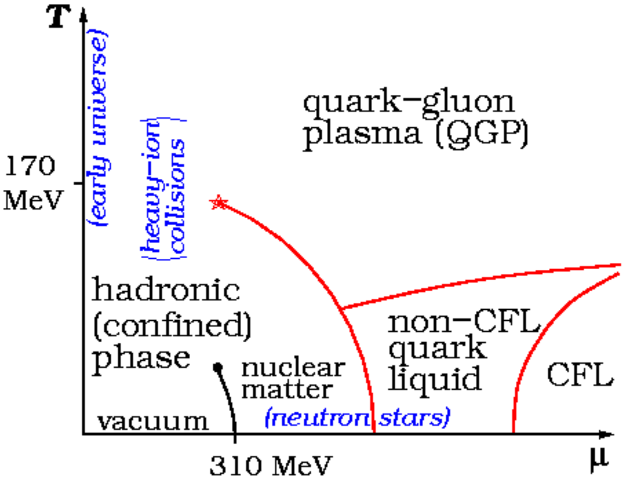
- Hadron resonance gas (Hagedorn): $\rho(E) \propto \exp(+cE)$
  $Z(T) = \int dE\ \rho(E)\ \exp(-E/T) \implies$ divergent for $T > 1/c$
- Asymptotic freedom: $g(T,\mu) \to 0$ for [very] high T or μ
  perturbation theory (at low order – Linde)

- high T: quark-gluon plasma
- high μ: $q-q$ attraction via 1 gluon exchange $\to$ color superconductivity



minimum phase diagram – phase transitions or crossovers?

QCD phase diagram according to Wikipedia



This talk is about: **hadron ↔ nuclear matter** transition
and $T = 0$ **nuclear interactions**

Nuclear physics from lattice QCD ?

- Fundamental QCD theory is known:

  $\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i(\not{D} + m_i)\psi_i$

  (gluons + quarks) confined into hadrons

- One should be able to *derive* interactions between hadrons from QCD

- Lattice QCD:

  only known non-perturbative gauge-invariant regulator of QCD

- Derive nuclear physics from lattice QCD ?

  All OK when quark density is zero (ie. $\mu = 0$)

  Non-zero quark density $\Longrightarrow$ **sign problem**

Why are we stuck at $\mu = 0$? The "sign problem"

$$Z = \int \prod dU \prod d\bar\psi d\psi \exp\left(-S_{YM} - \int d^3x \int_0^\beta d\tau \sum_{i=1}^{N_f} \bar\psi_i (\not{D} + m_i)\psi_i\right)$$

- quarks anti-commute $\rightarrow$ integrate analytically: $\det(\not{D}(U) + m + \mu\gamma_0)$

$$\gamma_5(i\not{p} + m + \mu\gamma_0)\gamma_5 = (-i\not{p} + m - \mu\gamma_0) = (i\not{p} + m - \mu^*\gamma_0)^\dagger$$

$$\boxed{\det \not{D}(\mu) = \det{}^* \not{D}(-\mu^*)}$$

det real only if $\mu = 0$ (or $i\mu_i$), otherwise can/will be complex

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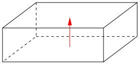
- The measure $\varpi \sim \det \not{D}$ *must be complex* to get correct physics



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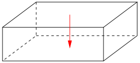

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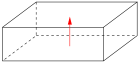

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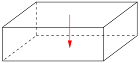

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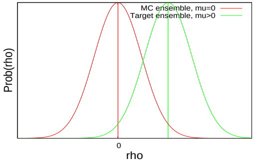
- Origin: $\mu \neq 0$ breaks charge conj. symm., ie. usually complex conj.
- Conversely: $\boxed{\mathrm{weight}(\{U\}) = \mathrm{weight}(\{U^*\}) \text{ real } \Rightarrow \mathrm{Re}\langle\rho_B\rangle = 0}$

Two difficulties: sign and overlap

MC ensemble has zero *average* baryon density ρ ⇒ exploit fluctuations in ρ



Each MC config has complex weight in target ensemble: sign problem.
→ noisy results

Larger volume.
Overlap problem becomes serious, starting with large-ρ tail
→ wrong results (Glasgow method) with finite sample

- Strategy: postpone inevitable [double] disaster $\sim \exp(-\frac{V}{T} f(\mu))$
- Physics (?) at small $\frac{\mu}{T}$, small V — low T inaccessible
- **Few-body physics** ok: $\rho_B \to 0$ as $V \to \infty$ Savage et al, Hatsuda et al
- Modify theory for milder sign problem? **strong coupling limit**

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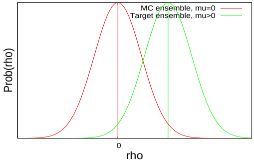

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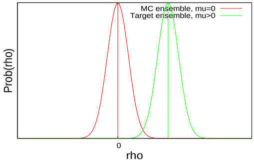

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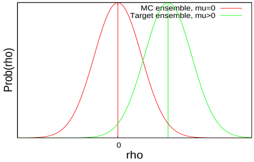

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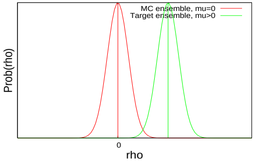

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Motivation (1)

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\rm gauge}) \propto \exp(-\frac{\beta_{\rm gauge}}{4N_c b_0})$

ie. $a \to 0$ when $\beta_{\rm gauge} \equiv \frac{2N_c}{g^2} \to +\infty$. Here $\boxed{\beta_{\rm gauge} = 0}$!!

- Lattice "infinitely coarse"
- Physics not universal

Nevertheless:

- Properties similar to QCD: confinement and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\rm gauge} > 0$
- When you can't find the solution to the sign problem,

think different

When $\beta_{\rm gauge} = 0$, sign problem is **manageable** $\to$ **full phase diagram**

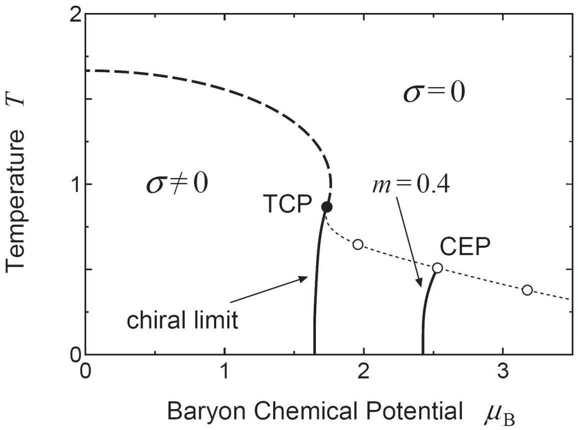
Motivation (2)

- 25^+ years of analytic predictions:
  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto
  $$T_c(\mu=0)=5/3,\ \ \mu_c(T=0)=0.66$$
  90's: Petersson et al., $1/g^2$ corrections
  00's: detailed (μ,T) phase diagram: Nishida, Kawamoto,...
  09: Ohnishi et al., Münster & Philipsen,...
  How accurate is mean-field ($1/d$) approximation?

- Almost no Monte Carlo crosschecks:
  89: Karsch-Mütter $\rightarrow$ MDP formalism $\rightarrow$ $\mu_c(T=0)\sim 0.63$
  92: Karsch et al. $T_c(\mu=0)\approx 1.40$
  99: Azcoiti et al., MDP ergodicity ??
  06: PdF-Kim, HMC $\rightarrow$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram according to Nishida (2004, mean field)



- Very similar to conjectured phase diagram of $N_f = 2$ **QCD**
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass $= M_{\text{proton}} \Rightarrow$ lattice spacing $a \sim 0.45$ fm

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U) + m)\psi)$, no plaquette term ($\beta_{\rm gauge} = 0$)

- One colored fermion field per site (6 d.o.f. – no Dirac indices, spinless)
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu} \eta_\nu(x)(U_\nu(x) - U^\dagger_\nu(x-\hat{\nu}))$, $\eta_\nu(x) = (-)^{x_1+..+x_{\nu-1}}$

$U(1)_V \times U(1)_A$ symmetry:

$$\left.\begin{array}{l}\psi(x) \to e^{i\theta}\psi(x) \\ \bar{\psi}(x) \to e^{-i\theta}\bar{\psi}(x)\end{array}\right\} \text{unbroken} \Rightarrow \text{quark number} \Rightarrow \text{chem. pot.}$$

$$\left.\begin{array}{l}\psi(x) \to e^{i\varepsilon(x)\theta}\psi(x) \\ \bar{\psi}(x) \to e^{i\varepsilon(x)\theta}\bar{\psi}(x) \\ \varepsilon(x) = (-)^{x_0+x_1+x_2+x_3}\end{array}\right\} \text{spont. broken } (m=0) \Rightarrow \text{quark condensate}$$

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- Chemical potential $\mu \quad \rightarrow \exp(\pm a\mu)U_{\pm 4}$

- Alternative 1: integrate over fermions
  $Z = \int \mathcal{D}U \ \det(\not{D}(U)+m) \quad \rightarrow \quad$ HMC, severe sign pb. for $\mu \neq 0$

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- Alternative 2: $\mathcal{D}U = \prod dU$ *factorizes* $\rightarrow$ integrate over links Rossi & Wolff

  $\rightarrow$ **Color singlet** degrees of freedom:

- **Monomer** (meson $\bar{\psi}\psi$) $M(x) \in \{0,1,2,3\}$
- **Dimer** (meson hopping), non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- **Baryon** hopping, oriented $\bar{B}B_\nu(x) \in \{0,1\}$ $\rightarrow$ *self-avoiding loops C*

Point-like, hard-core baryons in pion bath

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$$Z(m,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{m^{M(x)}}{M(x)!} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{n_\nu(x)!} \prod_{\text{loops}\,C} \rho(C)$$

with **constraint** $(M + \sum_{\pm\nu} n_\nu)(x) = 3 \ \forall x \notin \{C\}$

MDP Monte Carlo

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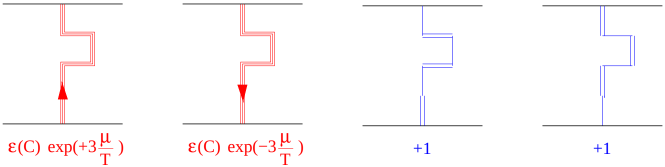
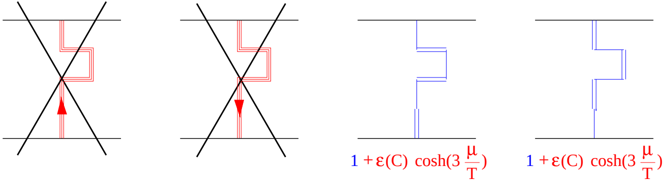


Karsch & Mütter: Resum into “MDP ensemble” $\rightarrow$ sign pb. eliminated at $\mu = 0$



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  monomer-changing update (Karsch & Mütter) restricted to $m \sim \mathcal{O}(1)$

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Solved with **worm algorithm** (Prokof'ev & Svistunov 1998)

Worm algorithm for MDP

- Sample $G(x,y)$ rather than Z, ie. add source and sink
- Monte Carlo: guided random walk of sink, ie. *local* steps ("**worm**")
- When $y = x$, contribution to Z $\rightarrow$ *global* change
- cf. "directed path" (Adams & Chandrasekharan) for $U(N)$

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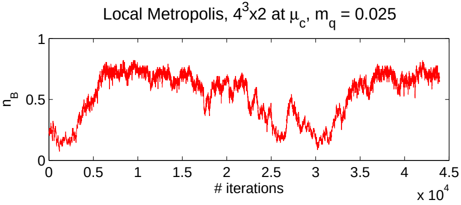
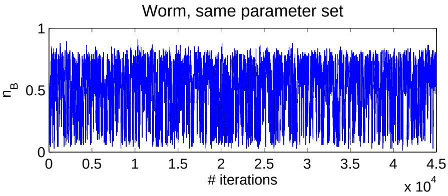
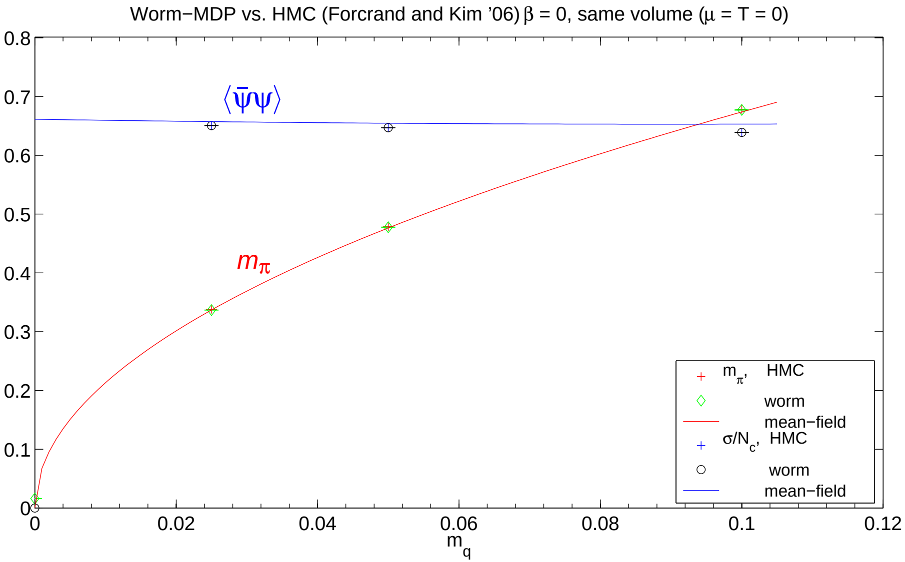
- Efficient even when $m_q = 0$

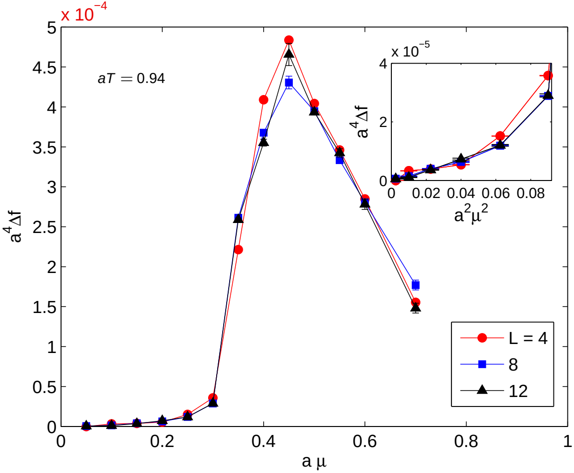




[Non-trivial] consistency check with HMC

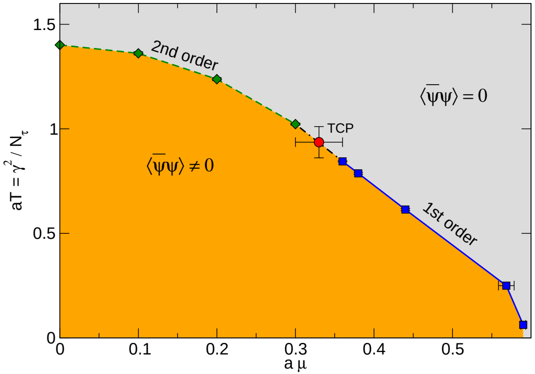


Sign problem? Monitor $-\frac{1}{V}\log\langle\text{sign}\rangle$



- $\langle\text{sign}\rangle = \frac{Z_{||}}{Z} \sim \exp(-\frac{V}{T}\Delta f(\mu^2))$ as expected; $\Delta f \sim \mu^2 + \mathcal{O}(\mu^4)$
- Can reach $\sim 16^3 \times 4\ \forall\mu$, ie. adequate

Phase diagram in the chiral limit



- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
- Mean field analysis: 2nd order at $\mu = 0$, $T_c = 5/3$
- If 2nd order, then expect 3d O(2) universality class
- Monte Carlo: 2nd order at $\mu = 0$ (Karsch et al, 1992)
- 1rst order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site $\Longrightarrow$ tricrit. pt. TCP

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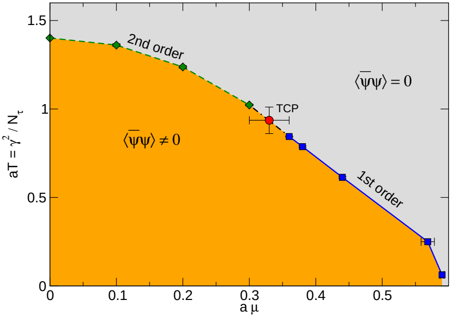
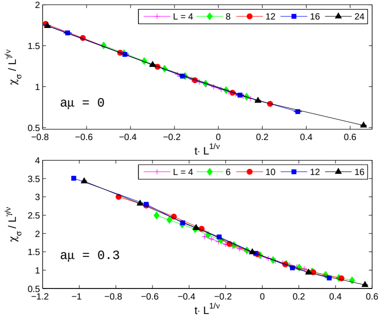


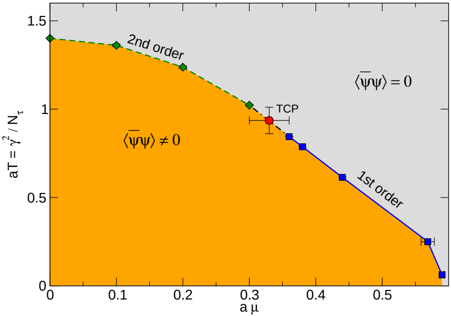


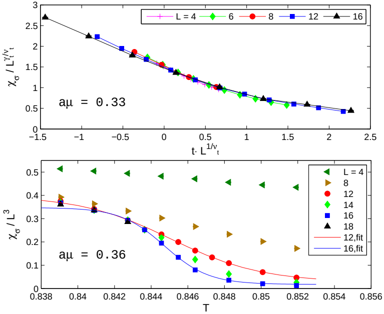
- $\chi_\sigma = \frac{1}{V}\frac{\partial^2}{\partial m_q^2}\log Z = \langle \sum_x \bar\psi\psi(x)\bar\psi\psi(0)\rangle \sim L^{\gamma/\nu}\tilde\chi(tL^{1/\nu})$

  $\rightarrow \quad \frac{\chi_\sigma}{L^{\gamma/\nu}}$ is *universal* function of $tL^{1/\nu}$

- Data collapse using 3d O(2) exponents for $a\mu = 0$ and 0.30

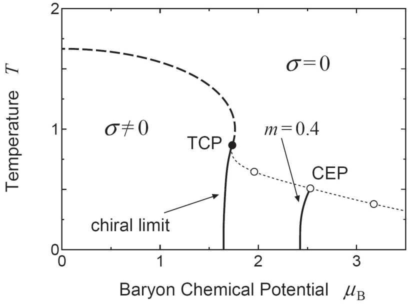
Phase diagram in the chiral limit

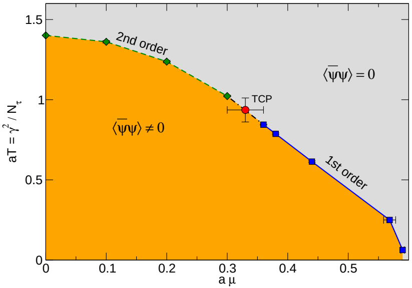




- Data collapse using mean-field exponents (d=3 is TCP upper crit. dim.) for $a\mu = 0.33$
- 1st-order Borgs-Kotecky: $Z(T) = \exp(-\frac{V}{T} f_1(T)) + c\exp(-\frac{V}{T} f_2(T))$ for $a\mu = 0.36$

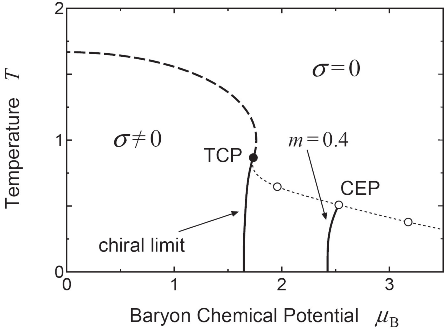
Compare with Nishida (2004) for $m_q = 0$

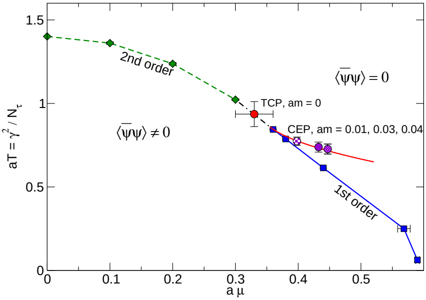




- TCP: $(\mu, T) = (0.33(3), 0.94(7))$ (Monte Carlo) vs $(0.577, 0.866)$ (mean-field)
- No reentrant phase diagram (caused by *decreasing* entropy in dense phase)
  cf. Clausius-Clapeyron: $\frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s} \rightarrow$ vertical at $T = 0$
- Beware of quantitative mean-field predictions for phase diagram

Phase diagram away from the chiral limit (in progress)





- Qualitatively similar to mean-field
- CEP moves fast with m_q, following tricritical scaling

Transition to nuclear matter: $T = 0, \mu = \mu_c$

Puzzle:

- Mean-field baryon mass is ≈ 3 $\Rightarrow$ expect $\mu_c = \frac{1}{3} F_B(T=0) \approx 1$
- Mean-field estimate $\mu_c \sim 0.55 - 0.66$ much smaller, ie. $\mu_c^B \sim 600$ MeV !

Mean field gives wrong M_B? wrong μ_c?

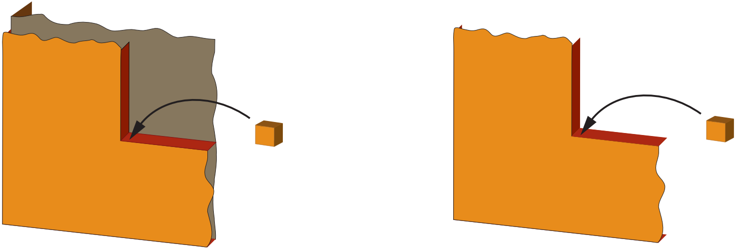
- Check M_B and μ_c by Monte Carlo $\longrightarrow$ ok (next slide)

Remaining explanation: nuclear attraction $\sim$ **1/3** baryon mass !!

Why so large ? Nuclear potential ? Nuclear spectroscopy ??

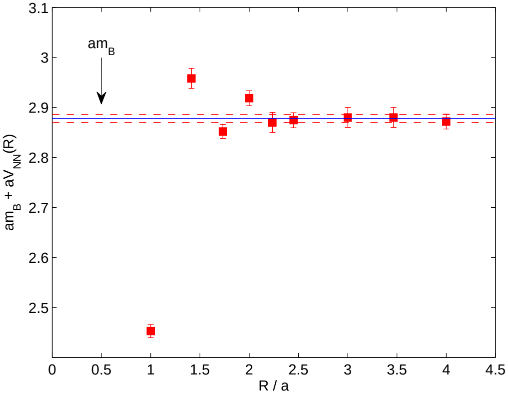
$\mu_c(T=0)$

- $T = 0$ dense phase is baryon crystal (1 baryon per site)
- μ_c is free energy necessary to add 1 baryon to dense phase



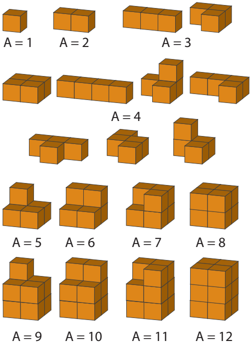
- Monte Carlo: $a\mu_c^B = 1.78(1)$ compared with $am_B = 2.88(1)$
- Each baryon binds to 3 nearest-neighbours $\rightarrow$ attraction
  $V_{NN}(r=a) \sim \frac{2.88-1.78}{3} a^{-1} \sim 120$ MeV !!
- Surface tension: first layer of dense phase $\rightarrow$ 2 nearest-neighbours only
  $$\sigma \approx \tfrac{1}{2}|V_{NN}(r=a)|a^{-2} \quad \sim (200\text{ MeV})^3$$
- Complete nuclear potential ?

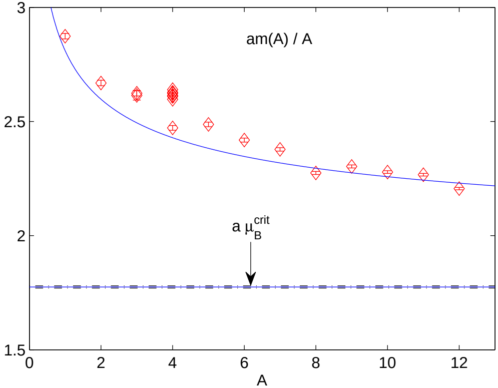
Nuclear potential



- Nucleons are point-like $\rightarrow$ no ambiguity with definition of static potential
- Nearest-neighbour attraction $\sim$ 120 MeV at distance $\sim$ 0.5 fm: cf. real world
  Baryon worldlines self-avoiding $\rightarrow$ no meson exchange here (just hard core)
  Attraction due to bath of neutral pions: cf. Casimir effect (see later)

Nuclear spectroscopy



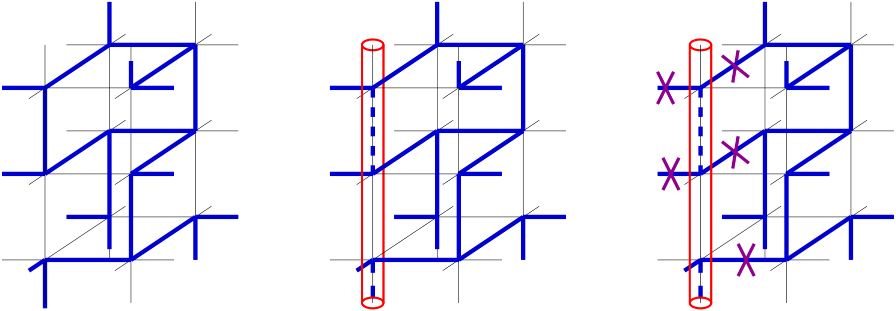


- Can compare masses of differently shaped "isotopes"
- $E(B=2) - 2E(B=1) \sim -0.4$, ie. "deuteron" binding energy ca. 120 MeV
- $am(A) \sim a\mu_B^{\rm crit} A + (36\pi)^{1/3} \sigma a^2 A^{2/3}$, ie. (bulk + surface tension)
  Bethe-Weizsäcker parameter-free ($\mu_B^{\rm crit}$ and σ fixed)
- "Magic numbers" with increased stability: $A = 4, 8, 12$

Nuclear attraction as *steric* effect

- Here, baryons make self-avoiding loops $\rightarrow$ no meson exchange
- Attraction comes from **rearrangement of pion bath**: 3 meson links per site

more time-like mesons near static baryon $\rightarrow$ higher energy



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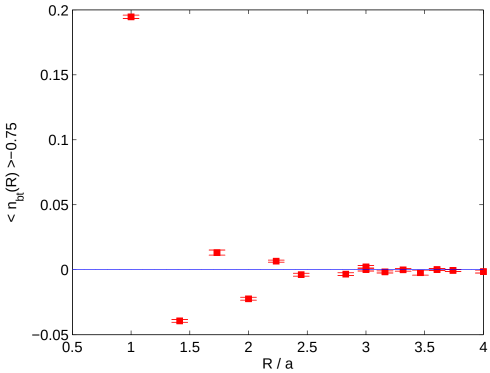


- $am_B \approx 2.88 = (3 - 0.75) + \Delta E_\pi$, ie. "valence" + "pion cloud"
- For 2 nearest-neighbour baryons, ΔE_π decreases $\rightarrow$ attraction

Conclusions

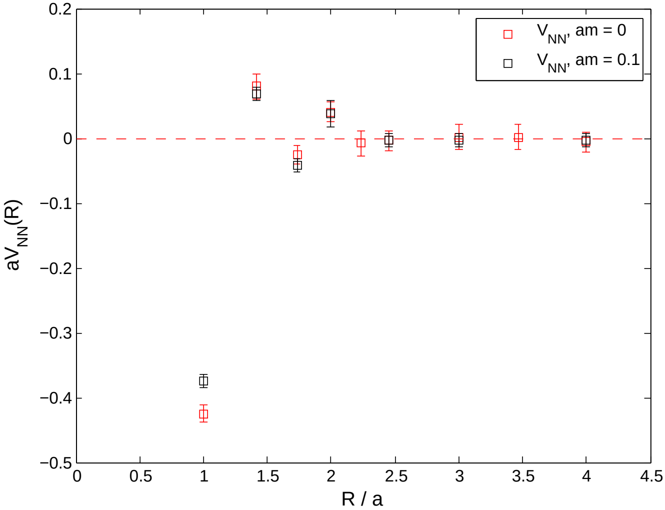
Summary

- Take mean-field results with a grain of salt
- “Clean-up” of phase diagram justified
- [Crude] nuclear matter from QCD
- “Understand” nuclear interaction as steric effect

Outlook

- Non-zero quark mass:
  - Critical end-point as a function of m_q
  - Nuclear potential & spectroscopy as a function of m_q
- Include second quark species $\rightarrow$ isospin (degenerate masses or not)
- Include $\mathcal{O}(\beta)$ effects ?

Backup slide: Influence of pion mass on nuclear potential



- Nearest-neighbour attraction weakens (as expected). Not much else.

Backup slide: $\mathcal{O}(\beta)$ effects