Subtractive renormalization of the NN interaction and deuteron electro-disintegration

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PRC. 77 014002(2008), PRC.80 034002, PRC.80 044002

Summary:PoS(CD09)064 (arXiv:0909.5414)

A simplified overview of nuclear physics

1 TeV

100 GeV Effective fundamental theories (QCD, QED, etc.)

Nuclear physics

Difficulty: Non-perturbative region

By spontaneous symmetry breaking

1 GeV

Lattice QCD

Chiral perturbation theory

100 MeV

NN data

Goal: Effective field theory for low energy NN interaction.

Application: Detail nuclear structure

Approach to NN interaction

Chiral EFT on NN sector:

- Lagrangian based on CHPT (πN,$\pi\pi$,perturbative).
- Main category:

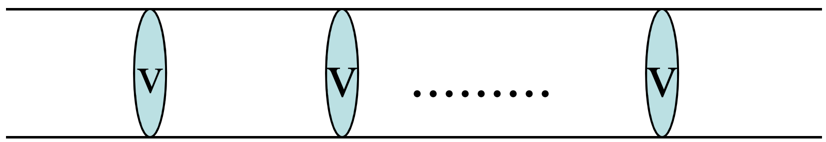
perturbative, non-perturbative.

(no iteration of all diagrams) Kaplan, Savage, Wise, Birse, Beane, etc.

(Lippmann-Schwinger eq.) Ordonez, Ray, van Klock, Epelbaum, Entem, Machleidt, Valderrama, Arriola, etc.

NN interaction and renormalization

Lippmann-Schwinger eq. (LSE)

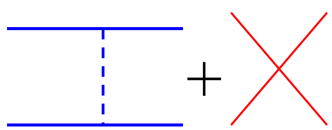


$$T(k,k;E) = \frac{e^{i\delta}\sin\delta}{-2\mu k}$$

$$T_l(k',k;E) = V_l(k',k;E) + \frac{2}{\pi}\int_0^{\Lambda} \frac{dp\,p^2 V_l(k',p;E)\,T_l(p,k;E)}{E - E_p + i\varepsilon}$$

V Includes long range part (pion exchange) + contact term

Cutoff in LSE



Expanded in power of momentum.

In s-waves, LO: $C_{S(T)}$.

Represent short range (high energy) physics.

$C_{S(T)}(\Lambda)$ are determined by fitting to physical observables.

A successful renormalization: For different Λ (or at least in some interval), we should be able to get the same physical result.

The goal of this work

- We want to evaluate whether (or under what conditions) χET, as an EFT (like QED or QCD), is really improved order by order, after renormalization.
  - * More and more counter terms (with free parameters fitted to data), which somehow make it difficult to see whether the improvement is coming from the inclusion of higher order or the fit.
  - * Many EFTs, each needs to be checked (RG) order by order. Pionful→~~analytic~~→need quick and good way to pin down LEC's.

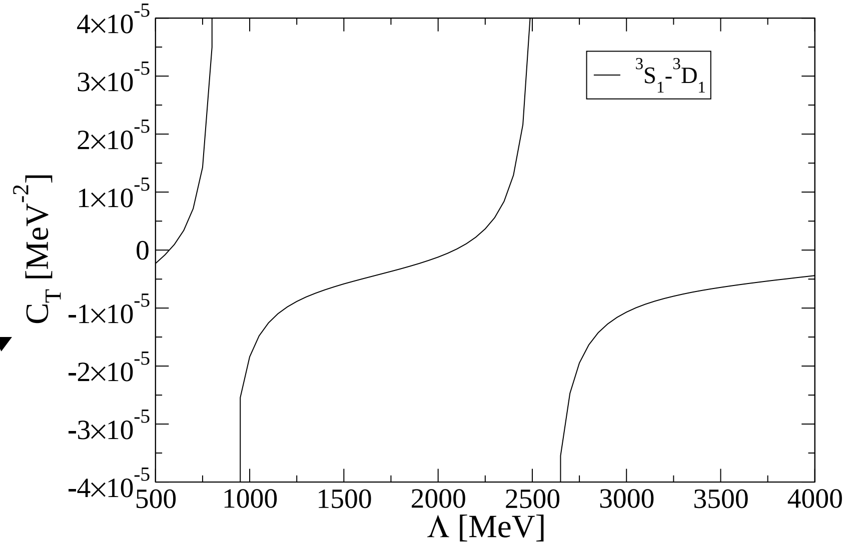
  1. What kind(order) of contact term should one adopt?
  2. Cutoff-indep. in phase shift $\overset{?}{\leftrightarrow}$ renormalize successfully.
  3. What is the highest Λ can one use?
- **We invent a subtractive renormalization scheme to achieve our goal. Advantage: go to high Λ, eliminate correlation b/w LECs.**

Determining LEC's in the contact terms

1. Fitting (Epelbaum, Machleidt, Valderrama, etc.)

- Previously, renormalize V by adjusting the unknown constants to fit data, e.g. fit scattering length, effective range or phase shifts.
- Problem: 1. fine-tuning. 2.

Problem of fitting



2. Subtraction method:

Leading order*: ***C. J. Yang, C. Elster, and D. R. Phillips, PRC. 77 014002(2008)**

- By subtracting two LSE to cancel the divergent part ($C_{S(T)}$).
- Direct input of the scattering length $a_{S,T}$.
- This was done by relating the on-shell $t(p_0,p_0;E)$ for an arbitrary E to the fully-off-shell t(p',p;0).

Method: Three steps

$$T(p,0;0) = V(p,0;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp'p'[\frac{V(p,p';0)+C}{-p'^2/2\mu}]T(p',0;0) \quad (1)$$

$$T(0,0;0) = \frac{f_0}{-2\mu k} = \frac{a}{2\mu}$$

$$T(0,0;0) = V(0,0;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp'p'[\frac{V(0,p';0)+C}{-p'^2/2\mu}]T(p',0;0) \quad (2)$$

$$(1)-(2): \; T(p,0;0) - \frac{a}{2\mu} = V(p,0;0) + \frac{2}{\pi}\int_0^{\Lambda} dp'\,p'^2[\frac{V(p,p';0)-V(0,p';0)}{-p'^2/2\mu}]T(p',0;0)$$

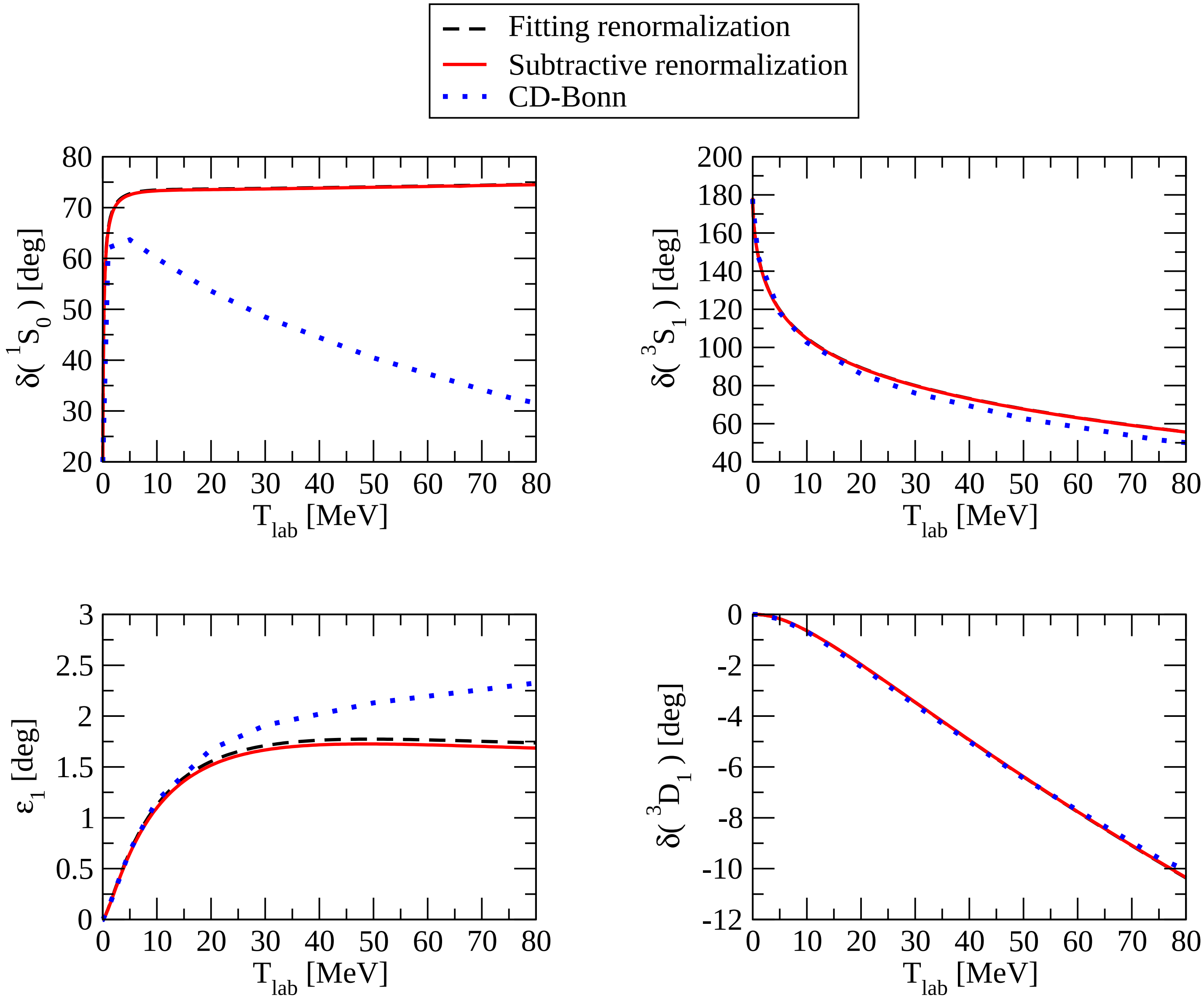
$$T(p,p^*;0) - \frac{a}{2\mu} = V(p,p^*;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp'\,p'^2[\frac{V(p,p';0)+C}{-p'^2/2\mu}]T(p',p^*;0) \quad (3)$$

$$T(0,p^*;0) - \frac{a}{2\mu} = V(0,p^*;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp'\,p'^2[\frac{V(0,p';0)+C}{-p'^2/2\mu}]T(p',p^*;0) \quad (4)$$

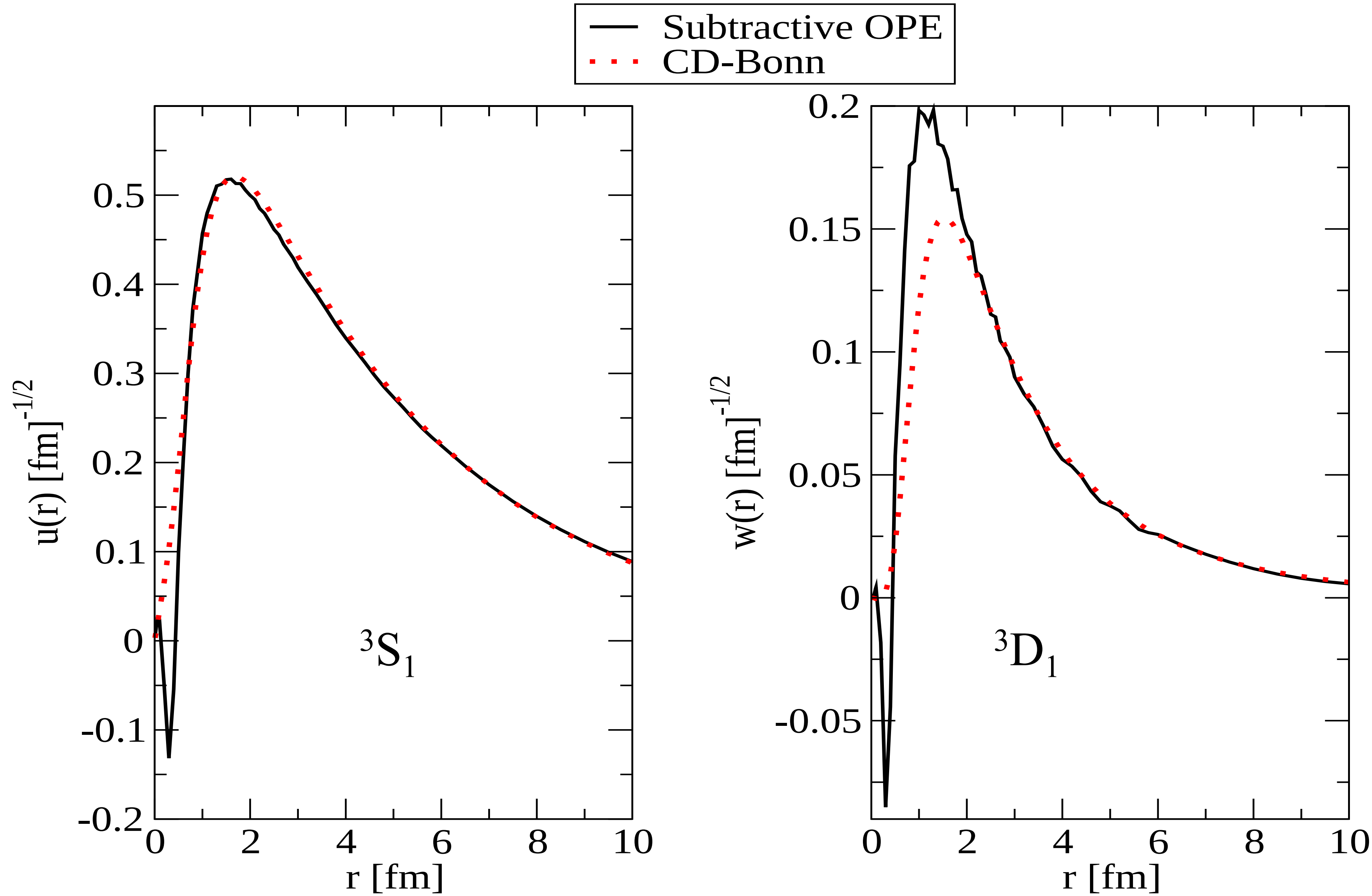
$$(3)-(4): T(p,p^*;0) - T(0,p^*;0) = V(p,p^*;0) - V(0,p^*;0) + \frac{2}{\pi}\int_0^{\Lambda} dp'\,p'^2[\frac{V(p,p';0)-V(0,p';0)}{-p'^2/2\mu}]T(p',p^*;0)$$

$Final$ step: $[1+T(0)(G(0)-G(E))]T(E) = T(0)$

LO Results


Fitting renormalization
Subtractive renormalization
CD-Bonn
δ(1S0) [deg]
δ(3S1) [deg]
ε1 [deg]
δ(3D1) [deg]
Tlab [MeV]

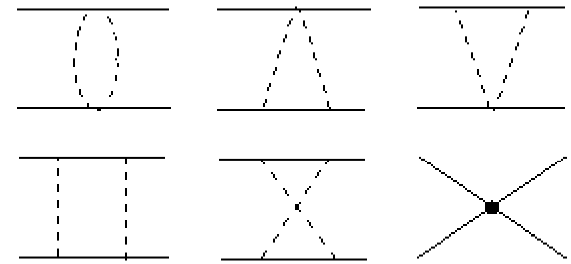
Wave function



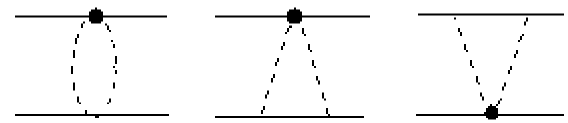
So subtraction works for the leading order.

Next: Go to NNLO

- In NLO(Q^2) and NNLO(Q^3) we have TPE, which diverges as $Q^{2(3)}$. => Include the O(Q^2) contact term to renormalize it.
- V=OPE+TPE+ λ+γ* [O(Q^2)].
- Further develop our subtraction technique to solve it.



NLO(Q^2)



NNLO(Q^3)

Use dimensional regularization (DR) or spectral function regularization (SFR) to regularize the diverge loop integral.

Contact terms: from LO to NLO/NNLO

In p-waves: none $\rightarrow$ Cpp'.

s-waves: $C_{S(T)} \rightarrow \lambda_{ll'}+\gamma_{ll'}$[O($Q^2$)].

Subtraction: (up to) NNLO for p-waves

Need to cancel out : C^{SJ}p'k. **Idea: get t(p',p;E) ← t(p',k;0) ← α_{ll}**

Note: $t_{l'l}(p',p;E) \sim (\ldots) p'^{l'} p^{l}$

Step 1 of subtraction:

Key: 1. Divide by p'k and then take limit. 2. $\lim_{k\to 0}[\frac{t(k,k;0)}{kk}] = \frac{\alpha_{11}}{M}$

$$\lim_{k\to 0}[\frac{t(p',k;0)}{p'k}] = \lim_{k\to 0}[\frac{v^{LR}(p',k)}{p'k} + C_{11}] + \frac{2}{\pi} M \lim_{k\to 0}[\frac{1}{p'k}\int_0^{\Lambda} \frac{dp'' p''^2 (v^{LR}(p',p'') + C_{11} p' p'') t(p'',k;0)}{-p''^2}] \quad (1)$$

$$\lim_{k\to 0}[\frac{t(k,k;0)}{kk}] = \lim_{k\to 0}[\frac{v^{LR}(k,k)}{kk} + C_{11}] + \frac{2}{\pi} M \lim_{k\to 0}[\frac{1}{kk}\int_0^{\Lambda} \frac{dp'' p''^2 (v^{LR}(k,p'') + C_{11} k p'') t(p'',k;0)}{-p''^2}] \quad (2)$$

$$p'[(1)-(2)] \Rightarrow \lim_{k\to 0}[\frac{t(p',k;0)}{k}] - \frac{\alpha_{11}}{M} p' = \lim_{k\to 0}[\frac{v^{LR}(p',k)}{k}] - p' \lim_{k\to 0}[\frac{v^{LR}(k,k)}{kk}]$$

$$+ \frac{2}{\pi} M \lim_{k\to 0}[\int_0^{\Lambda} \frac{dp'' p''^2 (v^{LR}(p',p'') - \lim_{k\to 0}[\frac{v^{LR}(k,p'')}{k}] p') \lim_{k\to 0}[\frac{t(p'',k;0)}{k}]}{-p''^2}]$$

get $\lim_{k\to 0}[\frac{t(p',k;0)}{k}]$.

Step 2 of subtraction:

Use result of step 1, i.e., $\lim_{k\to 0}[t(p',k;0)/k]$ as input.

Do similar subtraction again. $\Rightarrow$ *get* $\dfrac{t(p'',p';0)}{p'}$, or, $t(p'',p';0) \equiv t(0)$.

Final step:

Use $t(E) = t(0) + t(0)[g(E) - g(0)]t(E)$.

=> get on-shell t-matrix t(E).

Results at Leading order(LO):

See also: Nogga, Timmermans and van Kolck (2005), Valderrama (2006).

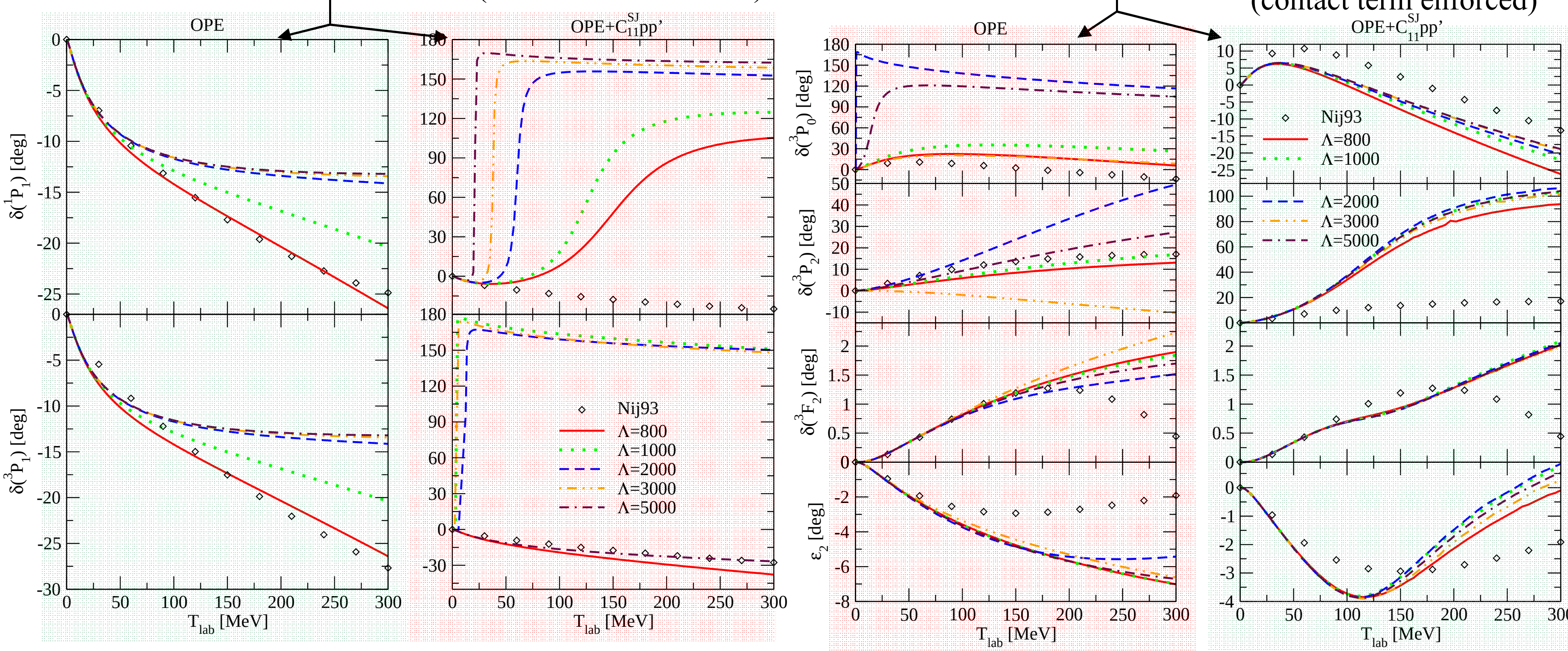
unrenormalized v.s. renormalized (Fail of Weinberg counting)

singular & repulsive

(contact term enforced)

singular & attractive

(contact term enforced)

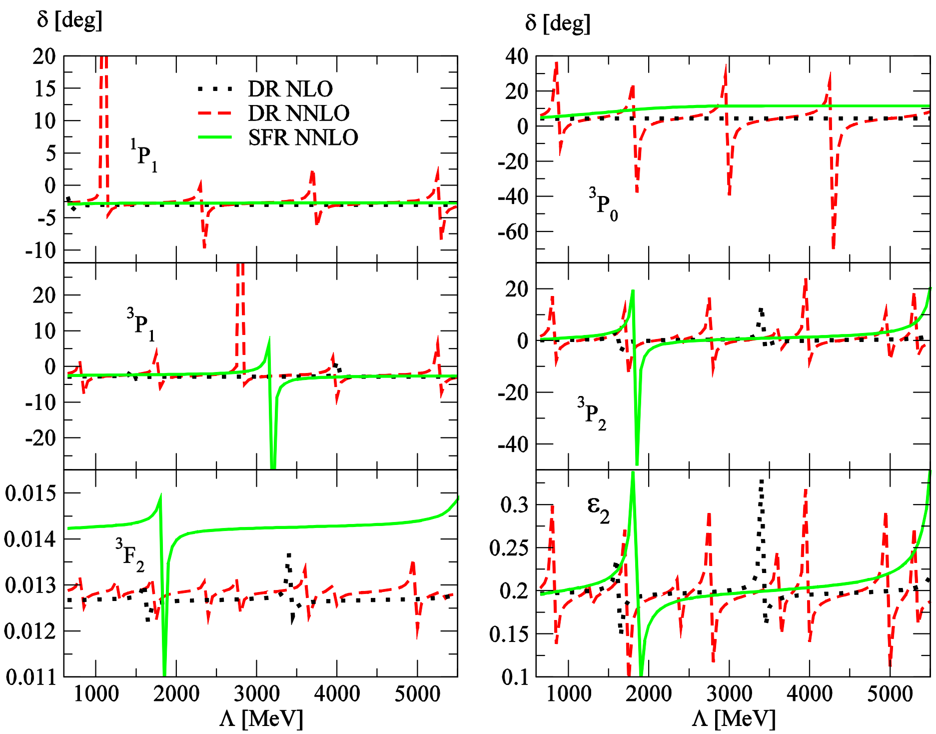


convergent | Divergent w.r.t. Λ | Divergent w.r.t. Λ | convergent

α_{11} from extracted value of NijmII & Reid93.

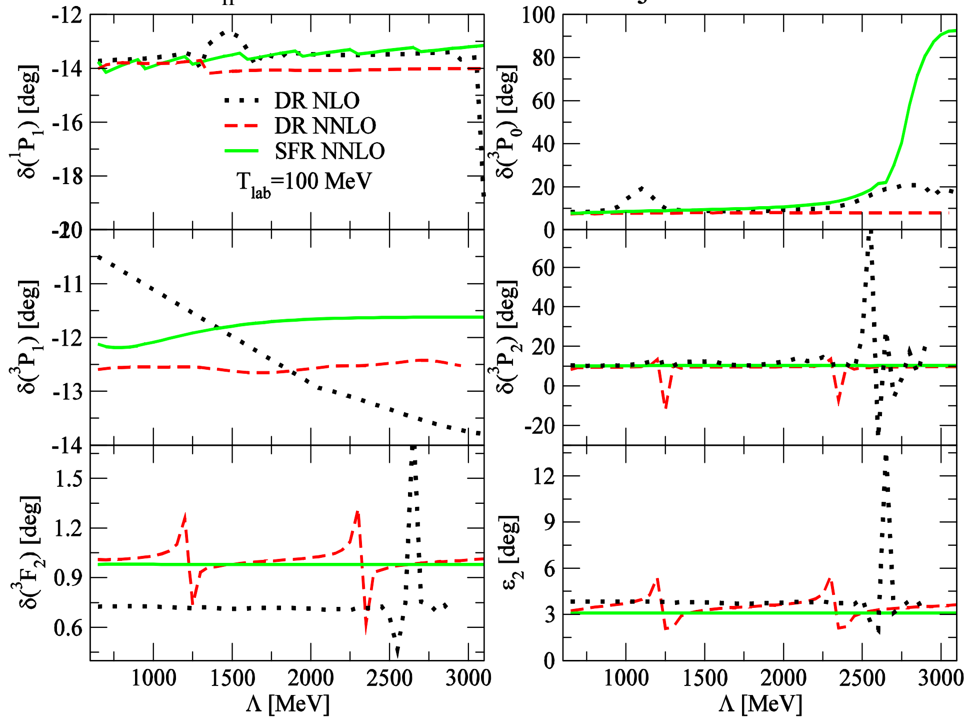
$r \to 0$ connection for the higher order

Un-renormalized



Renormalized

α_{ll} from extracted value of NijmII & Reid93.



	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$
OPE	U	R	U	R
NLO (DR)	U	U	R	R
NNLO (DR)	R	R	R	*
NLO (DR) + NNLO (SFR)	U	U	R	R
NNLO (SFR)	U	U	R	R

Need counter term to stabilize?

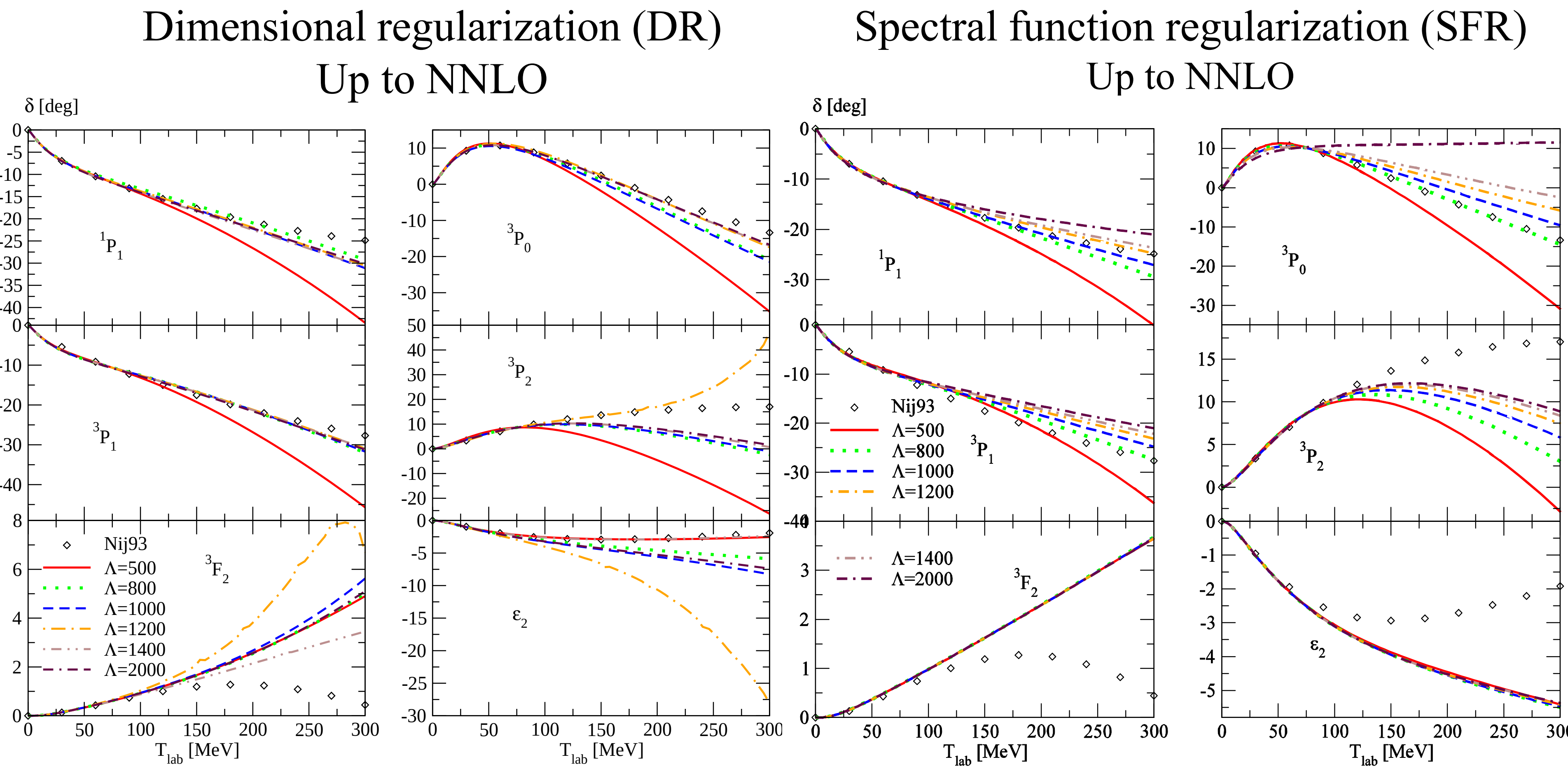
Attractive: Yes(R on the left).

Repulsive: No(U on the left).

(See: Valderrama, et al (2006) for same thing in S-waves.)

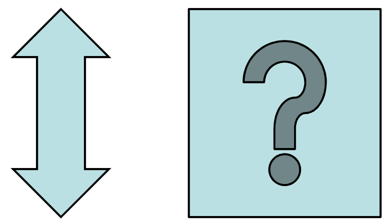
Best fit: by adjusting α^{SJ}

For DR TPE, need to vary α^{SJ} away from Nijm value up to 30% in some channels.



Even if we can fit to the data, does this necessarily mean that the renormalization is done successfully?

Cutoff-indep. in phase shift



Successful Renormalization.

α_{best} v.s. Λ (DR TPE)

Dimensional regularization (DR) up to NNLO

Λ[MeV]	$\alpha_{11}^{01}(^1P_1)$	$\alpha_{11}^{10}(^3P_0)$	$\alpha_{11}^{11}(^3P_1)$	$\alpha_{11}^{12}(^3P_2)$
500	3.6405	-4.3385	3.1326	-0.59589
800	3.6455	-4.8930	2.9524	-0.47614
1000	3.6593	-4.9346	2.9552	-0.47028
1200	3.6593	-4.5049	3.3613	-0.42356
1400	3.6732	-3.4098	4.1098	-0.45942
1600	3.6732	-2.5089	4.4564	-0.46142
1800	3.6732	-2.3425	4.0405	-0.46142
2000	3.6732	-2.9386	3.4819	-0.45642

Λ_C~ 1GeV

(above that, renormalization point dependence show up.)

Summary of p-waves results

Whether need contact term to reach Λ independence?

=> exactly depends on the singularity structure of $V(r\rightarrow 0)$.

Attractive: Yes.

Repulsive: No.

Successfully renormalized?

1. **There is a critical cutoff Λ_C~1 GeV for DR TPE up to NNLO, after that the contact term in LSE dominate the result.**
2. **Replacing the whole DR NNLO TPE by SFR brings Λ_C up to 2.5 GeV.**
3. **Λ_C is in the same order of Λ_χ (~1 GeV).**

Applications:

This scheme works for any contact term has the form $Cp^l p'^{l}$,
=> can be applied in D-waves and F-waves.

NNLO in S-waves

- Adopt more subtractions to perform the renormalization.
- In S-wave, contact term (up to Q^2) has the following structure:

$$\lambda + C_2(p^2 + p'^2)\text{.....................} \textit{for } {}^1S_0\text{. (two unknowns)} \;\; \Rightarrow \text{solve by 1 sub + 1 fit.}$$

$$\begin{pmatrix} \lambda + C_2(p^2 + p'^2) & \lambda_t p'^2 \\ \lambda_t p^2 & 0 \end{pmatrix} \text{....} \textit{for } {}^3S_1 - {}^3D_1\text{. (three unknowns)} \Rightarrow \text{solve by 2 sub + 1 fit.}$$

-

Method

- Combine previous ideas:

1. Use first subtraction to eliminate λ.
2. Use previous formula to relate t(E*) to t(E).
3. For $^3S_1$-$^3D_1$, apply the idea of dividing $p^l p'^{l'}$ in LSE to eliminate $\lambda_t p^2$.
4. Input: a_0, $\delta(E^*)$ for the singlet; with additional α_{20} for triplet.

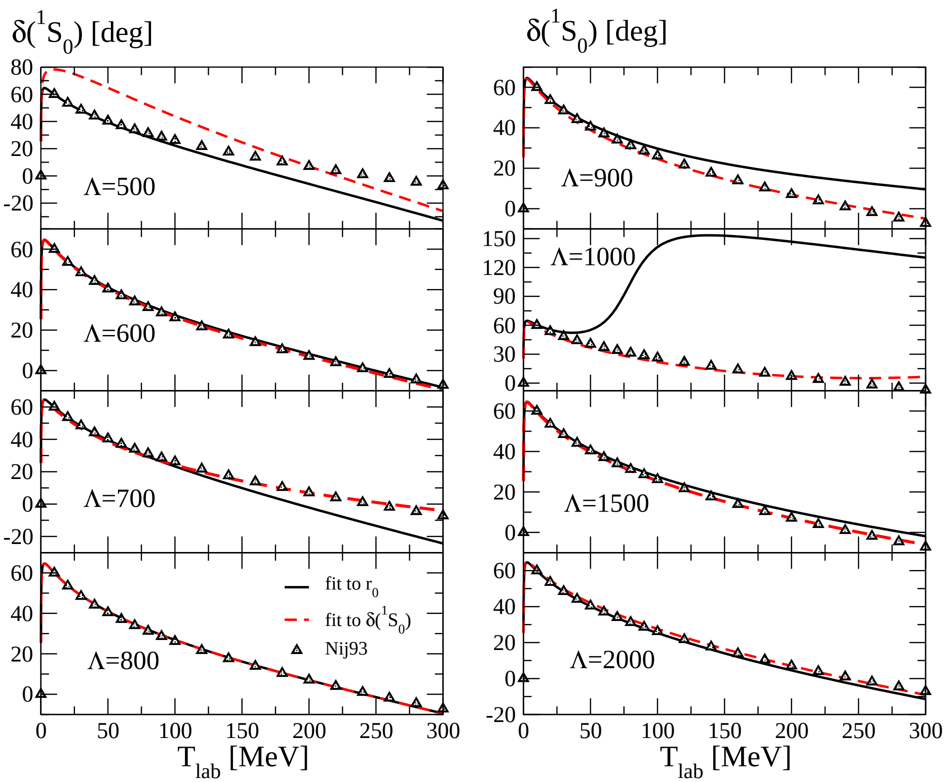
1 Subtraction + 1 fitting

- For contact term at NLO/NNLO, we need to replace one input by one fitting (due to the fact that $C_2(p^2+p'^2)$ term has *no* on-shell observable corresponds to it.)
- So, we need 1 fitting (fitted either to r_o or $\delta(E^*)$).

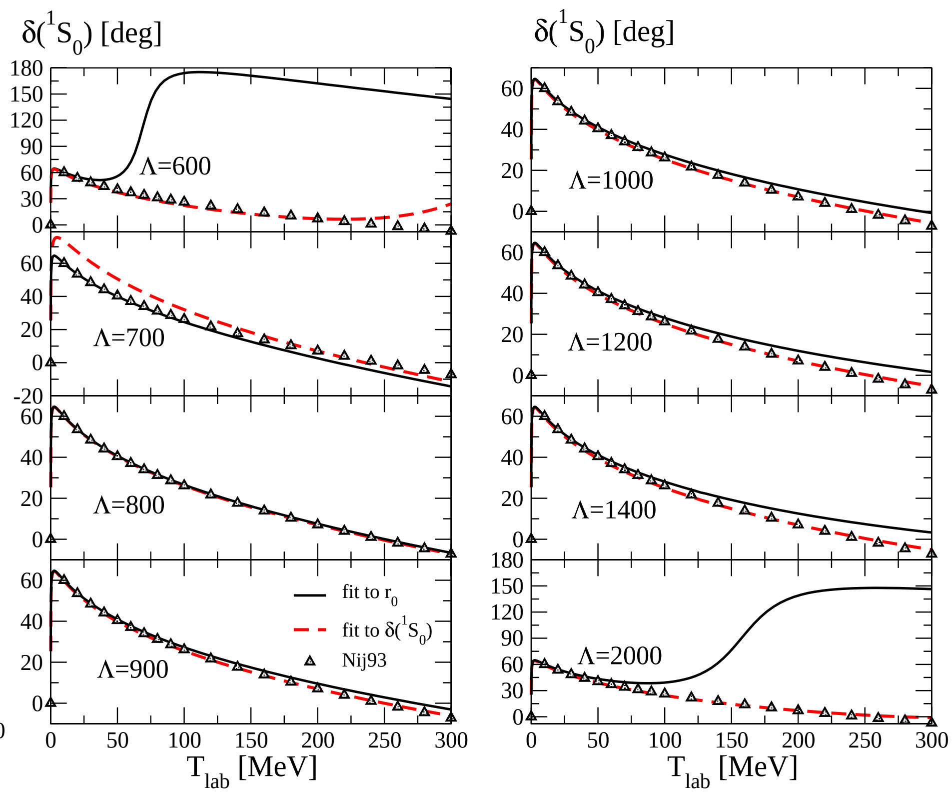
Results: Singlet S-wave

At higher Λ, has fitting point dependence.

DR NNLO(fails at 1GeV)

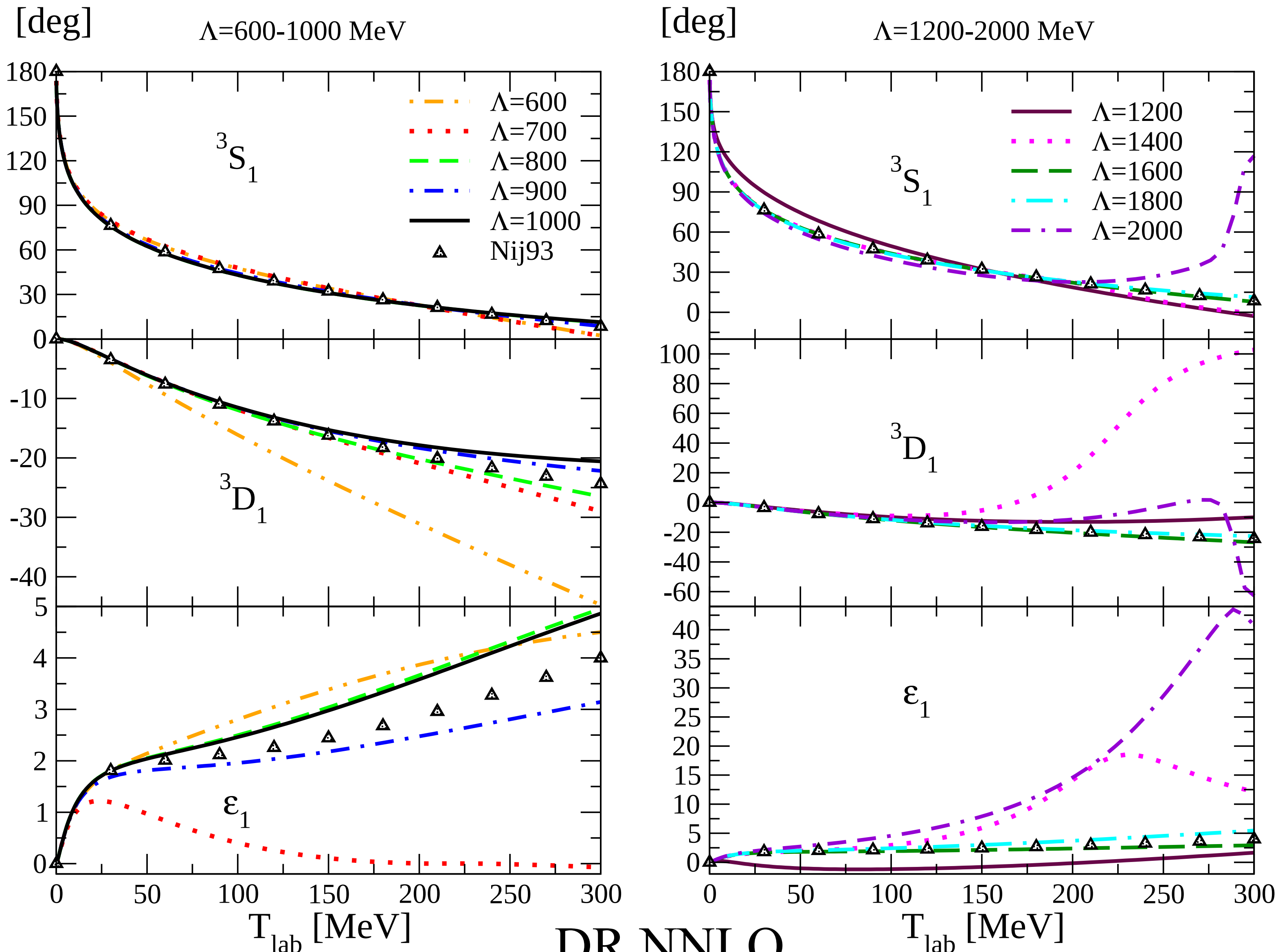


Full SFR NNLO(fails at 2 GeV)



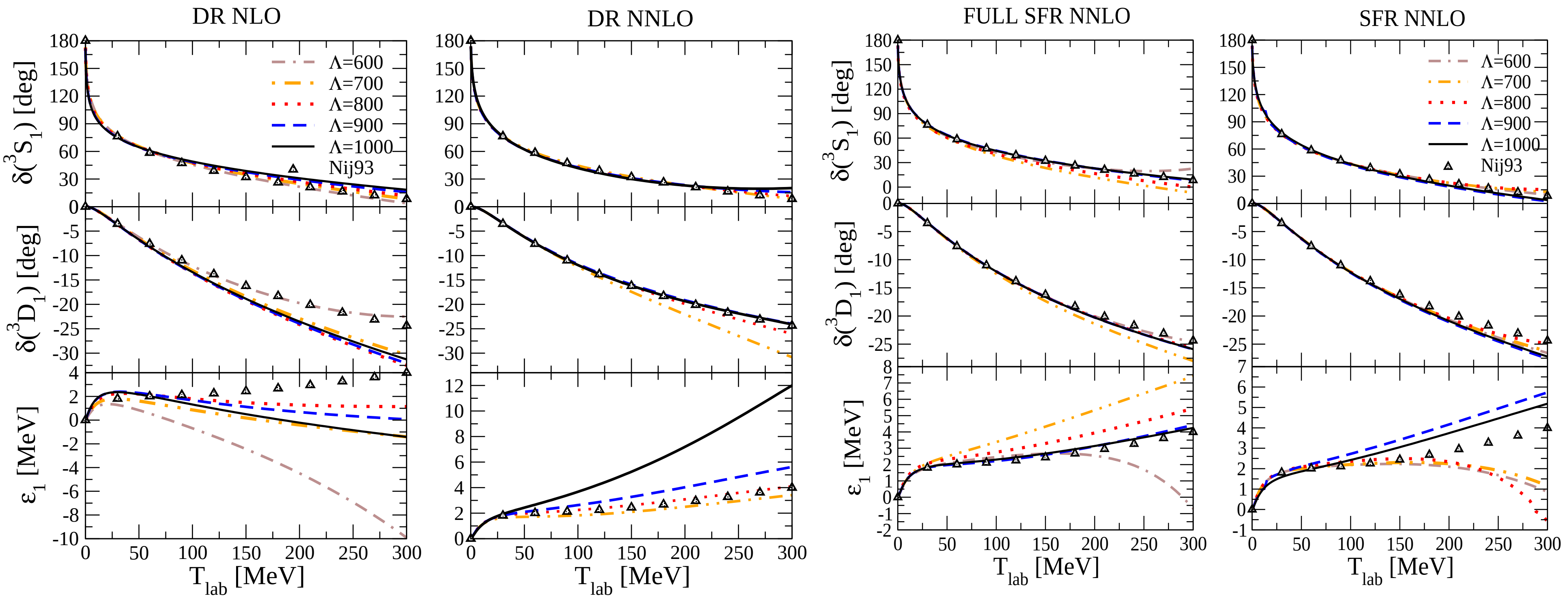
Λ cannot be too low (≤500 MeV) also.

Results: Triplet S-wave (2 sub+1 fitting)



DR NNLO

Allow all 3 constants to fit freely



3 fitting > 2 sub + 1 fitting .

=>The two low energy inputs prohibits the best fit.

Additional: Energy dependent contact term

- Replace (p^2+p'^2) by E.
- Why?

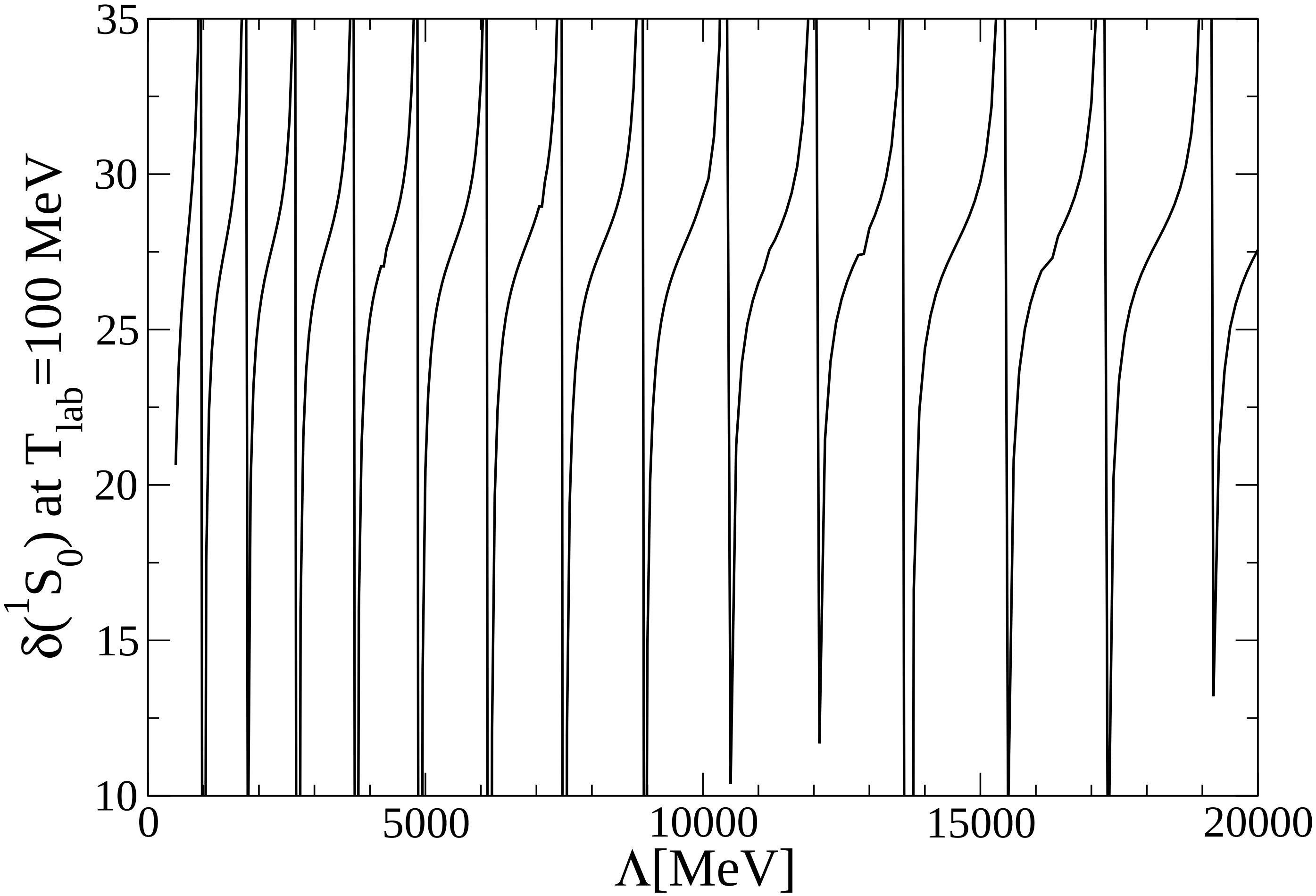
=>See whether the E-dep. term fit data better.

$$\lambda + \gamma E .... for\ {}^1S_0.\ \text{(two unknowns)} \quad \text{=> solve by 2 sub.}$$

$$\begin{pmatrix} \lambda + \gamma E & \lambda_t p'^2 \\ \lambda_t p^2 & 0 \end{pmatrix} .... for\ {}^3S_1 - {}^3D_1.\ \text{(three unknowns) => solve by 3 sub.}$$

Result: *When it fits, it fits better (than p-dep). But,*

Oscillation with respect to Λ



S-waves Summary

- **P-dep. contact term**:

  For $^1S_0$ the indep. of renormalization point breaks once Λ>1~1.2 GeV for DR NNLO, and Λ~2000 for SFR NNLO.

  For $^3S_1$-$^3D_1$, fit breaks down at about Λ~1.2 GeV in general.

- **E-dep. contact term**, there is oscillatory behavior.

1. Singlet channel: The first diverged phase shift appears at Λ~1000 MeV for DR NNLO

   Λ~2000 MeV for Full SFR NNLO.

2. Triplet channel: The first diverged phase shift appears at Λ~1200 MeV for DR NNLO and

   Λ~2300 MeV for Full SFR NNLO.

Conclusion on NN scattering

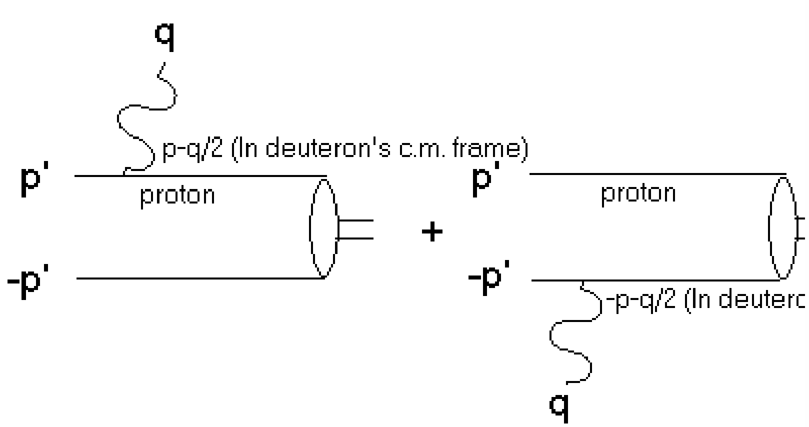
- Subtraction method provides:
  1. An easy way to go to high cutoff in LSE.
  2. A clean information of the dependence of results on the low energy observable (α^{SJ}).
- We found:
  1. **There is a critical cutoff Λ_C~1 GeV for DR TPE up to NNLO, after that the contact term in LSE dominate the result**.
  2. **Replacing the whole DR NNLO TPE by SFR brings Λ_C up to 2 GeV.**

Deuteron electro-disintegration

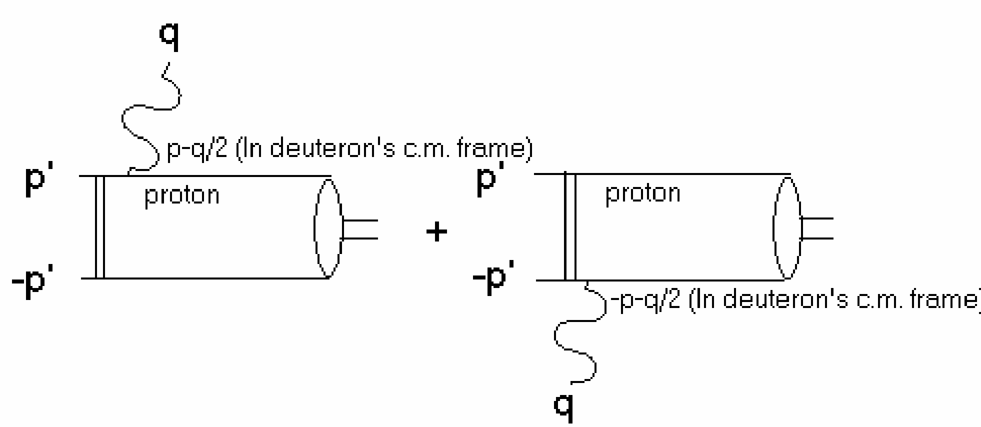
Motivation:

- Further test the wfs and t-matrices from χpt potential.
- Look for cutoff-dependence in other physical process.
- Compare results to traditional models.

Impulsive approximation



Final state interaction



Longitudinal response function

$$\frac{d\sigma}{d\Omega_e} = c\{\rho_L f_L + \rho_T f_T + \rho_{LT} f_{LT} + \rho_{TT} f_{TT} + h(\rho'_{LT} f'_{LT} + \rho'_T f'_T)\}.$$

Where h is the electron polarization degree, $\rho(f)_{L,T,LT,TT}$ describe the lepton(hadron) tensor.

$c = \frac{\alpha}{6\pi^2}\frac{k_2^L}{k_1^L q_\nu^4}$, where $k_{1,2}^L$ is the abs. value of the electron momentum in the Lab frame.

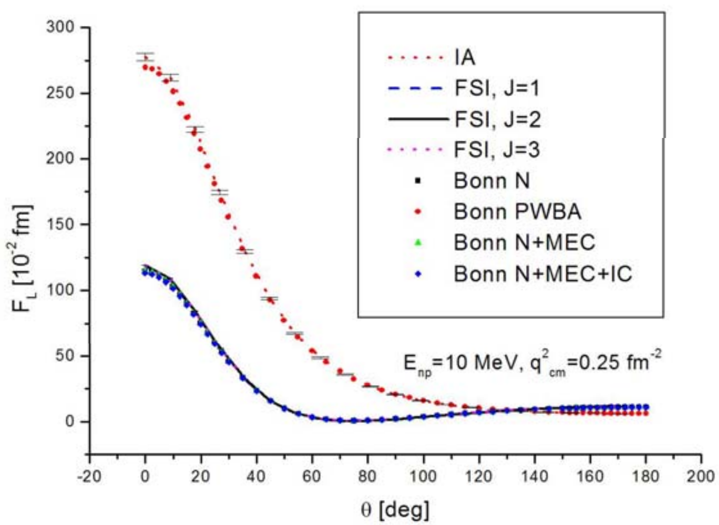
$$f_L = -\pi^2 2\alpha \mathrm{p}_{np} M_N \,|\, M_{T_f S' M_{s'}}(q,0;\omega)\,|^2$$

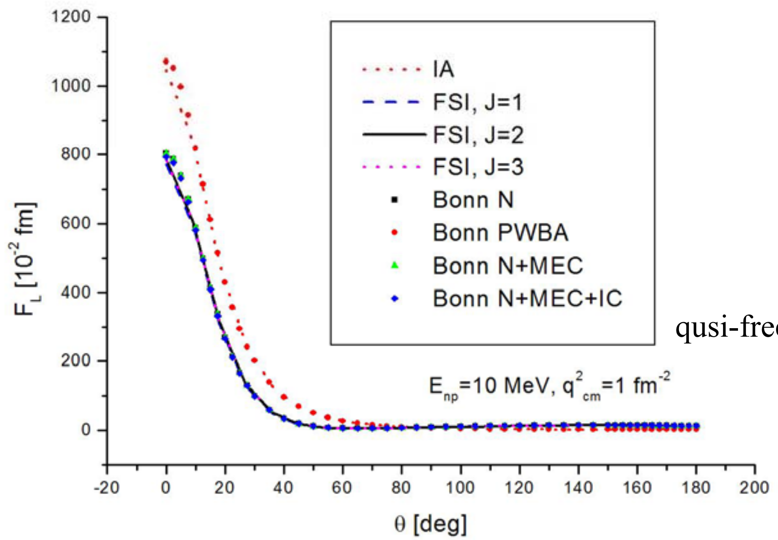
$$M_{T_f S' M_{s'}}(q,0;\omega) = <\mathbf{p}' S' M'_s T_f | J(\mathbf{q};0) | M> + <\mathbf{p}' S' M'_s T_f | t(E') G_0(E') J(\mathbf{q};0) | M>$$

Impulsive approximation (IA) Final state interaction (FSI)

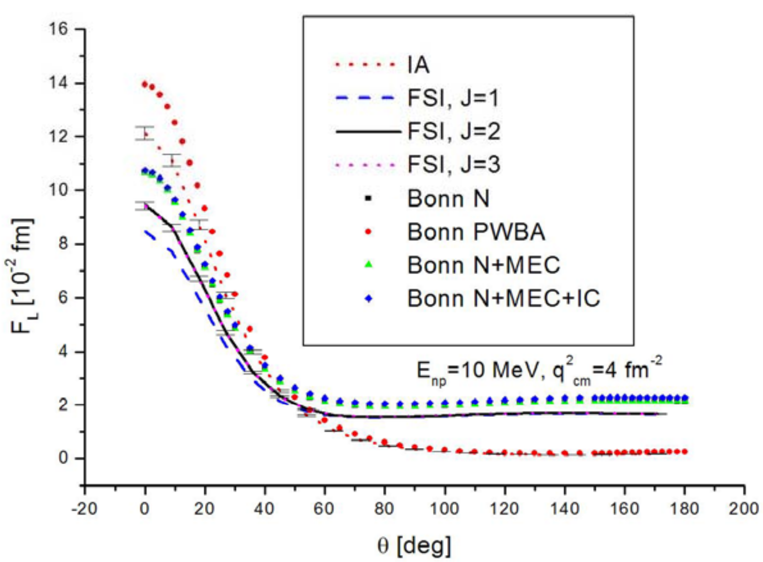
$\sim\Psi_{\mathbf{d}}(|\mathbf{p'}-\mathbf{q}/2|)$ $\sim\int d^3\mathbf{p}\; t(\mathbf{p}',\mathbf{p};E')(E'-\mathbf{p}^2/M+i\varepsilon)^{-1}\; \Psi_{\mathbf{d}}(|\mathbf{p}-\mathbf{q}/2|)$

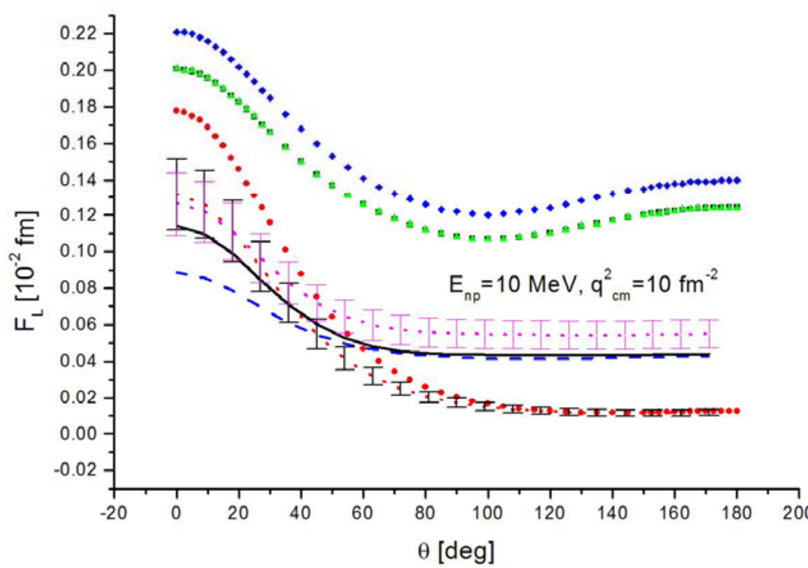
Results

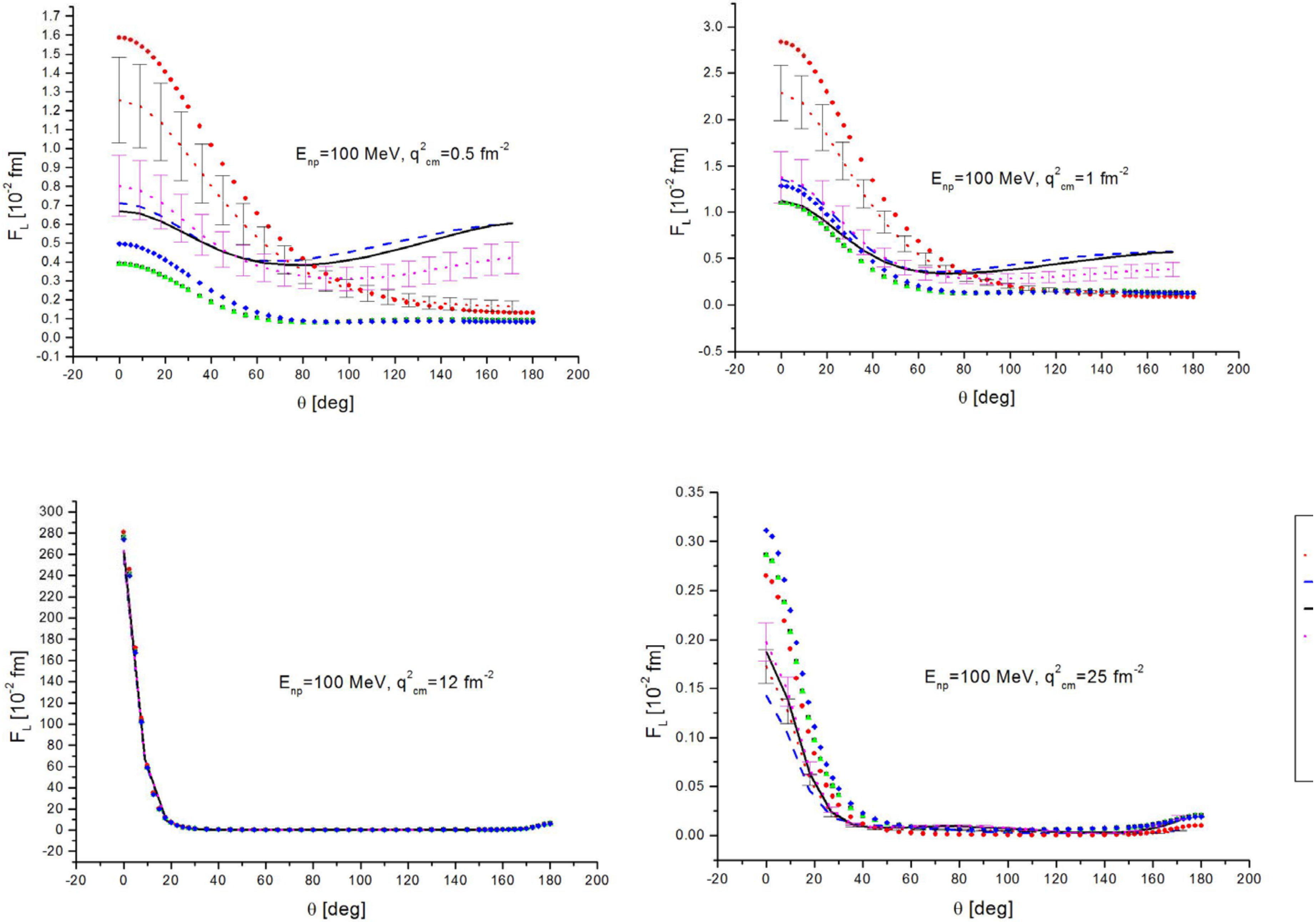




qusi-free ridge







E_np=100 MeV, q^2_cm=0.5 fm^-2
F_L [10^-2 fm]
θ [deg]
E_np=100 MeV, q^2_cm=1 fm^-2
F_L [10^-2 fm]
θ [deg]
E_np=100 MeV, q^2_cm=12 fm^-2
F_L [10^-2 fm]
θ [deg]
E_np=100 MeV, q^2_cm=25 fm^-2
F_L [10^-2 fm]
θ [deg]

Conclusion

E_{np} (MeV) / q^2_{cm} (fm^{-2})	10	30	60	100	120	160
0.25(q~100MeV)	√ √ √	√ ↕ √	↕ ↕ √	X X √	X X √	X X √
0.5(q~100MeV)	√ √ √	√ √ √	↕ √ √	X X √	X X √	X X √
1(q~100MeV)	√ √ X	√ √ ↕	↕ √ √	X X √	X X √	X X √
4(q~100MeV)	↕ ↕ ↕	√ √ X	√ √ X	↕ ↕ ↕	↕ ↕ ↕	↕ ↕ √
10(q~100MeV)	XX √	↕ X ↕	√ √ X	√ √ X	√ √ X	√ √ X
16(q~100MeV)	XX √	↕ X √	↕ ↕ ↕	√ √ X	√ √ X	√ √ X
25(q~100MeV)	XX √	XX √	XX √	X X ↕	↕ X ↕	↕ ↕ X

Λ-ind.? Match Bonn result? FSI important?

Future Works and application

- Apply the results for the deuteron electro-disintegration calculation.
- Use the same technique to evaluate the NN problem in perturbative way (with Bingwei Long).
- Evaluate the LEC's for the N^3LO:

=> eliminate the correlations between them cause by performing a best fit in the phase shift.

Application:

Our method enables us to obtain an EFT NN potential at "any" cutoff; it is faster (don't need to search for the L.E.C.'s) and with a more reasonable renormalization procedure.

$\Rightarrow$ This V_{EFT} can be used in further few body and nuclear structure calculation (evaluate Λ-dep., etc.).

THANK YOU!

Singular potential

Consider time-independent Schordinger eq. with $V = \frac{-\lambda}{r^n}$ as r~0:

$KE \sim \frac{\hbar^2}{mr^2}$, $PE \sim \frac{-\lambda}{r^n}$, Then for n>2, total E can takes arbitrarily large negative values for very small r.

=>Simplify the equation: (ignore m, $\hbar$, set $x = r/r_0$)

S wave, radial part with E=0: $u'' + \frac{\lambda}{r^n} u = 0$

For n=1, which is Coulomb, O.K.

For n=2, general solution is: $u = Ax^{1/2+\sqrt{\frac{1}{4}-\lambda}} + Bx^{1/2-\sqrt{\frac{1}{4}-\lambda}}$,

so for λ<1/4, by choosing A, B, we can make stationary state.

for λ>1/4, singular potential.

For n>3, singular potential.

$$T(0) = V + VG(0)T(0) \quad (5)$$

*Notation: $G(E) \equiv [E - H_0 + i\varepsilon]^{-1}$

$$T(E) \equiv T(p, p^*, E)$$

$$T = V + VGV + VGVGV + \ldots = V + VG(\underbrace{V + VGV + \ldots}_{T}) = V + VGT.$$

$$or, = V + (\underbrace{V + VGV + \ldots}_{T})GV = V + TGV$$

(5) becomes $T(0) = V + T(0)G(0)V$

$$T(E) = V + VG(E)T(E) = V(1 + G(E)T(E)),$$

$$\Rightarrow \quad T(E)(1 + G(E)T(E))^{-1} = V$$

$$T(0) = (1 + T(0)G(0))T(E)(1 + G(0)T(E))^{-1}$$

$$T(0)(1 + G(E)T(E)) = (1 + T(0)G(0))T(E)$$

$$T(0) + T(0)G(E)T(E) = T(E) + T(0)G(0)T(E)$$

$$T(E) = T(0) + T(0)[G(E) - G(0)]T(E)$$

$$[1 + T(0)(G(0) - G(E))]T(E) = T(0)$$

=> Get T(E) from T(0)

Chiral Effective Lagrangian

- Preserve symmetry, d.o.f., relevant for QCD at low energies (spontaneously breaking scale Λ~1 GeV). Not directly deduced from QCD.

$$L^{massless}_{QCD} = \bar{q} i\gamma^{\mu}(\partial_{\mu} + i g_s \frac{\lambda^{\alpha}}{2} G^{\alpha}_{\mu}) q - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\alpha\mu\nu} + L_{heavy\ quarks}$$

$$L_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \frac{1}{4} G^{\alpha}_{\mu\nu} G_{\alpha}^{\mu\nu} + L_{heavy\ quarks}$$

$$L^{(0)}_{Eff} = L_{\pi\pi} + L_{\pi N} + L_{NN} = \frac{1}{2}\partial_{\mu}\pi\cdot\partial^{\mu}\pi - \frac{1}{2} m_{\pi}^2 \pi^2 + \bar{N}[i\partial_0 + 2D^{-1}F_{\pi}^{-2}\tilde{t}\cdot(\pi\times\dot{\pi})$$

$$-M - 2D^{-1}F_{\pi}^{-1} g_A \tilde{t}\cdot(\sigma\cdot\nabla)\pi]N - \frac{1}{2}C_s(\bar{N}N)(\bar{N}N) - \frac{1}{2}C_T(\bar{N}\sigma N)(\bar{N}\sigma N)$$

- Expanded in power of "Q", "m".
- Non-renormalizable, fitted to data at each order, =>those fitting constants are called low energy constants (LEC's).