The dependence of few-nucleon bound states on light quark masses

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COULD FUNDAMENTAL "CONSTANTS" VARY OVER THE HISTORY OF THE UNIVERSE?

Theories unifying gravity with other interactions suggest the possibility of temporal and spatial variations of physical "constants" in the expanding universe.

Some evidence for variations in the fine structure constant α, strength of the strong interaction, and particle masses has been inferred from studies of big bang nucleosynthesis, quasar absorption spectra, and the Oklo natural nuclear reactor.

More generally, we might ask how much could "constants" change and still give us a universe similar to our own?

Program for studying the universe's dependence on the quark mass ($X_q = m_q/\Lambda_{QCD}$):

- Study how hadron masses depend on light quark mass
- Evaluate how nuclear binding depends on hadron masses
- Investigate consequences for big bang nucleosynthesis and "nuclear clocks"

Unanticipated application: qualitative guidance for extrapolation of nuclear physics results from Lattice QCD calculations.

HADRON MASS DEPENDENCE ON CURRENT-QUARK MASS

Prediction from a Dyson-Schwinger equation study of the sigma terms of light-quark hadrons:

V.V.Flambaum, A.Höll, P.Jaikumar, C.D.Roberts and S.V.Wright [Few-Body Syst. **38**, 31 (2006)]

$$\frac{\delta m_H}{m_H} = \frac{\sigma_H}{m_H}\frac{\delta m_q}{m_q} \qquad\qquad m_q = (m_u + m_d)/2$$

	π	ρ	ω	N	Δ
$\frac{\sigma_H}{m_H}$	0.498	0.030	0.043	0.064	0.041

In later work the ρ and ω values were reduced to 0.021 and 0.034, respectively.
Other models are possible, but we expect pion mass to vary most rapidly due to Gell-Mann-Oakes-Renner relation $m_\pi^2 = m_q$ and that other masses will vary in the same direction, i.e., all get larger or smaller together.

Nuclear Hamiltonian and hadron mass

To study the effect of δm_H on nuclear systems, we consider Hamiltonians of the form:

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ \ldots$$

We examine three interaction models:

- Argonne v_{28} (AV28): coupled-channels OPE with explicit Δ's fit to 1981 phase shifts
- Argonne v_{14} (AV14): nucleons-only with approximate TPE, phase-equivalent to AV28
- Argonne v_{18} (AV18): updated AV14 with charge-independence-breaking fit to 1993 data, weaker $f_{\pi NN}$, deeper well, stiffer core; supplement with Urbana IX (UIX) V_{ijk}

These potentials all have the operator structure:

$$v_{ij} = v_\gamma(r_{ij}) + \sum_p [v^p_\pi(r_{ij}) + v^p_I(r_{ij}) + v^p_S(r_{ij})] O^p_{ij} \ .$$

The number of operators O^p_{ij} is 28, 14, or 18 according to AVxx.

The NN one-pion-exchange (OPE) in all cases is:

$$v_\pi(NN \to NN) = \frac{f^2_{\pi NN}}{4\pi}\left(\frac{m_\pi}{m_s}\right)^2 \frac{m_\pi c^2}{3}[Y(m_\pi r)\sigma_1 \cdot \sigma_2 + T(m_\pi r)S_{12}](\tau_1 \cdot \tau_2)$$

$$Y(mr) = \frac{e^{-\mu r}}{\mu r}\xi(r) \qquad T(mr) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) Y(mr)\xi(r)$$

where m_s is a scaling mass and $\xi(r)$ is a short-range form factor.

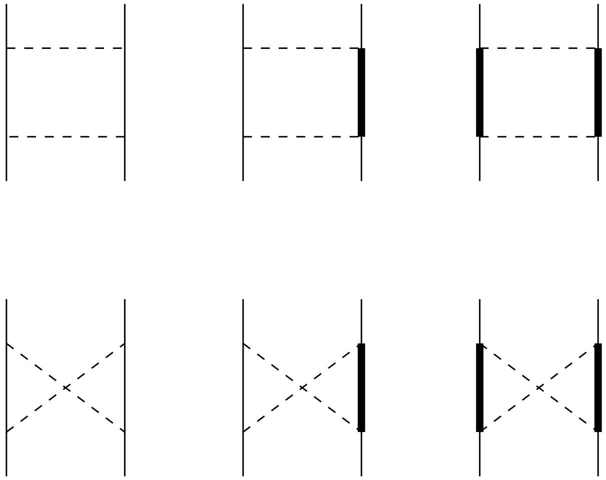
The AV28 model adds transition terms to states with intermediate Δ's

$$v_\pi(NN \to N\Delta) = \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi}\left(\frac{m_\pi}{m_s}\right)^2 \frac{m_\pi c^2}{3}[Y(m_\pi r)\sigma_1 \cdot \mathbf{S}_2 + T(m_\pi r)S^{II}_{12}](\tau_1 \cdot \mathbf{T}_2)$$

$$v_\pi(NN \to \Delta\Delta) = \frac{f^2_{\pi N\Delta}}{4\pi}\left(\frac{m_\pi}{m_s}\right)^2 \frac{m_\pi c^2}{3}[Y(m_\pi r)\mathbf{S}_1 \cdot \mathbf{S}_2 + T(m_\pi r)S^{III}_{12}](\mathbf{T}_1 \cdot \mathbf{T}_2)$$

with $\mathbf{S}_i$ ($\mathbf{T}_i$) the transition spin (isospin) operator that connects spin (isospin) $\frac{1}{2}$ and $\frac{3}{2}$ states.
AV28 also includes $V_{N\Delta\to\Delta N}$, $V_{N\Delta\to N\Delta}$, $V_{N\Delta\to\Delta\Delta}$, and $V_{\Delta\Delta\to\Delta\Delta}$ terms.

Smith & Pandharipande [Nucl. Phys. **A256**, 327 (1976)] showed the transition potentials effectively represent the time-ordered diagrams for a nonrelativistic interaction Lagrangian, with box diagrams approximating the 2nd Born terms to ∼10%, while cross-box diagrams largely cancel.



The intermediate- and short-range parts of the potentials are given by:

$$v_I^p(r_{ij}) = I^p T^2(m_\pi r) \qquad\qquad v_S^p(r_{ij}) = (S^p + Q^p r + R^p r^2) W(r)$$

$W(r)$ is a Woods-Saxon function and the CI operators are

$$O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes [1, \tau_i \cdot \tau_j]$$

The intermediate-range function $T^2(m_\pi r)$ gives the AV14 & AV18 models an approximate TPE character. This can be seen by using the transition potentials in a closure approximation:

$$v^{2\pi} = v^{2\pi}_{1\Delta} + v^{2\pi}_{2\Delta}$$

$$v^{2\pi}_{1\Delta} = \left[X^{II\dagger}_{ij}\tau_i \cdot \mathbf{T}^\dagger_j\right] \frac{-1}{\bar{E}_1 + (m_\Delta - m_N)} \left[X^{II}_{ij}\tau_i \cdot \mathbf{T}_j\right] + (i \leftrightarrow j)$$

$$v^{2\pi}_{2\Delta} = \left[X^{III\dagger}_{ij}\mathbf{T}^\dagger_i \cdot \mathbf{T}^\dagger_j\right] \frac{-1}{\bar{E}_2 + 2(m_\Delta - m_N)} \left[X^{III}_{ij}\mathbf{T}_i \cdot \mathbf{T}_j\right]$$

$$X^{II}_{ij} = \frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \left(\frac{m_\pi}{m_s}\right)^2 \frac{m_\pi c^2}{3} \left[Y(m_\pi r)\sigma_i \cdot \mathbf{S}_j + T(m_\pi r) S^{II}_{ij}\right]$$

$$X^{III}_{ij} = \frac{f^2_{\pi N\Delta}}{4\pi} \left(\frac{m_\pi}{m_s}\right)^2 \frac{m_\pi c^2}{3} \left[Y(m_\pi r)\mathbf{S}_i \cdot \mathbf{S}_j + T(m_\pi r) S^{III}_{ij}\right]$$

Sum over transition spin and isospin operators using rules analogous to standard Pauli spin operators:

$$\sigma \cdot \sigma = 3 \;\; ; \;\; \sigma \times \sigma = 2i\sigma \;\; ; \;\; \sigma \cdot \mathbf{A}\, \sigma \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{B} + i\sigma \cdot \mathbf{A} \times \mathbf{B} \, ;$$

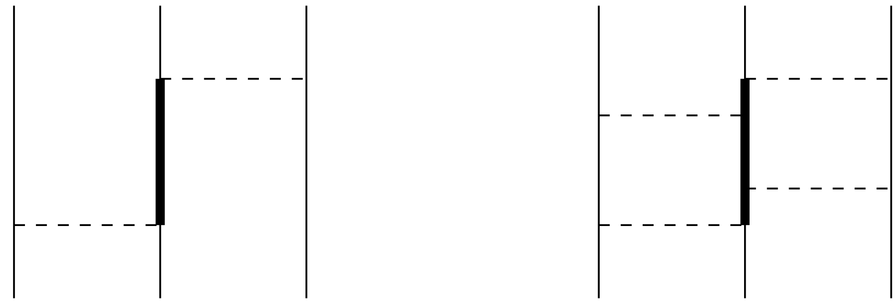
$$\mathbf{S}^\dagger \cdot \mathbf{S} = 2 \; ; \; \mathbf{S}^\dagger \times \mathbf{S} = -\frac{2}{3} i\sigma \;\; ; \;\; \mathbf{S}^\dagger \cdot \mathbf{A}\, \mathbf{S} \cdot \mathbf{B} = \frac{2}{3}\mathbf{A} \cdot \mathbf{B} - \frac{1}{3} i\sigma \cdot \mathbf{A} \times \mathbf{B} \, .$$

$$
\begin{aligned}
v^{2\pi} \quad &= \quad (8\,\chi_1 + \frac{32}{9}\chi_2)[T^2(m_\pi r) + \frac{1}{2}Y^2(m_\pi r)] \\
&+ \quad (\frac{8}{3}\chi_1 - \frac{16}{27}\chi_2)[T^2(m_\pi r) + \frac{1}{2}Y^2(m_\pi r)]\ (\tau_i \cdot \tau_j) \\
&- \quad (\frac{4}{3}\chi_1 - \frac{8}{27}\chi_2)[T^2(m_\pi r) - Y^2(m_\pi r)]\ (\sigma_i \cdot \sigma_j) \\
&- \quad (\frac{4}{9}\chi_1 + \frac{4}{81}\chi_2)[T^2(m_\pi r) - Y^2(m_\pi r)]\ (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \\
&+ \quad (\frac{4}{3}\chi_1 - \frac{8}{27}\chi_2)[T^2(m_\pi r) - Y(m_\pi r)T(m_\pi r)]\ S_{ij} \\
&+ \quad (\frac{4}{9}\chi_1 + \frac{4}{81}\chi_2)[T^2(m_\pi r) - Y(m_\pi r)T(m_\pi r)]\ S_{ij}(\tau_i \cdot \tau_j)
\end{aligned}
$$

$$
\begin{aligned}
\chi_1 \quad &= \quad \frac{-2}{\bar{E}_1 + (m_\Delta - m_N)} \left(\frac{f_{\pi NN} f_{\pi N\Delta}}{4\pi} \frac{m_\pi c^2}{3} \right)^2 \\
\chi_2 \quad &= \quad \frac{-1}{\bar{E}_2 + 2(m_\Delta - m_N)} \left(\frac{{f_{\pi N\Delta}}^2}{4\pi} \frac{m_\pi c^2}{3} \right)^2
\end{aligned}
$$

One could adjust $f_{\pi N\Delta}$, $\bar{E}_1$, and $\bar{E}_2$ to fit data when constructing a potential, or just fit the entire coefficient I^p_{ij} of each O^p_{ij}, which is the procedure for AV14 and AV18.

Transition potentials with closure approximation can also be used to construct $3N$ potential terms, including standard Fujita-Miyazawa force. Phenomenological short-range repulsive term added to Urbana models.



$$V_{ijk}^{2\pi} = A_{2\pi}^{PW}\left(\sum_{cyc}\{X_{ij},X_{ik}\}\{\tau_i\cdot\tau_j,\tau_i\cdot\tau_k\} + \frac{1}{4}[X_{ij},X_{ik}][\tau_i\cdot\tau_j,\tau_i\cdot\tau_k]\right)$$

$$X_{ij} = Y(m_\pi r_{ij})\sigma_i\cdot\sigma_j + T(m_\pi r_{ij})S_{ij}$$

$$A_{2\pi}^{PW} = \frac{-2/9}{\bar{E}_3 + (m_\Delta - m_N)}\left(\frac{f_{\pi NN}f_{\pi N\Delta}}{4\pi}\left(\frac{m_\pi}{m_s}\right)^2\frac{m_\pi c^2}{3}\right)^2$$

$$V_{ijk}^{R} = A_R\sum_{cyc}T^2(m_\pi r_{ij})T^2(m_\pi r_{ik})$$

Parameter $\bar{E}_3$ could be adjusted (along with $f_{\pi N\Delta}$ from NN potential) but instead the strengths $A_{2\pi}^{PW}$ and A_R were adjusted to fit ^{3}H, ^{4}He binding and nuclear matter saturation density in many-body calculations when used with given NN potential, e.g., AV14+UVII or AV18+UIX.

Baryon mass dependence

Baryon masses appear in the kinetic energy operator:

$$K_i = -\frac{\hbar^2}{2m_i}\nabla_i^2 + (m_i - m_N)c^2$$

with m_i being m_N or m_Δ as appropriate.

Also contribute through the NN–$N\Delta$–$\Delta\Delta$ coupled channels in AV28, e.g., in the closure approximation

$$\bar{E}_1 \approx \frac{\hbar^2 \bar{k}^2}{2m_N} + \frac{\hbar^2 \bar{k}^2}{2m_\Delta} \qquad \bar{E}_2 \approx 2\frac{\hbar^2 \bar{k}^2}{2m_\Delta}$$

Coupled-channels solution for AV28 handles this dependence exactly.

Approximate this effect in AV14 by altering intermediate-range attraction

$$\bar{I}^p \approx (1 + \delta_N + \delta_\Delta) I^p$$

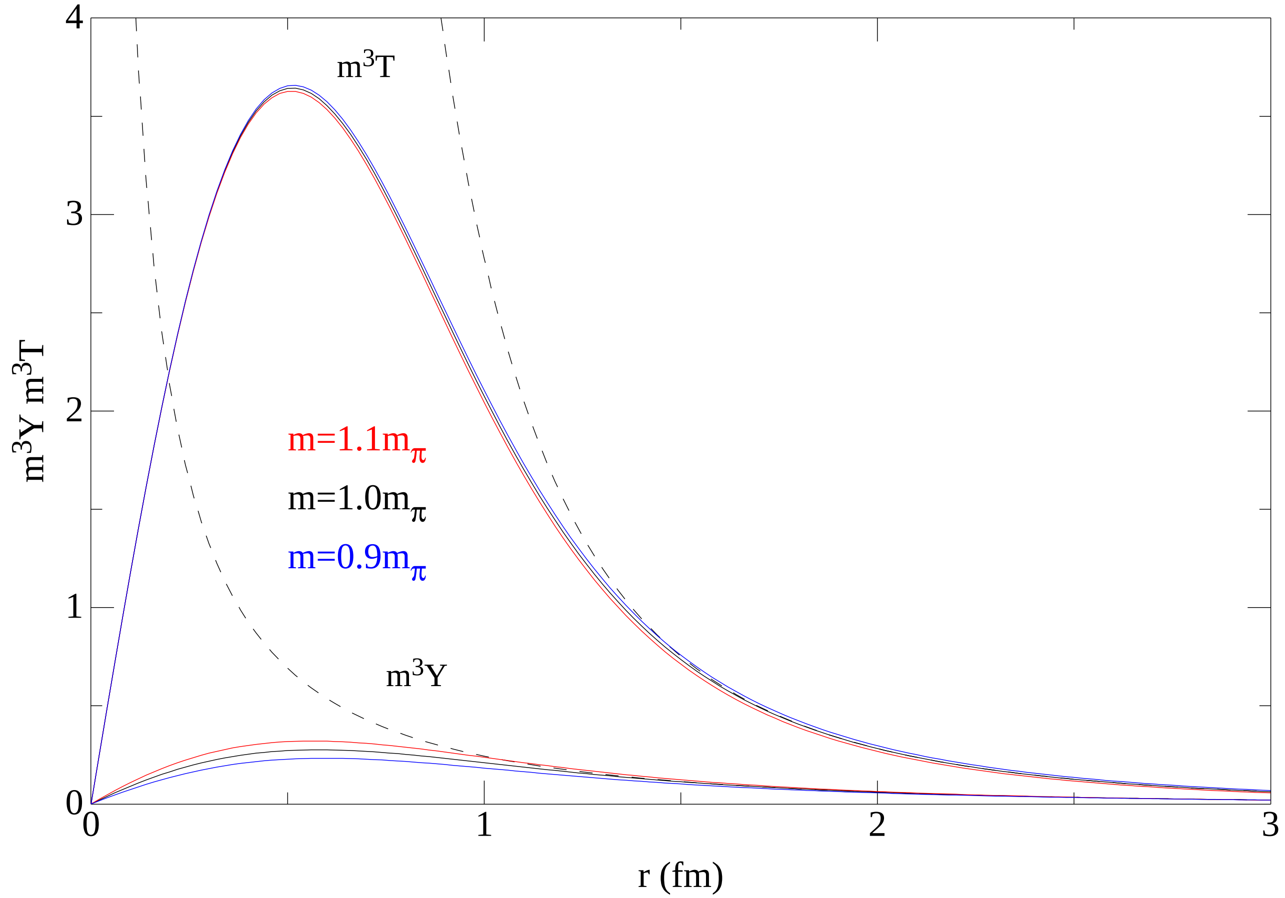
Fix δ_N and δ_Δ by requiring same mass dependence as phase-equivalent AV28.

$$\delta_N = 0.49\frac{\delta m_N}{m_N} \qquad \delta_\Delta = -0.57\frac{\delta m_\Delta}{m_\Delta}$$

Assume same factors are approximately correct modifications for AV18.

Pion mass dependence

Pion mass-dependence enters through $m_\pi^3 Y(m_\pi r)$ and $m_\pi^3 T(m_\pi r)$ in OPE and TPE, etc.
The scaling mass m_s is not changed, and possible variations in the coupling strength are neglected.
Same dependence as Beane & Savage [Nucl. Phys. **A713**, 148 (2003).]



Volume integral of (cutoff) $m^3 Y(mr) \approx$ constant with $r = 2/m$ crossing.
However, if m increases (cutoff) $m^3 T(mr)$ decreases for all r and vice versa.

Add pion mass dependence in 3-4 steps:

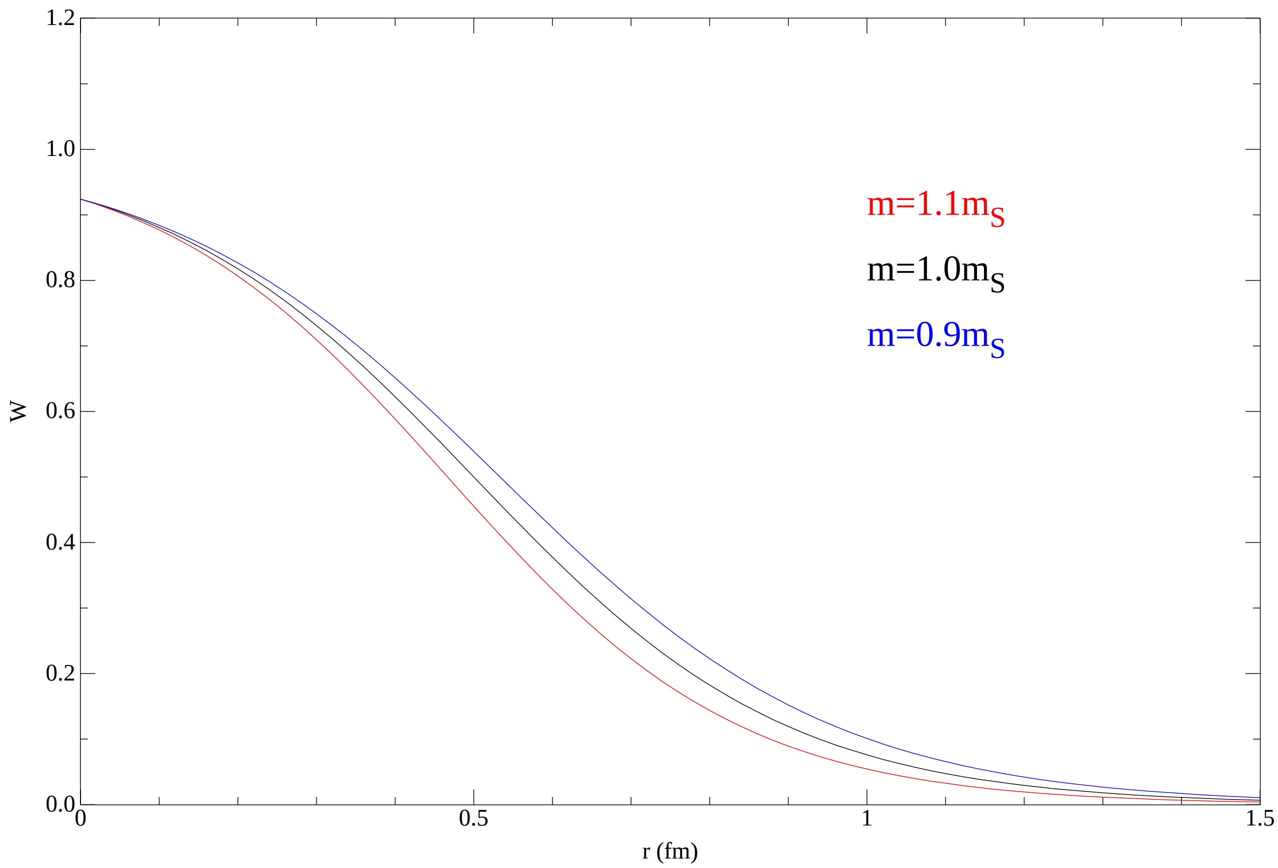
- change m_π in OPE only (including generalized OPE for AV28)
- add change in static TPE ($I^p O^p_{ij}$ for p=1,6)
- add change in all other TPE ($I^p O^p_{ij}$ for p=7,14)
- add change in $V^{2\pi}_{ijk}$ (no attempt to change V^R_{ijk})

HEAVY MESON MASS DEPENDENCE

No heavy mesons in AVxx models, so alter short-range Woods-Saxon $W(r)$ to mimic vector-meson Yukawa coupling $\propto m_V Y(m_V r)$ by changing range and strength of range parameters:

$$\frac{\delta r_0}{r_0} = \frac{\delta a}{a} = -\frac{2}{3}\frac{\delta m_V}{m_V}$$

$$W(r) = \frac{1}{1 + \exp[(r - r_0)/a]}$$



$W(r = 0)$ remains constant while volume integral varies as m_V^{-2}.

Energy calculations

Study deuteron and ^{1}S$_0$ virtual bound state by direct solution of two-body equations.

Study $A = 3 - 8$ nuclei using variational Monte Carlo (VMC) method:

Construct suitably parametrized trial wave functions Ψ_V and evaluate upper bound to ground-state energy:

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Simplified trial wave function:

$$|\Psi_V\rangle = [1 + \sum_{i<j<k} U_{ijk}][\mathcal{S} \prod_{i<j} (1 + U_{ij})] \prod_{i<j} f_{ij} |\Phi_A(JMTT_3)\rangle$$

$\Phi_A(JMTT_3)$ is antisymmetric product of single-particle functions coupled to given quantum numbers (translationally invariant with multiple spatial-symmetry components, but NOT harmonic oscillator)
f_{ij} are central (mostly short-ranged repulsion) correlations
U_{ij} are non-commuting 2-body correlations from v_{ij}

$$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O^p_{ij}$$

U_{ijk} are 3-body correlations from V_{ijk}

Energy results

VMC energies of light nuclei in MeV for the different Hamiltonians compared to experiment

	$^1S_0(np)$	^{2}H	^{3}H	^{3}He	^{4}He	^{5}He	^{6}Li	^{7}Li	^{8}Be
AV28	0.0661	-2.2250							
AV14	0.0663	-2.2250	-7.50	-6.88	-23.60	-21.26	-24.31	-28.31	-40.26
AV18+UIX	0.0665	-2.2246	-8.24	-7.49	-27.50	-25.26	-28.22	-33.33	-48.50
(GFMC)	0.0665	-2.2246	-8.46	-7.71	-28.33	-26.90	-31.10	-37.80	-54.40
Expt.		-2.2246	-8.48	-7.72	-28.30	-27.41	-31.99	-39.24	-56.50

Note $^1S_0$ virtual-bound-state energies are reported as positive quantity.

Evaluating mass dependence

Change hadron masses m_H one at a time $\pm 0.1\%$ and recalculate E. Two-nucleon cases evaluated exaclty. Multi-nucleon cases evaluated using same variational parameters and same Monte Carlo samples (from stored walk) to reduce sampling noise; correlation functions f_{ij}, U_{ij}, and U_{ijk} do change slightly because the altered v_{ij} and V_{ijk} are used as input to their construction.

Results are expressed as dimensionless derivatives:

$$\Delta\mathcal{E}(m_H) = \frac{\delta E/E}{\delta m_H/m_H}$$

They can be combined with any given model for the correlation between hadron and quark masses:

$$E(m_q) = E_0\left[1 + \sum_{m_H} \Delta\mathcal{E}(m_H)\frac{\delta m_H(m_q)}{m_H}\right]$$

TWO-NUCLEON RESULTS

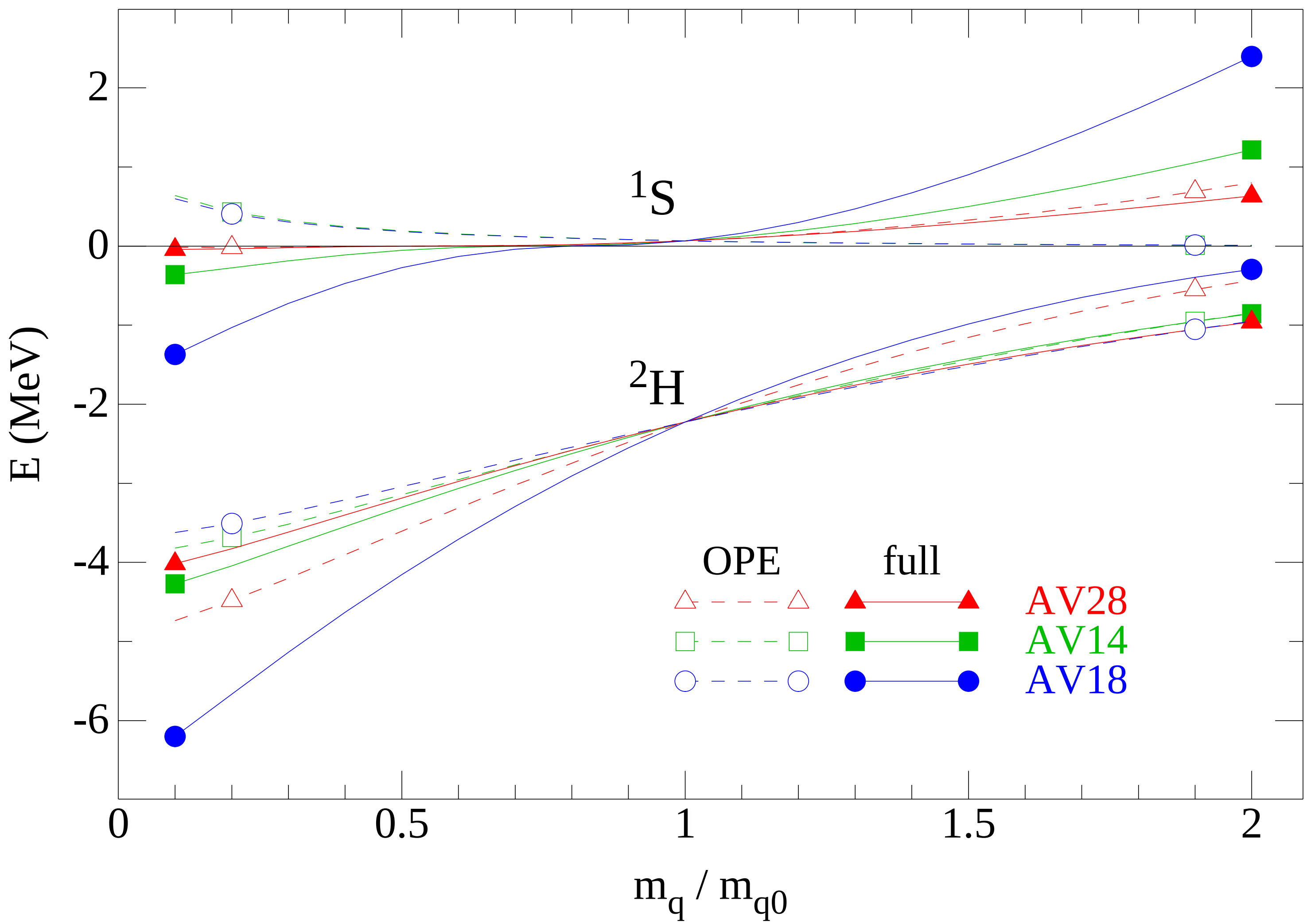
$\Delta\mathcal{E}(m_H)$ for the ${}^1\mathrm{S}_0(np)$ virtual bound state ϵ_v and the deuteron Q

m_H	$\Delta\epsilon_v$			ΔQ		
	AV28	AV14	AV18	AV28	AV14	AV18
m_N	-88.1	-32.6	-33.4	13.06	8.63	8.90
$m_N + \delta_N$		-91.2	-121.2		13.03	17.82
m_Δ	63.9			-5.15		
δ_Δ		68.1	102.2		-5.12	-10.36
m_π (OPE)	9.5	-4.1	-3.8	-2.23	-1.55	-1.40
m_π (+TPE-s)	24.4	35.5	53.0	-3.63	-4.02	-6.70
m_π (+TPE-L)				-4.02	-4.31	-6.74
m_V	-153.7	-245.0	-381.9	20.88	22.92	41.74

Dmitriev, Flambaum & Webb [Phys. Rev. D**69**, 063506 (2004)] find relation between ΔQ and $\Delta\epsilon_v$:

$$\frac{\Delta\epsilon_v(m_H)}{\Delta Q(m_H)} \approx -\frac{\sqrt{Q}}{\sqrt{\epsilon_v}}$$

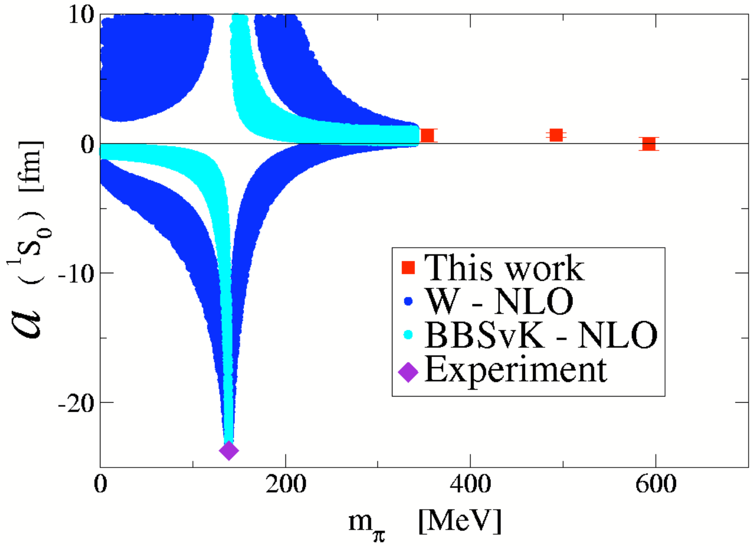
This is satisfied within factor of ~ 2 for all cases but m_π (OPE) for AV14 and AV18 which have wrong sign.

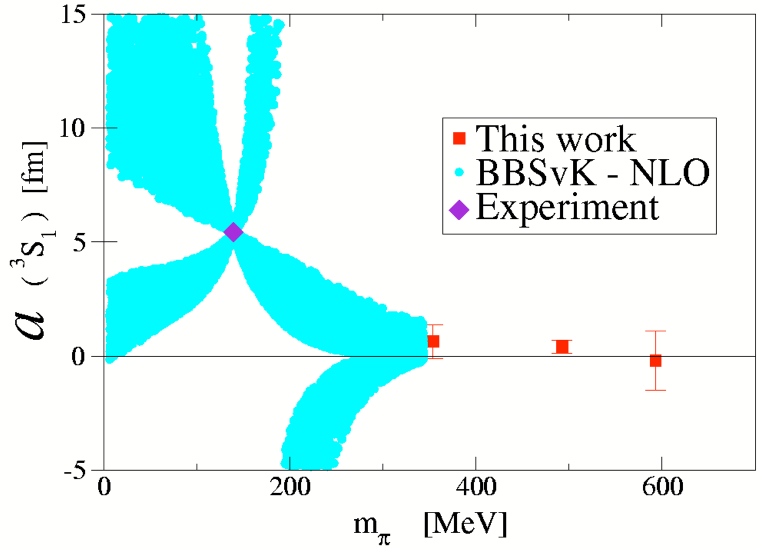


Dashed lines are m_π (OPE) only — Solid lines are "full" calculation with DSE $\delta m_H/m_H$:
AV28 includes m_N, m_Δ, m_π (+TPE-L), m_V; AV14 & AV18 include $m_N + \delta_N$, δ_Δ, m_π (+TPE-L), m_V.

NUCLEON-NUCLEON

Beane, Bedaque, KO, Savage hep-lat/0602010





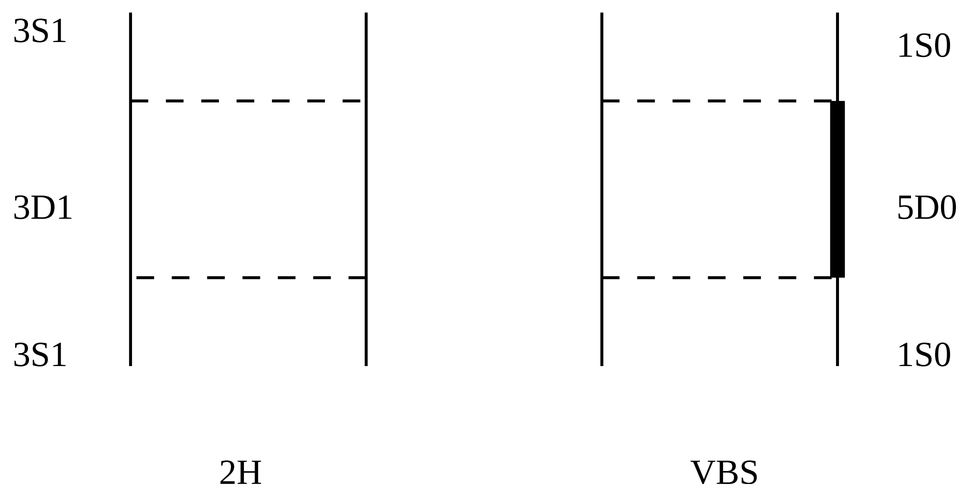
BBSvK: Beane Bedaque Savage van Kolck '02

W: Weinberg '90; Weingberg '91; Ordonez et.al '95

Fukugita et al. '95

Why should deuteron and virtual bound state behave the same?

Dominant effect is change of m_π in $T(m_\pi r)$ coupling ${}^3\mathrm{S}_1$-${}^3\mathrm{D}_1$ states in deuteron. Virtual bound state has no tensor force with nucleons only, but with Δ degree of freedom intermediate range attraction is largely attributable to ${}^1\mathrm{S}_0(NN)$-${}^5\mathrm{D}_0(N\Delta)$ coupling.

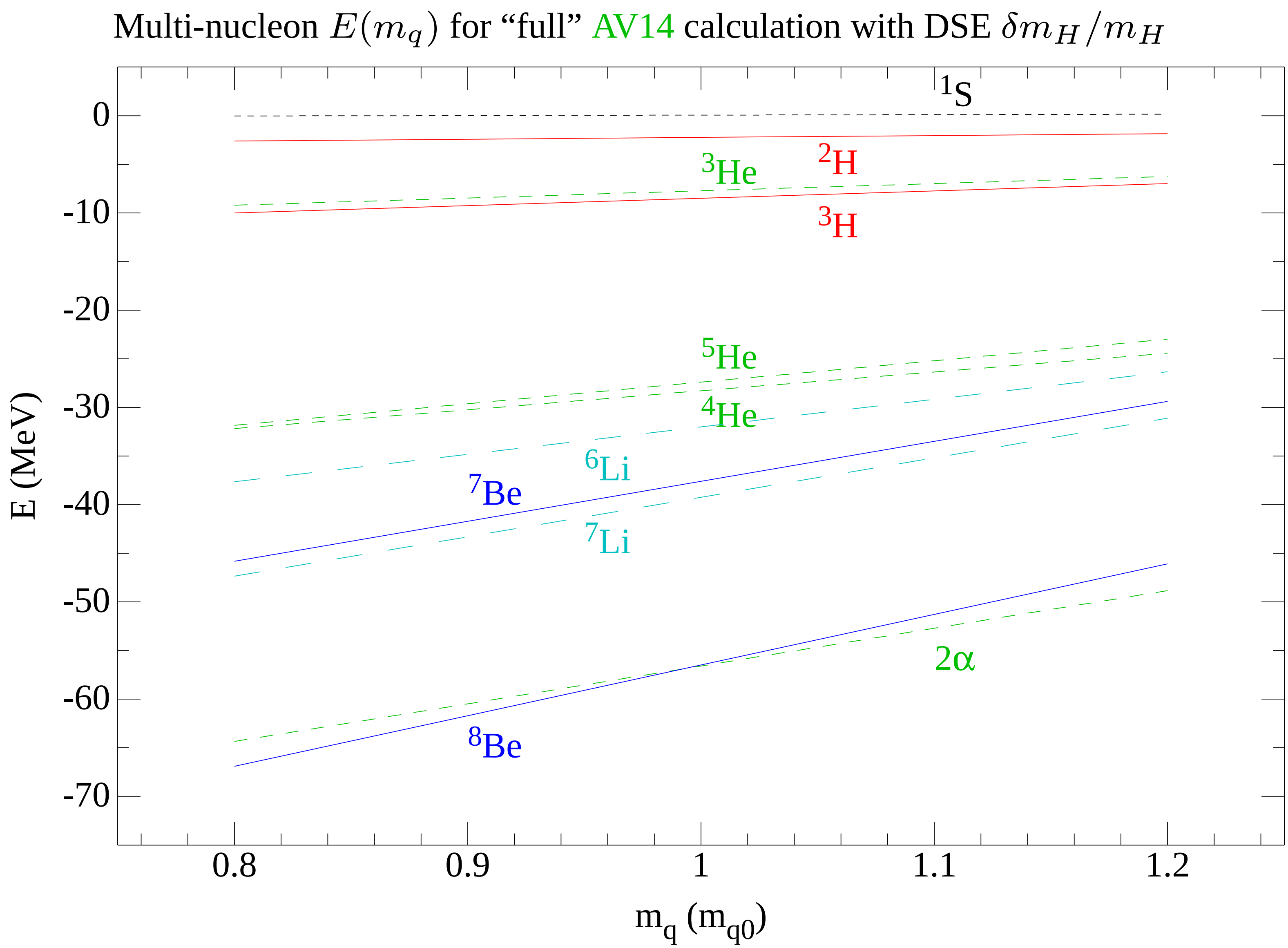


Multi-nucleon $\Delta\mathcal{E}(m_H)$ for AV14

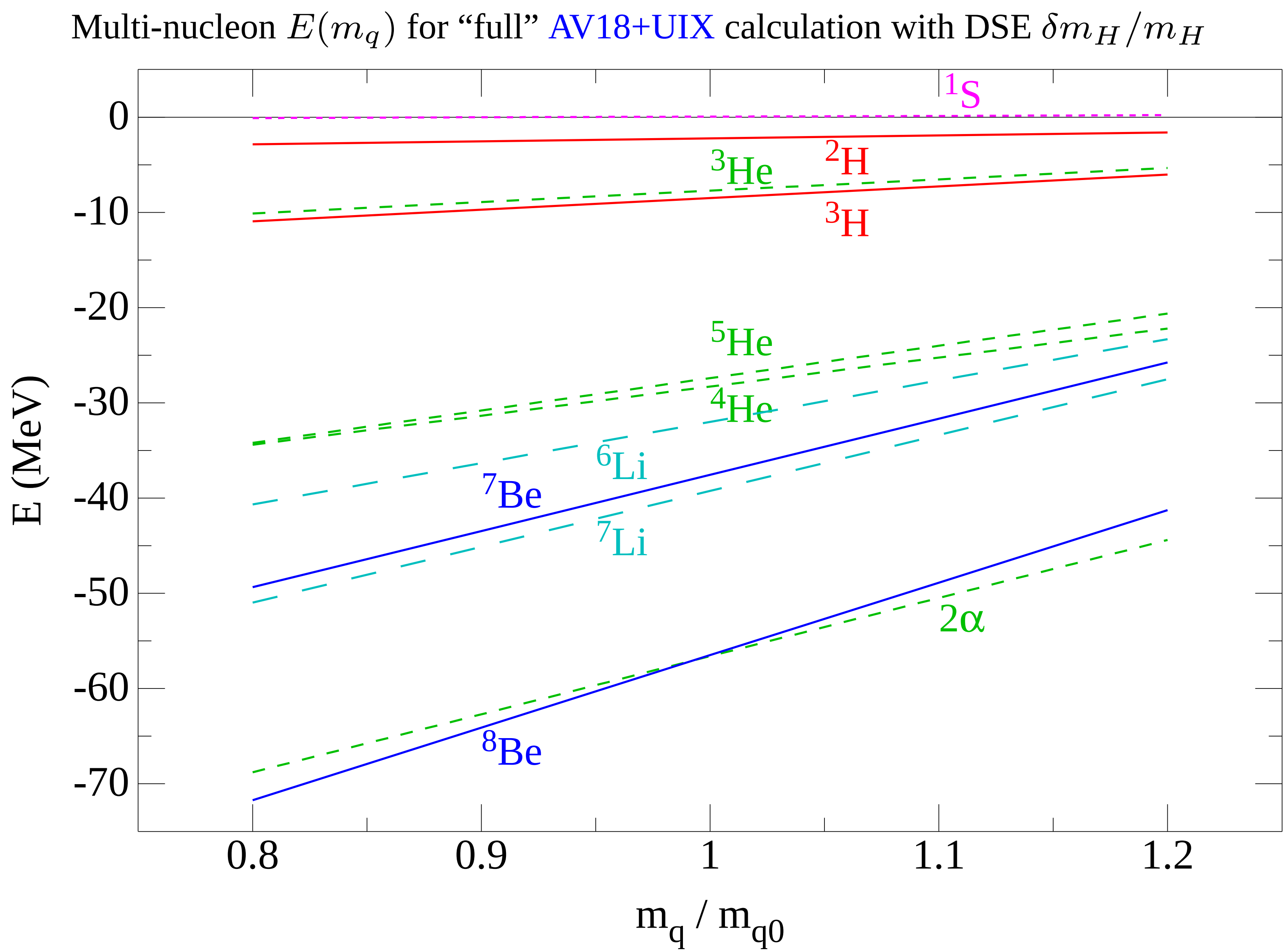
	^{3}H	^{3}He	^{4}He	^{5}He	^{6}Li	^{7}Li	^{8}Be
m_N	6.00	6.44	3.97	4.58	5.25	5.60	5.10
$m_N + \delta_N$	12.32	13.17	9.03	10.38	11.35	12.74	11.71
δ_Δ	-7.35	-7.82	-5.89	-6.74	-7.10	-8.31	-7.69
m_π (OPE)	-0.45	-0.50	-0.20	-0.24	-0.36	-0.30	-0.23
m_π (+TPE-s)	-4.35	-4.66	-3.33	-3.87	-4.19	-4.83	-4.38
m_π (+TPE-L)	-4.53	-4.85	-3.47	-4.04	-4.40	-5.06	-4.59
m_V	29.36	31.30	23.60	27.09	28.98	33.72	30.98

Multi-nucleon $\Delta\mathcal{E}(m_H)$ for AV18+UIX

	^{3}H	^{3}He	^{4}He	^{5}He	^{6}Li	^{7}Li	^{8}Be
m_N	6.07	6.54	3.99	4.51	5.12	5.24	4.81
$m_N + \delta_N$	16.56	17.73	11.86	13.31	14.41	15.53	14.36
δ_Δ	-12.20	-13.02	-9.16	-10.24	-10.80	-11.96	-11.11
m_π (OPE)	-0.37	-0.42	-0.19	-0.24	-0.36	-0.29	-0.23
m_π (+TPE-s)	-6.90	-7.38	-5.11	-5.82	-6.33	-6.95	-6.34
m_π (+TPE-L)	-6.87	-7.36	-5.06	-5.75	-6.24	-6.84	-6.24
m_π (+TNI)	-6.91	-7.40	-5.12	-5.82	-6.31	-6.91	-6.31
m_V	47.98	51.23	36.34	40.87	43.48	48.11	44.40



Multi-nucleon $E(m_q)$ for "full" AV14 calculation with DSE $\delta m_H/m_H$
E (MeV)
m_q (m_{q0})
0
-10
-20
-30
-40
-50
-60
-70
0.8
0.9
1
1.1
1.2
^{1}S
^{2}H
^{3}He
^{3}H
^{5}He
^{4}He
^{6}Li
^{7}Be
^{7}Li
2α
^{8}Be



Multi-nucleon $E(m_q)$ for "full" AV18+UIX calculation with DSE $\delta m_H / m_H$
E (MeV)
0
-10
-20
-30
-40
-50
-60
-70
0.8
0.9
1
1.1
1.2
m_q / m_{q0}
^{1}S
^{2}H
^{3}He
^{3}H
^{5}He
^{4}He
^{6}Li
^{7}Be
^{7}Li
2α
^{8}Be

Consequences for Big Bang

Dent, Stern, and Wetterich, [PRD **76**, 063513 (2007)] calculated sensitivity of BBN abundances for ^{2}H, ^{4}He, and ^{7}Li to variations in A=2-7 binding energies. Folding our results with theirs, we find these BBN abundances will be in much better agreement with the WMAP value of η (baryon to photon ratio) for $\delta X_q/X_q = K \cdot (0.013 \pm 0.02)$ where $K = \frac{\delta E/E}{\delta m_q/m_q} \sim 1.45$ is the total sensitivity to the light quark mass.

K	$^1S_0(np)$	^{2}H	^{3}H	^{3}He	^{4}He	^{5}He	^{6}Li	^{7}Li	^{7}Be	^{8}Be
AV28	4.5	-0.75								
AV14	7.3	-0.84	-0.89	-0.96	-0.69	-0.81	-0.89	-1.03	-1.09	-0.92
AV18+UIX	11.4	-1.39	-1.44	-1.55	-1.08	-1.24	-1.36	-1.50	-1.57	-1.35

This is equivalent to reducing the deuteron binding by 40 keV, which leads to a significant increase in the ^{7}Li abundance.

Flambaum & Wiringa, PRC **76**, 054002 (2007)

KEY REACTIONS IN BIG BANG

Nollett & Burles, PRD 61, 123505 (2000)

$p \leftrightarrow n$

$^1\mathrm{H}(n,\gamma)^2\mathrm{H}$

$^2\mathrm{H}(p,\gamma)^3\mathrm{He}$

$^2\mathrm{H}(d,n)^3\mathrm{He}$

$^2\mathrm{H}(d,p)^3\mathrm{H}$

$^3\mathrm{H}(d,n)^4\mathrm{He}$

$^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$

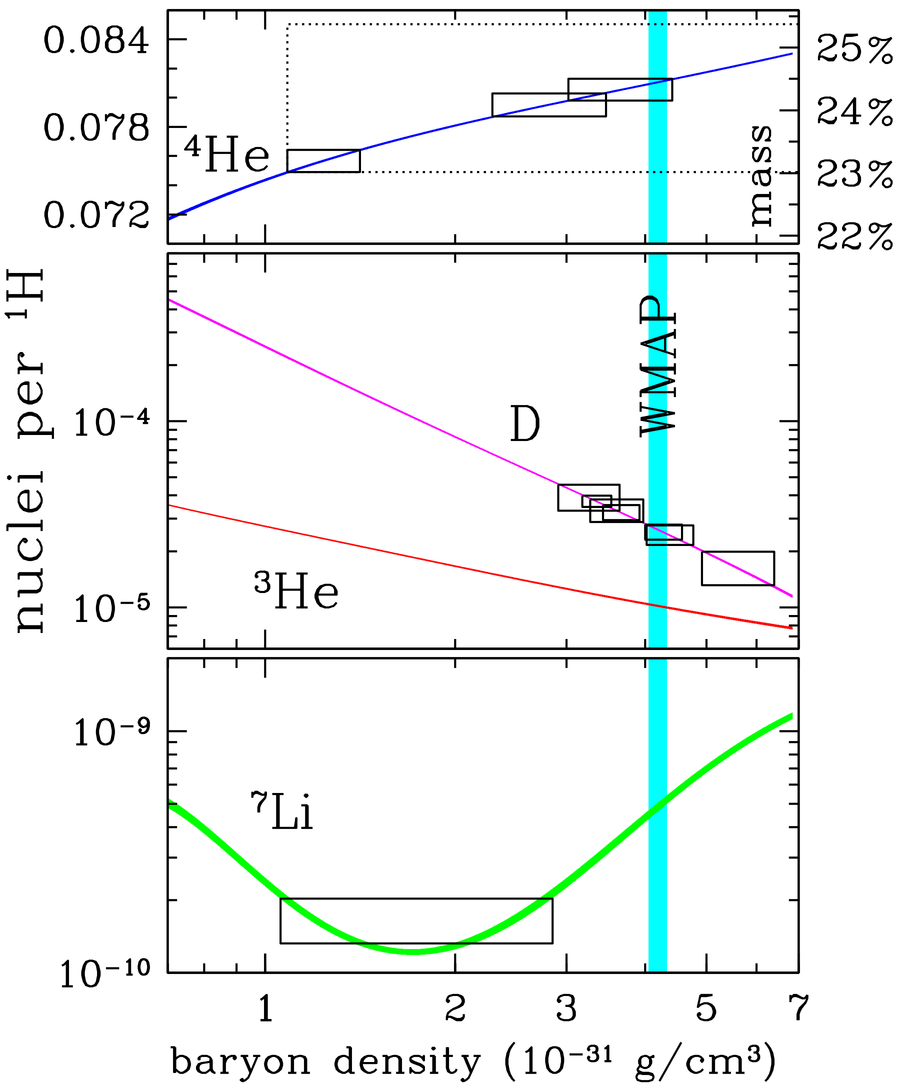
$^3\mathrm{He}(n,p)^3\mathrm{H}$

$^3\mathrm{He}(d,p)^4\mathrm{He}$

$^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$

$^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$

$^7\mathrm{Be}(n,p)^7\mathrm{Li}$



Nuclear Clocks

^{229}Th 7.6 eV transition to first excited state – amenable to present-day laser spectroscopy experiments to set limits to ongoing changes in X_q or α
^{150}Sm 0.1 eV compound resonance – can be used to set limits on variation over last 1.8 Gyr from Oklo natural nuclear reactor

These transitions are sensitive to changes in effective spin-orbit interaction, which may behave differently than overall binding.

Program for studying nuclear transition sensitivity:

- Study pairs of ground and excited states in light nuclei: $J = \frac{3}{2}^-, \frac{1}{2}^-$ in ^{5}He, ^{7}He, ^{7}Li, ^{9}Be
- Also evaluate sensitivity of nuclear radius to changes in m_q for atomic clocks

Flambaum & Wiringa, PRC **79**, 034302 (2009)

VMC energies for ground $\frac{3}{2}^-$ and excited $\frac{1}{2}^-$ (*) pairs of states in MeV compared to experiment

	^{5}He	^{5}He*	^{7}He	^{7}He*	^{7}Li	^{7}Li*	^{9}Be	^{9}Be*
Expt.	-27.41	-26.1	-28.83	-26.2	-39.24	-38.77	-58.16	-55.38
AV18+UIX	-25.26	-24.02	-21.77	-19.56	-33.33	-33.02	-45.39	-42.01
K	-1.24	-1.29	-1.93	-2.13	-1.50	-1.49	-1.59	-1.70
K_{so}	-0.27		-0.16		-2.58		-0.22	

After varying m_H and applying DSE values for m_q variation, we obtain $\delta E_{so}/E_{so} \approx -0.22\delta m_q/m_q$. This is in remarkable agreement with earlier estimates made using relativistic mean-field theory (σ- and ω-exchange) which obtained $\delta E_{so}/E_{so} \approx -0.2\delta m_q/m_q$.

Qualitatively, the VMC studies find that pion and vector meson variation contribute with the same sign for spin-orbit splittings, versus opposite signs for the total binding. Similarly, RMFT has σ- and ω-exchange working together for spin-orbit splittings instead of against each other for the total binding energy.

Combined with the overall binding sensitivity $\delta E/E \approx -1.45\delta m_q/m_q$ the consequent shift of nuclear resonances is $\delta E_r \approx 10\delta X_q/X_q$. The experimental limit on the shift in the 0.1 eV resonance in ^{150}Sm from the Oklo natural nuclear reactor is $|E_r| < 20$ meV, which can be interpreted to give the best terrestial limit on quark mass variation: $\delta X_q/X_q < 2.2 \times 10^{-18} y^{-1}$.

Atomic Clocks

Experiments can also look for variation in fundamental constants in microwave atomic clocks where the transition frequency depends on a probability of an electron to be inside a nucleus. For this purpose, we also evaluated dependence of the nuclear radius on quark mass variation, $K_r = \frac{\delta r/r}{\delta m_q/m_q}$

Experimental and calculated point proton rms radii for stable $A = 2 - 9$ nuclei

	^{2}H	^{3}H	^{3}He	^{4}He	^{6}He	^{6}Li	^{7}Li	^{7}Be	^{9}Be
Expt.	1.953	1.59	1.75	1.45	1.93	2.39	2.26	2.51	2.38
AV18+UIX	1.967	1.58	1.77	1.45	1.92	2.46	2.34	2.45	2.40
K_r	0.48	0.34	0.33	0.20		0.35	0.27	0.22	

After varying m_H and applying DSE values for m_q variation, we obtain $K_r \approx 0.3$. By contrast, the sensitivity of the ^{4}He radius to changes in the fine-structure constant α is $K_\alpha = \frac{\delta r/r}{\delta\alpha/\alpha} = 0.0034$

CONCLUSIONS

Dependence of nuclear binding on $m_\pi, m_V \approx m_\rho \approx m_\omega, m_N, m_\Delta$ calculated for $A \leq 9$ nuclei.

Results can be combined with any particular model for $\delta m_H/m_H$ variation as function of m_q.

Two-pion-exchange contributions are important.

With TPE, all nuclei, including ^{1}S virtual bound state, show same trends with variations in $\delta m_H/m_H$.

Instability of ^{8}Be vs. 2α appears to be greatest "fine-tuning" issue.

Consequences for big bang and stellar nucleosynthesis should be further explored.

Spin-orbit splittings have different sensitivity behavior than total binding, so very low-energy transitions may be good place to set limits on present time dependence of X_q