# EQUATION OF STATE OF THE UNITARY GAS


**DIAGRAMMATIC MONTE CARLO**

Kris Van Houcke
(UMass Amherst & U of Ghent)

Félix Werner (UMass Amherst)

Evgeny Kozik (ETH Zurich)

Boris Svistunov &
Nikolay Prokofev
(UMass Amherst)

**EXPERIMENT**

Andre Schirotzek
Ariel Sommer
Mark Ku
Martin Zwierlein
(MIT)



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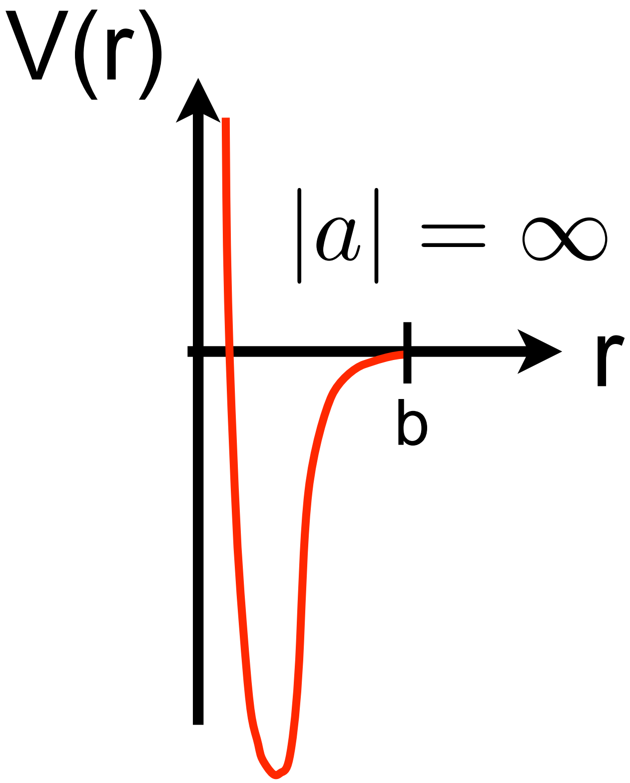
Seattle, March 17, 2010

# What is the unitary gas:

- Spin 1/2 fermions
- Interactions have
  - infinite scattering length
  - zero range

(3D)

Universality hypothesis:




Zero-range limit
$$\begin{cases} n^{-1/3} \gg b \\ \lambda \gg b \end{cases}$$

$$\lambda \equiv \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

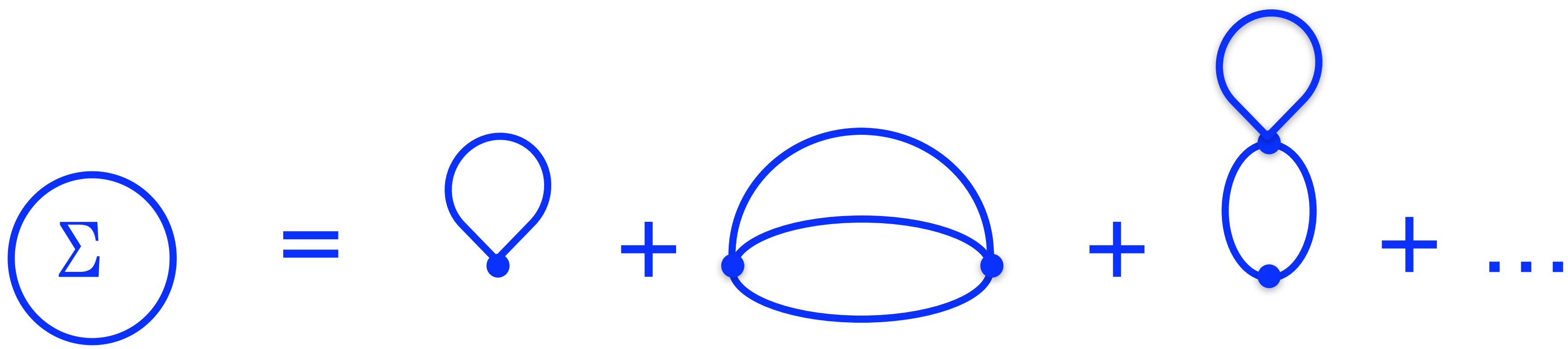
$\Rightarrow \quad \exists$ (quasi) equilibrium homogeneous phase properties independent on V(r)

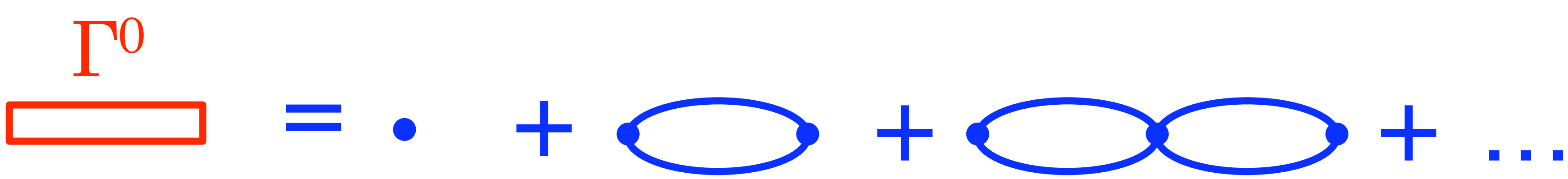
$N_\uparrow = N_\downarrow$

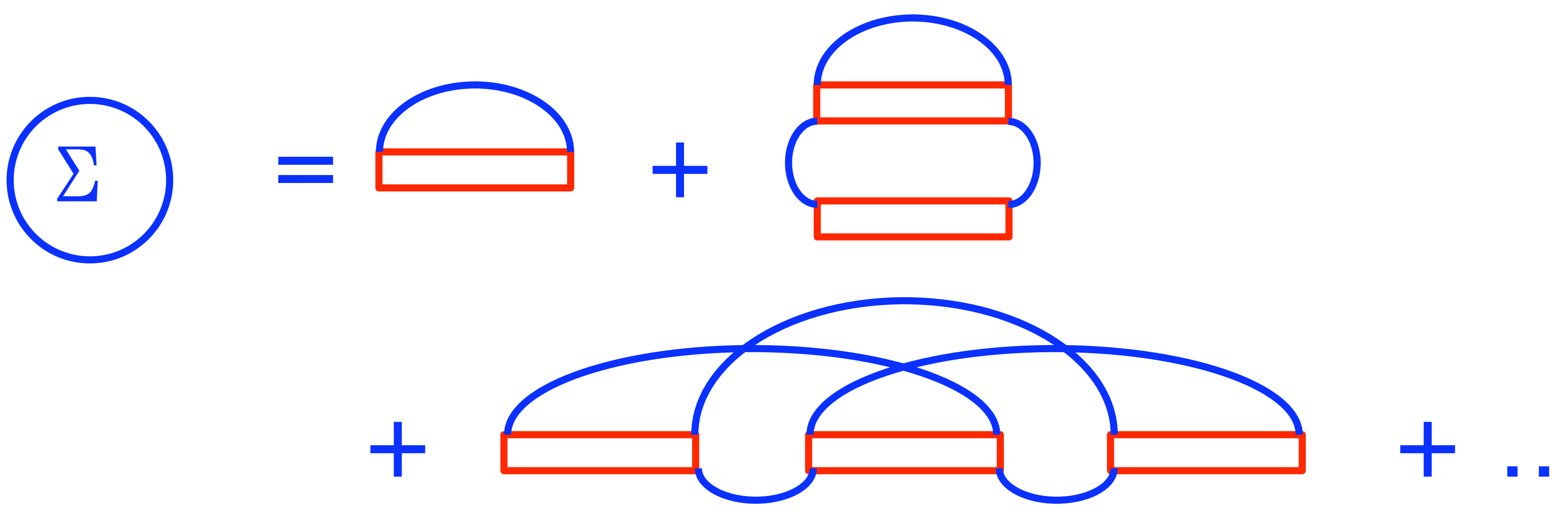
$$\frac{n(T,\mu)}{n_0(T,\mu)} = \text{universal function of } \beta\mu \qquad [\beta \equiv 1/T]$$

$$\begin{cases} V(r) = g_0\, \delta^3(\vec{r}) \\ |\vec{k}| < \Lambda \end{cases} \qquad \frac{1}{g_0} = \frac{m}{4\pi\hbar^2 a} - \int_{|\vec{k}|<\Lambda} \frac{d\vec{k}}{(2\pi)^3}\, \frac{m}{k^2}$$

Zero-range limit: $\Lambda \to \infty$ $\quad g_0(\Lambda)$ s.t. $a$ fixed



# Diagrammatic Monte-Carlo

Random walk in the space of
all possible diagram topologies
and all values of internal and external variables.
Each configuration is visited with a probability
proportional to the absolute value of its contribution
to $\Sigma(\vec{p}, \tau)$

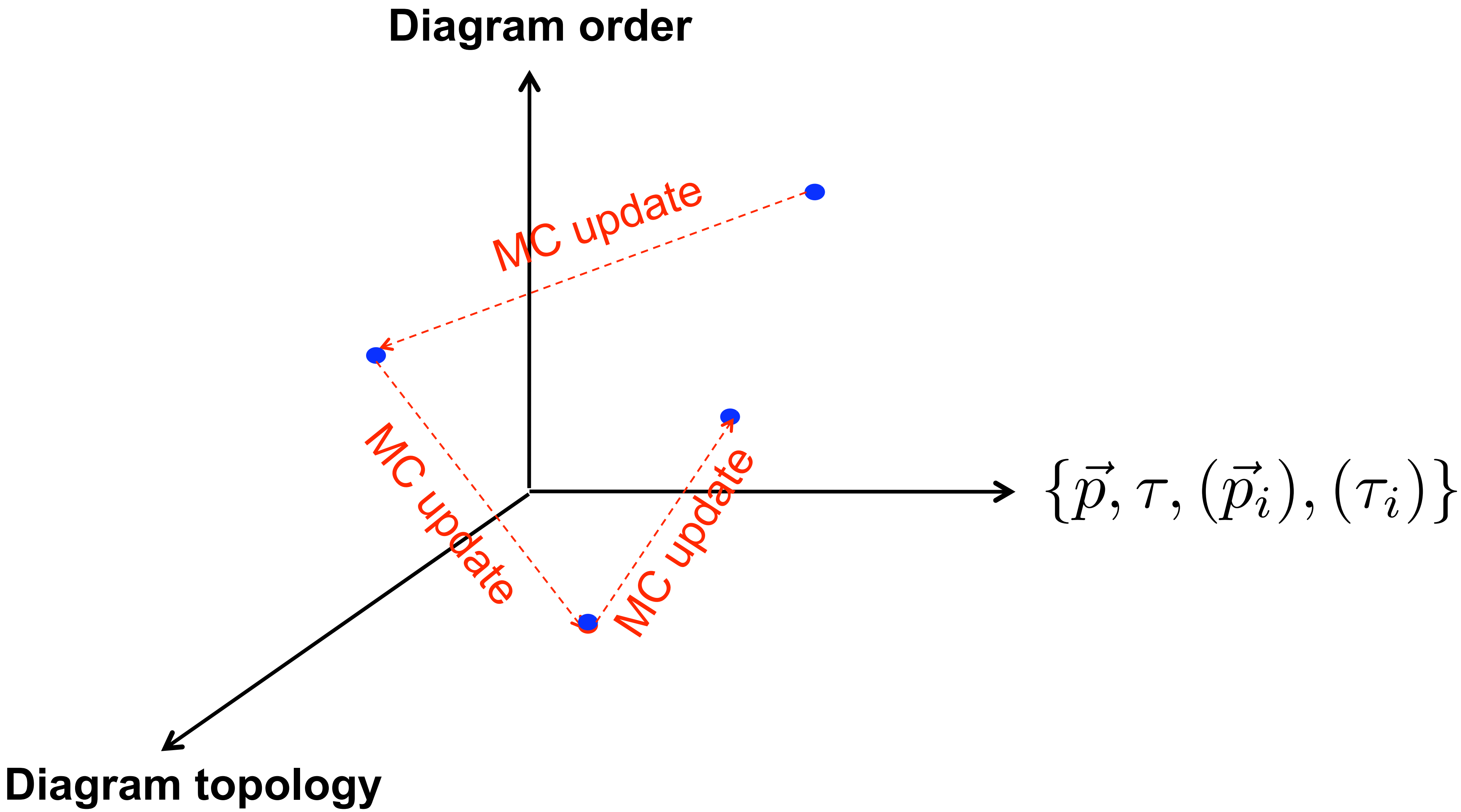
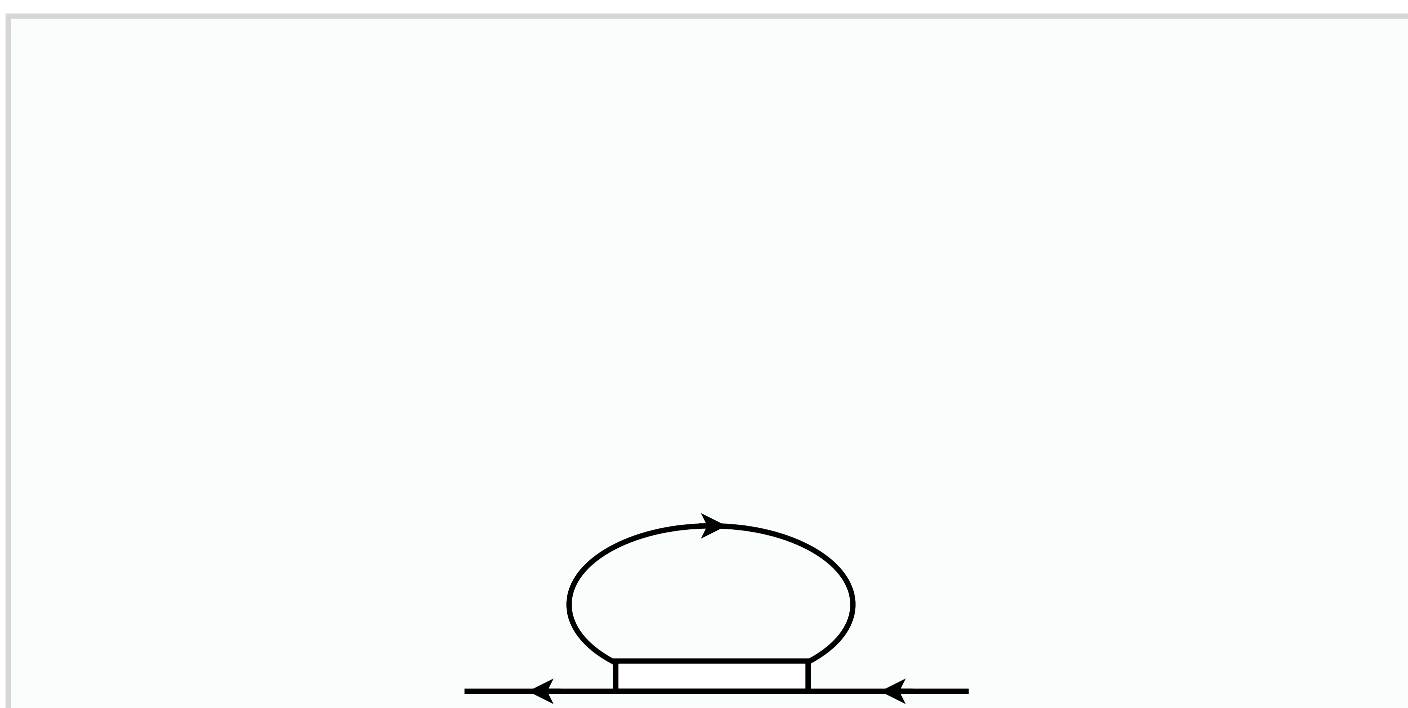


Diagram order
MC update
MC update
MC update
$\{\vec{p}, \tau, (\vec{p}_i), (\tau_i)\}$
Diagram topology



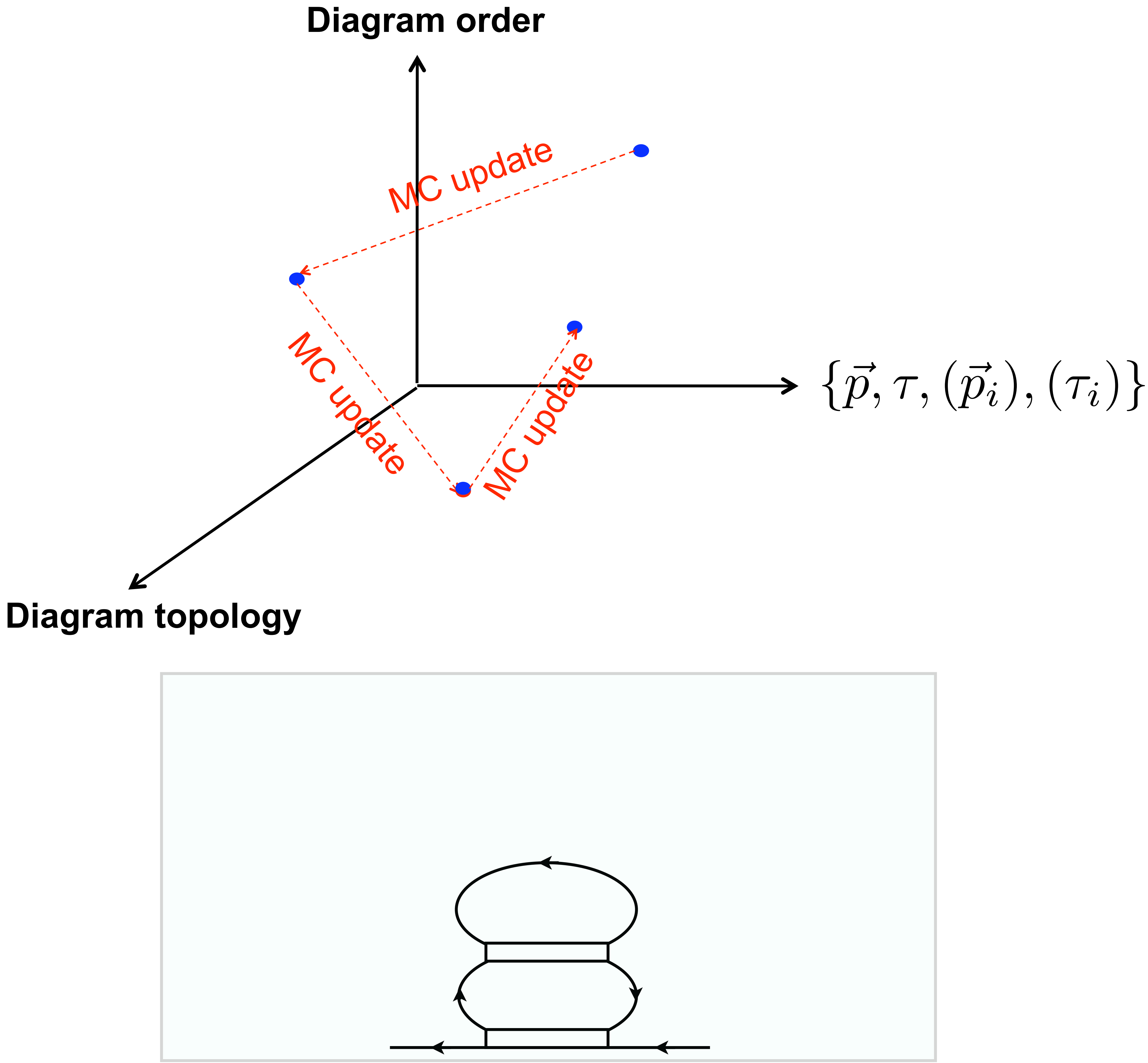


Diagram order
MC update
MC update
MC update
$\{\vec{p}, \tau, (\vec{p}_i), (\tau_i)\}$
Diagram topology

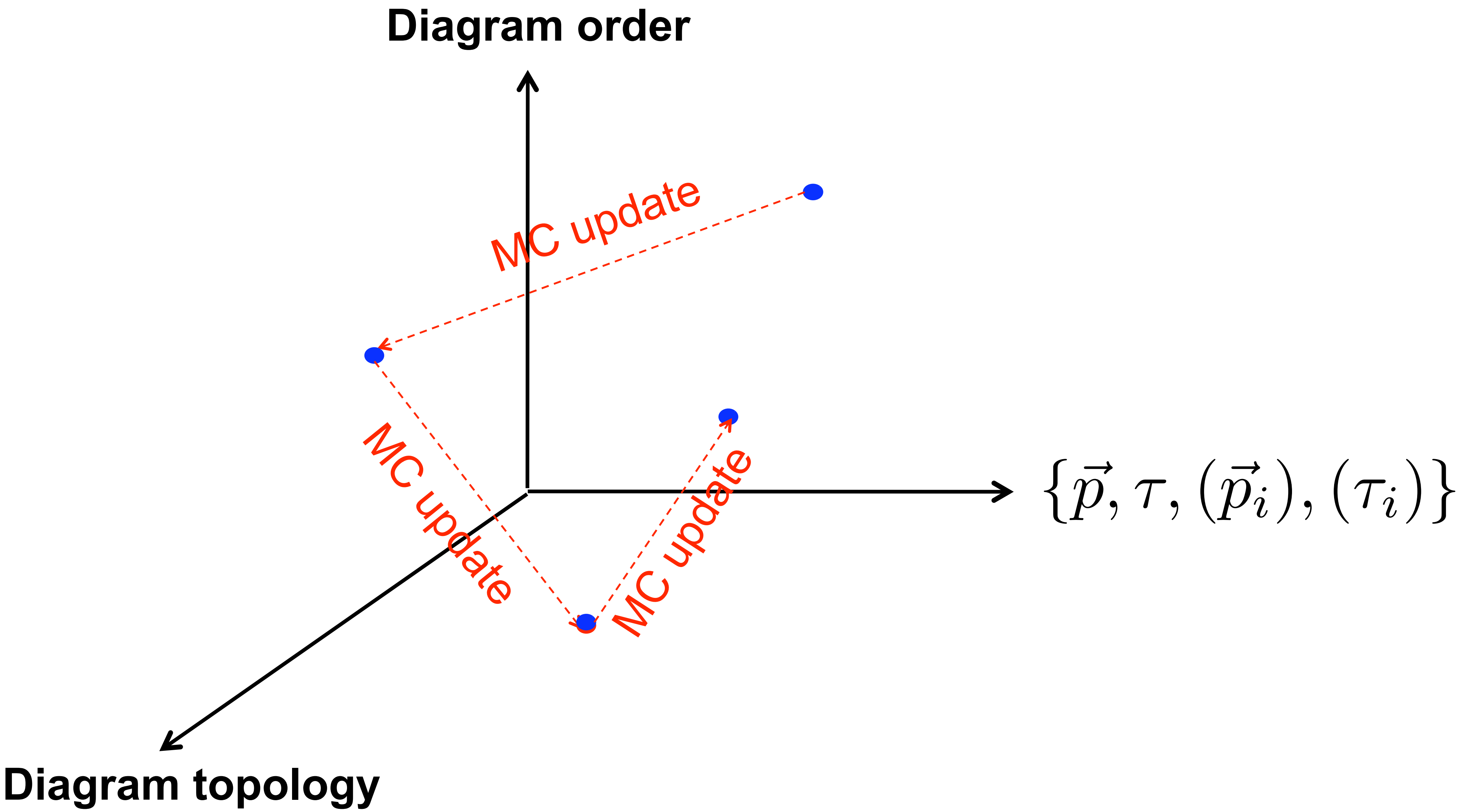
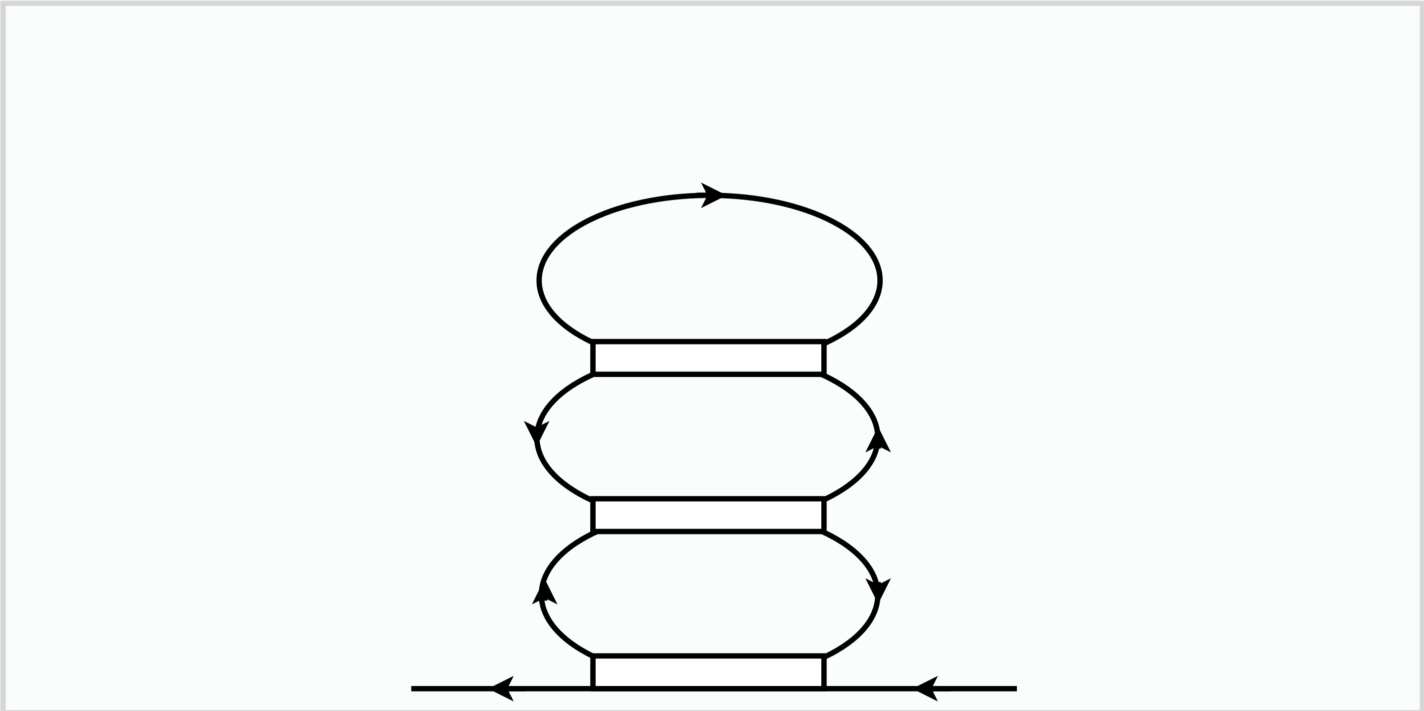


Diagram order
MC update
MC update
MC update
$\{\vec{p}, \tau, (\vec{p}_i), (\tau_i)\}$
Diagram topology

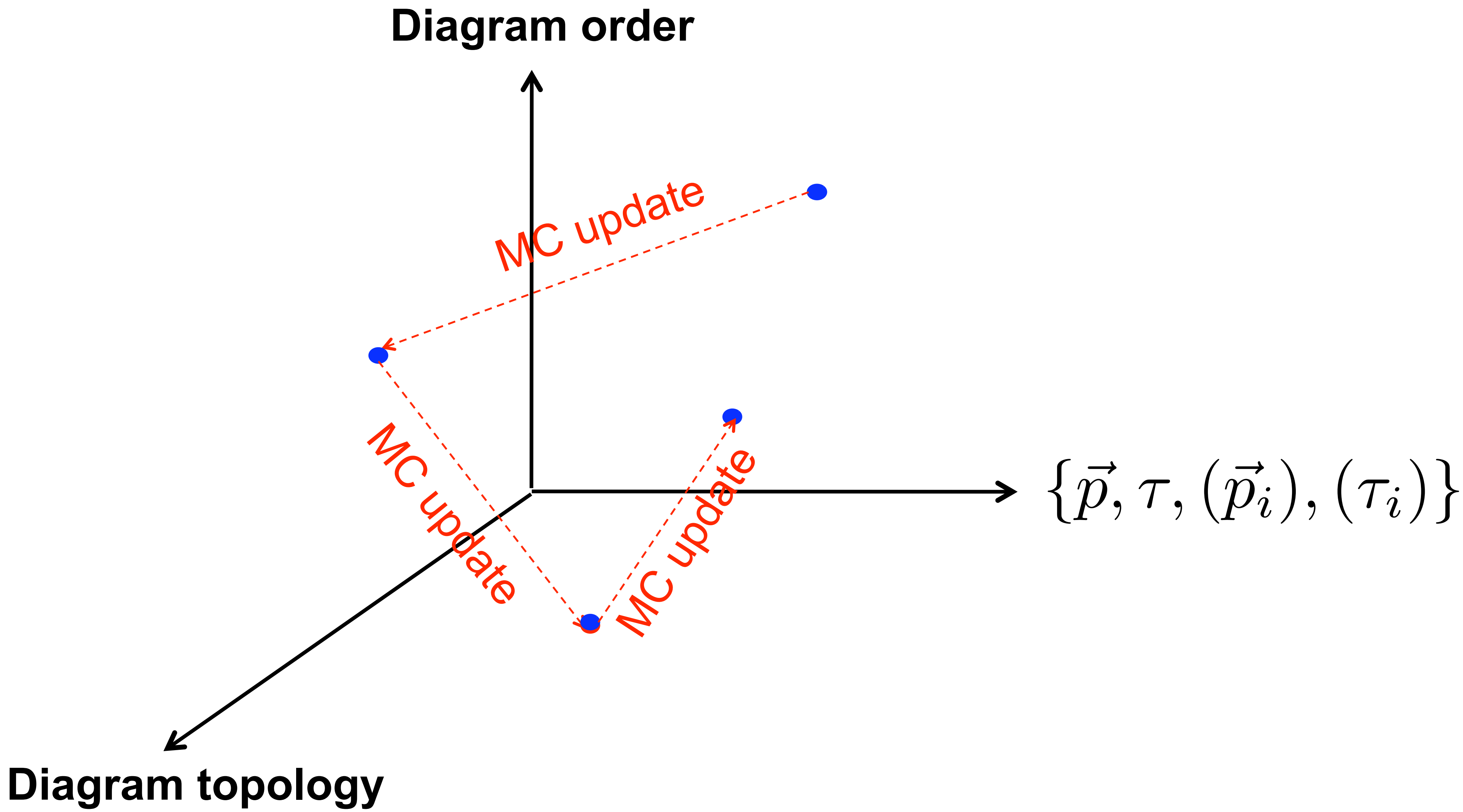
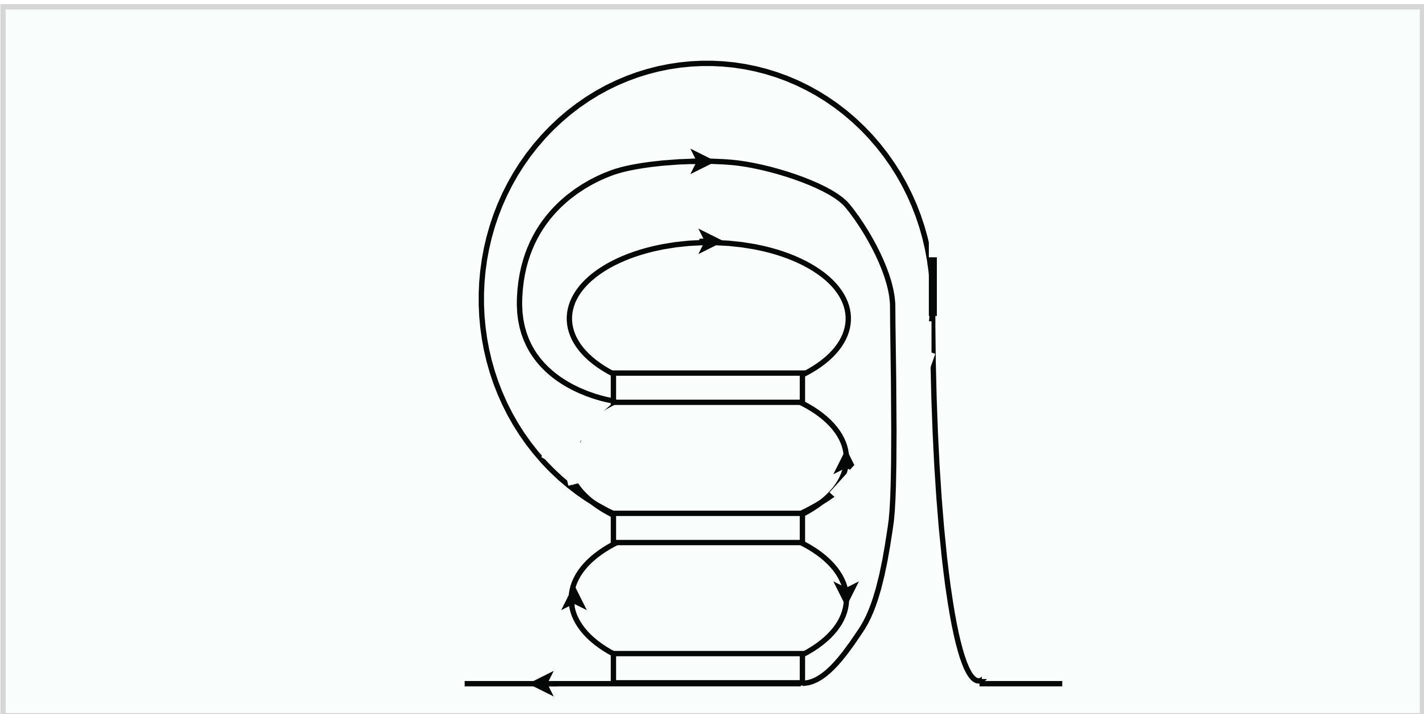
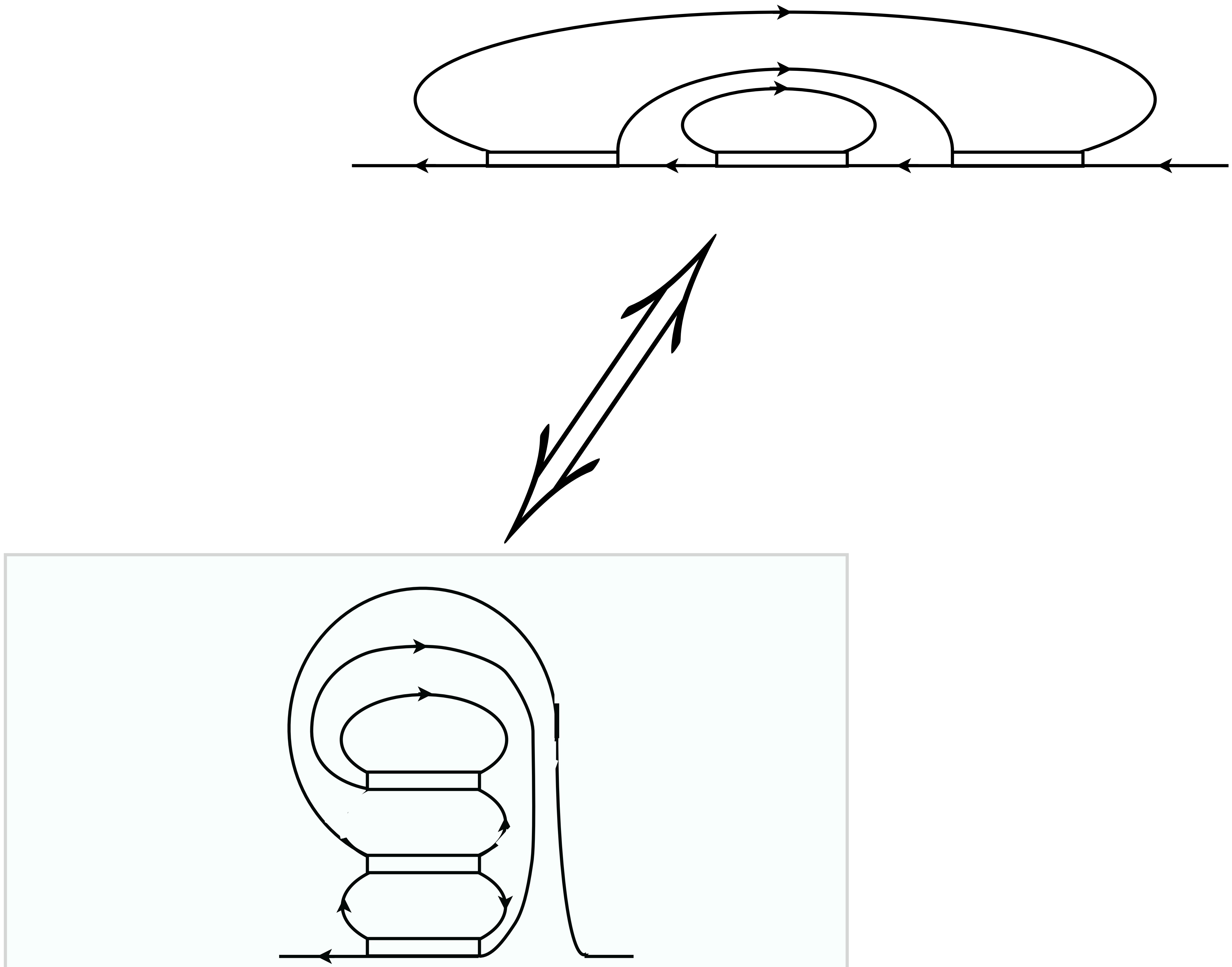
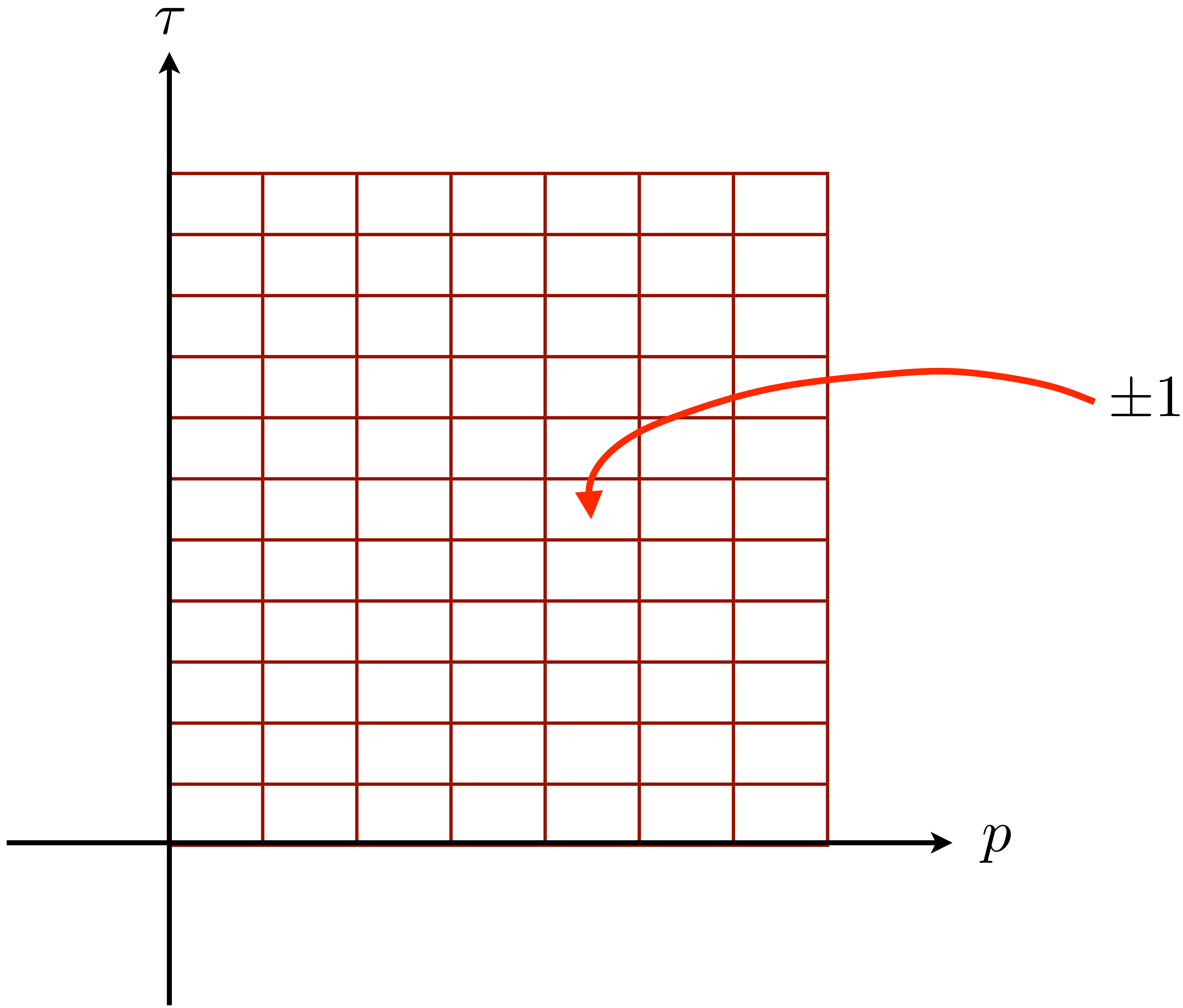


Diagram order
MC update
MC update
MC update
$\{\vec{p}, \tau, (\vec{p}_i), (\tau_i)\}$
Diagram topology





# after each MC update:




$\Rightarrow$ histogram for $\Sigma(p, \tau)$

## Previous applications of DiagMC:

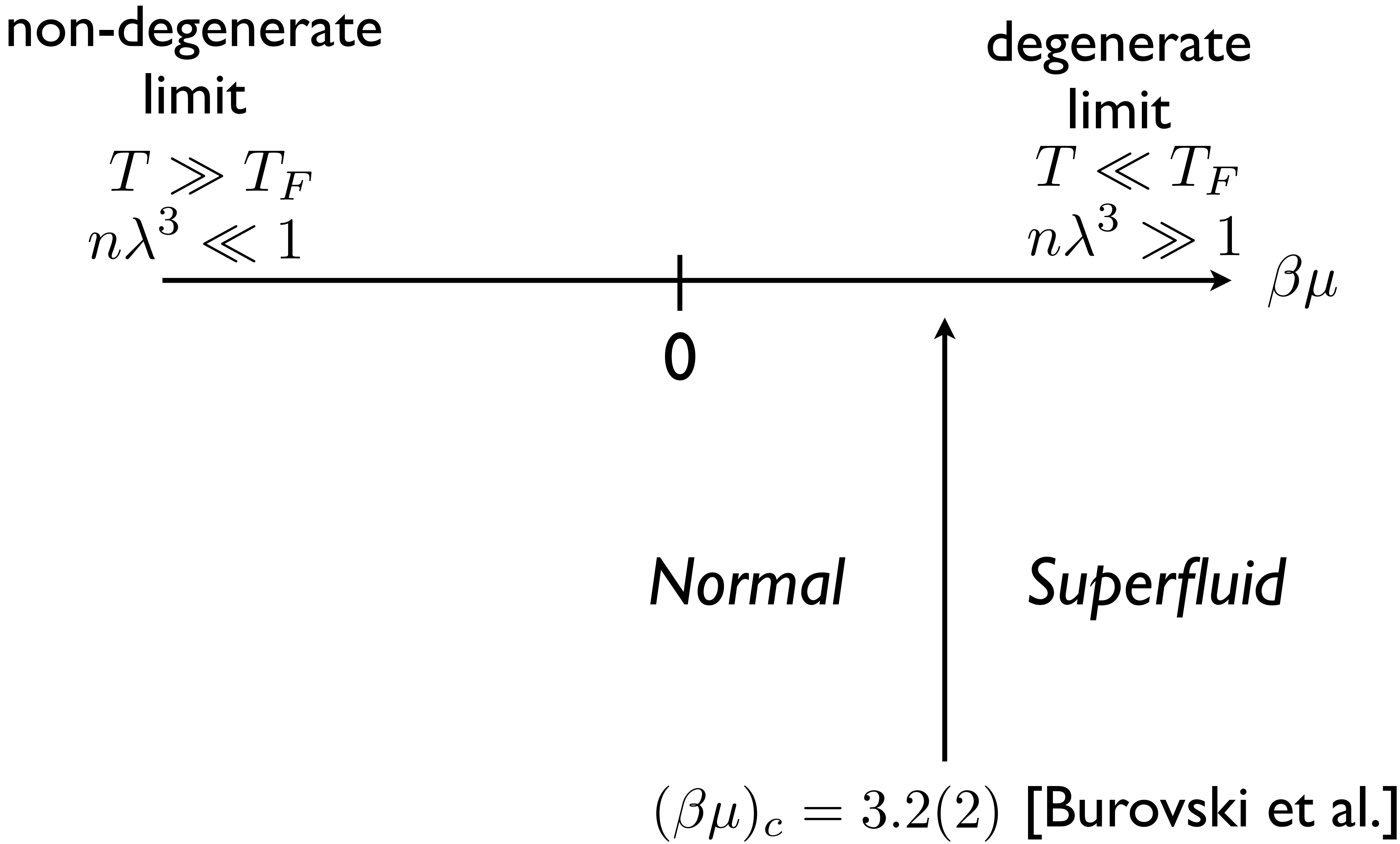
- Solution of Fermi−polaron problem
  [Prokofev&Svistunov, PRB 2008]

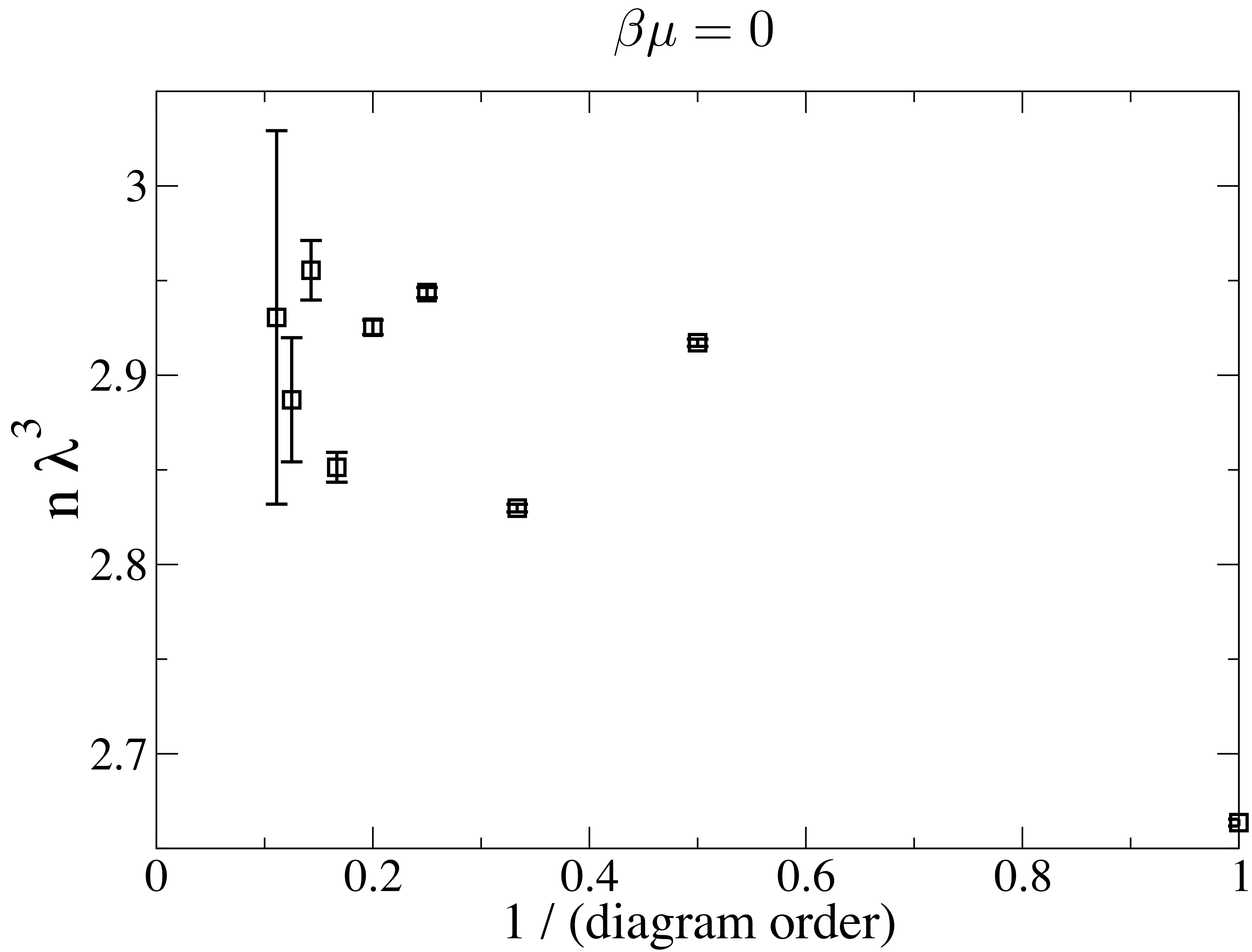
- Doped Hubbard model
  [Van Houcke et al., 2008; Kozik et al. arXiv 2009]

## *A new way to fight the sign problem:*

traditional QMC: error bars ~ $\exp\{\#\beta\,\mathrm{Volume}\}$

DiagMC: $\mathrm{Volume} = \infty$ error bars ~ $\exp\{\#\ \mathrm{diagram\ order}\}$


non-degenerate limit
$T \gg T_F$
$n\lambda^3 \ll 1$
degenerate limit
$T \ll T_F$
$n\lambda^3 \gg 1$
$\beta\mu$
0
*Normal*
*Superfluid*
$(\beta\mu)_c = 3.2(2)$ [Burovski et al.]



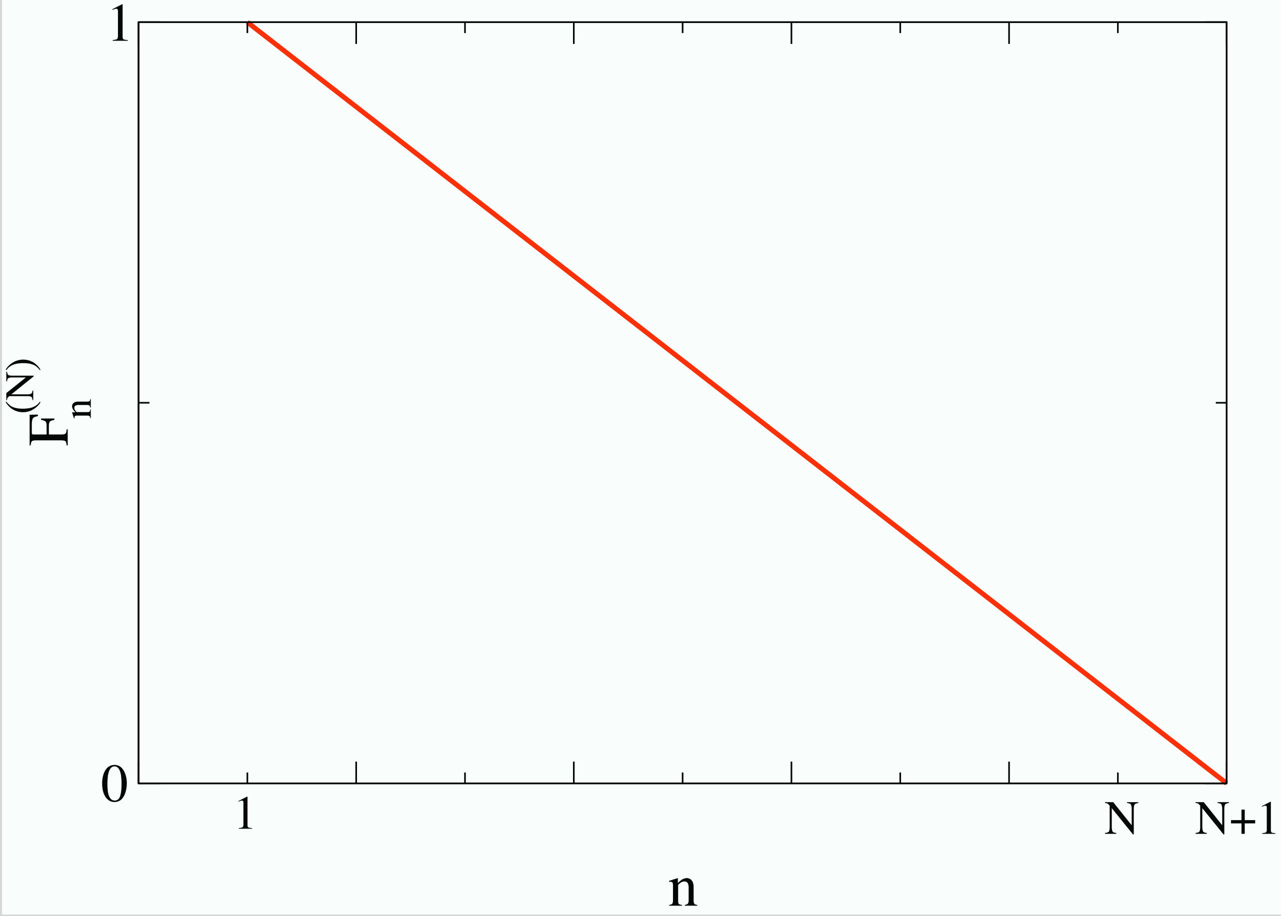
$\beta\mu = 0$
3
2.9
2.8
2.7
$n\,\lambda^3$
0
0.2
0.4
0.6
0.8
1
1 / (diagram order)

# Resummation

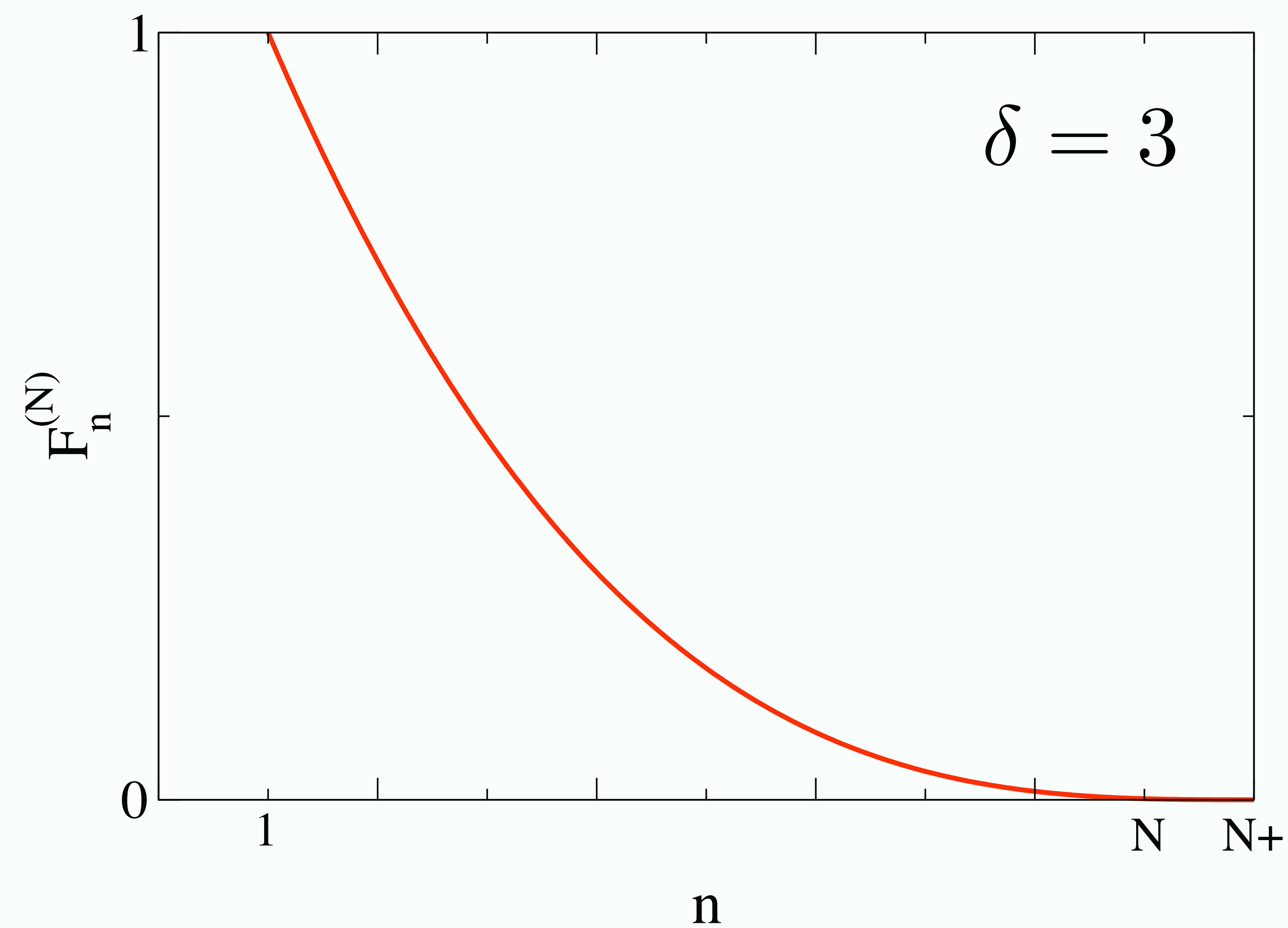
$$\Sigma^{\text{bare}}_{(N)}(p,\tau) = \sum_{n=1}^{N} \Sigma^{(n)}(p,\tau)$$

$$\Sigma^{\text{resummed}}_{(N)}(p,\tau) = \sum_{n=1}^{N} F_n^{(N)}\, \Sigma^{(n)}(p,\tau) \qquad F_n^{(N)} \underset{N\to\infty}{\longrightarrow} 1$$

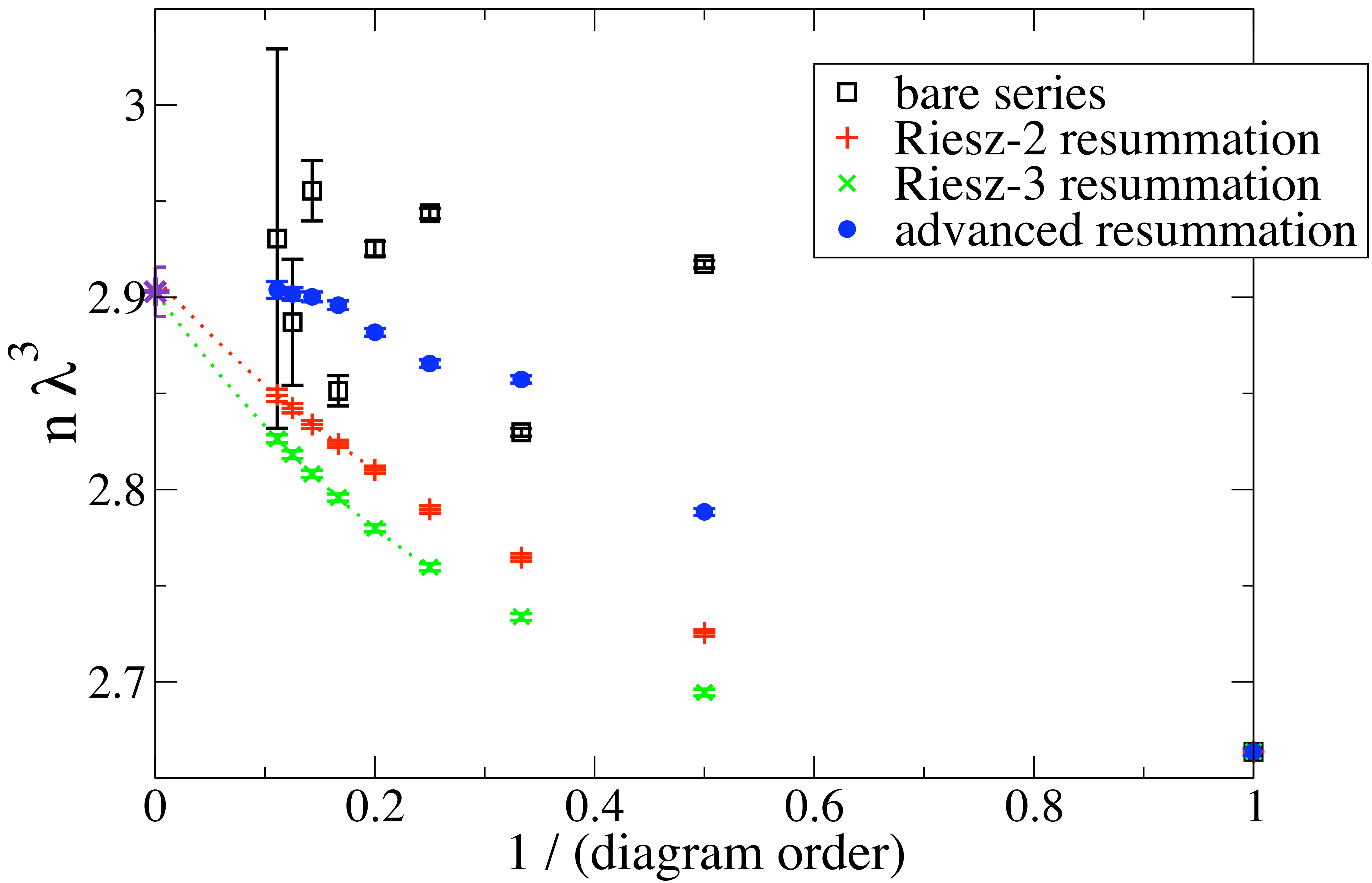
Cesaro: $F_n^{(N)} = 1 - \dfrac{n-1}{N}$



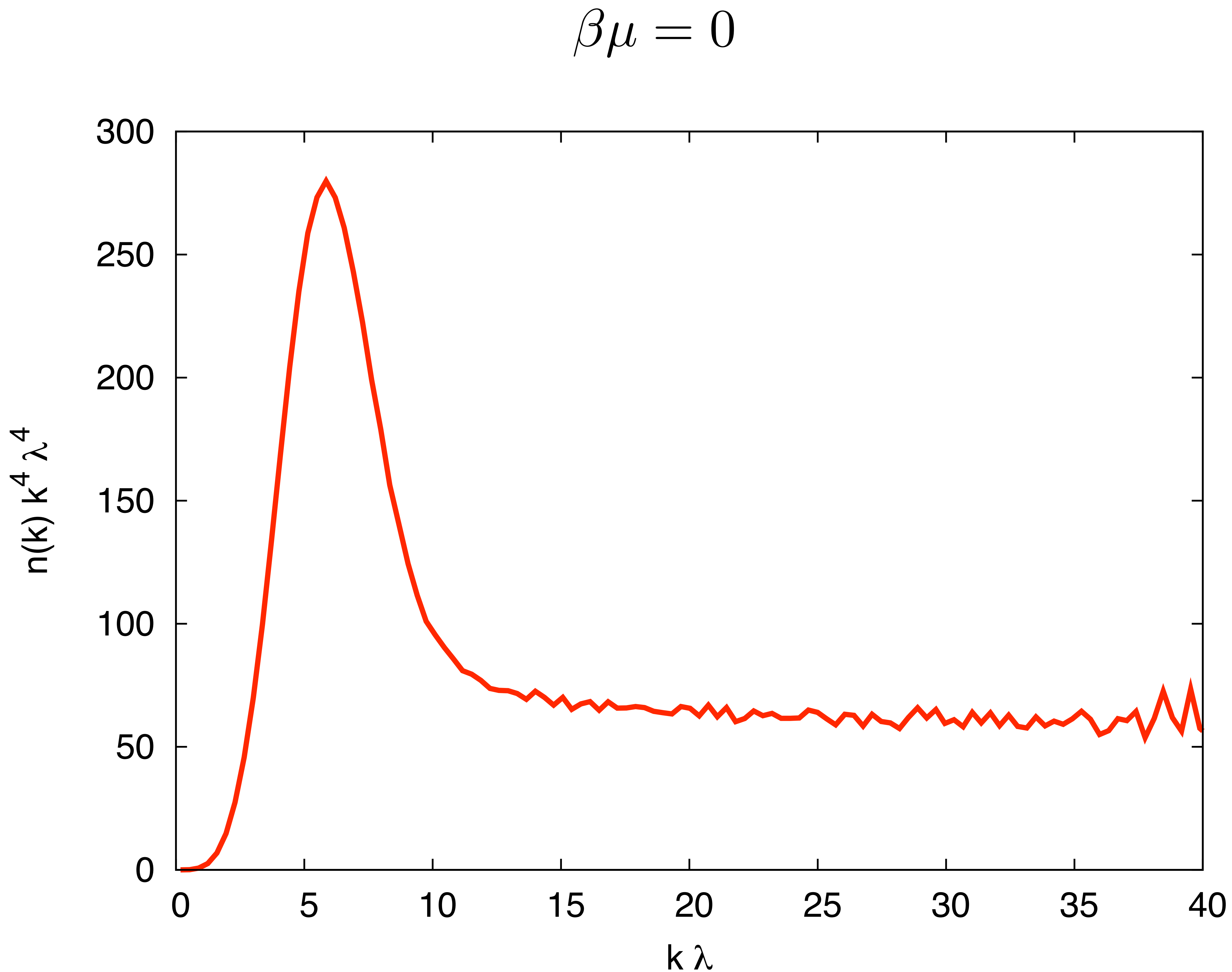

Riesz: $F_n^{(N)} = \left(1 - \dfrac{n-1}{N}\right)^{\delta}$



$\beta\mu = 0$

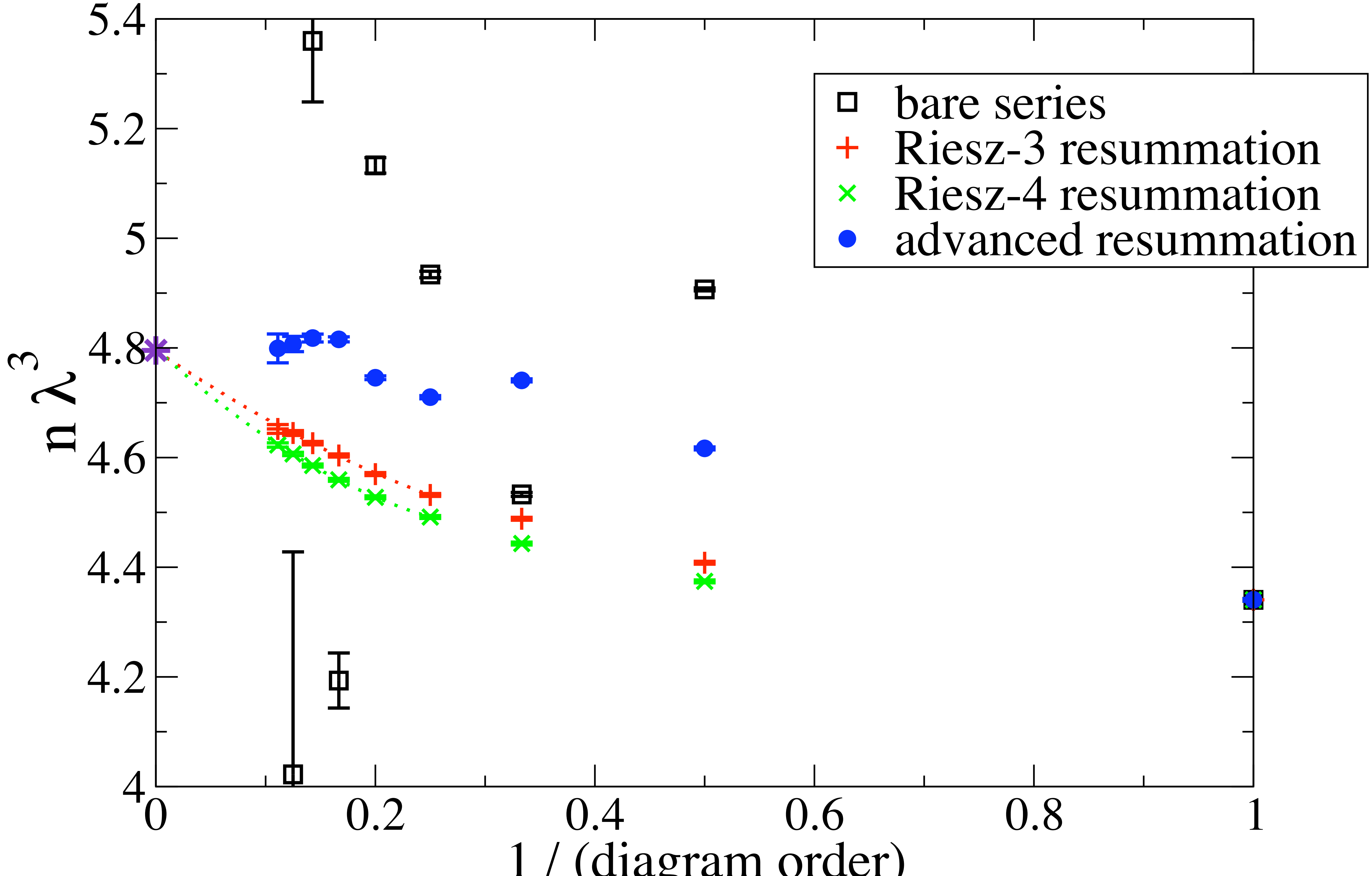



$\Rightarrow \; n\lambda^3 = 2.90(2) \quad$ i.e. $\quad T/T_F = 0.646(6)$

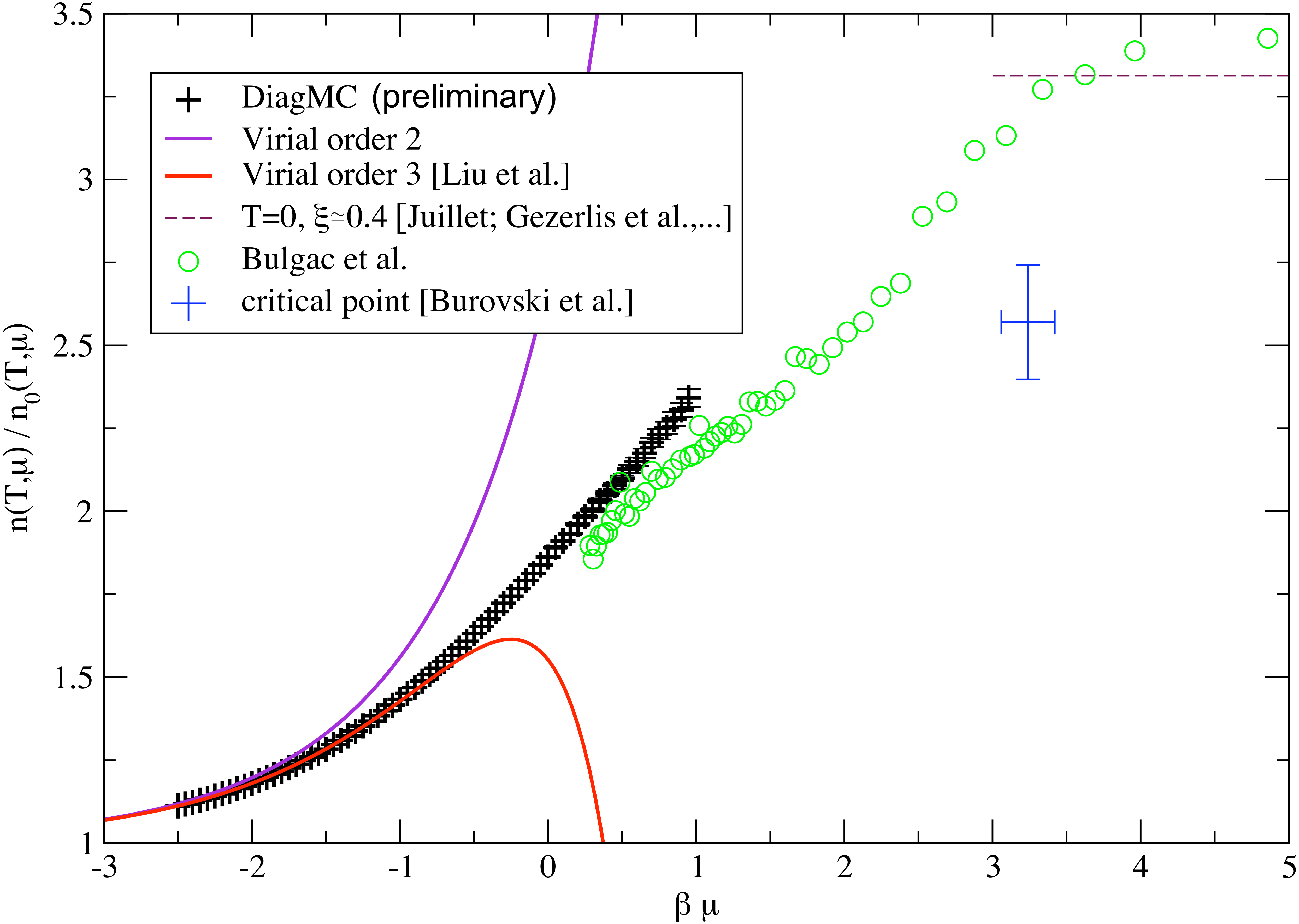


$\beta\mu = 0$
300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
k λ
n(k) k⁴ λ⁴

$\beta\mu = 0.5$

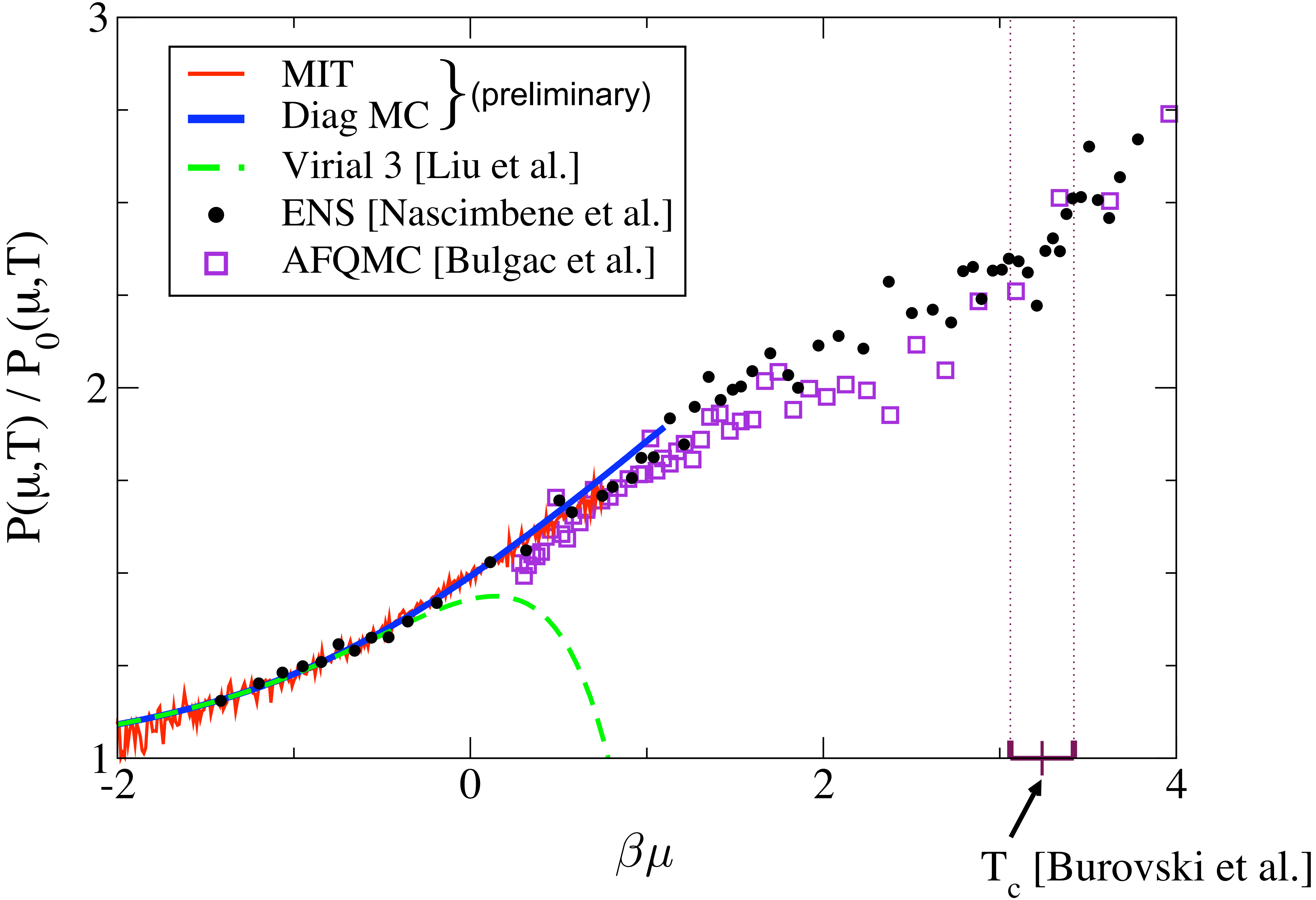



$\Rightarrow\ n\lambda^3 \simeq 4.8 \quad \text{i.e.} \quad T/T_F \simeq 0.46$

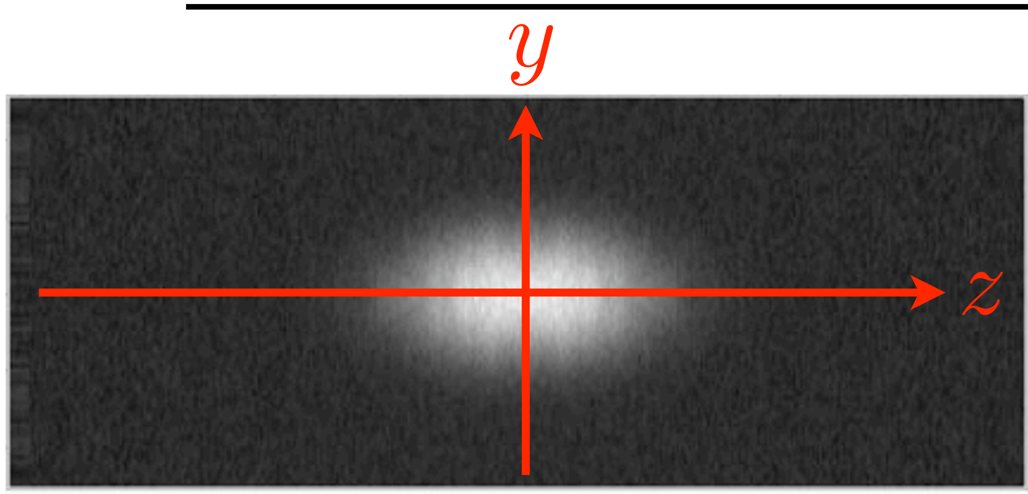


DiagMC (preliminary)
Virial order 2
Virial order 3 [Liu et al.]
T=0, ξ≃0.4 [Juillet; Gezerlis et al.,...]
Bulgac et al.
critical point [Burovski et al.]
n(T,μ) / n0(T,μ)
β μ
3.5
3
2.5
2
1.5
1
-3
-2
-1
0
1
2
3
4
5

# Comparison with experiments:



# How were these experimental EOS obtained:




$\rho = \sqrt{x^2 + y^2}$

trapping potential $V(\rho, z)$

local density $n_{3D}(\rho, z)$

$$\text{local density approximation}: \ n_{3D}(\rho, z) = n_{\text{homogeneous}}\left(T, \mu = \mu_g - V(\rho, z)\right)$$
$$p_{3D}(\rho, z) = p_{\text{homogeneous}}\left(T, \mu = \mu_g - V(\rho, z)\right)$$

$$n_{2D}(y, z) = \int_{-\infty}^{+\infty} dx \ n_{3D}(\rho = \sqrt{x^2 + y^2}, z) \xrightarrow{?} p_{\text{homogeneous}}(T, \mu)$$

__ENS protocol:__ [Ho&Zhou]

$$p_{3D}(\rho = 0, z) = \frac{m\omega_\rho^2}{2\pi} \int_{-\infty}^{+\infty} dy \, n_{2D}(y, z)$$ for harmonic trap

fitting the wings with virial (2nd order) $\longrightarrow$ $(T, \mu_g)$

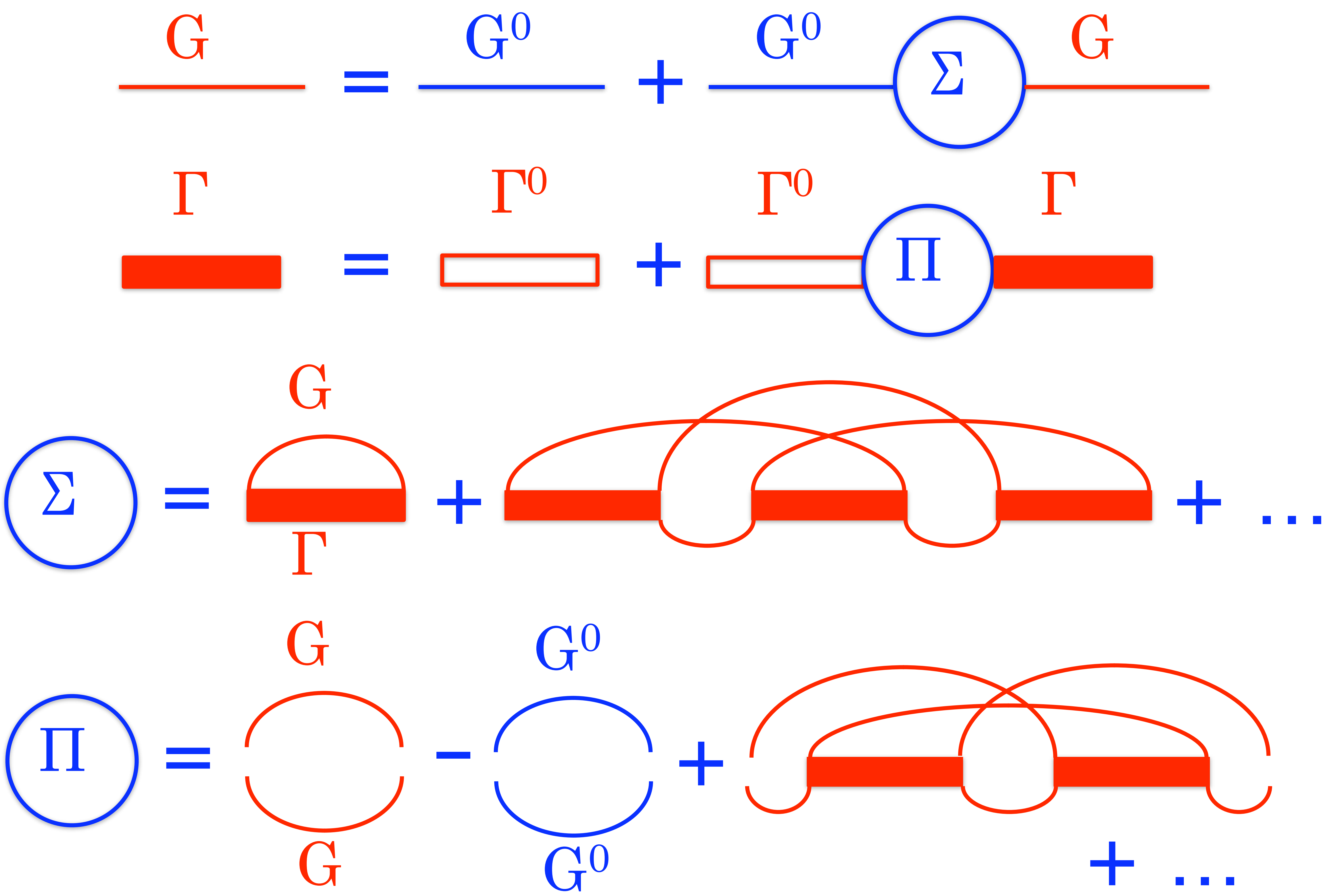
__our protocol:__

$$p_{3D}(\rho_0, z) = \frac{1}{\pi} \int_{\rho_0}^{\infty} dy \ n_{2D}(y, z) \left[ \frac{\frac{\partial V}{\partial \rho}(y, z)}{\sqrt{y^2 - \rho_0^2}} + \int_{\rho_0}^{y} d\rho \ \frac{\rho \frac{\partial V}{\partial \rho}(y, z) - y \frac{\partial V}{\partial \rho}(\rho, z)}{(y^2 - \rho^2)^{3/2}} \right]$$ for any trap

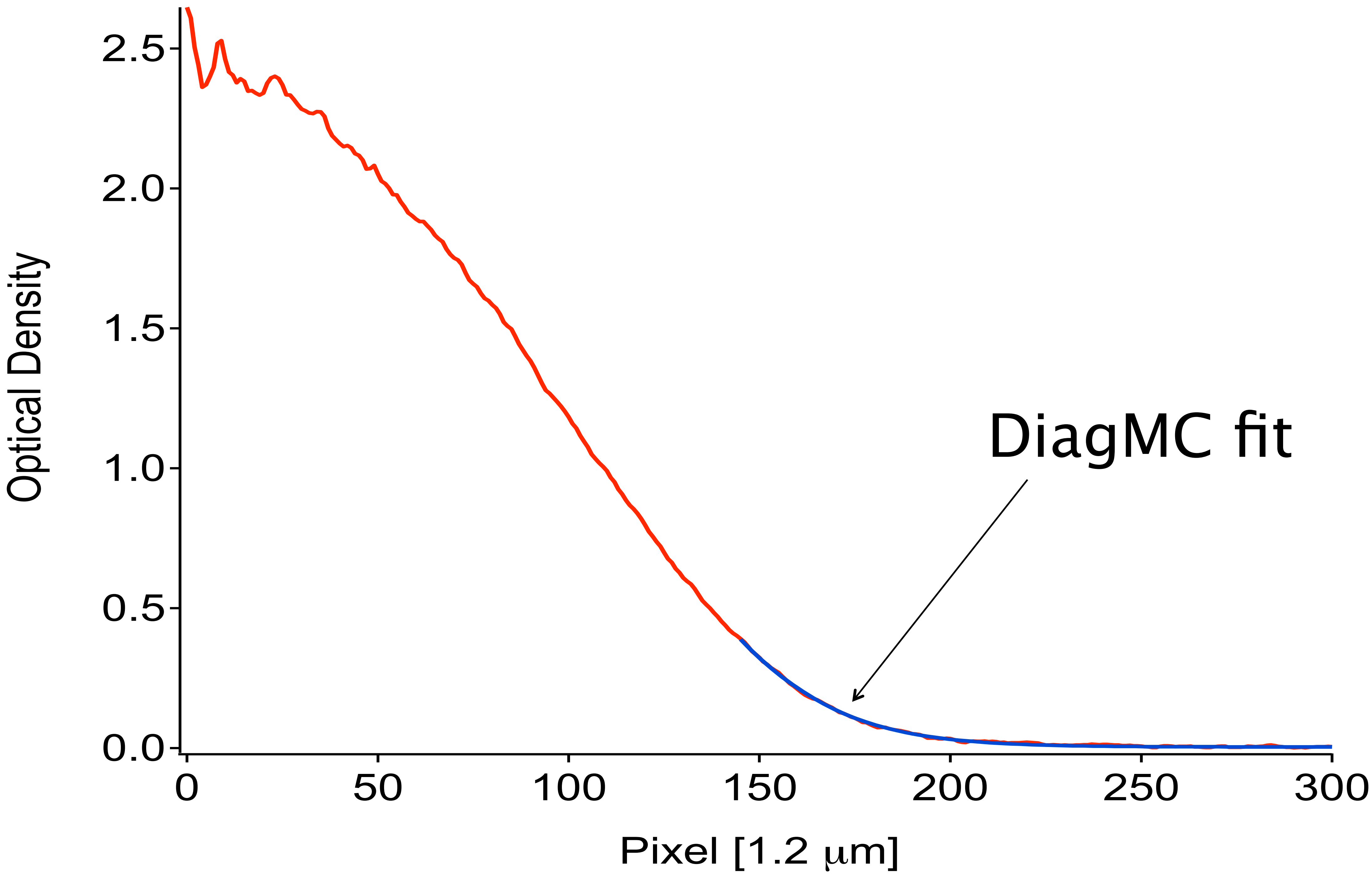
fitting the wings with virial (3rd order) $\longrightarrow$ $(T, \mu_g)$

# What we are working on

# Bold Diagrammatic Monte Carlo

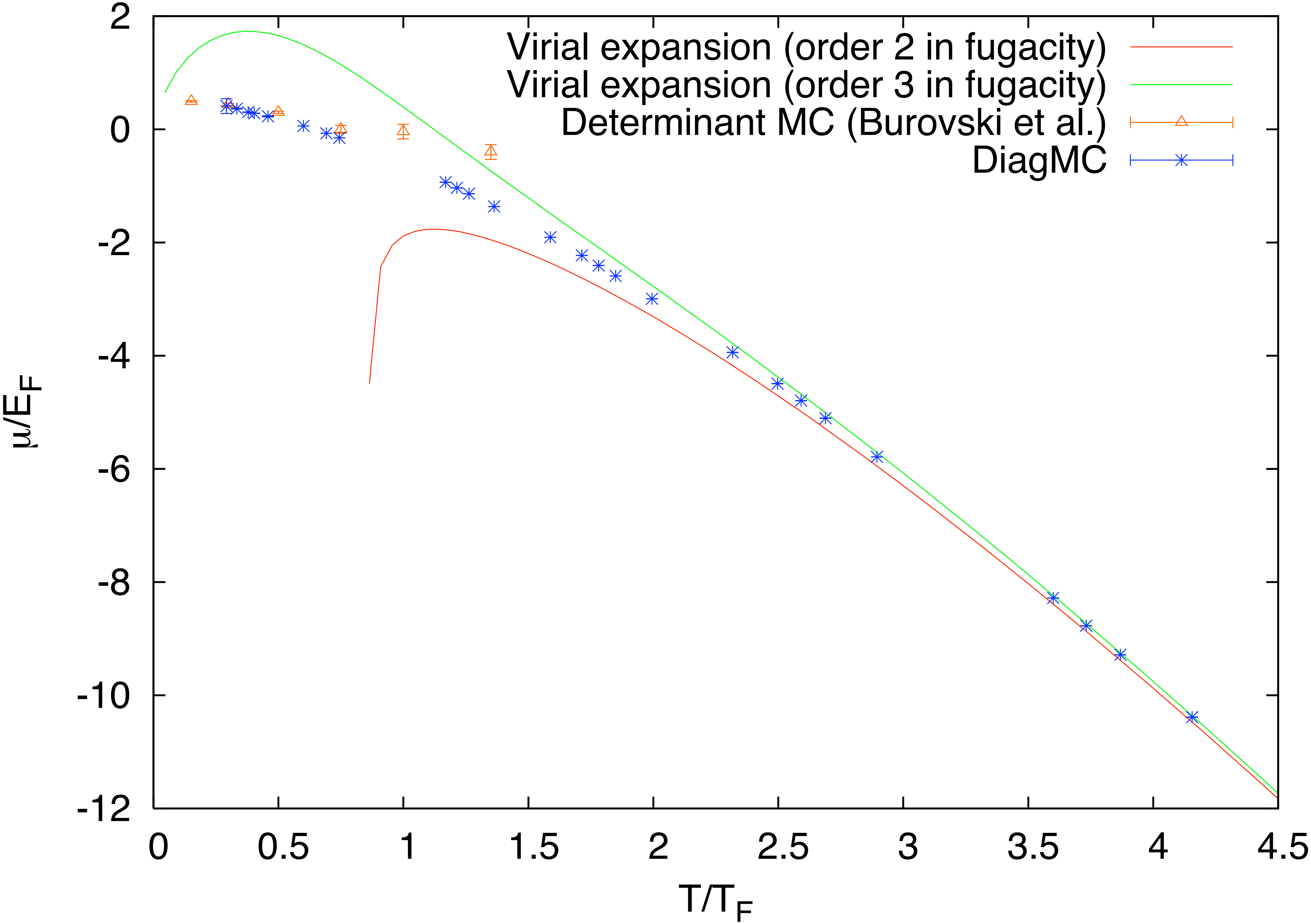


- Combine theory with experiment to obtain accurately EOS down to T=0



# Next projects:

- Finite scattering length
- Finite imbalance
- Doped Hubbard model



Virial expansion (order 2 in fugacity)
Virial expansion (order 3 in fugacity)
Determinant MC (Burovski et al.)
DiagMC
μ/E_F
T/T_F
2
0
-2
-4
-6
-8
-10
-12
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5