Nuclear Parity Violation and Lattice QCD

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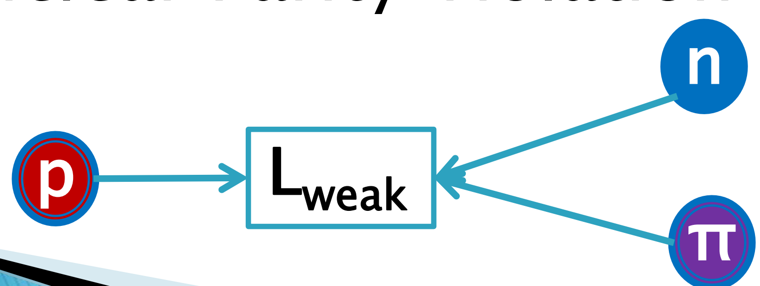
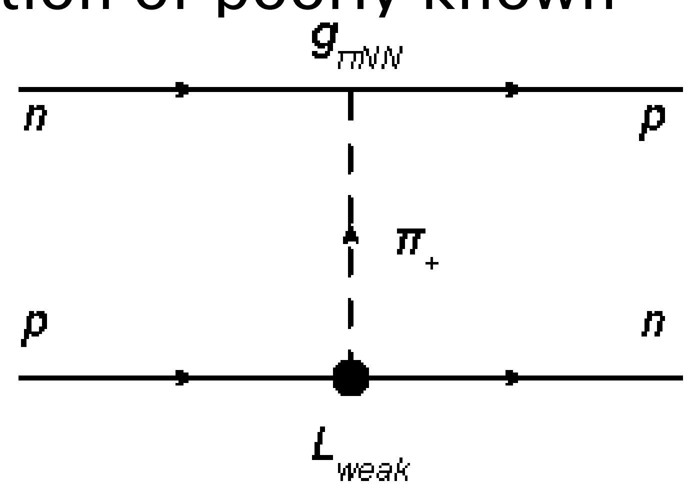
Fast Times for QCD

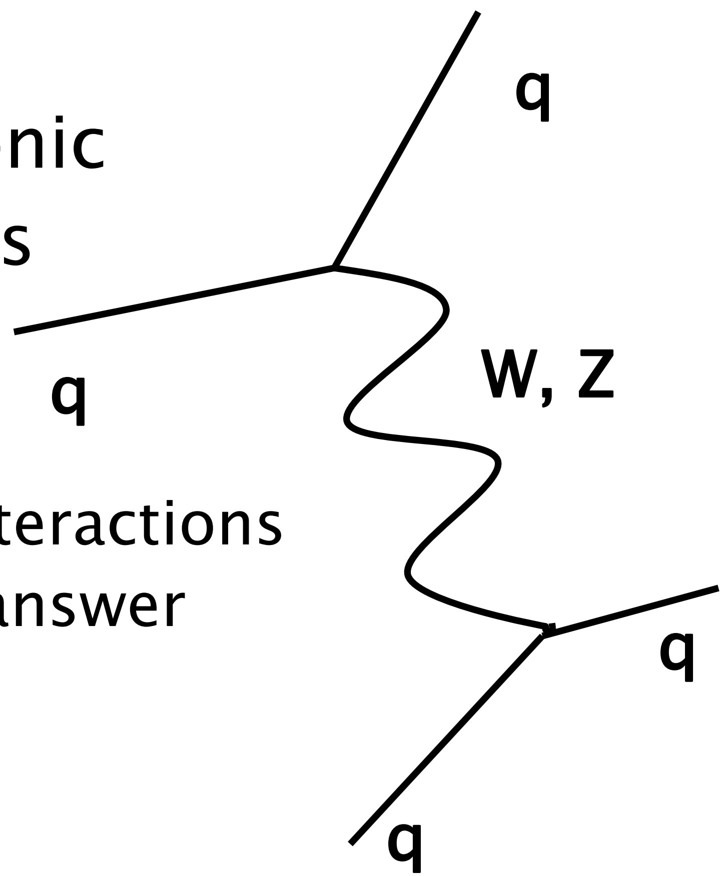
- Faster computers & better algorithms
  - Fully dynamical nonperturbative lattice calculations
  - Beginning to do precise calculations
    - Proton mass with ~0.1% error (NPLQCD)
    - ππ scattering length with ~1% error (NPLQCD)
    - But at unphysical pion mass and only certain calcs.
  - Starting to look at the calculation of poorly known observables
- Nuclear Parity Violation





Parity Violation

- Discovered in 1957 in beta and mu decays
- Weak force effect mediated by W or Z
- Tested extensively in leptonic and semileptonic processes
- What about the quarks?
  - Specifically neutral current interactions
  - NN interactions are the only answer
  - Hadronic PV much harder

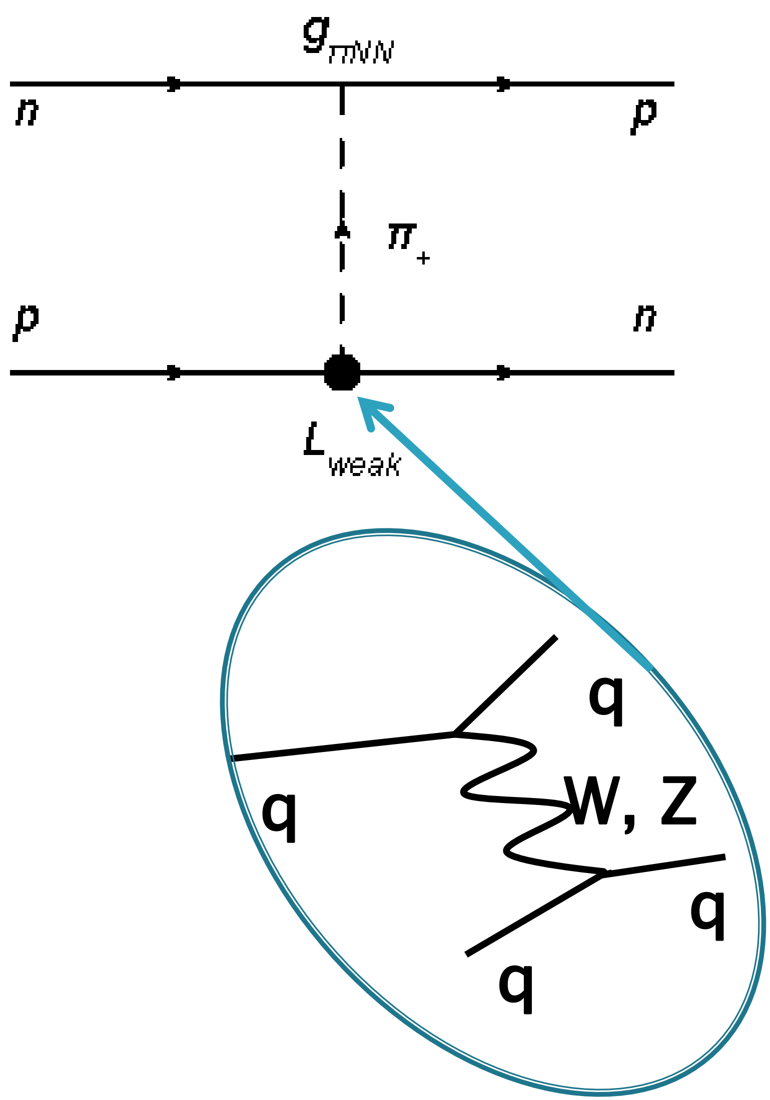


Nuclear Parity Violation

- Predicted by Feynman & Gell-Mann in 1958, confirmed experimentally in 1967 by Lobashov
- PV signal is dwarfed by QCD: $\mathcal{O}(10^{-7})$
- Experimental ways around this
- Large uncertainties and many-body effects

NN Parity Violating Interaction

- PV interaction ~ 0.002 fm
- NN force dominated by long-range interactions
- Hence: meson exchange models
- All weak physics encapsulated in weak vertex



Quark Level PV Operators

- Quark level operators known at W, Z scale
- Operator coefficients are scale-dependent
- Want operators at hadronic scale:
  - Integrate out W,Z
  - Run down to b-quark scale and integrate out
  - Run down to c-quark scale and integrate out
  - Run to hadronic scale

$$L_{PV} = -\frac{G_F \sin^2(\theta_w)}{3\sqrt{2}} \sum_i C_i(\lambda, m_c, m_b)\theta_i$$

$$\theta_1 = \overline{q}^a \gamma^\mu q_a \overline{q}^b \gamma_\mu \gamma_5 \tau_3 q_b$$

$$\theta_2 = \overline{q}^a \gamma^\mu q_b \overline{q}^b \gamma_\mu \gamma_5 \tau_3 q_a$$

$$\theta_3 = \overline{q}^a \gamma^\mu \gamma_5 q_a \overline{q}^b \gamma_\mu \tau_3 q_b$$

$$\theta_4 = \overline{q}^a \gamma^\mu \gamma_5 q_b \overline{q}^b \gamma_\mu \tau_3 q_a$$

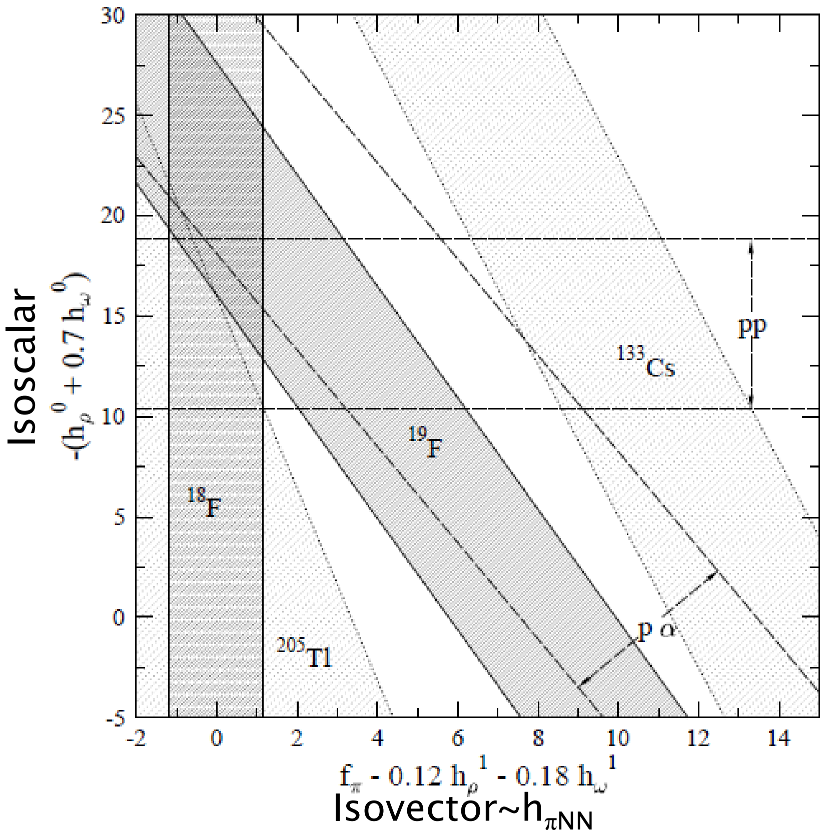
Hadron Level PV Interaction

- Leading order one-pion exchange

$$L_{weak}^{\Delta I=1} \sim h_{\pi NN}\left(\bar{p}n\pi^{+} - \bar{n}p\pi^{-}\right)$$
$$\sim h_{\pi NN} f_{\pi}\bar{N}\left(X_{L}^{3} - X_{R}^{3}\right)N$$

- Meson exchange model or ChPT
- $h_{\pi NN}$:
  - Encapsulates short-distance physics
  - Expected to dominate NN interaction (by dimensional analysis)
- This is the parameter we want to calculate!

Extracting $h_{\pi NN}$ from Experiment



From Haxton WC, Liu CP, and Ramsey-Musolf MJ, 2001, *Phys Rev Lett* 86, 5247-5250.

- NPDGamma (LANL & ORNL) want to extract at the 10% level.

Extracting $h_{\pi NN}$ from Theory

	Quark Model					χ-Soliton	QCD SR	
$\times 10^7$	DDH Range			Best	DZ	FCDH	KM	HHK	Lobov
h_π^1	0.0	$\leftrightarrow$	11.4	4.6	1.1	2.7	0.2	3.0	3.4

From CP Liu, J.Phys.Conf.Ser.69:012038,2007

- Large theoretical unknowns
- Fundamental problem is nonperturbative QCD
- Lattice QCD is an answer…
  - First proposed by Beane & Savage (2002)

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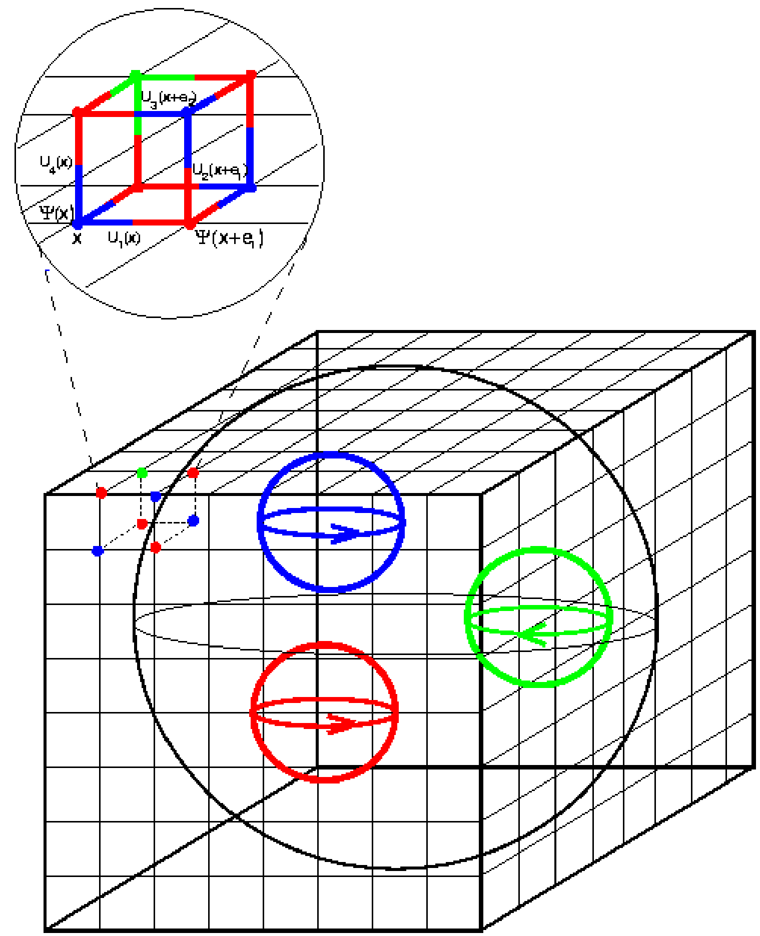
- Large theoretical unknowns
- Fundamental problem is nonperturbative QCD
- Lattice QCD is an answer…
  - First proposed by Beane & Savage (2002)
  - Unfortunate abbreviation!

Lattice QCD

- Only known way of calculating nuclear observables directly from the underlying theory
- Space and time discrete and finite

$$\Rightarrow \int [d\phi] \rightarrow \prod_n \int_{-\infty}^{\infty} d\phi_n$$

- Straightforward idea, but the devil is in the details!



What is Calculated

- Lattice correlation function:

$$C(x,y,z) = \langle 0 | \vartheta_1(x)\vartheta_2(y)\vartheta_3(z) | 0 \rangle$$
$$= \int [dq][d\bar{q}][dG]\vartheta_1(x)\vartheta_2(y)\vartheta_3(z)e^{-(S_g+S_q)}$$

$$C = \sum_{\vec{y},\vec{z}} C(0,y,z)$$
$$= \sum_{\vec{y},\vec{z}} \langle 0 | \vartheta_1(0)e^{-ipy}\vartheta_2(0)e^{ip(y-z)}\vartheta_3(0) | 0 \rangle$$

- Operators we want:

$$\vartheta_1 \sim proton$$

$$\vartheta_2 = \int d^3x\, L_{weak}(x)$$

$$\vartheta_3 \sim neutron + pion$$

Correlation Functions

- Lattice Correlation Function ≠ Matrix Element
- 3 point functions:

$$C_{3pt} \sim \langle 0 | \vartheta_p | p \rangle e^{-m_p t_1} \langle p | \int d^3 x L_{weak} | n\pi \rangle e^{-m_{n\pi}(t_2 - t_1)} \langle n\pi | \vartheta_{N\pi} | 0 \rangle$$

- 2 point function:

$$C_p \xrightarrow[t \to \infty]{} \left| \langle p | \vartheta_p | 0 \rangle \right|^2 e^{-m_p t}$$

$$C_{n\pi} \xrightarrow[t \to \infty]{} \left| \langle n\pi | \vartheta_{n\pi} | 0 \rangle \right|^2 e^{-m_{n\pi} t}$$

- Form ratios of correlation functions
  - Ratio as a function of time will plateau to the matrix element value

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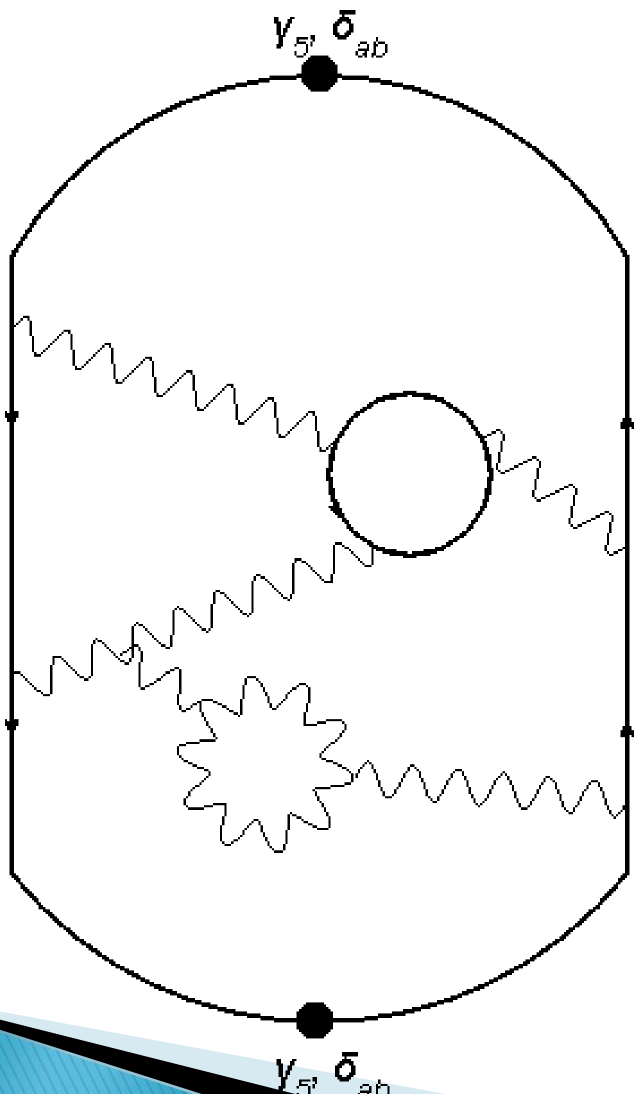
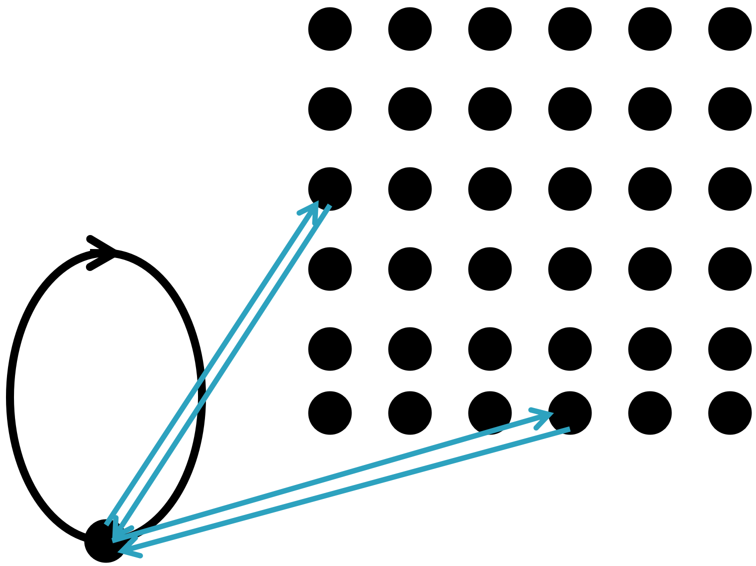
Steps to a Lattice Calculation



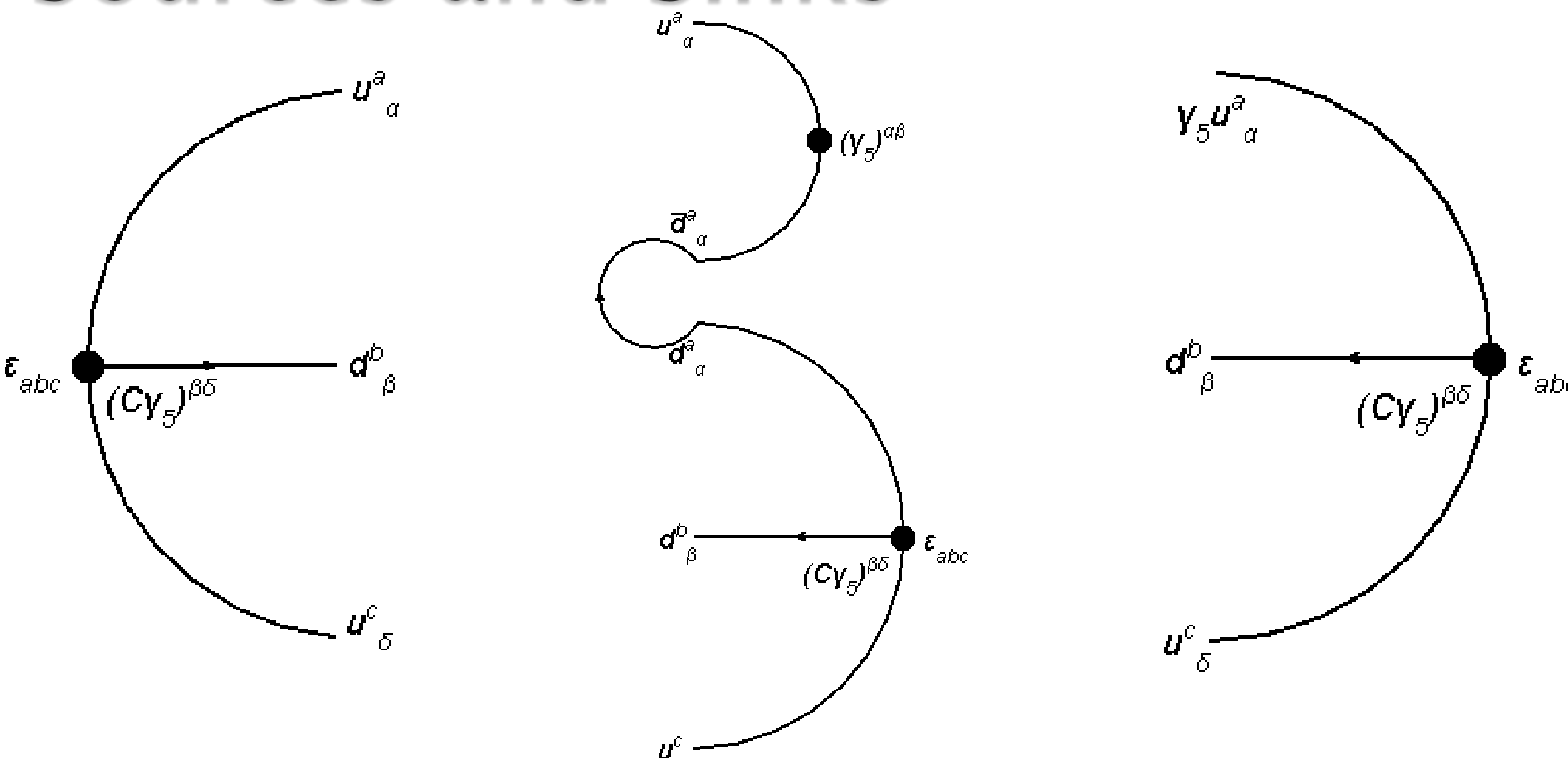
1. Gauge Configurations
2. Propagator Generation
3. Quark Contractions
4. Correlation Function Analysis

Disconnected Diagrams

- Quark loops
- Require all-to-all propagators
- Computationally expensive
- Can we avoid them?



Sources and Sinks

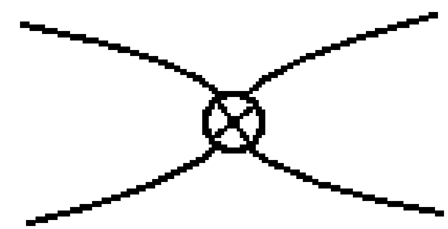


- Source for proton, sinks for neutron-pion
- Interpolating operators
- Produce more than just a proton/neutron-pi
- Parity projection

The Weak Operator

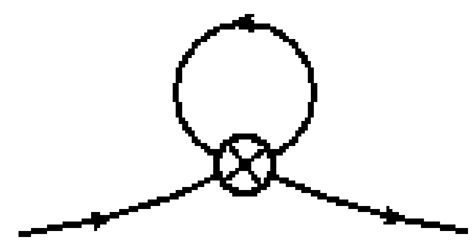
- 8 ops in SU(3), only 4 in SU(2)
- Three ways to put together:
  - Connected:

Sink $\bar{q}^{\alpha}\gamma^{\mu}q_{\alpha}\bar{q}^{\beta}\gamma_{\mu}\gamma_{5}\tau_{3}q_{\beta}$ Source



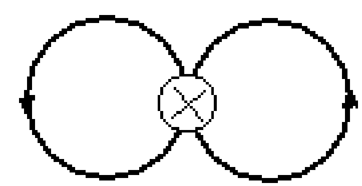
  - Disconnected:

Sink $\bar{q}^{\alpha}\gamma^{\mu}q_{\alpha}\bar{q}^{\beta}\gamma_{\mu}\gamma_{5}\tau_{3}q_{\beta}$ Source

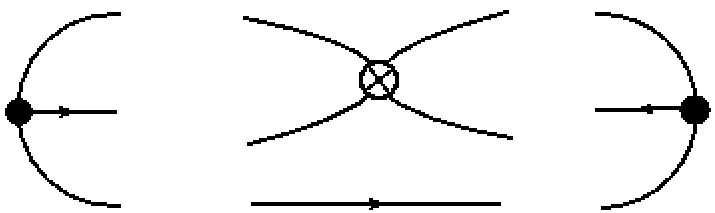


  - "I"-type

Sink $\bar{q}^{\alpha}\gamma^{\mu}q_{\alpha}\bar{q}^{\beta}\gamma_{\mu}\gamma_{5}\tau_{3}q_{\beta}$ Source



Propagators with a Weak Insertion



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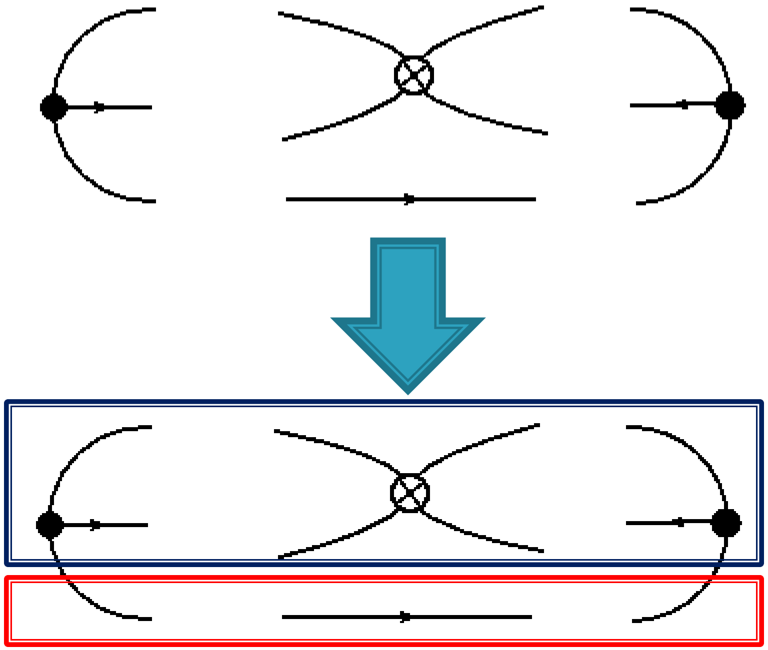

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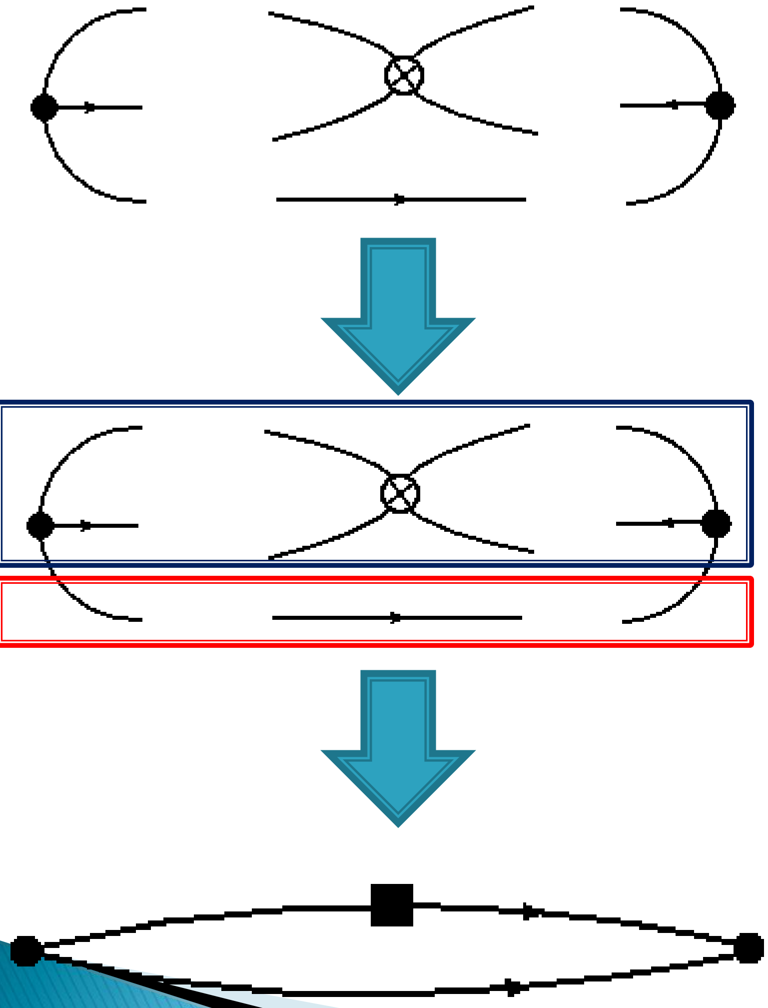
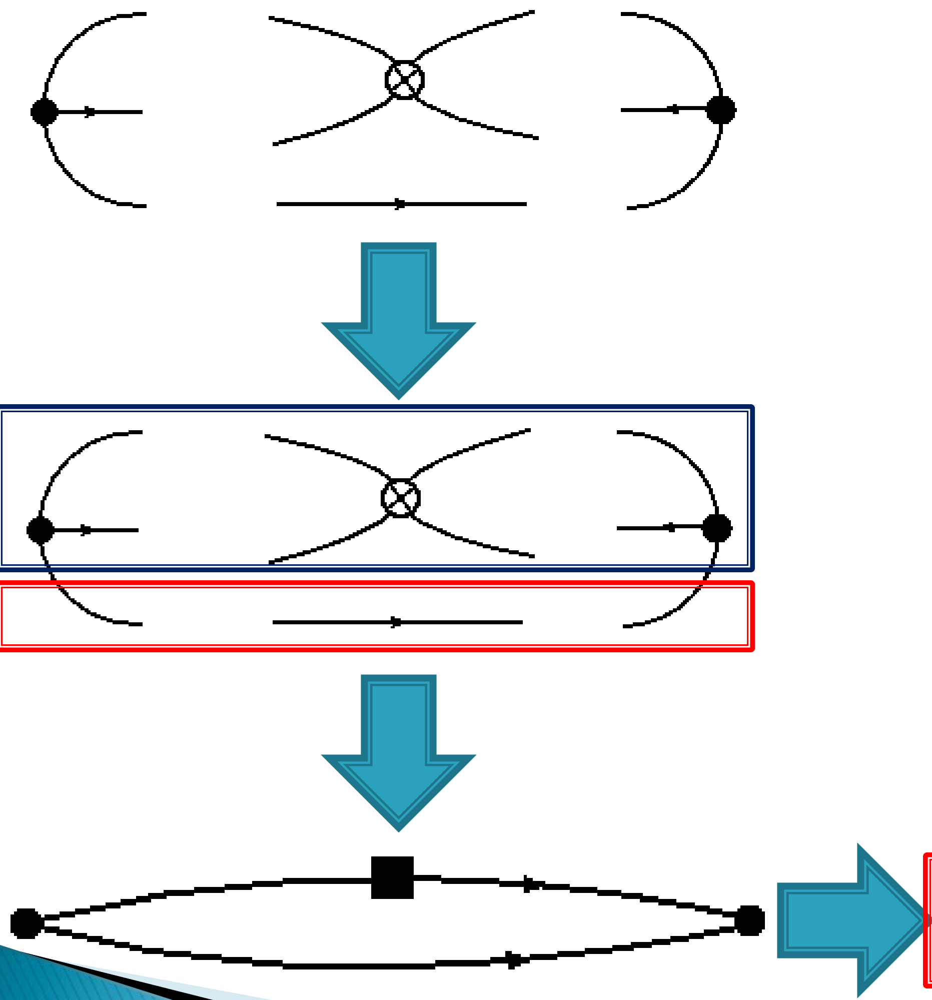


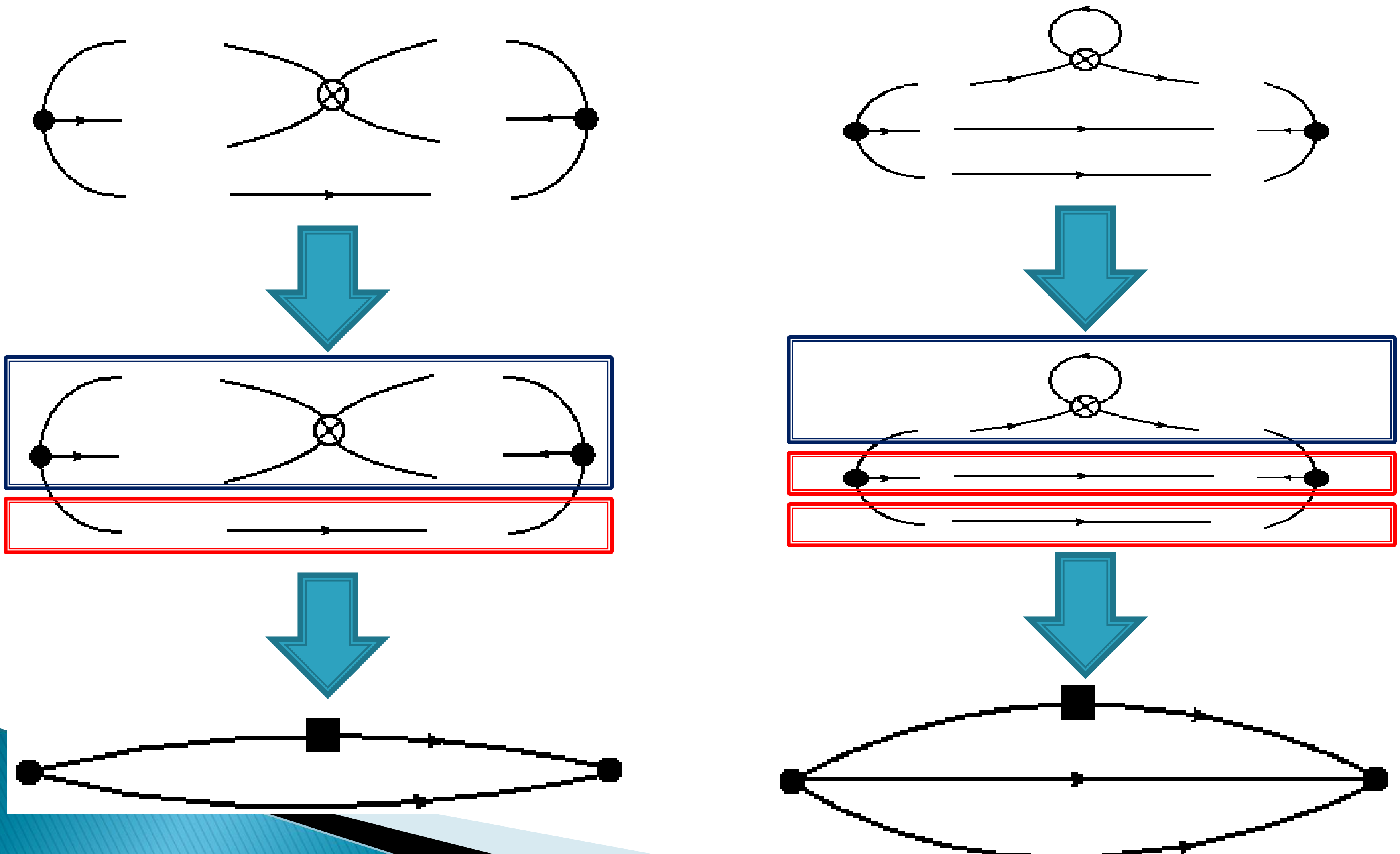
Propagators with a Weak Insertion



$$C(0,x,y) = \mathrm{tr}\left(U'[0,x,y]D[0,y]\right)$$

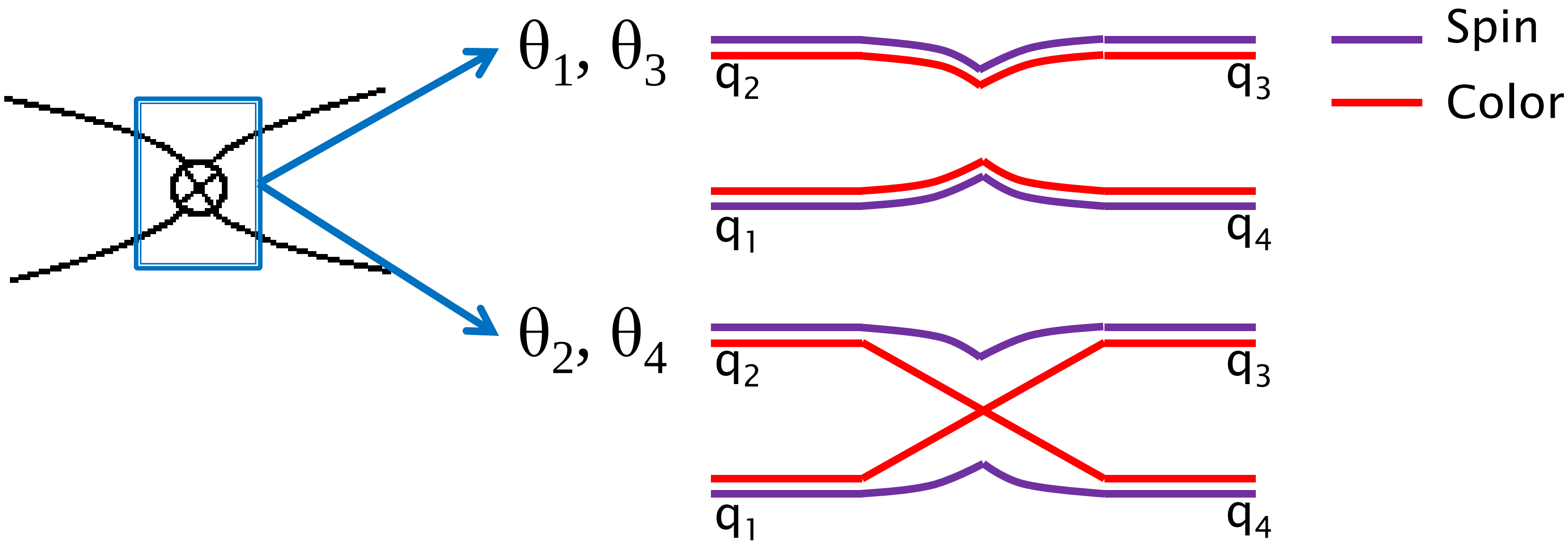
Propagators with a Weak Insertion

- Similarly for the disconnected types:



Propagators with a Weak Insertion

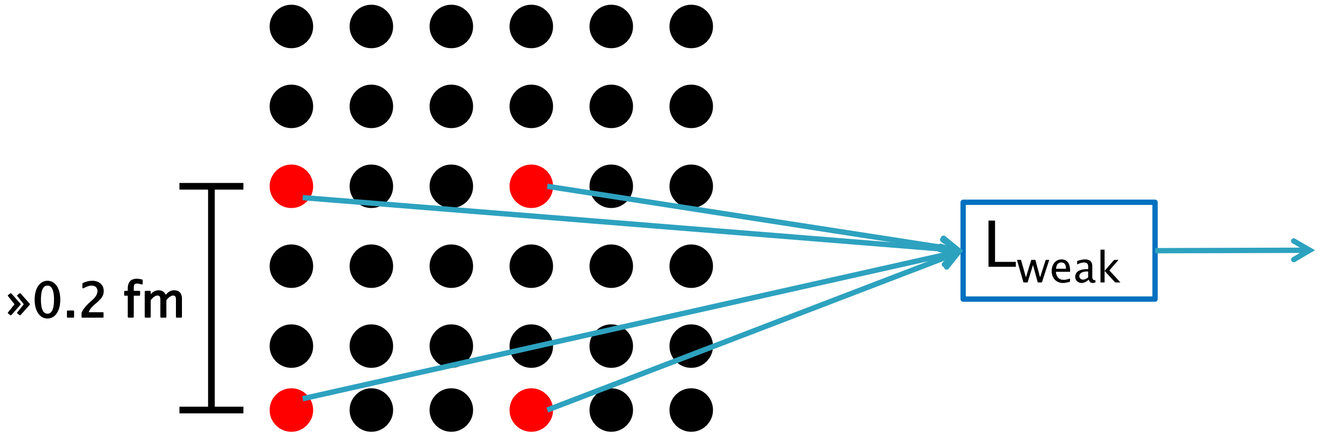
- 2 types of operators:



- First type allows matrix multiplication
- Second type:
  - Allows matrix multiplication in disconnected
  - Requires slower tensor manipulations w/connected

Additional Calculation Methods

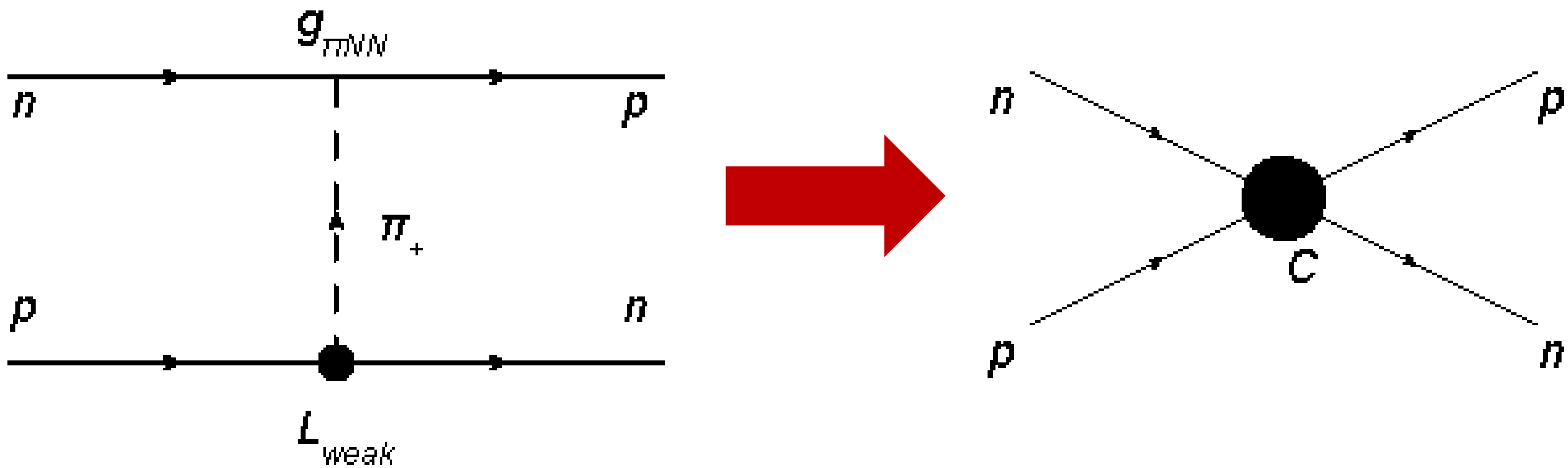
- Multiple sources on the same lattice
  - Nucleon wavelength ~ 0.2 fm
  - Allows multiple measurements on a single config.
  - Significant reduction in statistical errors
  - Used by Beane et al., Phys.Rev.D79:114502,2009.



Current Status

- Running code on $4^3 \times 32$ w/~30k configurations w/operator on a single timeslice
- Need to hammer out noise….
- Only hints of a signal thus far, but it is early.
- Stepping stone calculation:
    - Lattice spacing and size errors ~O(1)
    - Indicate necessary scaling

2 Nucleon Sector



- Need to understand 2 pt functions first!
- Very noisy (even compared with 1 baryon)
- No momentum injection though

$$C_{3pt} \sim \langle 0|\vartheta_{np,S}|pn\rangle e^{-m_{pn}t_1}\langle pn|\int d^3x L_{weak}|np\rangle e^{-m_{pn}(t_2-t_1)}\langle np|\vartheta_{np,P}|0\rangle$$

$$C_{pn} \xrightarrow[t\to\infty]{} \left|\langle pn|\vartheta_{pn}|0\rangle\right|^2 e^{-m_{pn}t}$$

Moving Forward

- Near future:
  - Calculation on small lattices with non-strange operators
    - Test scalability and provide ballpark figure for full calc.
- Farther ahead:
  - Full calculation including strange operators
  - Goal is to meet or beat NPDGamma (~10% errors)
  - A Peta or Exascale project…?