Testing quark forces in baryons using the 1/Nc expansion

How to match the quark model to the 1/Nc expansion of QCD

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* Relates properties of states in different SU(3) multiplets.
* The breaking of spin-flavor symmetry can be studied order by order in 1/Nc as an operator expansion.

Baryons in large Nc QCD

Witten '79

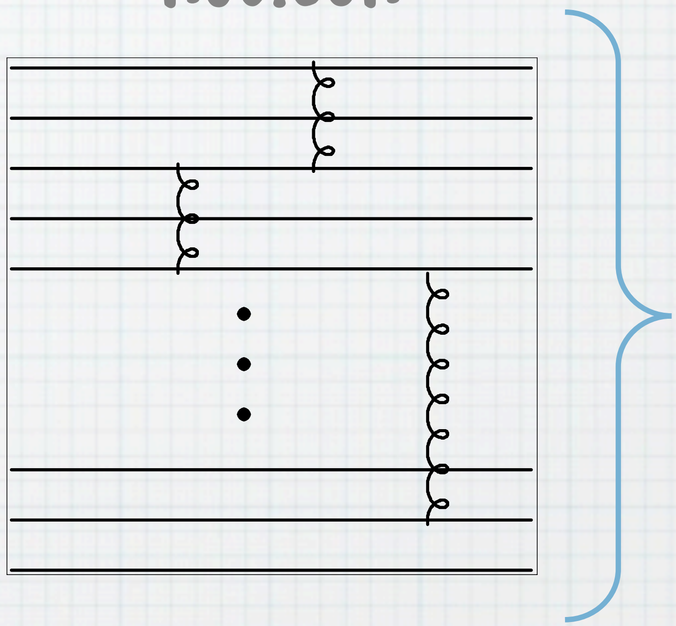
Large Nc counting rules

$$g \sim \frac{1}{\sqrt{N_c}}$$

m-body operator

$$\sim \frac{1}{N_c^{m-1}}$$

nucleon



Quark operator expansion

Ground state baryons:

R.F.Dashen, E.Jenkins and A.V.Manohar, PRD51, 3697 (1995)

Excited baryons:

J.L. Goity, PLB414, 140 (1997)
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Mass operator : $H_{mass} = \sum c_i O_i$

power counting
natural size

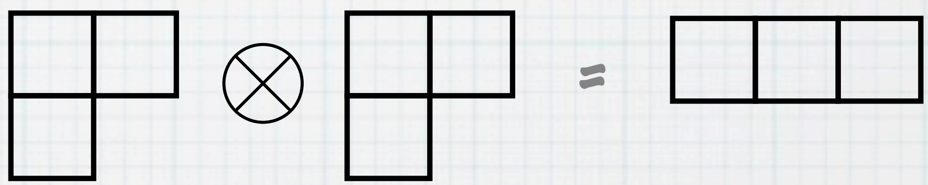
where c_i are numbers that must be fitted to data.

The N_J^* spectrum for L=1

$N_{3/2}$ (1700)

$N_{5/2}$ (1675)

$N_{1/2}$ (1650)



orbital spin-flavor

$N_{1/2}$ (1535)

$N_{3/2}$ (1520)

S=1/2 ,3/2 mixing

Example: The masses for L=1

Building blocks for the mass operator:

SU(4) generators acting on the core

$$S_c^i = \sum_{\alpha=1}^{N_c-1} s^i_{(\alpha)}, \quad T_c^a = \sum_{\alpha=1}^{N_c-1} t^a_{(\alpha)}, \quad G_c^{ia} = \sum_{\alpha=1}^{N_c-1} s^i_{(\alpha)} t^a_{(\alpha)}$$

and on the excited quark

s^i, t^a, g^{ia} spin-flavor $\quad$ l^i orbital degrees of freedom

The leading order

CCGL operators

$$\mathcal{O}_1 = N_c \mathbf{1}\,, \qquad \mathcal{O}_2 = l^i s^i\,, \qquad \mathcal{O}_3 = \frac{1}{N_c} l^{(2)ij} g^{ia} G_c^{ja}\,.$$

with

$$l^{(2)ij} = \tfrac{1}{2}\{l^i, l^j\} - \tfrac{1}{3} l^2 \delta^{ij}$$

and

$$\langle G_c^{ja} \rangle \propto N_c$$

$$\hat{M} = C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + C_3 \mathcal{O}_3 + \mathcal{O}(1/N_c)$$

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flavor exchange

with

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and

$$\langle G_c^{ja} \rangle \propto N_c$$

$$\hat{M} = C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + C_3 \mathcal{O}_3 + \mathcal{O}(1/N_c)$$

Mixing matrices

$$\mathbf{M}_{N_{1/2}} = \begin{pmatrix} C_1 N_c - \frac{2}{3} C_2 & -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 \\ -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 & C_1 N_c - \frac{5}{6} C_2 - \frac{5}{16} C_3 \end{pmatrix},$$

$$\mathbf{M}_{N_{3/2}} = \begin{pmatrix} C_1 N_c + \frac{1}{3} C_2 & -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 \\ -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 & c_1 N_c - \frac{1}{3} C_2 + \frac{1}{4} C_3 \end{pmatrix},$$

$$\mathbf{M}_{N_{5/2}} = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3 .$$

two mixing angles $\theta_{1/2}, \theta_{3/2}$.

We expect 5, but we only get 3 different masses !

C.S, D.Pirjol, Phys.Rev.D67, 096009 (2003)
T.Cohen, R.Lebed, Phys.Rev.Lett. 91, 012001, (2003)

$$M_0 = C_1 N_c - C_2 - \frac{5}{8} C_3$$

$$M_1 = C_1 N_c - \frac{1}{2} C_2 + \frac{5}{16} C_3$$

$$M_2 = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3$$

$$\mathbf{M}_{N_{1/2}} = \begin{pmatrix} C_1 N_c - \frac{2}{3} C_2 & -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 \\ -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 & C_1 N_c - \frac{5}{6} C_2 - \frac{5}{16} C_3 \end{pmatrix},$$

$$\mathbf{M}_{N_{3/2}} = \begin{pmatrix} C_1 N_c + \frac{1}{3} C_2 & -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 \\ -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 & c_1 N_c - \frac{1}{3} C_2 + \frac{1}{4} C_3 \end{pmatrix},$$

$$\mathbf{M}_{N_{5/2}} = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3 .$$

The N* spectrum for L=1

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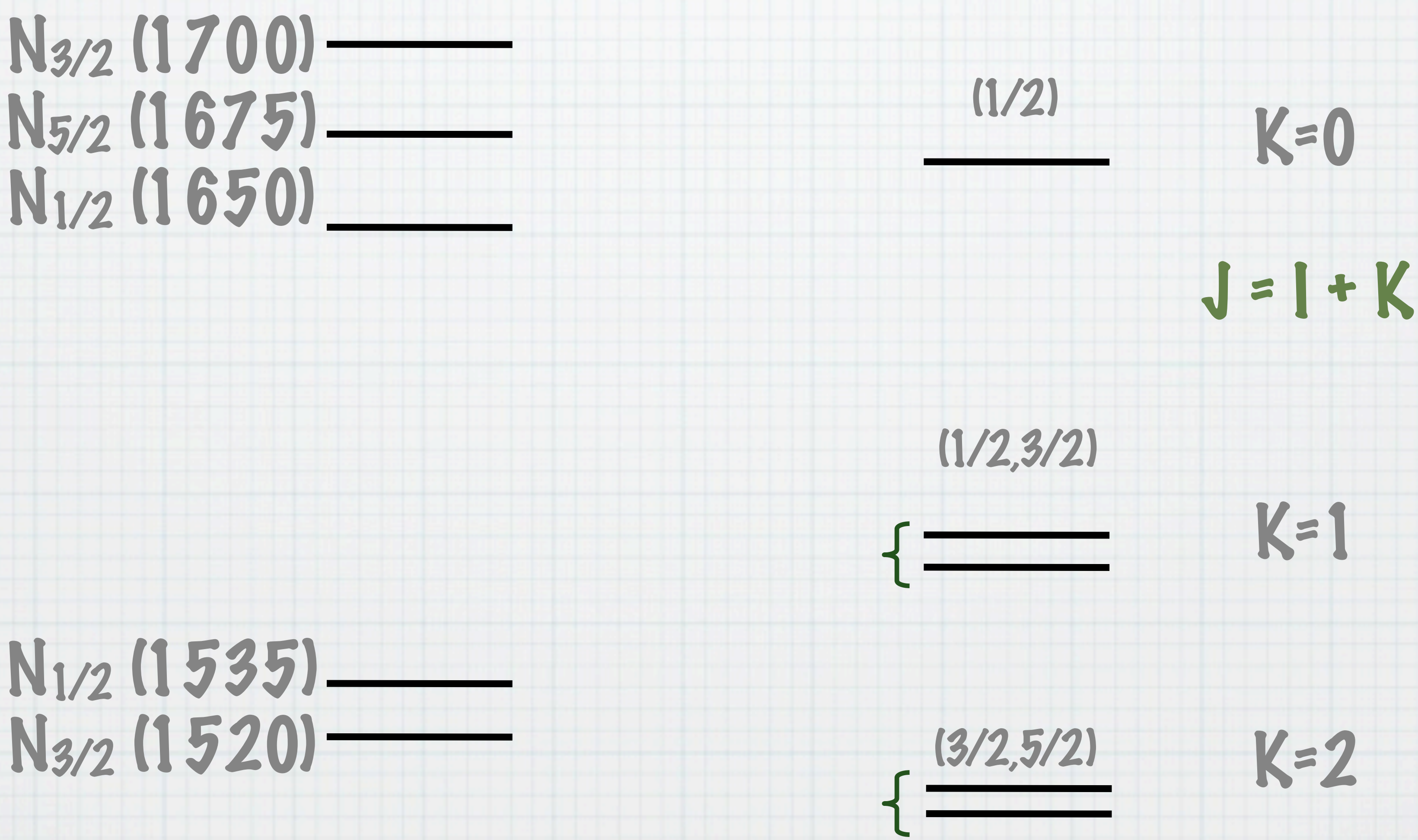
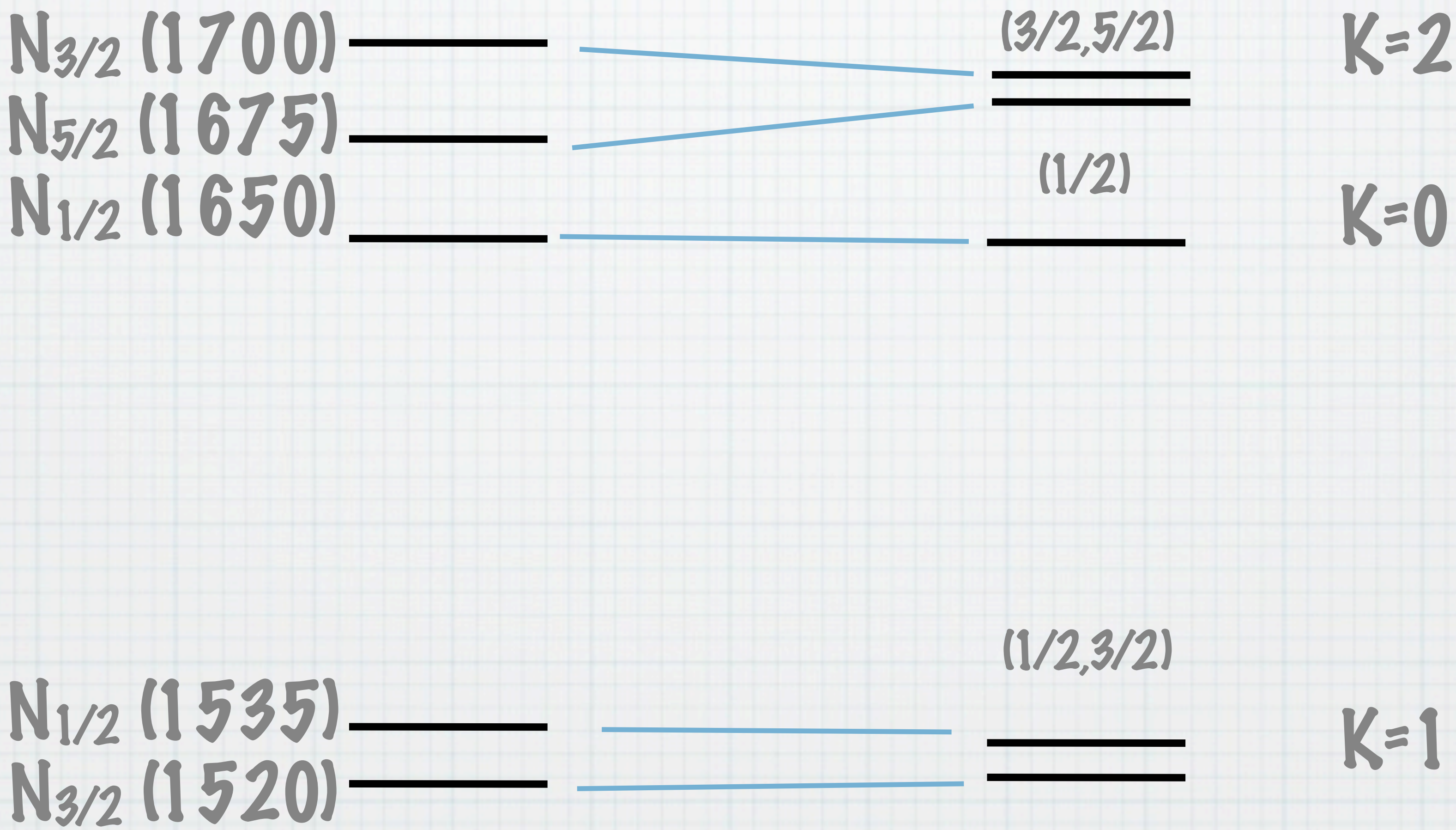
The N* spectrum for L=1

Spectrum in the $N_c \longrightarrow \infty$ limit



The N* spectrum for L=1



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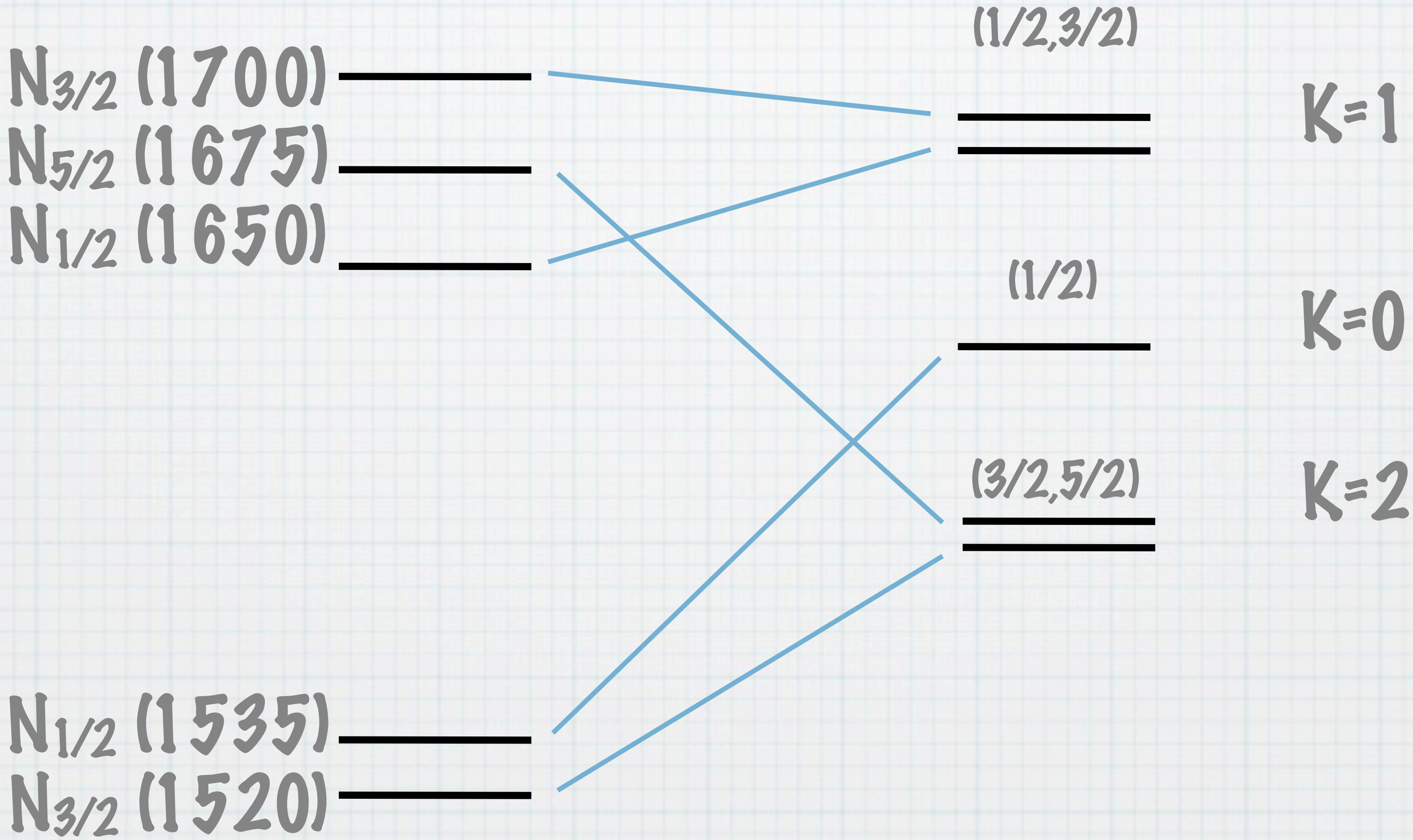

SU(4) algebra

$$
\begin{aligned}
[S_i, S_j] &= i\epsilon_{ijk} S_k \\
[T_a, T_b] &= i\epsilon_{abc} T_c
\end{aligned}
$$

$$
\begin{aligned}
[S_i, G_{ja}] &= i\epsilon_{ijk} G_{ka} \\
[T_a, G_{ib}] &= i\epsilon_{abc} G_{ic}
\end{aligned}
$$

$$
\left[G_{ia}, G_{jb}\right] = \frac{i}{4}\delta_{ij}\epsilon_{abc}T_c + \frac{i}{4}\epsilon_{ijk}\delta_{ab}S_k
$$

SU(4)c algebra

$$X^0_{ia} \equiv \lim_{N_c \to \infty} \frac{G_{ia}}{N_c}$$

$$[S_i, S_j] = i\epsilon_{ijk} S_k$$

$$[T_a, T_b] = i\epsilon_{abc} T_c$$

$$\left[S_i, X^0_{ja}\right] = i\epsilon_{ijk} X^0_{ka}$$

$$\left[T_a, X^0_{ib}\right] = i\epsilon_{abc} X^0_{ic}$$

$$\left[X^0_{ia}, X^0_{jb}\right] = 0$$

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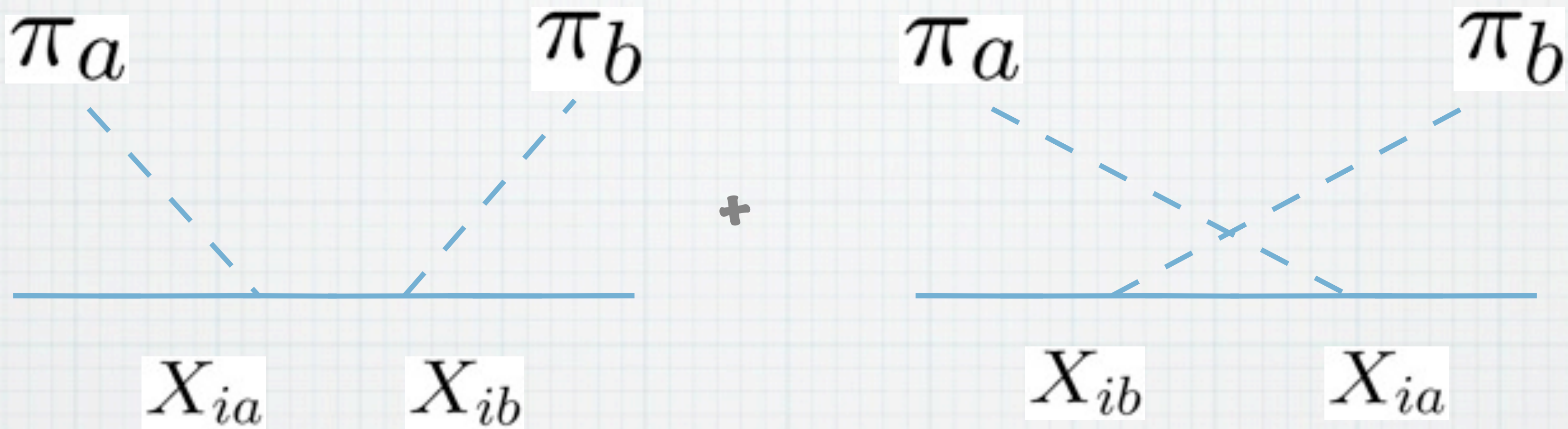
Consistency relations



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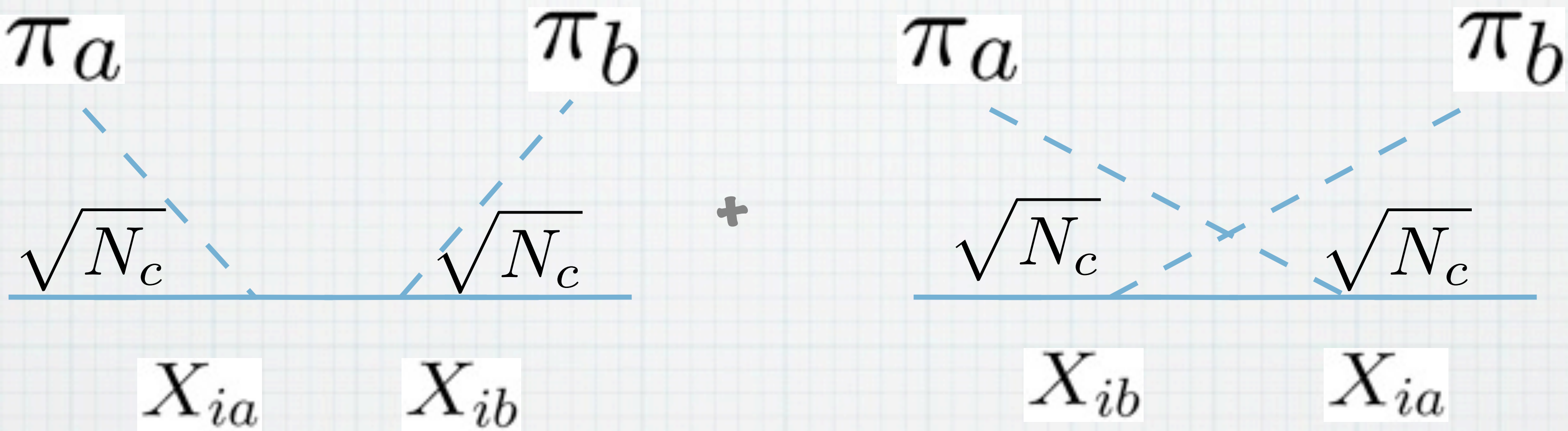
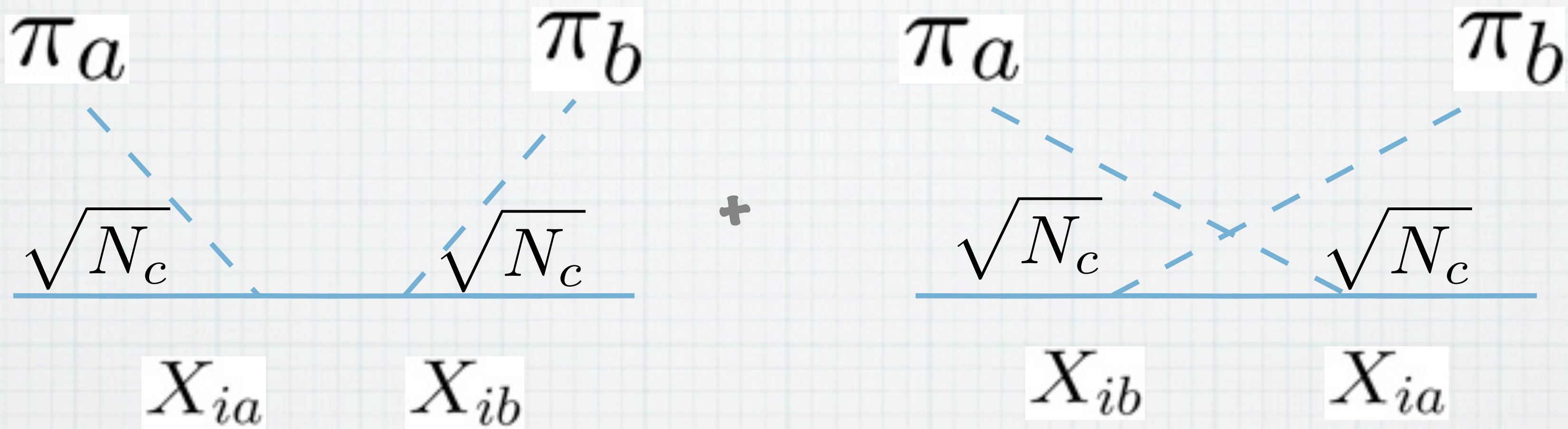


Consistency relations

Amplitudes are finite !



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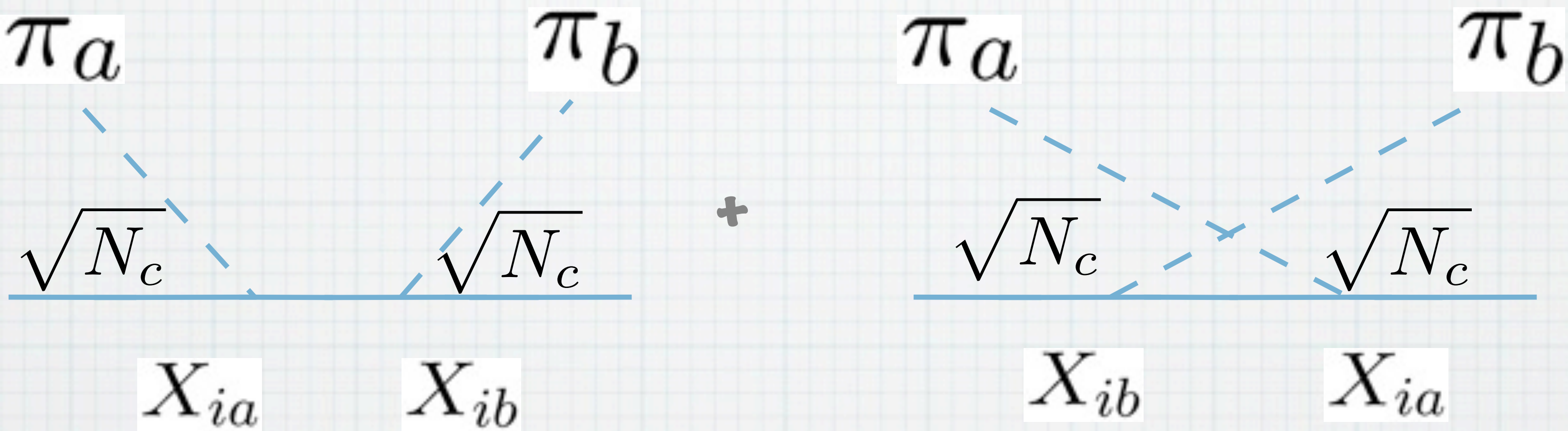

$$[X^0_{ia}, X^0_{jb}] = 0 \;\; (\approx \frac{1}{N_c})$$

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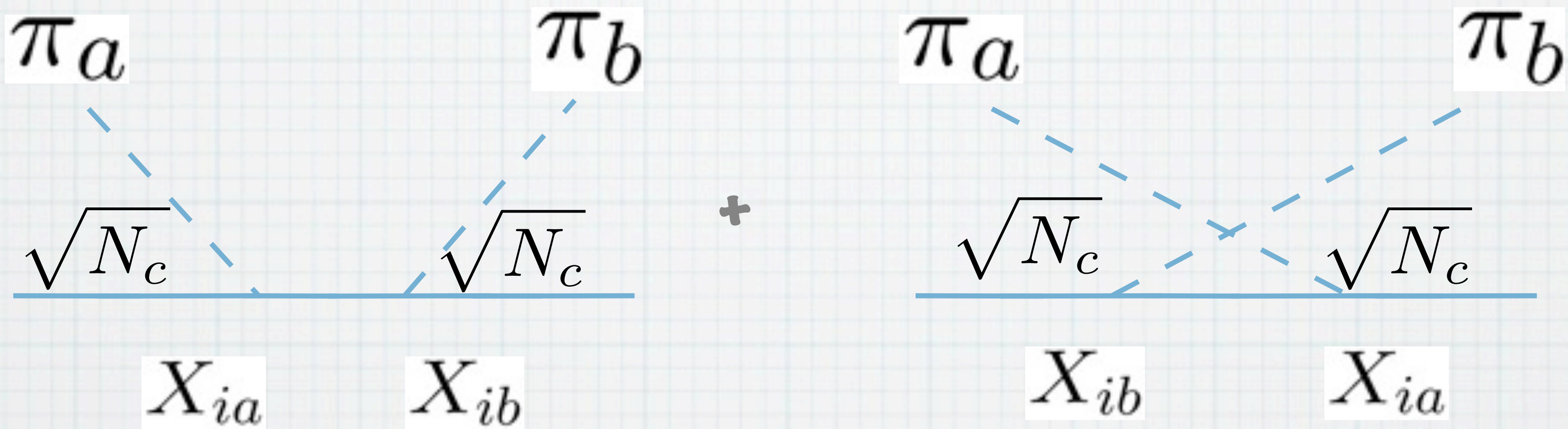

$$\Rightarrow \quad [X^0_{ia}, X^0_{jb}] = 0 \;\; (\approx \frac{1}{N_c})$$

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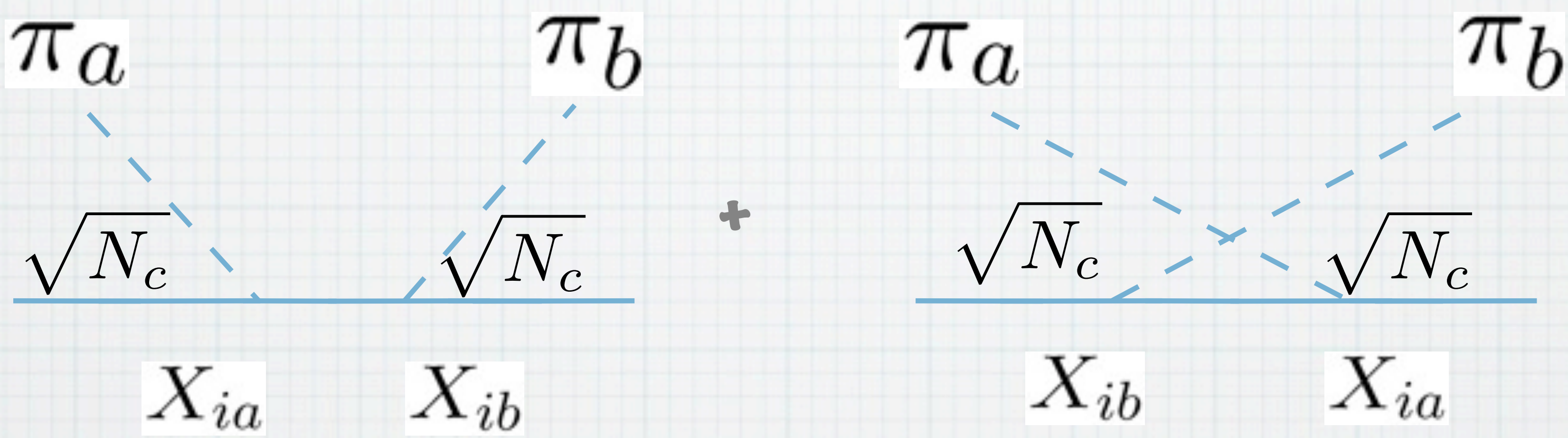
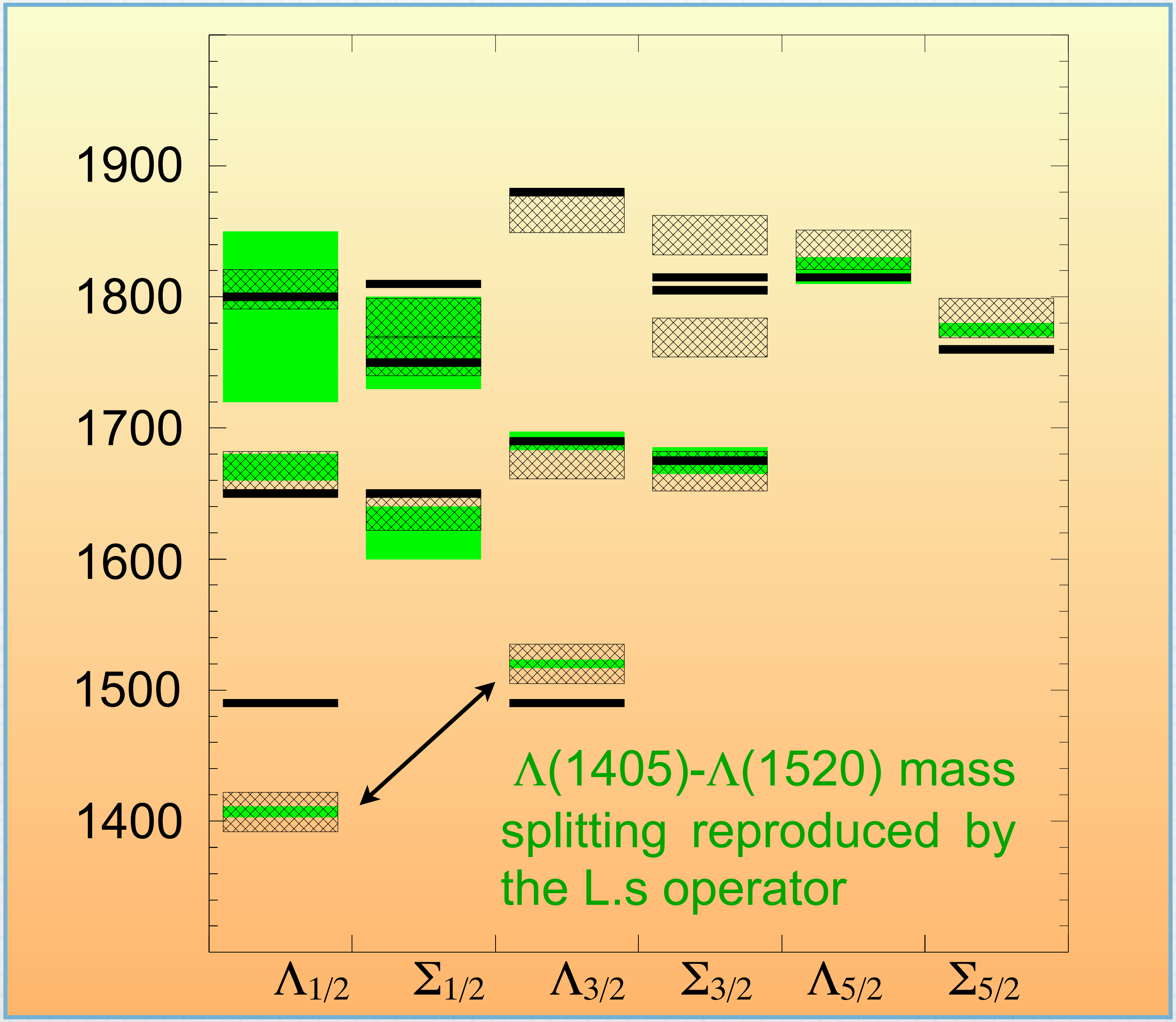


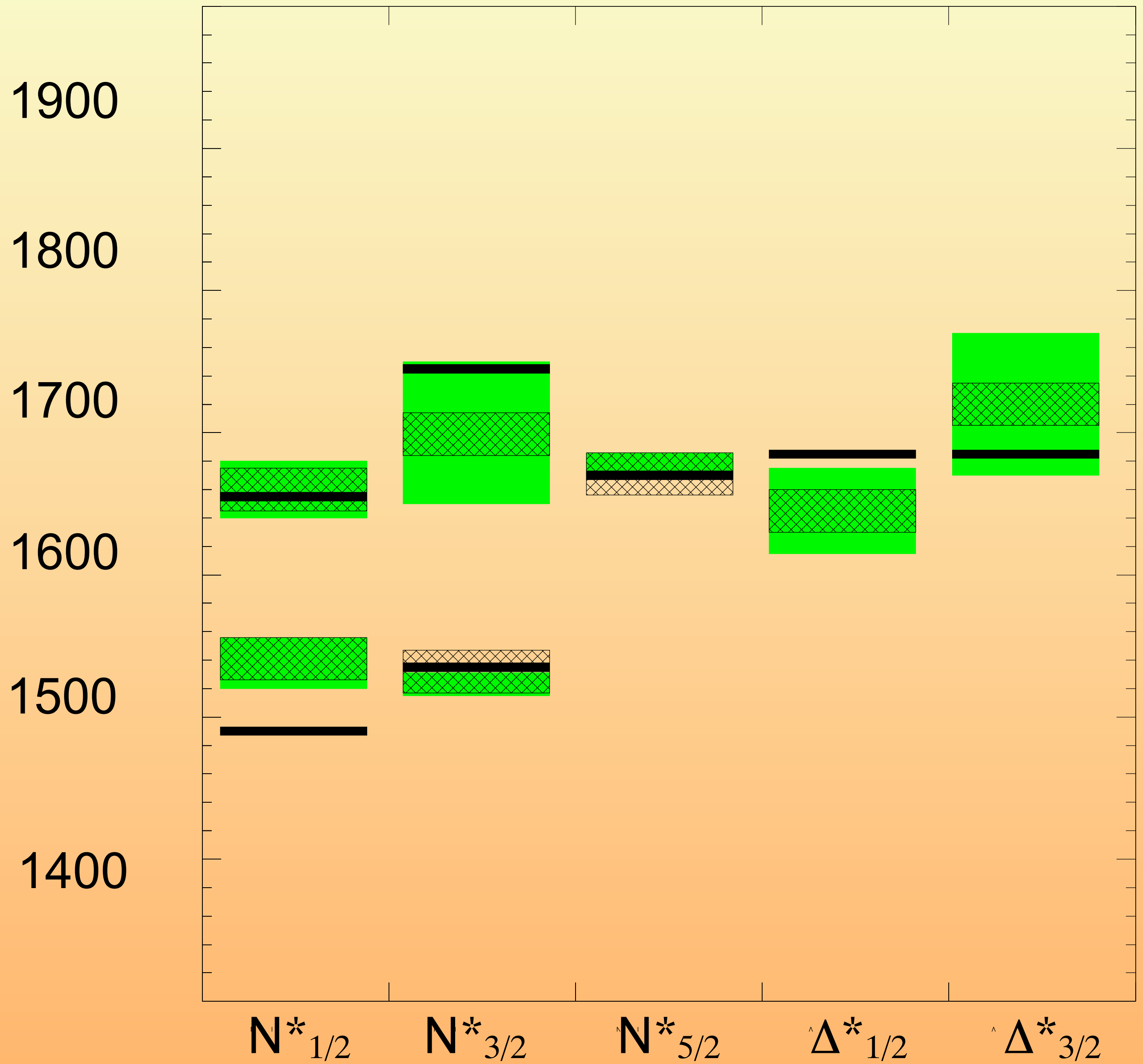
$$\left[X^0_{ia}, X^0_{jb}\right] \quad = \quad 0 \;\left(\approx \frac{1}{N_c}\right)$$

R.F.Dashen, E.Jenkins and A.V.Manohar, PRD49, 4713 (1994) .

Including 1/Nc and SU(3)



Including 1/Nc and SU(3)



C.S., J.L.Goity and N.N.Scoccola, PRL88, 102002 (2002).

Operator	Fitted coef. [MeV]
$O_1 = N_c\ 1$	$c_1 =$ 449 $\pm$ 2
$O_2 = l_h\ s_h$	$c_2 =$ 52 $\pm$ 15
$O_3 = \frac{3}{N_c}\ l^{(2)}_{hk}\ g_{ha}\ G^c_{ka}$	$c_3 =$ 116 $\pm$ 44
$O_4 = \frac{4}{N_c+1}\ l_h\ t_a\ G^c_{ha}$	$c_4 =$ 110 $\pm$ 16
$O_5 = \frac{1}{N_c}\ l_h\ S^c_h$	$c_5 =$ 74 $\pm$ 30
$O_6 = \frac{1}{N_c}\ S^c_h\ S^c_h$	$c_6 =$ 480 $\pm$ 15
$O_7 = \frac{1}{N_c}\ s_h\ S^c_h$	$c_7 =$ -159 $\pm$ 50
$O_8 = \frac{1}{N_c}\ l^{(2)}_{hk} s_h\ S^c_k$	$c_8 =$ 6 $\pm$ 110
$O_9 = \frac{1}{N_c^2}\ l_h\ g_{ka}\{S^c_k, G^c_{ha}\}$	$c_9 =$ 213 $\pm$ 153
$O_{10} = \frac{1}{N_c^2} t_a\{S^c_h, G^c_{ha}\}$	$c_{10} =$ -168 $\pm$ 56
$O_{11} = \frac{1}{N_c^2}\ l_h\ g_{ha}\{S^c_k, G^c_{ka}\}$	$c_{11} =$ -133 $\pm$ 130
$\bar{B}_1 = t_8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 =$ -81 $\pm$ 36
$\bar{B}_2 = T^c_8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 =$ -194 $\pm$ 17
$\bar{B}_3 = \frac{1}{N_c}\ d_{8ab}\ g_{ha}\ G^c_{hb} + \frac{N_c^2-9}{16\sqrt{3}N_c^2(N_c-1)} O_1 +$	
$+ \frac{1}{4\sqrt{3}(N_c-1)} O_6 + \frac{1}{12\sqrt{3}} O_7$	$d_3 =$ -150 $\pm$ 301
$\bar{B}_4 = l_h\ g_{h8} - \frac{1}{2\sqrt{3}} O_2$	$d_4 =$ -82 $\pm$ 57

SU(3) breaking

Basic questions

* Is this operator basis general enough ?
* What is the physics hidden in the coeff. ?
* How can we match quark models to the 1/Nc expansion ?

C.S and Dan Pirjol, Phys.Rev. D78, 034026 (2008)

Recent work:

* Theory: Matching the quark model to the 1/Nc expansion

  L.Galeta, D.Pirjol, C.S., Phys.Rev.D80, 116004 (2009).

* Phenomenology: Two new relations for L=1 excited baryons

  D.Pirjol, C.S., Phys.Rev.Lett.102:152002(2009).

The Isgur-Karl model revisited

L.Galeta, D.Pirjol, C.S., Phys.Rev. D80, 116004 (2009)

$$\mathcal{H}_{IK} = H_0 + \mathcal{H}_{\rm hyp}$$

confining potential + hyperfine interaction:

$$\mathcal{H}_{\rm hyp} = A \sum_{i<j} \Big[\frac{8\pi}{3} \vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) + \frac{1}{r_{ij}^3} (3 \vec{s}_i \cdot \hat{r}_{ij}\ \vec{s}_j \cdot \hat{r}_{ij} - \vec{s}_i \cdot \vec{s}_j) \Big]$$

Matching: find c_i and O_i

$$\langle B | \mathcal{H}_{\rm hyp} | B \rangle = \sum_i c_i \langle \Phi(JSI) | O_i | \Phi(JSI) \rangle$$

IK model interactions

$$\mathcal{H}_{\rm hyp} = \sum_{i<j} \mathcal{R}_{ij} \cdot \mathcal{O}_{ij}$$

spatial

$$R_{ij} = \frac{8\pi}{3} A\delta^{(3)}(\vec{r}_{ij}) \,,$$

$$Q_{ij}^{ab} = \frac{A}{r_{ij}^3}(3\hat{r}_{ij}^a\hat{r}_{ij}^b - \delta^{ab}) \,,$$

spin-flavor

$$O_{ij} = s_i \cdot s_j \,,$$

$$O_{ij}^{ab} = \frac{1}{2}(s_i^a s_j^b + s_i^b s_j^a)$$

Using the permutation group S_3

$$\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \frac{1}{3}\langle\mathcal{R}^S\rangle\langle\mathcal{O}^S\rangle + \frac{1}{3}\langle\mathcal{R}^{MS}\rangle\langle\mathcal{O}^{MS}\rangle$$

$$\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \frac{1}{3}\langle R_S\rangle\Big(\frac{1}{2}\vec{S}^2 - \frac{9}{8}\Big) + \frac{1}{3}\langle R_{MS}\rangle\Big(-\vec{S}^2 + 3\vec{s}_1 \cdot \vec{S}_c + \frac{9}{4}\Big)$$

+ tensor interaction

IK model matching

$$\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \sum_i c_i \langle \Phi(JSI)|O_i|\Phi(JSI)\rangle$$

The result of the matching is

$$\hat{M} = c_0 + aS_c^2 + bL_2^{ab}\{S_c^a\,, S_c^b\} + cL_2^{ab}\{s_1^a\,, S_c^b\}$$

$$\begin{aligned} a &= \frac{1}{2}\langle R_S\rangle\,, \\ b &= \frac{1}{12}\langle Q_S\rangle - \frac{1}{6}\langle Q_{MS}\rangle\,, \\ c &= \frac{1}{6}\langle Q_S\rangle + \frac{1}{6}\langle Q_{MS}\rangle\,. \end{aligned}$$

for a contact interaction:

$$\langle R_{MS}\rangle = -\langle R_S\rangle$$

The operators that appear are O_1, O_6, O_8, O_{17}

IK mass matrix

$$
\begin{aligned}
M_{1/2} &= \begin{pmatrix} c_0 + a & -\frac{5}{3}b + \frac{5}{6}c \\ -\frac{5}{3}b + \frac{5}{6}c & c_0 + 2a + \frac{5}{3}(b+c) \end{pmatrix}, \\
M_{3/2} &= \begin{pmatrix} c_0 + a & \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c \\ \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c & c_0 + 2a - \frac{4}{3}(b+c) \end{pmatrix}, \\
M_{5/2} &= c_0 + 2a + \frac{1}{3}(b+c)\,, \\
\Delta_{1/2} &= \Delta_{3/2} = c_0 + 2a\,.
\end{aligned}
$$

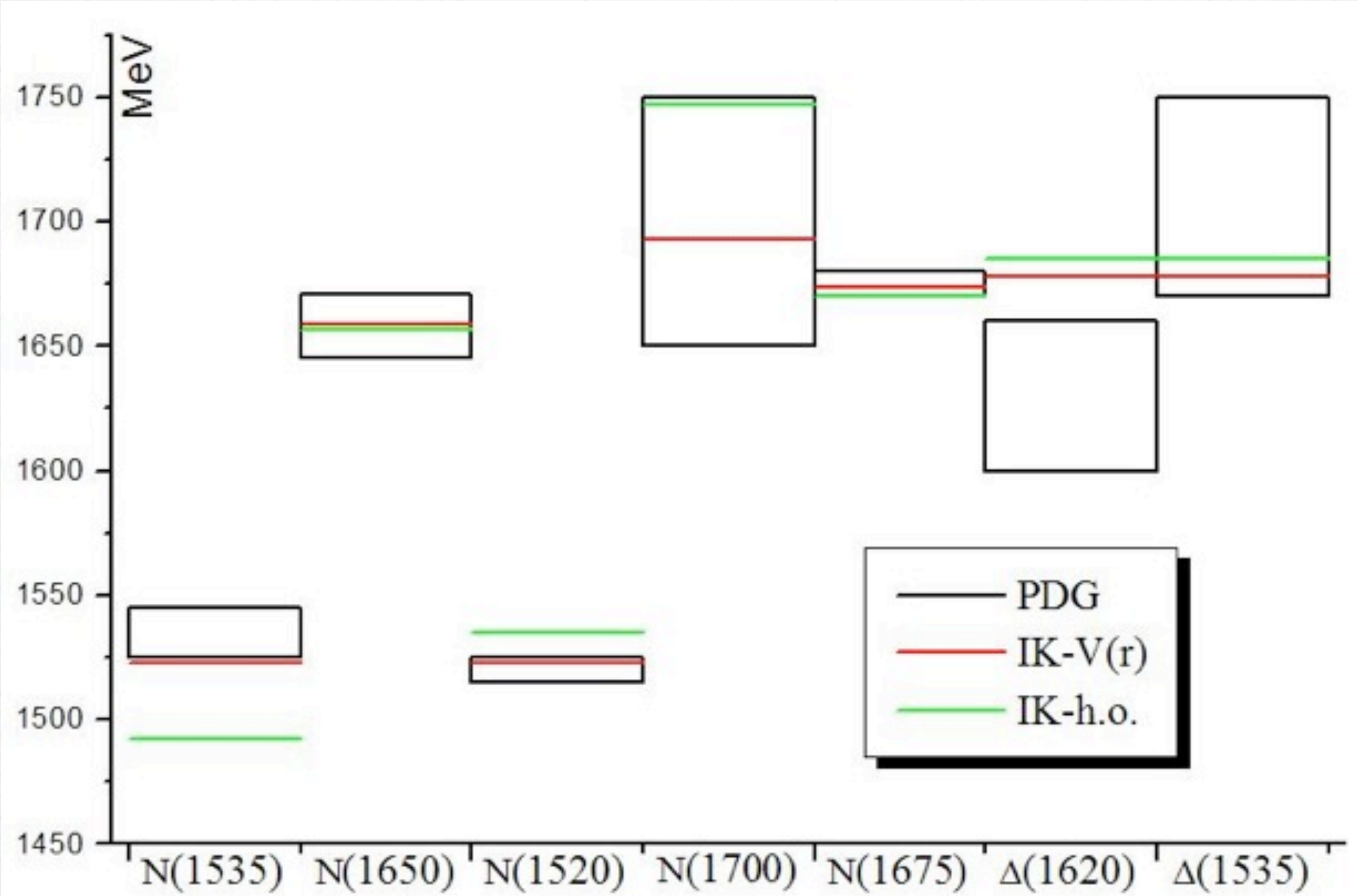
The spatial integrals can be computed explicitly in the original IK model with harmonic confinement:

$$
a = \frac{1}{2}\delta\,, \qquad b = \frac{1}{20}\delta\,, \qquad c = -\frac{1}{5}\delta\,.
$$

$$
\delta = M_\Delta - M_N \sim 300 \text{ MeV}
$$

ground states

IK-V(r) vs. IK-h.o.



MeV
1750
1700
1650
1600
1550
1500
1450
N(1535)
N(1650)
N(1520)
N(1700)
N(1675)
Δ(1620)
Δ(1535)
PDG
IK-V(r)
IK-h.o.

One gluon exchange model

The QCD Breit-Wigner interaction

$$\mathcal{H} = \sum_{i<j} V_{ij} \qquad V = V_{ss} + V_q + V_{so}$$

$$V_{ss} = \frac{16\pi\alpha_s}{9}\frac{1}{m_i m_j}\vec{s}_i \cdot \vec{s}_j \delta(\vec{r}_{ij})$$ Spin-spin

$$V_q = \frac{2\alpha_s}{3r_{ij}^3}\frac{1}{m_i m_j}\Big[\frac{3}{r_{ij}^2}(\vec{r}_{ij}\cdot\vec{s}_i)(\vec{r}_{ij}\cdot\vec{s}_j) - (\vec{s}_i\cdot\vec{s}_j)\Big]$$ Quadrupole

$$V_{so} = \frac{\alpha_s}{3r_{ij}^3}\Big[\frac{1}{m_i^2}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i - \frac{1}{m_j^2}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j$$ Spin-orbit

$$+\frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j - \frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i\Big]$$

One gluon exchange

more general

$$V_{ss} = \sum_{i<j=1}^{N} f_0(r_{ij}) \vec{s}_i \cdot \vec{s}_j$$

$$V_q = \sum_{i<j=1}^{N} f_2(r_{ij}) \left[3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \right]$$

$$V_{so} = \sum_{i<j=1}^{N} f_1(r_{ij}) \Big[(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j$$

$$+2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \Big]$$

One pion exchange

$$V_{ss} = \sum_{i<j=1}^{N} g_0(r_{ij})\vec{s}_i \cdot \vec{s}_j t_i^a t_j^a$$

$$V_q = \sum_{i<j=1}^{N} g_2(r_{ij})\Big[3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j)\Big] t_i^a t_j^a$$

$$V_{so} = \sum_{i<j=1}^{N} g_1(r_{ij})\Big[(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j$$

$$+2(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - 2(\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j\Big] t_i^a t_j^a$$

The permutation group S_N

L=1 states = $|S\rangle = |MS\rangle \times |MS\rangle$

orbital

$\hat{O} = \hat{O} \times \hat{O}$

spin-flavor

The permutation group S_N

L=1 states = $|S\rangle = |MS\rangle \times |MS\rangle$

orbital

spin-flavor

$\hat{O} = \hat{O} \times \hat{O}$

$$\langle S \,|\, \underbrace{\hat{O} \times \hat{O}}_{S} \,|\, S \rangle$$

The permutation group S_N

L=1 states = |S> = |MS> x |MS>

orbital → |MS>, Ô

spin-flavor → |MS>, Ô

$\hat{O} = \hat{O} \times \hat{O}$

$\langle S | \underbrace{\hat{O} \times \hat{O}}_{S} | S \rangle$

$\langle MS | \hat{O} | MS \rangle \longrightarrow MS \times MS = S + MS + E + A'$

The permutation group S_N

L=1 states = $|S\rangle = |MS\rangle \times |MS\rangle$

$$\langle S | \underbrace{\hat{O} \times \hat{O}}_{S} | S \rangle$$

orbital

spin-flavor

$$\hat{O} = \hat{O} \times \hat{O}$$

$$\langle MS | \hat{O} | MS \rangle \longrightarrow MS \times MS = \underbrace{S + MS + E + A'}_{\hat{O}}$$

The permutation group S_N

L=1 states = $|S\rangle = |MS\rangle \times |MS\rangle$

orbital

spin-flavor

$\hat{O} = \hat{O} \times \hat{O}$

$\langle S | \underbrace{\hat{O} \times \hat{O}}_{S} | S \rangle$

$\langle MS | \hat{O} | MS \rangle \longrightarrow MS \times MS = \underbrace{S + MS + E + A'}_{\hat{O}}$

Name	Partition	Dim	Character$\chi^{(R)}_{1^\alpha,2^\beta,3^\gamma,\ldots}$
S	$[N]$	1	1
MS	$[N-1,1]$	$N-1$	$\alpha-1$
E	$[N-2,2]$	$\frac{1}{2}N(N-3)$	$\frac{1}{2}(\alpha-1)(\alpha-2)+\beta-1$
A'	$[N-2,1,1]$	$\frac{1}{2}(N-1)(N-2)$	$\frac{1}{2}(\alpha-1)(\alpha-2)-\beta$

The permutation group S_N

L=1 states = |S> = |MS> x |MS>

orbital

spin-flavor

Ô= Ô x Ô

<S | Ô x Ô | S>

(Ô x Ô = S)

<MS | Ô | MS> → MS x MS = S + MS + E + A′

(S + MS + E + A′ → Ô)

Name	Partition	Dim	Character $\chi^{(R)}_{1^\alpha,2^\beta,3^\gamma,\ldots}$
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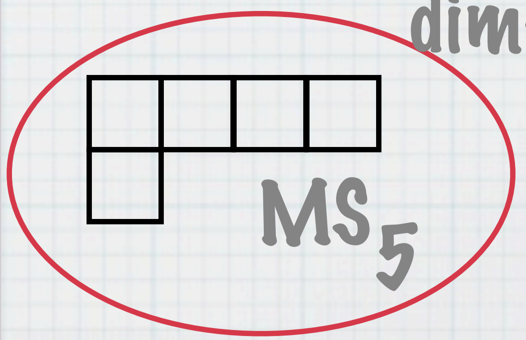
dim=4



Two body operators

Example: the spin-spin interaction $f_0(r_{ij})\ s_i \cdot s_j$

"Symmetric" two body operators:

$\{O_{ij}\} = S + MS + E$ $\quad$ $\text{dim} = \dfrac{N(N-1)}{2}$

$\langle S \mid \hat{O} \times \hat{O} \mid S \rangle$, with $\underbrace{\hat{O} \times \hat{O}}_{S}$

$S = S \times S + MS \times MS + E \times E$

$f(r_{ij})$ $\quad$ $s_i \cdot s_j$

Matching

Example: the spin-spin interaction $O_{ij} = f_0(r_{ij})\ s_i \cdot s_j$

$$\langle \mathcal{O}^{MS} \rangle = \frac{1}{N-2} \langle \Phi(SI) | -\vec{S}^{\,2} + N \vec{s}_1 \cdot \vec{S}_c + \frac{3}{4} N | \Phi(SI) \rangle$$

$$\langle \mathcal{R}^{MS} \rangle = (N-2)(\mathcal{I}_{\rm dir} - \mathcal{I}_s) - 2\mathcal{I}_{\rm exc}$$

To extract the leading order , we use the Hartree picture

$$\mathcal{I}_s = \langle ss | f_0 | ss \rangle$$

$$\mathcal{I}_{dir} = \langle sp | f_0 | sp \rangle$$

$$\mathcal{I}_{exc} = \langle sp | f_0 | ps \rangle$$

Overlap integrals

Gluon exchange model

$$M = c_0 \mathbf{1} - g^2 \left(a\vec{S}^2 + b\vec{s}_1 \cdot \vec{S}_c + cL^{ij}_{(2)} S^i S^j + dL^{ij}_{(2)} S^i_c s^j_1 + e\vec{L} \cdot \vec{S} + f\vec{L} \cdot \vec{s}_1 \right)$$

$S = S_c + s_1$

$$a = \frac{1}{2}\mathcal{I}_s \qquad d = -\mathcal{K}_{dir}$$

$$b = \mathcal{I}_{dir} - \mathcal{I}_s \qquad e = \frac{3}{2}\mathcal{J}^S_{\rm dir} + \frac{1}{2}\mathcal{J}^A_{dir}$$

$$c = \mathcal{O}(1/N) \qquad f = N\left(\frac{3}{2}\mathcal{J}^S_{\rm dir} - \frac{1}{2}\mathcal{J}^A_{dir}\right)$$

Pion exchange model

$$M = c_0\mathbf{1} + \frac{g_A^2}{f_\pi^2}\Big(a\vec{S}_c^2 + b\vec{s}_1\cdot\vec{S}_c + ct_1^aT_c^a + dL_{(2)}^{ij}g_1^{ia}G_c^{ja} + eL_{(2)}^{ij}\{S_c^i, S_c^j\} + f\vec{L}\cdot\vec{S}_c + gL^it_1^aG_c^{ia} + hL^ig_1^{ia}T_c^a\Big)$$

$$a = -\frac{1}{4}\mathcal{I}_s$$

$$b = c = -\frac{1}{4}\mathcal{I}_{dir}$$

$$d = -2\mathcal{K}_{dir}$$

$$e = \mathcal{O}(1/N)$$

$$f = \mathcal{O}(1/N)$$

$$g = \frac{3}{2}\mathcal{J}_{\rm dir}^S + \frac{1}{2}\mathcal{J}_{dir}^A$$

$$h = \frac{3}{2}\mathcal{J}_{\rm dir}^S - \frac{1}{2}\mathcal{J}_{dir}^A$$

Pion exchange model

$$M = c_0\mathbf{1} + \frac{g_A^2}{f_\pi^2}\Big(a\vec{S}_c^2 + b\vec{s}_1\cdot\vec{S}_c + ct_1^aT_c^a + dL_{(2)}^{ij}g_1^{ia}G_c^{ja} + eL_{(2)}^{ij}\{S_c^i, S_c^j\}$$
$$+ f\vec{L}\cdot\vec{S}_c + gL^it_1^aG_c^{ia} + hL^ig_1^{ia}T_c^a\Big)$$

flavor index

$$a = -\frac{1}{4}\mathcal{I}_s$$

$$b = c = -\frac{1}{4}\mathcal{I}_{dir}$$

$$d = -2\mathcal{K}_{dir}$$

$$e = \mathcal{O}(1/N)$$

$$f = \mathcal{O}(1/N)$$

$$g = \frac{3}{2}\mathcal{J}_{\rm dir}^S + \frac{1}{2}\mathcal{J}_{dir}^A$$

$$h = \frac{3}{2}\mathcal{J}_{\rm dir}^S - \frac{1}{2}\mathcal{J}_{dir}^A$$

A different approach: Consider all possible 2-body interactions

D.Pirjol, C.S., Phys.Rev.Lett.102:152002(2009)

The most general 2-body quark interaction

Complete list of 2-body spin-flavor quark operators, and their projection onto irreps of $S_3^{\rm sp-fl}$

Operator	$\mathcal{O}_{ij}$	O_S	O_{MS}
Scalar	1	1	$-$
	$t_i^a t_j^a$	$T^2 - 3C_2(F)$	$T^2 - 3t_1 T_c - 3C_2(F)$
	$\vec{s}_i \cdot \vec{s}_j$	$\vec{S}^2 - \frac{9}{4}$	$\vec{S}^2 - 3\vec{s}_1 \cdot \vec{S}_c - \frac{9}{4}$
	$\vec{s}_i \cdot \vec{s}_j t_i^a t_j^a$	$G^2 - \frac{9}{4}C_2(F)$	$3g_1 G_c - G^2 + \frac{9}{4}C_2(F)$
Vector (symm)	$\vec{s}_i + \vec{s}_j$	$\vec{L} \cdot \vec{S}$	$3\vec{L} \cdot \vec{s} - \vec{L} \cdot \vec{S}$
	$(\vec{s}_i + \vec{s}_j) t_i^a t_j^a$	$\frac{1}{2}L^i \{G^{ia}, T^a\} - C_2(F) L^i S^i$	$-\frac{1}{2}L^i S_c^i + L^i g_1^{ia} T_c^a + L^i t_1^a G_c^{ia}$
Vector (anti)	$\vec{s}_i - \vec{s}_j$	$-$	$3\vec{L} \cdot \vec{s} - \vec{L} \cdot \vec{S}$
	$(\vec{s}_i - \vec{s}_j) t_i^a t_j^a$	$-$	$L^i g_1^{ia} T_c^a - L^i t_1^a G_c^{ia}$
Tensor (symm)	$\{s_i^a, s_j^b\}$	$L_2^{ij} \{S^i, S^j\}$	$3L_2^{ij} \{s^i, S_c^j\} - L_2^{ij} \{S^i, S^j\}$
	$\{s_i^a, s_j^b\} t_i^c t_j^c$	$L_2^{ij} \{G^{ia}, G^{ja}\}$	$L_2^{ij} g_1^{ia} G_c^{ja} - \frac{1}{4} L_2^{ij} \{S_c^i, S_c^j\}$
Tensor (anti)	$[s_i^a, s_j^b]$	$-$	0
	$[s_i^a, s_j^b] t_i^c t_j^c$	$-$	0

$$C_2(F) = \frac{F^2 - 1}{2F}$$

[Contribute on S, MS states] [Contribute on MS states]

Basis of operators induced by 2-body quark operators

spin-spin

$$O_1 = T^2$$

$$O_2 = \vec{S}_c^2$$

$$O_3 = \vec{s}_1 \cdot \vec{S}_c$$

spin-orbit

$$O_4 = \vec{L} \cdot \vec{S}_c$$

$$O_5 = \vec{L} \cdot \vec{s}_1$$

$$O_6 = L^i t_1^a G_c^{ia}$$

$$O_7 = L^i g_1^{ia} T_c^a$$

tensor

$$O_8 = L_2^{ij} \{S_c^i, S_c^j\}$$

$$O_9 = L_2^{ij} s_1^i S_c^j$$

$$O_{10} = L_2^{ij} g_1^{ia} G_c^{ja}$$

Hadronic mass matrix (L=1 states)

$$N_i^* = \sum_{i=0}^{10} \hat{M}_{ij} c_j$$

$$N_i^* = (N_{1/2}, N'_{1/2}, N_{1/2} - N'_{1/2}, N_{3/2}, N'_{3/2}, N_{3/2} - N'_{3/2}, N_{5/2}, \Delta_{1/2}, \Delta_{3/2}, \Lambda_{1/2}, \Lambda_{3/2})$$

Valid for any model with only 2-body quark interactions.

The rank of M is 9, this leads to 2 mass relations:

Two new mass relations

#1 requires only isospin symmetry

$$\frac{1}{2}(N(1535)+N(1650))+\frac{1}{2}(N(1535)-N(1650))(3\cos 2\theta_{N1}+\sin 2\theta_{N1})$$
$$-\frac{7}{5}(N(1520)+N(1700))+(N(1520)-N(1700))\Big[-\frac{3}{5}\cos 2\theta_{N3}+\sqrt{\frac{5}{2}}\sin 2\theta_{N3}\Big]=-2\Delta_{1/2}+2\Delta_{3/2}-\frac{9}{5}N_{5/2}$$

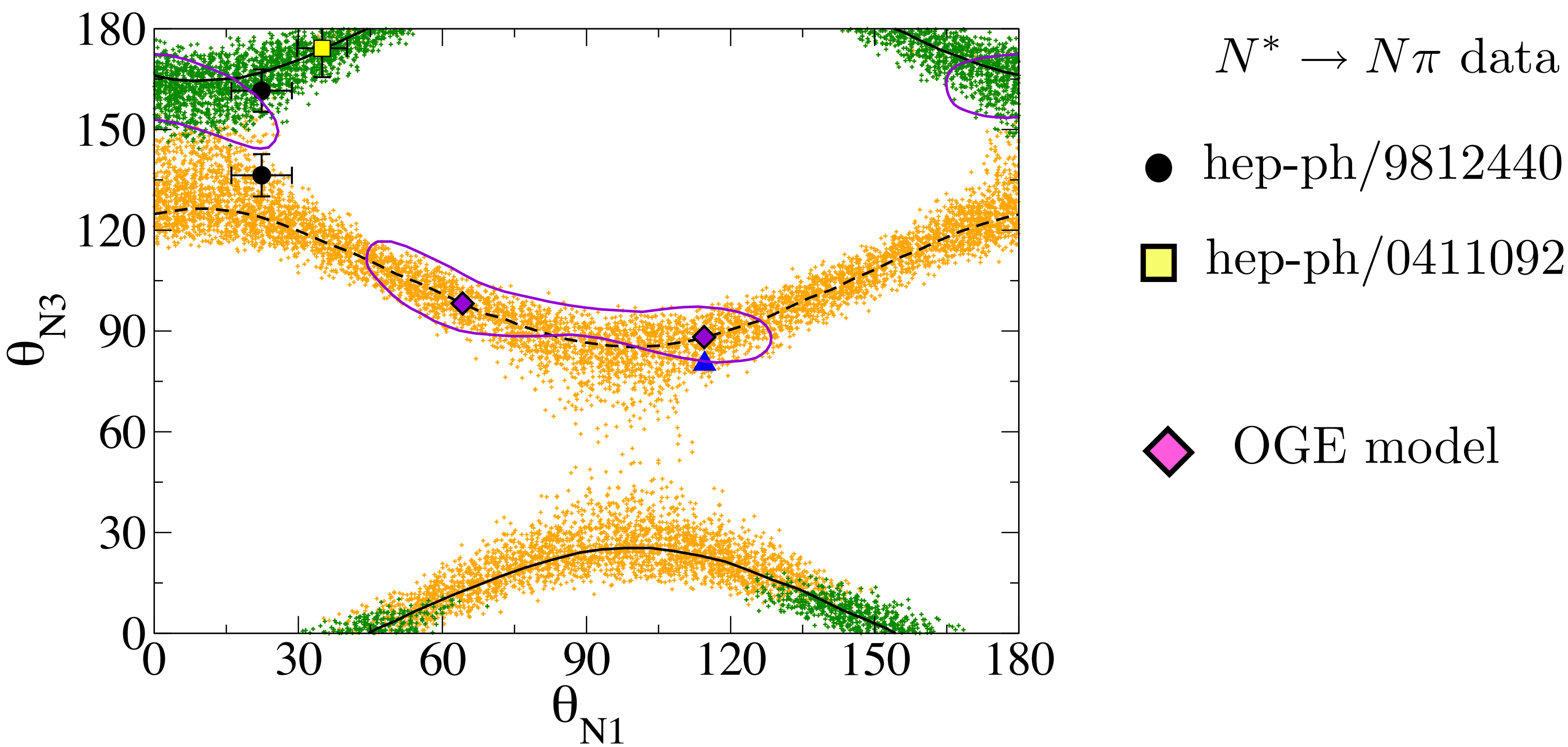
#2 includes SU(3) breaking

$$\begin{aligned}\bar{\Lambda} \;=\; & \frac{1}{6}(N(1535)+N(1650))+\frac{17}{15}(N(1520)+N(1700))-\frac{3}{5}N_{5/2}(1675)-\Delta_{1/2}(1620)\\ & -\frac{1}{6}(N(1535)-N(1650))(\cos 2\theta_{N1}+\sin 2\theta_{N1})+(N(1520)-N(1700))(\frac{13}{15}\cos 2\theta_{N3}-\frac{1}{3}\sqrt{\frac{5}{2}}\sin 2\theta_{N3})\end{aligned}$$

First universal relation

Correlation between mixing angles in the J=1/2 and 3/2 sectors

Requires only isospin symmetry



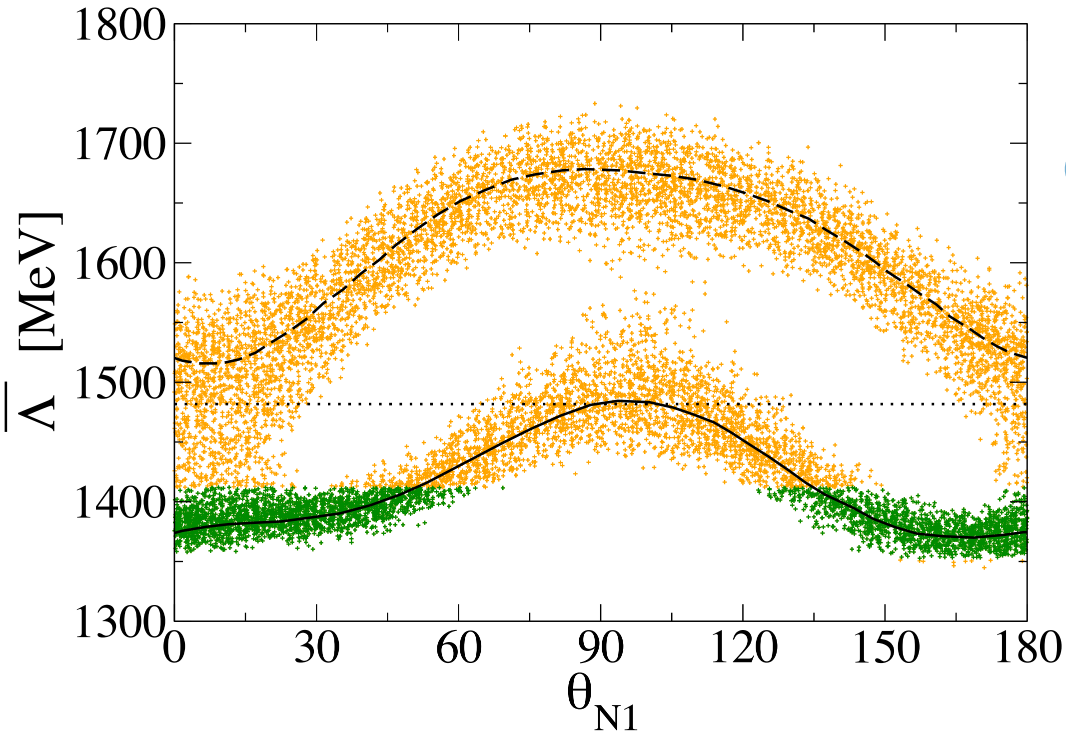
- Compare with direct determinations of the mixing angles -> test for the presence of 3-body quark interactions
- Tests for specific models of 2-body quark interactions, e.g. OGE model

Second universal relation

Expresses the spin-weighted SU(3) singlet mass

Takes into account SU(3) symmetry breaking

$$\bar{\Lambda} = \frac{1}{3}\Lambda_{1/2} + \frac{2}{3}\Lambda_{3/2}$$



$$\langle\Lambda\rangle^{\rm exp}_{\rm av} = 1481 \pm 3.1 \text{ MeV}$$

● $\langle\Lambda\rangle - \langle\Lambda\rangle^{\rm exp} = 100 \pm 30 \text{ MeV}$

Combining the two universal relations gives a determination of the mixing angles from hadron masses alone

Summary

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* The coefficients of the 1/Nc expansion can be matched to spatial integrals in quark models
* Flavor dependent interactions are required.
* Two new mass relations for N*'s that constrain the mixing angles and test for 3-body forces