### Testing quark forces in baryons using the 1/Nc expansion

How to match the quark model to the 1/Nc expansion of QCD

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- $*$  In the Nc  $\rightarrow \infty$  limit baryons fall into irreps of the contracted spin-flavor algebra SU(2 Nf)c .
- \* Relates properties of states in different SU(3) multiplets.
- \* The breaking of spin-flavor symmetry can be studied order by order in 1/Nc as an operator expansion.

# Baryons in large Nc QCD

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## Quark operator expansion

Ground state baryons:

R.F.Dashen, E.Jenkins and A.V.Manohar, PRD51, 3697 (1995)

C.E. Carlson, C.D. Carone, J.L. Goity and R.F. Lebed , PRD59, 114008 (1999) J.L. Goity, PLB414, 140 (1997) Excited baryons:

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#### Mass operator :  $H_{mass} = \sum c_i O_i$  power counting natural size

where c<sub>i</sub> are numbers that must be fitted to data.

 $N_{3/2}$  (1700) — N<sub>5/2</sub> (1675) — N<sub>1/2</sub> (1650)

### $\Box \otimes \begin{array}{ccc} \parallel & \cdot \end{array}$

orbital spin-flavor

N<sub>1/2</sub> (1535) — N<sub>3/2</sub> (1520) –

S=1/2 ,3/2 mixing

## Example: The masses for L=1

#### Building blocks for the mass operator:

SU(4) generators acting on the core

$$
S_c^i=\sum_{\alpha=1}^{N_c-1} s_{(\alpha)}^i, \ \ T_c^a=\sum_{\alpha=1}^{N_c-1} t_{(\alpha)}^a, \ \ G_c^{ia}=\sum_{\alpha=1}^{N_c-1} s_{(\alpha)}^i t_{(\alpha)}^a
$$

#### and on the excited quark

 $s^i, t^a, g^{ia}$  spin-flavor  $l^i$  orbital degrees of freedom

## The leading order

#### CCGL operators

$$
\mathcal{O}_1 = N_c \mathbf{1} \,, \qquad \mathcal{O}_2 = l^i s^i \,, \qquad \mathcal{O}_3 = \frac{1}{N_c} l^{(2)ij} g^{ia} G_c^{ja} \,.
$$

with 
$$
l^{(2)ij} = \frac{1}{2} \{l^i, l^j\} - \frac{1}{3} l^2 \delta^{ij}
$$

and

$$
\overline{\langle G^{ja}_c \rangle \propto N_c}
$$

 $\hat{M} = C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + C_3 \mathcal{O}_3 + \mathcal{O}(1/N_c)$ 

## The leading order

#### CCGL operators

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$$
\nwith\n
$$
l^{(2)ij} = \frac{1}{2} \{l^i, l^j\} - \frac{1}{3} l^2 \delta^{ij}
$$
\n\nflavor exchange\n
$$
\boxed{\langle G_c^{ja} \rangle \propto N_c}
$$
\n
$$
\hat{M} = C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + C_3 \mathcal{O}_3 + \mathcal{O}(1/N_c)
$$

### Mixing matrices

 $\overline{\phantom{0}}$ 

$$
\begin{array}{rcl}\n\mathbf{M}_{N_{1/2}} & = & \begin{pmatrix} C_1 N_c - \frac{2}{3} C_2 & -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 \\ -\frac{1}{3\sqrt{2}} C_2 - \frac{5}{8\sqrt{2}} C_3 & C_1 N_c - \frac{5}{6} C_2 - \frac{5}{16} C_3 \end{pmatrix} \\
\mathbf{M}_{N_{3/2}} & = & \begin{pmatrix} C_1 N_c + \frac{1}{3} C_2 & -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 \\ -\frac{\sqrt{5}}{6} C_2 + \frac{\sqrt{5}}{16} C_3 & c_1 N_c - \frac{1}{3} C_2 + \frac{1}{4} C_3 \end{pmatrix} \,, \\
\mathbf{M}_{N_{5/2}} & = & C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3 \,. \end{array}
$$

two mixing angles  $\,\theta_{1/2}$  ,  $\theta_{3/2}$  .

╭

#### We expect 5, but we only get 3 different masses! C.S, D.Pirjol, Phys.Rev.D67, 096009 (2003) T.Cohen, R.Lebed, Phys.Rev.Lett. 91, 012001, (2003)

$$
M_0 = C_1 N_c - C_2 - \frac{5}{8} C_3
$$
  
\n
$$
M_1 = C_1 N_c - \frac{1}{2} C_2 + \frac{5}{16} C_3
$$
  
\n
$$
M_2 = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3
$$

 $\mathbf{M}_{N_{1/2}}$ 

 $\mathbf{M}_{N_{3/2}}$ 

$$
=\left(\begin{array}{ccc}C_{1}N_{c}-\frac{2}{3}C_{2}&-\frac{1}{3\sqrt{2}}C_{2}-\frac{5}{8\sqrt{2}}C_{3}\\ -\frac{1}{3\sqrt{2}}C_{2}-\frac{5}{8\sqrt{2}}C_{3}&C_{1}N_{c}-\frac{5}{6}C_{2}-\frac{5}{16}C_{3}\\\ \end{array}\right)\\ =\left(\begin{array}{ccc}C_{1}N_{c}+\frac{1}{3}C_{2}&-\frac{\sqrt{5}}{6}C_{2}+\frac{\sqrt{5}}{16}C_{3}\\\ -\frac{\sqrt{5}}{6}C_{2}+\frac{\sqrt{5}}{16}C_{3}&c_{1}N_{c}-\frac{1}{3}C_{2}+\frac{1}{4}C_{3}\end{array}\right)\,,
$$

$$
\mathbf{M}_{N_{5/2}} = C_1 N_c + \frac{1}{2} C_2 - \frac{1}{16} C_3.
$$

 $N_{3/2}$  (1700) — N<sub>5/2</sub> (1675) — N1/2 (1650)

N<sub>1/2</sub> (1535) — N<sub>3/2</sub> (1520) –

#### Spectrum in the  $N_c \rightarrow \infty$  limit







### SU(4) algebra

 $[S_i, S_j] = i\epsilon_{ijk}S_k$ <br>  $[T_a, T_b] = i\epsilon_{abc}T_c$ 

$$
[S_i, G_{ja}] = i\epsilon_{ijk}G_{ka}
$$
  

$$
[T_a, G_{ib}] = i\epsilon_{abc}G_{ic}
$$

$$
\begin{bmatrix} G_{ia}, G_{jb} \end{bmatrix} = \frac{i}{4} \delta_{ij} \epsilon_{abc} T_c + \frac{i}{4} \epsilon_{ijk} \delta_{ab} S_k
$$

SU(4)c algebra

$$
\boxed{X_{ia}^0 \equiv \lim_{N_c \to \infty} \frac{G_{ia}}{N_c}}
$$



$$
\begin{bmatrix} S_i, X_{ja}^0 \end{bmatrix} = i\epsilon_{ijk}X_{ka}^0
$$
  

$$
\begin{bmatrix} T_a, X_{ib}^0 \end{bmatrix} = i\epsilon_{abc}X_{ic}^0
$$

$$
\left[X^0_{ia}, X^0_{jb}\right] = 0
$$



$$
\boxed{X_{ia}^0 \equiv \lim_{N_c \to \infty} \frac{G_{ia}}{N_c}}
$$



$$
\begin{bmatrix} S_i, X_{ja}^0 \end{bmatrix} = i\epsilon_{ijk} X_{ka}^0
$$
  

$$
\begin{bmatrix} T_a, X_{ib}^0 \end{bmatrix} = i\epsilon_{abc} X_{ic}^0
$$

 $\left[X^0_{ia},X^0_{jb}\right] \quad = \quad 0$ 



Consistency relations  $\pi_b$  $\pi_a$  $\pi_b$  $\pi_a$ +  $\sqrt{N_c}$  $\sqrt{N_c}$  $\sqrt{N_c}$  $'N_c$  $X_{ib}$  $X_{ia}$  $X_{ia}$  $X_{ib}$ 

Amplitudes are finite !

+



 $\pi_h$  $\pi_a$  $\sqrt{N_c}$  $\sqrt{N_c}$ 

 $X_{ib}$  $X_{ia}$ 

Amplitudes are finite !



Amplitudes are finite !



Amplitudes are finite !



## Including 1/Nc and SU(3)



### Including 1/Nc and SU(3)



#### C.S., J.L.Goity and N.N.Scoccola, PRL88, 102002 (2002).



SU(3) breaking



### \* Is this operator basis general enough? What is the physics hidden in the coeff. ?

\* How can we match quark models to the 1/Nc expansion ?

C.S and Dan Pirjol, Phys.Rev. D78, 034026 (2008)



#### \* Theory: Matching the quark model to the 1/Nc expansion

L.Galeta, D.Pirjol, C.S., Phys.Rev.D80, 116004 (2009).

 $*$  Phenomenology: Two new relations for L=1 excited baryons D.Pirjol, C.S., Phys.Rev.Lett.102:152002(2009).

#### The Isgur-Karl model The Islamic model is defined by the Islamic model is defined by the Islamic model is a set of the problem. In the state of the stat II. The Isque Karl model The Islands is defined to the participant is defined to the participant is defined by the quark Hamiltonian is defined to the contract of the II. The learne V arl model  $t_{\text{max}}$  i,  $\frac{1}{2}$  under the reducible reducible representation 3 under the permutation 3 under t

L.Galeta, D.Pirjol, C.S., Phys.Rev. D80, 116004 (2009)

$$
\mathcal{H}_{IK}=H_0+\mathcal{H}_{\rm hyp}
$$

confining potential + hyperfine interaction: confining potential + hyperfine interaction. confining potential + hyperfine interaction:

i

 $\frac{1}{\sqrt{2\pi}}$  contains the contains of the contains of the quark fields, and is the quark field  $\frac{i}{i}$  is given by  $\frac{i}{i}$  is given by  $r_{ij}$  is given by  $r_{ij}$  $\mathcal{H}_{\mathrm{hyp}} = A \sum_{\sigma}$  $i{<}j$  $\lceil \frac{8\pi}{2} \rceil$ 3  $\vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) +$ 1  $r^3_{ij}$  $\left(3\vec{s}_i\cdot\hat{r}_{ij}\ \vec{s}_j\cdot\hat{r}_{ij}-\vec{s}_i\cdot\vec{s}_j\right)\right]$  $\frac{1}{\sqrt{2}}$ rij = # r<sup>i</sup> −# r<sup>j</sup> is the distance between  $d\mathcal{H}_{\text{hyp}} = A \sum_i \left[ \frac{8\pi}{2} \vec{s}_i \cdot \vec{s}_j \delta^{(3)}(\vec{r}_{ij}) + \frac{1}{2} (3\vec{s}_i \cdot \hat{r}_{ij} \vec{s}_j \cdot \hat{r}_{ij} - \vec{s}_i \cdot \vec{s}_j) \right]$ Matching: find c<sub>i</sub> and O<sub>i</sub>

 $\vert$   $\vert$   $\langle$  *L*  $\mathcal{L}$  $\overline{\phantom{a}}$  $\mathcal{I}$  $\ket{C_i|B} = \sum_{i} c_i \langle \Phi_i |$ 1  $\overline{U}$  $\langle SI \rangle |O_i| \Phi (JSI) \rangle$  $\langle B|H_1|B\rangle = \sum c_i \langle \Phi(ISI)|O_i|\Phi(ISI)\rangle$ interaction between two dipoles. This interaction  $\boldsymbol{i}$  $\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \sum$  $c_i \langle \Phi (JSI) | O_i | \Phi (JSI) \rangle$ 

s<sup>i</sup> · #

r<sup>j</sup> is the distance between

 $\frac{1}{2}$ 

rij =

#

r<sup>i</sup> −#

#### IK model interactions we repeat the convenience of the readership spin-flavor operators. Both can also ca  $\mathcal{S}$

system of the L  $\sim$  1 negative particle particle particle particle particle particle particle particle permutation

where  $R$  is a coordinate  $R$  is a coordinate orbital operators acting on the coordinates of the quarks i, and Oij are  $\alpha$ 

$$
\mathcal{H}_{\text{hyp}} = \sum_{i < j} \mathcal{R}_{ij} \cdot \mathcal{O}_{ij}
$$
\nspatial\n
$$
R_{ij} = \frac{8\pi}{3} A \delta^{(3)}(\vec{r}_{ij}), \qquad \qquad O_{ij} = s_i \cdot s_j,
$$
\n
$$
Q_{ij}^{ab} = \frac{A}{r_{ij}^3} (3 \hat{r}_{ij}^a \hat{r}_{ij}^b - \delta^{ab}), \qquad Q_{ij}^{ab} = \frac{1}{2} (s_i^a s_j^b + s_i^b s_j^a)
$$

Weise the repeat to ties exern C Using the permutation group S3

$$
\langle B|\mathcal{H}_{\text{hyp}}|B\rangle = \frac{1}{3}\langle \mathcal{R}^{S}\rangle \langle \mathcal{O}^{S}\rangle + \frac{1}{3}\langle \mathcal{R}^{MS}\rangle \langle \mathcal{O}^{MS}\rangle
$$

$$
\langle B|\mathcal{H}_{\text{hyp}}|B\rangle = \frac{1}{3}\langle R_{S}\rangle \Big(\frac{1}{2}\vec{S}^{2} - \frac{9}{8}\Big) + \frac{1}{3}\langle R_{MS}\rangle \Big(-\vec{S}^{2} + 3\vec{s}_{1} \cdot \vec{S}_{c} + \frac{9}{4}\Big)
$$
  
+ tensor interaction

2

2

, S<sup>b</sup>

tion are written in terms of the matrix of the spin-flavor of the spin-flavor of spin-flavor of spin-flavor operators taken between  $\overline{a}$ 4 tensor in } era .<br>|
| 3  $+$  tensor interaction  $+$ be tensor interaction tensor interaction

 $v$  the states of  $\mu$  and  $\sigma$  arbitrary  $\sigma$  arbitrary  $\sigma$  arbitrary  $\sigma$  arbitrary  $\sigma$  arbitrary  $\sigma$  arbitrary  $\sigma$  $\frac{1}{2}$  $t$  is the reducible reducible reducible reducible reducible representation  $\mathbf{z}$  under the permutation  $\mathbf{z}$ Wednesday, April 28, 2010 **Composition of Oil into its interventation** we are permutations of the permutation group of the Wednesday, April 28, 2010

#### IK model matching decomposition of Oij into irreducible representations of S3, the permutation group of three US GLOUGI GLATGOING in the IK model depends only on three unknown orbital overlap integrals, plus an additive ιχίου του κατά του κ<br>12|χj (1m) = 10|χj 3

be expressed in the spin-flavor of the matrix of the spin-flavor of the spin-flavor of the spin-flavor of the  $\epsilon$ 

2(!QS" + !QMS") !QS" + !QMS"

1

!QS" + !QMS" 2!QS" − !QMS"

Using the relation in Eq. (13), one finds the most general mass of the most general mass operator in the most general mass of  $\epsilon$ 

 $\langle B|\mathcal{H}_{\rm hyp}|B\rangle = \sum$ i  $c_i \langle \Phi(JSI)|O_i|\Phi(JSI)\rangle$  $\sum_{i=1}^n \sum_{i=1}^n \frac{1}{(i-1)(i-1)}$  $c$  relationship constant constant  $c$  $\langle B | H, | B \rangle = \sum c \langle \Phi( J S I) | O | \Phi( J S I) \rangle$ −χ2, and is thus odd under a permutation of the quarks 1, 2. This implies that χ2(ri)

where coult of the matching is contained matrix elements of the matching is  $\mathbf{F}_{\text{max}}$ a the tensor of the characterity to  $\mathcal{S}$ The result of the matching is

 $\overline{O}$ 

$$
\hat{M} = c_0 + aS_c^2 + bL_2^{ab} \{ S_c^a \, , S_c^b \} + cL_2^{ab} \{ s_1^a \, , S_c^b \}
$$

$$
a = \frac{1}{2} \langle R_S \rangle, \qquad \text{for a contact interaction:}
$$
  
\n
$$
b = \frac{1}{12} \langle Q_S \rangle - \frac{1}{6} \langle Q_{MS} \rangle, \qquad \langle R_{MS} \rangle = - \langle R_S \rangle
$$
  
\n
$$
c = \frac{1}{6} \langle Q_S \rangle + \frac{1}{6} \langle Q_{MS} \rangle.
$$

The operators that appear are  $0<sub>1</sub>$   $0<sub>6</sub>$   $0<sub>9</sub>$ Evaluating the matrix elements using the tables in Ref. [4] we find the following explicit The operators that appear are  $O_1$ ,  $O_6$ ,  $O_8$ ,  $O_{17}$ 

11<br>|-<br>|

 $\overline{O}$ 

Wednesday, April 28, 2010 Wednesday, April 28, 2010

IK mass matrix Evaluating the matrix elements using the tables in Ref. [4] we find the following explicit result for the mass matrix The matrix element in Eq. (55) can be computed straightforwardly with the result

11

"QMS". (27) CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CO<br>"QMS" COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY CONTROL COMPANY

<sup>√</sup>2<sup>π</sup> . (57)

1

$$
M_{1/2} = \begin{pmatrix} c_0 + a & -\frac{5}{3}b + \frac{5}{6}c \\ -\frac{5}{3}b + \frac{5}{6}c & c_0 + 2a + \frac{5}{3}(b+c) \end{pmatrix},
$$
  
\n
$$
M_{3/2} = \begin{pmatrix} c_0 + a & \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c \\ \frac{\sqrt{10}}{6}b - \frac{\sqrt{10}}{12}c & c_0 + 2a - \frac{4}{3}(b+c) \end{pmatrix},
$$
  
\n
$$
M_{5/2} = c_0 + 2a + \frac{1}{3}(b+c),
$$
  
\n
$$
\Delta_{1/2} = \Delta_{3/2} = c_0 + 2a.
$$

In the study in the next Section we study the implications of the implication of the implications of t  $T_{\text{initial}}$  is a relation and constructed and computed among matrix  $T_{\text{initial}}$ The spatial integrals can be computed explicitly in the original IK model with harmonic confinement:

$$
a=\frac{1}{2}\delta\,,\qquad b=\frac{1}{20}\delta\,,\qquad c=-\frac{1}{5}\delta\,.
$$

 $\delta = M_\Delta - M_N \sim 300 \text{ MeV}$  . (22) ground states ground states

integrals conductions conduct beyond the wednesday, April 28, 2010 Wednesday, April 28, 2010 **and A and Strong is fixed by one single constant of the L** and Strong Wednesday, April 28, and the single single and an overall additional and the single single single single single single single  $\theta$  are vector is useful to discuss the alternative approach to discuss the alternative approach to the  $1/2$ 

since the angular ρ integration vanishes " <sup>1</sup>

11|Q<sup>33</sup>

12|Ψ<sup>ρ</sup>





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### One gluon exchange model

The QCD Breit-Wigner interaction

$$
\mathcal{H} = \sum_{i < j} V_{ij} \qquad \qquad V = V_{ss} + V_q + V_{so}
$$

$$
V_{ss} = \frac{16\pi\alpha_s}{9} \frac{1}{m_i m_j} \vec{s}_i \cdot \vec{s}_j \delta(\vec{r}_{ij})
$$
 Spin-spin  
\n
$$
V_q = \frac{2\alpha_s}{3r_{ij}^3} \frac{1}{m_i m_j} \left[ \frac{3}{r_{ij}^2} (\vec{r}_{ij} \cdot \vec{s}_i)(\vec{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \right]
$$
Quadrupole  
\n
$$
V_{so} = \frac{\alpha_s}{3r_{ij}^3} \left[ \frac{1}{m_i^2} (\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - \frac{1}{m_j^2} (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \right]
$$
Spin-orbit  
\n
$$
+ \frac{2}{m_i m_j} (\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_j - \frac{2}{m_i m_j} (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_i \right]
$$

#### One gluon exchange spin interaction (*Vss*), the quadrupole interaction *V<sup>q</sup>* and the spin-orbit terms *Vso*. We write  $m$ ore general form a slightly more general form as  $[7]$

In the nonrelativistic limit, the two-body interaction *Vij* contains three terms: the spin-

more general

$$
V_{ss} = \sum_{i < j=1}^{N} f_0(r_{ij}) \vec{s}_i \cdot \vec{s}_j
$$

*N*

*N*

$$
V_q = \sum_{i < j = 1}^{\cdots} f_2(r_{ij}) \Big[ 3(\hat{r}_{ij} \cdot \vec{s}_i)(\hat{r}_{ij} \cdot \vec{s}_j) - (\vec{s}_i \cdot \vec{s}_j) \Big]
$$

$$
V_{so} = \sum_{i < j=1}^{N} f_1(r_{ij}) \Big[ (\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j \Big]
$$

We would like to decompose the Hamiltonian *H* into a sum of term transforming according

$$
+2(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j-2(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j\Big]
$$

to irreps of the permutation group acting on the permutation group action of flavor degrees of  $\alpha$  spin-flavor degrees of  $\alpha$  spin-flavor degrees of  $\alpha$  spin-flavor degrees of  $\alpha$  spin-flavor degrees of  $\alpha$  spin-flav Wednesday, April 28, 2010

#### One pion exchange +2(! *rij* × *p* ! *<sup>i</sup>*) *·* ! *s<sup>j</sup>* − 2(! *rij* × *p* !*<sup>j</sup>* ) *·* !

 $V_{ss} = \sum g_0(r_{ij}) \vec{s_i} \cdot \vec{s_j} t_i^a t_j^a$ *N*  $i < j = 1$  $\frac{a}{j}$  $V_q$  =  $\sum$ *N*  $i < j = 1$  $g_2(r_{ij})$  $\sqrt{ }$  $3(\hat{r}_{ij}\cdot\vec{s}_i)(\hat{r}_{ij}\cdot\vec{s}_j) - (\vec{s}_i\cdot\vec{s}_j)$  $\overline{1}$  $t^a_i t^a_j$  $\begin{array}{c} a \ j \end{array}$  $V_{so}$  =  $\sum$ *N*  $i$   $\lt$ *i*=1  $g_1(r_{ij})$  $\sqrt{ }$  $(\vec{r}_{ij} \times \vec{p}_i) \cdot \vec{s}_i - (\vec{r}_{ij} \times \vec{p}_j) \cdot \vec{s}_j$  $+2(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j-2(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j$  $\overline{1}$  $t^a_i t^a_j$ *j*

*V* =

One pion exchange

## The permutation group SN

#### L=1 states =  $|S\rangle$  =  $|MS\rangle$  x  $|MS\rangle$  $\hat{0} = \hat{0} \times \hat{0}$ orbital spin-flavor



#### The permutation group SN L=1 states =  $|S\rangle$  =  $|MS\rangle$  x  $|MS\rangle$  $\hat{0} = \hat{0} \times \hat{0}$ orbital spin-flavor <S | Ô x Ô | S> S

 $\langle MS | \hat{0} | MS \rangle$   $\longrightarrow$   $MS \times MS = S + MS + E + A'$ 



#### The permutation group SN **THE PERMUTATION CONTRACTOR** In this Appendix we give a few details about the permutation group of *N* objects *S<sup>N</sup>* ,

#### $I=1$  states =  $|S\rangle$  =  $|MS\rangle$  x  $|MS\rangle$

and its irreducible representations which are used in the main text. It is not the main text. It is no main te

**second reports in the second reports in the second reports in the second reports in the second reports in the** 

 $\hat{O} = \hat{O} \times \hat{O}$  spin-flavor We list also the characters for the conjugacy class 1<sup>α</sup>2<sup>β</sup>3<sup>γ</sup> *· · ·* containing α 1-cycles, β 2-cycles,

*<sup>i</sup>*=1 *n<sup>i</sup>* = *N*. Each <S | Ô x Ô | S>

S

Ô

#### etc. Using these expressions and the orthogonality theorem for characters, the projection of any operator onto these irreps of *S<sup>N</sup>* can be computed explicitly.  $MS \times MS = S + MS + E + A'$



partition can be represented as a Young diagram with *n*<sup>1</sup> boxes on the first row, *n*<sup>2</sup> on the

 $\hat{0} = \hat{0} \times \hat{0}$ 

The choice of a basis for a basis for a given in the choice of a several possible choices are adopted and several<br>Weblatted the choices are adopted and several possible choices are adopted and several possible choices are Wednesday, April 28, 2010

#### The permutation group SN **THE PERMUTATION CONTRACTOR** In this Appendix we give a few details about the permutation group of *N* objects *S<sup>N</sup>* , and its irreducible representations which are used in the main text. It is not the main text. It is no main te  $I=1$  states =  $|S\rangle$  =  $|MS\rangle$  x  $|MS\rangle$ *<sup>i</sup>*=1 *n<sup>i</sup>* = *N*. Each <S | Ô x Ô | S>

#### **second reports in the second reports in the second reports in the second reports in the second reports in the**  $\hat{O} = \hat{O} \times \hat{O}$  spin-flavor We list also the characters for the conjugacy class 1<sup>α</sup>2<sup>β</sup>3<sup>γ</sup> *· · ·* containing α 1-cycles, β 2-cycles,  $\hat{0} = \hat{0}$  x  $\hat{0}$

partition can be represented as a Young diagram with *n*<sup>1</sup> boxes on the first row, *n*<sup>2</sup> on the

#### etc. Using these expressions and the orthogonality theorem for characters, the projection of any operator onto these irreps of *S<sup>N</sup>* can be computed explicitly.  $MS \times MS = S + MS + E + A'$

Ô

 $MS<sub>5</sub>$ 

dim=4

S



The choice of a basis for a basis for a given in the choice of a several possible choices are adopted and several<br>Weblatted the choices are adopted and several possible choices are adopted and several possible choices are Wednesday, April 28, 2010

## Two body operators

Example: the spin-spin interaction  $f_n(r_{ij})$  si. sj

"Symmetric" two body operators :

 ${0_{ii}} = S + MS + E$ N(N-1)  $dim = \frac{1}{2}$ 



#### Matching The spin-flavor operator transforming in the *MS* irrep is *<sup>O</sup>MS <sup>k</sup>* <sup>=</sup> !*<sup>N</sup>*

*s*1−!

! *· <sup>S</sup>*

<sup>2</sup> *{S<sup>a</sup>*

and the corresponding orbital operator is *<sup>R</sup>MS <sup>j</sup>*=2*,j*!=*k*[*f*0(*r*1*<sup>j</sup>* ) <sup>−</sup> *<sup>f</sup>*0(*rkj* )]. Example: the spin-spin interaction  $O_{ij}$  =  $f_o(r_{ij})$  s<sub>i</sub>. s<sub>j</sub>

$$
\langle \mathcal{O}^{MS} \rangle = \frac{1}{N-2} \langle \Phi(SI) | -\vec{S}^2 + N\vec{s}_1 \cdot \vec{S}_c + \frac{3}{4} N | \Phi(SI) \rangle
$$
  

$$
\langle \mathcal{R}^{MS} \rangle = (N-2)(\mathcal{I}_{\text{dir}} - \mathcal{I}_s) - 2\mathcal{I}_{\text{exc}}
$$

 $\overline{\phantom{a}}$ To extract the leading order , we use the Hartree picture

*b* = *Idir* − *I<sup>s</sup>*

 $\mathcal{I}_{dir} = \langle \xi \rangle$ 2*N<sup>c</sup>*  $\mathcal{I}_{dir} = \langle sp | f_0 | sp \rangle$ **Overlap integrals**  $\mathcal{I}_s = \langle ss|f_0|ss \rangle$  $\mathcal{I}_{exc} = \langle sp|f_0|ps \rangle$ 

 $G$ *luon exchange model* Gluon exchange potential: *a* = 1 2 *Is* **a**  $\mathbf{a}$ 1 2 *av*ige 7 2 *Is*

3

*N|*Φ(*SI*)# (18)

1

 $\frac{1}{1}$ 

 $M = c_0 1 - g^2$  $\sqrt{ }$  $a \vec{S}^2 + b \vec{s}_1 \cdot \vec{S_c} + c L^{ij}_{(2)} S^i S^j + d L^{ij}_{(2)} S^i_c s^j_1 + e \vec{L} \cdot \vec{S} + f \vec{L} \cdot \vec{s}_1$  $\overline{ }$  $S^i S^j + dL_{(2)}^{ij} S_c^i s_1^j S^i S^j + dL_{(2)}^{ij} S_c^i s_1^j -$ 



*a* = 1 2 *Is*  $b = \mathcal{I}_{dir} - \mathcal{I}_{s}$ *c*  $\left(\frac{1}{2}, \frac{1}{2}\right)$  $\frac{a}{a}$  $\frac{1}{2}$  $\overline{L}_s$  $\overline{1}$  $\overline{I}$  $b = \mathcal{I}_{dir} - \mathcal{I}_{dir}$  $c$  =  $\mathcal{O}(1/N)$  $d = -\mathcal{K}_{dir}$ *e* =  $\frac{1}{2}$  $\begin{array}{c} \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \end{array}$  $\frac{J}{\text{dir}}$  +  $\frac{1}{2}$  $\frac{J}{4}$  *dir dir* (20)  $\sqrt{2}$  dir  $2^{\circ}$ *dir*"  $d = -K$ <sub>dir</sub> *e* = 3 2  $\mathcal{J}^S_{\rm dir} +$ 1 2  $\mathcal{J}_{dir}^{A}$  $\left(\frac{3}{2}\right)$  $f = N\left(\frac{3}{2}\mathcal{J}_{\text{dir}}^S - \frac{1}{2}\mathcal{J}_{\text{dir}}^A\right)$ *d* = −*Kdir e* =  $\overline{\Omega}$  $\frac{1}{2}$  $\mathcal{J}^S$ <sup>+</sup>  $\overline{\phantom{a}}$  $\frac{1}{2}$  $J_A^A$  $/3$ 2  $\mathcal{J}^{S}_{\rm dir} - \frac{1}{2}$ 2  $\mathcal{J}_{dir}^{A}$ 

Goldstone boson exchange potential:

1

!

*aS* !2

*<sup>c</sup>* + *b*!

*s*<sup>1</sup> *· S*

!

*<sup>c</sup>* + *ct*

*a* <sup>1</sup>*T <sup>a</sup>*

*<sup>c</sup>* + *dLij*

(2)*gia*

<sup>1</sup> *Gja*

*<sup>c</sup>* + *eLij*

*d* = −*Kdir d* = −*Kdir* Wednesday, April 28, 2010**M**  $\frac{1}{2}$  +  $\$ 

#### *Pion exchange model* <u>न</u><br>२ *J S* dir <sup>−</sup> <sup>1</sup> **2** *J A dir*" Goldstone boson exchange potential: *f* = *N*  $\overline{\phantom{a}}$ *J S* dir <sup>−</sup> <sup>1</sup> 1  $P_{\text{data}}$ Goldstone boson exchange potential: 2<br>2 *J S* dir <sup>−</sup> <sup>1</sup>  $\overline{\phantom{a}}$ *J A dir*" Goldstone boson exchange potential: *f* = *N* 2 *J S* dir <sup>−</sup> <sup>1</sup> **2** *J A dir*" Goldstone boson exchange potential: *f* = *N dir*" Goldstone boson exchange potential: **b**  $\alpha$   $\alpha$ 4 *Idir <sup>b</sup>* <sup>=</sup> *<sup>c</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup> 4 *Idir <sup>b</sup>* <sup>=</sup> *<sup>c</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup> 4 *Idir*

2

*J S*

dir +

2

*J A*

*<sup>a</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup>

*dir* (20)

*dir* (20)

**e** = **e** = **e** = **e** = **e** = **e** = **e** 

*J A*

2

2

2

2

2

2

*J S*

dir +

 $M$  =  $c_0 1 + \frac{g_A^2}{f^2}$ *A*  $f_{\pi}^2$ !  $a \vec{S}_{c}^2 + b \vec{s}_1 \cdot \vec{S}_{c} + c t_{1}^{a} T_{c}^{a} + d L_{(2)}^{ij} g_{1}^{ia} G_{c}^{ja} + e L_{(2)}^{ij} \{S_{c}^{i}\, , S_{c}^{j}\}$ " *<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup> *A f* 2 π  $\overline{a}$  $M \;\; = \;\; c_0 {\bf 1} + \frac{g_A}{f^2} \Bigl(a \vec{S}_c^2 + b \vec{s}_1 \cdot \vec{S}_c + c t_1^a T_c^a + d L_{(2)}^{ij} g_1^{ia} G_c^{ji} \Bigr)$ *A f* 2  $\overline{\phantom{a}}$  $\overline{\phantom{a}}$  $M \;\; = \;\; c_0 {\bf 1} + \frac{g_A^2}{c^2} \Bigl( a \vec{S}_c^2 + b \vec{s}_1 \cdot \vec{S}_c + c t_1^a T_c^a + d L_{(2)}^{ij} g_1^{ia} G_c^j \Bigr)$ .<br>.<br>. *<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup> *A f* 2 π |
|- $A \hspace{.2cm} = \hspace{.2cm} c_0 {\bf 1} + \frac{g_A^2}{r^2} \Big( a \vec{S}_c^2 + b \vec{s}_1 \cdot \vec{S}_c + c t_1^a T_c^a + d L_{(2)}^{ij} g_1^{ia} G_c^{ji} \Big)$  $\vec{F}$   $\vec{G}$   $\vec{G}$ *<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup> *A f* 2 π !  $A = c_0 1 + \frac{g_A^2}{4} \left( a \vec{S}^2 + b \vec{S}_1 \cdot \vec{S} + c t^a T^a + dL^{ij} a^{ia} G^{ji} \right)$ *c*  $\frac{1}{2}$  $^{0}1^{1}c$   $^{0}w$ <sub>(2)</sub>91  $^{0}c$  $\frac{d}{dz} + dL^{ij}_{(2)}g_1^{ia}G_c^{ja}$ 

 $+ f\vec{L} \cdot \vec{S_c} + gL^i t_1^a G_c^{ia} + hL^i g_1^{ia} T_c^a$  $+ f\vec{L}\cdot\vec{S_c} + gL^i$  $+f\vec{L}\cdot\vec{S}_c+gL^it_1^c$  $+ f\vec{L} \cdot \vec{S_c} + gL^i$ *e* = *O*(1*/N*)  $\frac{1}{2}$   $\frac{nL}{g_1}$   $\frac{1}{c}$ 



*f* = *O*(1*/N*) Wednesday, April 28, 2010

*Pion exchange model* <u>न</u><br>२ *J S* dir <sup>−</sup> <sup>1</sup> **2** *J A dir*" Goldstone boson exchange potential:  $M$  =  $c_0 1 + \frac{g_A^2}{f^2}$ *A*  $f_{\pi}^2$ ! *aS* !2 *<sup>c</sup>* + *b*! *s*<sup>1</sup> *· S* ! *<sup>c</sup>* + *ct a* <sup>1</sup>*T <sup>a</sup> <sup>c</sup>* + *dLij* (2)*gia* <sup>1</sup> *Gja <sup>c</sup>* + *eLij* (2)*{S<sup>i</sup> <sup>c</sup> , S<sup>j</sup> c }*  $+ f\vec{L}\cdot\vec{S_c} + gL^i t_1^a G_c^{ia} + hL^i g_1^{ia} T_c^a$ " *<sup>b</sup>* <sup>=</sup> *<sup>c</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup>  $\frac{y}{x}$ *f* = *N*  $\overline{\phantom{a}}$ *J S* dir <sup>−</sup> <sup>1</sup> 1  $P_{\text{data}}$ Goldstone boson exchange potential:  $\overline{a}$ *aS* !2 *<sup>c</sup>* + *b*! *s*<sup>1</sup> *· S* ! *<sup>c</sup>* + *ct a* <sup>1</sup>*T <sup>a</sup> <sup>c</sup>* <sup>+</sup> *dLij* (2)*gia* <sup>1</sup> *Gja <sup>c</sup>* <sup>+</sup> *eLij* (2)*{S<sup>i</sup> <sup>c</sup> , S<sup>j</sup> c }*  $+f\vec{L}\cdot\vec{S_c} + g\vec{L}^i$  $t^c$  $a = -\frac{1}{4}$ 4 *Is b* =  $c = -\frac{1}{4}$  $\mathbf{f}$ *Idir d* = −2*Kdir e* = *O*(1*/N*) 2<br>2 *J S* dir <sup>−</sup> <sup>1</sup>  $\overline{\phantom{a}}$ *J A dir*" Goldstone boson exchange potential:  $\overline{\phantom{a}}$  $M \;\; = \;\; c_0 {\bf 1} + \frac{g_A^2}{c^2} \Bigl( a \vec{S}_c^2 + b \vec{s}_1 \cdot \vec{S}_c \not\!\! + \vec{c} t_1^a T_c^a + d L_{(2)}^{ij} g_1^{ia} G_c^j \Bigr)$  $+f\vec{L}\cdot\vec{S_c} + g\vec{L}^i$ .<br>.<br>.  $a = -\frac{1}{4}$  $\overline{A}$  $a = -\frac{1}{4}\mathcal{I}_s$  $b = c = -\frac{1}{4}$ 4  $b = c = -\frac{1}{4}\mathcal{I}_{dir}$  $d = -2\mathcal{K}_{dir}$  $e = O(1/N)$ *f* = *N* 2 *J S* dir <sup>−</sup> <sup>1</sup> **2** *J A dir*" Goldstone boson exchange potential: |
|- $A \hspace{.2cm} = \hspace{.2cm} c_0 {\bf 1} + \frac{g_A^2}{c^2} \Big( a \vec{S}_c^2 + b \vec{s}_1 \cdot \vec{S}_c \bigg/ \hspace{-3.5cm} + \hspace{-3.5cm} c t_1^a T_c^a + d L_{(2)}^{ij} g_1^{ia} G_c^{ji} \bigg.$  $+f\vec{L}\cdot\vec{S_c} + g\vec{L}^i$ "  $\frac{1}{4}$ *Is* 4 *Idir f* = *N dir*" Goldstone boson exchange potential: !  $A = c_0 1 + \frac{g_A^2}{4} \left( a \vec{S}^2 + b \vec{S} \cdot \vec{S} \right) + c_0 a T^a + dI^{ij} a^{ia} G^{ji}$  $+ f\vec{L} \cdot \vec{S} + a\vec{L}$  $^{0}1^{1}c$   $^{0}w$ <sub>(2)</sub>91  $^{0}c$  $\frac{1}{a}$  =  $\frac{1}{a}$  $\frac{1}{\sqrt{2}}$  $\tau$ *b* =  $c = -\frac{1}{4}$  $\frac{1}{4}$ *Idir*  $d = -2\mathcal{K}_{dir}$  $e = O(1/N)$ **b**  $\alpha$   $\alpha$ 4 *Idir e* = *O*(1*/N*)  $f = O(1/N)$ <u> $\frac{1}{2}$ </u>  $\overline{a}$  $J_{\text{dir}} +$  $\frac{1}{1}$  $\overline{a}$  $J_{dir}$ *h* =  $\frac{1}{2}$  $\overline{\phantom{a}}$  $\mathcal{J}_{\text{dir}}^S - \frac{1}{2} \mathcal{J}_{di}^A$  $\overline{a}$ The 1*/N<sup>c</sup>* expansion: *<sup>b</sup>* <sup>=</sup> *<sup>c</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup> 4 *Idir <sup>b</sup>* <sup>=</sup> *<sup>c</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup>  $+ dL_{(2)}^{ij}g_1^{ia}G_c^{ja}$  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $f = O(1/N)$  $g =$ 3 2  $\mathcal{J}^S_{\rm dir} +$ 1 2  $\mathcal{J}_{dir}^{A}$ *h* =  $\frac{3}{5}$  $\frac{2}{2}$  $\mathcal{J}^{S}_{\text{dir}} - \frac{1}{2}$  $\frac{2}{2}$  $\mathcal{J}_{dir}^A$ *<sup>a</sup>* <sup>=</sup> <sup>−</sup><sup>1</sup> 4 *Idir*  $+ dL^{ij}_{(2)}g_1^{ia}G_c^{ja}$  $+mL$   $g_1$   $I_c$  $g =$ e<br>3  $\overline{2}$  $\mathcal{J}_{\rm dir}^S$  + 1  $\overline{2}$  $\mathcal{J}_{dir}^{A}$ *h* = 3 2  $\mathcal{J}^{S}_{\rm dir} - \frac{1}{2}$ 2  $\mathcal{J}_{dir}^{A}$ The 1*/N<sup>c</sup>* expansion: flavor index

**e** = **e** = **e** = **e** = **e** = **e** = **e** 

*J A*

2

2

2

2

2

2

*J S*

dir +

2

*J S*

dir +

2

*J A*

*dir* (20)

*dir* (20)

*f* = *O*(1*/N*) Wednesday, April 28, 2010

*<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup>

*<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup>

*<sup>M</sup>* <sup>=</sup> *<sup>c</sup>*0<sup>1</sup> <sup>+</sup> *<sup>g</sup>*<sup>2</sup>

*A*

*A*

*A*

*A*

*f* 2 π

*f* 2  $\overline{\phantom{a}}$ 

*f* 2 π

*f* 2 π

### A different approach: Consider all possible 2-body interactions

D.Pirjol, C.S., Phys.Rev.Lett.102:152002(2009)

### The most general 2-body quark interaction

Complete list of 2-body spin-flavor quark operators, and their projection onto irreps of  $\overline{S_3^{\mathrm{sp-fl}}}$ 



 $C_2(F) = \frac{F^2 - 1}{2F}$ 

[Contribute on S, MS states] [Contribute on MS states]

#### Basis of operators induced by 2-body quark operators

 $O_1 = T^2$  $O_2 = \vec{S}_c^2$ c  $O_3 = \vec{s}_1 \cdot \vec{S}$  $\overline{c}$ 

 $O_4 = \vec{L} \cdot \vec{S}$ c  $O_5 = \vec{L} \cdot \vec{s}_1$  $O_6 = L^i t_1^a G_c^{ia}$  $O_7=L^i g_1^{ia}T^a_c$ spin-spin spin-orbit tensor

 $O_8 = L_2^{ij} \{ S_c^i, S_c^j \}$  $O_9=L_2^{ij}s_1^i s_c^j$  $O_{10}=L_2^{ij}g_1^{ia}G_c^{ja}$ 

Hadronic mass matrix (L=1 states)

 $\hat{m}_i^* = \sum \hat{M}_{ij} c_j$ 10  $i=0$ 

 $N_i^* = (N_{1/2}, N'_{1/2}, N_{1/2} - N'_{1/2}, N_{3/2}, N'_{3/2}, N_{3/2} - N'_{3/2}, N_{5/2}, \Delta_{1/2}, \Delta_{3/2}, \Lambda_{1/2}, \Lambda_{3/2})$ 

Valid for any model with only 2-body quark interactions.

#### Two new mass relations !"<sup>1</sup> |
|-FIG. 1: Left: correlation in the co quark model with the model with the model and contact general 2-body quark interactions. The model of the model of black dots correspond to the mixing angles (θN1, θN3) = experience (θN1, θN3) = experience (θN3) = experience  $\alpha$  volotions of the strong decay in Ref. [11]. The second point is a second point in Ref. [11]. The second point is a second factored by a 1/Nc analysis of photoproduction and

#### $\mathbf{A}$  experience  $\mathbf{A}$  and  $\mathbf{A}$  $\mathbf{h}_{\mathbf{X}}$ #1 requires only isospin symmetry

1 2  $(N(1535) + N(1650)) +$ 1 2  $(N(1535) - N(1650))(3\cos 2\theta_{N1} + \sin 2\theta_{N1})$  $-\frac{7}{5}$ 5  $(N(1520) + N(1700)) + (N(1520) - N(1700)) \Big[-\frac{3}{5}\Big]$ 5  $\cos 2 \theta_{N3} +$  $\sqrt{5}$ 2  $\left[\sin 2 \theta_{N3} \right] = -2 \Delta_{1/2} + 2 \Delta_{3/2} - \frac{9}{5}$  $\frac{1}{6}(N(1535) + N(1650)) + \frac{1}{6}(N(1535) - N(1650))(3 \cosh^{-1}$ taining only  $2$  body interactions. This correlations is correlated interactions. This correlation holds in  $\mathcal{E}_\text{max}$  $a_n = \frac{7}{N(1520)} + N(1700) + (N(1520) - N(1700))$  $\left[-\frac{1}{5}(N(1320) + N(1700)) + (N(1320) - N(1700))\right]$  $\ddotsc$   $\ddotsc$   $\ddotsc$  $N_1 + \sin 2\theta N_1$ ,  $\frac{1}{2}$  $\left[1\right]$   $\left[1\right]$   $\left[2\right]$   $\left[1\right]$   $\left[1\right]$ 

#### <u>17</u> #2 includes SU(3) breaking

the mass operator to order O(1/Nc) [9, 10] is generated by the operators in Eq. (3). An example of an operator

 $\frac{1}{2}$ 

Right: prediction for the spin-weighted Λο mass in the SU(3) mass in the SU(3) mass in the SU(3) mass in the SU(3) limit as a function of the limit as a function of the limit angle, corresponding to the limit  $\sim$ the two solutions for two solutions for determining to green points correspond to the green points correspond to

This expression and the mixing angles and the mixing angles and the mixing angles and the mixing angles and the

FIG. 1: Left: correlation in the (θN<sup>1</sup>, θN<sup>3</sup>) plane in the quark model with the most general 2-body quark interactions.  $R = \frac{1}{\sqrt{2}}$  mass in the spin-weighted  $\frac{1}{\sqrt{2}}$  mass in the SU(3)  $\frac{1}{\sqrt{2}}$  mass in the SU(3)  $\frac{1}{\sqrt{2}}$ 

$$
\bar{\Lambda} = \frac{1}{6}(N(1535) + N(1650)) + \frac{17}{15}(N(1520) + N(1700)) - \frac{3}{5}N_{5/2}(1675) - \Delta_{1/2}(1620)
$$

$$
-\frac{1}{6}(N(1535) - N(1650))(\cos 2\theta_{N1} + \sin 2\theta_{N1}) + (N(1520) - N(1700))(\frac{13}{15}\cos 2\theta_{N3} - \frac{1}{3}\sqrt{\frac{5}{2}}\sin 2\theta_{N3})
$$

5

6 (2Λ1/2 + 4Λ3/2)

of the strong decays in Ref. [11]. The second point is second point in Ref. [11]. The second point is a second point is a second point is a second point in  $\mathcal{I}^{\mathcal{I}}$ 

On the same plane plot we show also the values of the values of the mixing angles obtained in several analyses of the N<sup>∗</sup> → Nπ strong decays and N<sup>∗</sup> hadron masses. The two

tudes Ref. [12]. The year of the year

and the triangle gives the angles corresponding to the so-

triangle) are compatible with the ranges design the ranges  $\mathcal{C}$ 

, 80.2 °C in 1980, and 1980, a

favored by a 1/Nc analysis of photoproduction and photoproduction and photoproduction and tudes Ref. [12]. The yellow square corresponds to the

of the quark interaction, which can discriminate between

values used in Ref. [9, 10] (θn3) = (35.0 (θn3) = (35.

 $N_{5/2}$  .

 $\cdot$ 

 $\frac{1}{2}$ , which  $\frac{1}{2}$ 

, 174.2◦),

Wednesday, April 28, 2010 mixing angles, corresponding angles, corresponding to the entries  $M$ ednoode $\nu$ , April 28, 2010 Wednesday, April 28, 2010

0  $\overline{a}$ 60 90 |
| |-<br>|-|
|-

90 !

### First universal relation

Correlation between mixing angles in the J=1/2 and 3/2 sectors Requires only isospin symmetry



- Compare with direct determinations of the mixing angles -> test for the presence of 3-body quark interactions •
- Tests for specific models of 2-body quark interactions, e.g. OGE model •

### Second universal relation

 $\bar{\Lambda} =$ 

1

3

 $\Lambda_{1/2}$  +

2

3

 $\Lambda_{3/2}$ 

Expresses the spin-weighted SU(3) singlet mass Takes into account SU(3) symmetry breaking



Combining the two universal relations gives a determination of the mixing angles from hadron masses alone





#### The coefficients of the 1/Nc expansion can be matched to spatial integrals in quark models



- The coefficients of the 1/Nc expansion can be matched to spatial integrals in quark models
- Flavor dependent interactions are required.



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- Flavor dependent interactions are required.

 $*$  Two new mass relations for  $N^*$ 's that constrain the mixing angles and test for 3-body forces