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Few-nucleon contributions to π-nucleus scattering

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INT Program on „Simulations and Symmetries: Cold Atoms, QCD, and Few-hadron Systems“

- Motivation
- Q- versus W-counting: a closer look to the deuteron
- Numerical approach
- 3-nucleon corrections
- Results for π-^{3}He
- 4-nucleon corrections
- Results for π^--^{4}He
- Conclusions & Outlook

S. Liebig, V. Baru, F. Ballout, C. Hanhart, AN, arXiv:1003.3826

April 14, 2010

Motivation

JÜLICH FORSCHUNGSZENTRUM

π-N scattering length ⟷ pattern of chiral symmetry and its breaking

LO ChPT (PCAC, Weinberg, 1966)

$$a^{(+)} = 0 \qquad a^{(-)} = \frac{1}{8\pi(1+\frac{m_\pi}{m_N})}\frac{m_\pi}{{f_\pi}^2} \approx 90\cdot 10^{-3} {m_\pi}^{-1}$$

Higher order results have been calculated

Experimentally not so well known,

requires very accurate determination of scattering lengths in two isospin channels, e.g.

$$a^{(-)} = \frac{1}{2}\left(a_{\pi^-p} - a_{\pi^-n}\right) \qquad a^{(+)} = \frac{1}{2}\left(a_{\pi^-p} + a_{\pi^-n}\right) \approx 0$$

Most accurate determination from pionic atoms using

$$a_{\pi^-A} = \left(\frac{1+m_\pi/m_N}{1+m_\pi/Am_N}\right)\left(Aa^{(+)} + 2T_3a^{(-)}\right)$$

+ IV corrections + **few-nucleon corrections**

(for IV corrections see Hoferichter et al., NPA 2010, Baru et al. arXiv:1003.4444 [nucl-th])

Motivation

ChPT calculation of few-nucleon corrections (Weinberg, 1992)

$$a_{\pi A} \propto \langle \Psi_A | \, \hat{O} \, | \Psi_A \rangle$$

perturbative expansion of the transition operator $\hat{O}$

phenomenological few-nucleon wave functions
or chiral wave functions (Beane at al., 1998) $|\Psi_A\rangle$

Result (for A=2): some of the few-nucleon corrections are smaller than expected
but they are in general somewhat larger than expected

Is this a systematic deviation from the power counting?

Beane at al., 2003 : consequence of a second scale entering

$$q_d = \sqrt{m_N |E_d|} \approx 45 \text{ MeV} \ll m_\pi$$

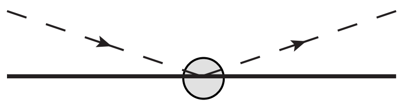
→ **„Q-counting“**

→ consequences for error estimates,
power counting in complex nuclei ?

Contributions to π-^{2}H scattering

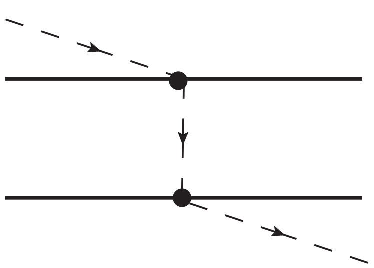
„Weinberg-counting“ - typical momenta $\propto m_\pi$

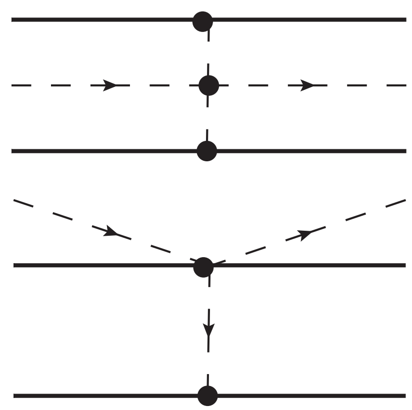
1-nucleon contributions



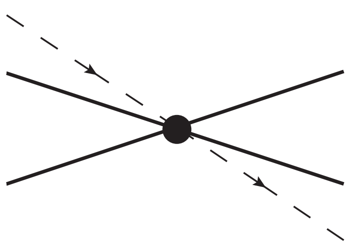
$$a_{\pi-A} = \left(\frac{1+m_\pi/m_N}{1+m_\pi/Am_N}\right)\left(Aa^{(+)} + 2T_3 a^{(-)}\right) \qquad \propto \frac{m_\pi}{{f_\pi}^2}$$

2-nucleon contributions (Beane et al., 2003)



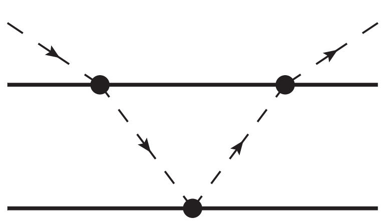


$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$$ (Weinberg, 1992; Beane et al. 1998)



~~$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^3\right)$~~ (isovector)

$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$$ (isoscalar)



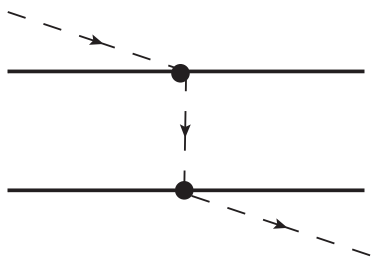
$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$$

no $\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^3\right)$ contributions

boost & dispersive corrections almost cancel each other (Lensky et al. 2007, Baru et al. 2008)

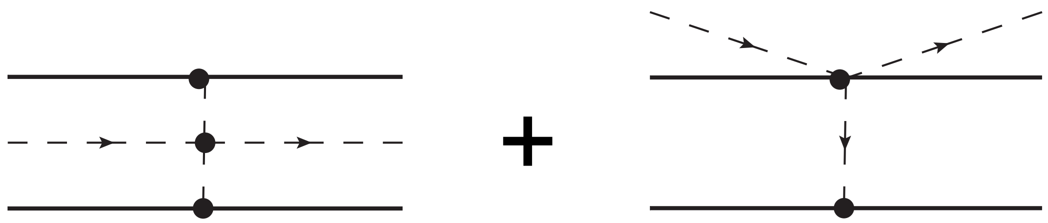
Numerical results (Beane et al.)

2-nucleon contributions have different sizes than expected by power counting

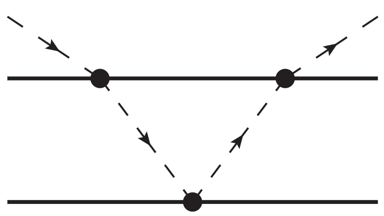


$\propto 20 \cdot 10^{-3}\ m_\pi{}^{-1}$

$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$



$< 1 \cdot 10^{-3}\ m_\pi{}^{-1}$



$\propto 2 \cdot 10^{-3}\ m_\pi{}^{-1}$

$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$

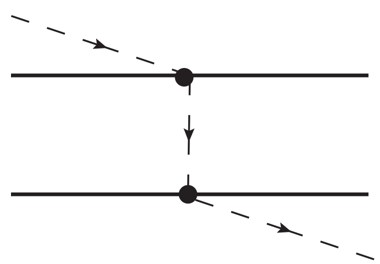
Solution of Beane at al.

assume that the typical momentum is the binding momenta of the deuteron

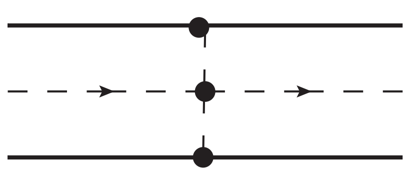
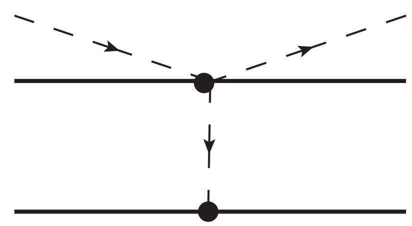
$$q_d = \sqrt{m_N |E_d|} \approx 45 \text{ MeV} \ll m_\pi \quad \rightarrow \quad \frac{q}{\Lambda} \propto \left(\frac{m_\pi}{\Lambda}\right)^2$$

Q-counting

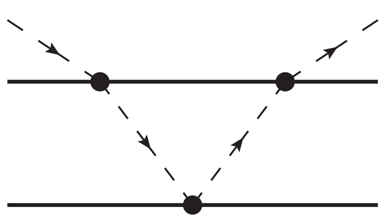
Numerical results seem to fit expectations!



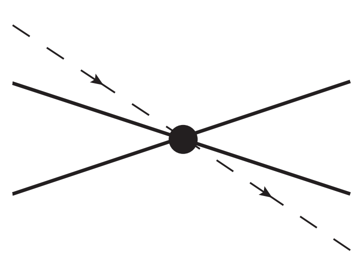
$\propto 20 \cdot 10^{-3}\ {m_\pi}^{-1}$ $\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$ $\mathcal{O}(Q^3)$

 + 

$< 1 \cdot 10^{-3}\ {m_\pi}^{-1}$ $\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$ $\mathcal{O}(Q^7)$



$\propto 2 \cdot 10^{-3}\ {m_\pi}^{-1}$ $\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$ $\mathcal{O}(Q^6)$



? $\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$ $\mathcal{O}(Q^7)$

First counter term is very much suppressed compared to few-nucleon contributions

Boost corrections are of the order $\mathcal{O}(Q^5)$ (Is the kinetic energy measurable?)

Is Weinberg counting restored for ^{3}He, ^{4}He ?

Physical & unphysical deuterons

LO chiral interaction

$$V_{LO} = -\left(\frac{g_a}{2F_\pi}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+m_\pi^2}\,\tau_1\cdot\tau_2 + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

small deuteron binding energy is fine-tuning effect

→ undo fine-tuning of C_S and fit to range of binding energies (here C_T=0)

E_d[MeV]	C_S[GeV^{-2}]
0.002	-71.7689
0.005	-72.1400
0.01	-72.5574
0.02	-73.1460
0.05	-74.3088
0.1	-75.6126
0.2	-77.4471
0.5	-81.0703
1.0	-85.1564
5.0	-103.225
10.0	-118.793
20.0	-147.032
30.0	-178.149
40.0	-217.147
50.0	-271.056

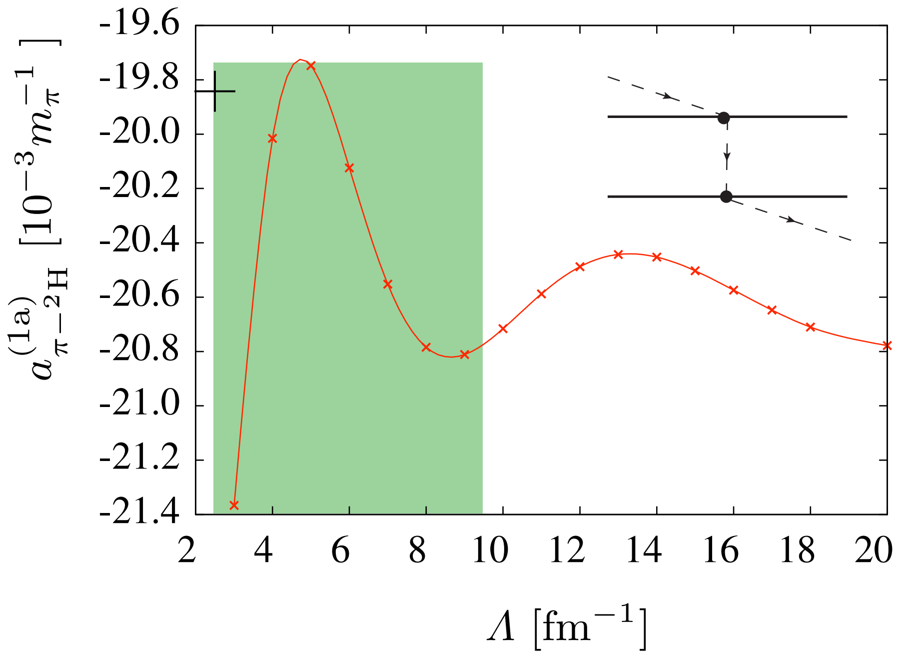
large range of cutoffs in LO to study

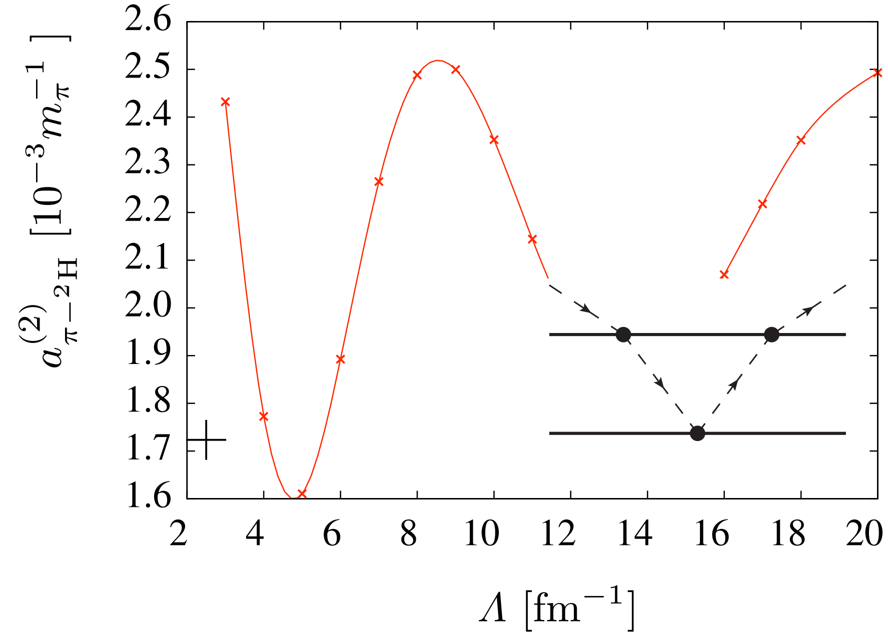
→ size of first ππ4N contribution

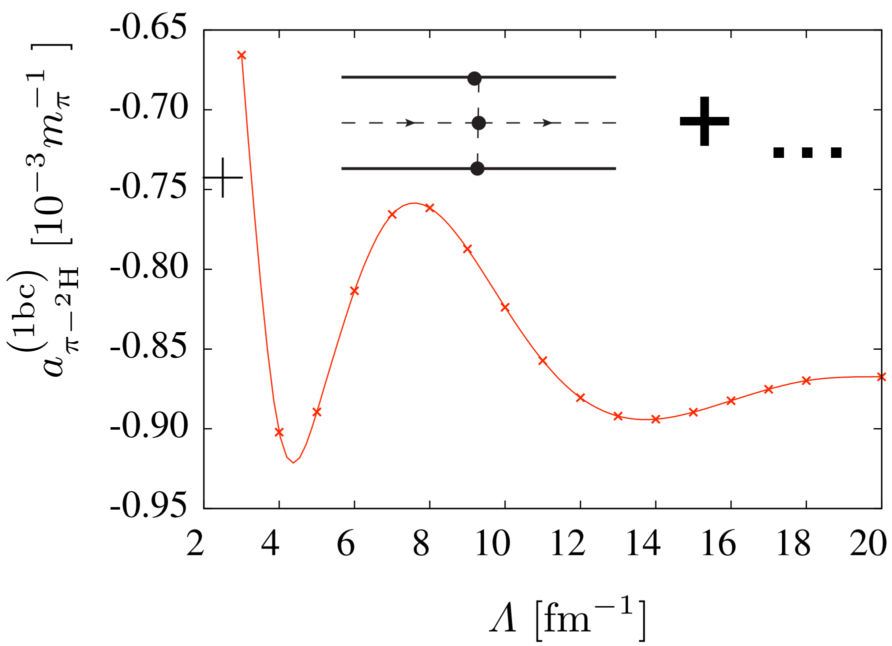
Λ[fm^{-1}]	C_S[GeV^{-2}]
3.0	-34.2225
4.0	48.1751
5.0	562.089
10.0	-50.9683
20.0	-92.1179

C_S is in general of natural size (≈ 100 GeV^{-2})
except close to new spurious bound states

Cutoff dependence - π-^{2}H





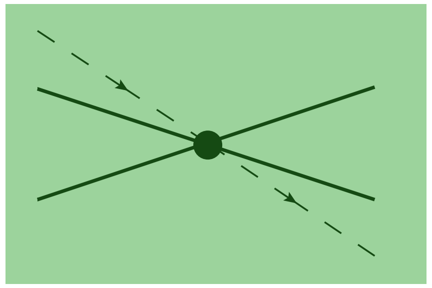


few-nucleon contributions are independent of the cutoff

(see also AN, Hanhart, 2005,
Pavon Valderrama, Ruiz Arriola, 2006
Platter, Phillips, 2006)

cutoff variation most significant for „double scattering“

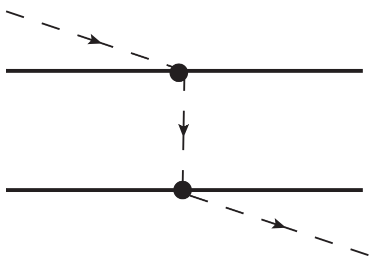
→ estimate of short range contribution $\approx 1 \cdot 10^{-3}\ m_\pi{}^{-1}$



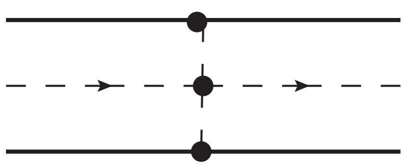
naive estimate agrees (Weinberg)

Energy dependence - π-^{2}H

Q-counting (W-counting $\propto const$)

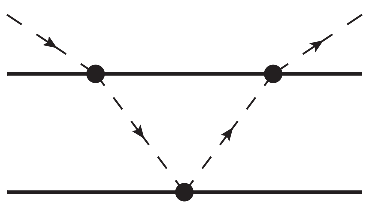


$$\propto \frac{{m_\pi}^2}{q^2}$$



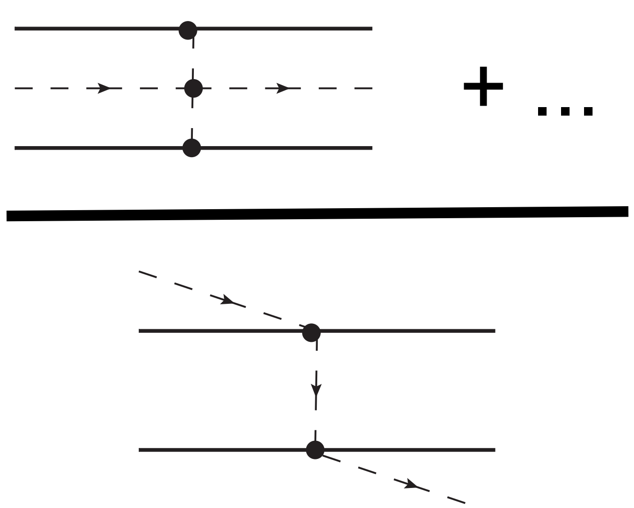
+ ...

$$\propto \frac{q^2}{{m_\pi}^2}$$

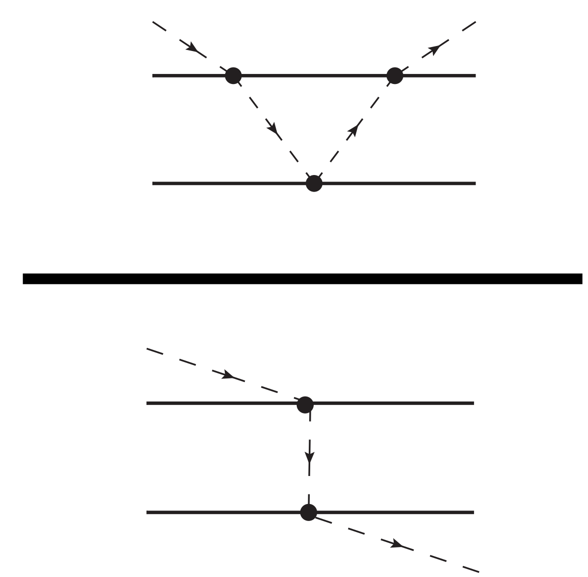


$$\propto \frac{m_\pi}{q}$$

→ Q-counting: ratios should have distinct binding energy dependence

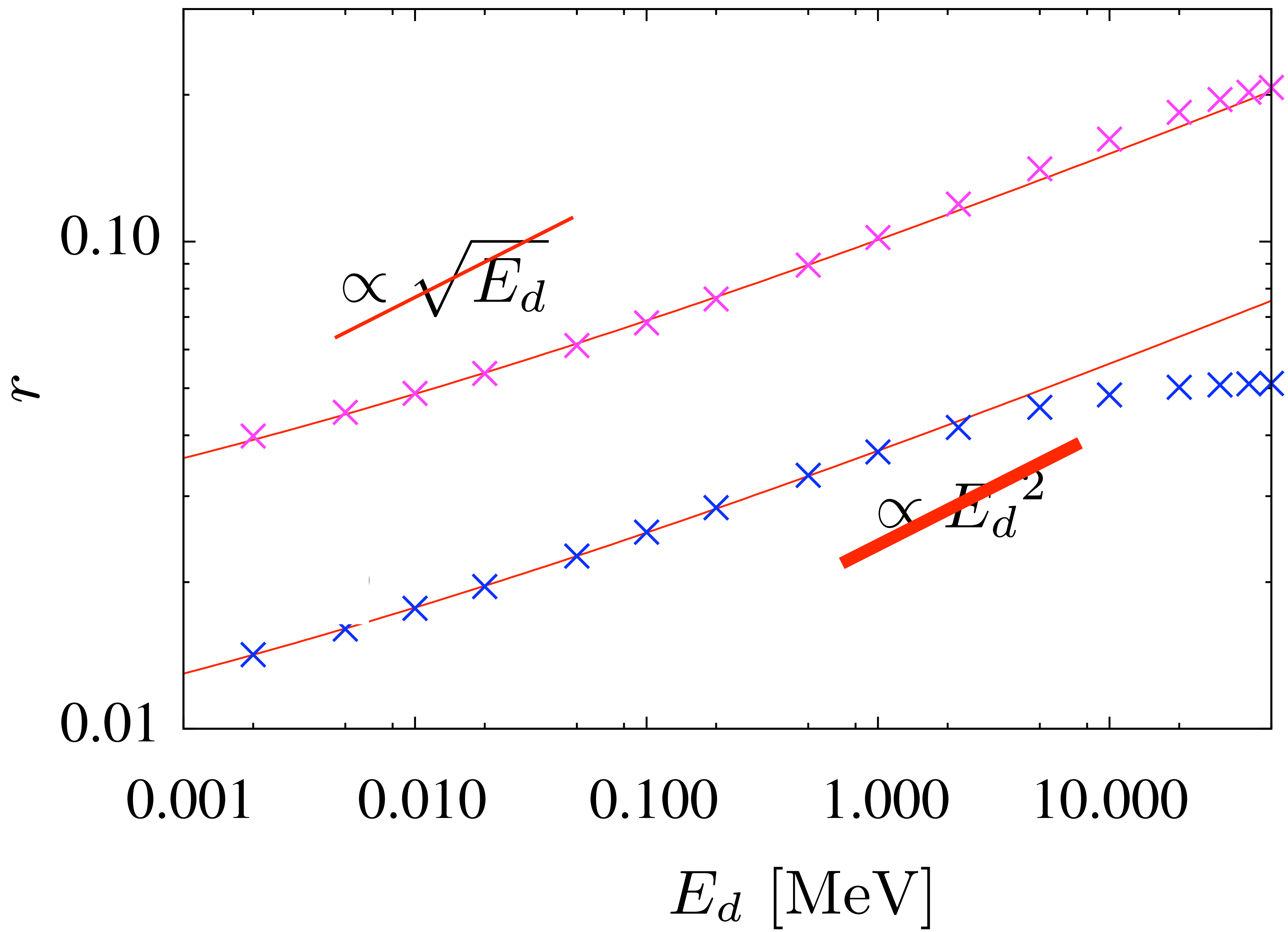


$$\propto {E_d}^2$$



$$\propto \sqrt{E_d}$$

Energy dependence - π-^2H



Explicit calculation strongly disagrees with Q-counting expectations

→ **Q-counting is not realized in low energy pion scattering**

Toy model - π-^{2}H: Hulthén wf

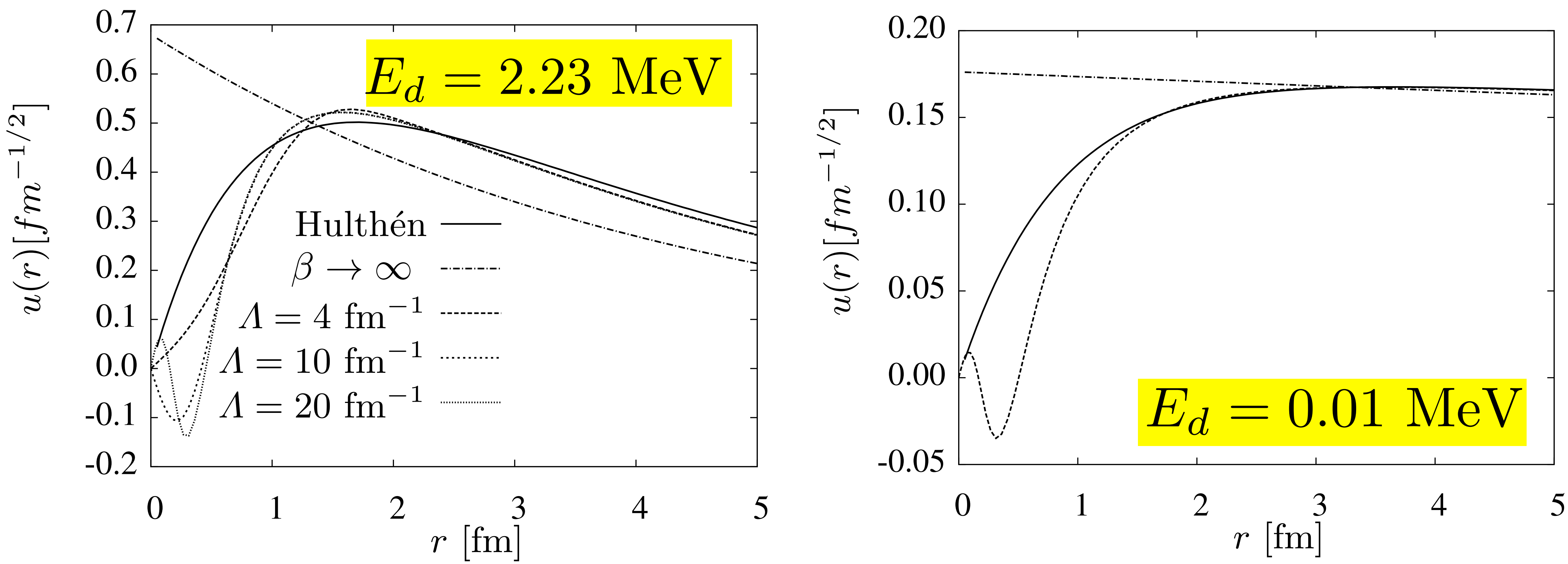
binding momentum is not the only scale entering the deuteron wf

Hulthén wf allows one to incorporate second scale

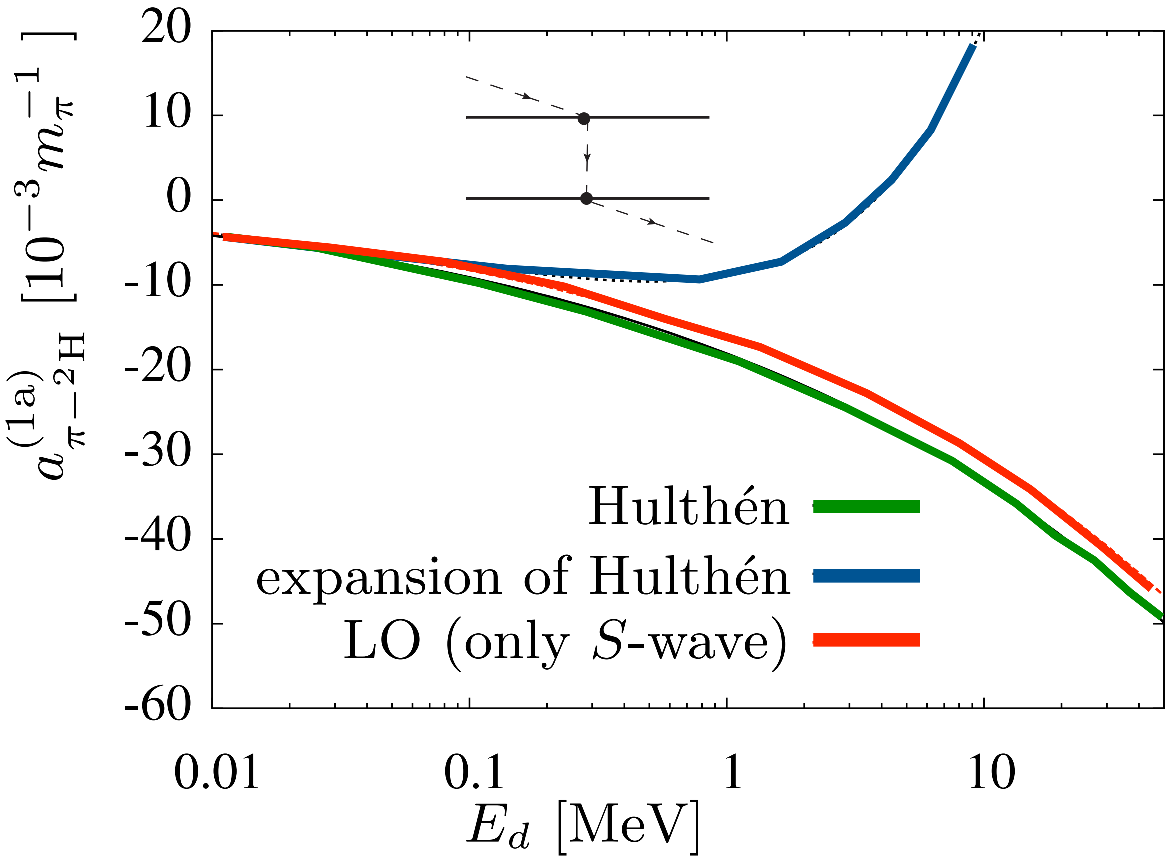
$$\psi(\vec{p}) = N(\gamma,\beta)\,\frac{1}{\vec{p}^{\,2}+\beta^2}\,\frac{1}{\vec{p}^{\,2}+\gamma^2}$$

fit to normalized s-wave part of the LO wave function

second scale is binding energy independent → $\beta = 1.7 m_\pi$



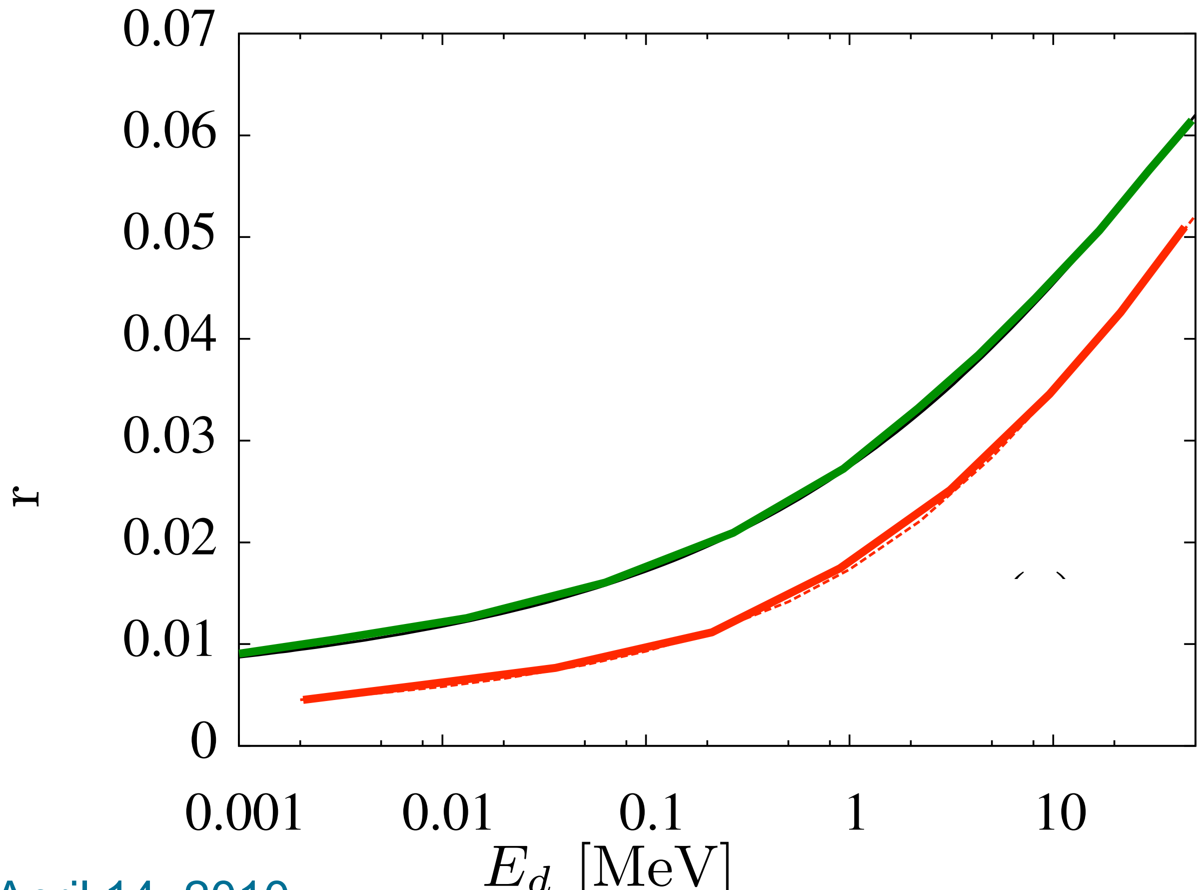
Toy model - π-²H: Hulthén wf



$$a^{(1a)}_{\pi-^2\mathrm{H}} = \kappa\, x \frac{(1+x)}{(1-x)^2} \ln\left(\frac{4x}{(1+x)^2}\right)$$

$$\kappa = \beta \frac{1}{8\pi^2\,(1+m_\pi/2m_N)} \frac{m_\pi^2}{f_\pi^4} \qquad x = \gamma/\beta$$

$$\beta \to \infty \quad \Rightarrow \quad a^{(1a)}_{\pi-^2\mathrm{H},\mathrm{LO}} = \kappa\, x\, \ln(4x)$$



- for double scattering diagram:

  Hulthén wf is a reasonable approximation

- energy dependence is logarithmic

- ratio is qualitatively reproduced

  (if restricted to S-wave)

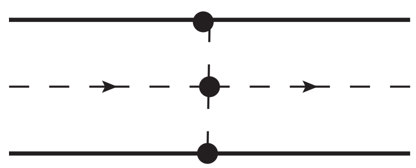
- „contact wf“ is not a good approximation

  convergence of HPEFT breaks down much earlier than expected

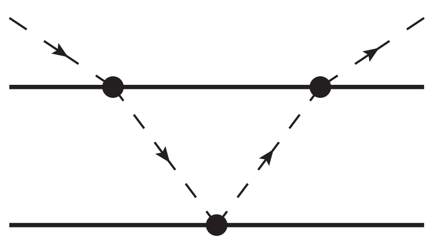
Triple scattering enhancement

Q-counting is not confirmed!

How to explain deviations of the numerical results from W-counting expectations?



$+ \ldots$ → probably accidental cancelations



look at the corresponding loop function

$$I_0(\omega, v\cdot Q, Q^2) = \frac{1}{i}\int \frac{d^d l}{(2\pi)^d}\,\frac{1}{v\cdot l - \omega - i\epsilon}\,\frac{1}{{m_\pi}^2 - l^2 - i\epsilon}\,\frac{1}{{m_\pi}^2 - (l-Q)^2 - i\epsilon}$$

(Dmitrasinovic et al., 1999)

→ $$I_0(m_\pi, 0, -\vec{q}^{\,2}) = \frac{1}{8|\vec{q}|} + \delta I_0 \gg \frac{1}{8\pi^2|\vec{q}|}$$ naive dimensional analysis

→ **π^2 enhancement compared to power counting from integrable singularities**

Similar topologies lead to similar enhancements in NN interaction,
nucleon form factor, π^0 photoproduction

(Friar et al., 2003; Becher et al., 1999; Bernard et al., 1991)

π scattering on A=3 and A=4

same strategy as before

obtain wave functions from solutions of the Schrödinger equation

(or Faddeev/Yakubovsky equations)

fold with π scattering operator (based on W-counting)

following calculations are based on a set of wf

LO to check dependence on cutoffs

$\Lambda[\mathrm{fm}^{-1}]$	$C_S\,[\mathrm{GeV}^{-2}]$	$C_T\,[\mathrm{GeV}^{-2}]$
2.0	-83.6941	2.63787
3.0	-29.0931	16.3942
4.0	86.3303	52.6427
5.0	-435.354	-122.611
10.0	-39.8356	6.36715
20.0	-66.2861	-4.03158

NLO/N^{2}LO consistent with operator

3nf in N^{2}LO

binding energies well described

models include 3nf's

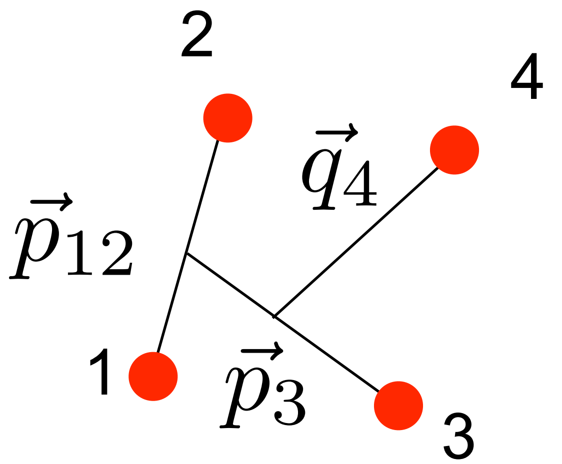
describe binding energies well

	Λ / $\tilde{\Lambda}$	$B(^3\mathrm{He})$	$B(^4\mathrm{He})$
LO	2.0 / –	11.042	39.88
LO	3.0 / –	6.878	20.25
LO	4.0 / –	6.068	17.08
LO	5.0 / –	5.987	16.48
LO	10.0 / –	5.611	15.05
LO	20.0 / –	5.429	—
NLO	400/500	7.678	28.57
NLO	550/500	6.991	24.38
NLO	550/600	7.051	24.72
NLO	400/700	7.699	28.77
NLO	550/700	7.090	24.94
N^{2}LO	450/500	7.717	28.04
N^{2}LO	600/500	7.740	28.11
N^{2}LO	550/600	7.722	28.28
N^{2}LO	450/700	7.726	27.65
N^{2}LO	600/700	7.808	28.57
CD-Bonn	—	7.719	28.28
AV18	—	7.736	28.36
Expt.	—	7.718	28.30

Numerical approach

- Most involved calculation: 4-nucleon operator in 4-nucleon system
- Avoid partial wave decompostion (tedious!)
- Need to calculate expectation value

$$
\begin{aligned}
\langle \hat{O} \rangle &= \sum_{\alpha\alpha'} \int d^3p_{12} d^3p_3 d^3q_4 d^3p'_{12} d^3p'_3 d^3q'_4 \langle \Psi | \vec{p}_{12} \vec{p}_3 \vec{q}_4 \alpha \rangle \, \langle \ldots | \hat{O} | \ldots \rangle \, \langle \vec{p}_{12}{}' \vec{p}_3{}' \vec{q}_4{}' \alpha' | \Psi \rangle \\
&= \sum_{\alpha\alpha'} \int d^3p_{12} d^3p_3 d^3q_4 d^3p'_{12} d^3p'_3 d^3q'_4 w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4) \\
&\qquad \frac{\langle \Psi | \vec{p}_{12} \vec{p}_3 \vec{q}_4 \alpha \rangle \, \langle \ldots | \hat{O} | \ldots \rangle \, \langle \vec{p}_{12}{}' \vec{p}_3{}' \vec{q}_4{}' \alpha' | \Psi \rangle}{w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4)}
\end{aligned}
$$



- ^{4}He wave function $\Psi \left(\vec{p}_{12} \vec{p}_3 \vec{q}_4, \alpha \right)$
- spin-isospin channels $|\alpha\rangle \equiv |m_1 m_2 m_3 m_4 \, m_1^t m_2^t m_3^t m_4^t \rangle$
- 4NF matrix element $\langle \vec{p}_{12} \vec{p}_3 \vec{q}_4 \alpha | \hat{O} | \vec{p}_{12}{}' \vec{p}_3{}' \vec{q}_4{}' \alpha' \rangle$ generated using ***Maple/Mathematica***

→

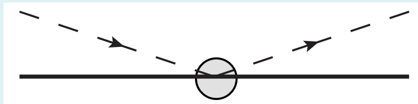
- Metropolis walk for evaluation based on weight function

$$
w(p_{12}, p_3, q_4; p'_{12}, p'_3, q'_4) \propto \prod_{\substack{i=12,3,4,\\ 12',3',4'}} \frac{1}{(p_i + C_i)^{n_i}}
$$

Contributions to π-^{3}He scattering

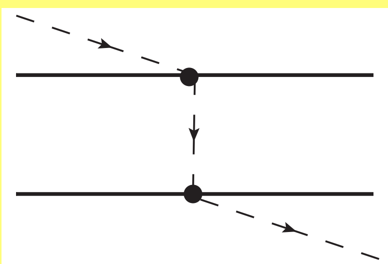
„Weinberg-counting" - typical momenta $\propto m_\pi$

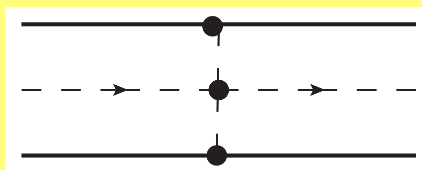
1-nucleon contributions



$$a_{\pi-A} = \left(\frac{1+m_\pi/m_N}{1+m_\pi/Am_N}\right)\left(Aa^{(+)} + 2T_3 a^{(-)}\right) \quad \propto \frac{m_\pi}{{f_\pi}^2}$$

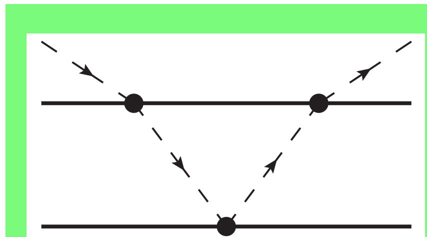
2-nucleon contributions



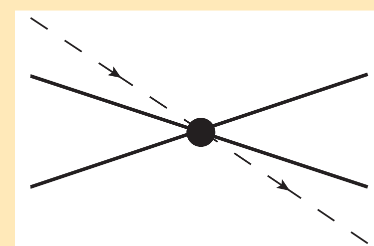


+ ...

$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$$



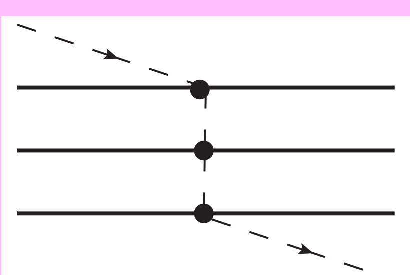
$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4 \pi^2\right)$$

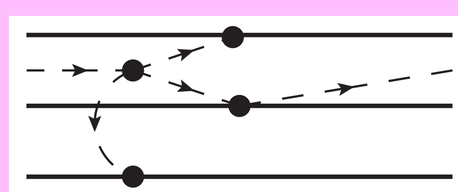


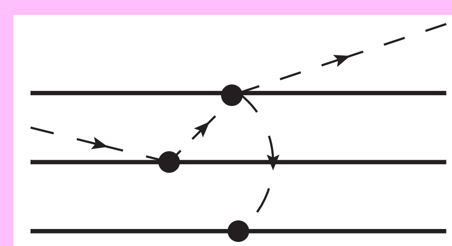
$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^3\right)$$ (isovector)

$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$$ (isoscalar)

3-nucleon contributions





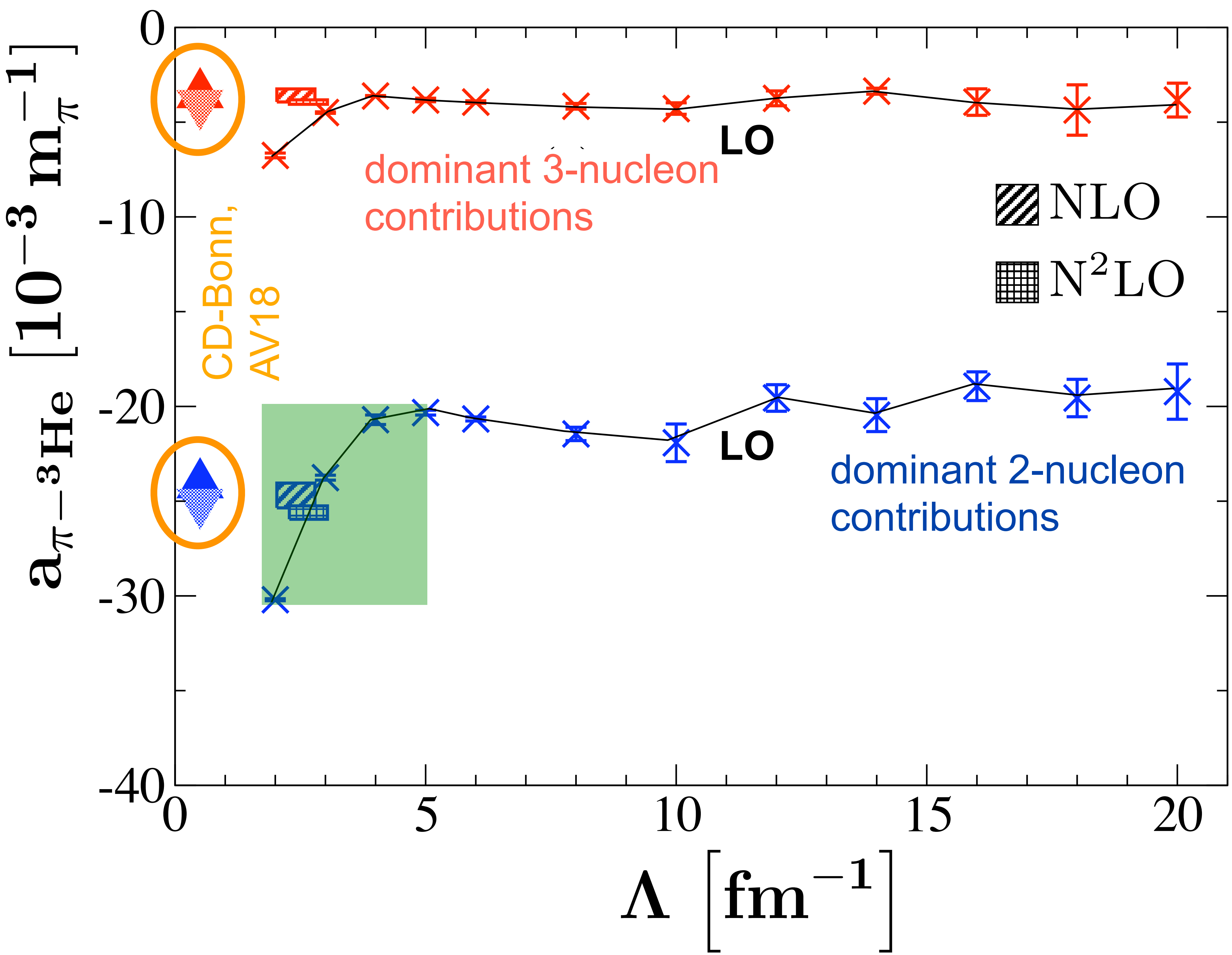


$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$$

+ 5 more

Does the explicit calculation support this power counting?

Cutoff dependence for π-^{3}He

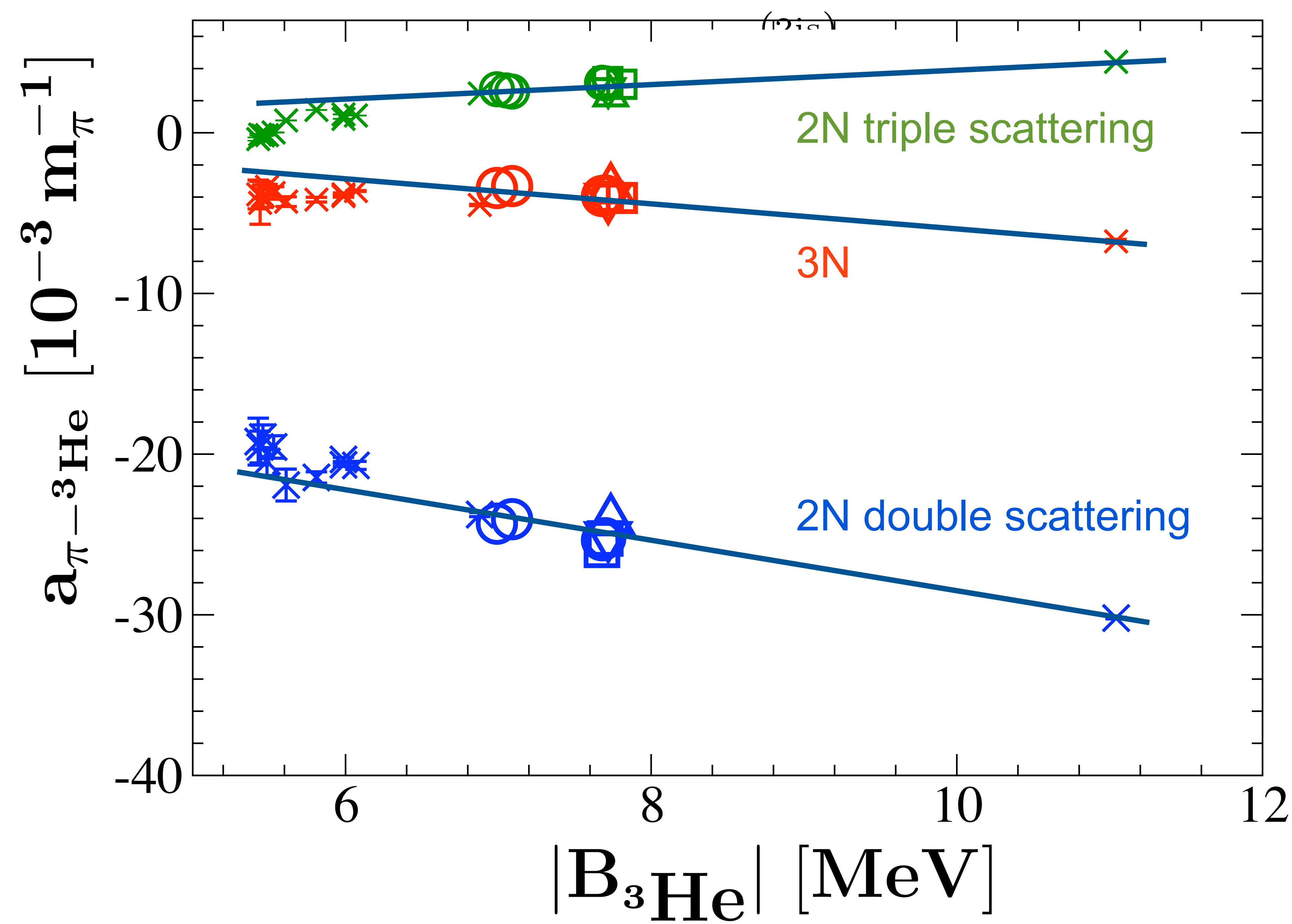


- no unexpected cutoff dependences for LO wave functions
- cutoff dependence seems to be relevant: much larger variation than for deuteron

Binding energy dependence

Bulk of cutoff dependence is result of low order NN interaction

→ binding energy dependence

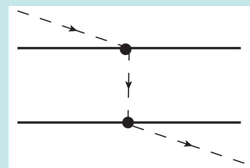
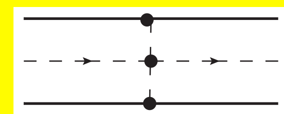
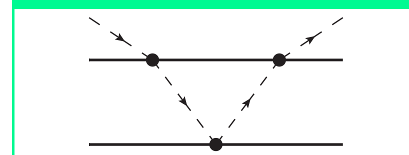
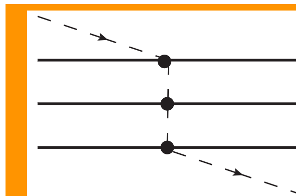
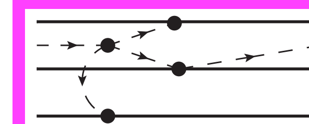


→

Estimate cutoff dependence from variation of N^2LO interactions

Results for π-^{3}He

- results do not strongly dependent on cutoff when binding energies are OK
- counter term estimate based on naive dimensional analysis larger
- cutoff dependence might result in low estimate since leading few-nucleon contribution is isoscalar

	$\Lambda/\tilde{\Lambda}$						
CD Bonn	—	−25.08(6)	−0.329(1)	2.769(2)	0.890(1)	−4.020(73)	−0.789(6)
AV18	—	−24.13(9)	−0.884(1)	2.286(3)	0.788(1)	−3.536(36)	−0.728(4)
N^2LO	450/500	−25.76(4)	0.642(1)	3.189(2)	0.987(1)	−3.979(7)	−0.721(2)
N^2LO	600/500	−25.60(5)	−0.021(1)	3.039(3)	0.779(1)	−3.826(58)	−0.496(4)
N^2LO	550/600	−25.55(2)	0.233(1)	3.104(3)	0.952(1)	−4.057(75)	−0.708(4)
N^2LO	450/700	−25.25(4)	0.611(1)	3.104(1)	1.052(1)	−4.038(15)	−0.806(1)
N^2LO	600/700	−25.51(7)	0.094(1)	3.022(2)	0.985(1)	−4.056(53)	−0.734(2)

$$\rightarrow \quad a^{(2N+3N)}_{\pi-^3\mathrm{He}} = ((-25.6 + 4.0 + 0.3) + (-4.0 - 0.7) \pm 7) \cdot 10^{-3} m_\pi^{-1} = (-26.0 \pm 7) \cdot 10^{-3} m_\pi^{-1}.$$

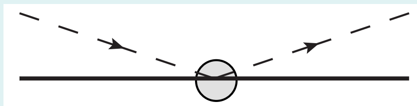
$\tilde{a}^{(+)} = (1 \pm 1) \times 10^{-3} m_\pi^{-1}, \quad a^{(-)} = (86.5 \pm 1.2) \times 10^{-3} m_\pi^{-1}$ (from Hoferichter et al., 2009)

$$\rightarrow \quad a^{(1N+2N+3N)}_{\pi-^3\mathrm{He}} = (62 \pm 4 (\pm 7)) \times 10^{-3} m_\pi^{-1}$$

	$a_{\pi-^3\mathrm{He}}\ [10^{-3}\, m_\pi^{-1}]$
R. Abela *et al.*	56 ± 6
G. R. Mason *et al.*	43 ± 5
I. Schwanner *et al.*	41 ± 4

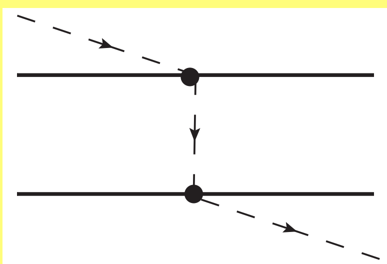
Contributions to π-⁴He scattering

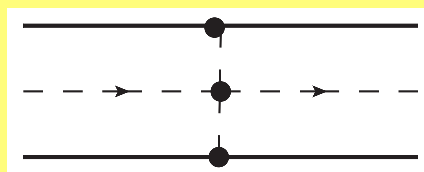
1-nucleon contributions



$$a_{\pi-A} = \left(\frac{1+m_\pi/m_N}{1+m_\pi/Am_N}\right)\left(Aa^{(+)} + 2T_3 a^{(-)}\right)$$

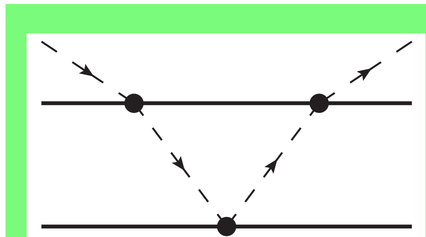
2-nucleon contributions



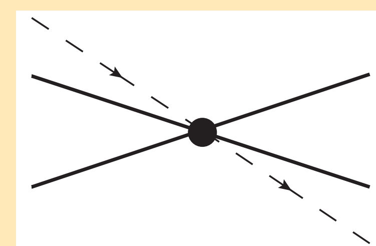


+ ...

$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^2\right)$$



$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4 \pi^2\right)$$

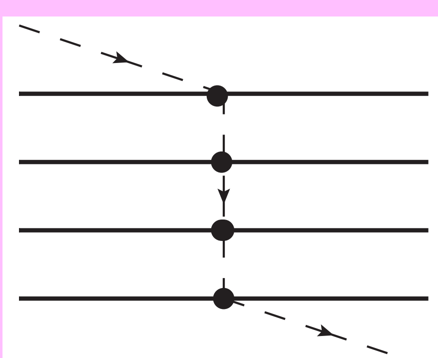


~~$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^3\right)$$~~ (isovector)

$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^4\right)$$ (isoscalar)

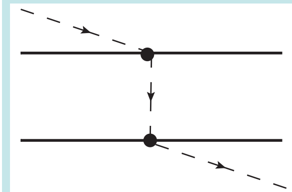
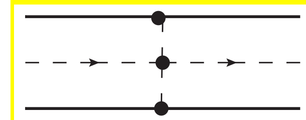
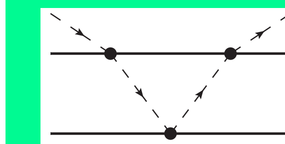
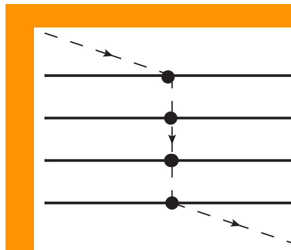
~~3-nucleon contributions~~ (isovector)

4-nucleon contributions (incomplete)



$$\mathcal{O}\left(\left(\frac{m_\pi}{\Lambda_\chi}\right)^6\right)$$

Results for π-^{4}He

	$\Lambda/\tilde{\Lambda}$				
AV18	—	-49.5(7)	-1.29(2)	5.00(5)	2.73(84)
N^2LO	450/500	-54.4(4)	1.92(1)	6.98(2)	3.00(20)
N^2LO	600/500	-52.0(8)	-0.09(2)	6.16(3)	2.13(11)
N^2LO	550/600	-52.7(6)	0.50(1)	6.42(3)	2.31(31)
N^2LO	450/700	-52.7(7)	1.81(1)	6.68(3)	2.56(10)
N^2LO	600/700	-53.9(8)	0.36(1)	6.34(2)	2.81(17)

- cutoff dependence in line with naive expectations
  (below 5 % of leading 2N contribution)
- 2N contribution only twice as large as for the deuteron
  (np and pp/nn pairs enter with opposite sign)
- 4N term finally comparable to counter term contribution (can be neglected)

- **few-nucleon contributions larger than naively expected**

Relative scaling of few-nucleon diagrams

JÜLICH FORSCHUNGSZENTRUM

- relative suppression of two-nucleon contributions are completely in agreement with expansion parameter $\frac{m_\pi}{\Lambda_\chi} \approx \frac{1}{5}$ & naive dimensional analysis

(cutoff dependence, triple scattering)

- few-nucleon contributions are suppressed by much less than expected
- suppression is comparable for 1N:2N:3N:4N

	$\Lambda/\tilde{\Lambda}$	$a^{(2N)}_{\pi-{}^3He}/a^{(1N)}_{\pi-{}^3He}$	$a^{(3N)}_{\pi-{}^3He}/a^{(2N)}_{\pi-{}^3He}$	$a^{(4N)}/a^{(3N)}$
AV18	—	0.222	0.194	0.307
N^2LO	450/500	0.212	0.222	0.297
N^2LO	600/500	0.220	0.194	0.239
N^2LO	550/600	0.215	0.224	0.225
N^2LO	450/700	0.207	0.236	0.245
N^2LO	600/700	0.216	0.224	0.266

systematically understandable?

impact for other few-nucleon operators / three-nucleon interactions?

Conclusions & Outlook

- systematic overview of few-nucleon contributions to πA scattering at low energies
- π-^{2}H
  - no significant cutoff dependence of the most prominent diagrams
  - binding momentum is not driving the power counting (no Q-counting)
  - toy model based on Hulthén wf indicates early breakdown of Hπ-EFT
  - enhancement of specific topolgy could be visible in other processes
- π-^{3}He
  - complete set of 3-nucleon diagrams
  - leading 3-nucleon diagrams are isovector
  - no significant cutoff dependence
  - prediction of π-^{3}He scattering length up to the accuracy possible
- π-^{4}He
  - studied the probably most important 4-nucleon contribution
  - accurate results for all relevant few-nucleon contributions
  - isospin violation needs to be take into account
  - check systematics of π-^{2}H, probably no further constraints on π-N

- naive counting good for estimate of subleading A-nucleon contributions
- relative suppression of A and A+1-nucleon contributions smaller than expected