Universal few/many-body physics in mixed dimensions

Yusuke Nishida (MIT)

March 16, 2010 @ INT program

"Simulations and Symmetries:
Cold Atoms, QCD, and Few-hadron Systems"

Introduction

Remarkable progress in ultracold atoms are owing to the tunability of various parameters

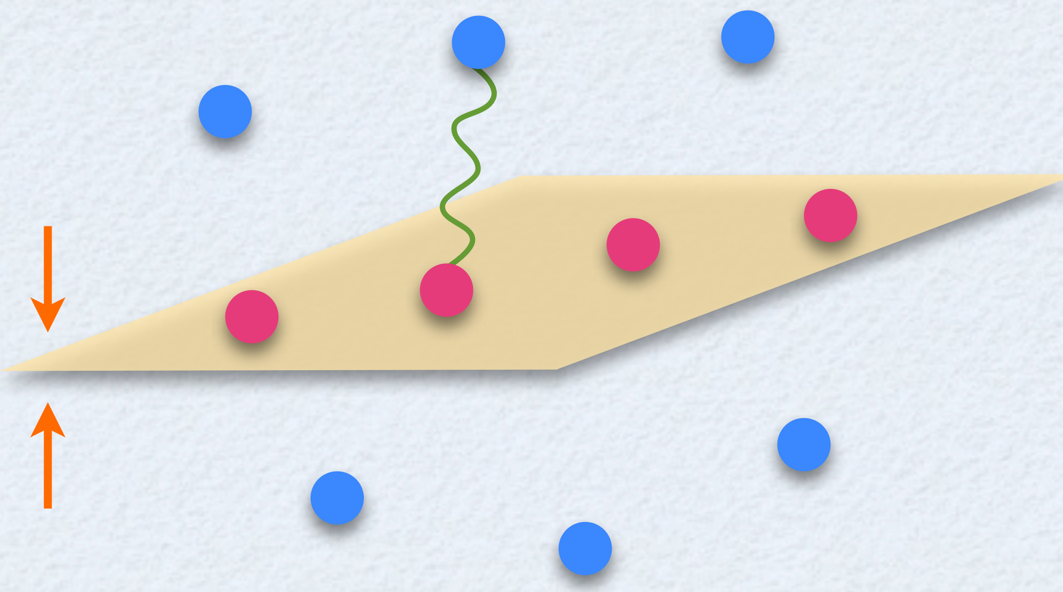
- Interaction strength by Feshbach resonances
  - Superfluid-Mott insulator transition in Bose gases
  - Collapse & phase separation in Bose-Fermi mixtures
  - BCS-BEC crossover in Fermi gases
  - Efimov effect
- Dimensionality of space by strong optical lattices
  - 3D : BCS-BEC crossover and etc.
  - 2D : Berezinsky–Kosterlitz–Thouless transition
  - 1D : Tomonaga-Luttinger liquids
  - ... and more?

Mixed dimensions

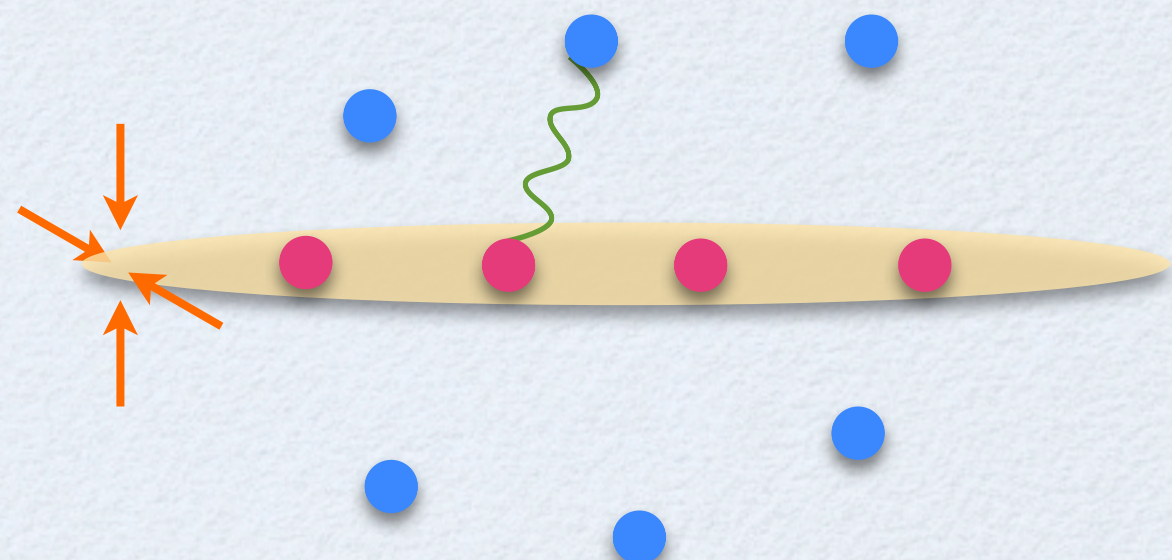
2-species mixture of A atom ● & B atom ●

confine "only" A atoms in lower dimensions (2D or 1D)

2D-3D mixture



1D-3D mixture



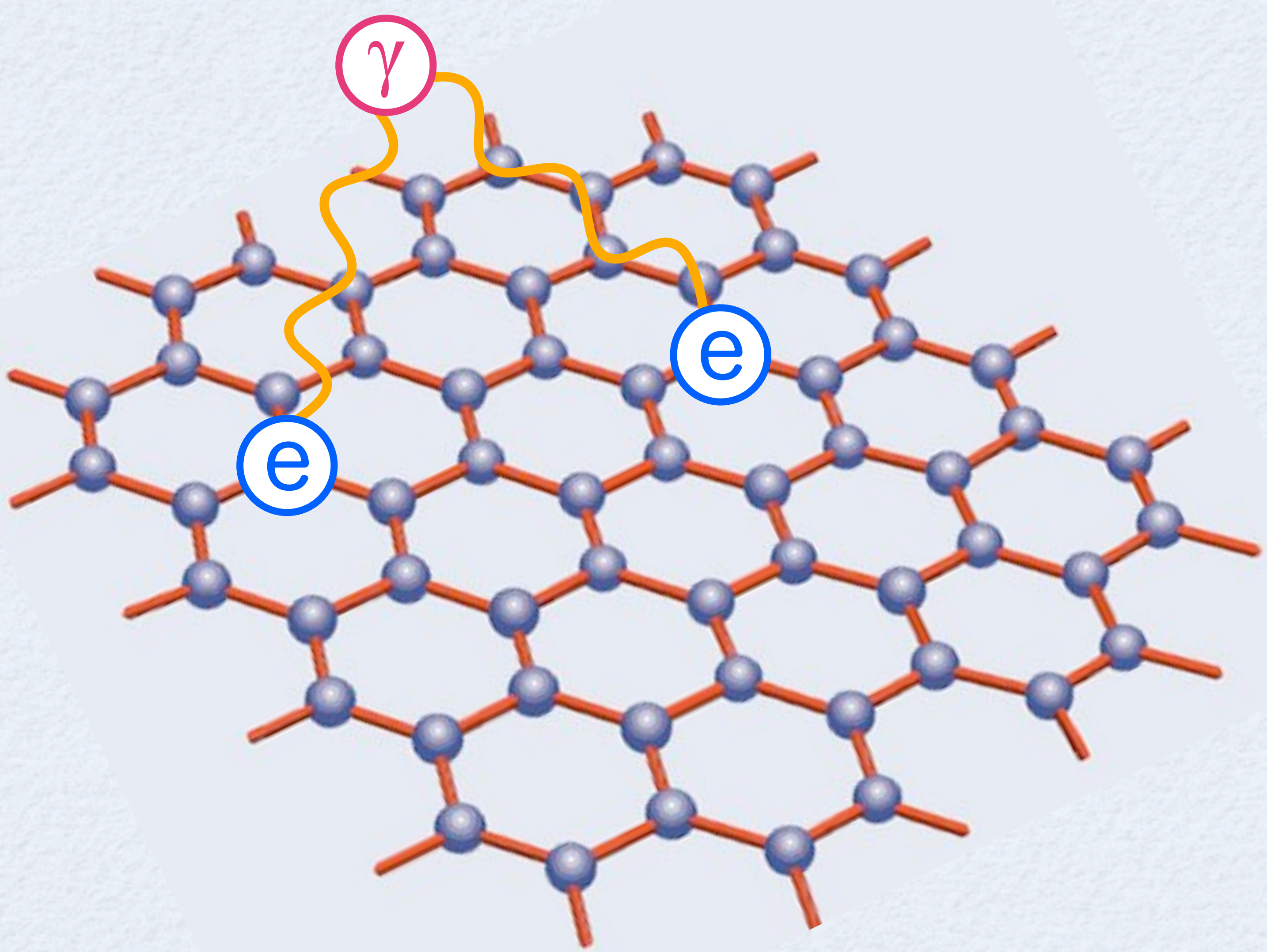
Choose laser frequency close to the resonance of A atoms
but far from the resonance of B atoms with low intensity

Brane world in cold atoms?

Idea of mixed dimensions often appears in physics ...

graphene in condensed matter

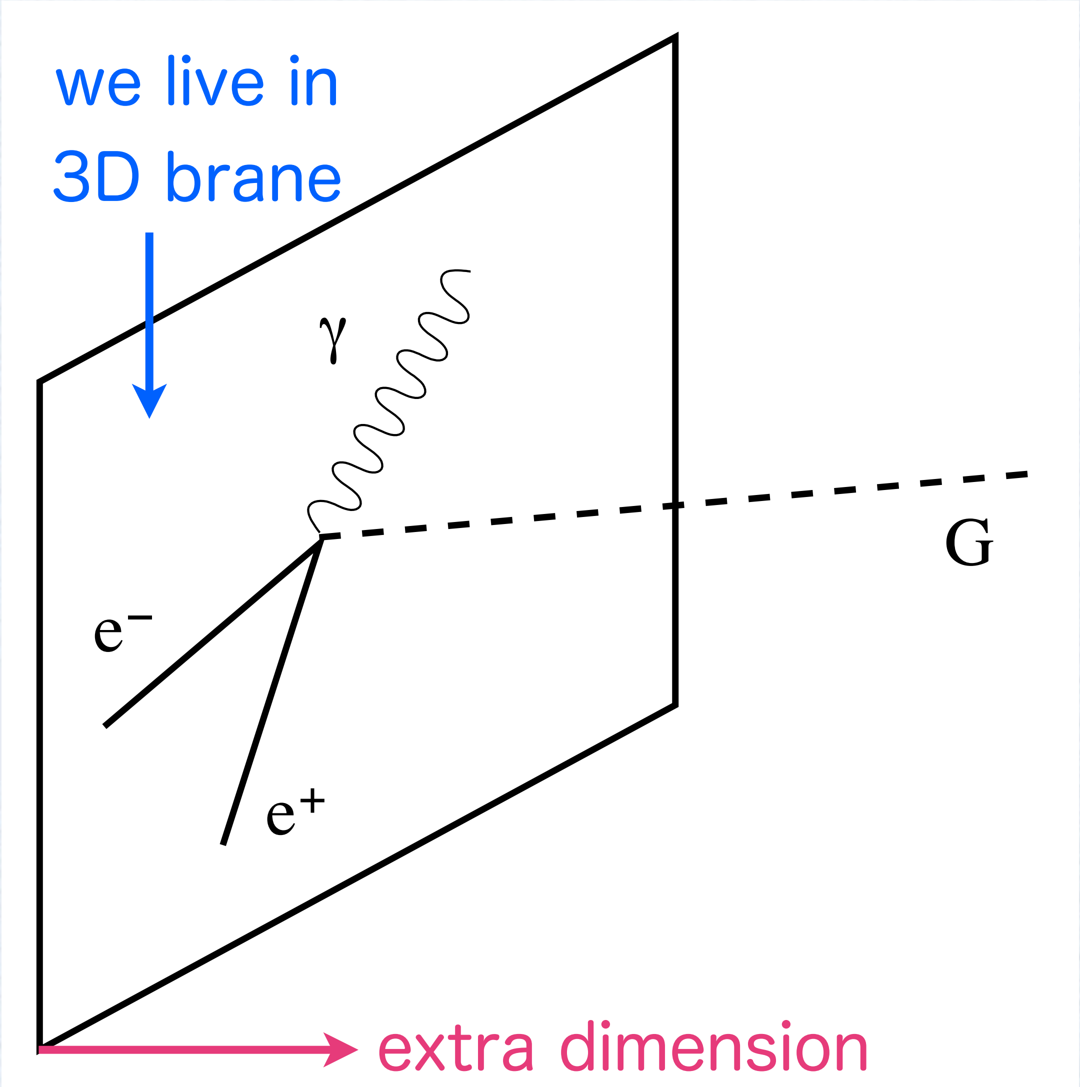
Photons in 3D induce 3D Coulomb int. between electrons confined in 2D



physicsworld.com (2007)

brane world in cosmology

We live in 3D brane but gravitons in extra dimensions



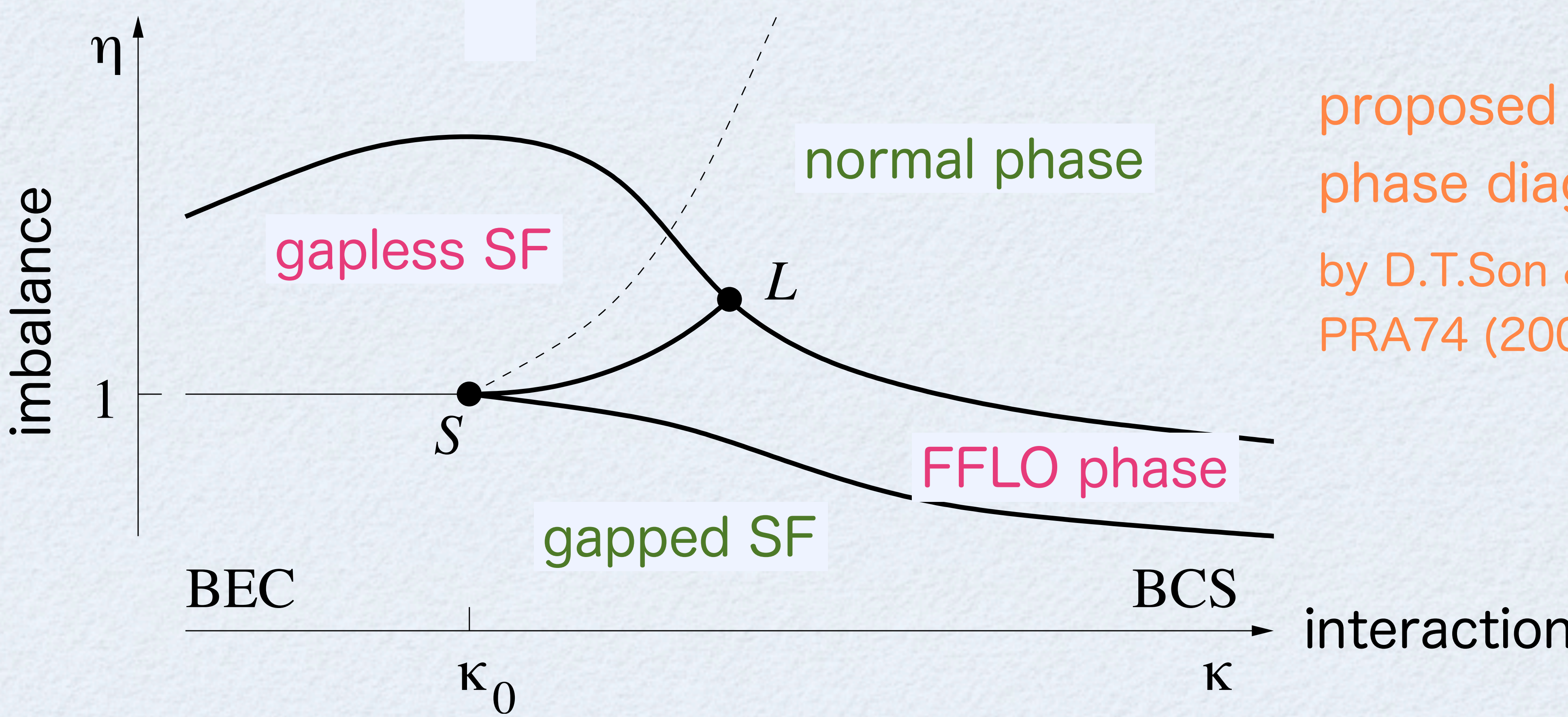
M. Cavaglia, Int. J. Mod. Phys. A (2003)

Mixed D = new type of imbalance

Fermi gas with equal number of ↑ and ↓ fermions

➡ BCS-BEC crossover (s-wave superfluid for any coupling)

But if we introduce an imbalance ($n_\uparrow \neq n_\downarrow$) ...



proposed T=0 phase diagram

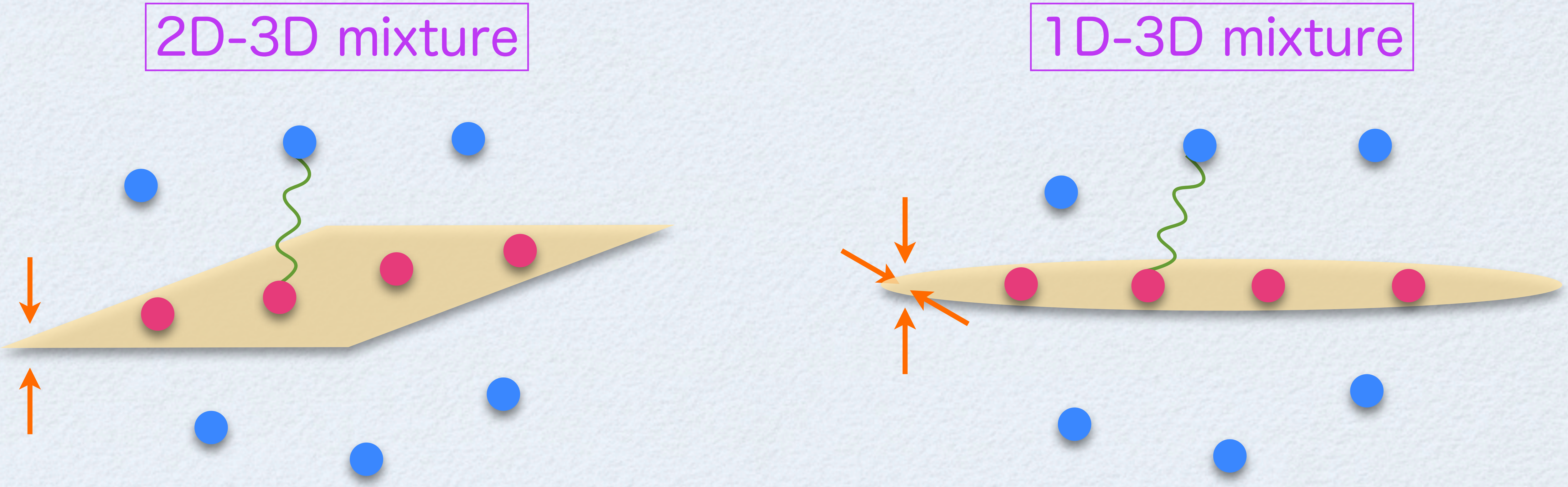
by D.T.Son & M.A.Stephanov PRA74 (2006)

Imbalance leads to new and rich physics

- density imbalance ($n_\uparrow \neq n_\downarrow$)
- mass imbalance ($m_K \neq m_{Li}$)
- dimensionality of space ($d_K \neq d_{Li}$)

Rich physics in mixed dimensions

Rich few-body & many-body physics
can be realized in mixed dimensions



- Modification of scattering properties

  ➡ confinement-induced 2-body & 3-body resonances

- Interesting and rich many-body phase diagram

  ➡ induced s-wave & p-wave superfluidity,
  dimer BEC, (stable) trimer Fermi gas, ...

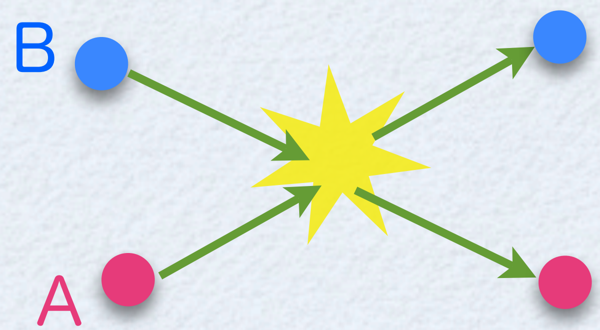
Table of contents

Few-body physics in mixed D

2-body scattering

Effective scattering length

2-body scattering in 3D



$$H_{3\mathrm{D}} = -\frac{\boldsymbol{\nabla}^2_{\boldsymbol{x}_A}}{2m_A} - \frac{\boldsymbol{\nabla}^2_{\boldsymbol{x}_B}}{2m_B} + V_a(\boldsymbol{x}_A, \boldsymbol{x}_B)$$

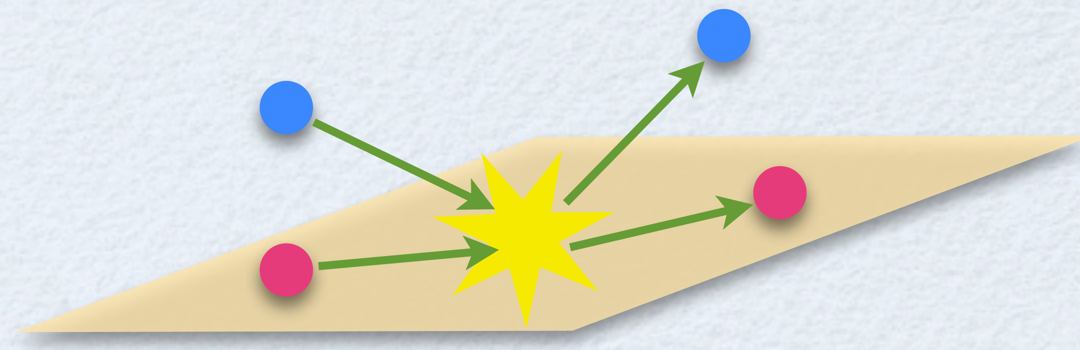
$$\Psi(\boldsymbol{x}_A, \boldsymbol{x}_B) \to \frac{1}{|\boldsymbol{x}_A - \boldsymbol{x}_B|} - \frac{1}{a}$$

3D scattering length "a"

tunable by Feshbach resonance

$$\mathcal{A}_{3\mathrm{D}}(k) \propto \frac{1}{ik - \frac{1}{a} + \cdots}$$

2-body scattering in mixed D



$$H_{\mathrm{mixed\,D}} = H_{3\mathrm{D}} + \frac{1}{2} m_A \omega^2 z_A^2$$

$$\Psi(\boldsymbol{x}_A, \boldsymbol{x}_B) \to \left[\frac{1}{||\boldsymbol{x}_A - \boldsymbol{x}_B||} - \frac{1}{a_{\mathrm{eff}}}\right] e^{-z_A^2/2l^2}$$

Effective scattering length "a_{eff}"

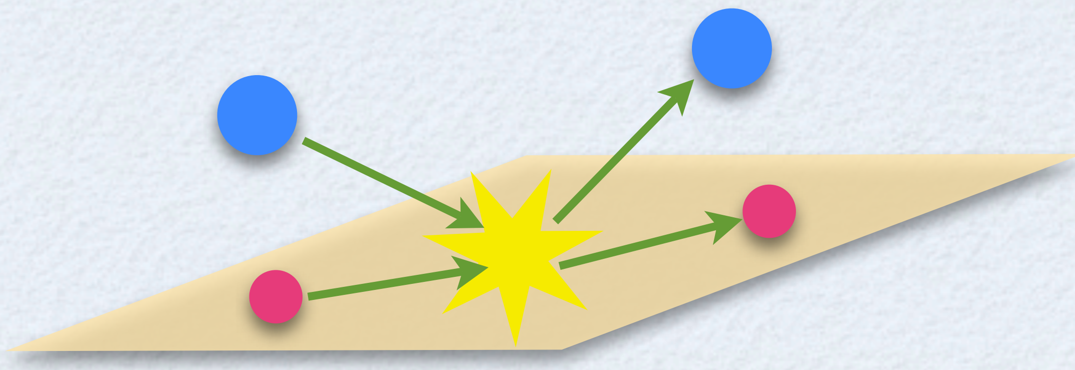
depending on "a" and "$l = (m_A\omega)^{-1/2}$"

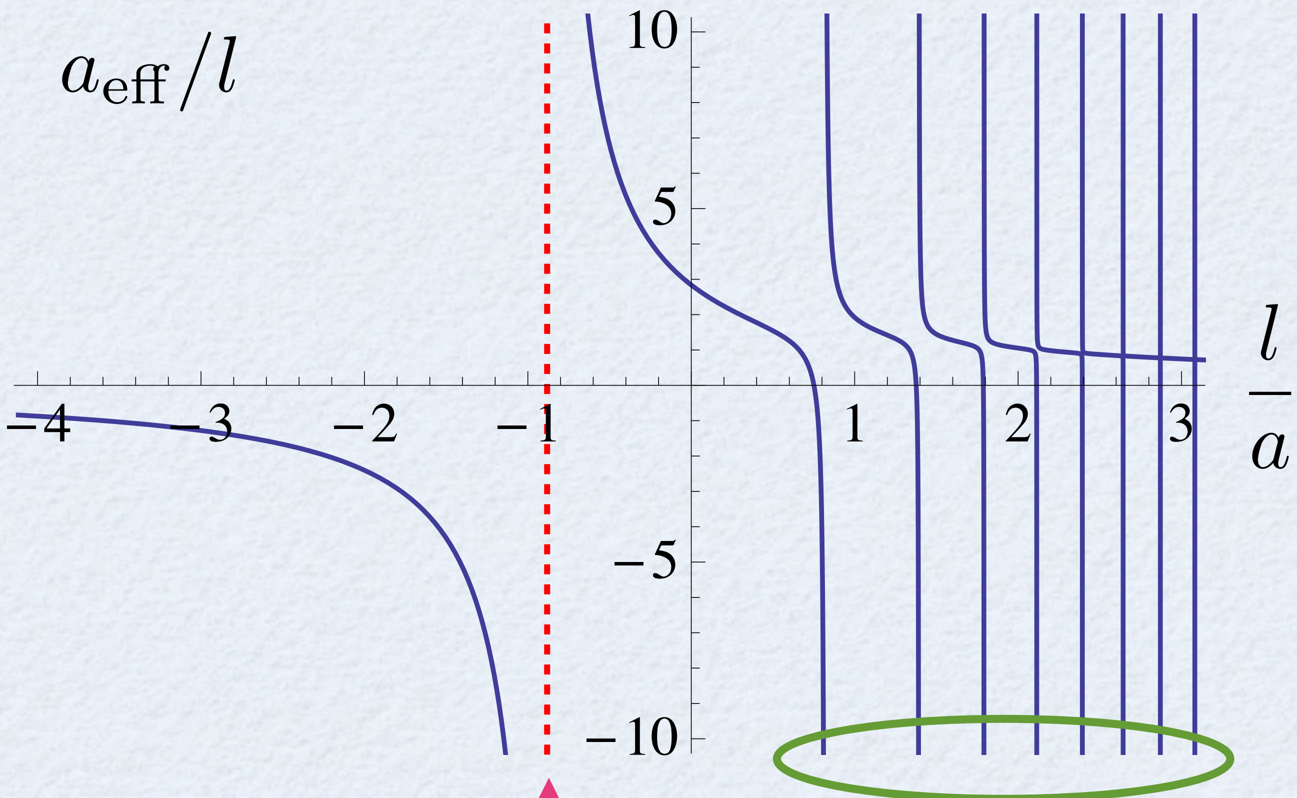
$$\mathcal{A}_{\mathrm{mixed\,D}}(k) \propto \frac{1}{ik - \frac{1}{a_{\mathrm{eff}}} + \cdots}$$

Resonances in mixed D ($m_A/m_B=0.15$)

2D-3D mixture

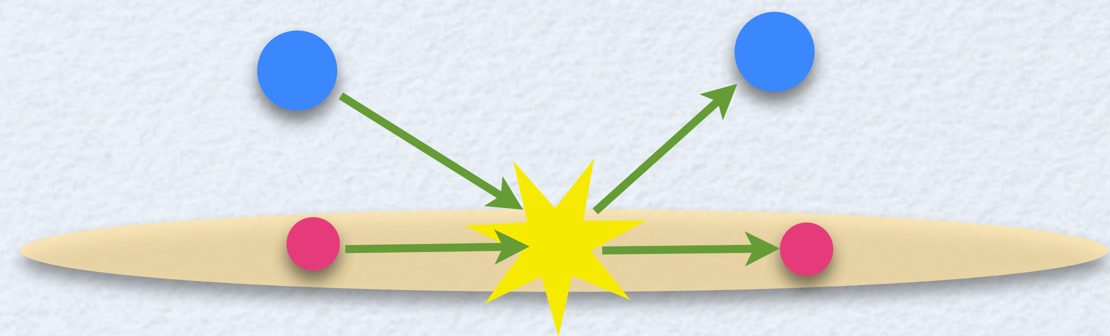
A=^{6}Li in 2D & B=^{40}K in 3D

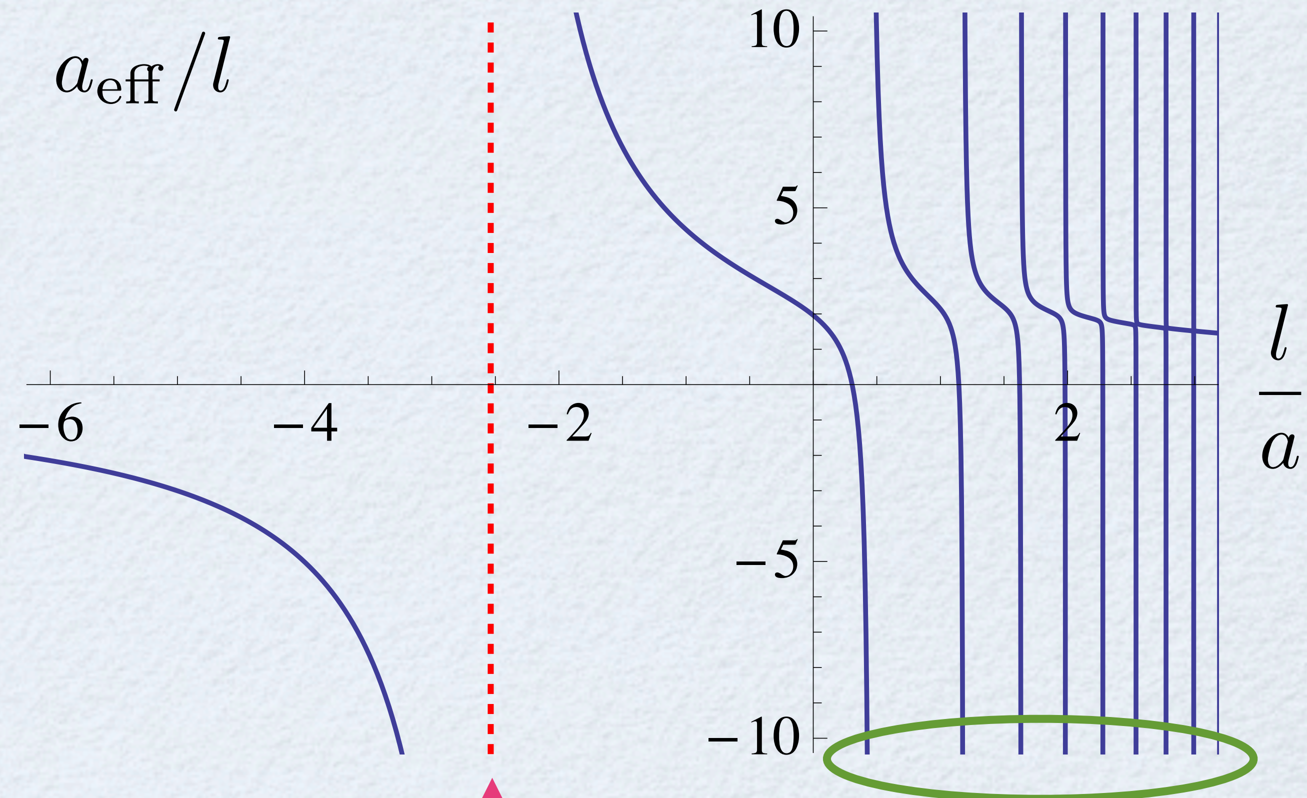




1D-3D mixture

A=^{6}Li in 1D & B=^{40}K in 3D





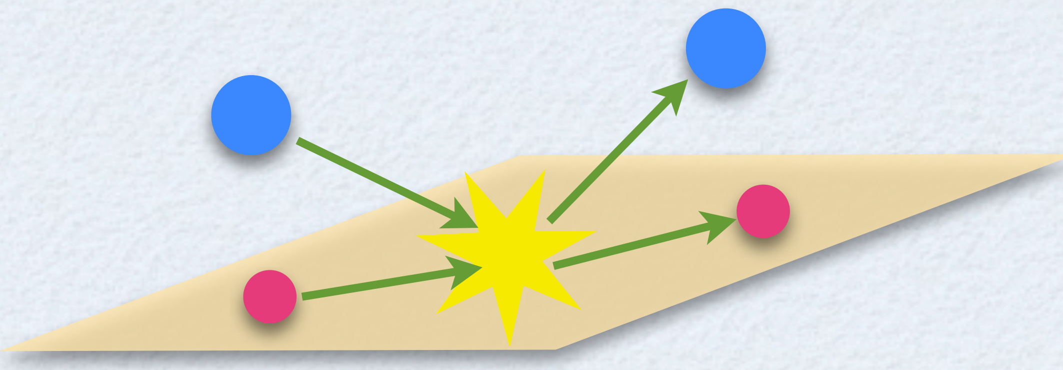
Infinite series of confinement-induced resonances

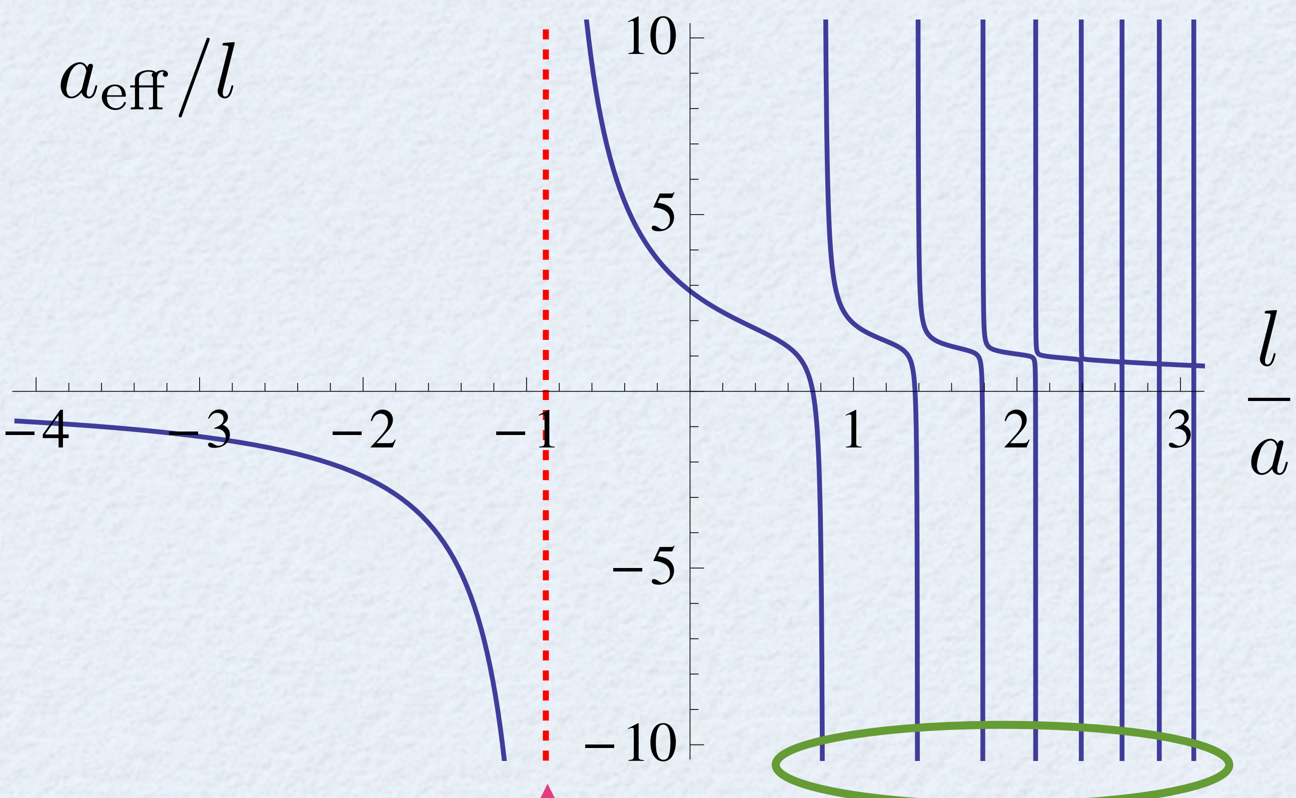
Resonance is shifted to “a<0” side

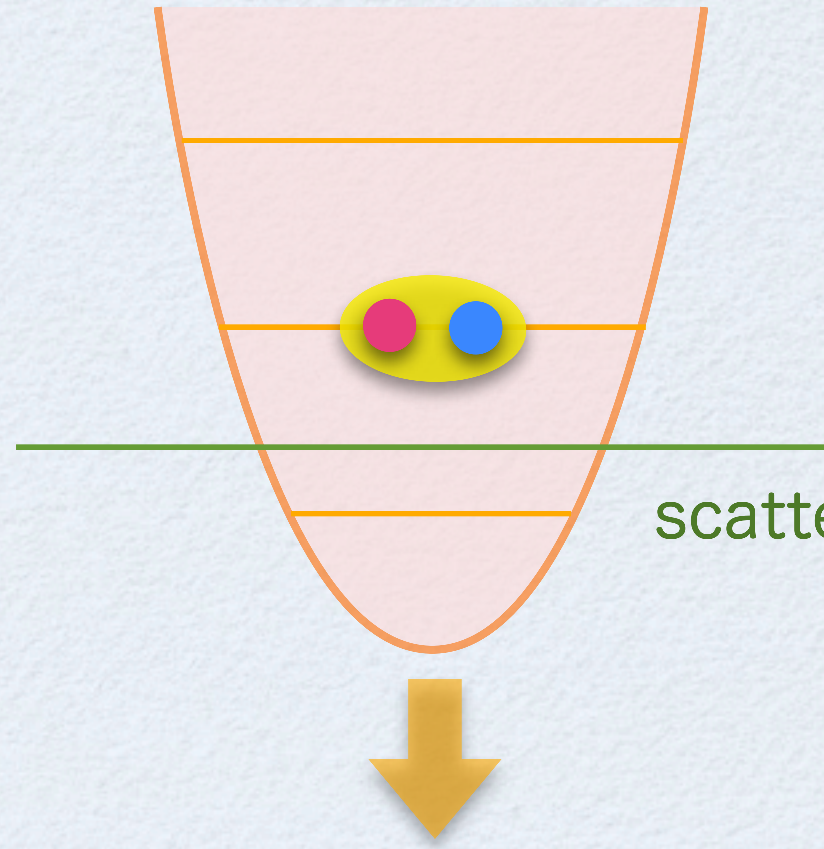
Resonances in mixed D $(m_A/m_B=0.15)$

2D-3D mixture

A=^{6}Li in 2D & B=^{40}K in 3D







$$\left(2n+\frac{1}{2}\right)\hbar\omega_M-\frac{\hbar^2}{ma^2}=\frac{1}{2}\hbar\omega_A$$

See also 0D-3D case;
P. Massignan & Y. Castin, PRA (2006)

Infinite series of confinement-induced resonances

Resonance is shifted to “a<0” side

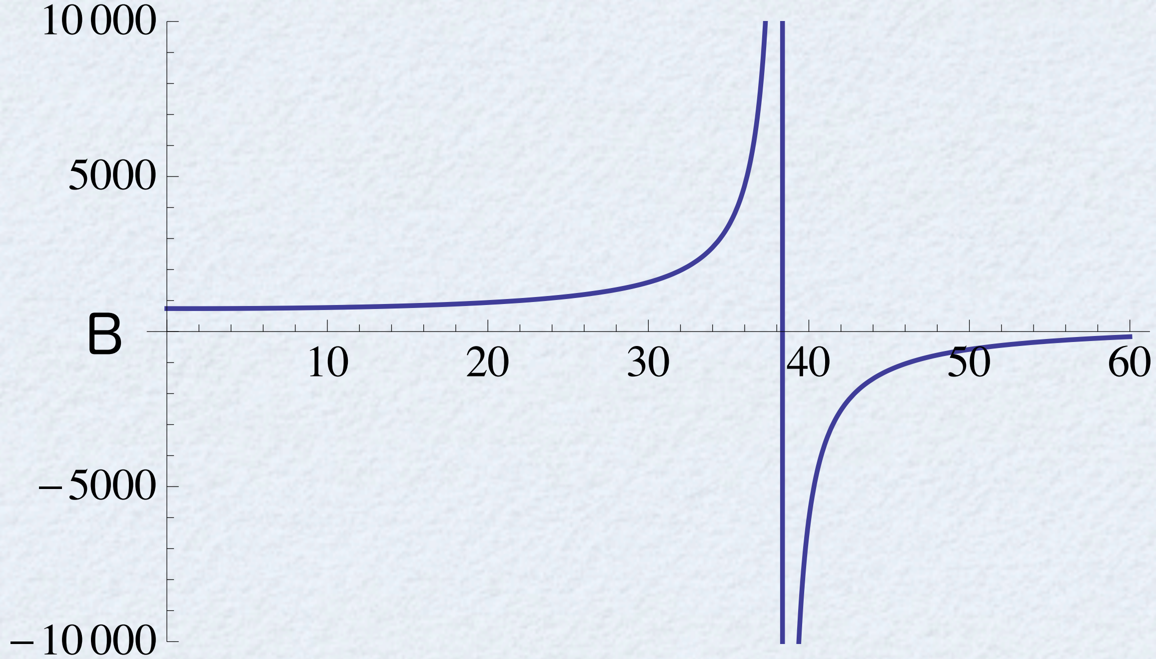
First experiment @ Florence

Scattering in mixed dimensions with ultracold gases

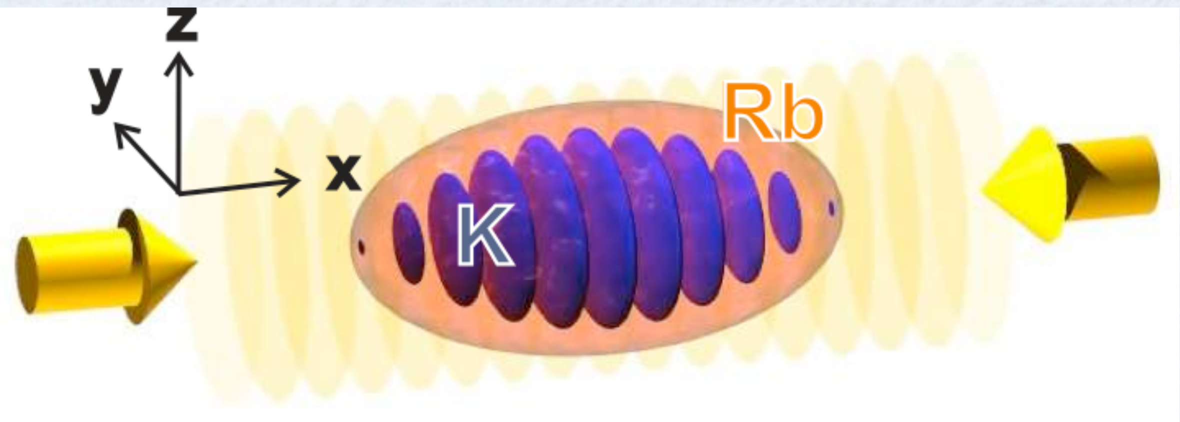
G. Lamporesi[1], J. Catani[1,2], G. Barontini[1], Y. Nishida[3], M. Inguscio[1,2], and F. Minardi[1,2]
[1]*LENS - European Laboratory for Non-Linear Spectroscopy and Dipartimento di Fisica, Università di Firenze, via N. Carrara 1, I-50019 Sesto Fiorentino, Italy*
[2]*INO-CNR, via G. Sansone 1, I-50019 Sesto Fiorentino, Italy*
[3] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Dated: January 31, 2010)

First experiment @ Florence

3D scattering length a/a_0
between A=^{41}K & B=^{87}Rb

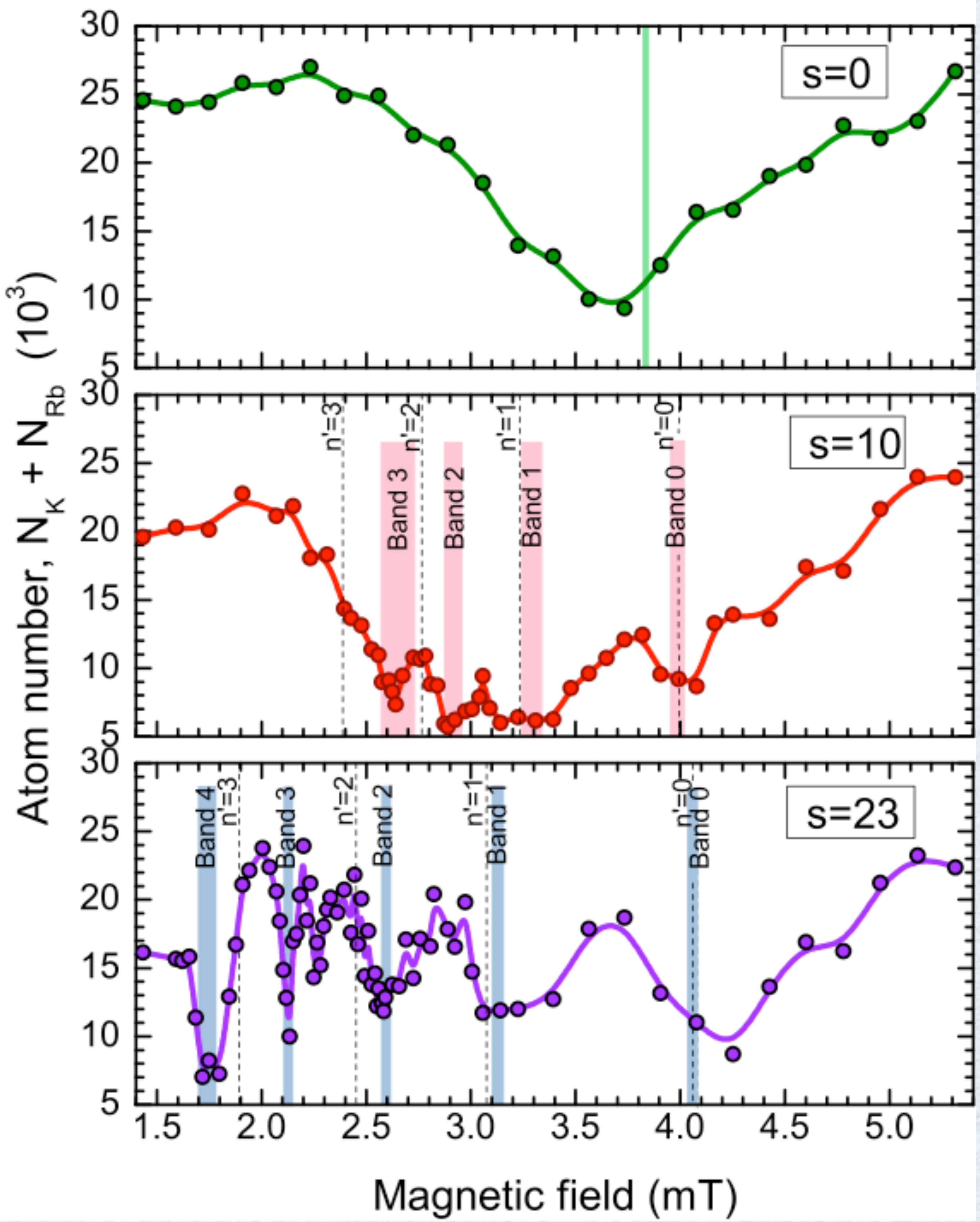


38.4G



A=^{41}K in 2D & B=^{87}Rb in 3D

G. Lamporesi et al, arXiv:1002.0114



Universality

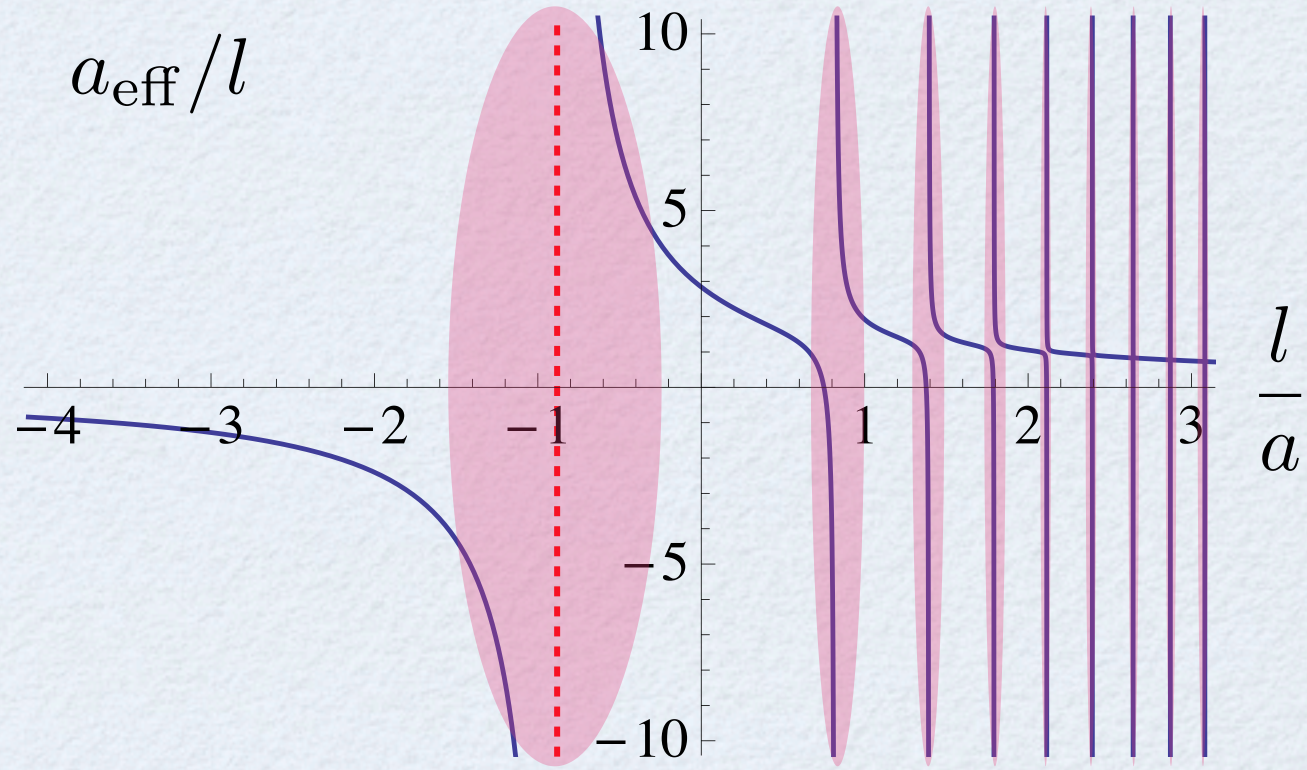
When "$a_{\rm eff} \gg l$", the confinement length "l" can be neglected

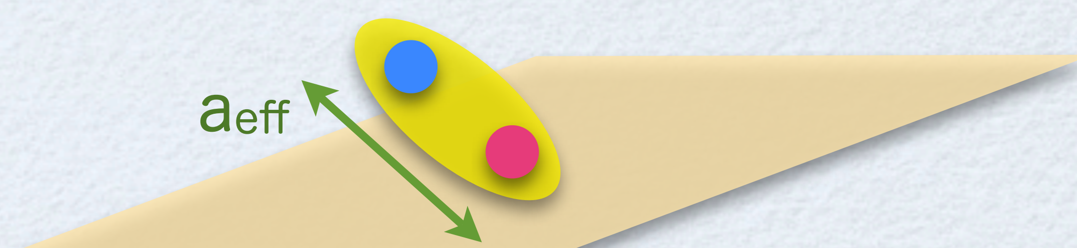
Then the system is universal, being characterized only by "$a_{\rm eff}$"

E.g., binding energy of AB dimer

$$E_{\rm dimer} = -\frac{1}{2m_{AB}a_{\rm eff}^2} \quad \text{for} \quad a_{\rm eff} > 0$$

(Cf. universality in 3D required "$a \gg r_0$")



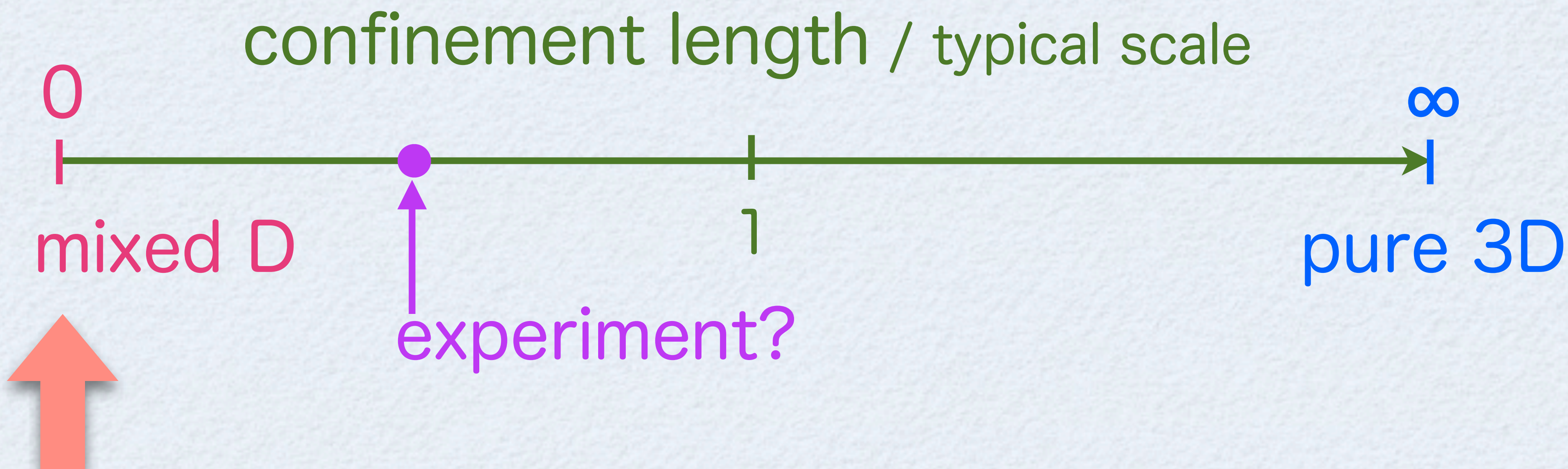


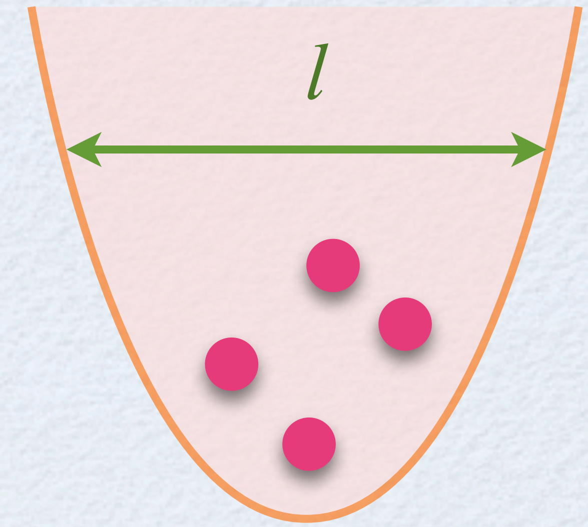
- $a_{\rm eff}<0$: weak attraction ("BCS") side
- $|a_{\rm eff}| \to \infty$: resonant (unitarity) limit with scale inv.
- $a_{\rm eff}>0$: strong attraction ("BEC") side with AB bound state $E_{\rm dimer} = -\frac{1}{2m_{AB}a_{\rm eff}^2}$

Universal limit

At finite density & temperature, the universality requires

a_{eff} & any other scales » l





We will work in the "universal limit" $l \to 0$

$$H_{\text{mixed D}} \Rightarrow -\sum_{i=1}^{N_A} \frac{\nabla^2_{\tilde{\boldsymbol{x}}_{A_i}}}{2m_A} - \sum_{j=1}^{N_B} \frac{\nabla^2_{\boldsymbol{x}_{B_j}}}{2m_B} + V_{a_{\mathrm{eff}}}$$

$$\tilde{\boldsymbol{x}}_A = \begin{cases} (x, y) & \text{for 2D} \\ (z) & \text{for 1D} \end{cases}$$

$$\boldsymbol{x}_B = (x, y, z)$$

($\tilde{\boldsymbol{x}}_A$ is 2D or 1D coordinate while $\boldsymbol{x}_B$ is 3D coordinate)

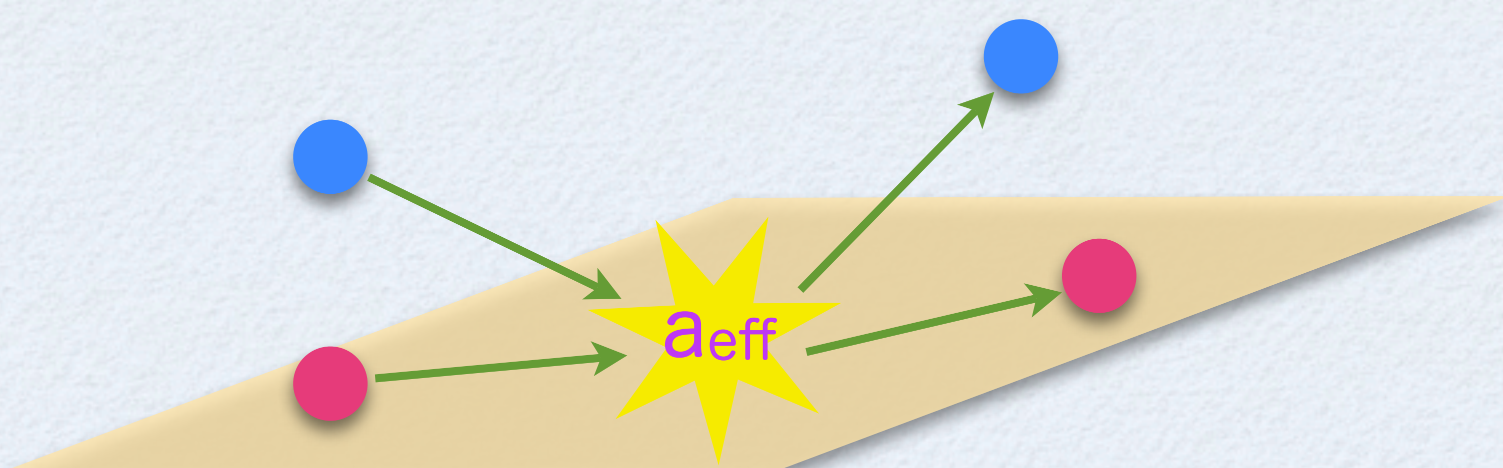
Field theoretical description

$$H = \int d\boldsymbol{x}\, \psi_A^\dagger(\boldsymbol{x}) \left(-\frac{\boldsymbol{\nabla}^2}{2m_A} - \mu_A \right) \psi_A(\boldsymbol{x})$$ ← A atoms ● in 2D

$$+ \int d\boldsymbol{x} dz\, \psi_B^\dagger(\boldsymbol{x}, z) \left(-\frac{\boldsymbol{\nabla}^2 + \nabla_z^2}{2m_B} - \mu_B \right) \psi_B(\boldsymbol{x}, z)$$ ← B atoms ● in 3D

$$+ g_0(a_{\text{eff}}) \int d\boldsymbol{x}\, \psi_A^\dagger(\boldsymbol{x}) \psi_B^\dagger(\boldsymbol{x}, 0) \psi_B(\boldsymbol{x}, 0) \psi_A(\boldsymbol{x})$$

scattering of A and B atoms occurring in 2D plane (z=0)



- μ_A and μ_B control the densities of A and B atoms

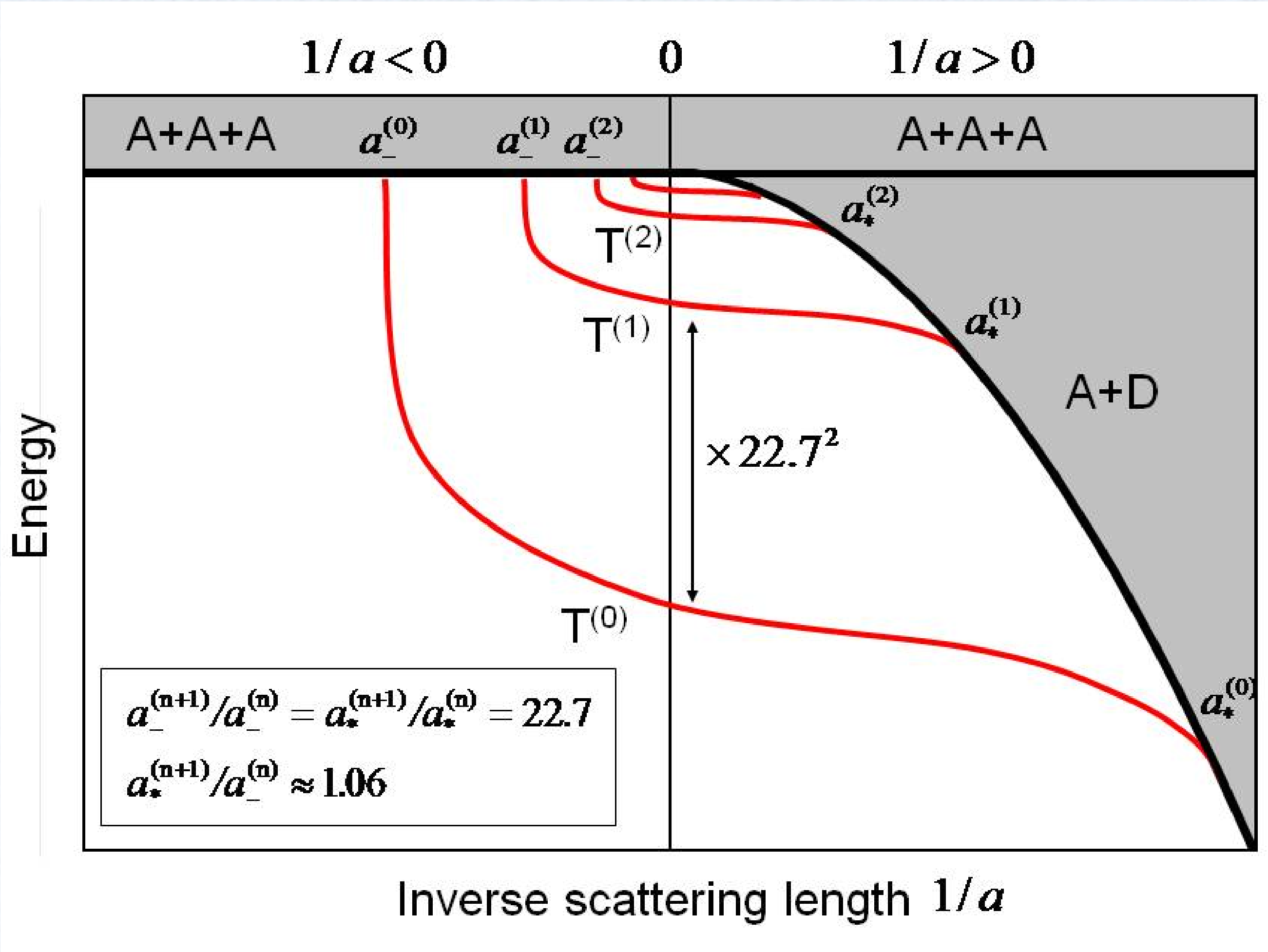
Few-body physics in mixed D

3-body scattering

Efimov effect in 3D

When 2 atoms resonantly interact, 3 atoms form Efimov trimers with a geometric spectrum $E_n/E_{n+1}|_{a\to\infty} = \lambda^2$

3 bosons

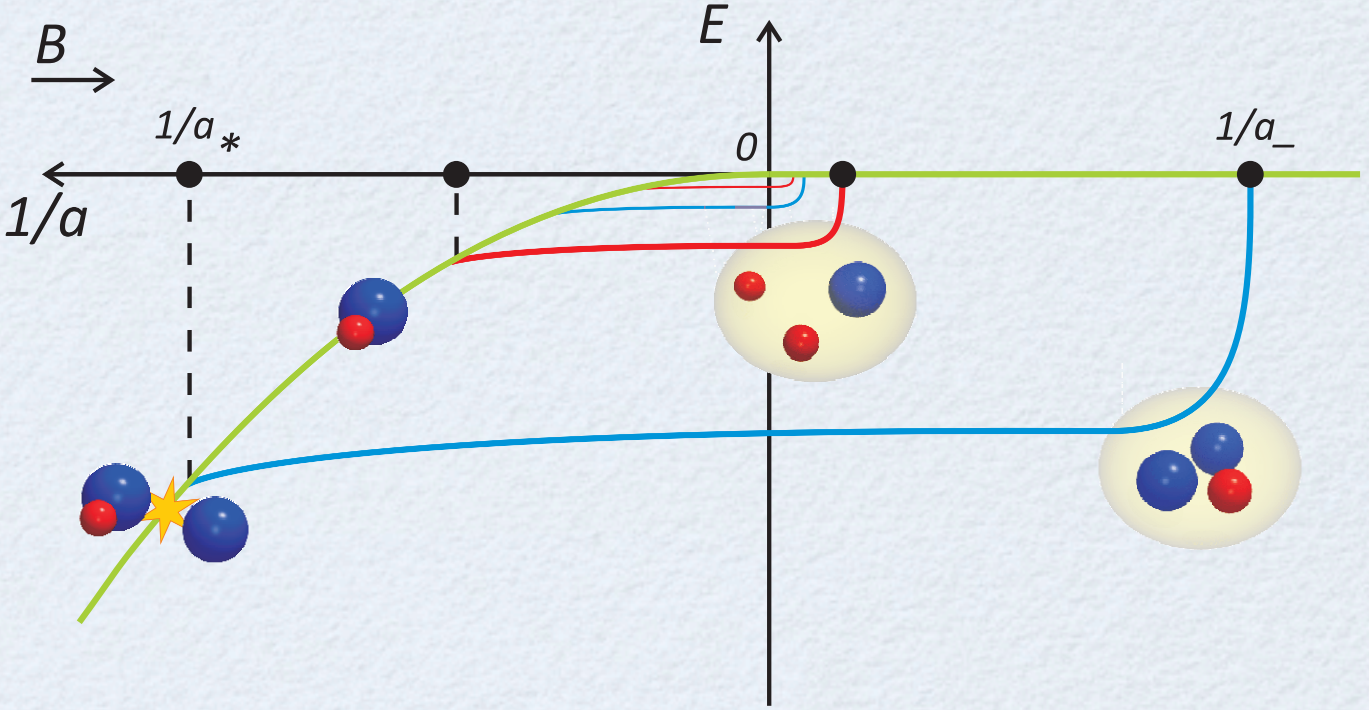


$a_-^{(n+1)}/a_-^{(n)} = a_*^{(n+1)}/a_*^{(n)} = 22.7$

$a_*^{(n+1)}/a_-^{(n)} \approx 1.06$

Innsbruck group, Nature (2006)

heteronuclear system



$\lambda_{RbRbK} = 131$

$\lambda_{KKRb} = 3.48\times10^5$

Florence group, PRL (2009)

Efimov effect exists only in 3D (2.3<d<3.8) but not in 2D or 1D ...

How about mixed dimensions ?

3-body problems in mixed D

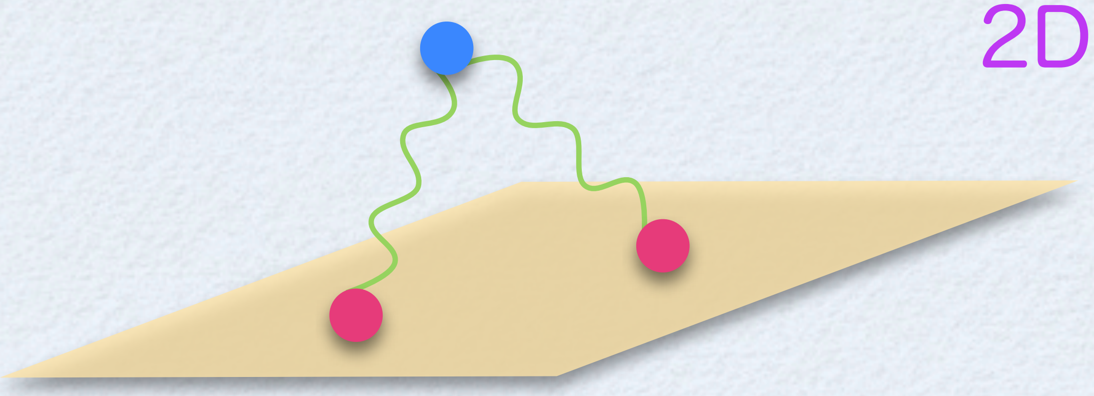
$$\left[-\sum_{i=1}^{N_A}\frac{\nabla^2_{\tilde{\boldsymbol{x}}_{A_i}}}{2m_A}-\sum_{j=1}^{N_B}\frac{\nabla^2_{\boldsymbol{x}_{B_j}}}{2m_B}+V_{a_{\mathrm{eff}}}\right]\Psi=E\Psi \qquad \text{with} \qquad N_A+N_B=3$$

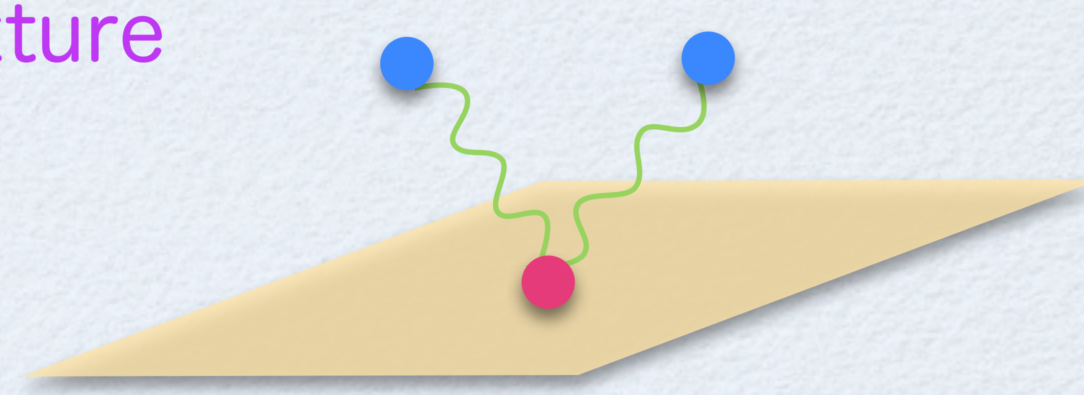
4 types of 3-body problems...

N_A=2 in 2D/1D & N_B=1 in 3D

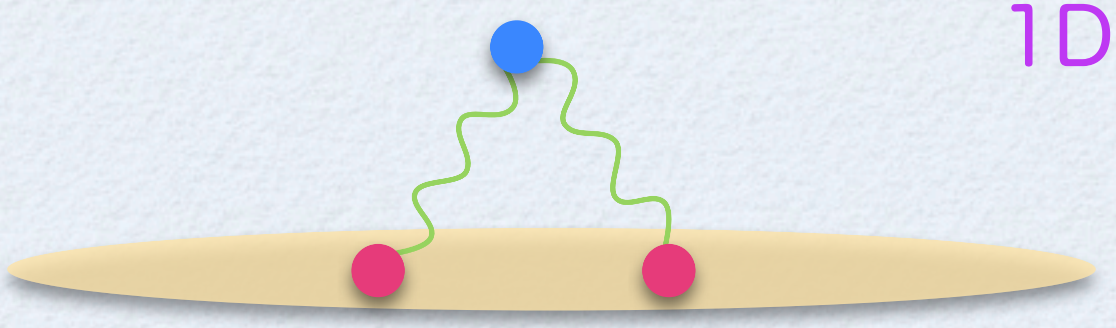
N_A=1 in 2D/1D & N_B=2 in 3D

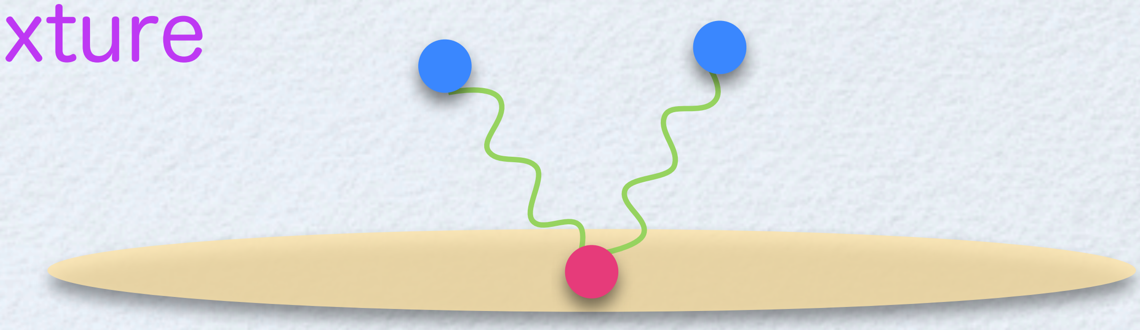
2D-3D mixture





1D-3D mixture



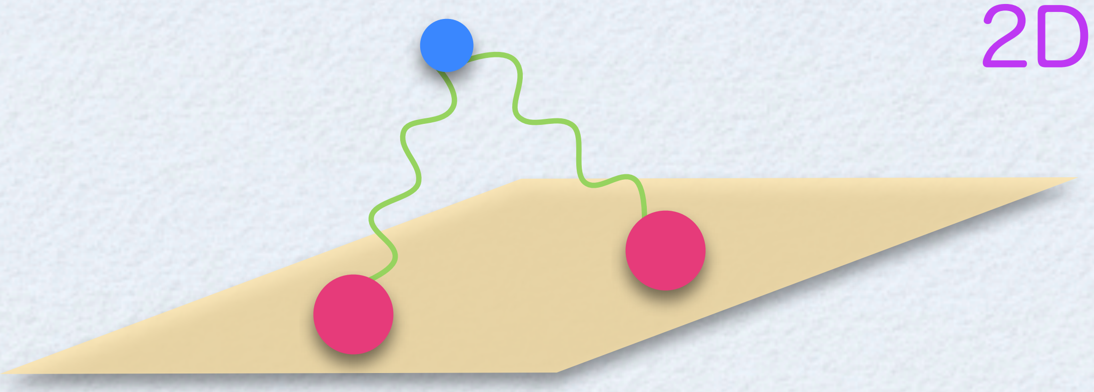


Are there Efimov trimer states in mixed dimensions? → Yes!

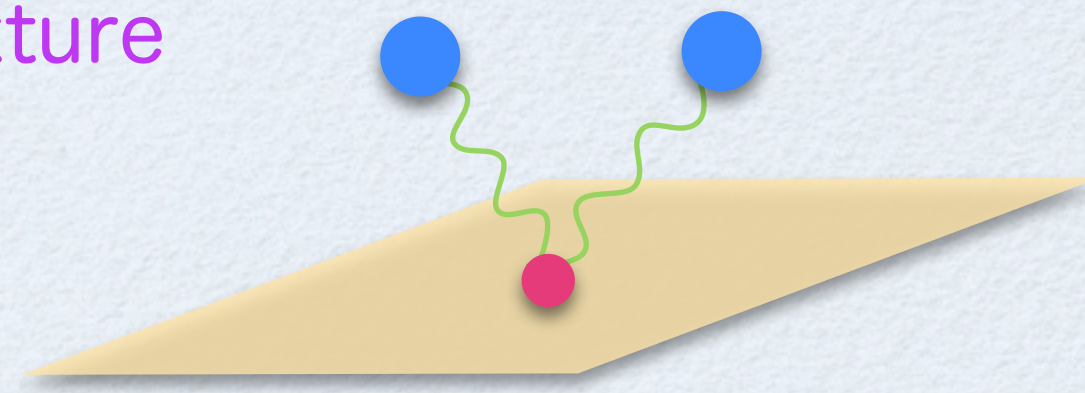
Efimov effect in mixed D

- If majority atoms are bosons, Efimov effect occurs for any m_A/m_B
- If majority atoms are fermions, Efimov effect occurs for ...

2D-3D mixture

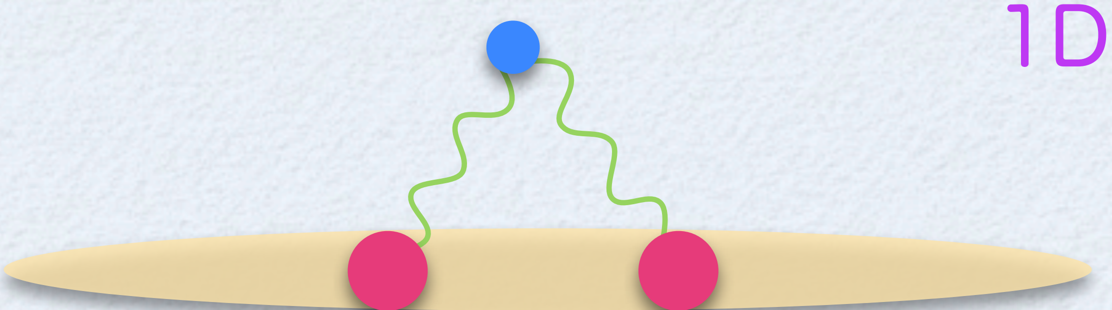


$m_A/m_B > 6.35$

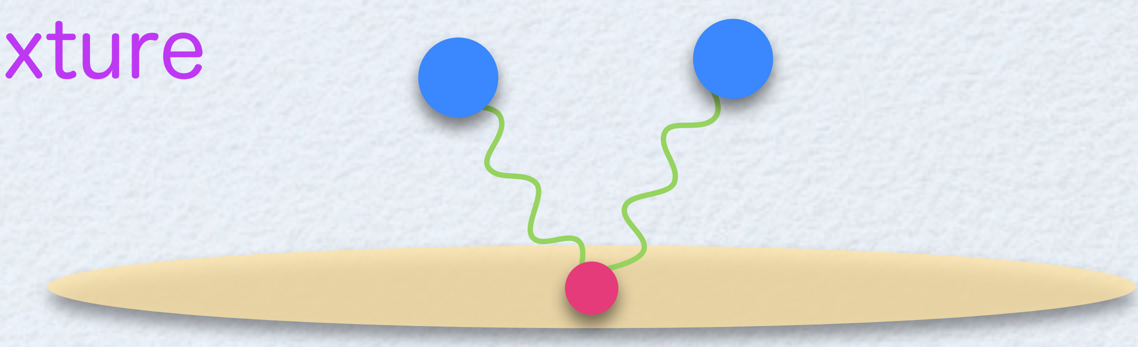


$m_A/m_B < 0.0351$

1D-3D mixture



$m_A/m_B > 2.06$



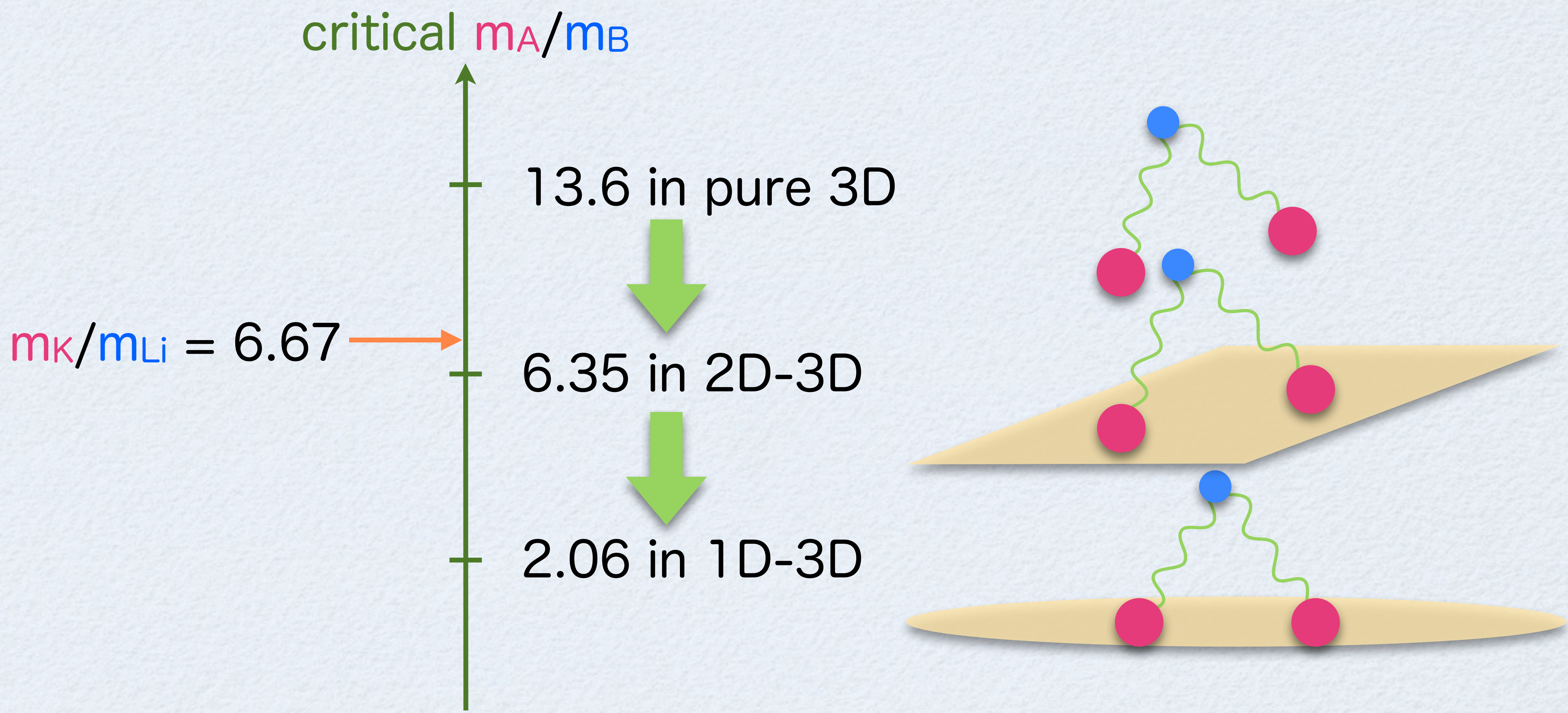
$m_A/m_B < 0.00646$

Compare those critical mass ratios with 3D values:

$m_A/m_B > 13.6$ $m_A/m_B < 0.0735$

Implication for ^{40}K-^{6}Li mixture

Interesting possibility in Fermi-Fermi mixture of A=^{40}K and B=^{6}Li when ^{40}K is confined in lower dimensions ...



Confinement induces the Efimov effect !!!

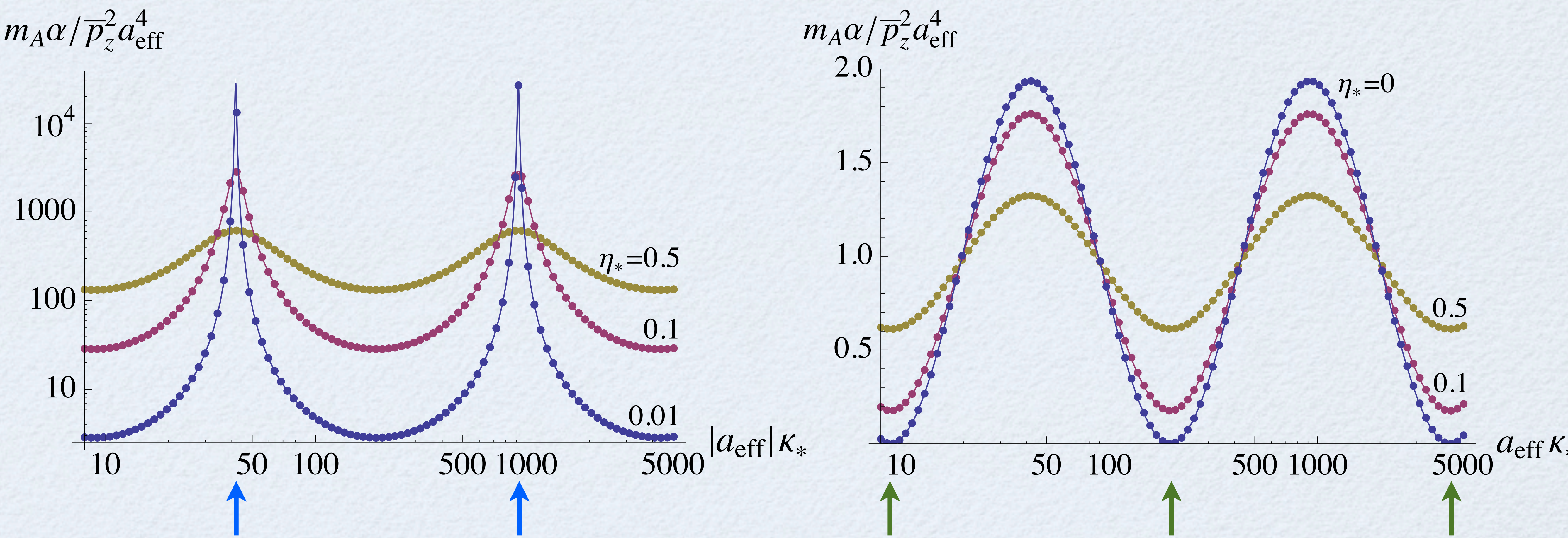
Such trimers are long lived : $\tau^{-1} \sim \frac{1}{mr_0^2}\left(\frac{r_0}{l}\right)^{4.39} \ll \varepsilon_{\mathrm{trimer}} \sim \frac{1}{ml^2}$

3-body recombination rate

3-body recombination (A+A+B->A+AB) results in atom losses

$$\dot{n}_A \approx -2\,\alpha\,n_A^2 n_B$$

Its rate constant α has the characteristic log-periodic behaviors with the scaling factor $\lambda = 22.0$ for A=^{40}K in 1D & B=^{6}Li in 3D



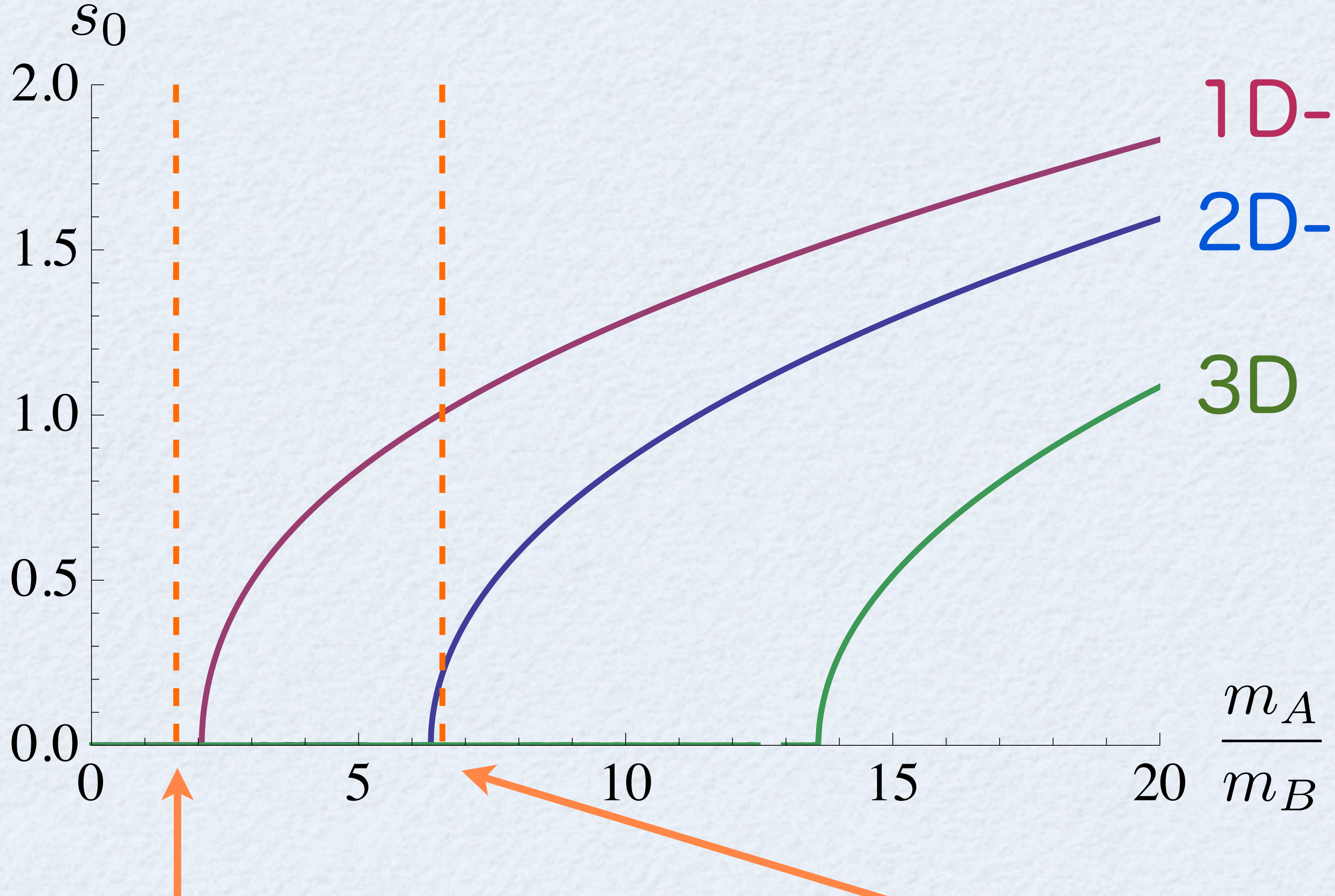
induced Efimov resonances for $a_{\rm eff}<0$ deconstructive interferences for $a_{\rm eff}>0$

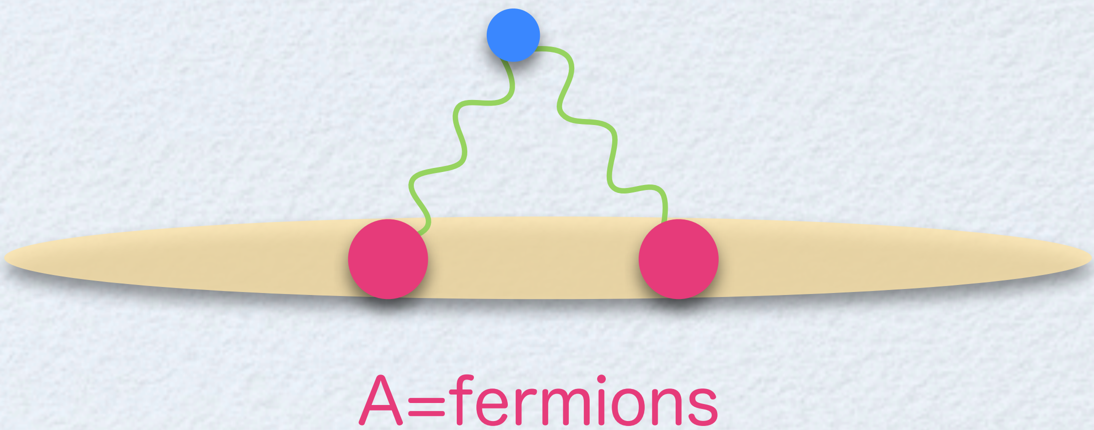
Efimov parameter $\kappa_* \approx 1.91/l$ & width parameter $\eta_* \sim (r_0/l)^{2.39} \ll 1$

If observed, the first evidence of the Efimov effect in fermions !!!

Scaling factor for fermions

Scaling factor is expressed by $\lambda = e^{\pi/s_0}$ with $E_n/E_{n+1} = \lambda^2$





For A=^{6}Li & B=^{40}K

pure 3D : no Efimov

2D-3D : no Efimov

1D-3D : no Efimov

For A=^{40}K & B=^{6}Li

pure 3D : no Efimov

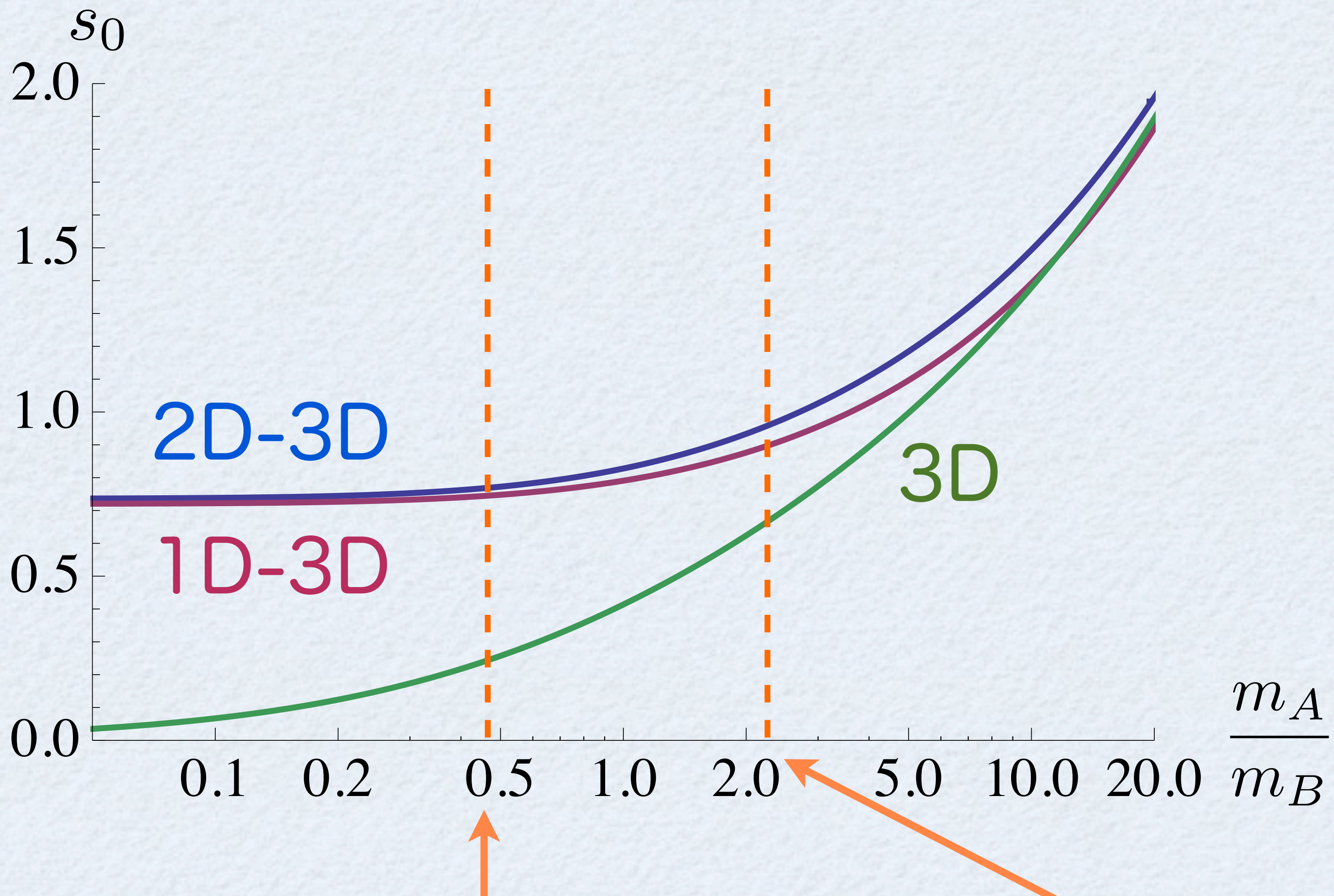
2D-3D : $\lambda = 1.78 \times 10^5$

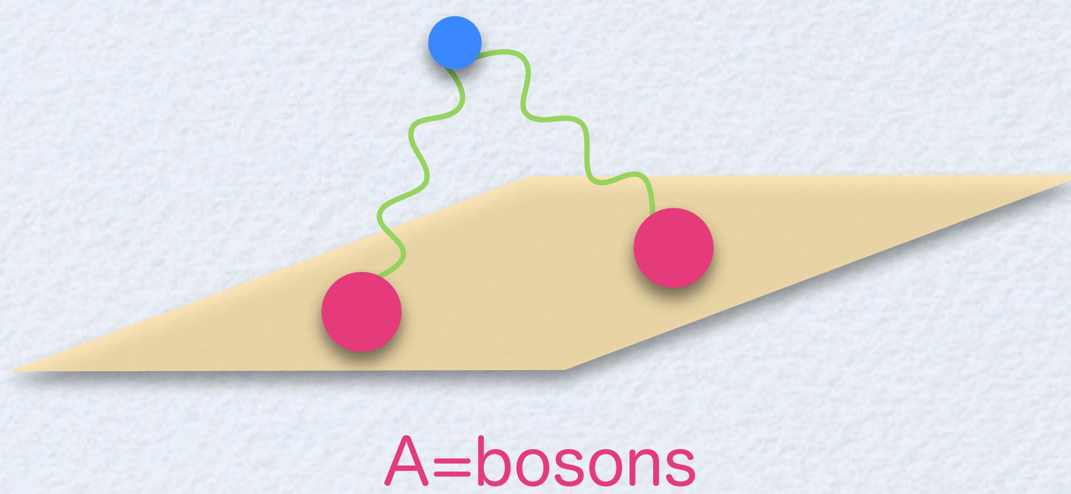
1D-3D : $\lambda = 22.0$

Confinement induces the Efimov effect !

Scaling factor for bosons

Scaling factor is expressed by $\lambda = e^{\pi/s_0}$ with $E_n/E_{n+1} = \lambda^2$





For A=^{41}K & B=^{87}Rb	For A=^{87}Rb & B=^{41}K
pure 3D : λ = 3.48×10^5	pure 3D : λ = 131
2D-3D : λ = 59.3	2D-3D : λ = 27.7
1D-3D : λ = 67.6	1D-3D : λ = 34.6

Confinement greatly reduces the scaling factor !

Table of contents

Many-body physics in mixed D

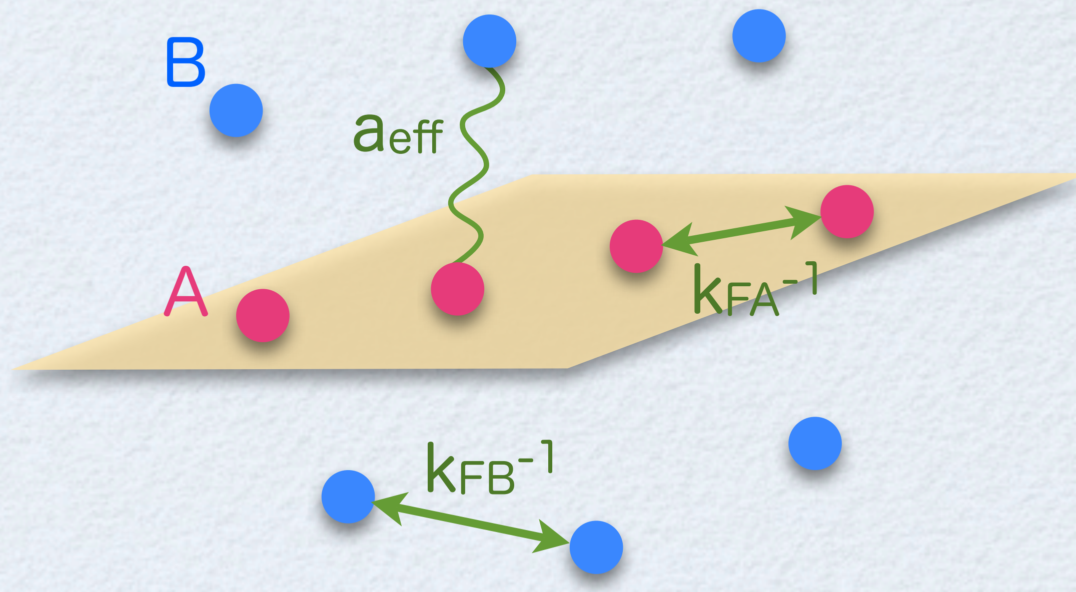
single-layer Fermi gas

Fermi gas in 2D-3D mixture

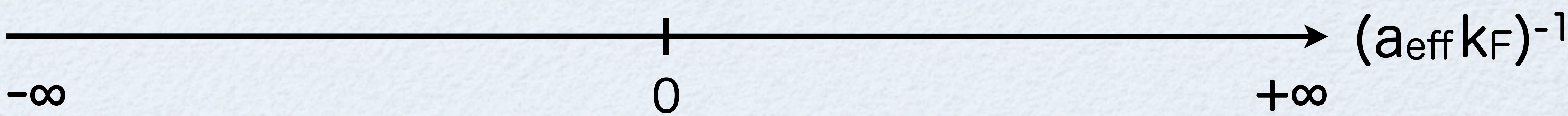
Fermi-Fermi mixture (e.g. ^{40}K and ^{6}Li) in 2D-3D mixed dimensions

parameters of the system

- a_{eff}
- $k_{FA} \equiv (4\pi n_A)^{1/2}$
- $k_{FB} \equiv (6\pi^2 n_B)^{1/3}$
- $m_A/m_B < 6.35$
- $T=0$



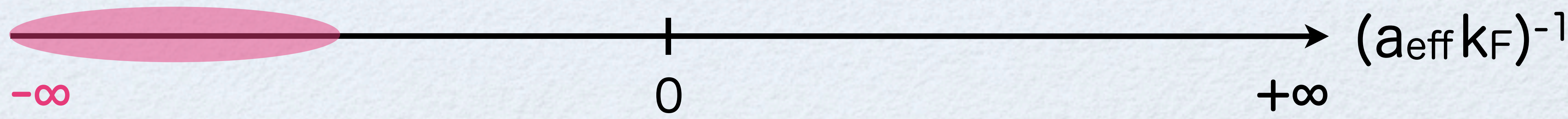
Investigate the phase diagram in terms of $(a_{\mathrm{eff}} k_F)^{-1}$ [$k_F \sim k_{FA} \sim k_{FB}$]



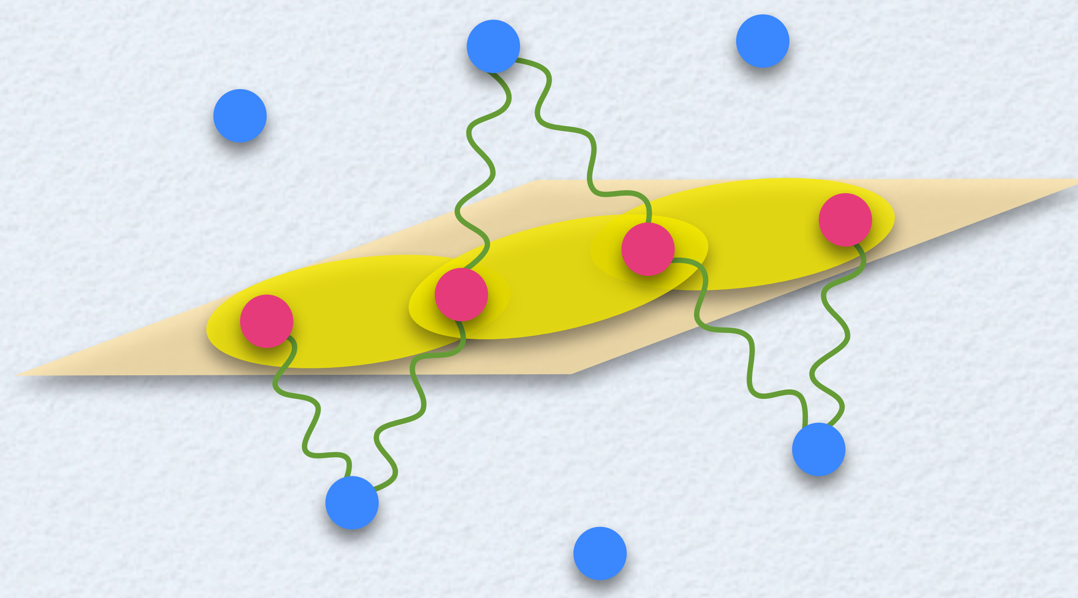
- $a_{\mathrm{eff}} k_F \to -0$: weak attraction ("BCS") limit
- $|a_{\mathrm{eff}} k_F| \to \infty$: resonant (unitarity) limit with scale inv.
- $a_{\mathrm{eff}} k_F \to +0$: strong attraction ("BEC") limit

  with AB dimer $E_{\mathrm{dimer}} = -\dfrac{1}{2m_{AB}a_{\mathrm{eff}}^2}$

Weak attraction (BCS) limit



- AB pairing does not take place due to the mismatch of 2D and 3D Fermi surfaces
- Instead B atoms in 3D induce an effective attraction between A atoms in 2D

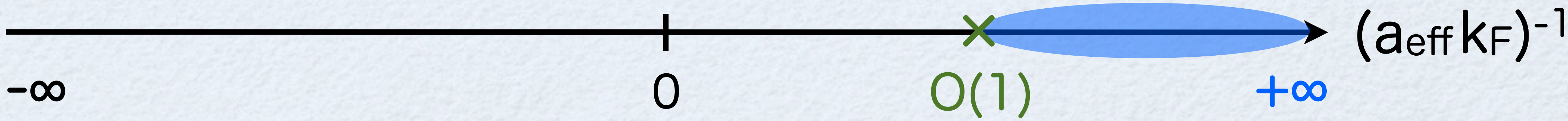


$$V_{\rm ind}(r) = -\frac{a_{\rm eff}^2}{m_{AB}} \frac{\sin(2k_{{\rm F}B} r) - 2k_{{\rm F}B} r \cos(2k_{{\rm F}B} r)}{4\pi r^4} + O(a_{\rm eff}^3)$$

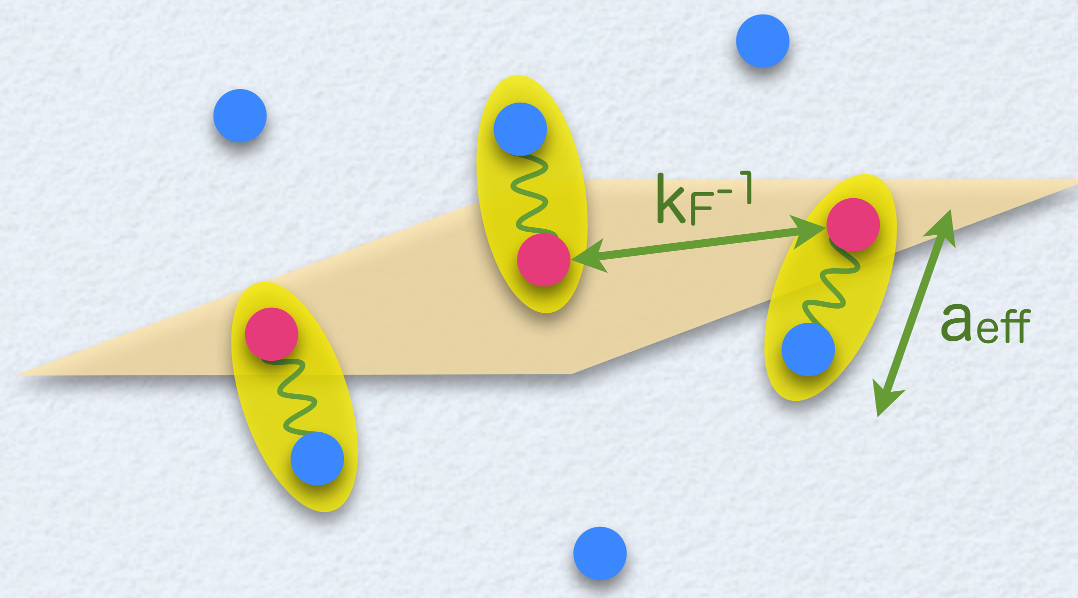
2D p-wave pairing between A atoms $\frac{\Delta(\boldsymbol{p})}{\varepsilon_{\rm FA}} \propto (\hat{p}_x + i\hat{p}_y)\, e^{-\#/(a_{\rm eff} k_{\rm FB})^2}$

(Cf. If B=bosons, $V_{\rm ind}(r) = -\frac{4\pi n_B a_{\rm eff}^2}{m_{AB}} \frac{e^{-\sqrt{2}r/\xi_B}}{r}$ with $\xi_B = \frac{1}{\sqrt{8\pi a_B n_B}}$)

Strong attraction (BEC) limit



- A atoms in 2D capture B atoms from 3D to form dimers



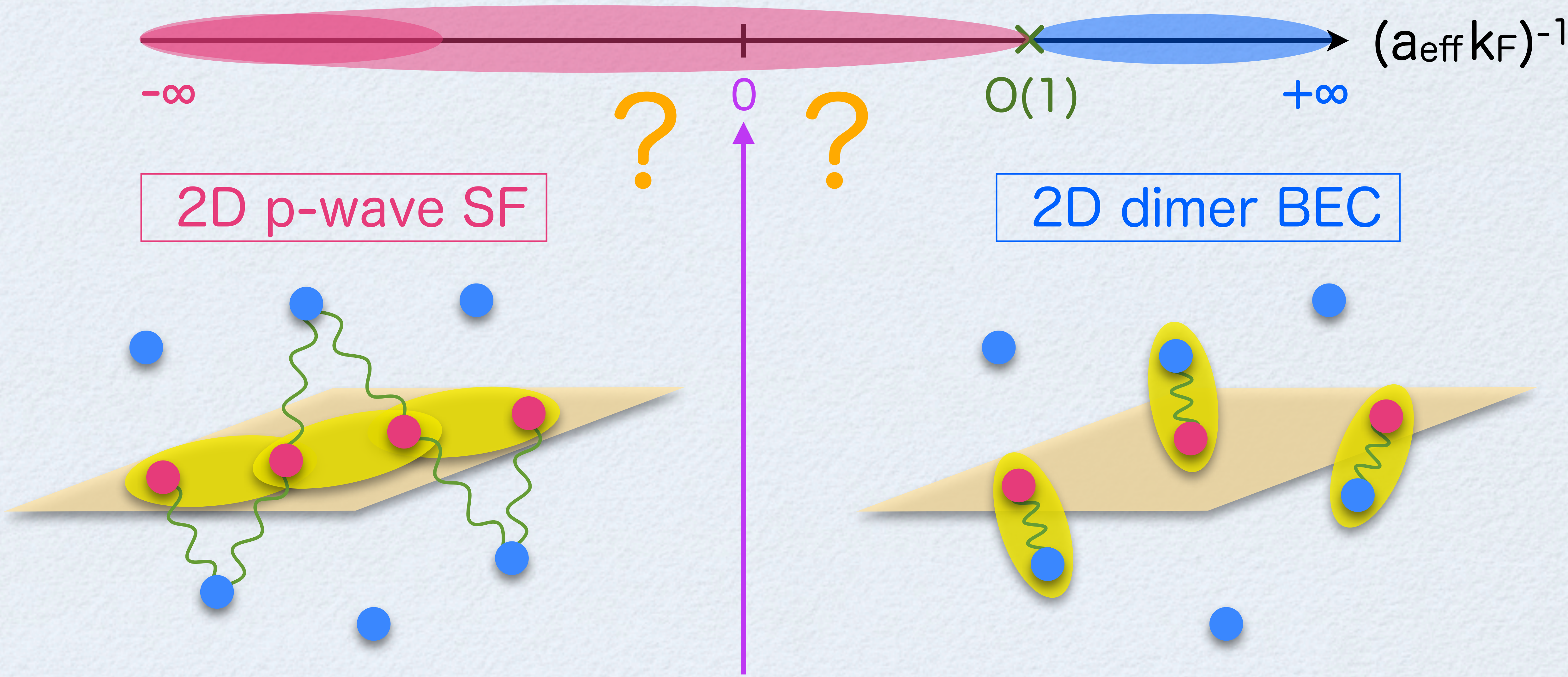
- When $a_{\rm eff} k_F < O(1)$, dimer size < mean distance

Dimer BEC in 2D

$$T_{\rm BKT} \to \frac{2\pi n_d}{M} \ln^{-1}\left(-\frac{380}{4\pi} \ln n_d a_{\rm eff}^2\right)$$

N. Prokof’ev, et al., PRL (01), PRA (02)

Possible phase diagram



If $p_x + ip_y$ pairing extends to the unitarity limit : $a_{\mathrm{eff}} k_F \to \infty$

$$\frac{\Delta(\boldsymbol{p})}{\varepsilon_{\mathrm{F}A}} \propto (\hat{p}_x + i\hat{p}_y)\, e^{-\#/(a_{\mathrm{eff}} k_{\mathrm{F}B})^2} \to (\hat{p}_x + i\hat{p}_y) \times O(1)$$

- Majorana fermions @ vortices
- Non-Abelian statistics
- Topological quantum computation ...

N.Read & D.Green, PRB (2000)
A.Y.Kitaev, AnnPhys (2003)

Many-body physics in mixed D

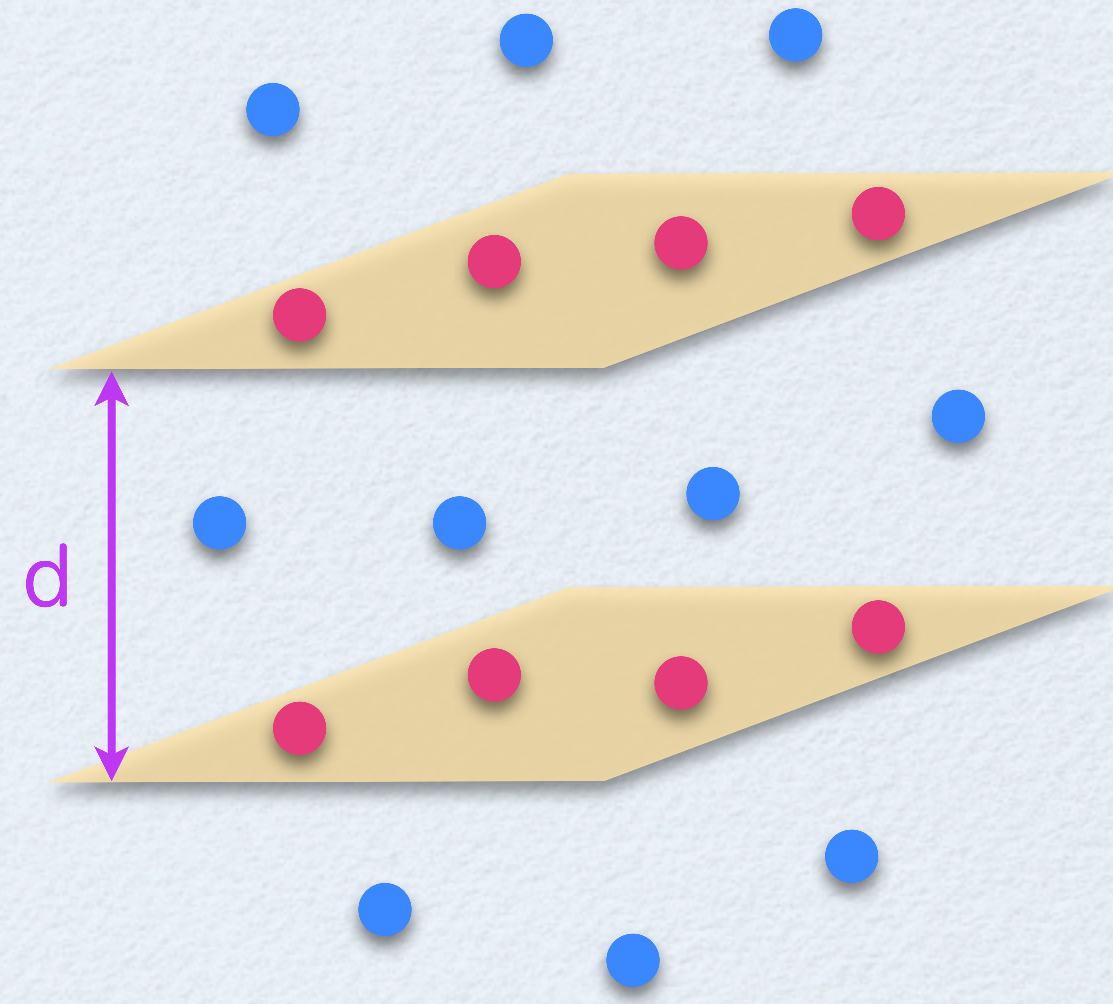
double-layer Fermi gas

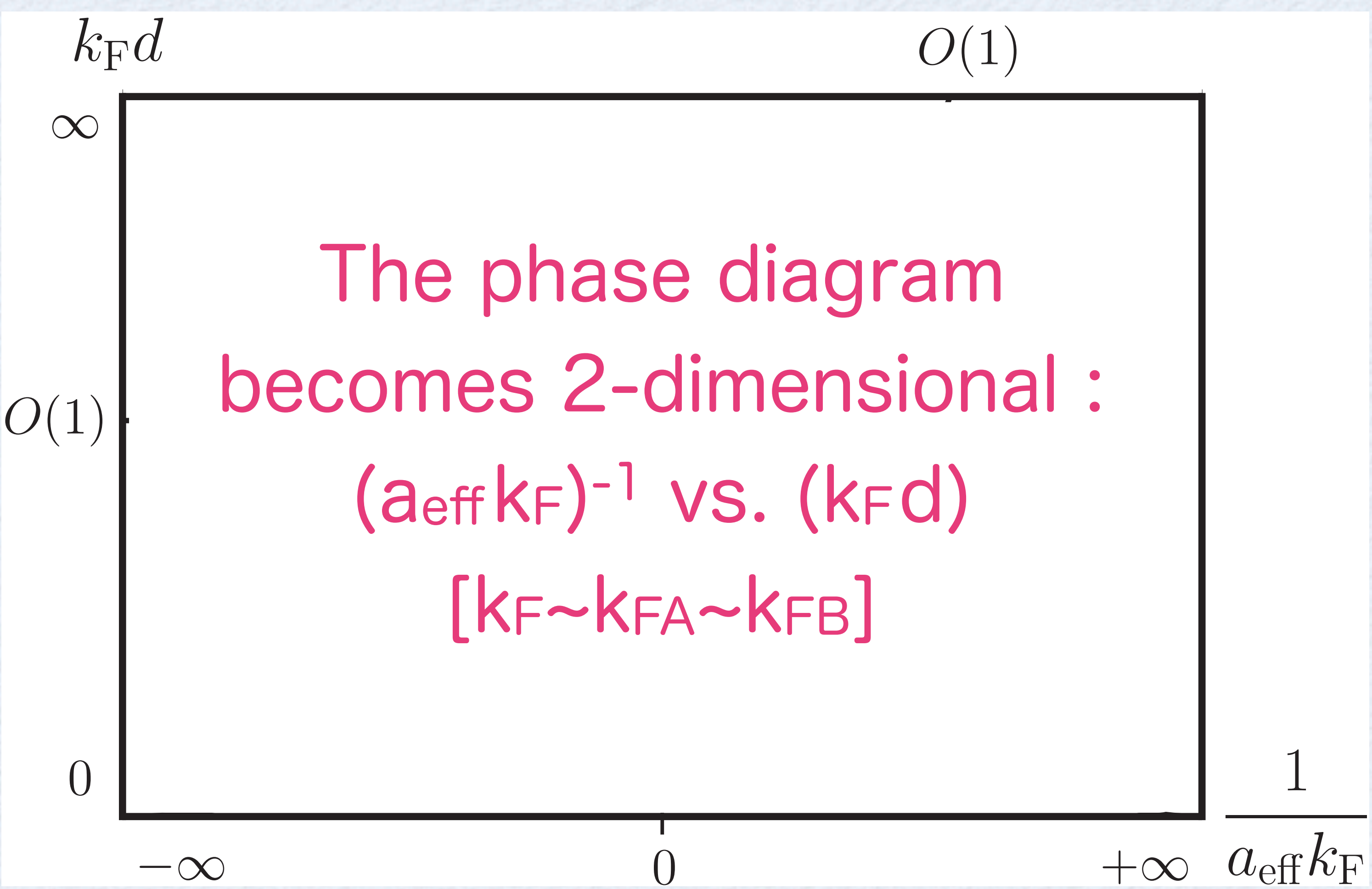
Bilayer Fermi gas

Optical lattice creates many layers ...

parameters of the system

- a_{eff}
- $k_{FA} \equiv (4\pi n_A)^{1/2}$
- $k_{FB} \equiv (6\pi^2 n_B)^{1/3}$
- $m_A/m_B < 6.35$
- T=0
- d





Interlayer correlation induced by B atoms would lead to rich physics

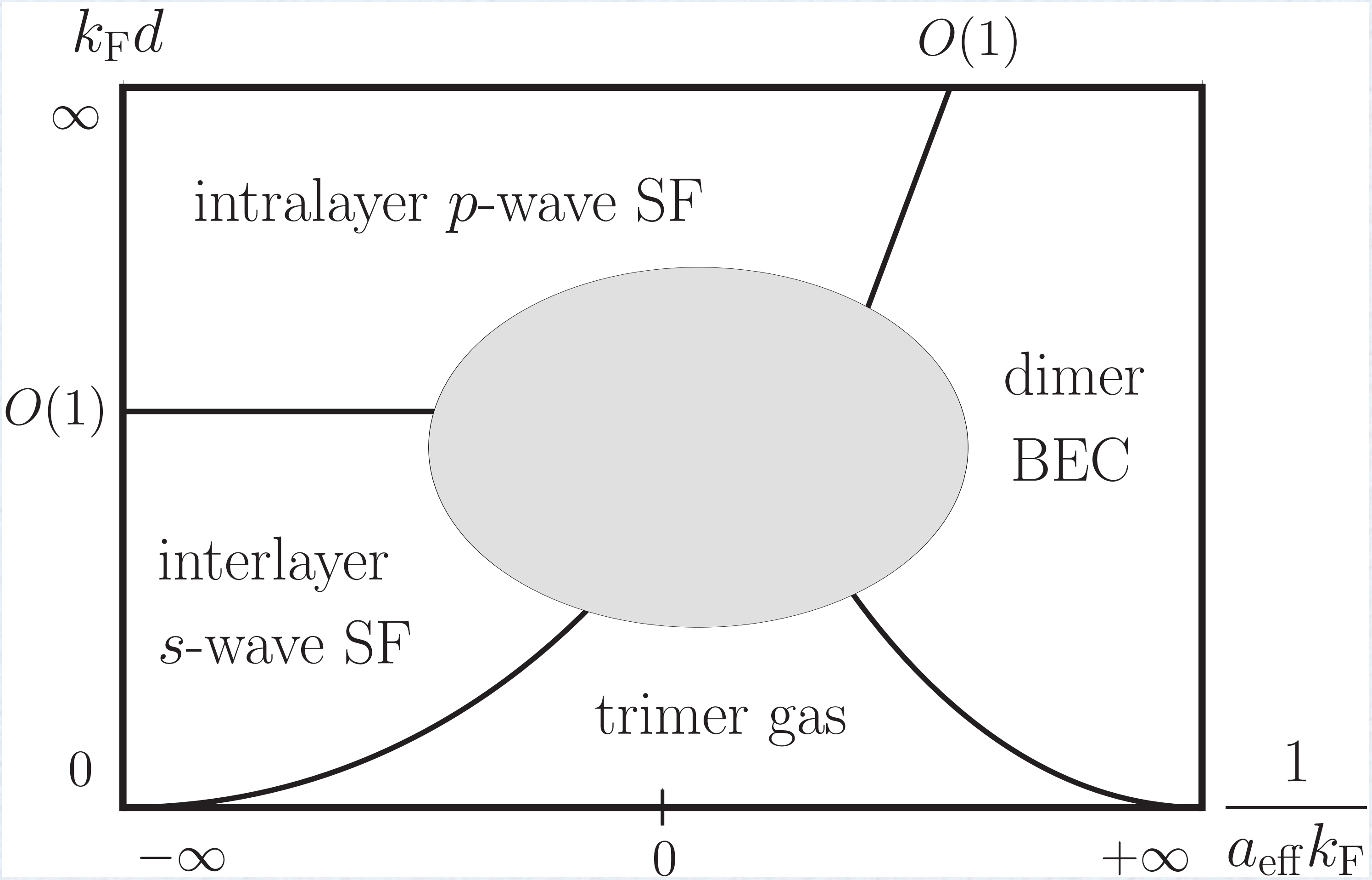
(even without tunneling)

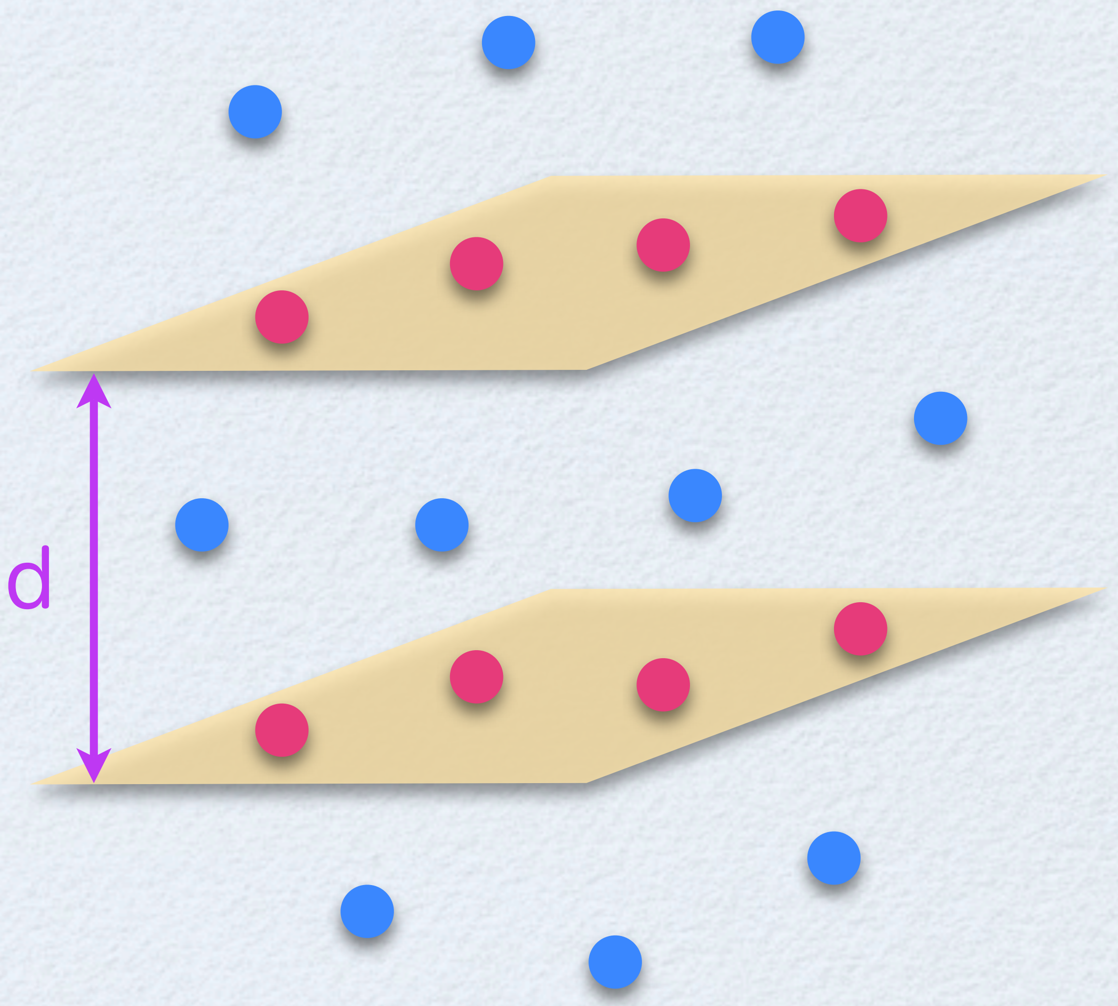
Bilayer Fermi gas

Optical lattice creates many layers ...

parameters of the system

- a_{eff}
- $k_{FA} \equiv (4\pi n_A)^{1/2}$
- $k_{FB} \equiv (6\pi^2 n_B)^{1/3}$
- $m_A/m_B < 6.35$
- T=0
- d



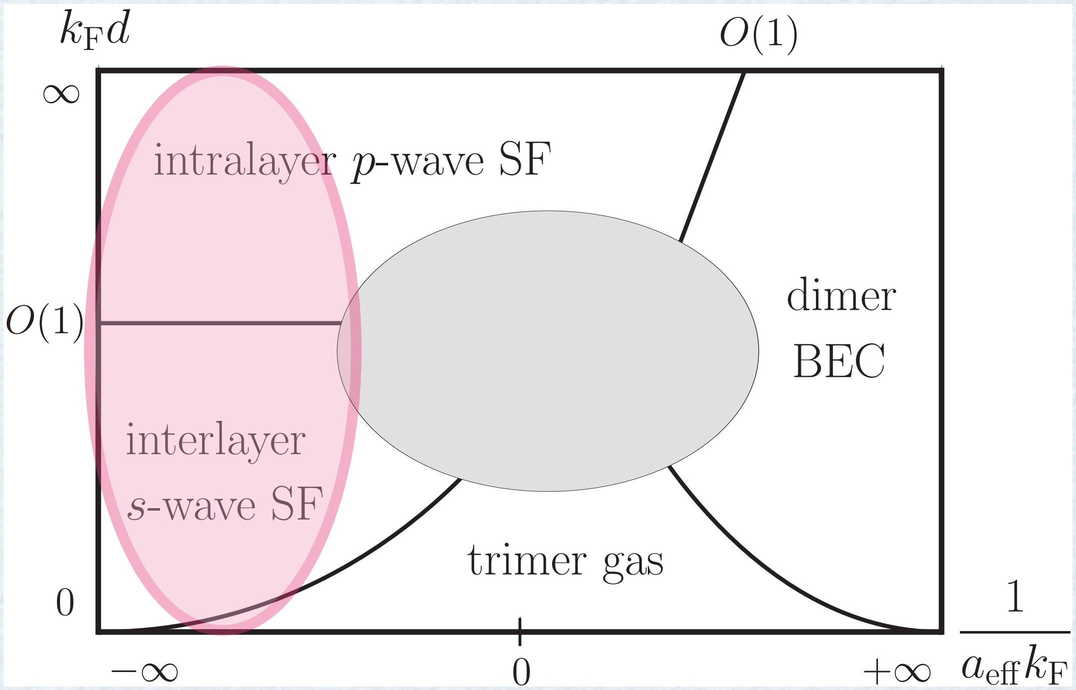


Interlayer correlation induced by B atoms would lead to rich physics

(even without tunneling)

Weak attraction (BCS) limit

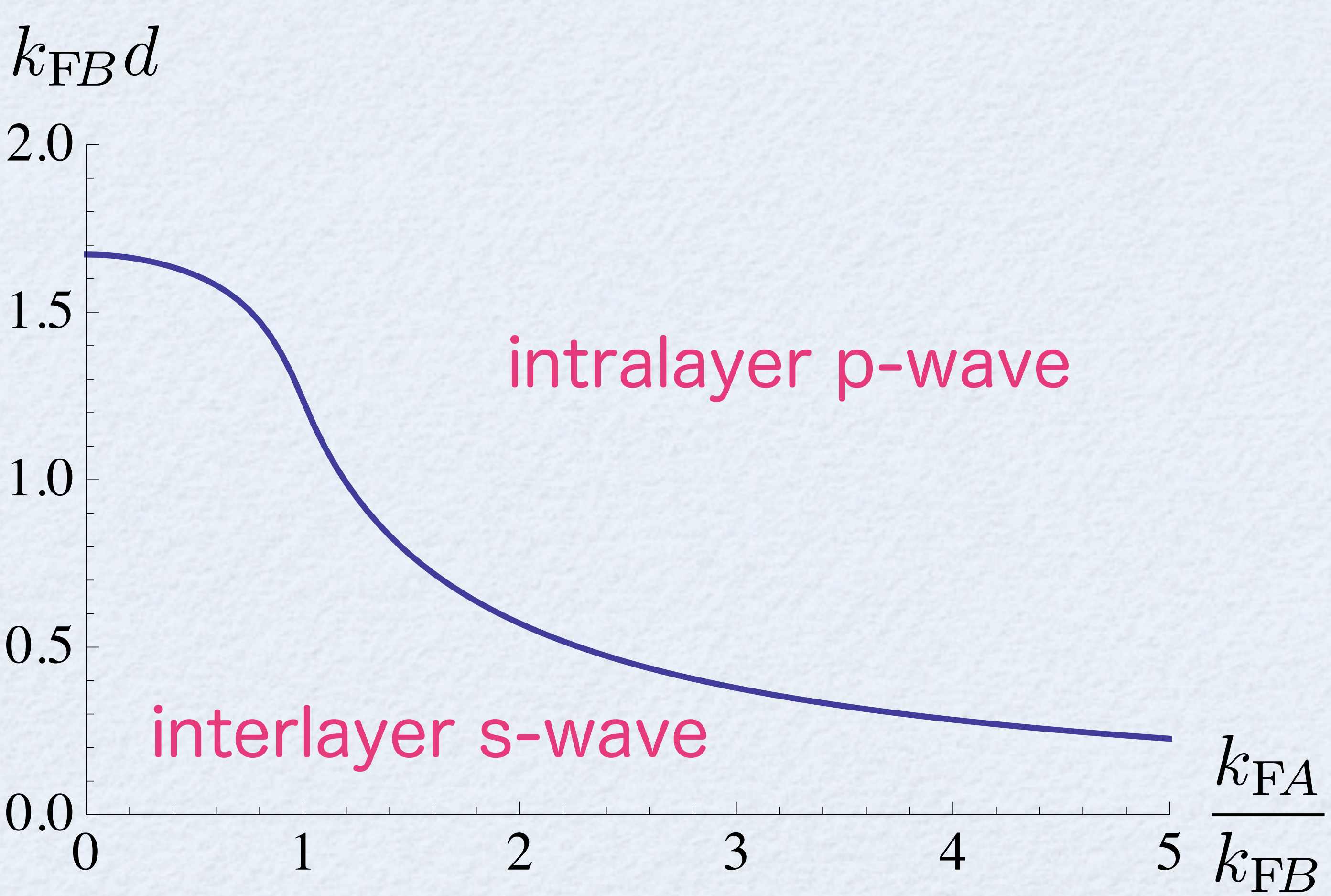
- B atoms in 3D induce $V_{ind}(r)$ between A atoms in 2D

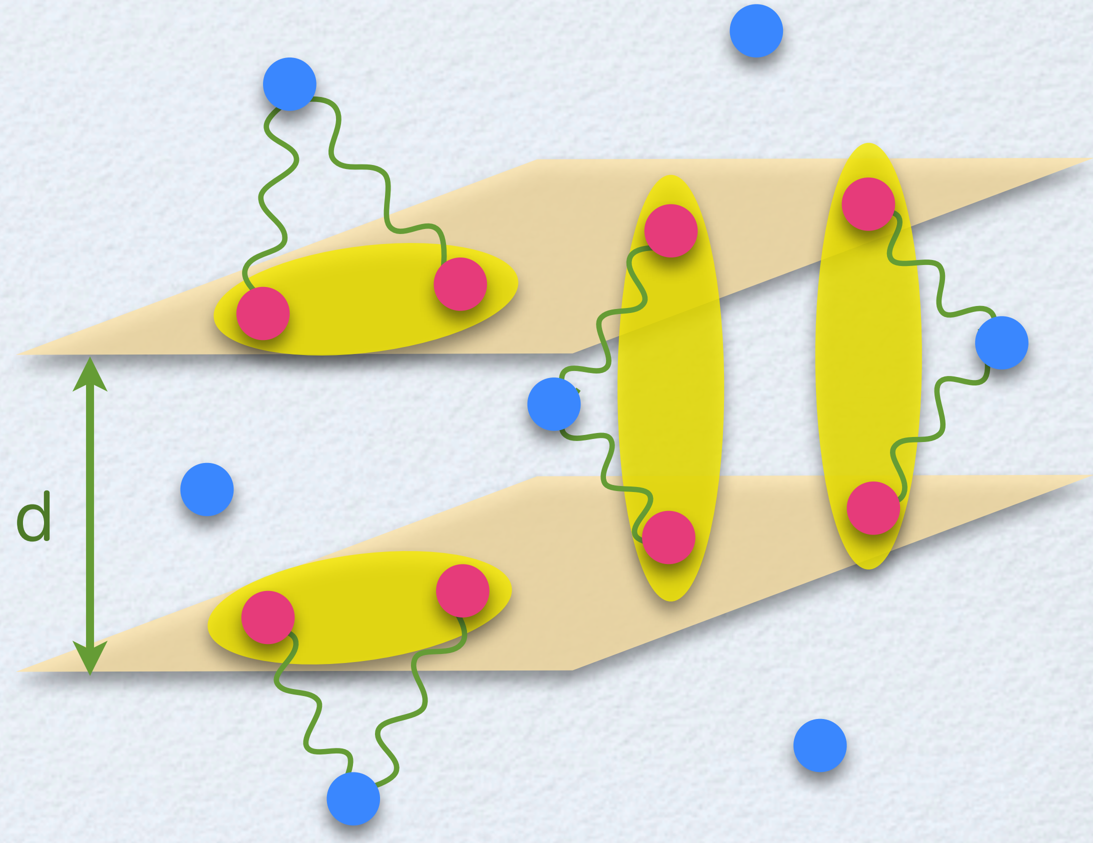


→ P-wave pairing of A atoms in the same layer for large d

or

→ S-wave pairing of A atoms in different layers for small d



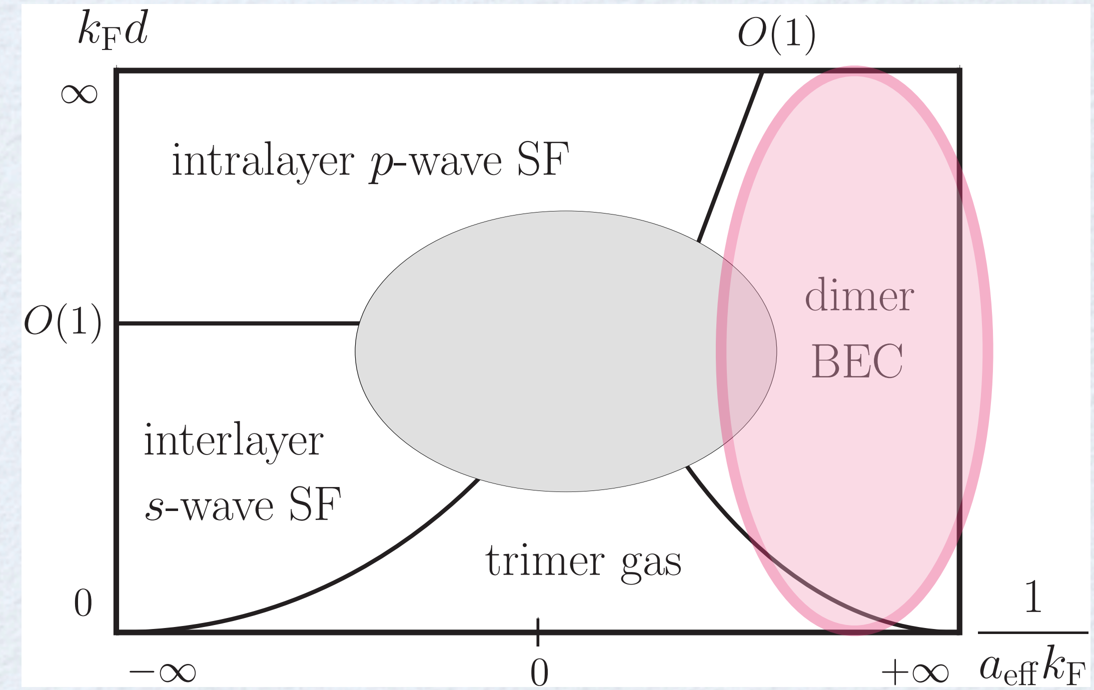


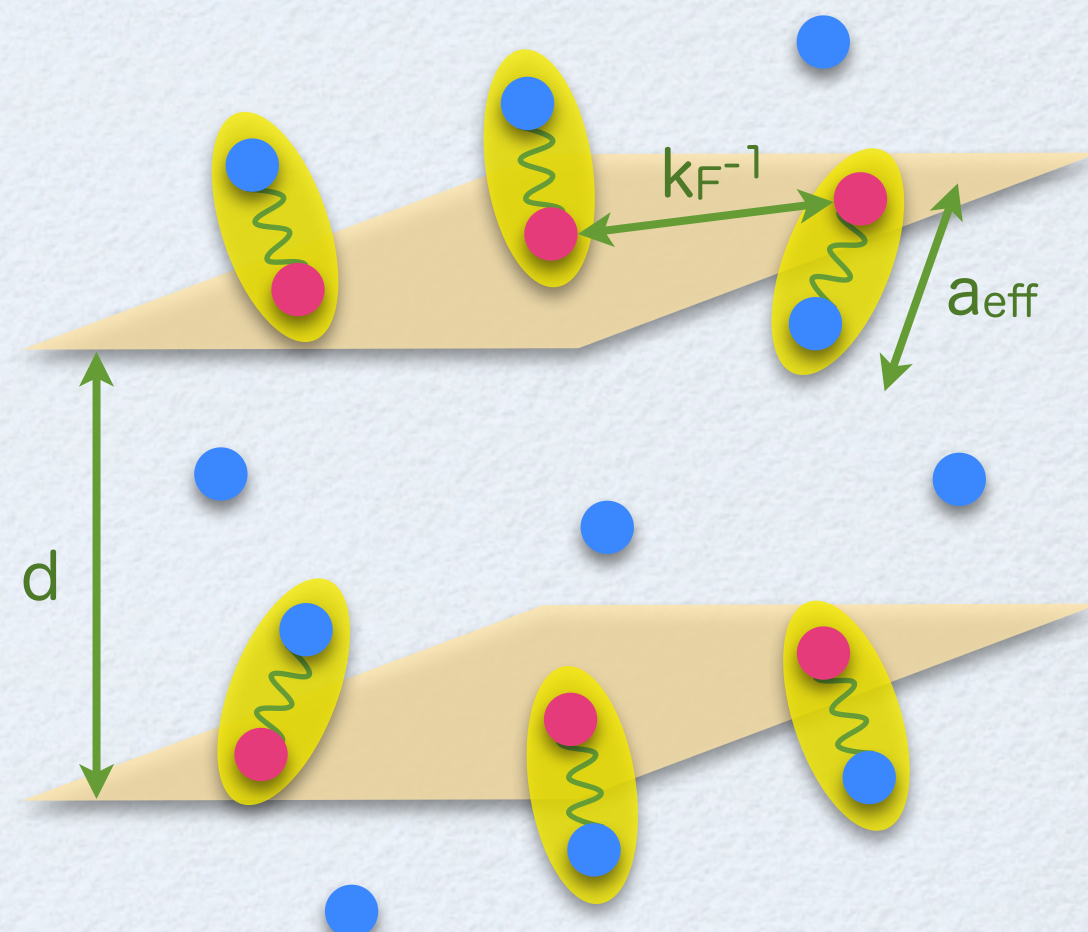
Strong attraction (BEC) limit

- A atoms in 2D capture B atoms from 3D to form dimers
- When $a_{\text{eff}} k_F < O(1)$ & $a_{\text{eff}} < d$, dimer size < mean distance

Dimer BEC in each 2D layer

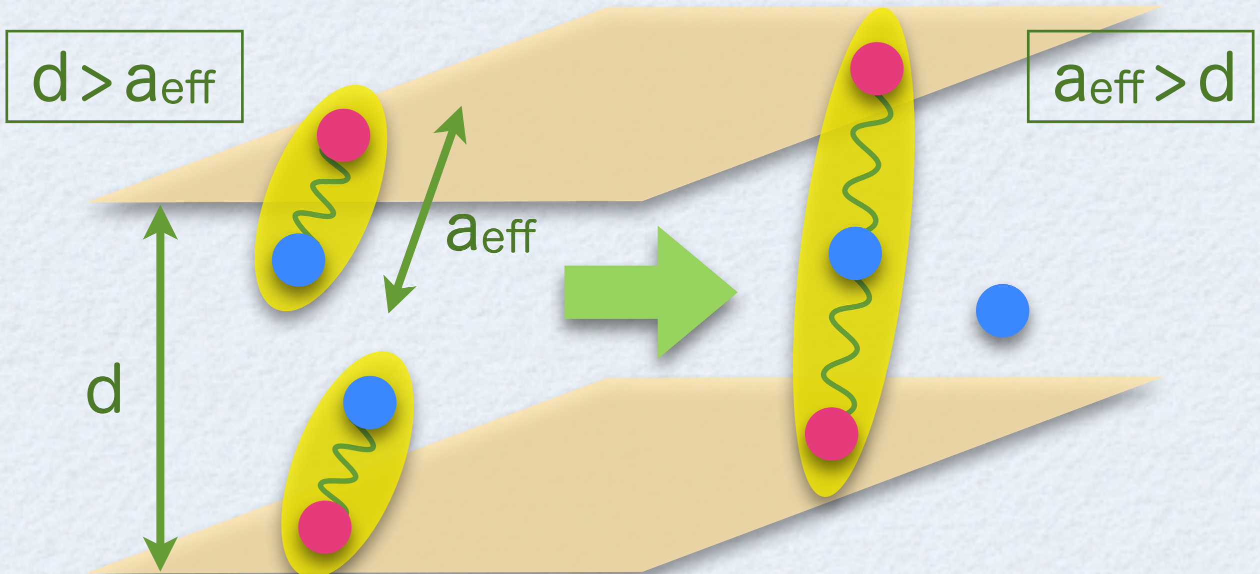
- 2 layered BECs are coupled via the residual B atoms in 3D

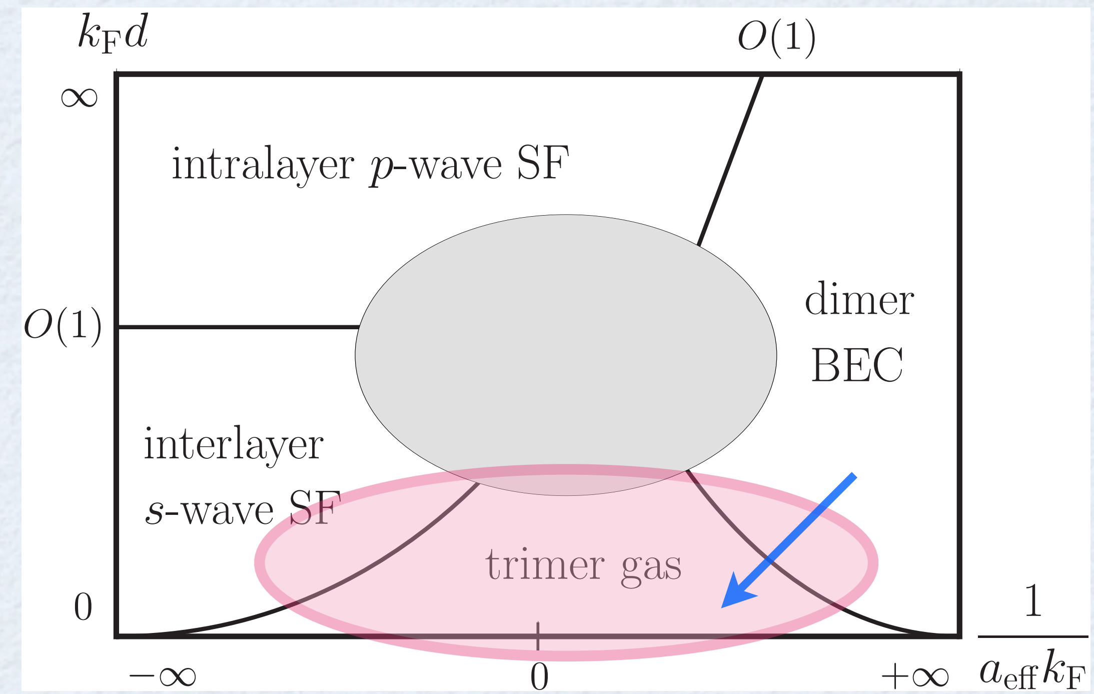




Interlayer trimer formation

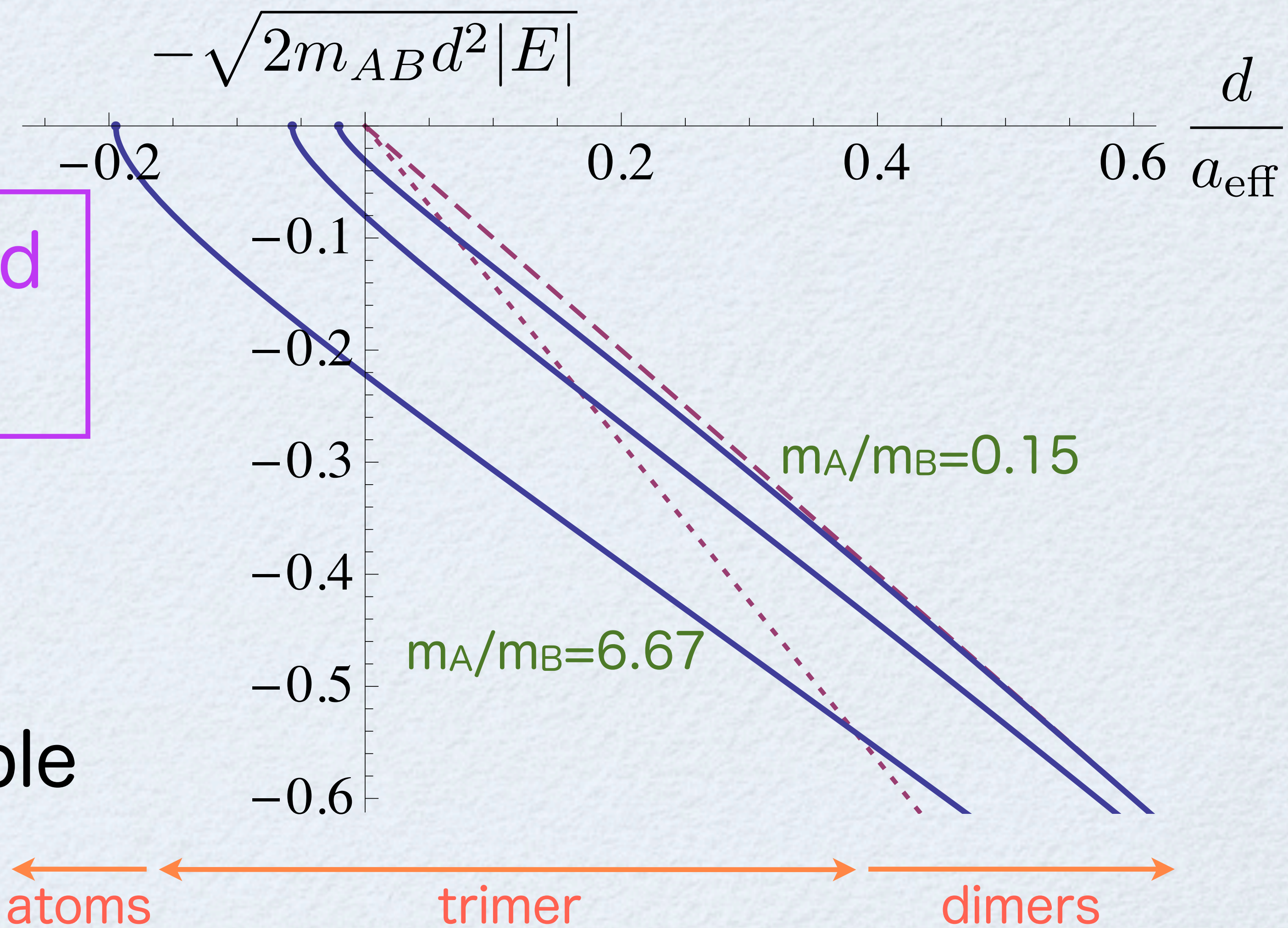
- When 2 dimers in different layers overlap "$a_{eff}>d$", they form a trimer!





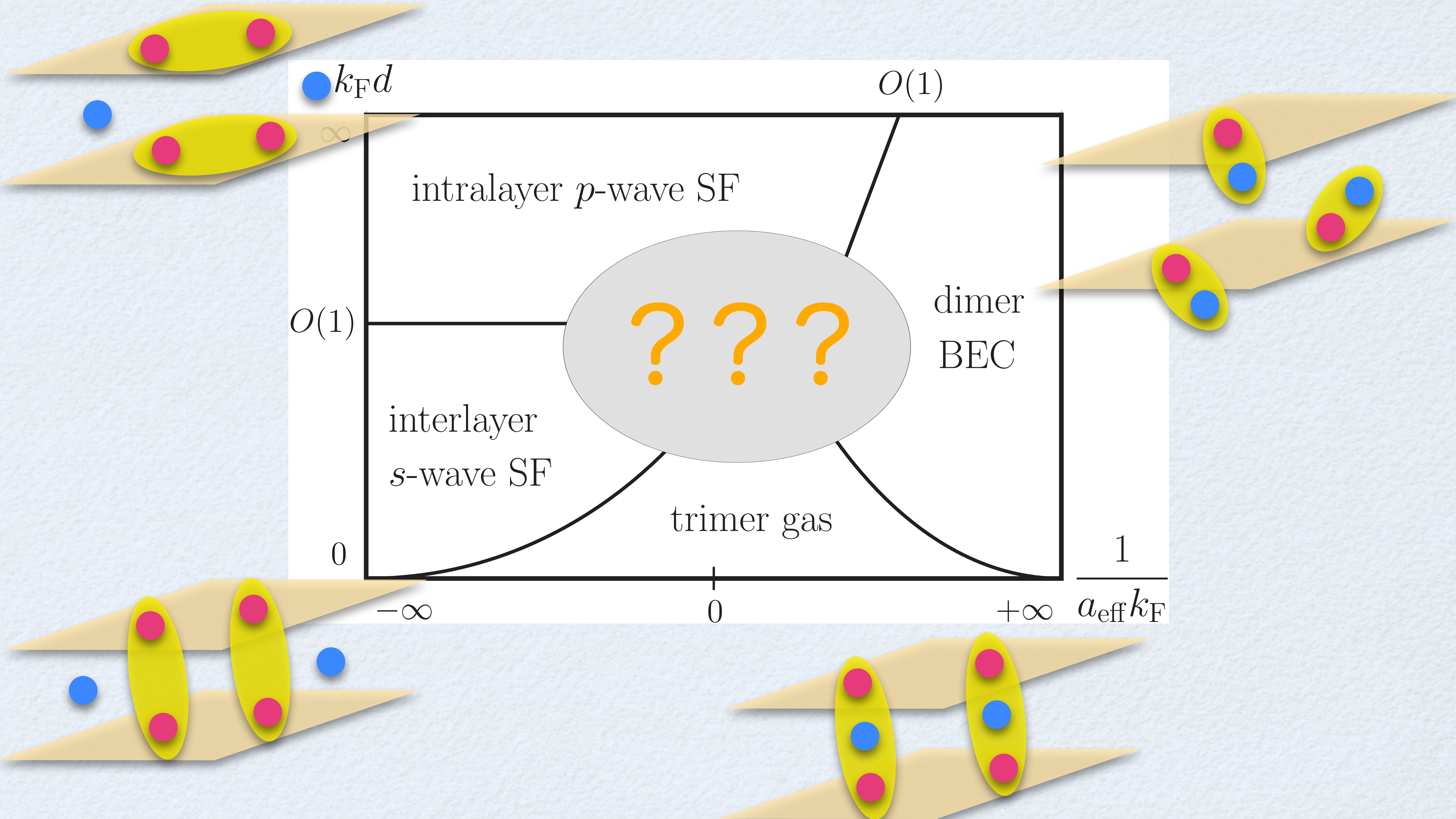
Interlayer AAB trimer is formed for "$O(-1)<d/a_{eff}<O(1)$"

- A single trimer state is stable!
  (as far as tunneling is negligible)
- RF spectroscopy will be possible
- Efimov spectrum

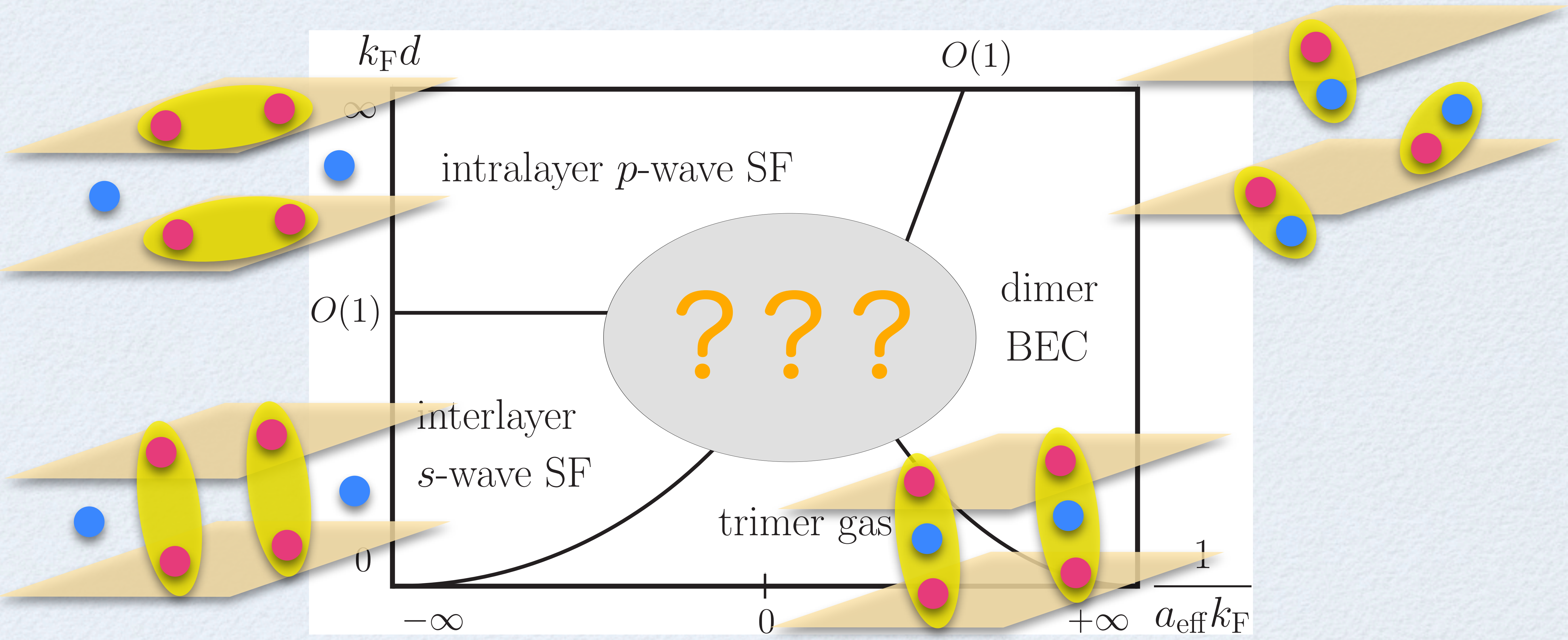


Phases of bilayer Fermi gas

Very rich but "minimal" phase diagram!!!



Possible many-body approaches



- mean-field approximation
- ε expansion & large-N expansion
- Monte-Carlo simulation
- experiment using ^{40}K-^{6}Li mixture

Summary

Mixed dimensions = New arena of universal physics !

- confinement-induced 2-body resonances (observed)
- 3-body (Efimov) resonances, critical mass ratio for fermions
- rich phase diagram including dimer BEC, trimer Fermi gas, intralayer p-wave & interlayer s-wave superfluidity, ...

Idea of mixed dimensions can be extended to Bose-Fermi mixtures, Bose-Bose mixtures, multilayer geometry, multiwire geometry

→ New interesting research direction !

References

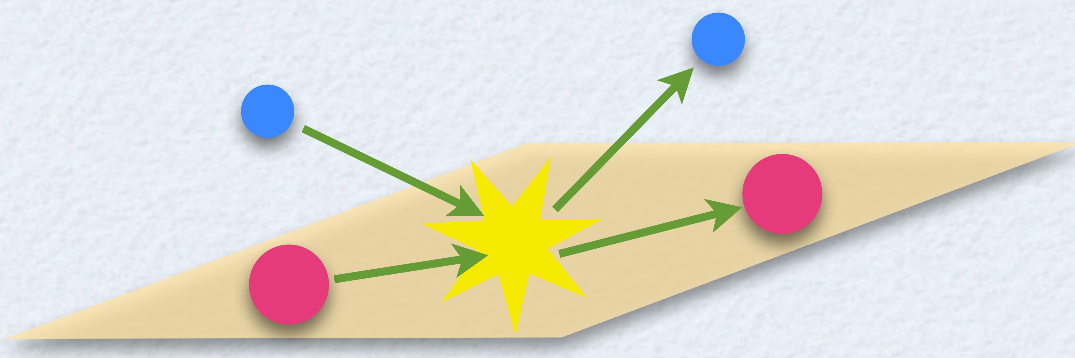
Mixed dimensions : Y.N. & S.Tan, PRL 101, 170401 (2008)
Efimov effect : Y.N. & S.Tan, PRA 79, 060701(R) (2009)
P-wave superfluidity : Y.N., Ann. Phys. 324, 897 (2009)
Bilayer Fermi gas : Y.N., arXiv:0906.4584

Backup slides

Resonances in mixed D $(m_A/m_B=6.67)$

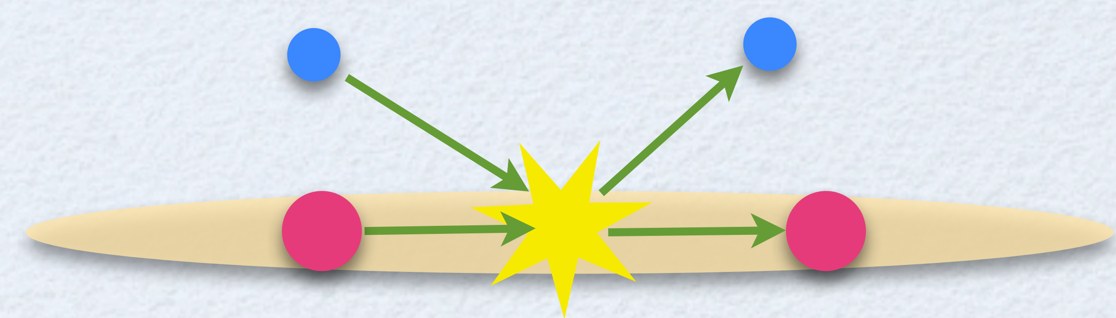
2D-3D mixture

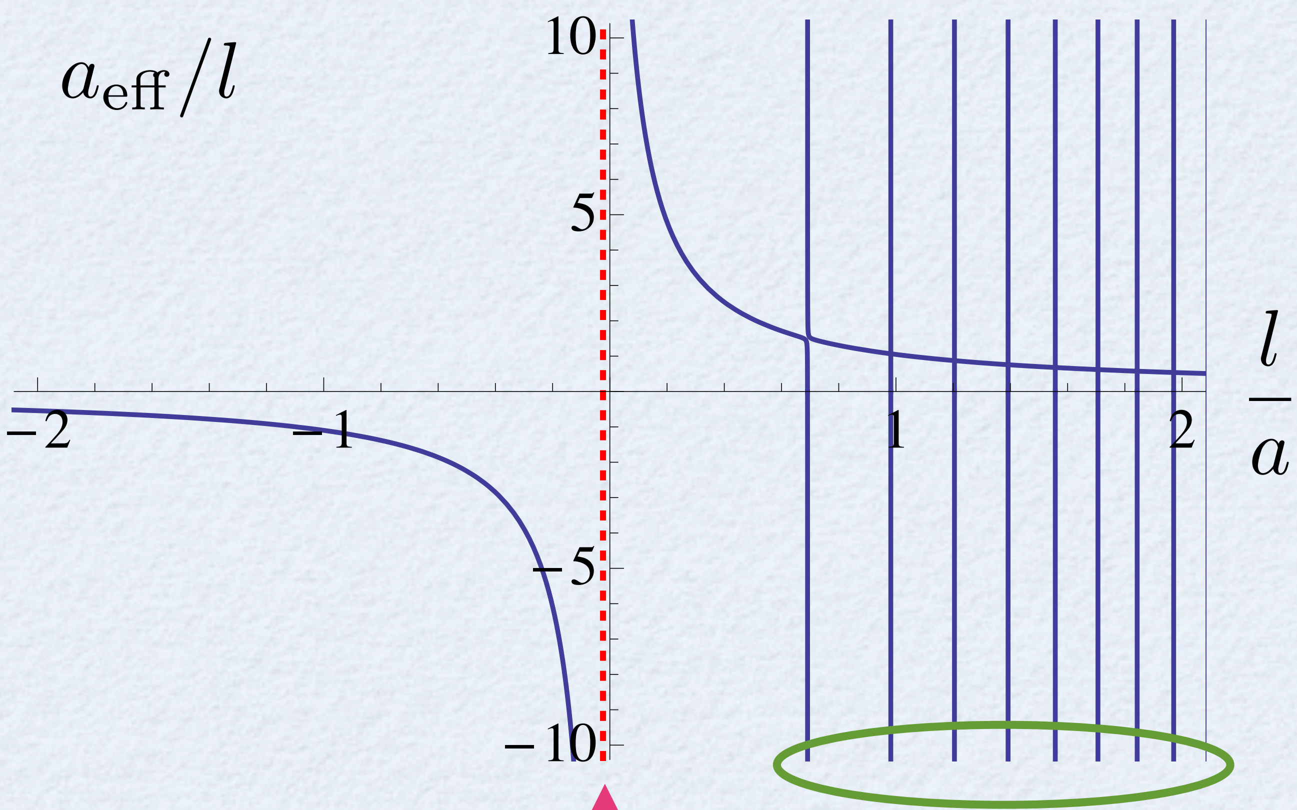
A=^{40}K in 2D & B=^{6}Li in 3D

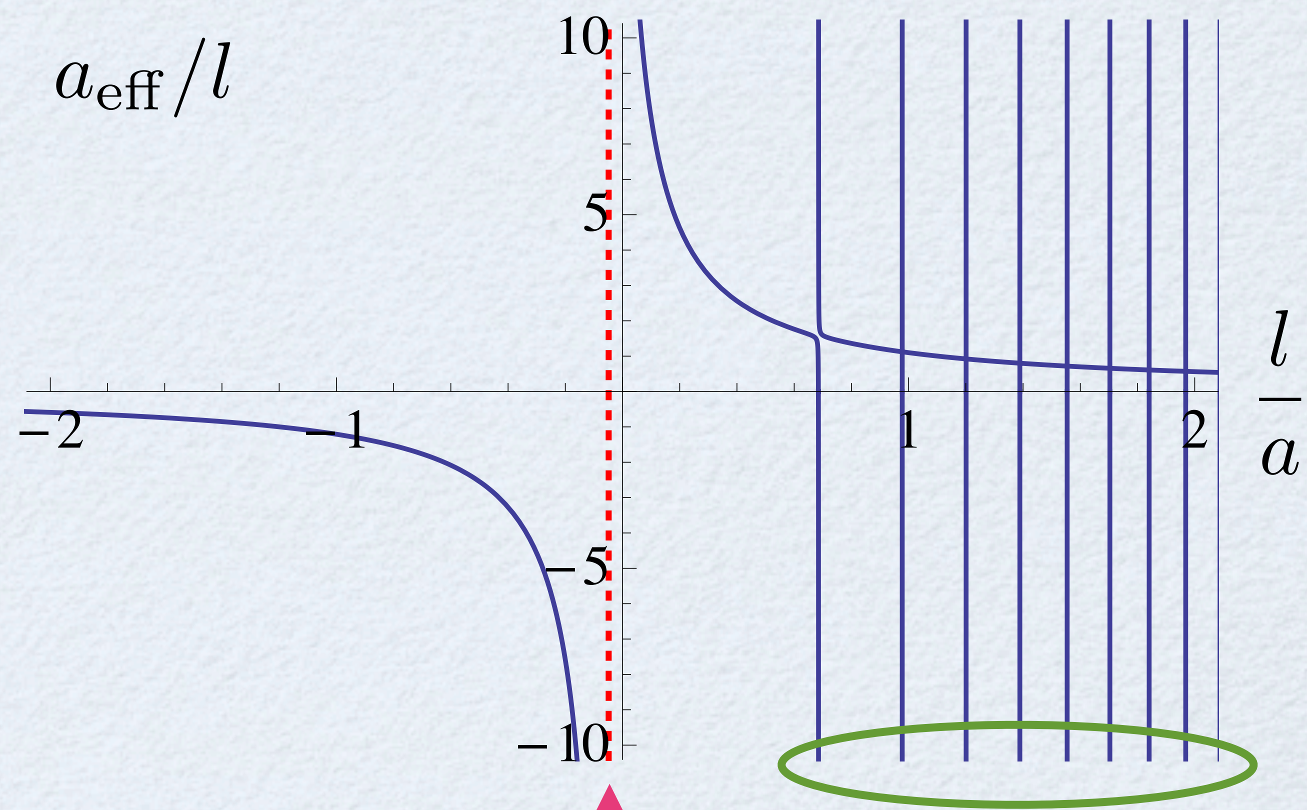


1D-3D mixture

A=^{40}K in 1D & B=^{6}Li in 3D





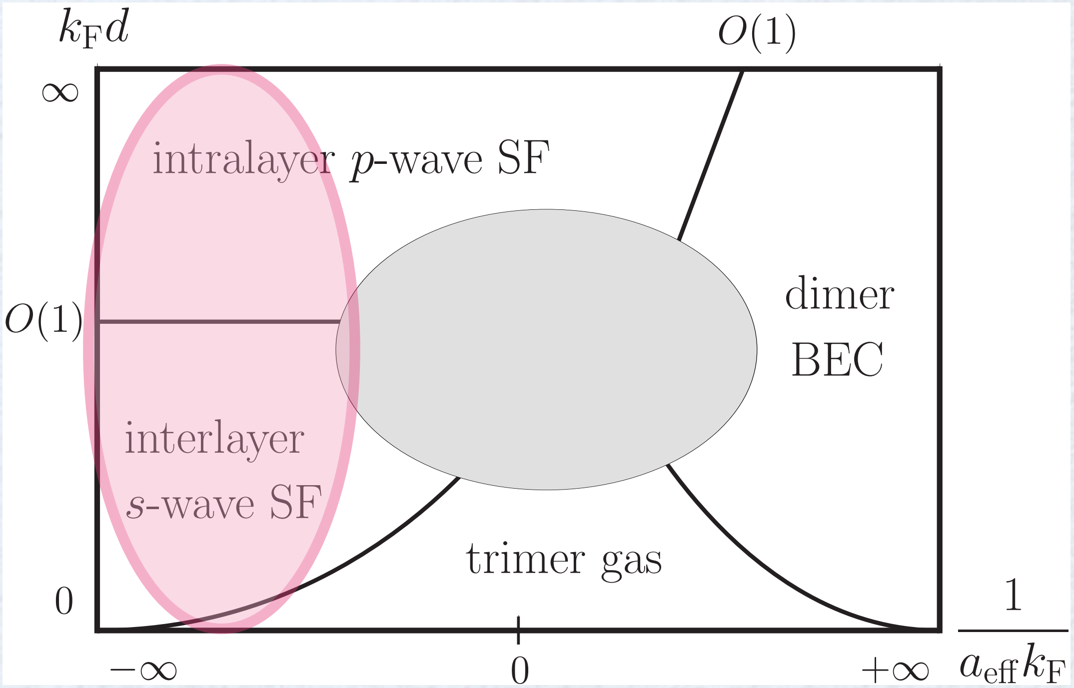


Infinite series of confinement-induced resonances

Resonance is shifted to "a<0" side

Weak attraction (BCS) limit

- B atoms in 3D induce $V_{ind}(r)$ between A atoms in 2D

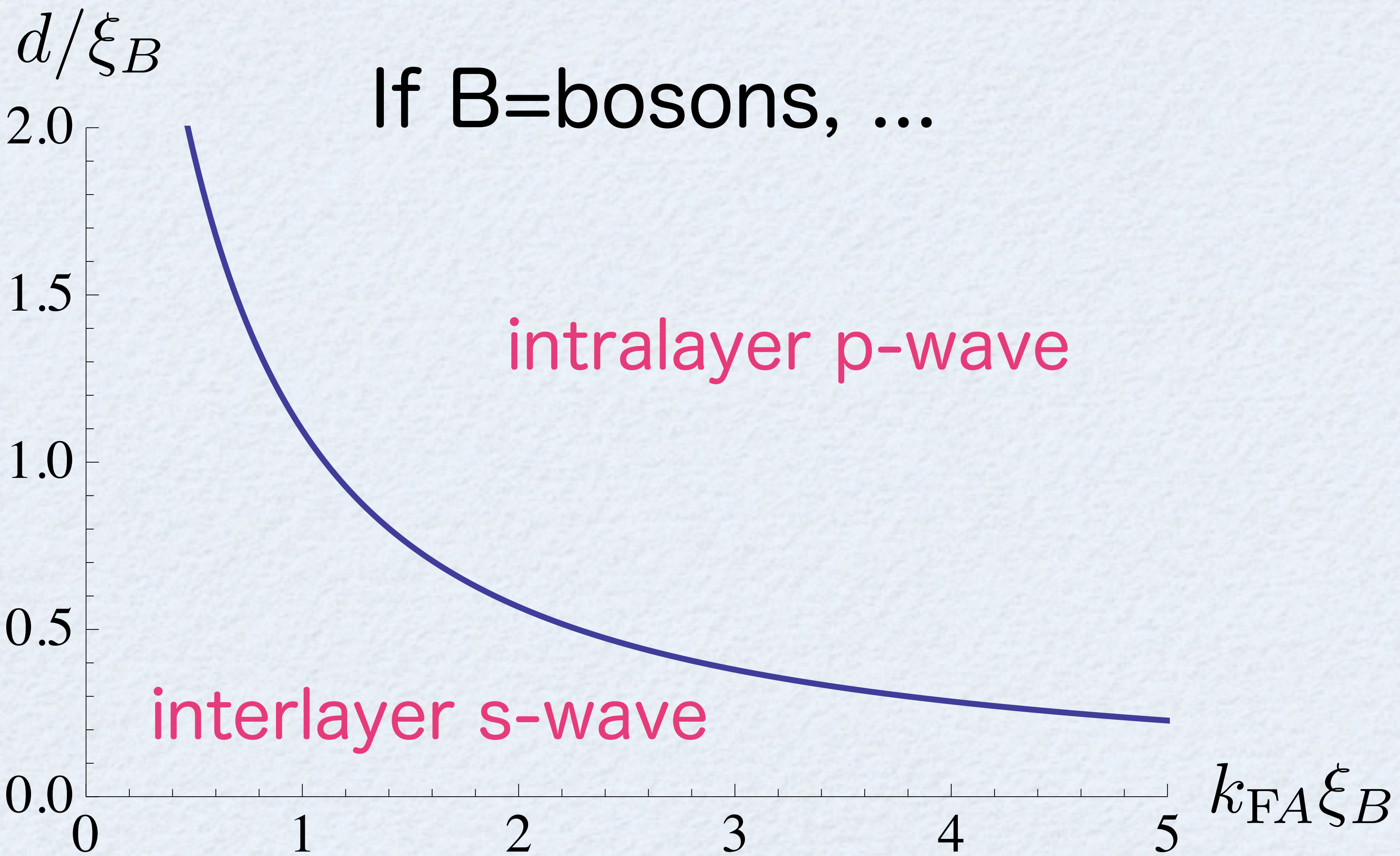


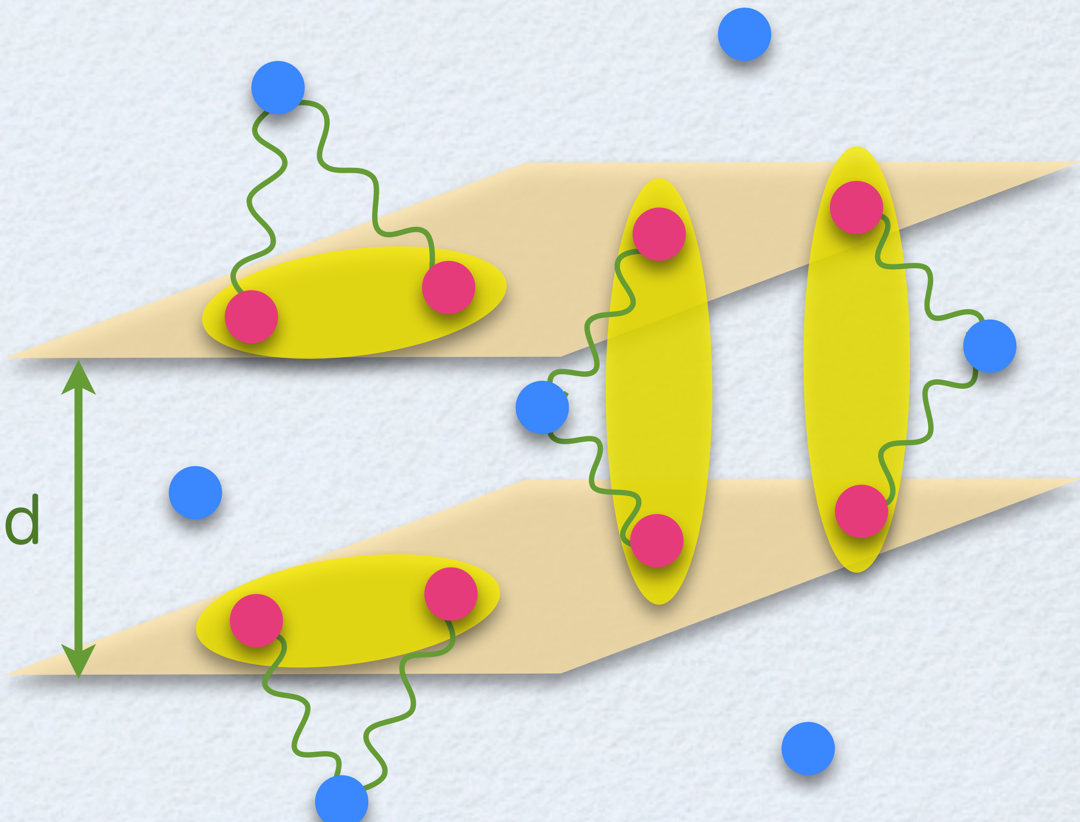
→ P-wave pairing of A atoms in the same layer for large d

or

→ S-wave pairing of A atoms in different layers for small d

If B=bosons, ...

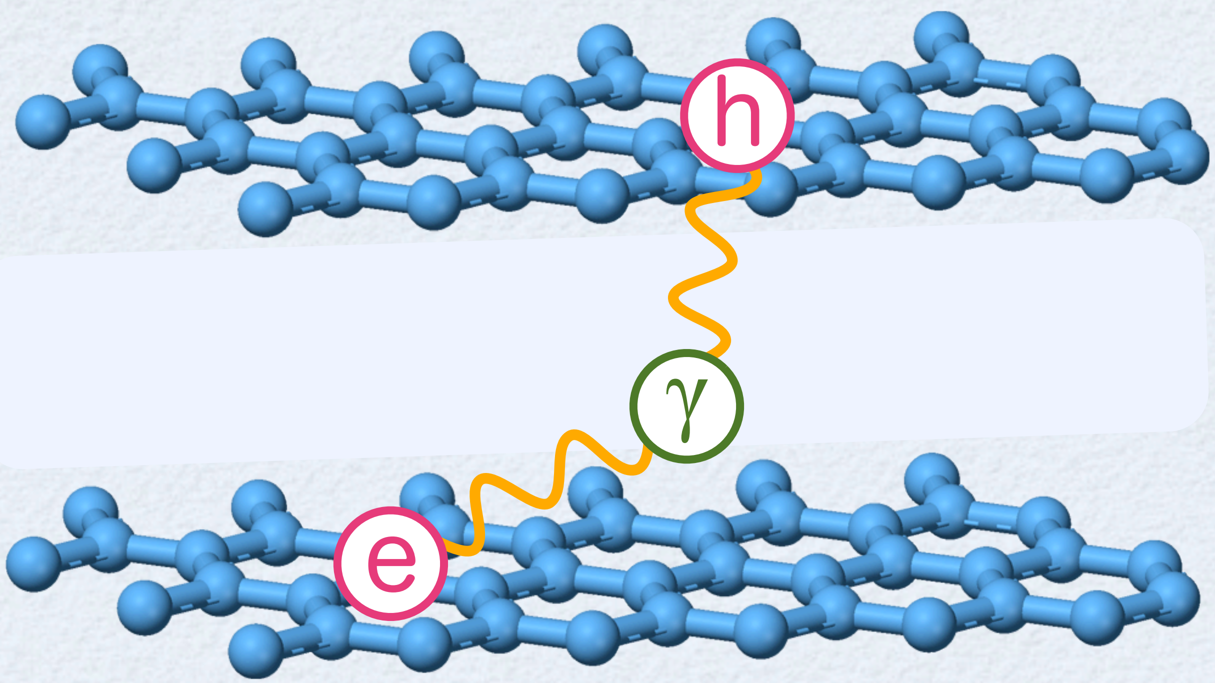




Other bilayer systems

bilayers in condensed matter

- bilayer semiconductors
- bilayer quantum hall systems
- bilayer graphenes ...

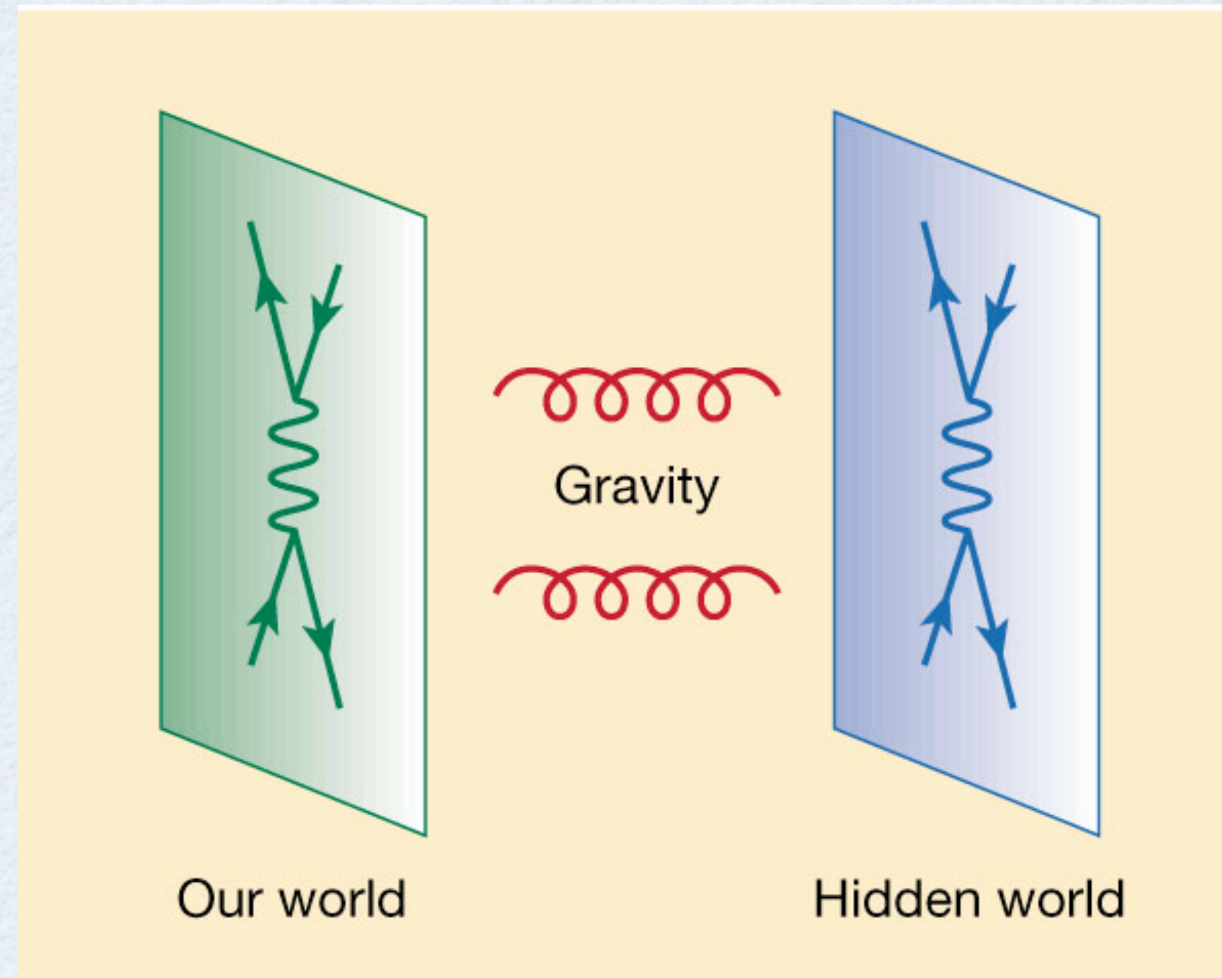


interplay between

intralayer & interlayer correlation

➡ interlayer exciton condensation

bilayer of the universe?



J. Gauntlett, Nature 404, 28 (2000)