Second Application:
Trapped Unitary Fermions

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Trapped Unitary Fermions

- Relevant for trapped cold atom systems
- Results from other theory calculations available

Benefits of the SHO

- Choice of source/sink less crucial
- Tunable scales to explore finite volume, finite lattice spacing effects
  - $\omega b_t \longrightarrow$ temporal discretization errors
  - $b_s/L_0 \longrightarrow$ spatial discretization errors
  - $L_0/L \longrightarrow$ finite volume errors

$$L_0 = (mk)^{-1/4} \qquad \omega = \sqrt{k/m}$$

Temporal Errors

Discretization errors set by choice of transfer matrix

Simplest choice to implement:

$$1 - Vb_t \to 1 - V_{int}b_t - V_{SHO}$$

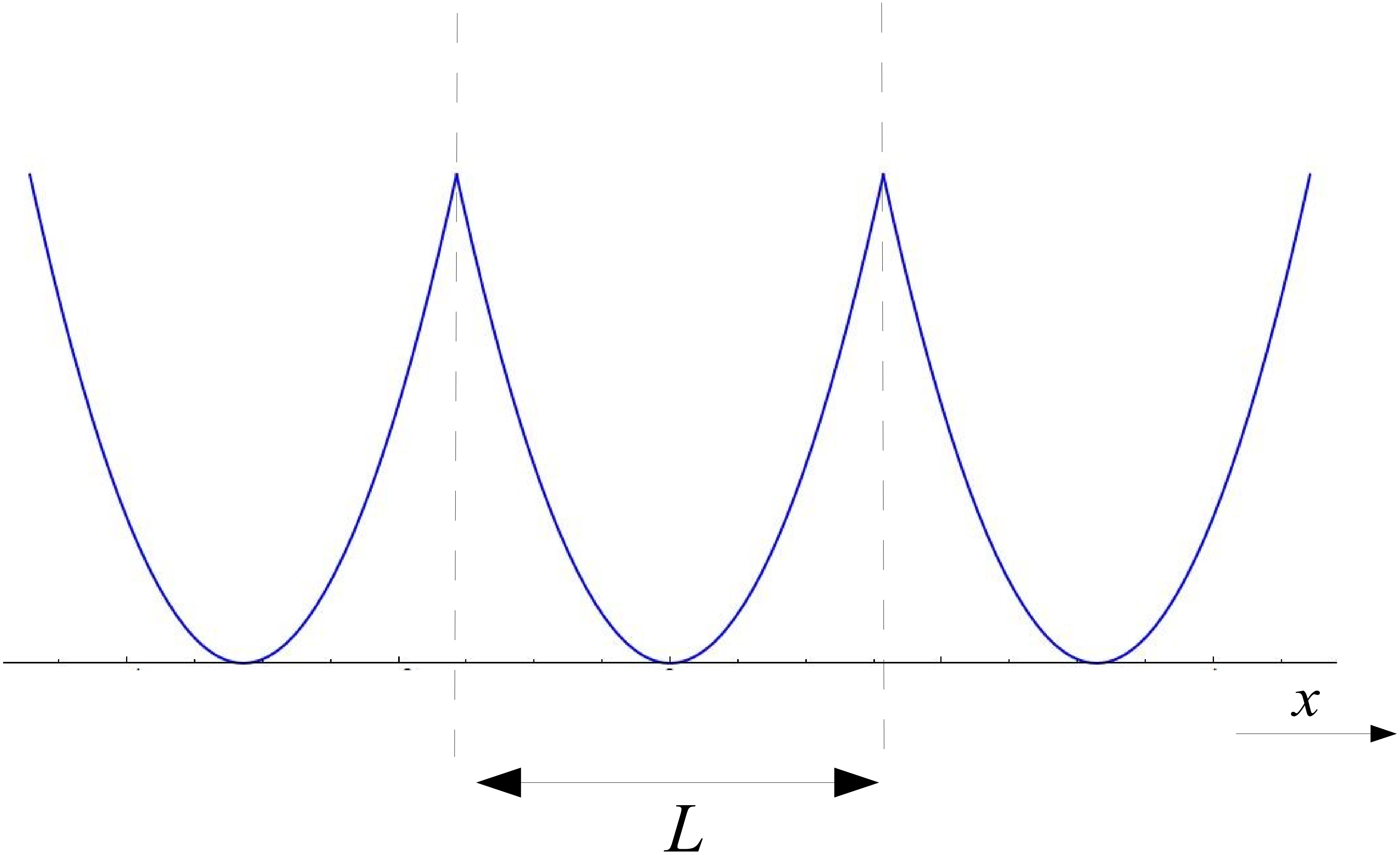
So far, smallest errors seen with

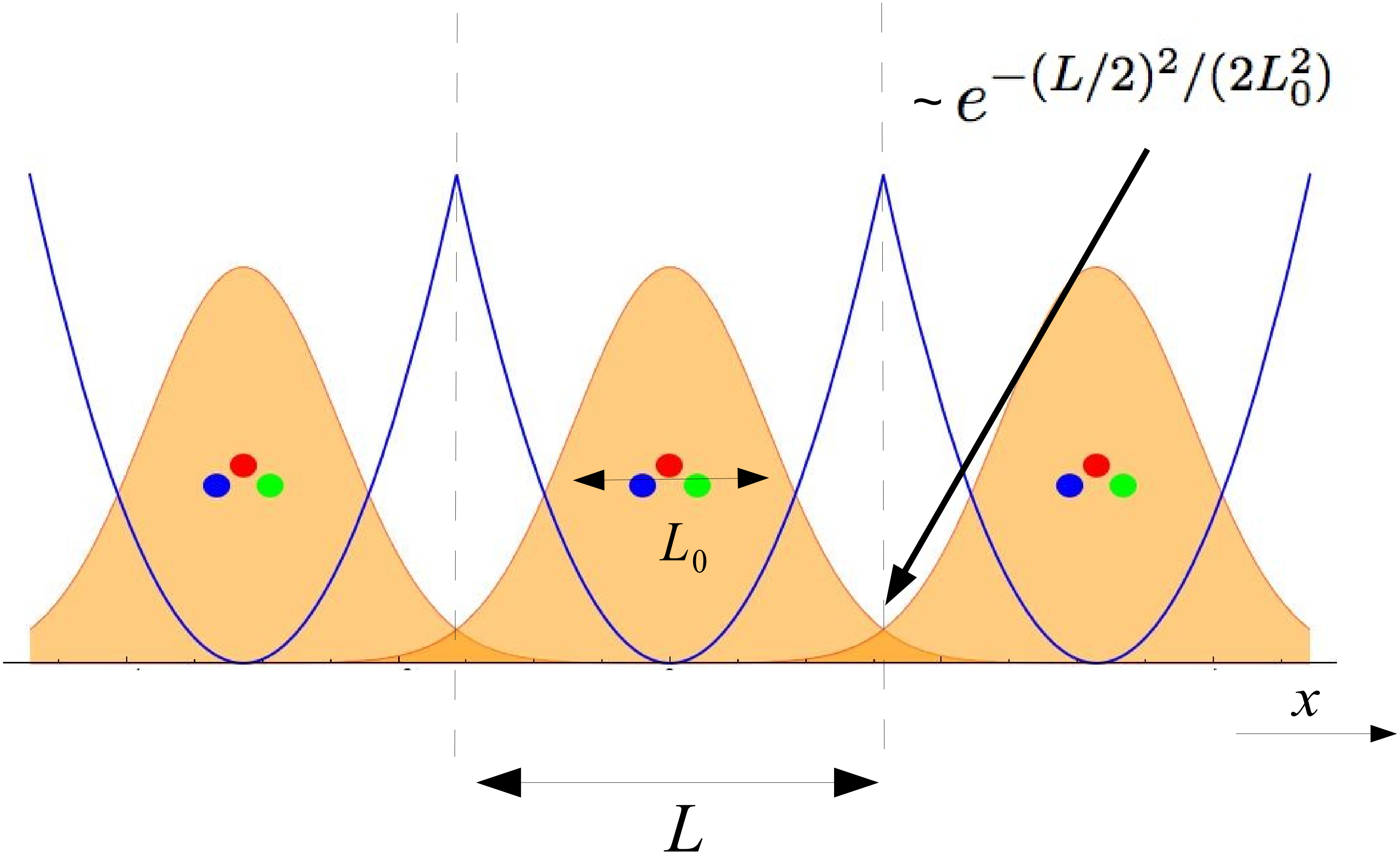
$$V_{SHO} = 1 - e^{-\frac{b_t}{2} m\omega^2 \sum_{i=1}^{3} (L_i/2 - x_i)^2}$$

Ensure small b_t errors by choosing small ω

Spatial Errors

Position space potential: PeriodicBC

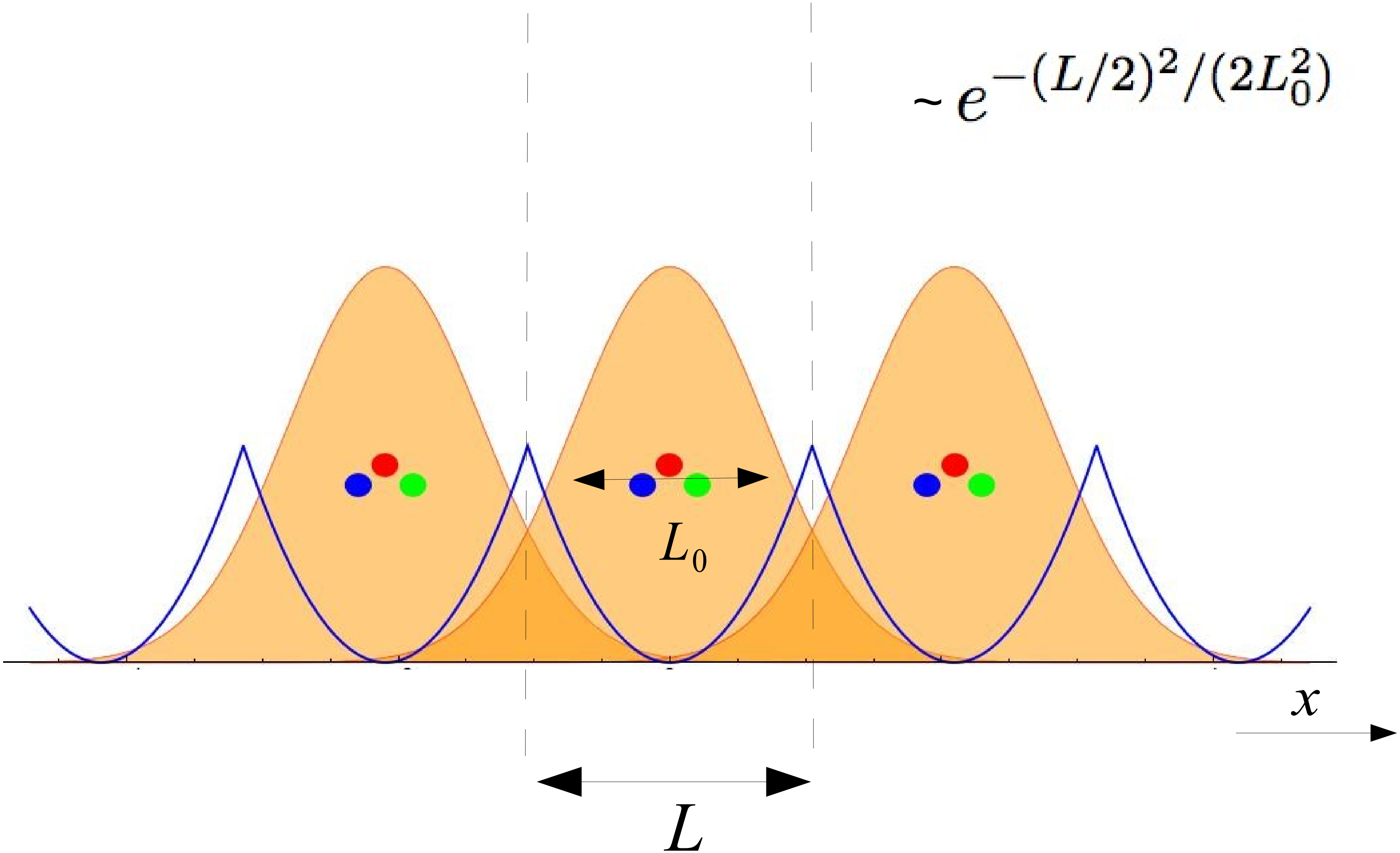




$\sim e^{-(L/2)^2/(2L_0^2)}$
L_0
x
L

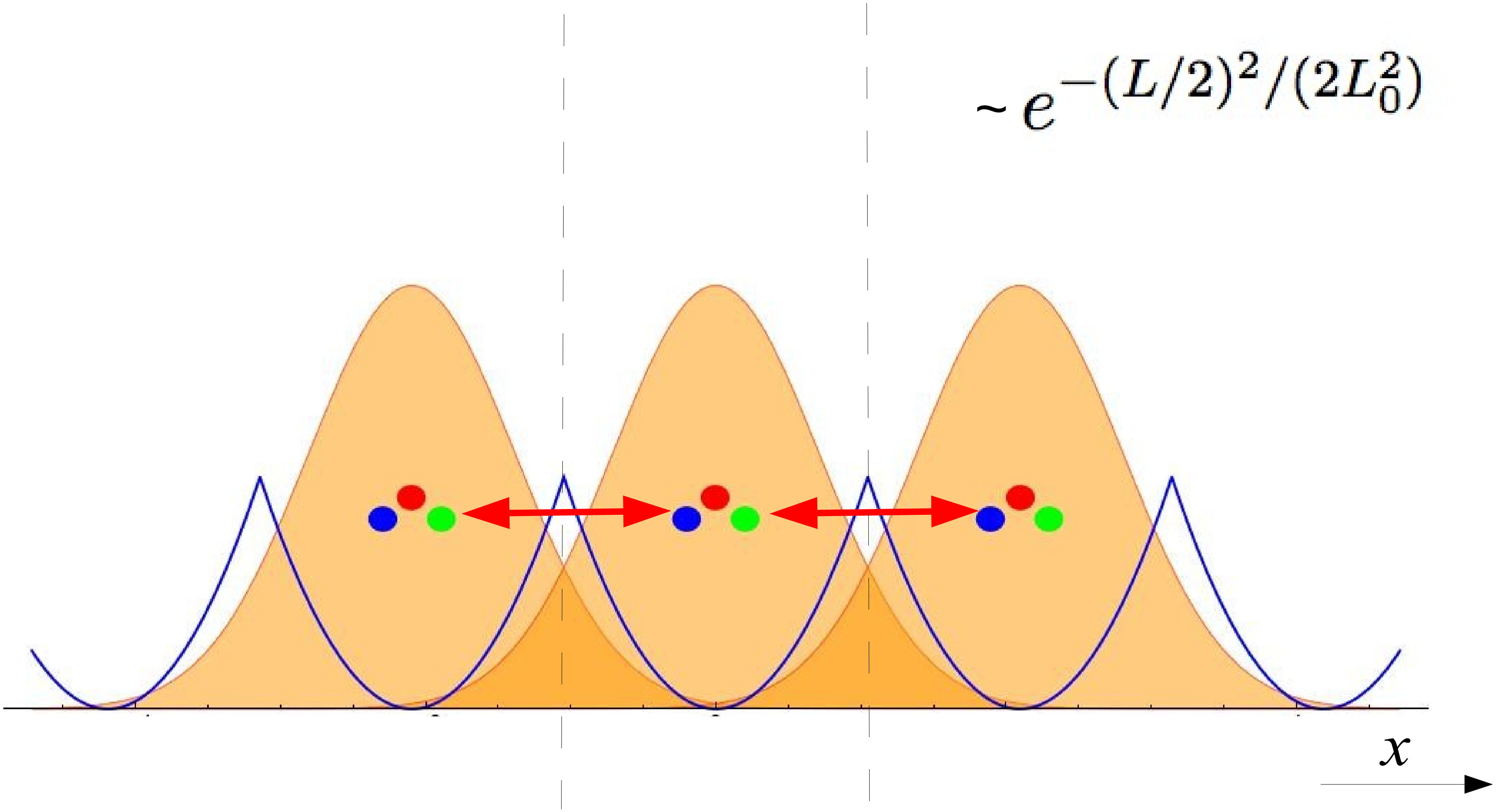
(ω fixed)

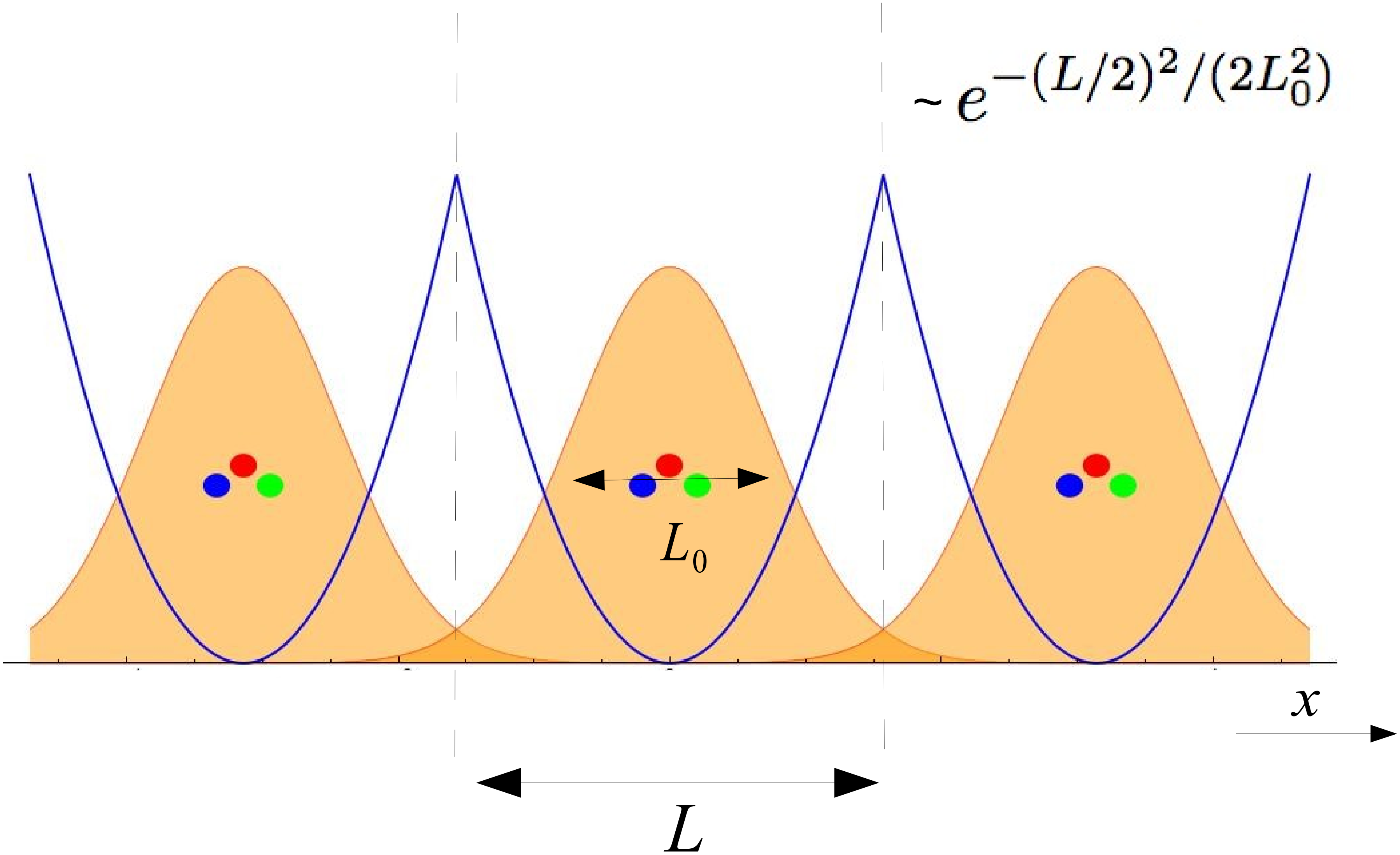
Reduce L, keep L_0 fixed



(ω fixed)

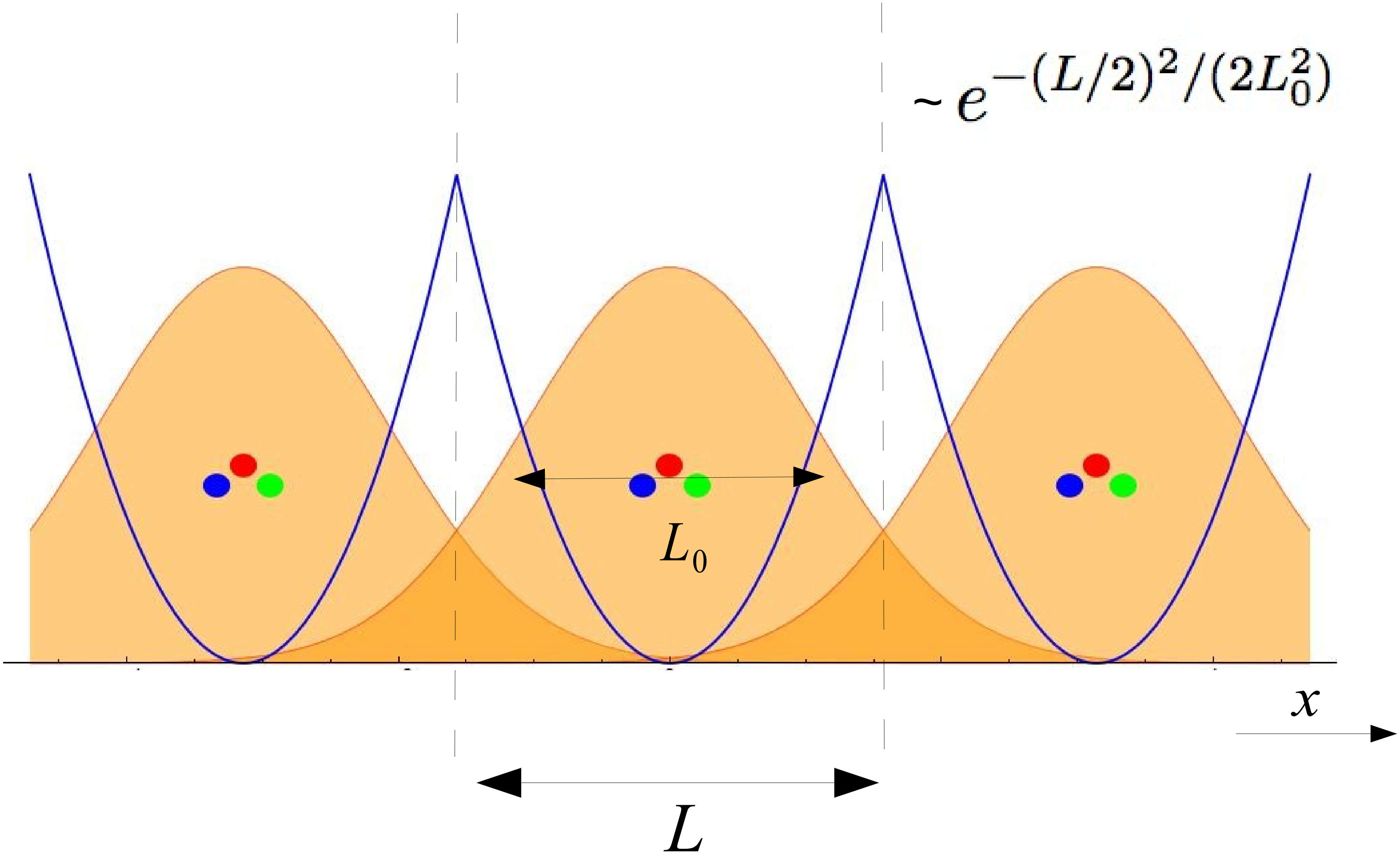
Interactions with image charges lower energy



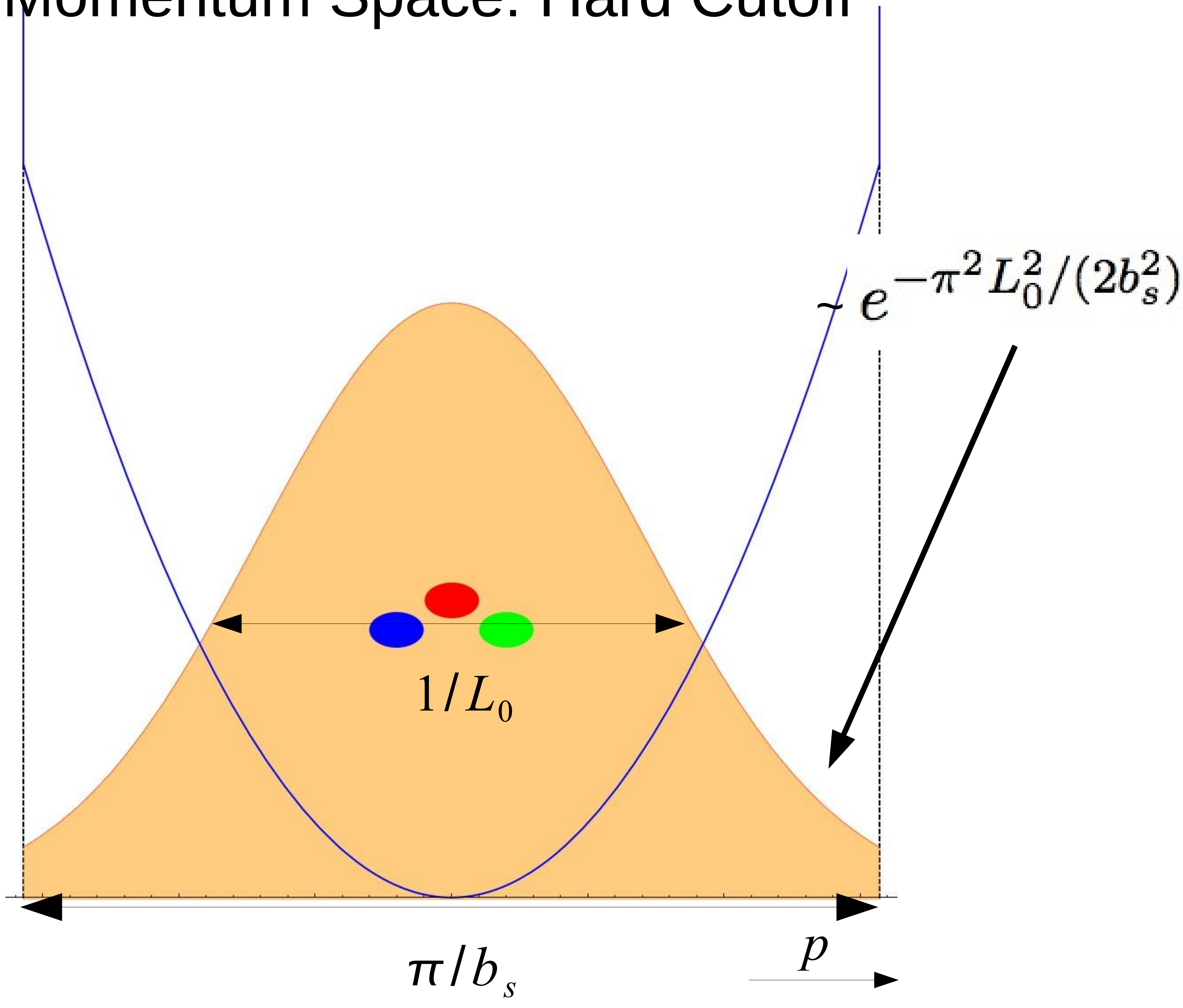

~ $e^{-(L/2)^2/(2L_0^2)}$
L_0
x
L

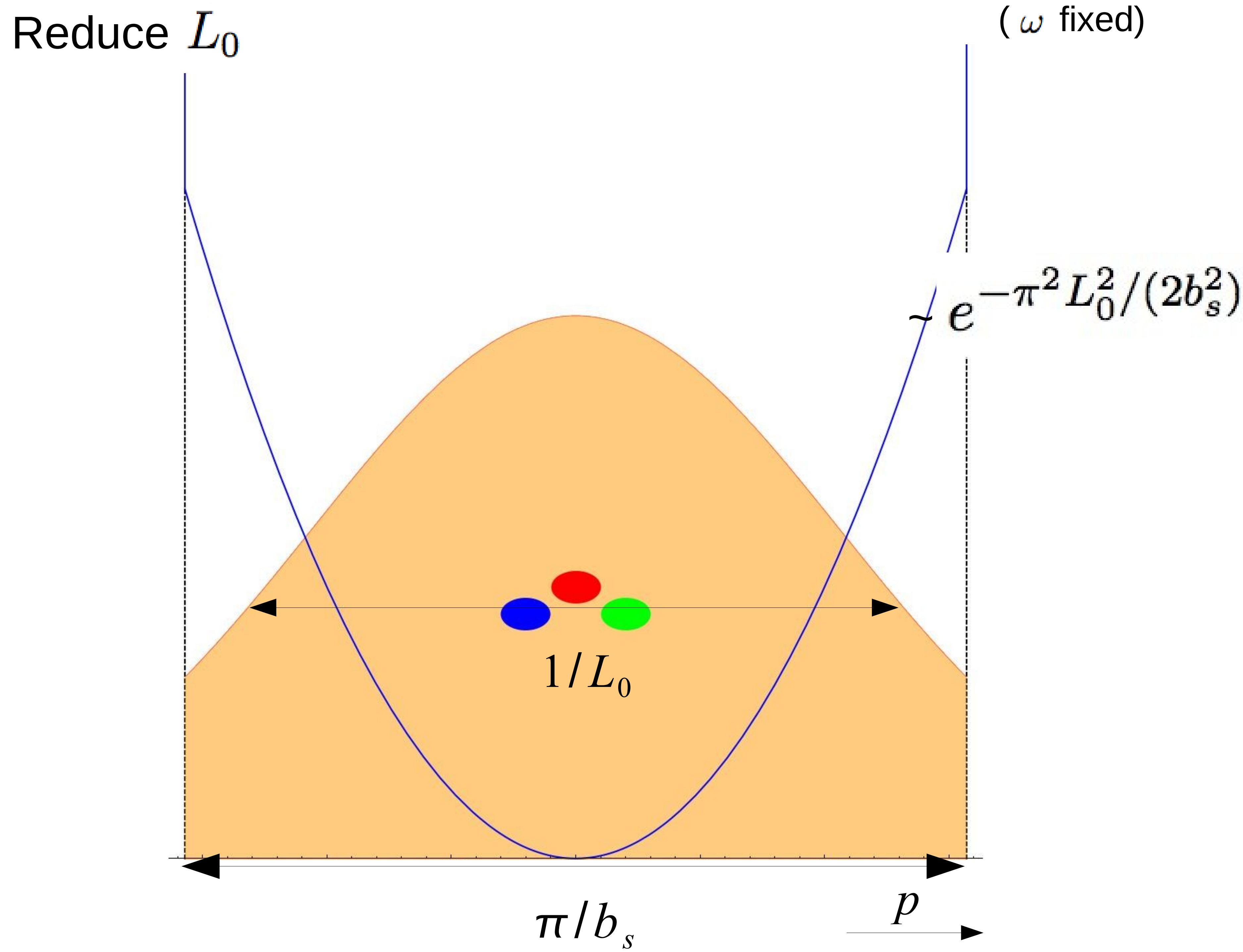
(ω fixed)

Increase L_0, keep L fixed



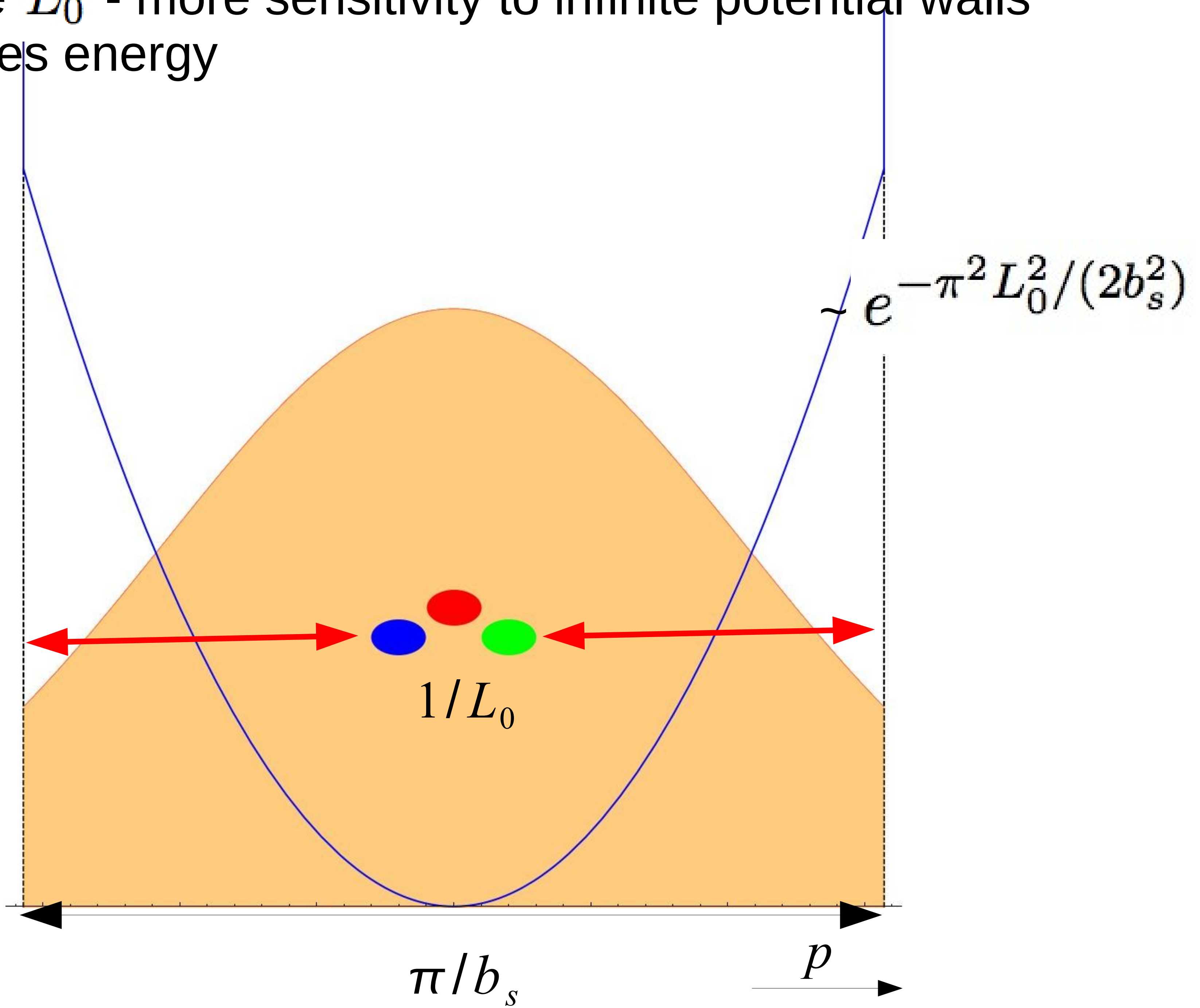
Momentum Space: Hard Cutoff





Reduce L_0
(ω fixed)
$\sim e^{-\pi^2 L_0^2/(2b_s^2)}$
$1/L_0$
π/b_s
p

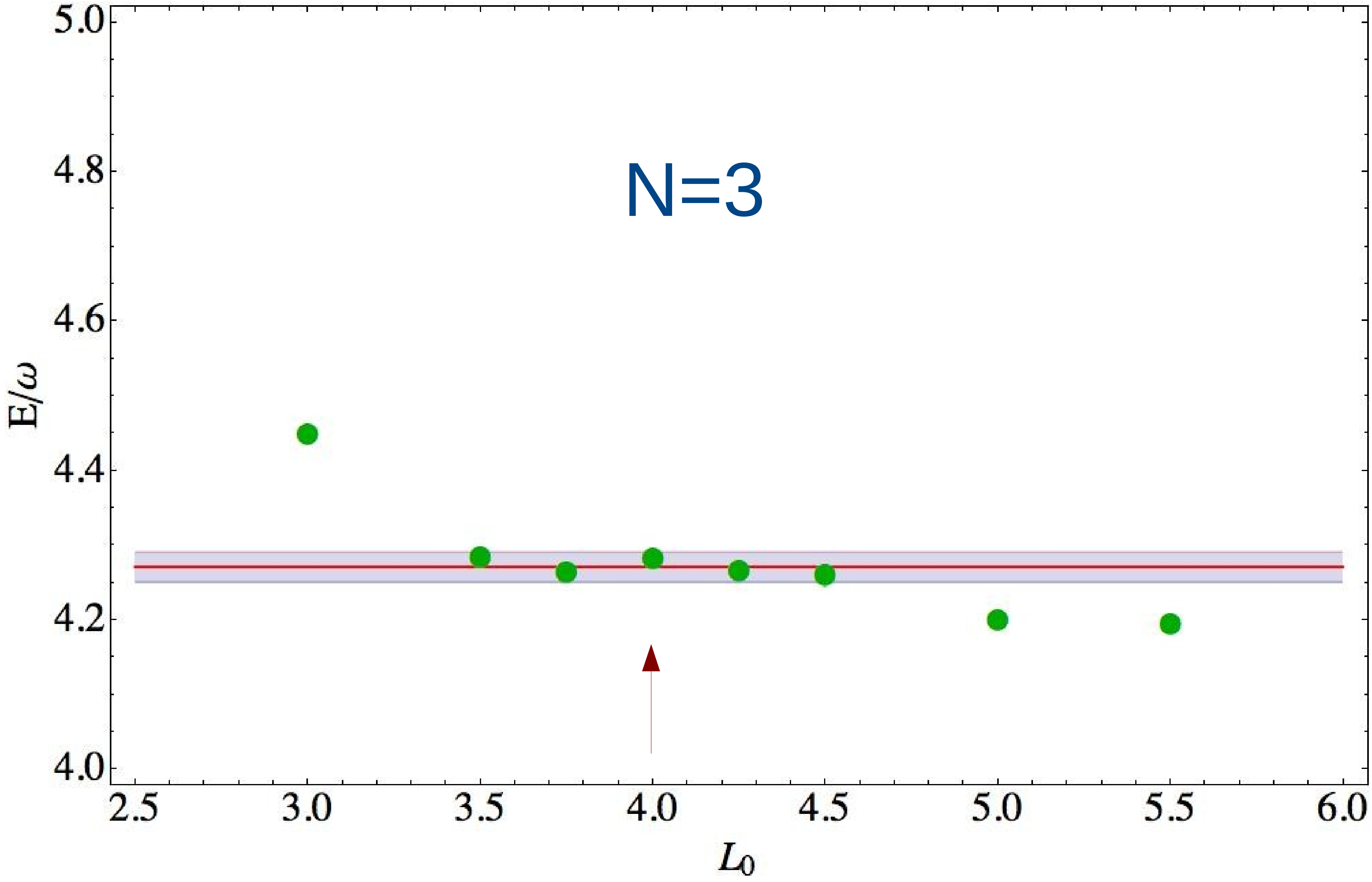
Reduce L_0 - more sensitivity to infinite potential walls increases energy



Spatial Errors

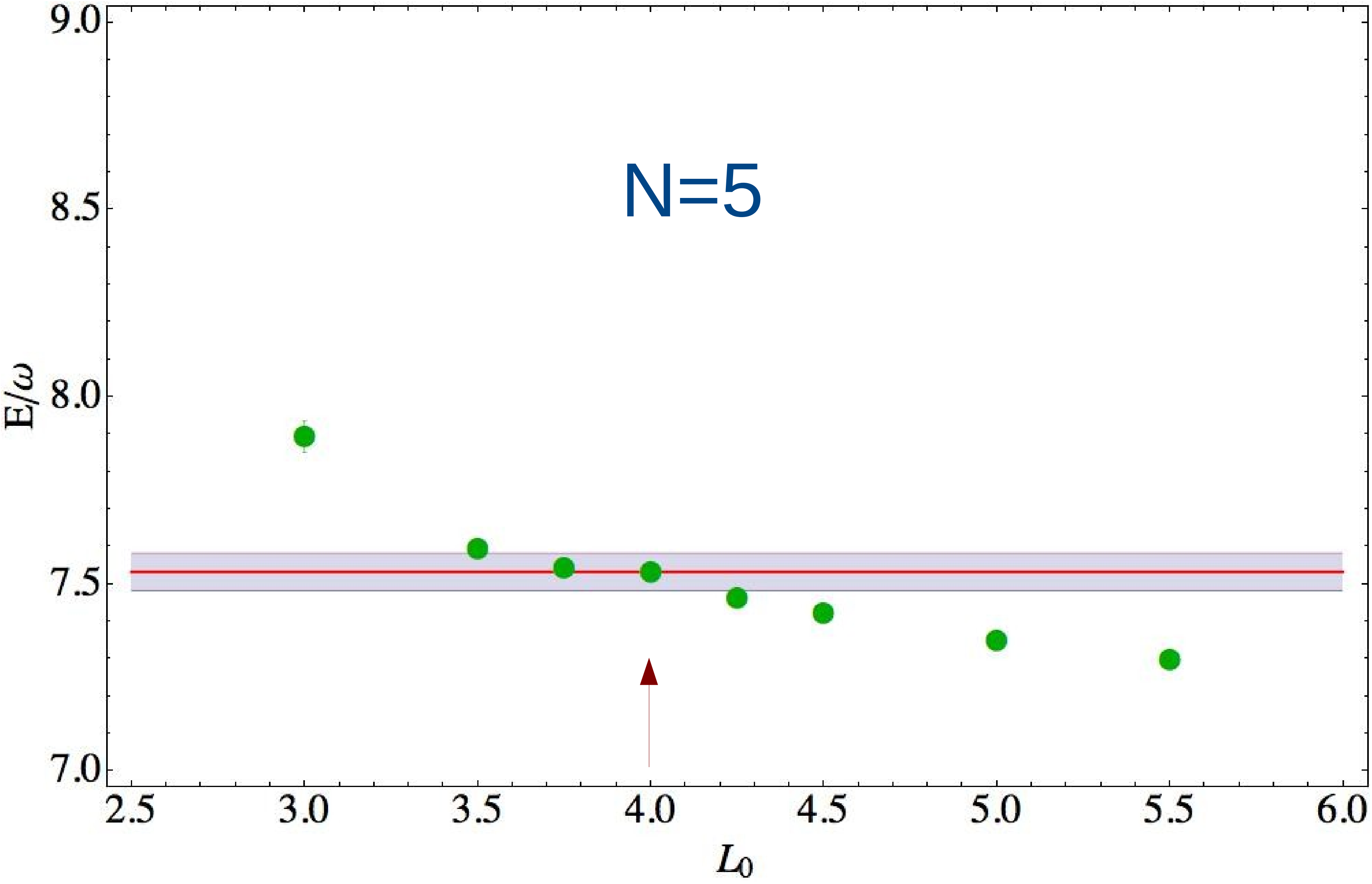
- Both finite volume and spatial discretization errors affected by changing L_0
  - Finite volume errors push energy down for large L_0
  - Discretization errors push energy up for small L_0
- Performed tests at various values of L_0 to choose ideal value

L = 32
ω = 0.013



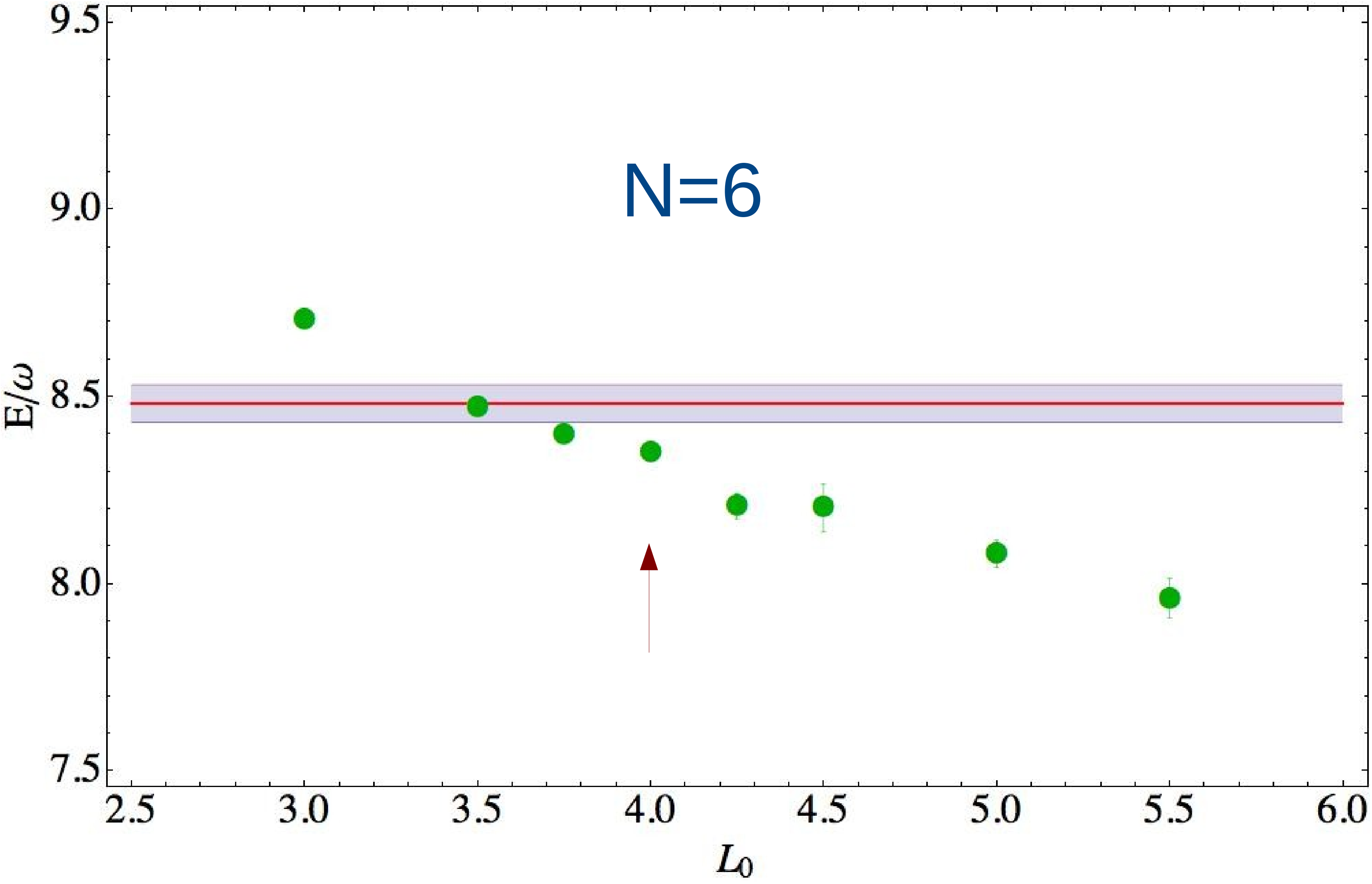
D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

$L = 32$
$\omega = 0.013$



D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

L = 32
ω = 0.013

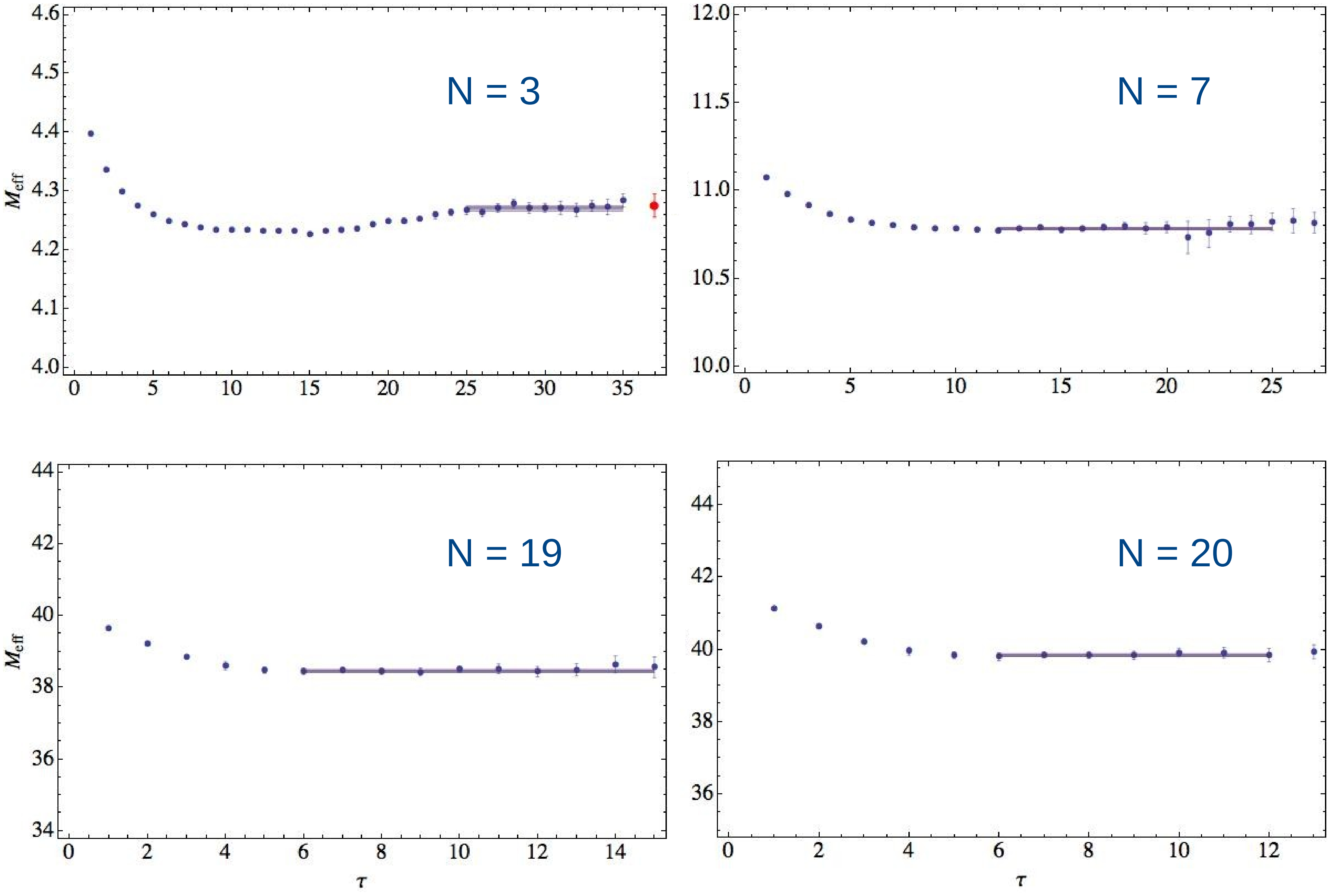


D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

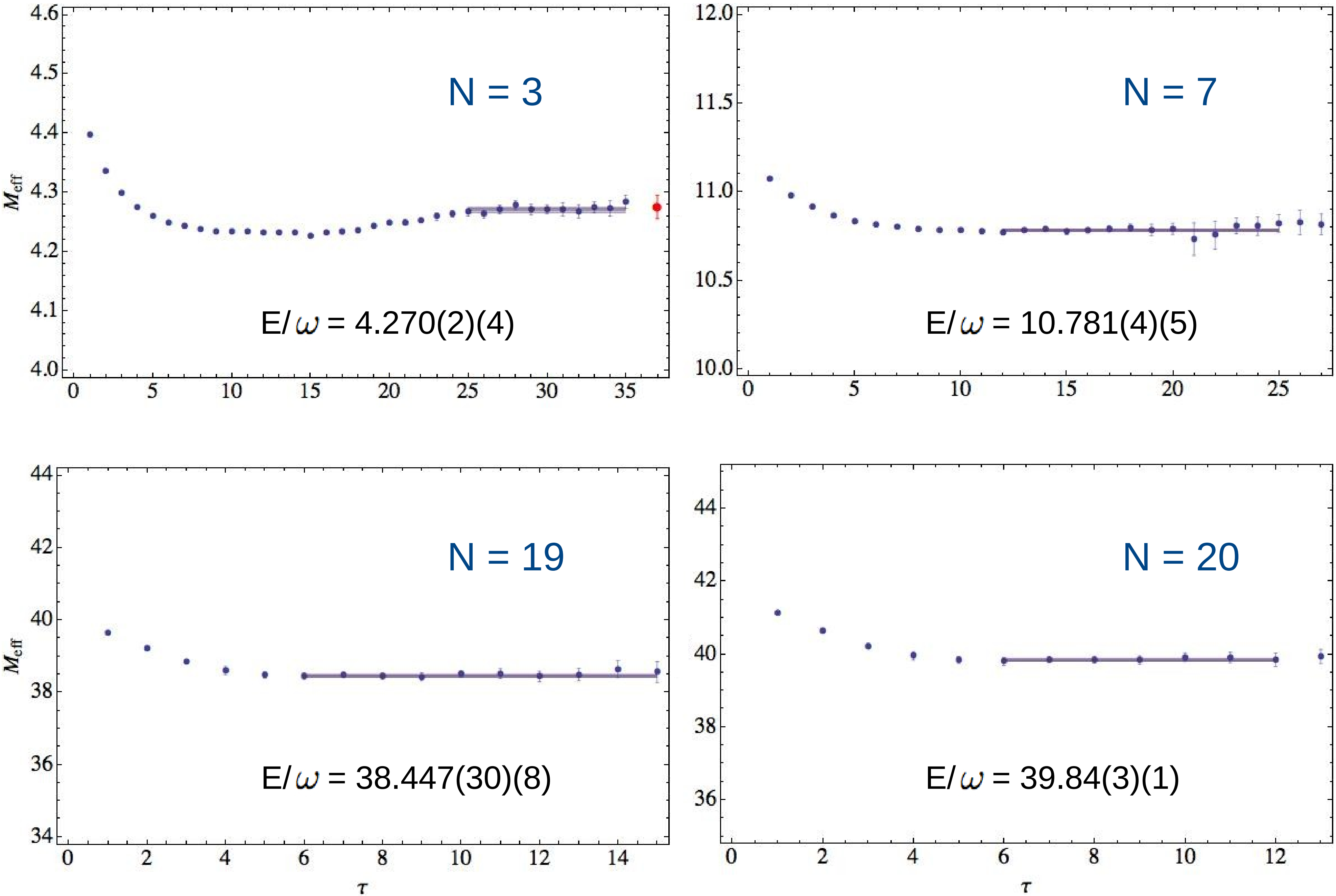
Calculation Details

- Parameters:
  - ω = 0.013
  - L_0 = 4.0
  - L = 24, 32, 48
- Coefficients tuned using Luscher method
- Wavefunctions chosen to be paired SHO states

$$\psi_{PAIR} = \frac{D(p/2b)}{p/2b} \qquad b = 1/(2L_0^2)$$

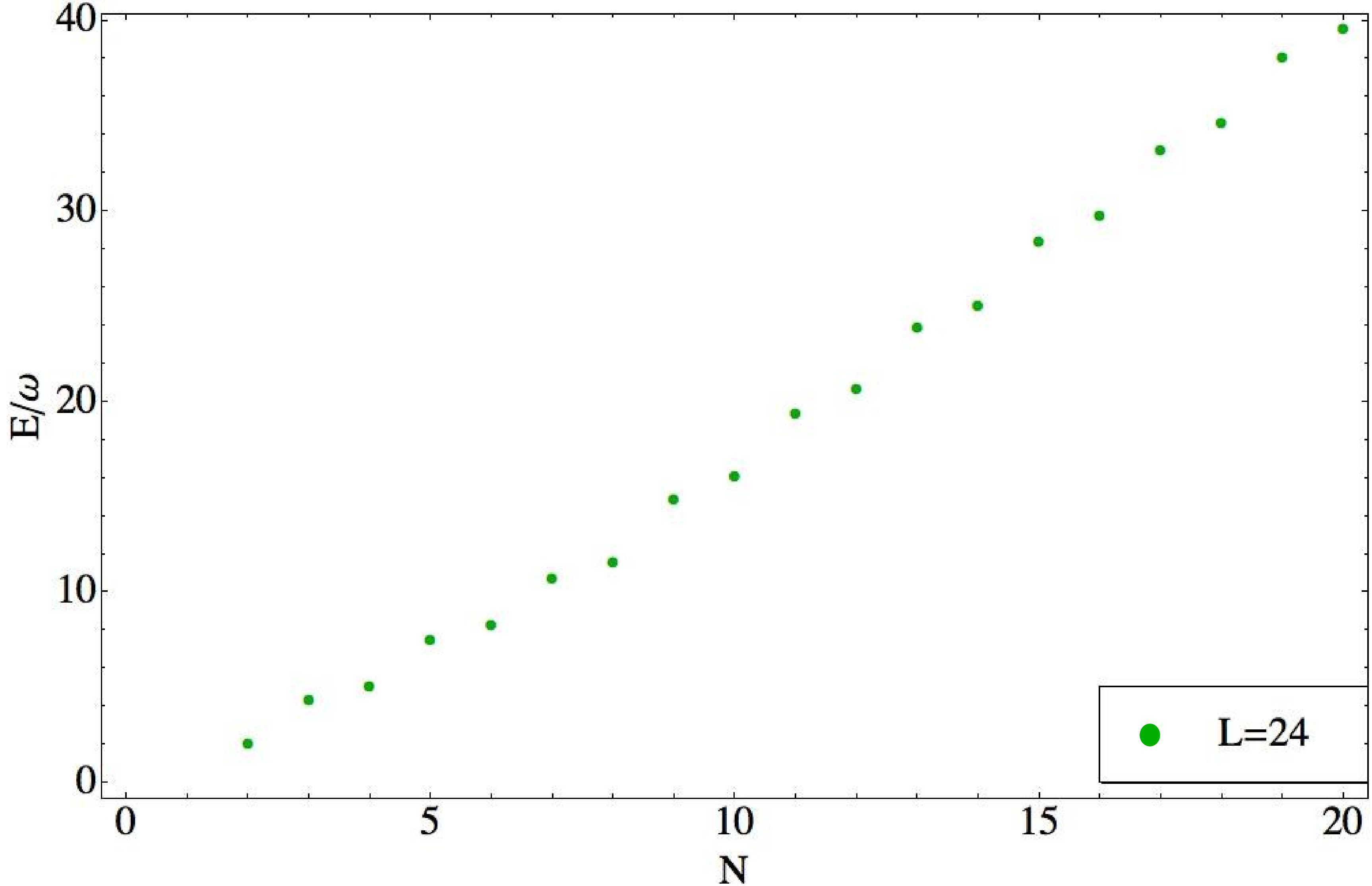


L = 32

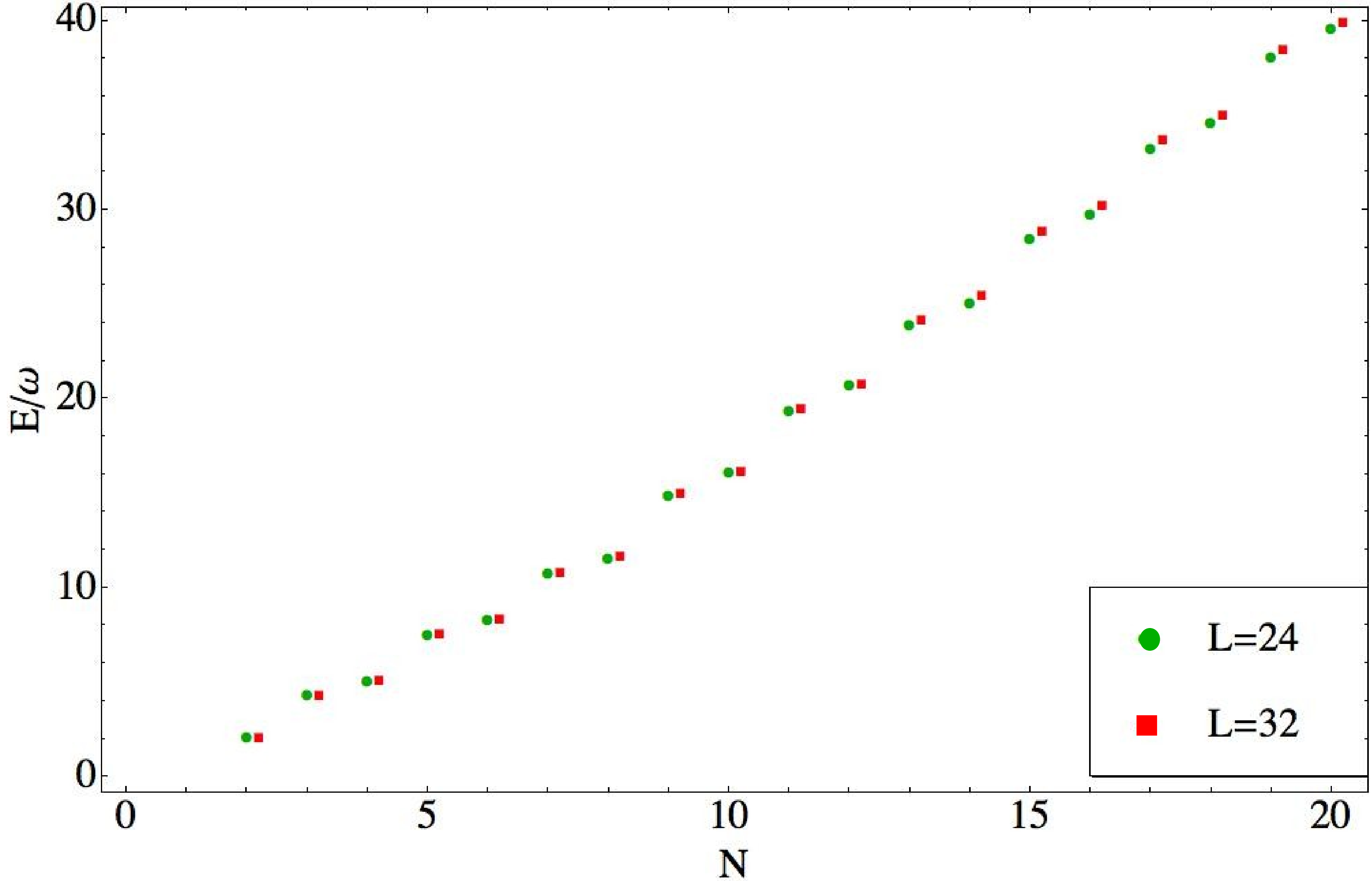


(preliminary)

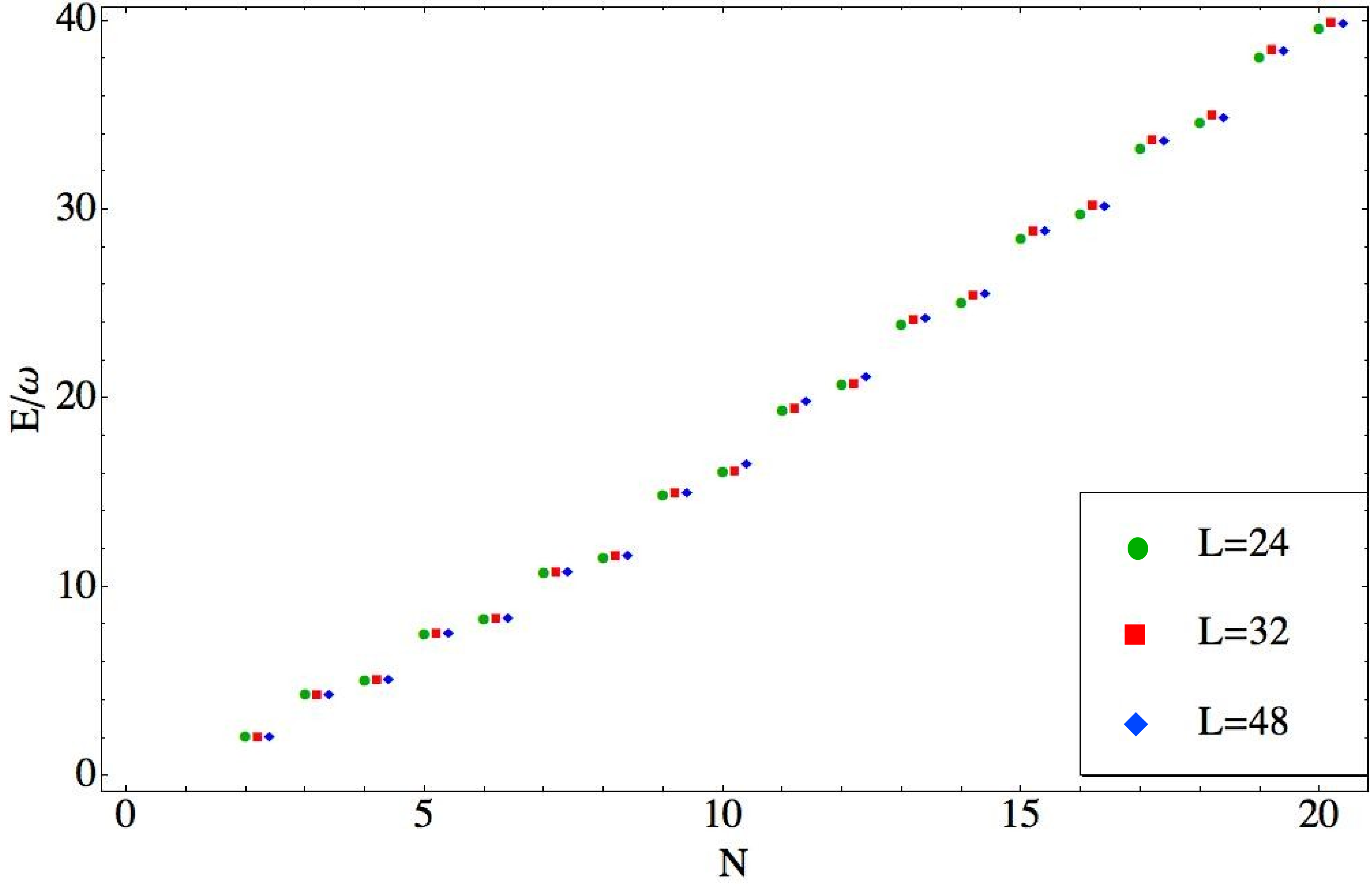
L = 24



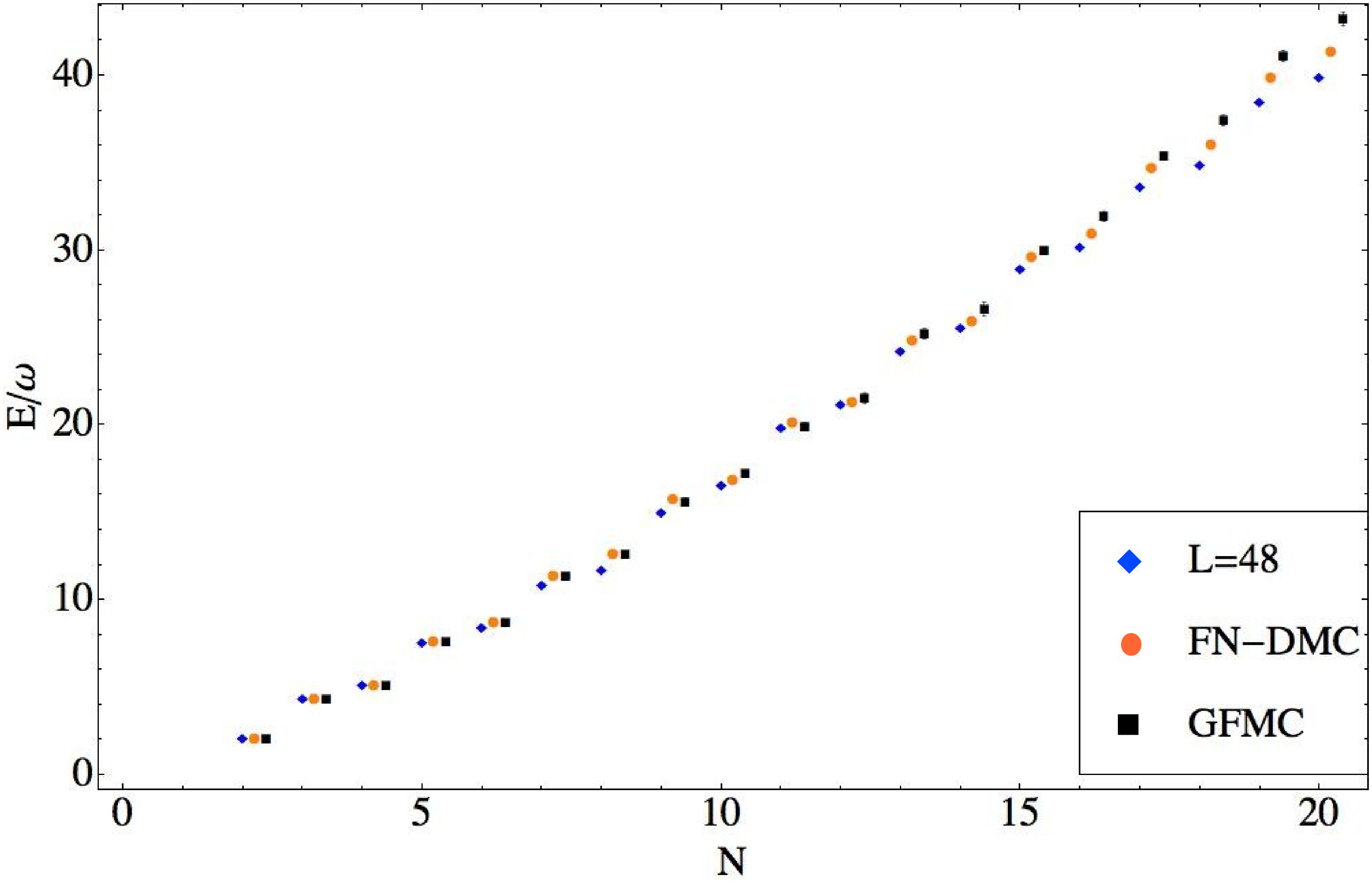
L = 32



L = 48

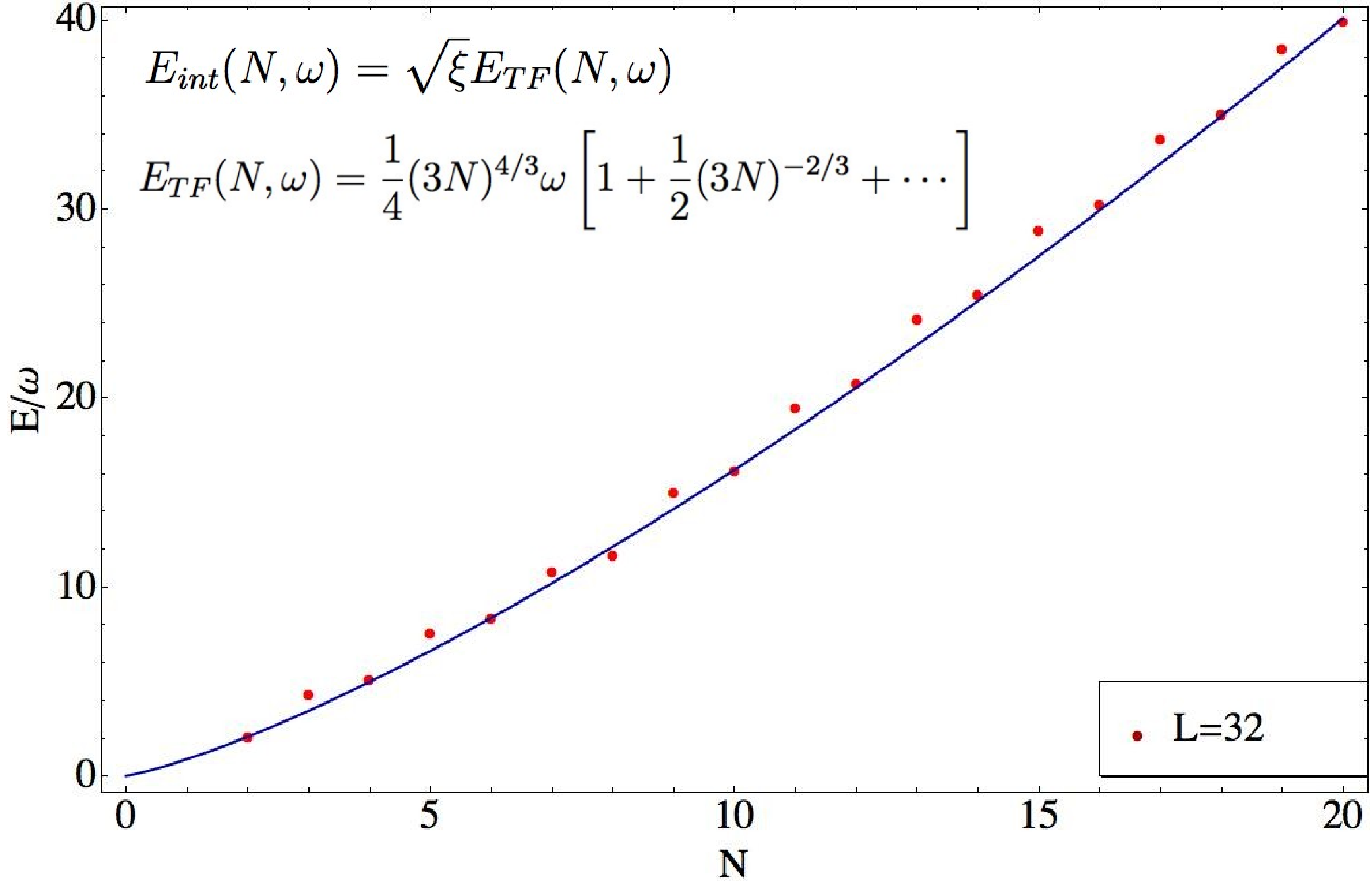


Comparison

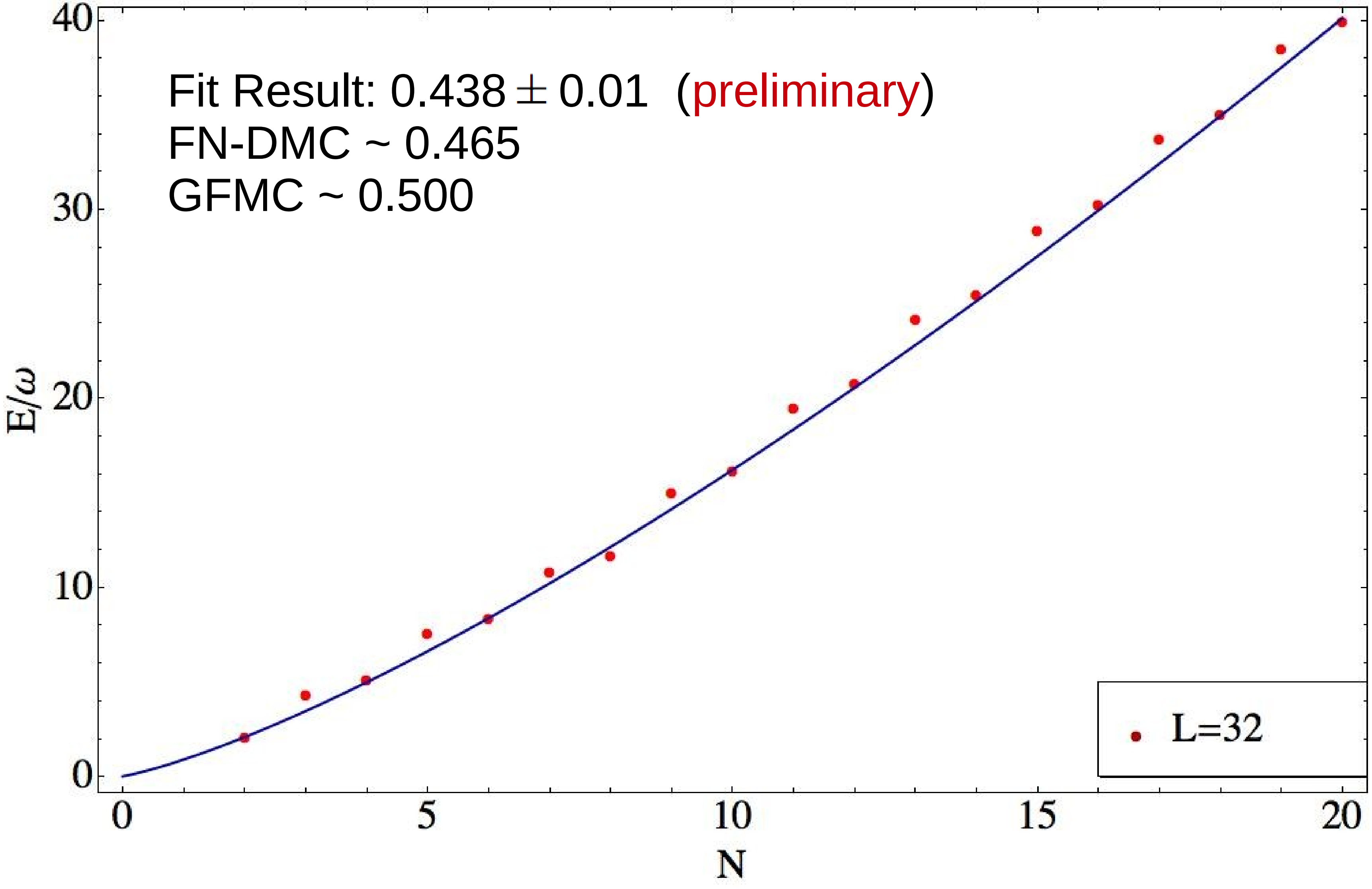


- FN-DMC: D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734
- GFMC: S. Y. Chang and G. F. Bertsch, arXiv:physics/0703190

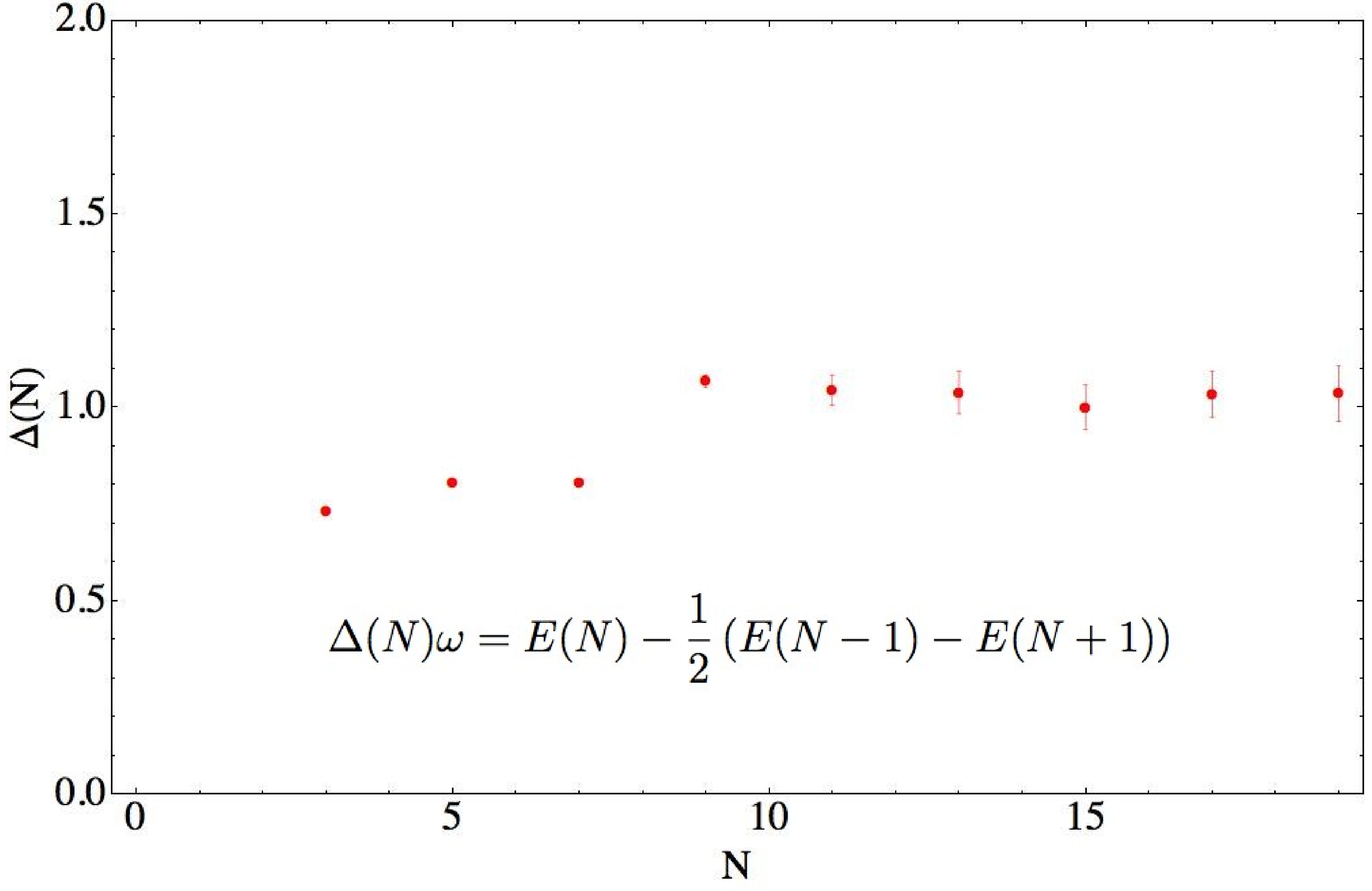
Bertsch Parameter



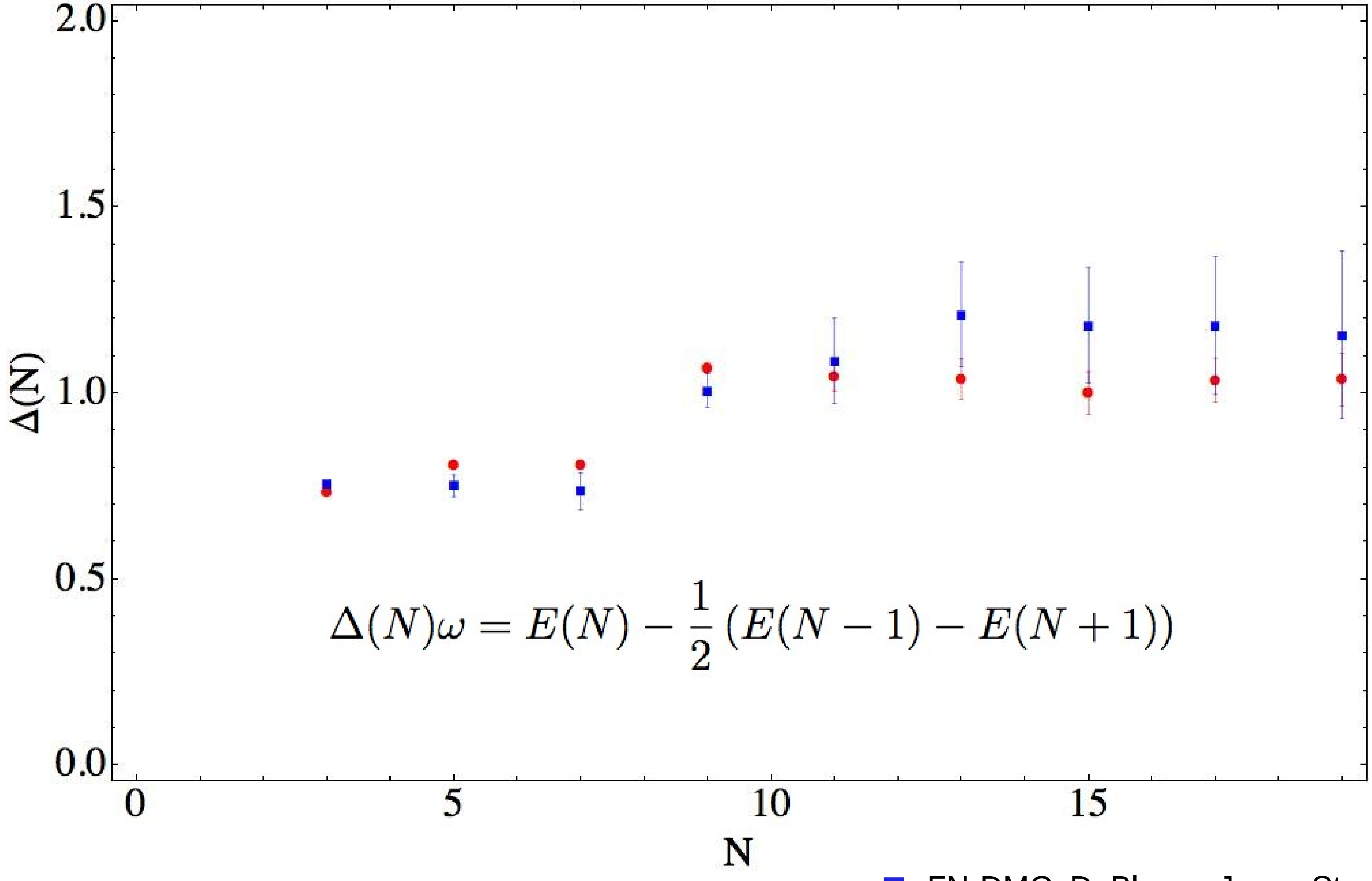
Bertsch Parameter



Gap


2.0
1.5
1.0
0.5
0.0
Δ(N)
0
5
10
15
N
$\Delta(N)\omega = E(N) - \frac{1}{2}\left(E(N-1) - E(N+1)\right)$

Gap



■ FN-DMC: D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

SHO Conclusions

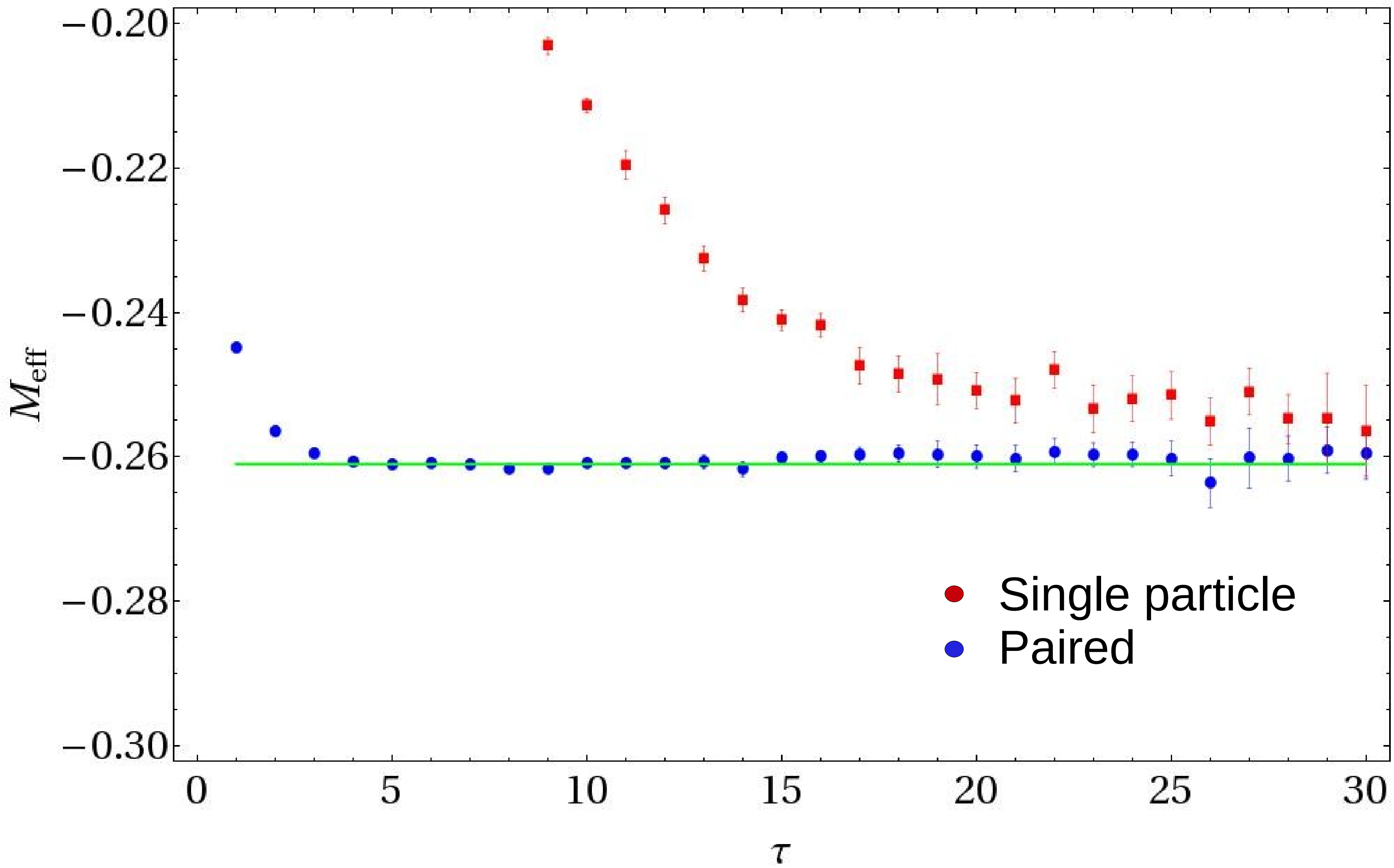
- Tunable scales can be used to control finite volume and discretization errors
- Clean signal allows for high-precision results
- Unitary fermion results consistent for small N, lower values for large N
- Trap confinement may be useful for studying bound states

Future Directions:
Steps Toward Nuclei

Steps Toward Nuclei

- Tuning method – in principle allows for any $p \cot \delta$ desired
- Adding a second auxiliary field allows you to tune both ${}^1S_0$ and ${}^3S_1$ channels
- Choose physical a, r_0 for each channel

Deuteron



−0.20
−0.22
−0.24
−0.26
−0.28
−0.30
M_{eff}
0
5
10
15
20
25
30
τ
Single particle
Paired

Problems with Tuning

- Tuning for physical channels gives complex couplings
- Only resolved for small mass (large temporal lattice spacing)
- Tuning no longer improved with higher dimension operators

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- Tuning for physical channels gives complex couplings
- Only resolved for small mass (large temporal lattice spacing)
- Tuning no longer improved with higher dimension operators
- Wigner bound?

Possible Solutions

- Take $b_s > r_0$
- Use cutoff in momentum space as physical scale
- Use KSW expansion
  - LO: tune $p \cot \delta$ to give physical scattering length, add 3-body interaction non-perturbatively
  - NLO: add r_0 perturbatively

Three Body Interactions

$$y \varphi_3 \psi_t^* \psi_{t+1}$$

$$\varphi_3 \in Z_3$$

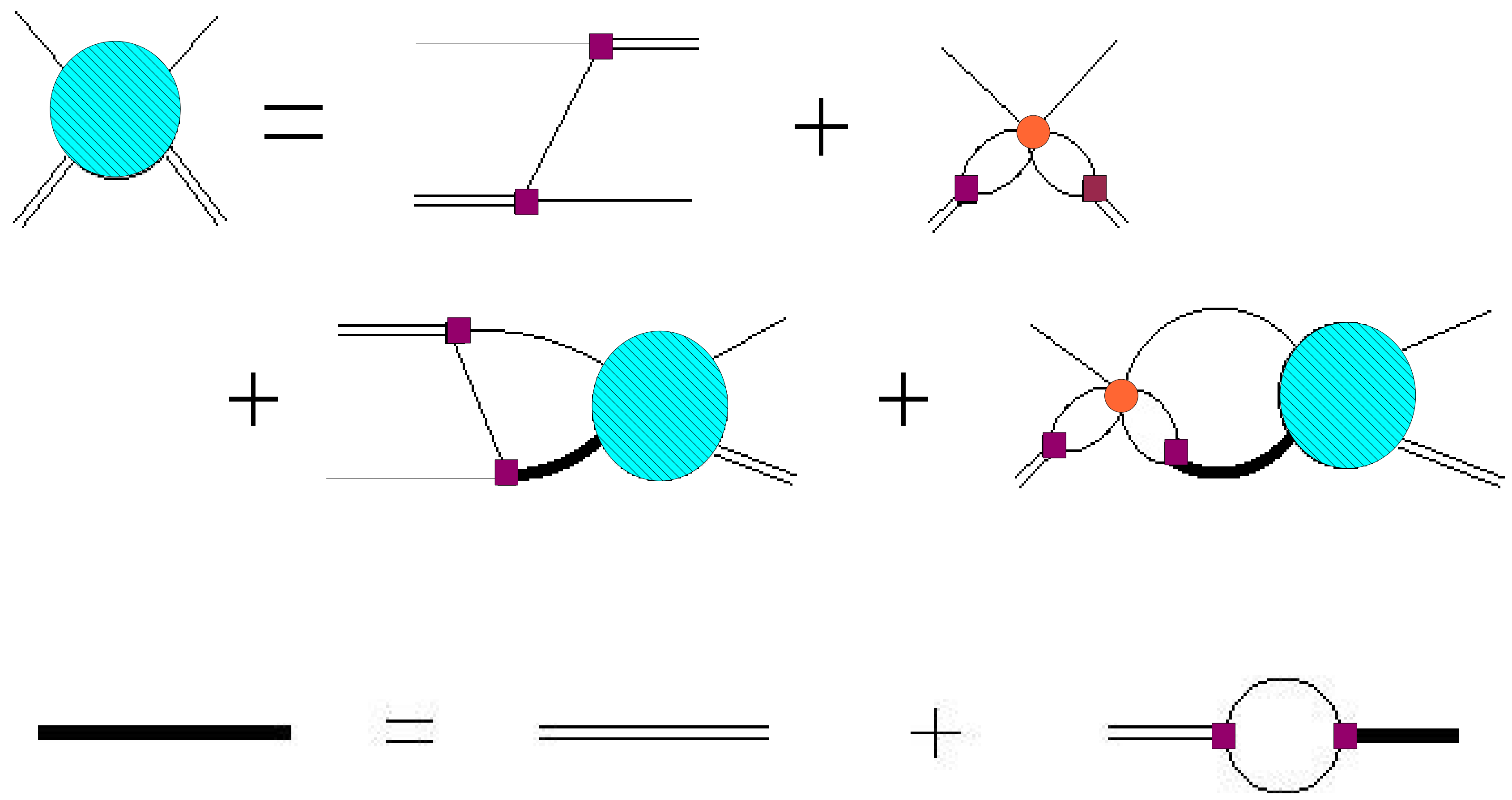
Three Body Interactions

$$y\varphi_3\psi_t^*\psi_{t+1}$$

$$\varphi_3 \in Z_3$$

- Complex field → Noise!
- How bad is it?
  - Performed checks at various values of y
  - Ok for y $\lesssim$ 1

Tuning – Solve Integral Equation



Conclusions

- Lattice method provides computationally efficient calculations of large systems of interacting fermions
- Promising results for fermions in a box and in a harmonic potential show method can give high precision results
- First steps toward calculations of properties of nuclei underway

Conclusions

- Lattice method provides computationally efficient calculations of large systems of interacting fermions
- Promising results for fermions in a box and in a harmonic potential show method can give high precision results
- First steps toward calculations of properties of nuclei underway
- Much of this work performed on New York Blue/L

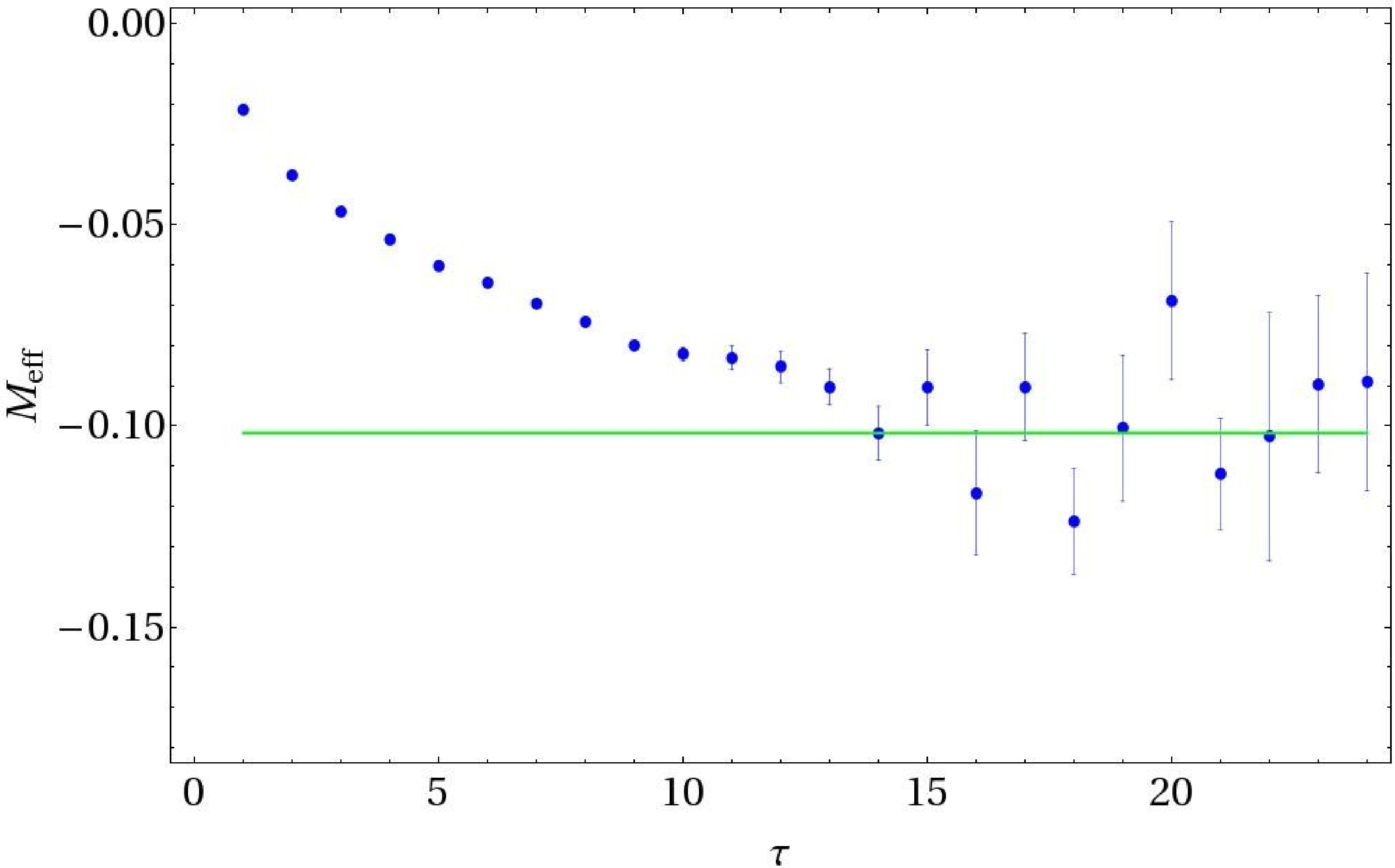
A=f+G A

$$f_p = \kappa^2 \left(\frac{e^{-iE_D} e^{\frac{3p^2}{4M}}}{1 - e^{iE_D} e^{\frac{p^2}{4M}}} + \frac{y^3}{\kappa^2} g(p,0) \right)$$

$$G_{pq} = \kappa^2 \frac{e^{iE_D} e^{-\frac{3p^2}{4M}} e^{-\frac{q^2}{2M}}}{1 + \kappa^2 g(p,q)} \left(\frac{e^{-iE_D} e^{\frac{3p^2}{4M}}}{1 - e^{\frac{q^2}{M}} e^{iE_D} e^{\frac{pq}{M}}} + \frac{y^3}{\kappa^2} g(p,q) \right)$$

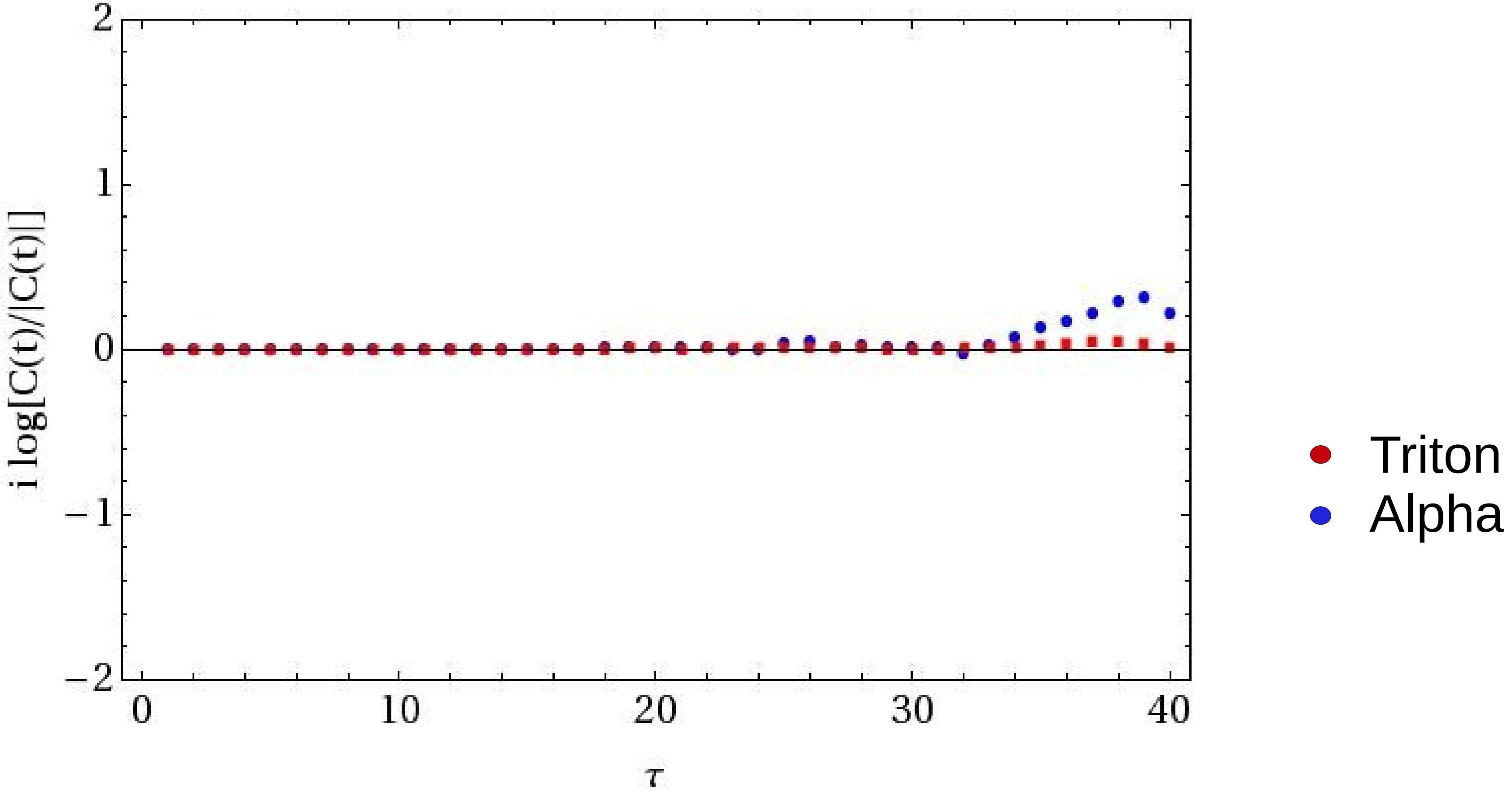
$$g(p,q) \equiv \frac{1}{V} \sum_{\{|q^{'}|\}} \frac{\mathcal{R}(|q^{'}|)}{1 - e^{iE_D} e^{-\frac{3p^2}{4M}} e^{\frac{3q^2}{4M}} e^{\frac{q^{'2}}{M}}},$$

Alpha



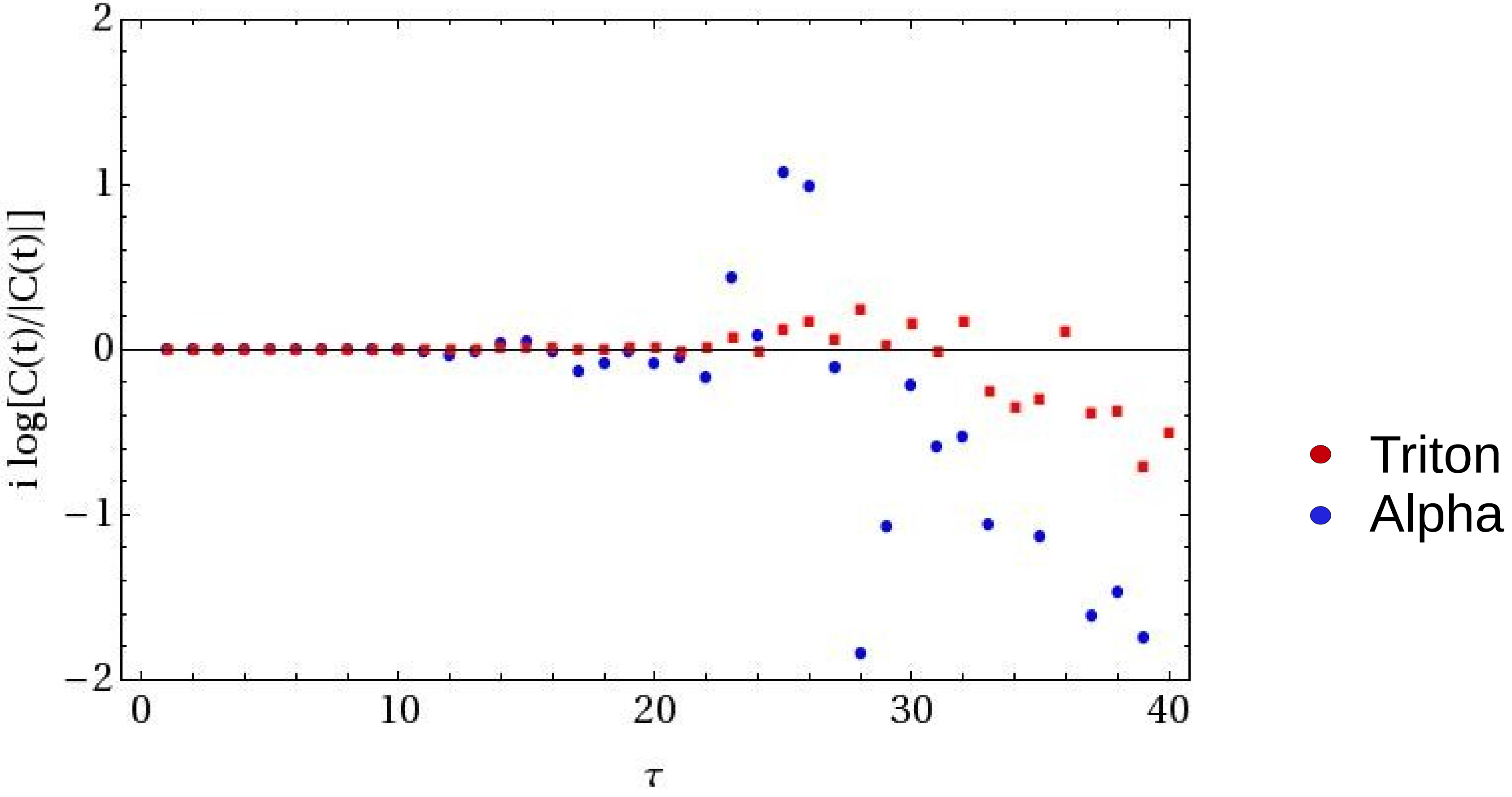
y = 0.0

Phase



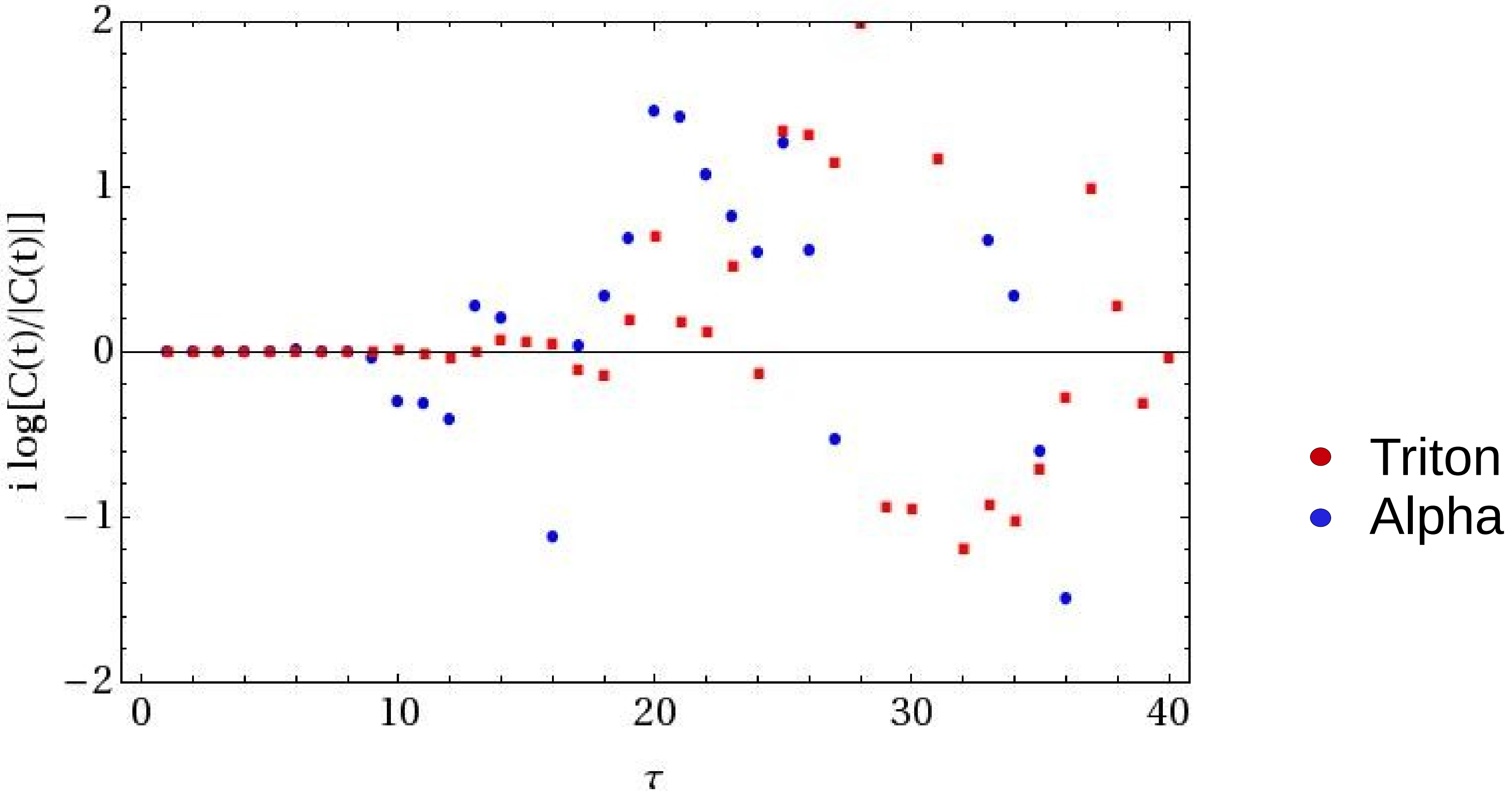
y = 0.1

Phase

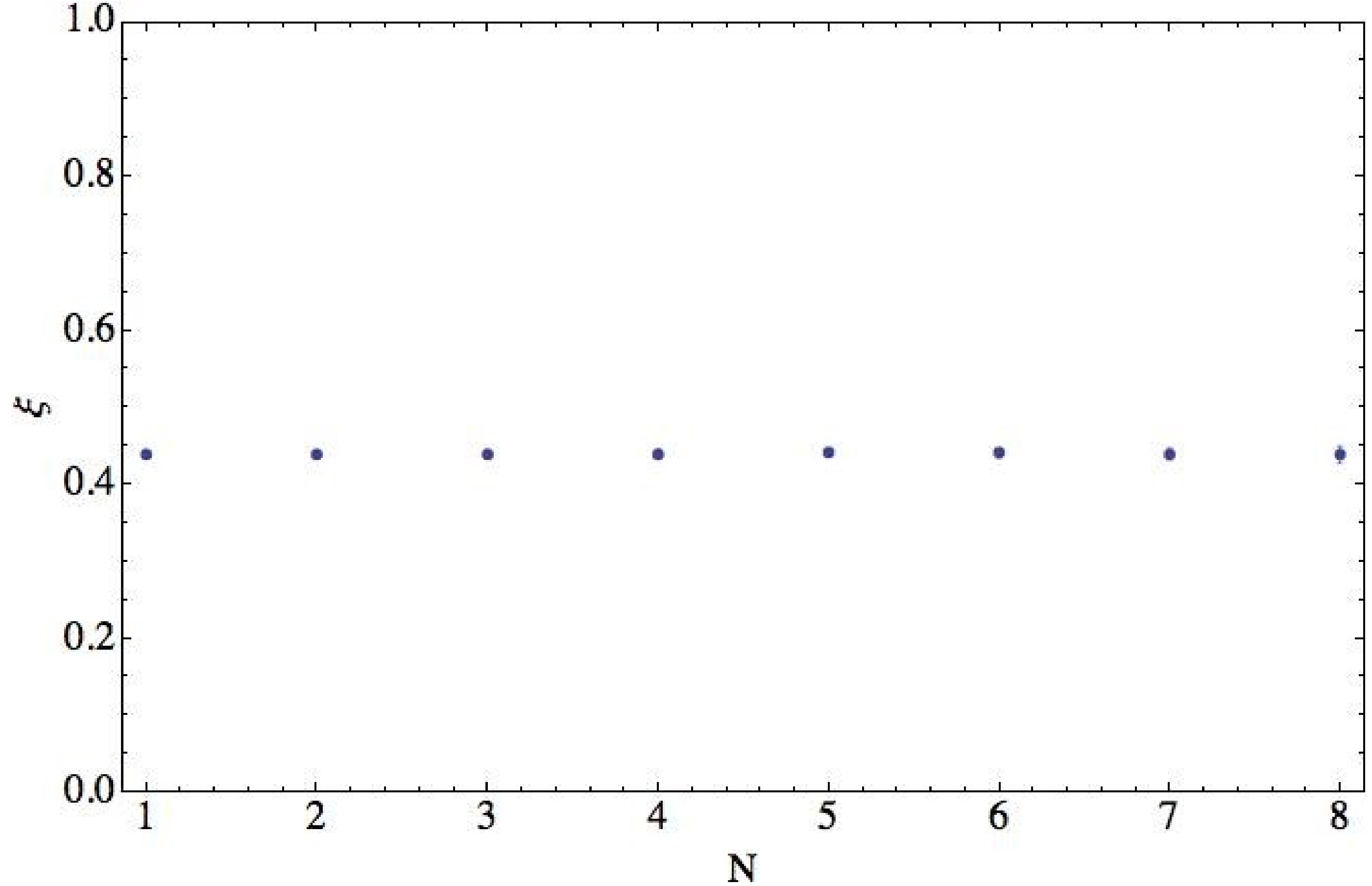


y = 0.5

Phase



y = 1.0



1.0
0.8
0.6
0.4
0.2
0.0
ξ
1
2
3
4
5
6
7
8
N

Temporal Errors

Some improvement found for

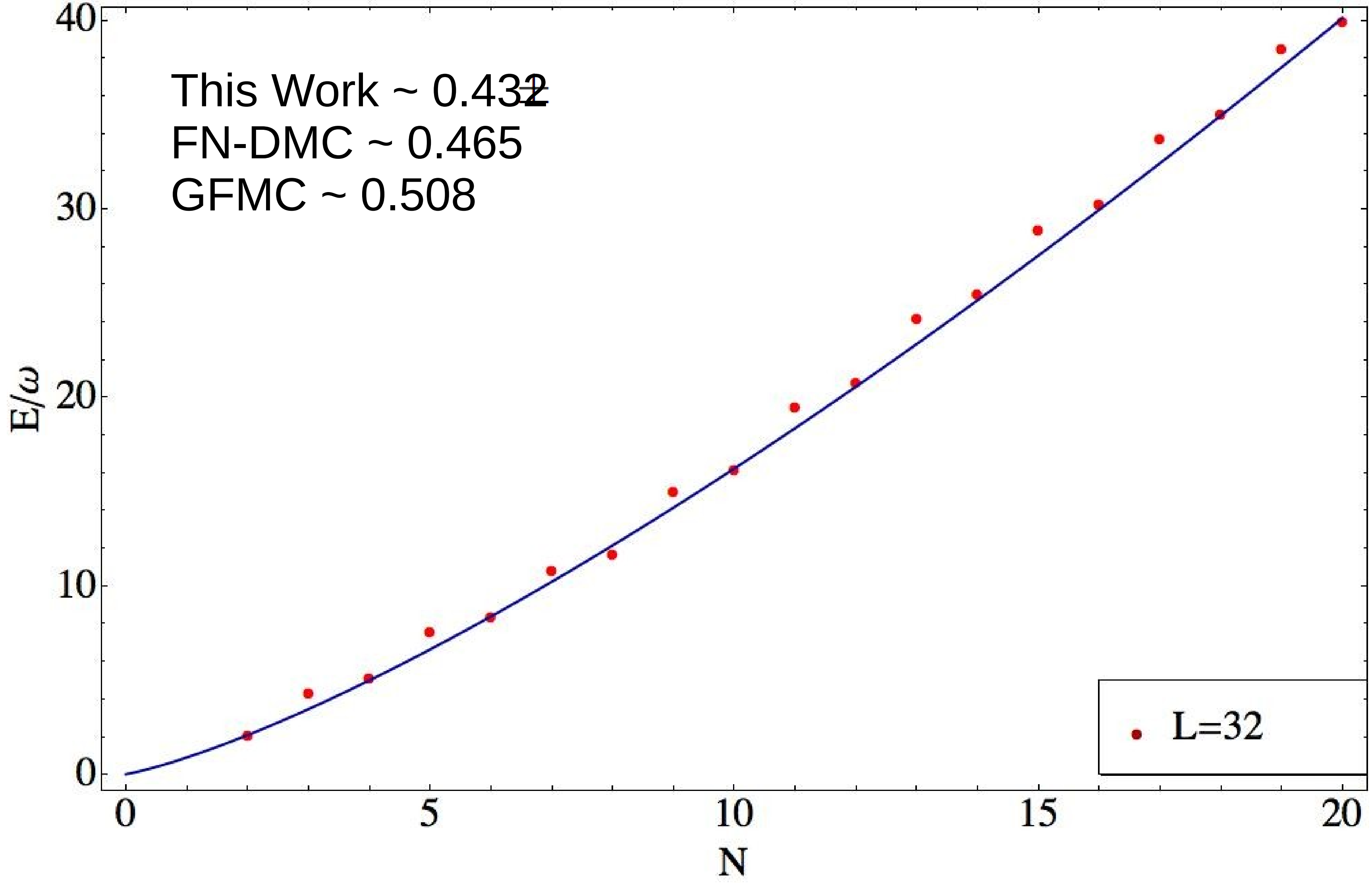
$$V_{SHO} b_t \rightarrow 1 - e^{V_{SHO} b_t}$$

Can also tune “bare” ω to eliminate b_t errors to all orders for non-interacting fermions

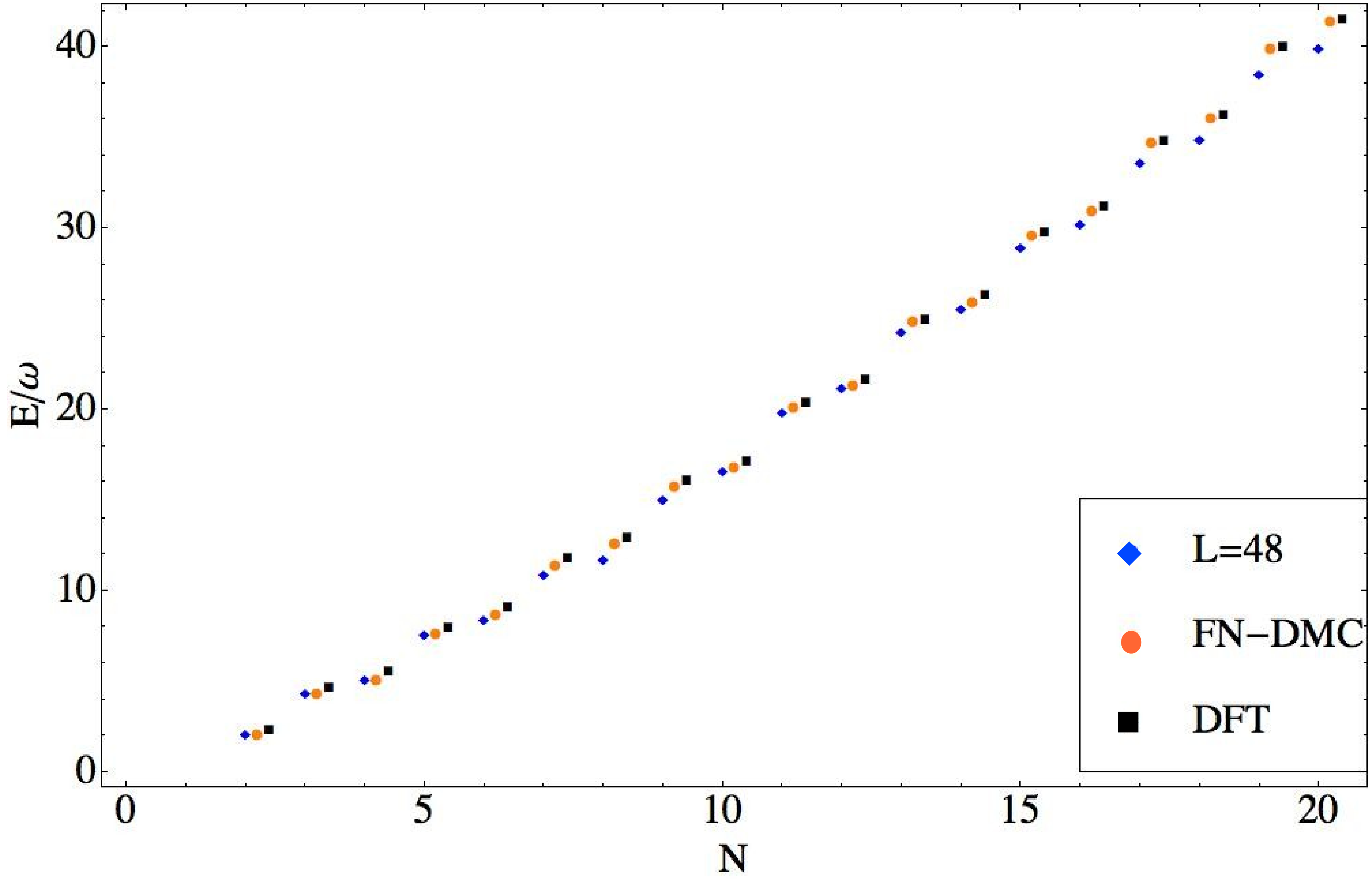
$$(\omega')^2 = \frac{\cosh^{-1}[1 + \omega^2 b_t^2/2])^2}{b_t^2}$$

Ensure small b_t errors by choosing small ω

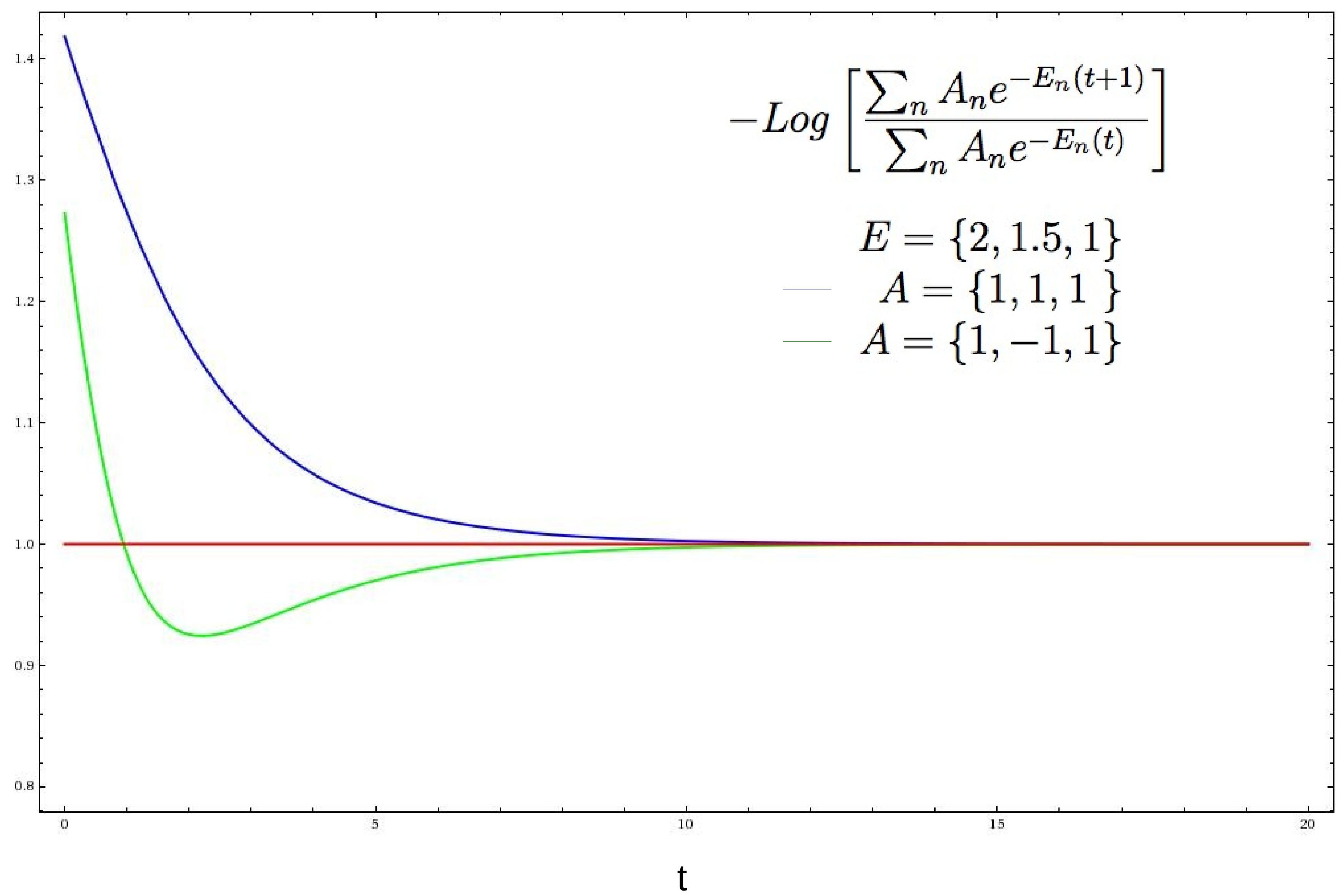
Bertsch Parameter



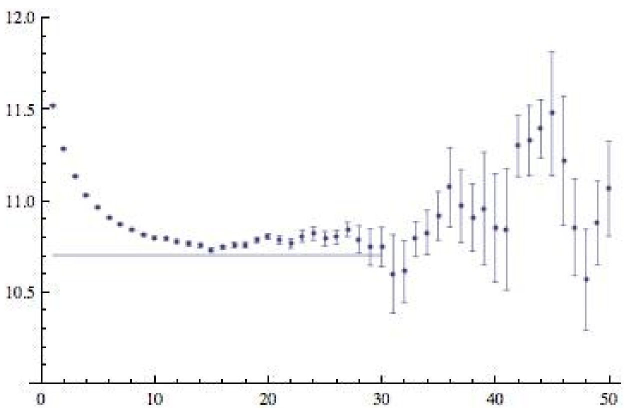
Other Methods

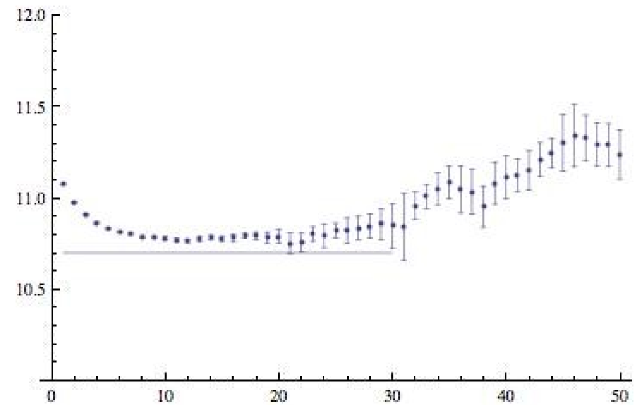


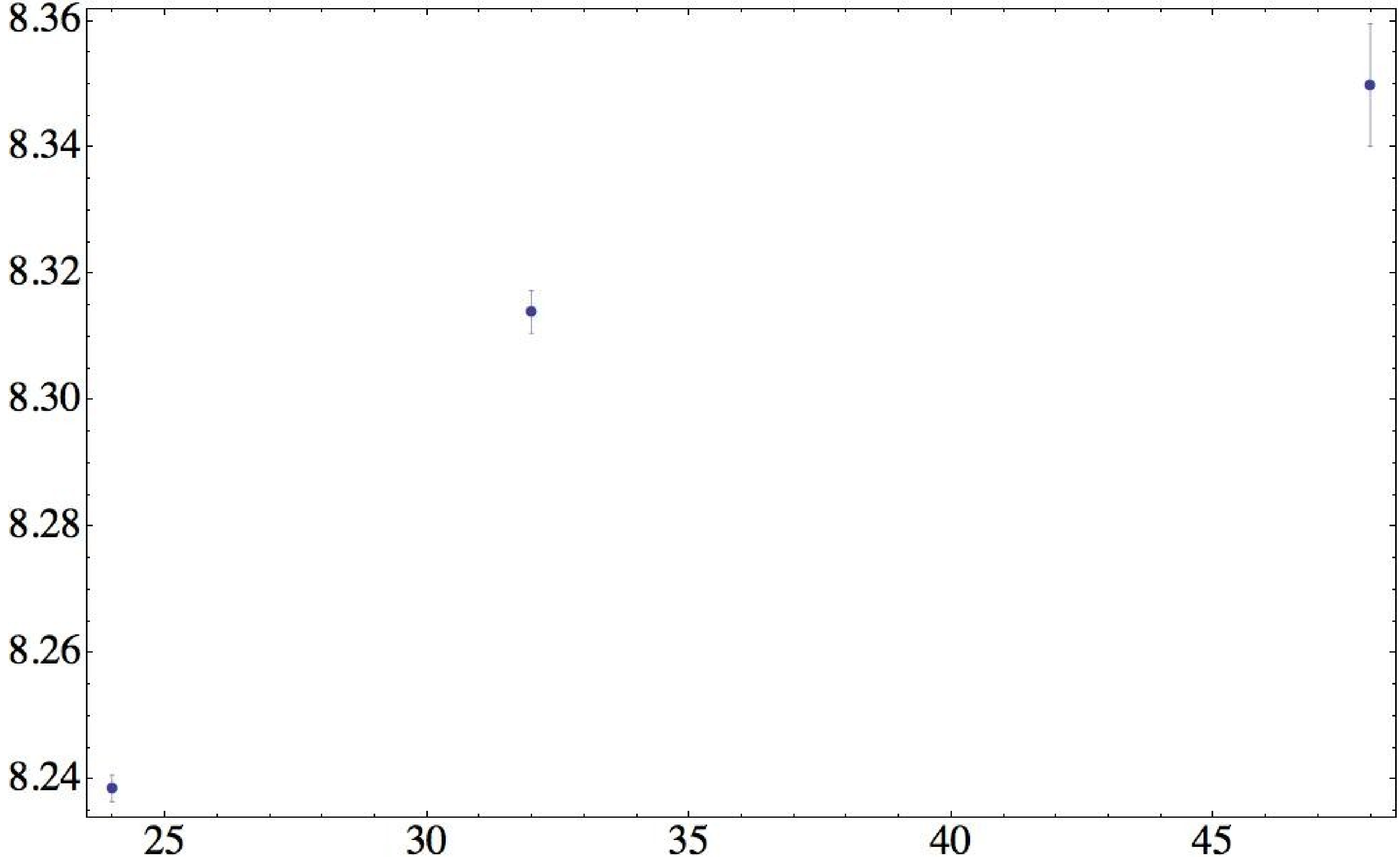
- FN-DMC: D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734
- DFT: A. Bulgac, arxiv:cond-mat/0703526

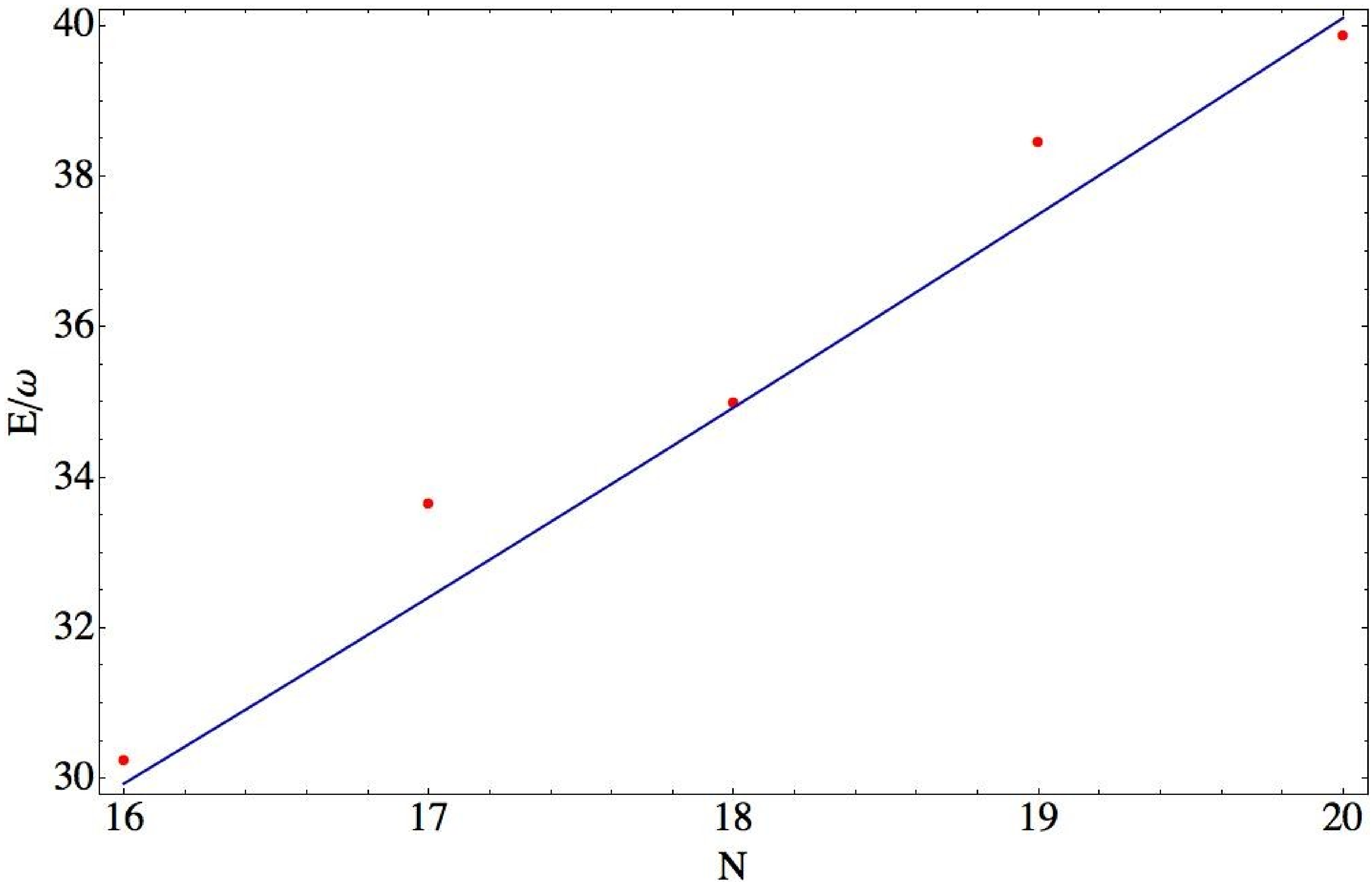


$-Log\left[\frac{\sum_n A_n e^{-E_n(t+1)}}{\sum_n A_n e^{-E_n(t)}}\right]$
$E = \{2, 1.5, 1\}$
$A = \{1, 1, 1\}$
$A = \{1, -1, 1\}$
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
5
10
15
20
t


12.0
11.5
11.0
10.5
0
10
20
30
40
50


12.0
11.5
11.0
10.5
0
10
20
30
40
50


8.36
8.34
8.32
8.30
8.28
8.26
8.24
25
30
35
40
45


E/ω
N
16
17
18
19
20
30
32
34
36
38
40