Pion-nucleon scattering around the delta-isobar resonance

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What do we really do

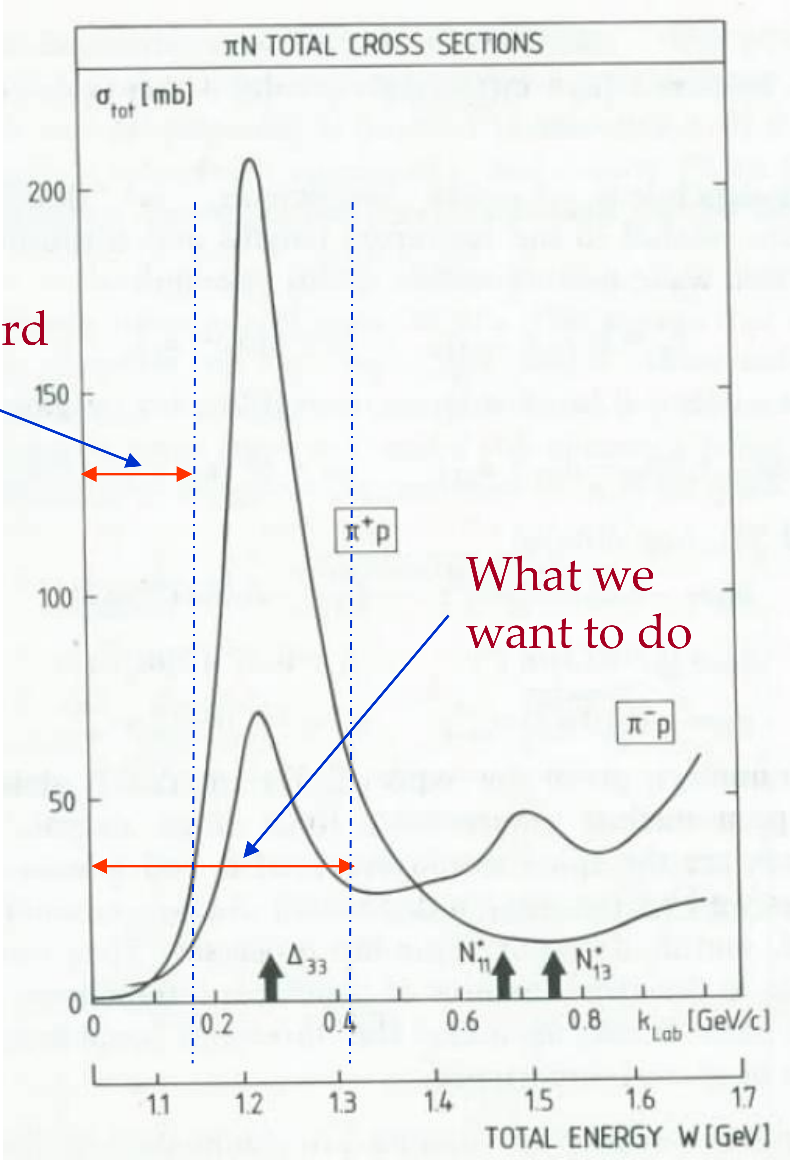
Fettes & Meissner 2001...

Standard ChPT

What we want to do

$\Delta(1232)\ J^P = \frac{3}{2}^+$

Isospin 3/2



Outline

Why delta? why EFT?

Standard ChPT power counting

Power counting of resumming delta

Some technicalities

P-wave phase shifts

Summary

DOFs of nuclear physics

Baryons		Mesons (mediating long-range forces)	
N	940MeV	π	140MeV
Δ	1232MeV	σ	(5~6)00MeV
$N^{\star}$	1440MeV (Roper)	ρ	770MeV
⋮		ω	782MeV
		⋮	

N, Δ excitations

break-down scale

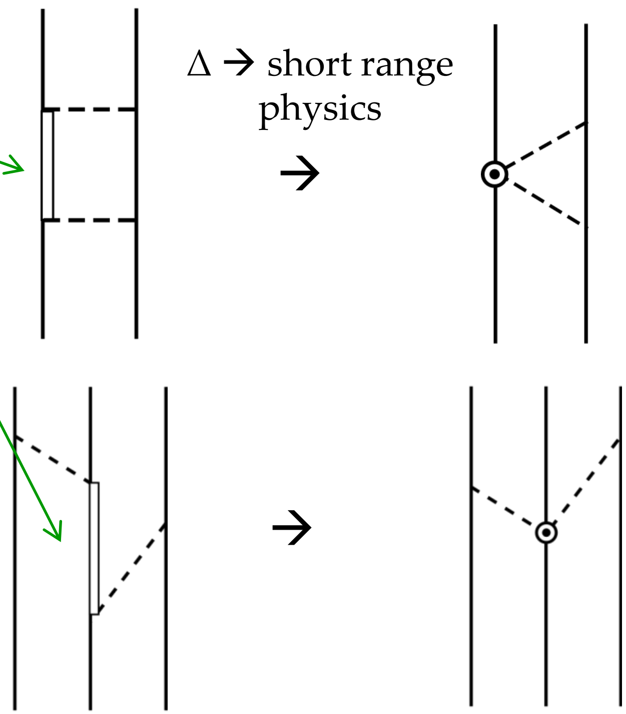
"most conventioal" chiral EFT

π N Δ $N^{\star}$

Delta in nuclear forces

$$\delta = m_\Delta - m_N \simeq 300\text{MeV} \simeq 2m_\pi \ll m_\sigma$$

Delta-ful | Delta-less



Delta as an explicit DOF

→ better convergence of EFT expansions, at the price of more DOFs

Delta becomes short-range physics

EFT expansion in powers of $m_\pi / \delta \sim 0.5$

Why effective field theory ?

Several non-EFT or quasi-EFT approaches describe pion-nucleon data well

- K-matrix based models
- Unitarized meson-exchange
- Unitarized chiral perturbation theory (ChPT)

  ......

But, with a proper EFT

- Model-independent, **controlled** low-energy approximation to QCD
- Provides inputs to other nuclear reactions through low-energy constants (**LECs**)
- Natural framework to take data from **lattice QCD**

Energy levels by EFT $\xrightarrow{\text{LQCD data}}$ LECs $\xrightarrow{\text{EFT in cont.}}$ nuclear reactions

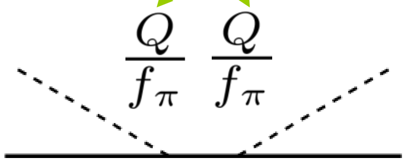
"Standard" ChPT counting

Q : generic momenta

--- πN scattering

chiral symmetry → derivative couplings

$\frac{Q}{f_\pi^2}$:

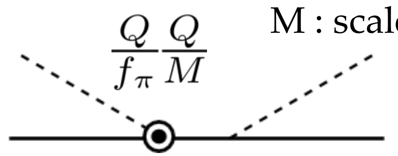


nucleon prop. $\sim 1/Q$

……

M : scale of underlying theory

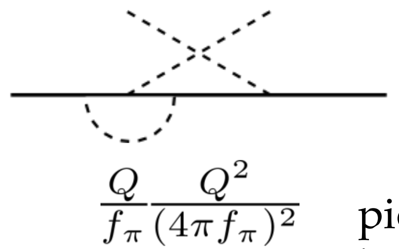
$\frac{Q}{f_\pi^2}\frac{Q}{M}$:



one more derivative

……

$\frac{Q}{f_\pi^2}\frac{Q^2}{M^2}$:



……

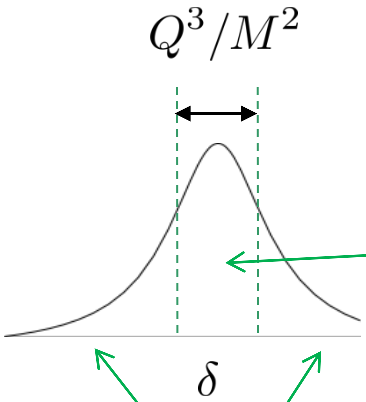
pion loop suppressed by $Q^2/(4\pi f_\pi)^2$

naturalness → $M \sim 4\pi f_\pi \simeq 1.2\mathrm{GeV}$

But, perturbative series does not generate resonances → $M < 4\pi f_\pi$

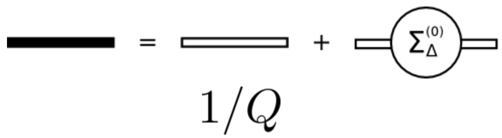
Power counting ingredients

Pascalutsa & Phillips (2003)
nucleon Compton scattering
Bedaque, Hammer, UvK(2002)
n-alpha scattering



(1) resum Δ pole diags. within the resonance window

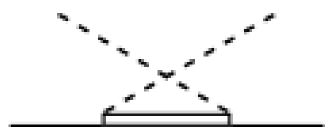
not necessary to resum



$\sim \frac{M^2}{Q^3}$

(2) dressed Δ propagator enhanced by $\sim M^2/Q^2$ relative to bare one

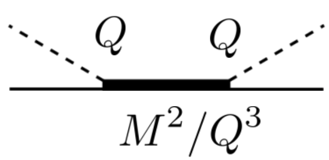
(3) u-channel not enhanced



$\sim \frac{Q^2}{E+\delta} \sim Q$

Background vs. Pole

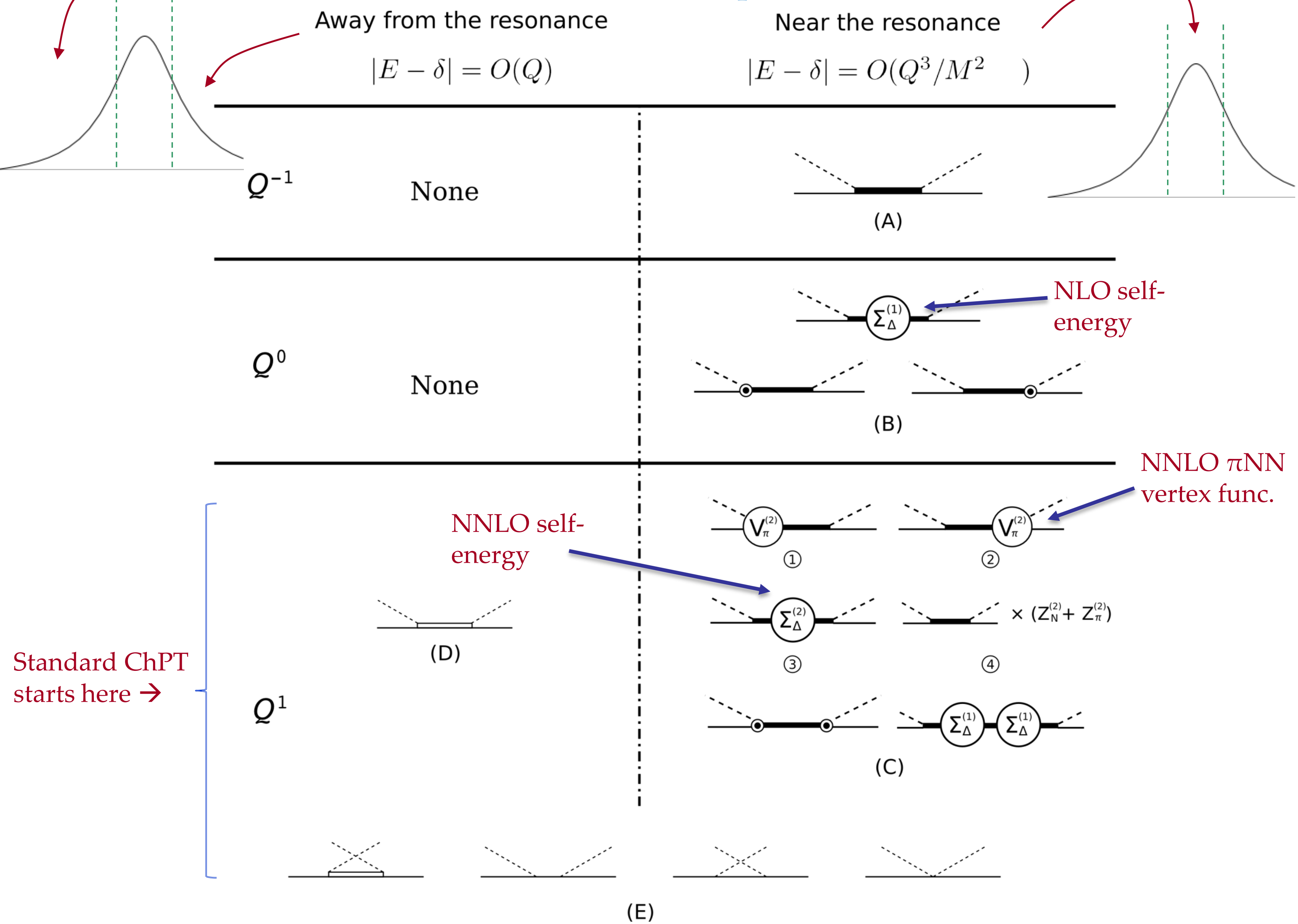
LO pi-N scattering :



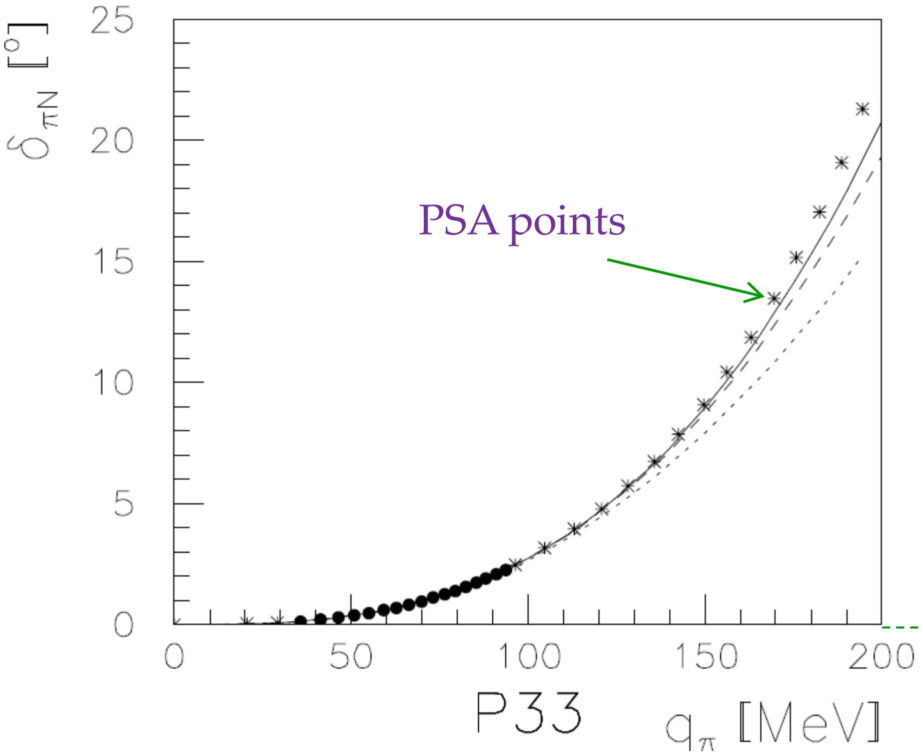
$\sim Q^{-1}$ Breit-Wigner

Higher orders → systematic improve over Breit-Wigner

Road map



Away from the resonance
$|E - \delta| = O(Q)$
Near the resonance
$|E - \delta| = O(Q^3/M^2)$
Q^{-1}
None
(A)
Q^0
None
NLO self-energy
(B)
Q^1
NNLO self-energy
NNLO πNN vertex func.
①
②
③
④
$\times (Z_N^{(2)} + Z_\pi^{(2)})$
(C)
(D)
Standard ChPT starts here →
(E)



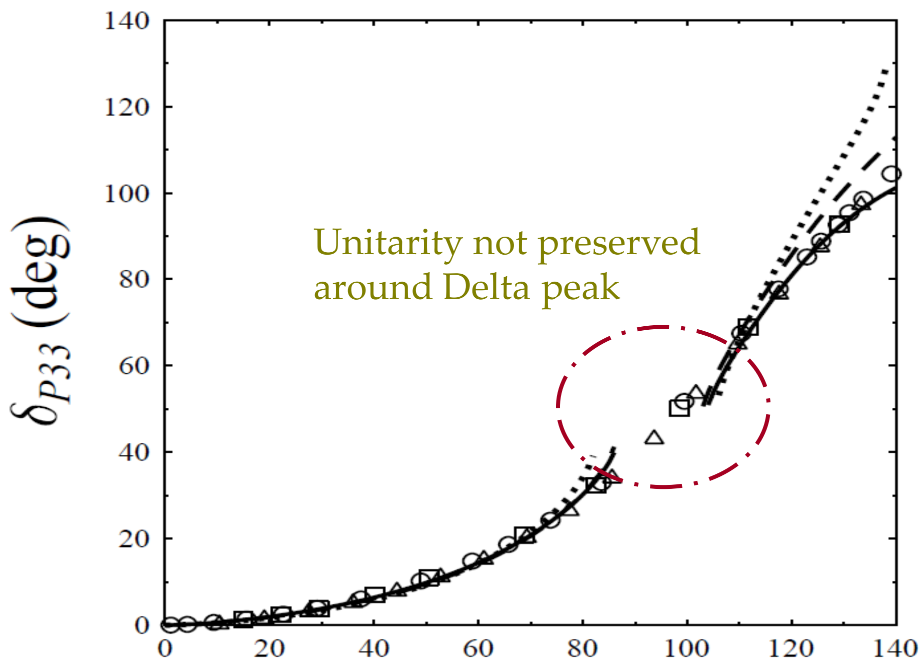
Δ
300

Fettes & Meissner (2001)
Delta-ful, w/o resummation → can't get the pole

Solid line : $\mathcal{O}(p^3)$

q_π lab momentum

$W_{CM} \sim 1232$ MeV $\Leftrightarrow q_\pi \sim 297$ MeV



Tang & Ellis (1998)
Delta-ful, with resummation

Curves : different fits $\mathcal{O}(p^4)$

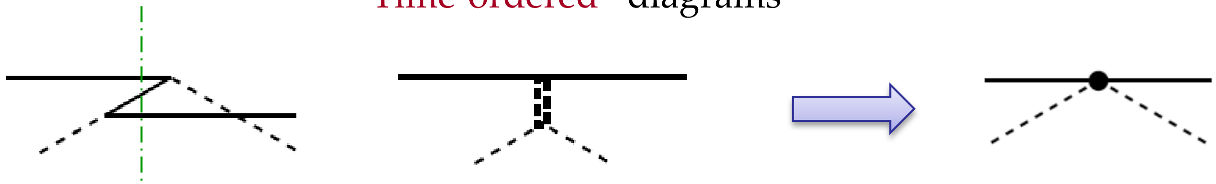
"standard" counting with delta prop. dressed to only LO

imbalance of accuracies in $\pi N \Delta$ vertex & delta prop

T_π CM pion kin. eng.

- Heavy-baryon ChPT

"Time-ordered" diagrams



$N - \bar{N}$ pair

$\sim 2m_N$ above threshold

Non-Goldstone boson with

$m \geqslant m_\sigma$

Integrating out anti-N

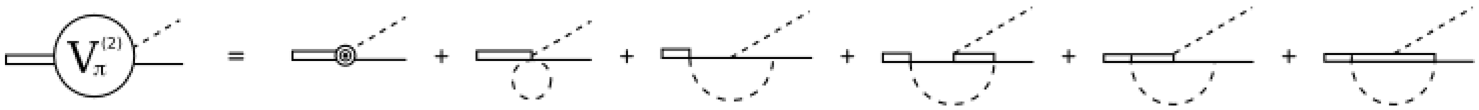
→constraint of micro. causality indistinguishable from other short-distance physics

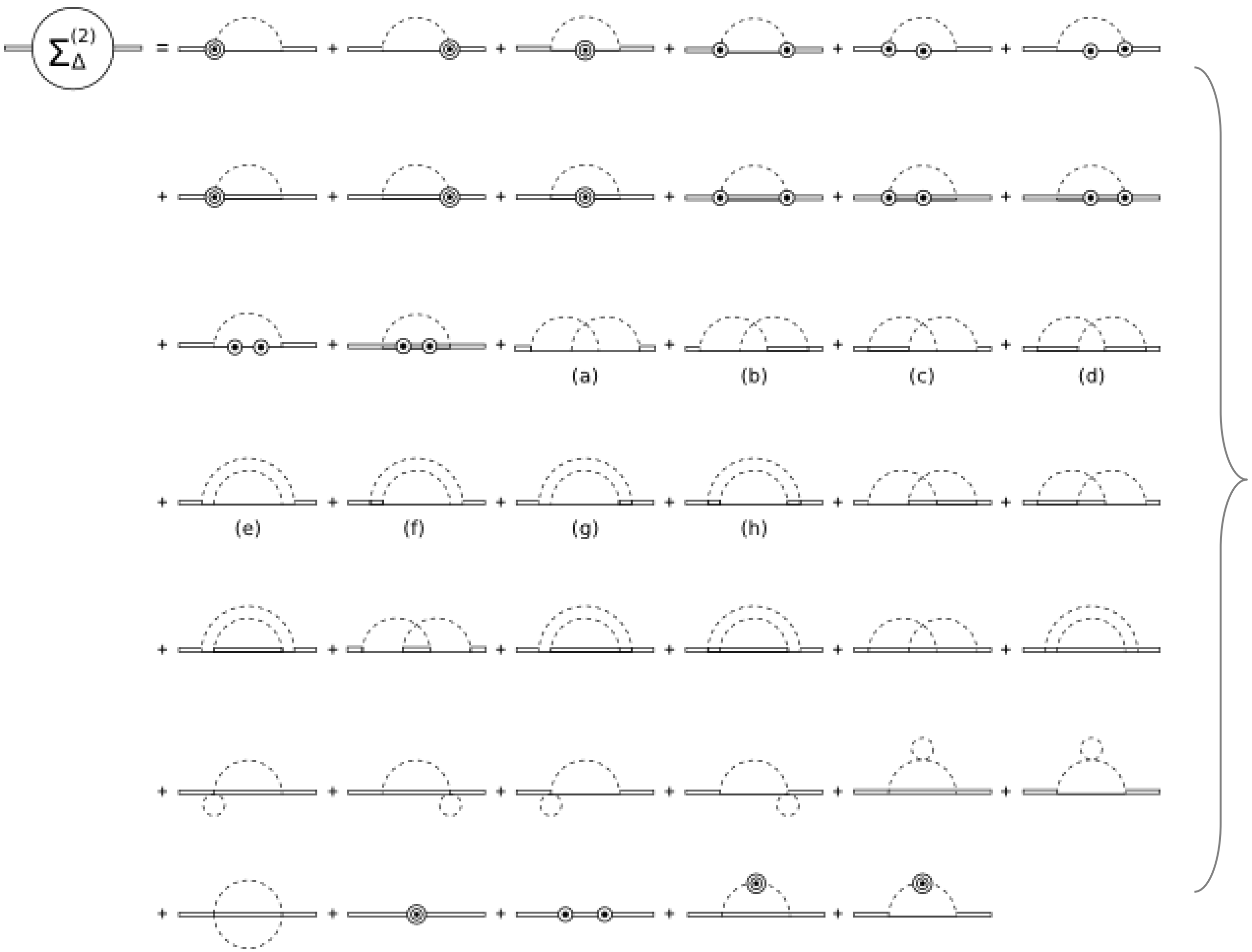
- Why explicity delta field ?

Resumming p-wave contact int. seems to break RG inv.

$\sim \nabla^2$ mass dim. too high to faciliate renormalization

Time to work...





Only the imaginary part needed

deal with two-loop graphs

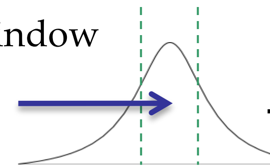
Define δ

$$S^{-1} = E - \delta + \Sigma(E) \qquad S =$$

$$\delta - \mathrm{Re}\Sigma(\delta) \to \delta \quad \text{so that } \mathrm{Re}\Sigma(\delta) = 0$$

with a narrow resum window

$$|E-\delta| \sim Q^3/M^2$$



$$\to \Sigma(E) = \Sigma(\delta) + \underbrace{(E-\delta)\Sigma'(\delta)}_{\mathcal{O}(Q^2/M^2)\text{ smaller}} + \cdots$$

$$S^{-1} = E - \delta + \mathrm{Im}\Sigma^{(0)}(\delta) + \mathrm{Im}\Sigma^{(1)}(\delta) - Z_\Delta^{(2)}(E-\delta) + (E-\delta)\,\mathrm{Im}\Sigma^{(0)}(\delta) + \mathrm{Im}\Sigma^{(2)}(\delta) + \cdots$$

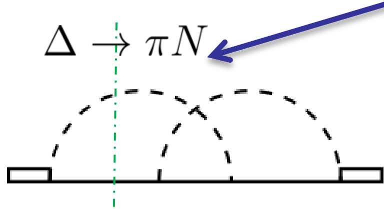
$$Z_\Delta^{(2)} = -\mathrm{Re}\Sigma^{(0)\prime}(\delta)$$

Δ field ren. const. , divergent

optical theorem

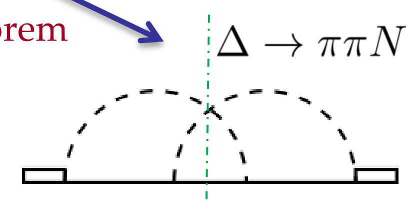
$\Delta \to \pi N$

$$\frac{Q^5}{M^4}$$



pion momenta $Q \sim \delta$

$\Delta \to \pi\pi N$

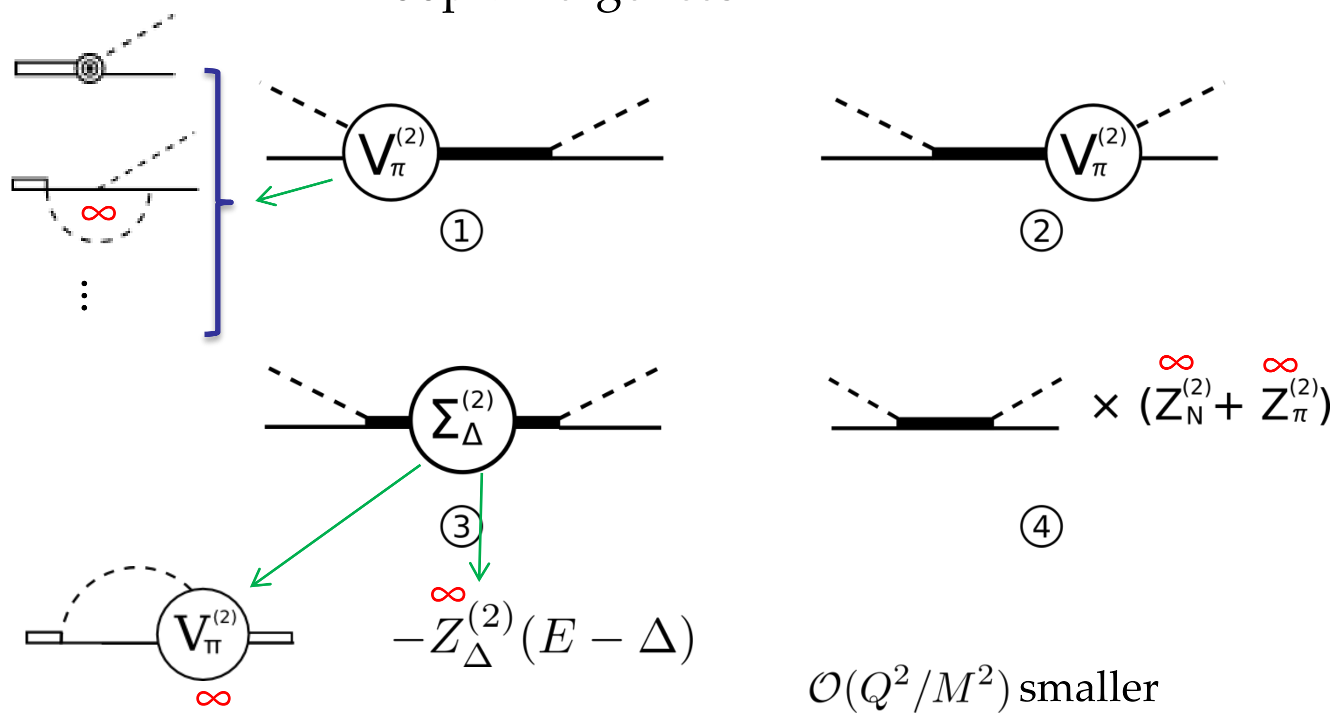


→ can be ignored

$$\tilde{Q} \sim 40\text{MeV} \ll \delta$$

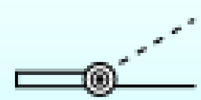
NNLO Renormalization

∞ : loop divergences



$\mathcal{O}(Q^2/M^2)$ smaller

$$|E-\delta| \sim Q^3/M^2 \rightarrow V_\pi(E) = V_\pi(\delta) + \underbrace{(E-\delta)V'_\pi(\delta)}_{} + \cdots$$

→All ∞ absorbed into δ, h_A, and 

Effective Lagrangian

f_π : pion decay constant

g_A : nucleon axial coupling

g_A^Δ : $\pi\Delta\Delta$ coupling

$\boldsymbol{T}(\vec{S})$: 2×4 matrices in isospin (spin) space

$$D = 1 + \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\,, \quad \vec{\boldsymbol{D}} = \frac{\vec{\nabla}\boldsymbol{\pi}}{2f_\pi} + \cdots\,, \quad \vec{\mathscr{D}}N = \vec{\nabla}N + \cdots$$

$$\begin{aligned}\mathcal{L}^{(0)} &= 2f_\pi^2\boldsymbol{D}^2 - \frac{1}{2D}m_\pi^2\boldsymbol{\pi}^2 + N^\dagger i\mathscr{D}_0 N + g_A N^\dagger\boldsymbol{\tau}\vec{\sigma}N\cdot\cdot\vec{\boldsymbol{D}} \\ &+\Delta^\dagger\left(i\mathscr{D}_0-\delta\right)\Delta + g_A^\Delta\Delta^\dagger\boldsymbol{t}^{(\frac{3}{2})}\vec{S}^{(\frac{3}{2})}\Delta\cdot\cdot\vec{\boldsymbol{D}} + h_A\left(N^\dagger\boldsymbol{T}\vec{S}\Delta + H.c.\right)\cdot\cdot\vec{\boldsymbol{D}} + \cdots\,,\end{aligned}$$

iso-vector contraction

spatial-vector contraction

$\pi N\Delta$ couplings

constrained by Lorentz inv.

$$\mathcal{L}^{(1)} = \frac{1}{2m_N}\left(N^\dagger\vec{\mathscr{D}}^2N + \Delta^\dagger\vec{\mathscr{D}}^2\Delta\right) - \frac{h_A}{m_N}\left(iN^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\Delta + H.c.\right)\cdot\boldsymbol{D}_0 + \cdots\,,$$

Cohen, Friar, Miller, & van Kolck (1996)....

- Remove redundancy due to integrations by part, EOM
- Next-to-leading $\pi N\Delta$ coupling : pure relativistic correction
- No new low-energy constant (LEC)

Effective Lagrangian

- Next-to-next-to-leading $\pi N\Delta$ couplings : 1 LEC

1/m² relativistic corrections

$$\mathcal{L}^{(2)} = -\frac{\Delta m_\pi^2}{2D^2}\boldsymbol{\pi}^2 - \frac{\delta}{2m_N^2}\Delta^\dagger\vec{\mathscr{D}}^2\Delta + \frac{h_A}{2m_N^2}\left[\left(N^\dagger \boldsymbol{T}\vec{S}\vec{\mathscr{D}}^2\Delta - N^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\vec{\mathscr{D}}\Delta\right) + H.c.\right]\cdot\cdot\vec{\boldsymbol{D}}$$
$$+\frac{h_A}{8m_N^2}\left[\left(\delta_{lm}N^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\Delta + 3N^\dagger\boldsymbol{T}S_l\mathscr{D}_m\Delta + 2\epsilon_{ijl}N^\dagger\boldsymbol{T}\Omega_{im}\mathscr{D}_j\Delta\right) + H.c.\right]\cdot\mathscr{D}_l\boldsymbol{D}_m$$
$$+d\,\frac{m_\pi^2}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\left(N^\dagger\boldsymbol{T}\vec{S}\Delta + H.c.\right)\cdot\cdot\vec{\boldsymbol{D}} + \cdots \qquad (16)$$

Results

- LO : recovering the Breit-Wigner formula

P_{33} partial-wave amplitude (P-wave, I = 3/2, & J=3/2)

$$T_{P_{33}}^{\mathrm{LO}} = -\frac{\gamma^{(0)}(\delta)}{E-\delta+i\gamma^{(0)}(\delta)/2}$$

$$\begin{aligned}\gamma^{(0)}(\delta) &= \mathcal{N}(k_\delta)\frac{h_A^2 k_\delta^3}{24\pi f_\pi^2} \\ &= \frac{h_A^2}{24\pi f_\pi^2}\frac{\left(\delta^2-m_\pi^2\right)^{\frac{3}{2}}}{16\left(\delta+m_N\right)^5}\left[\left(\delta+2m_N\right)^2-m_\pi^2\right]^{\frac{3}{2}}\left[\left(\delta+m_N\right)^2+m_N^2-m_\pi^2\right]\end{aligned}$$

with $\mathcal{N}(k) \equiv \dfrac{\sqrt{k^2+m_N^2}}{\sqrt{k^2+m_N^2}+\sqrt{k^2+m_\pi^2}}$

2 free parameters : h_A, δ

leading $\pi N \Delta$ coupling

- NLO : vanishes in CMS

Results

- NNLO : beyond the Breit-Wigner formula

$$T_{P_{33}}^{\mathrm{LO+NNLO}} = -\frac{\Gamma(E)}{E-\delta+i\Gamma(E)/2}\left[1+iT_B(E)\right]+T_B(E)$$

$$\Gamma(E) \equiv \frac{[h_A(1+\varkappa)]^2}{24\pi f_\pi^2}k^3\mathcal{N}(k)$$

Δ axial coupling

$$\begin{aligned}\varkappa &= \frac{d}{h_A}m_\pi^2 + \frac{k_\delta^2}{(4\pi f_\pi)^2}\left\{\left[\left(\frac{2}{3}g_A^2+\frac{1}{54}h_A^2+\frac{27}{128}{g_A^\Delta}^2\right)-\frac{15}{108}h_A^2\left(1+\frac{m_\pi^2}{k_\delta^2}\right)\right]\right.\\ &\quad\times\left(1+\frac{m_\pi^2}{k_\delta^2}\right)^{-\frac{1}{2}}\ln\left(\frac{\sqrt{1+(m_\pi/k_\delta)^2}-1}{\sqrt{1+(m_\pi/k_\delta)^2}+1}\right)\\ &\quad\left.+\pi\left(\frac{27}{128}{g_A^\Delta}^2-\frac{2}{3}g_A^2\right)\frac{m_\pi^3}{k_\delta^3}\left(1+\frac{m_\pi^2}{k_\delta^2}\right)^{-\frac{1}{2}}\right\}\end{aligned}$$

NNLO $\pi N\Delta$ coupling

$$T_B(E) = \frac{k^3}{24\pi f_\pi^2}\left(4\frac{g_A^2}{E}+\frac{1}{9}\frac{h_A^2}{E+\delta}\right)$$

non-resonant “background”

1 free parameters at NNLO : $\varkappa$

Results

- NNLO : non-delta P-waves P_{11} , P_{13} , & P_{31}

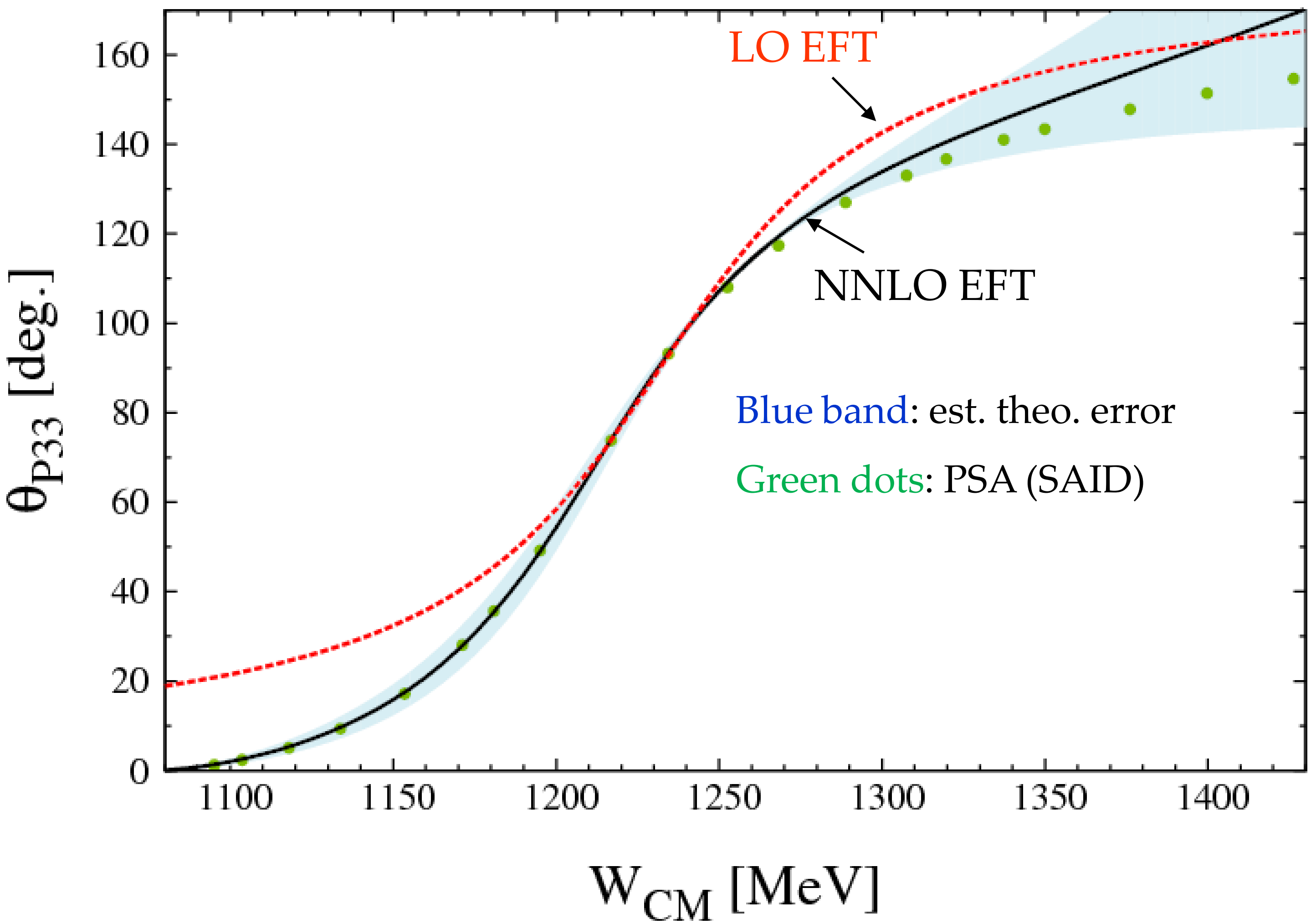
$$T_{P_{13}}^{\mathrm{NNLO}} = T_{P_{31}}^{\mathrm{NNLO}} = \frac{1}{4} T_{P_{11}}^{\mathrm{NNLO}} = -\frac{k^3 \mathcal{N}(k)}{12\pi f_\pi^2} \left(\frac{g_A^2}{E} - \frac{2}{9} \frac{h_A^2}{E+\delta} \right)$$

contribution of the delta

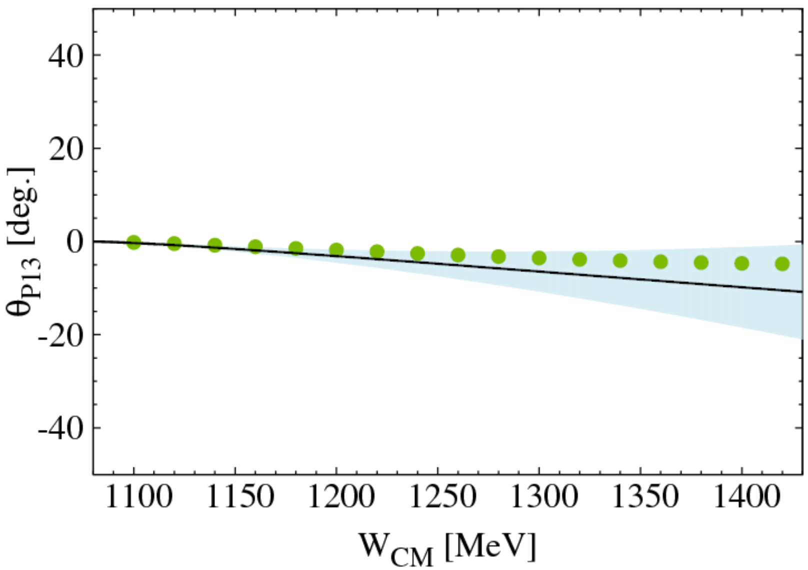
Large N_c limit: $h_A^2 = \frac{9}{2} g_A^2$

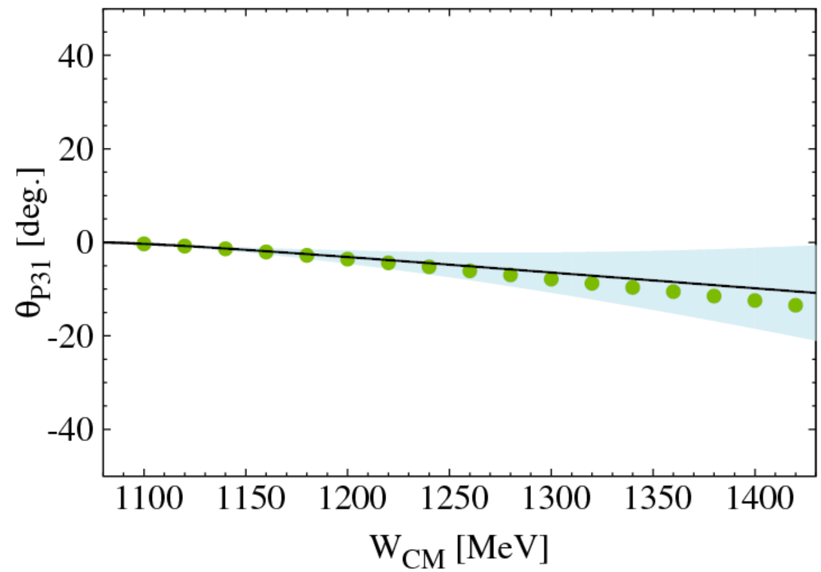
P_{33} phase shifts

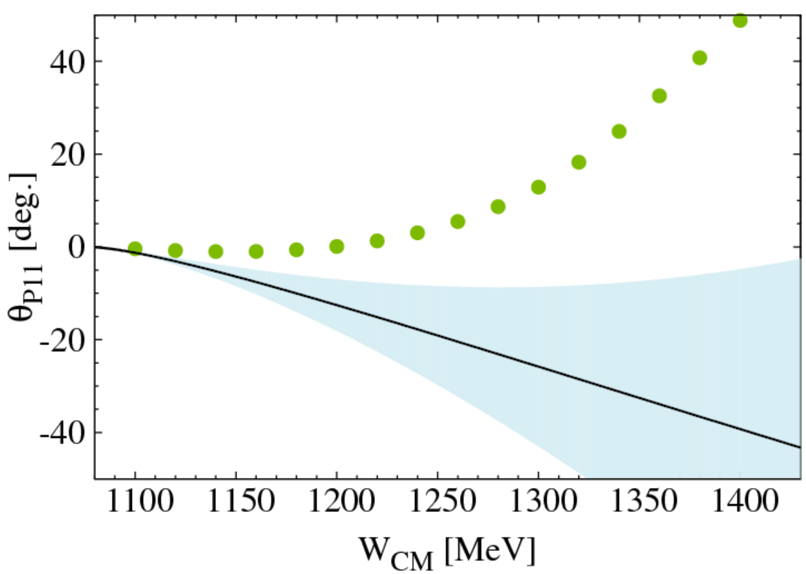
Fitted to the phase-shift analysis of SAID (George Washington group)



other P-waves







Still at O(Q)

Roper N*(1440)

Including Roper (N*)

A **preliminary** study before looking into the Roper region

consider intermediate kinematic domain : $W_{\mathrm{CM}} \sim m_\Delta$ or $E \sim \delta$

$$\delta \equiv m_\Delta - m_N \sim 300\mathrm{MeV}$$

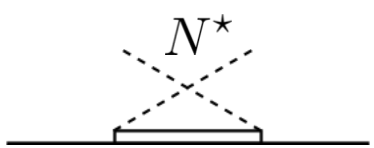
$$\rho \equiv m_{N^\star} - m_N \sim 500\mathrm{MeV}$$

Q

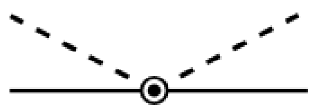
$$\frac{1}{E-\rho} \sim \frac{1}{Q}$$

→ P11

Q^2



$$\frac{1}{E+\rho} \sim \frac{1}{M}$$



→ All S, P-waves

To answer : How much the Roper helps ?

Outlook & Summary

Model-independent framework to study pion-nucleon system

Power counting that unifies **pole** and **background** contributions

Extracting delta-related LECs → other nuclear reactions, eg. nucl-force

Pushing for higher-order calculations

To the Roper : N*(1440)