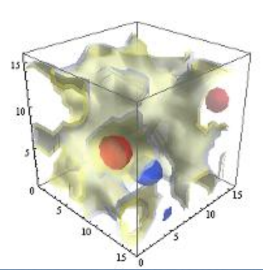
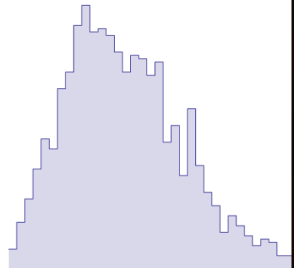
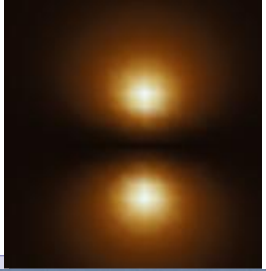
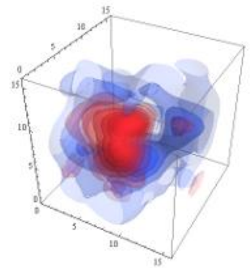
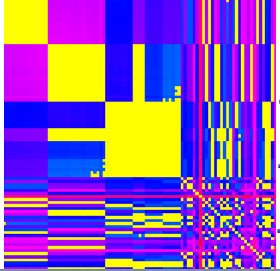
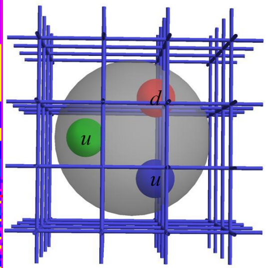
      

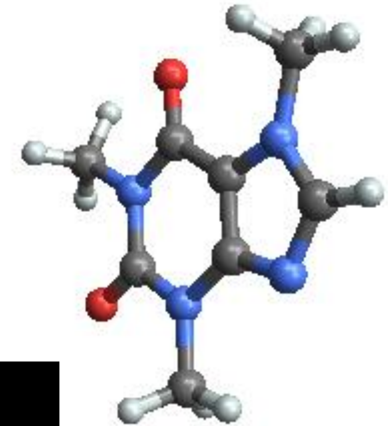
Lattice QCD and Hadron Structure

Huey-Wen Lin
University of Washington

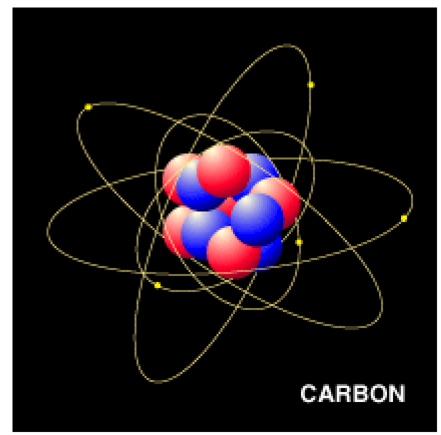
Human Exploration

- Matter has many layers of structure

10^{-9} m

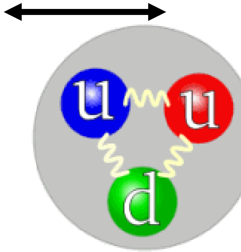


10^{-2} m

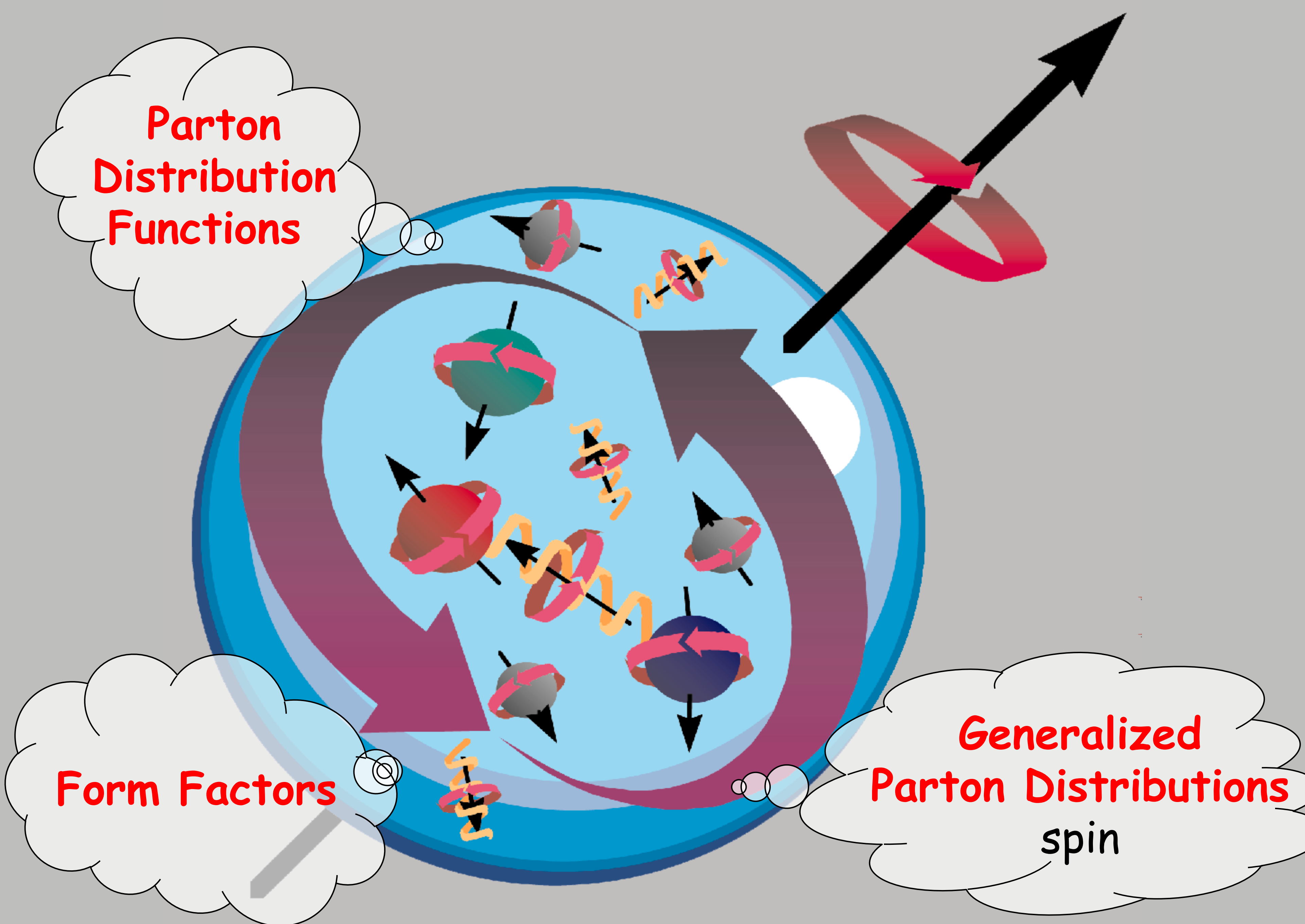


10^{-15} m

Proton



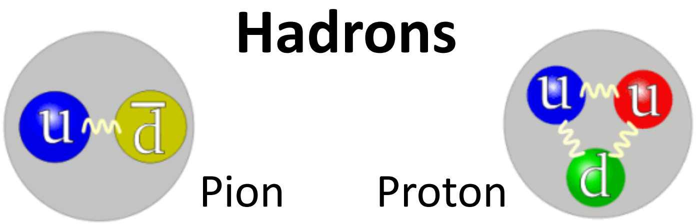
- The scientific cycle


Parton Distribution Functions
Form Factors
Generalized Parton Distributions
spin

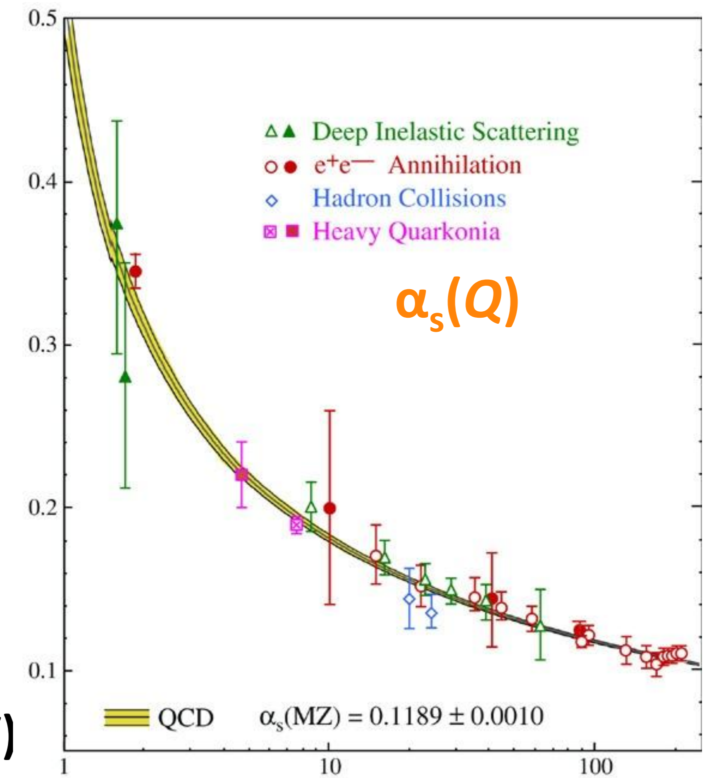
Quantum Chromodynamics

§ The strong interactions of quarks and gluons (SU(3) gauge)

- “Confinement”
  no free quarks allowed



- “Asymptotic freedom”
  weak interactions at large energies



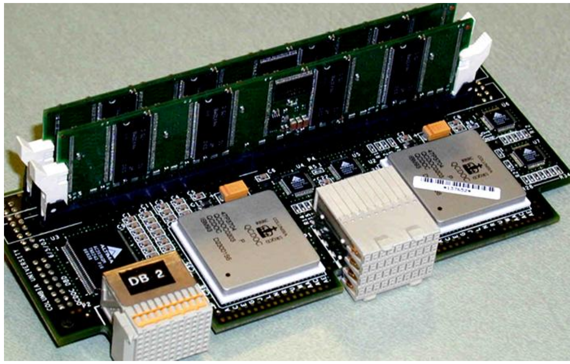
The Nobel Prize in Physics 2004

Difficulties at Low Energy

- Strong interactions make analytic calculation impossible

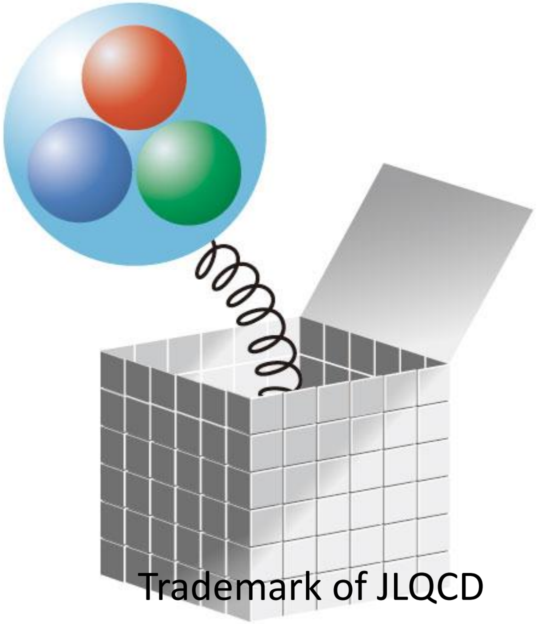
- Direct QCD calculation is desired
  → Lattice QCD

→ **010010101010 111010…** →



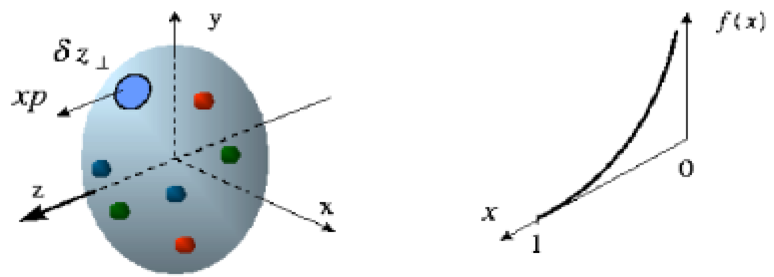
Outline

§ The tool = Lattice Gauge Theory

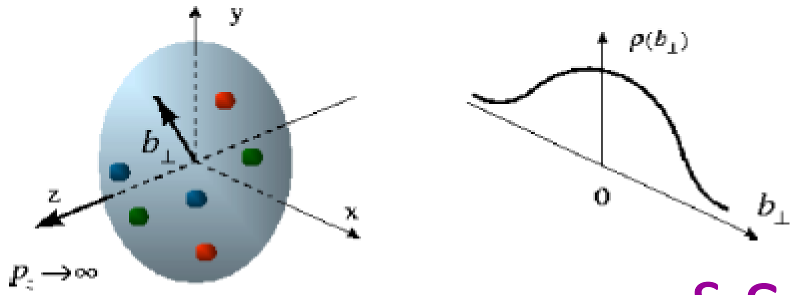

Trademark of JLQCD

§ Topics in Hadron Structure

Parton distribution functions



Form factors



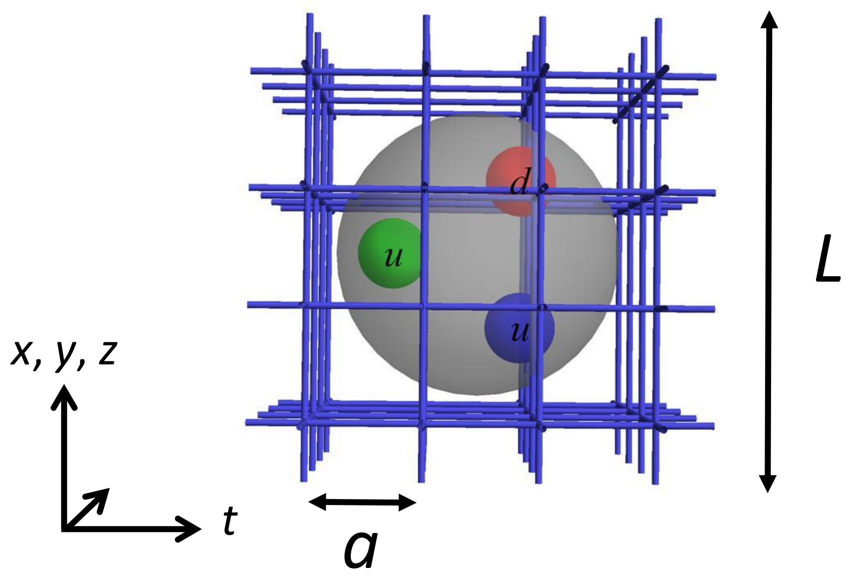
§ Summary and Outlook

QCD

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\,\mathcal{L}^{\rm QCD}(\overline{\psi},\psi,A)}$$

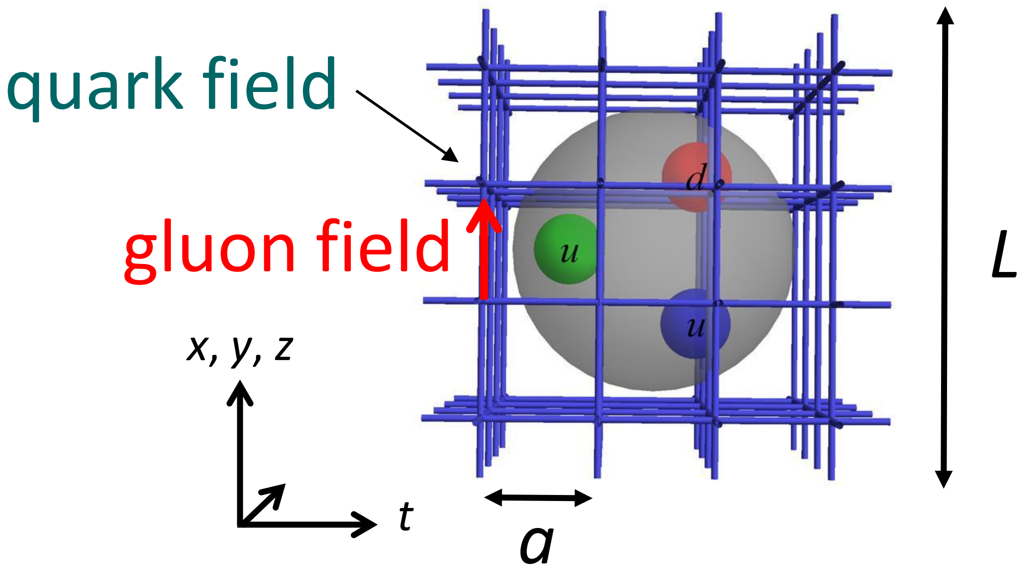
Lattice QCD

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\, \mathcal{L}^{\rm QCD}(\overline{\psi},\psi,A)}$$



Lattice QCD

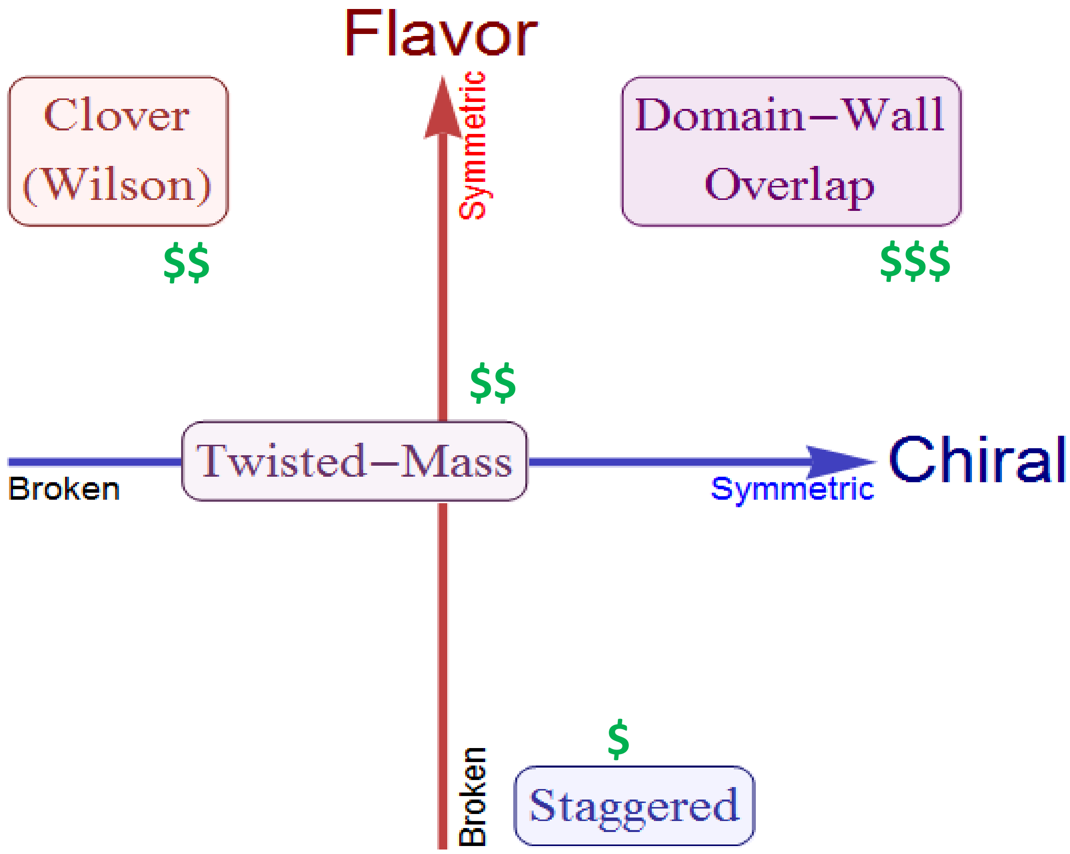
$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\,\mathcal{L}^{\mathrm{QCD}}(\overline{\psi},\psi,A)}$$



Actions

§ Guided by Symanzik Improvement (order in *a*)

- Gauge sector: $O(a^2)$-improved
- Fermion sector: $O(a)$-improved



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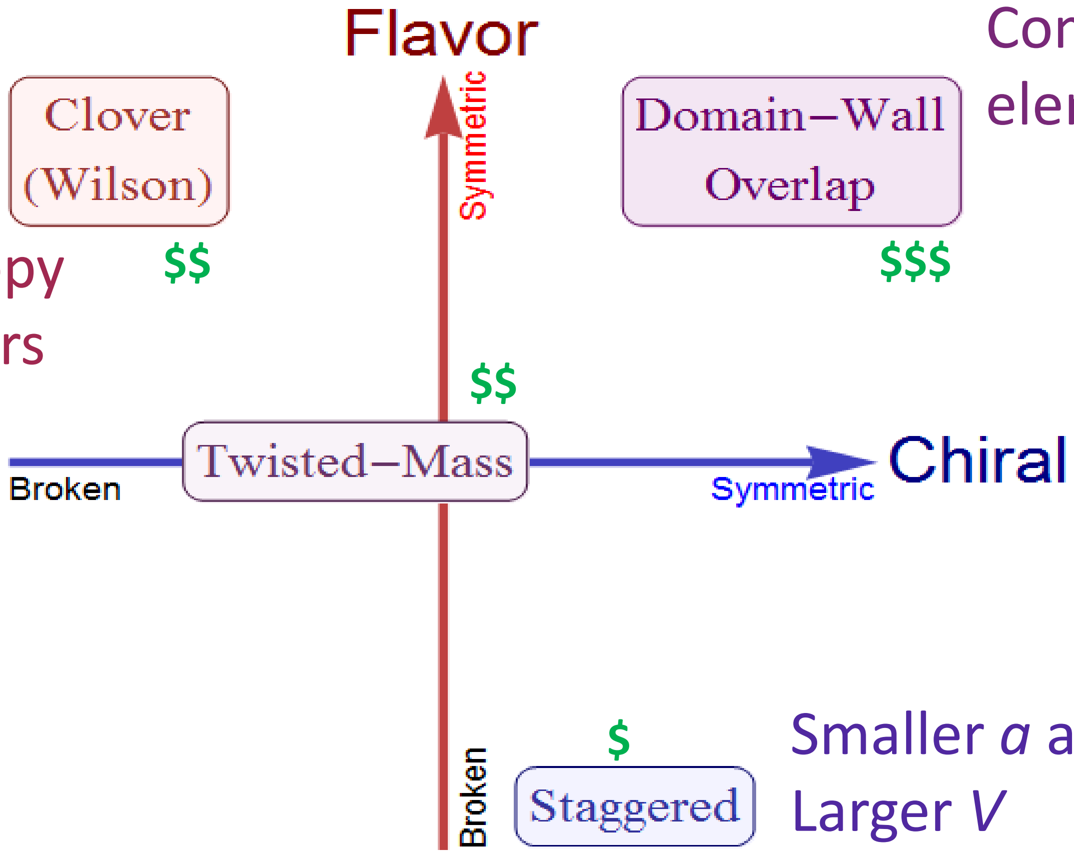

Complicated matrix elements, mixings

Spectroscopy
Form factors

Smaller a and m_π
Larger V

Actions

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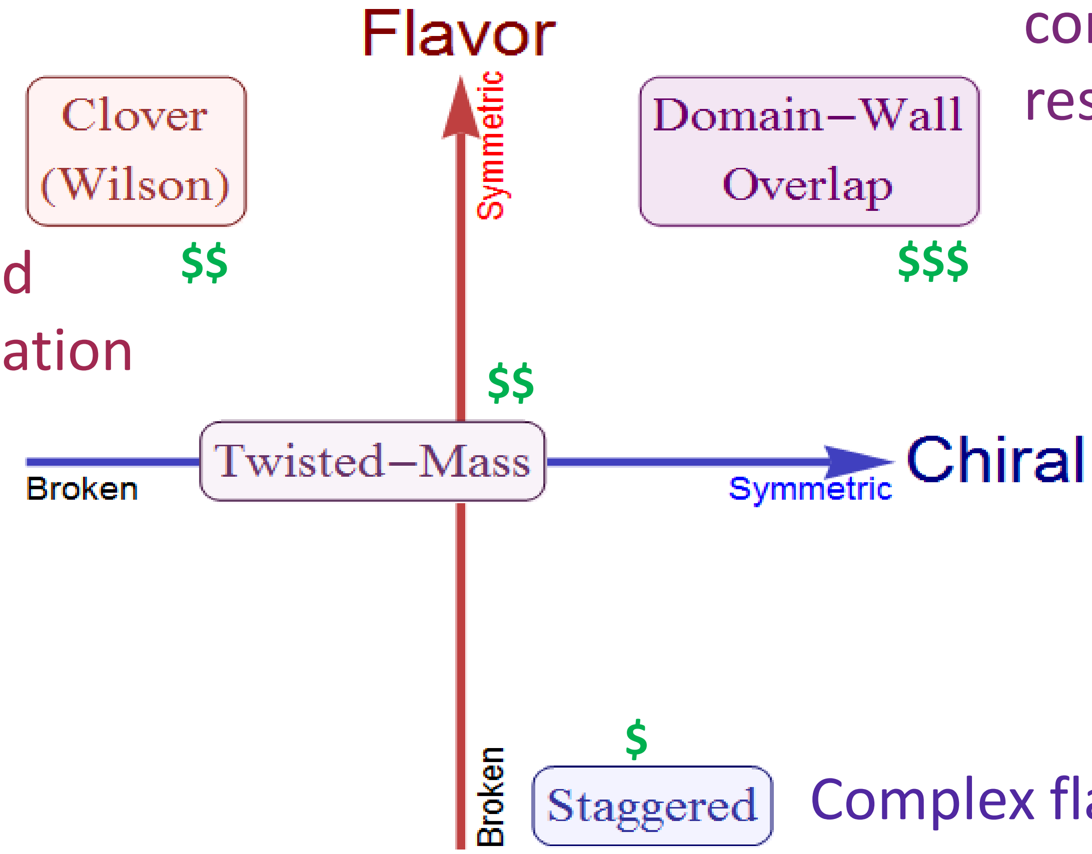

Needs a lot more computational resources

Mixings and renormalization

Complex flavor mixing!

Actions

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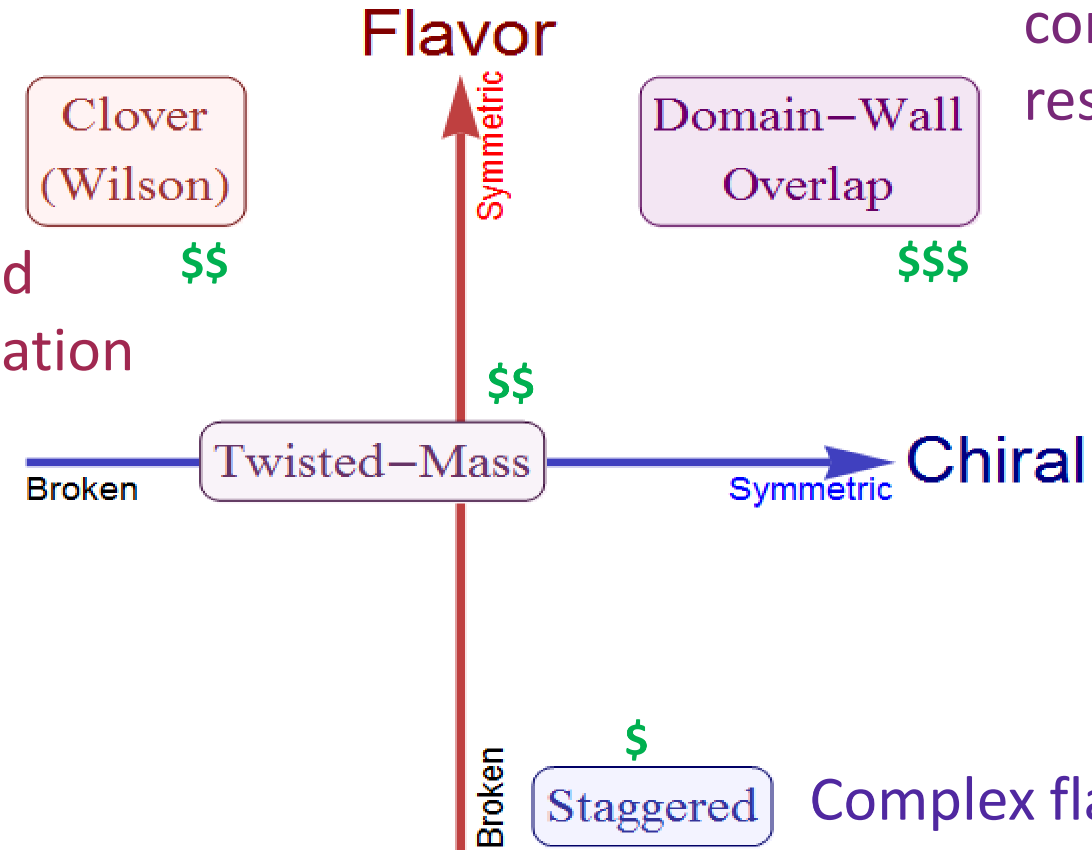


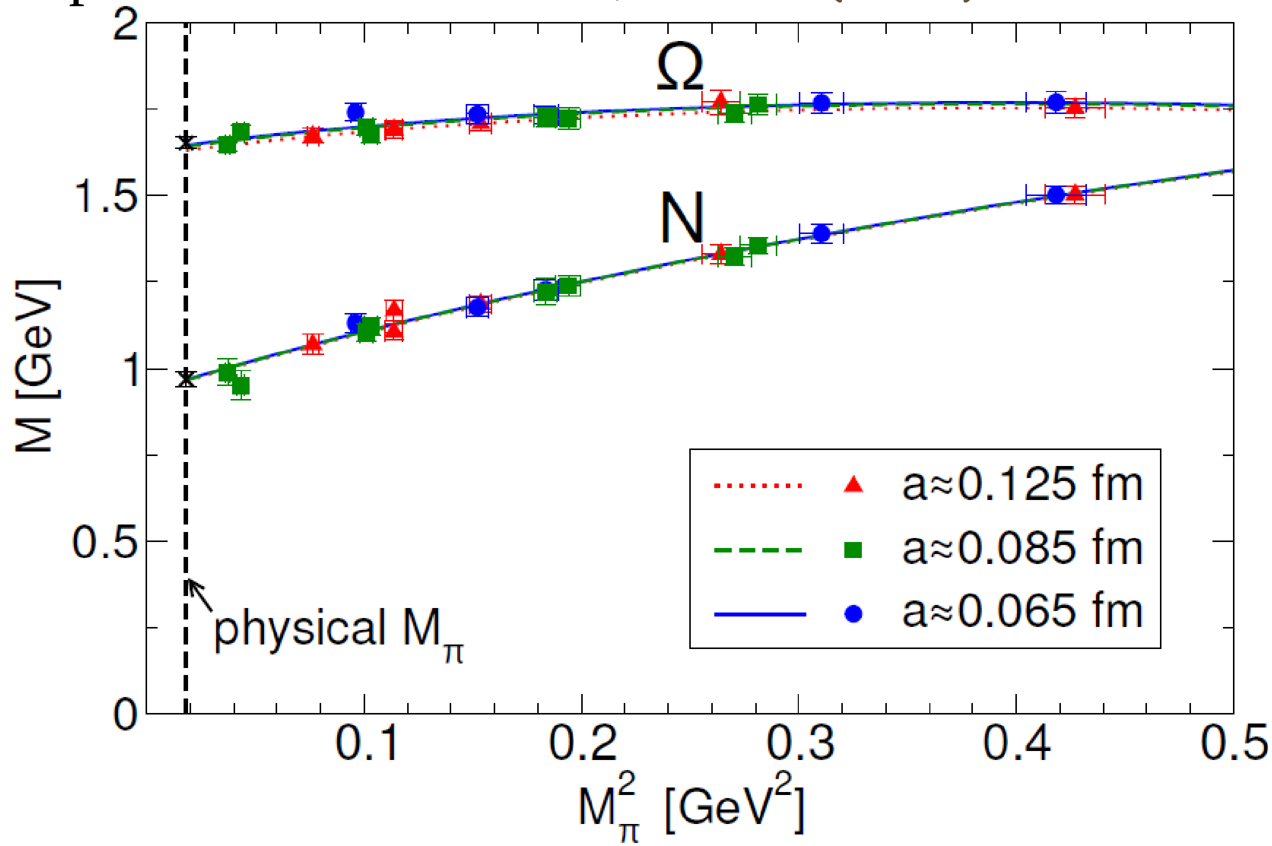
Mixings and renormalization

Complex flavor mixing!

The Dark Side…

§ Currently, not running with the physical pion mass

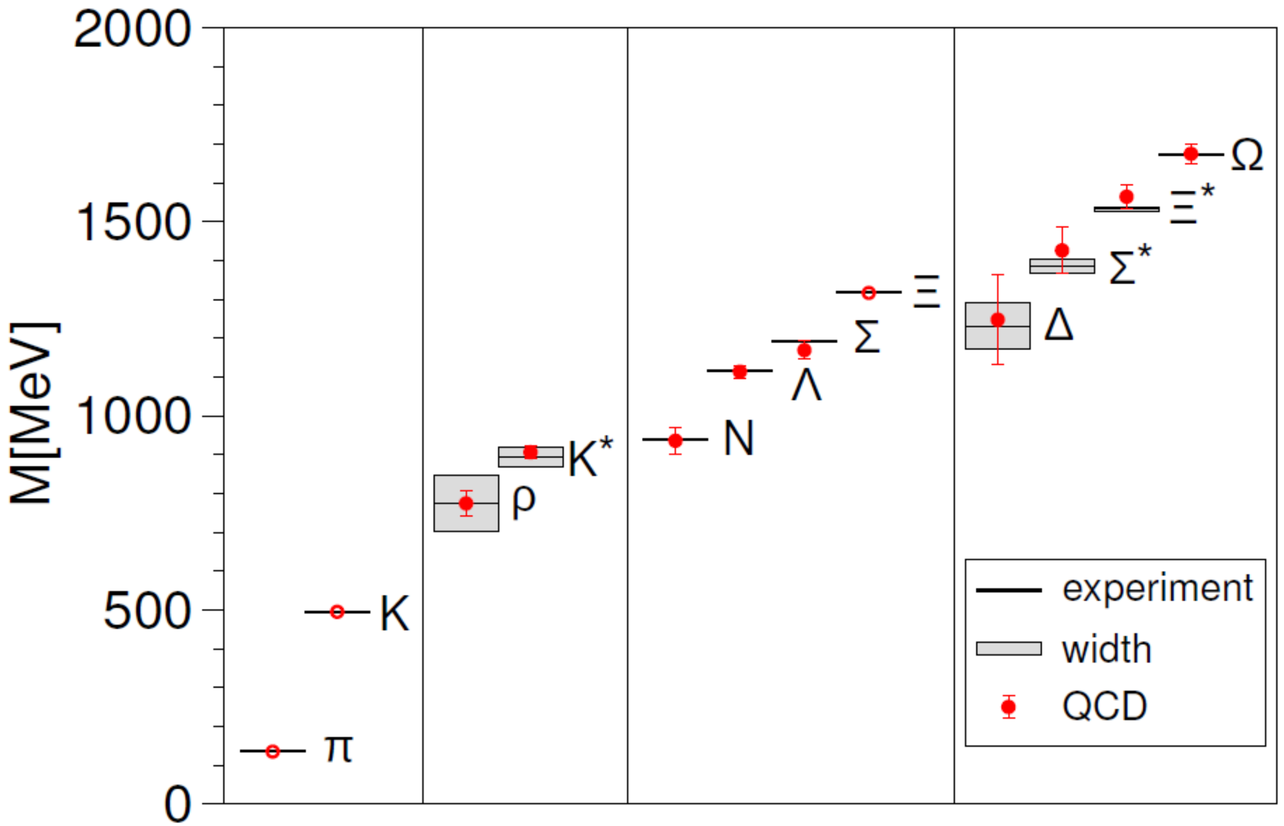
- Lighter quark simulations require $$$
- Example: BMW Collaboration, Science (2008)



Lattice in the News

§ Post-dictions of well known quantities

- Example: BMW Collaboration, Science 2008

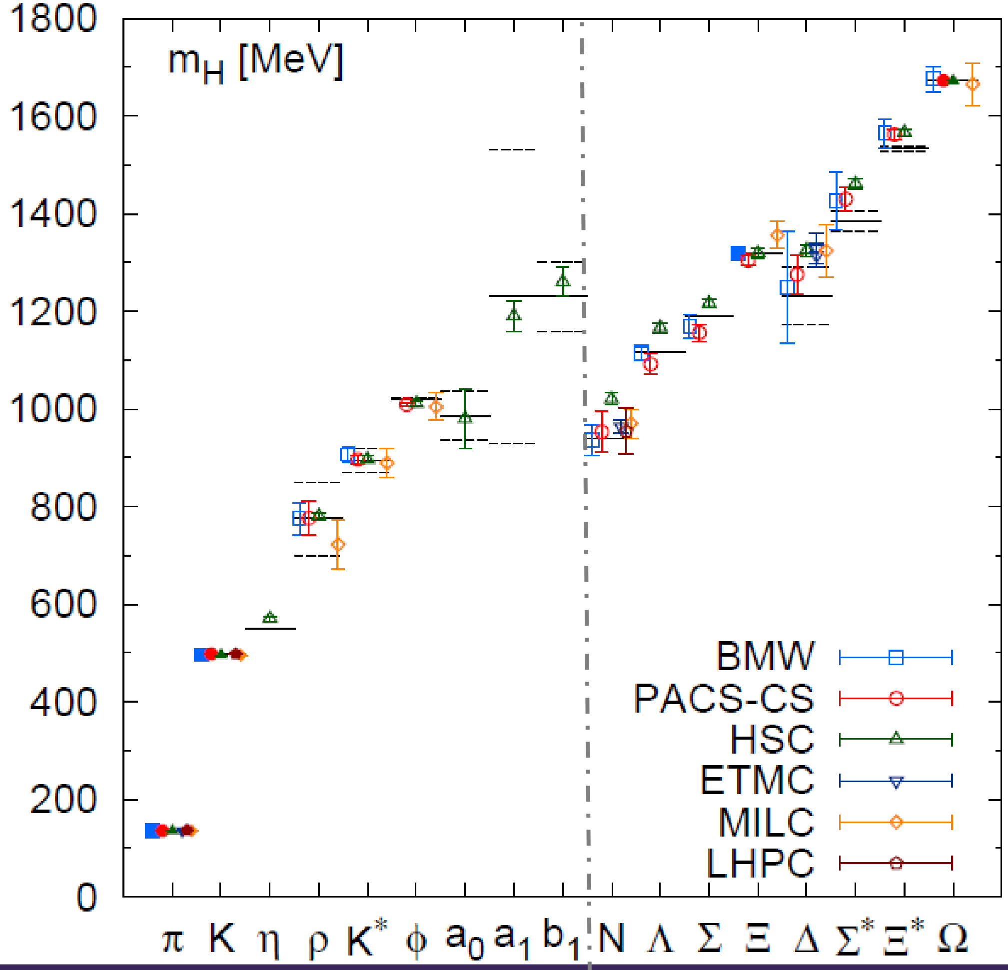


§ Proves all the systematics are under control

From Lattice 2009

- Post-dictions of well known quantities
- Consistent results from various actions/groups

- BMW
  $n_f = 2{+}1$ Clover, $m_\pi \geq 190$ MeV
- PACS-CS
  $n_f = 2{+}1$ Clover, $m_\pi \geq 160$ MeV
- HSC
  $n_f = 2{+}1$ anisoClover, $m_\pi \geq 370$ MeV
- ETMC
  $n_f = 2$ Twisted Wilson, $m_\pi \geq 300$ MeV
- MILC
  $n_f = 2{+}1$ Staggered, $m_\pi \geq 240$ MeV
- LHPC
  $n_f = 2{+}1$ DWF/Stag., $m_\pi \geq 300$ MeV

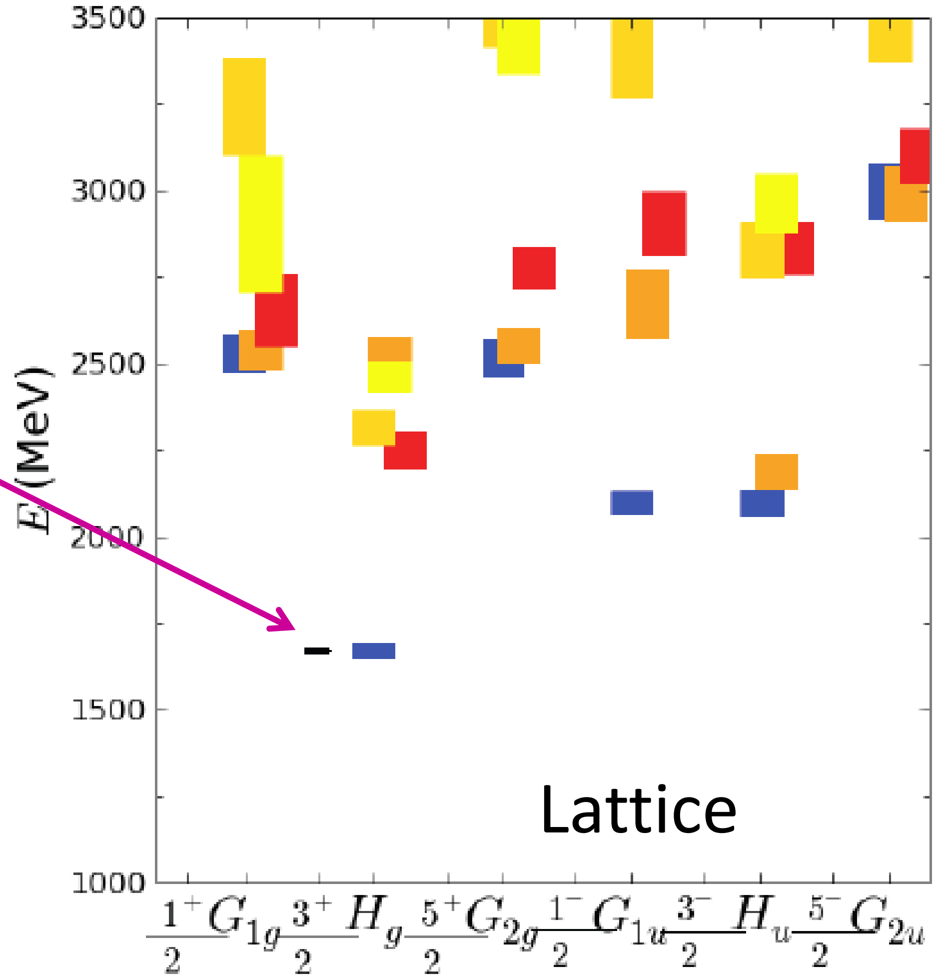


Prediction

§ Omega spectroscopy

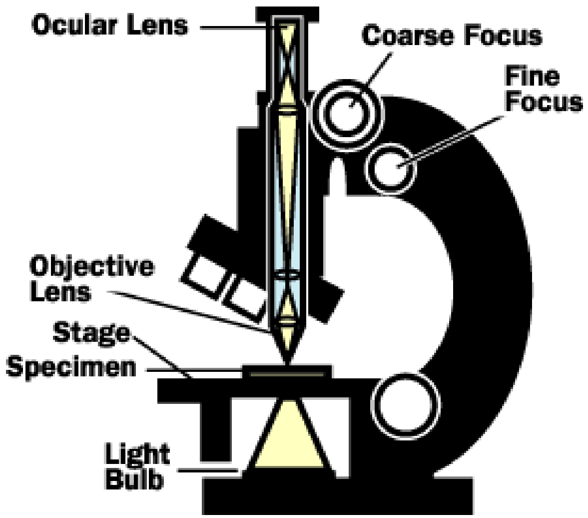
From Particle Data Book

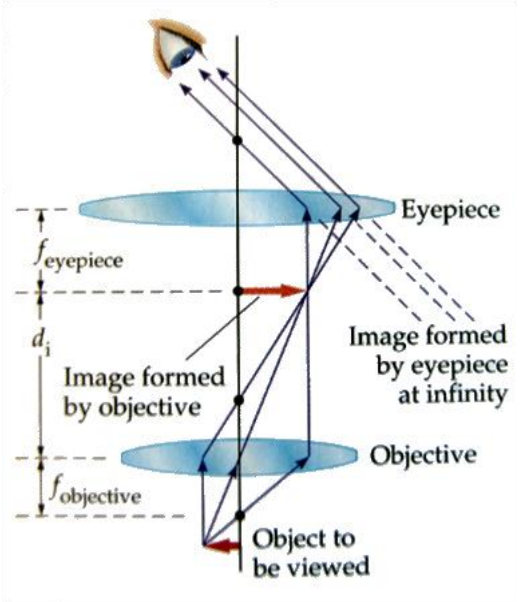
Ω BARYONS ($S = -3$, $I = 0$)		
Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**



HSC, 1004.5072

Probing Insights into Hadrons

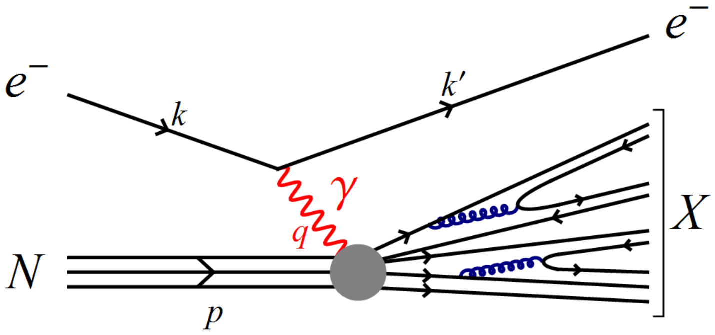


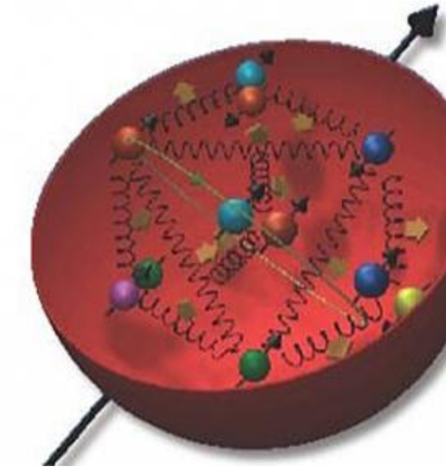


Probing Insights into Hadrons

BNL

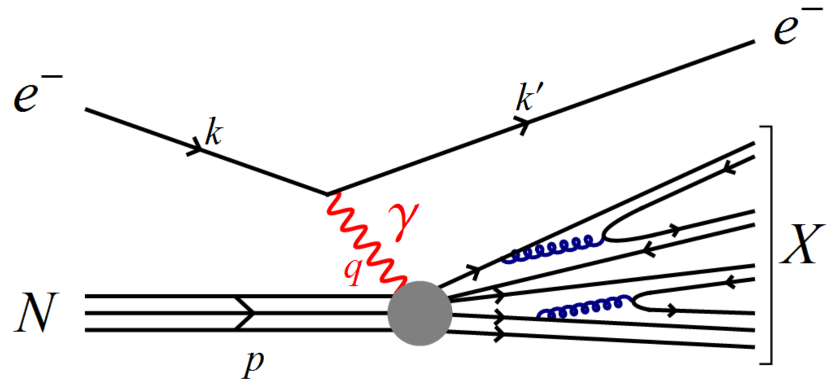
JLab





Parton Distribution Function

- Deep inelastic scattering
- Probing nucleon structure



$$\sigma \sim L^{\mu\nu} W_{\mu\nu},$$

$$W_{\mu\nu} = i \int d^4 x e^{iqx} \langle N | T\{J^\mu(x), J^\nu(0)\} | N \rangle$$

- The symmetric, unpolarized, spin-averaged

$$W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \left(p^\mu - \frac{\nu}{q^2} q^\mu\right)\left(p^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x, Q^2)}{\nu}$$

- The anti-symmetric, polarized

$$W^{[\mu\nu]}(x, Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{s_\sigma}{\nu}(g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot s p_\sigma}{\nu^2} g_2(x, Q^2)\right)$$

Moments of the Structure Function

- No light-cone operator directly calculated on the lattice
- Operator product expansion
  - Polarized

$$2\int dx\, x^n g_1(x,Q^2) = \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}$$

$$2\int dx\, x^n g_2(x,Q^2) = \frac{n}{(n+1)}\sum_{q=u,d}\Big[2e_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))d_n^q(\mu) + e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}\Big]$$

  - Unpolarized

$$2\int dx\, x^{n-1} F_1(x,Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q$$

$$\int dx\, x^{n-2} F_2(x,Q^2) = \sum_{q=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_q$$

- e_1, e_2, c_1, c_2 are Wilson coefficients
- $\langle x^n\rangle_q$, $\langle x^n\rangle_{\Delta q}$, d_n are forward nucleon matrix elements

Green Functions

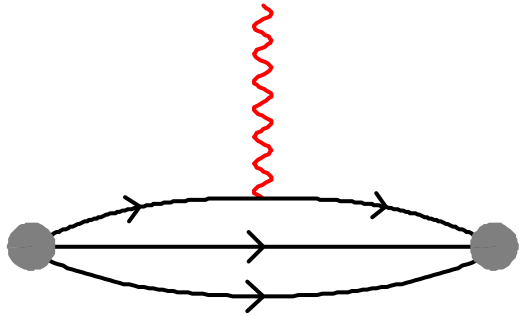
- Three-point function with connected piece only

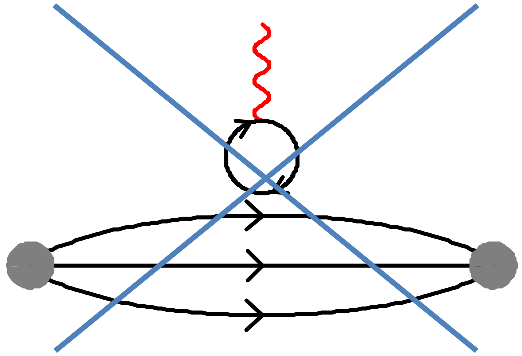
$$C_{3\text{pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

O: $V_\mu = \bar{q}\gamma_\mu q$, $A_\mu = \bar{q}\gamma_\mu\gamma_5 q$, or others

$J = \epsilon^{abc}[q_1^{a\mathrm{T}}(x)C\gamma_5 q_2^b(x)]q_1^c(x)$

- Two topologies:



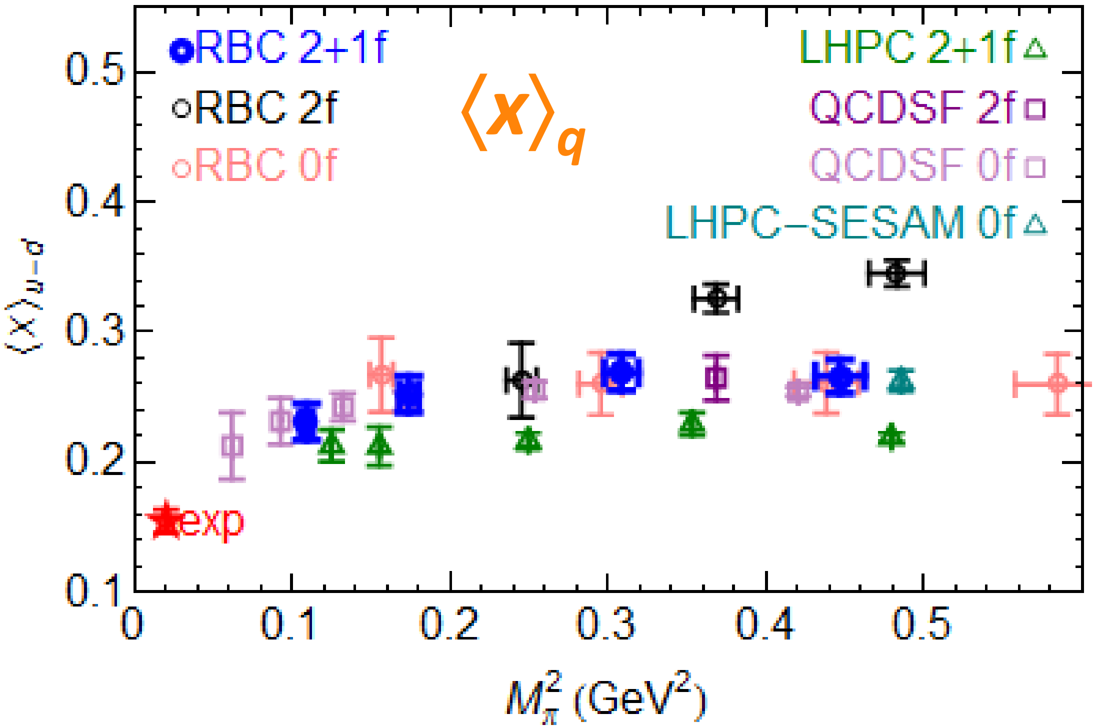


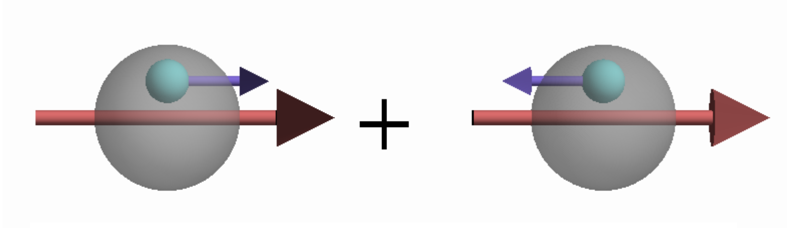
- Isovector quantities O^{u-d}

disconnected diagram cancelled

Nucleon Structure Functions

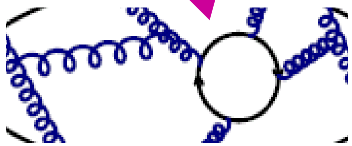
§ The first moment of the quark momentum fraction





$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n} q - trace$$

2+1f : u/d + s
2f : u/d
0f : Ø



HWL et al., Phys. Rev. D78, 014505 (2008) and RBC/UKQCD arXiv:1003.3387[hep-lat];
K. Orginos et al., Phys.Rev.D73:094507 (2005);
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Nucleon Structure Functions

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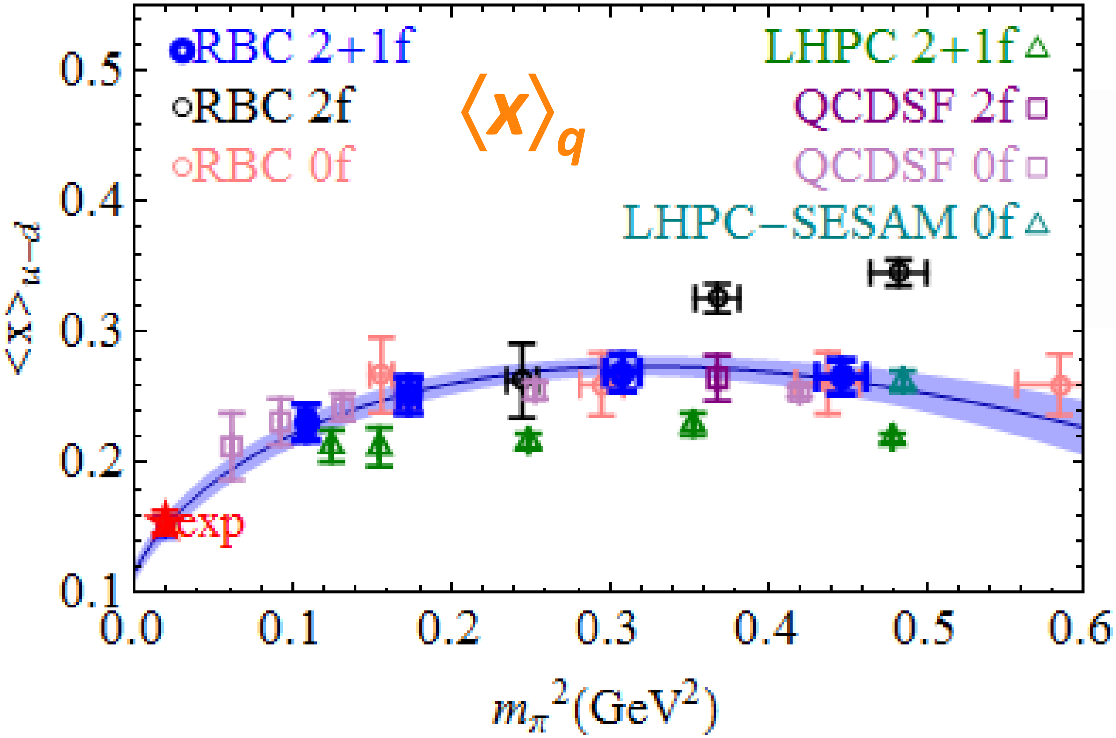

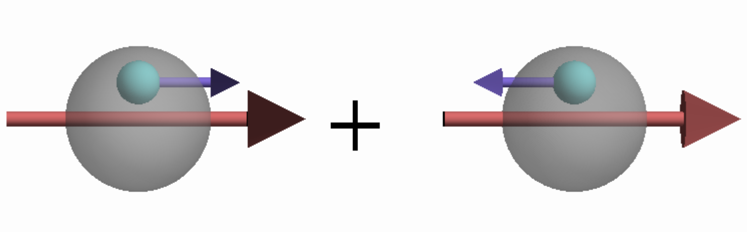

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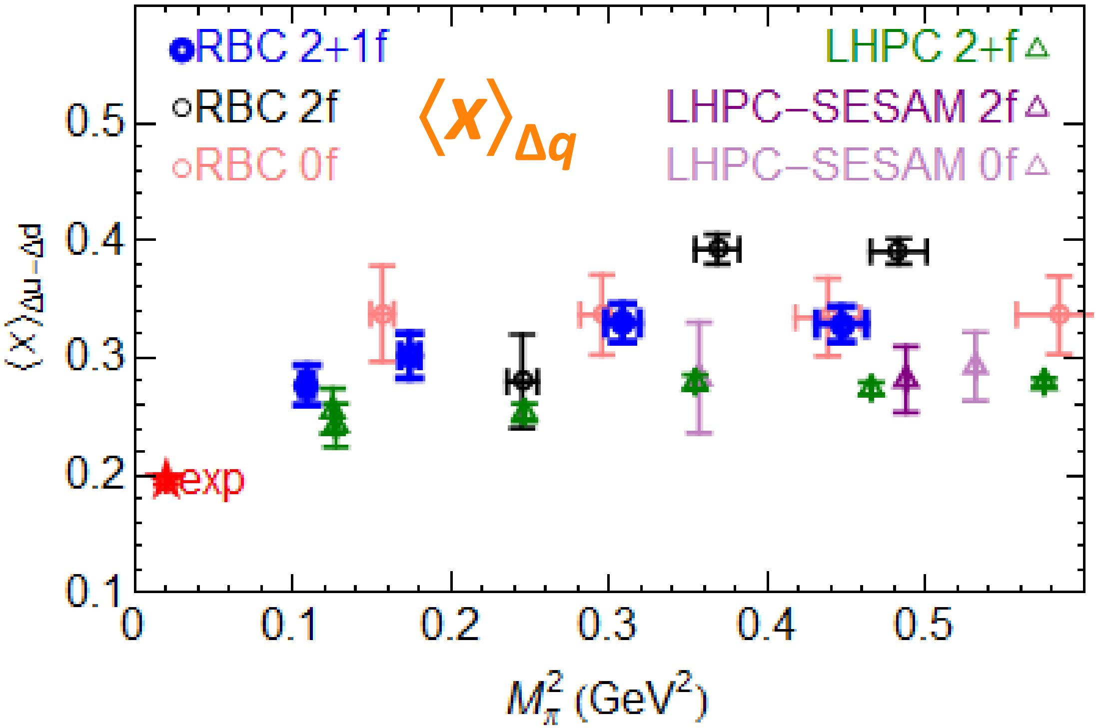
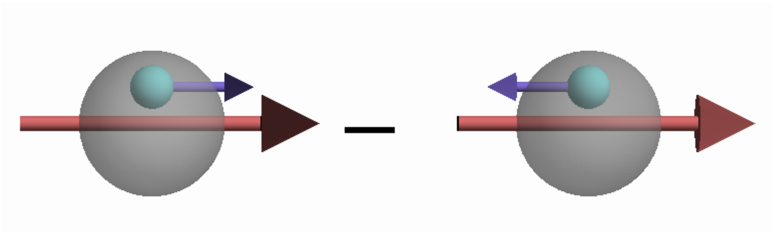
Nucleon Structure Functions

§ The first moment of the quark helicity distribution





$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_\sigma\gamma_5 \overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}\, q - trace$$

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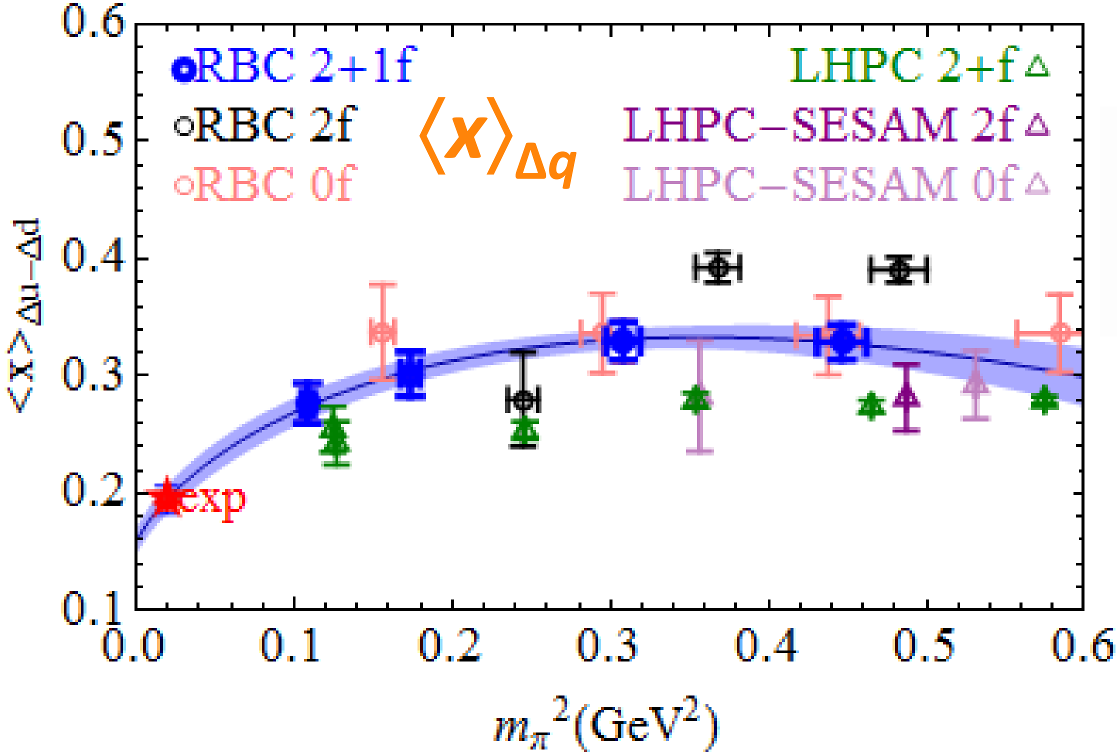

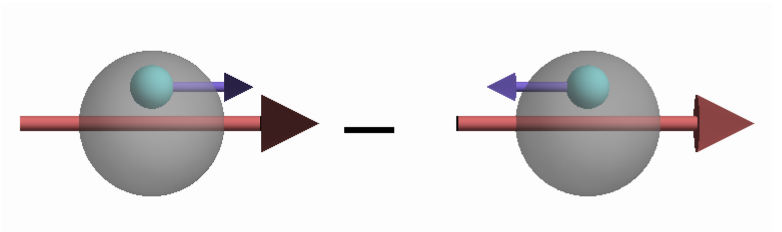

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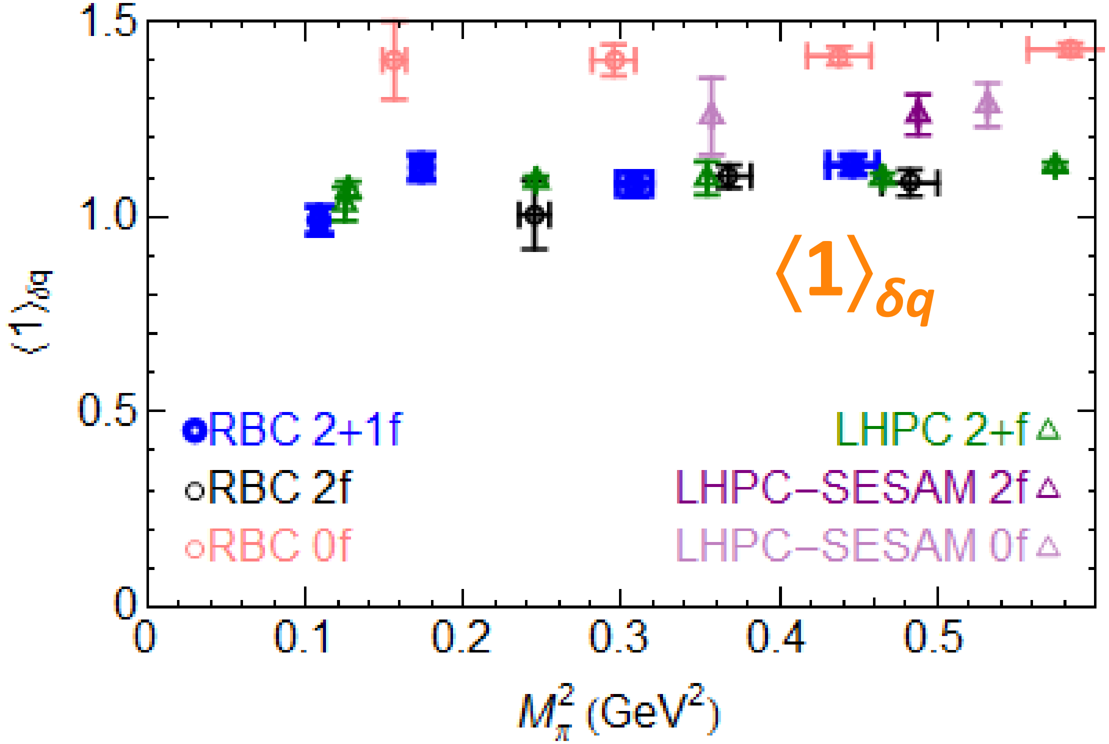
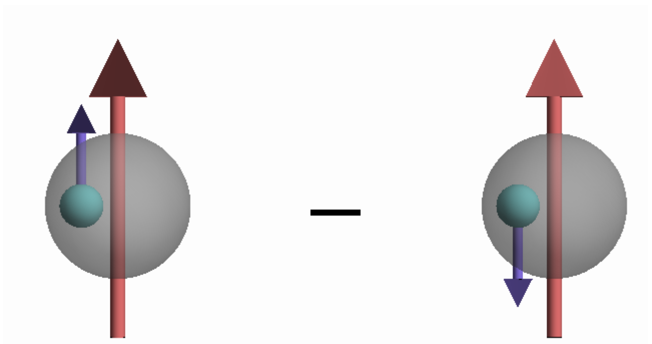
Nucleon Structure Functions

§ World data: the zeroth moment of the transversity





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Nucleon Structure Functions

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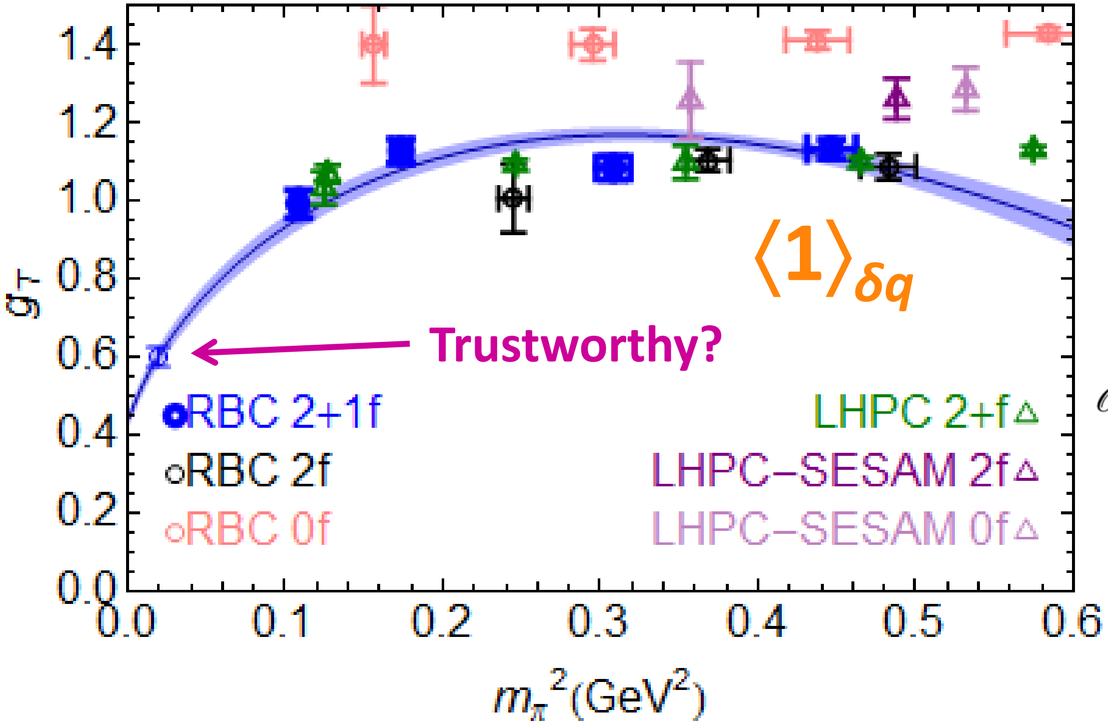
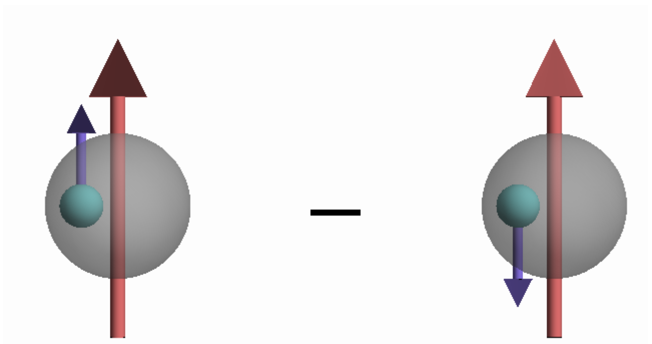




$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \left(\frac{i}{2}\right)^n \bar{q}\sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n} q - trace$$

HWL et al., Phys. Rev. D78, 014505 (2008) and RBC/UKQCD arXiv:1003.3387[hep-lat];
K. Orginos et al., Phys.Rev.D73:094507 (2005);
LHPC, arXiv:1001.3620[hep-lat]
D. Dolgov et al., Phys. Rev. D66, 034506 (2002);
M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;

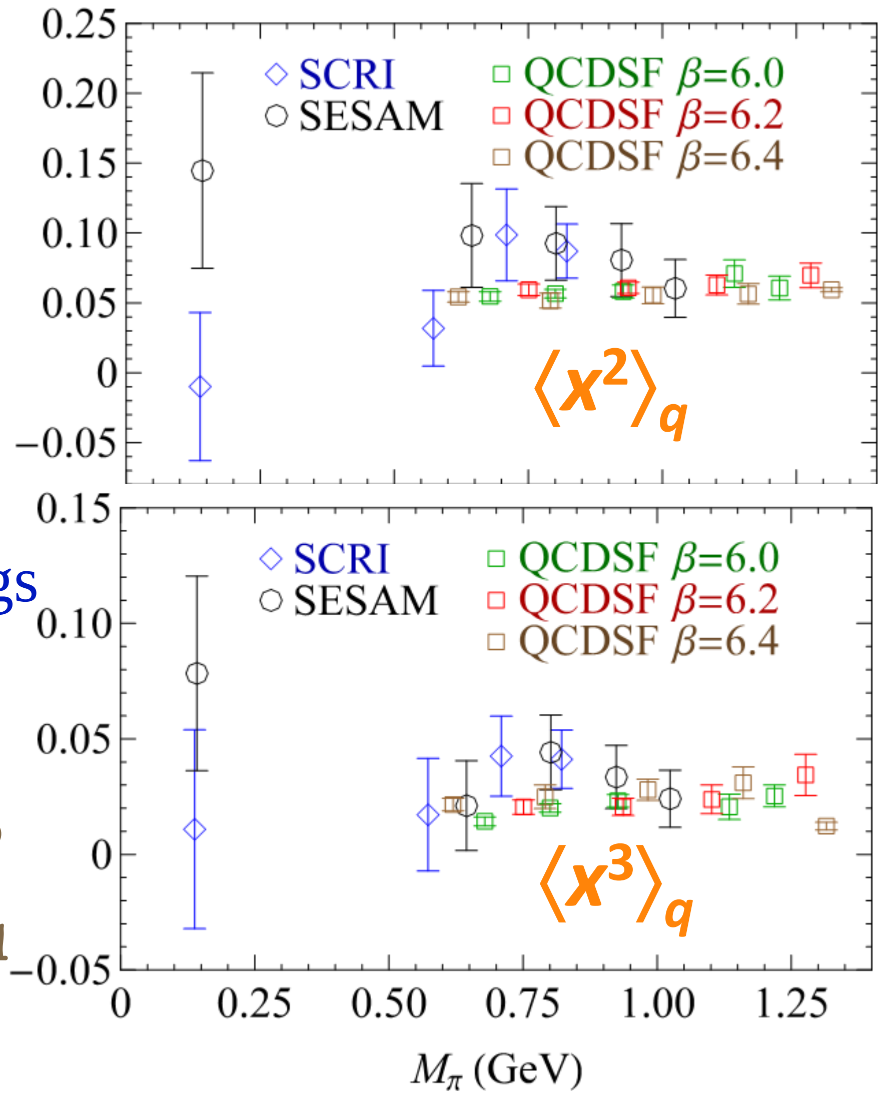
Nucleon Structure Functions

§ Higher moments?
Yes

Isovector unpolarized moments

- LHPC (SCRI, SESAM):
  2f, Wilson and clover
- QCDSF:
  0f clover, multiple lattice spacings

D. Dolgov et al., Phys. Rev. D66, 034506 (2002);
M. Gockeler et al. Phys. Rev. D71, 114511 (2005).



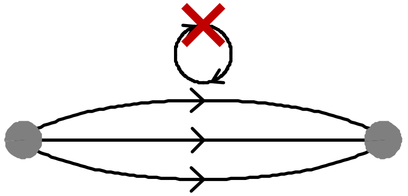
Nucleon Structure Functions

- § Higher moments?
  Yes.... But at $n \geq 4$: mixing with lower-dimension operators
  - Getting around
  - Direct calculation

W. Detmold et. al. Phys.Rev.D73:014501 (2006)

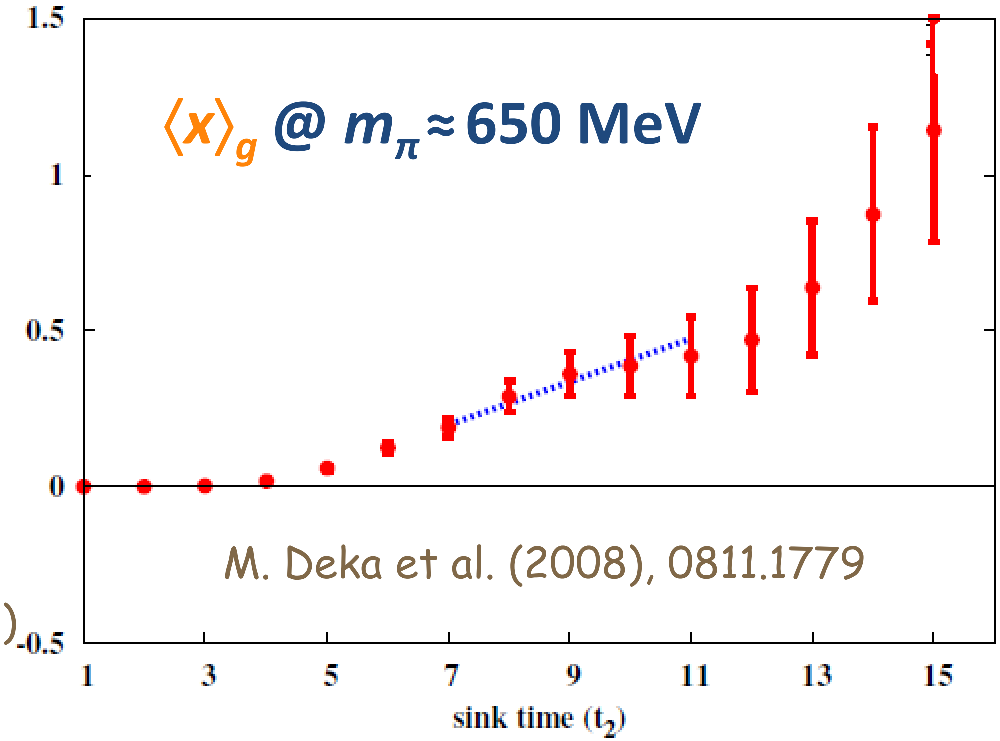
K. Liu, Phys.Rev.D62:074501 (2000)

- § Gluon structure such as $\langle x \rangle_g$?
  $\approx 2\sigma$ away from zero
- § "Disconnected"?



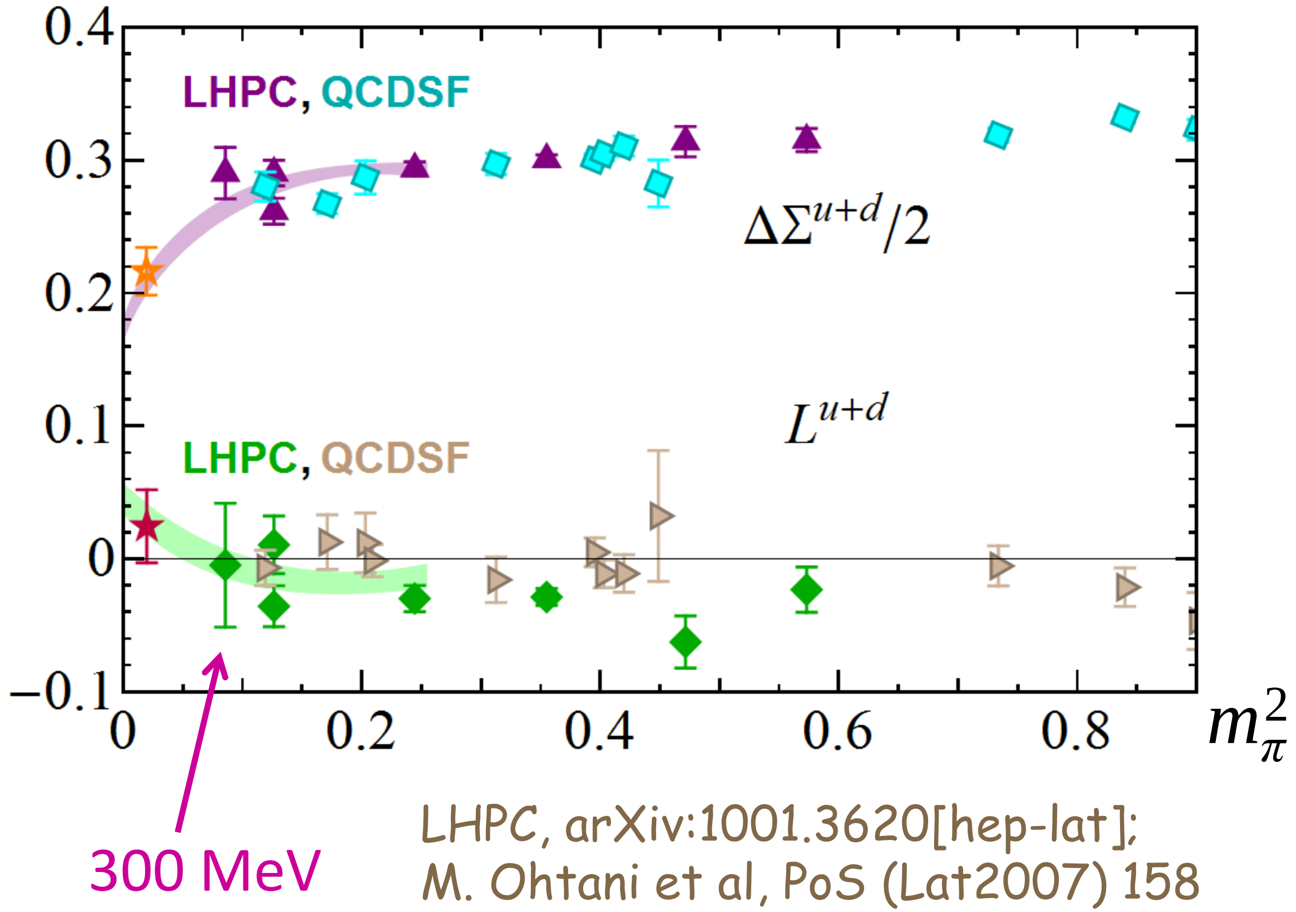
5σ $\langle x \rangle_{u,d}$
$\langle x^2 \rangle \approx 0$

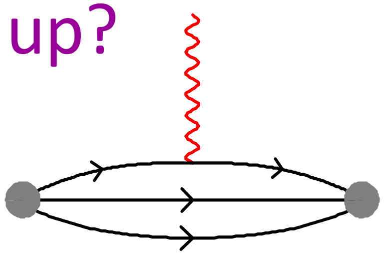
χQCD, Phys.Rev.D79:094502 (2009)



Origin of Nucleon Spin

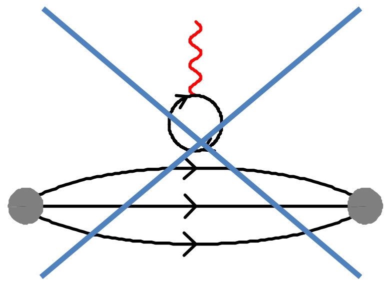
- Nucleon total spin is ½, but how does it add up?
- Quark contribution





$\Delta\Sigma$: spin

L: orbital angular momentum



LHPC, arXiv:1001.3620[hep-lat];
M. Ohtani et al, PoS (Lat2007) 158

Origin of Nucleon Spin

- Nucleon total spin is ½, but how does it add up?
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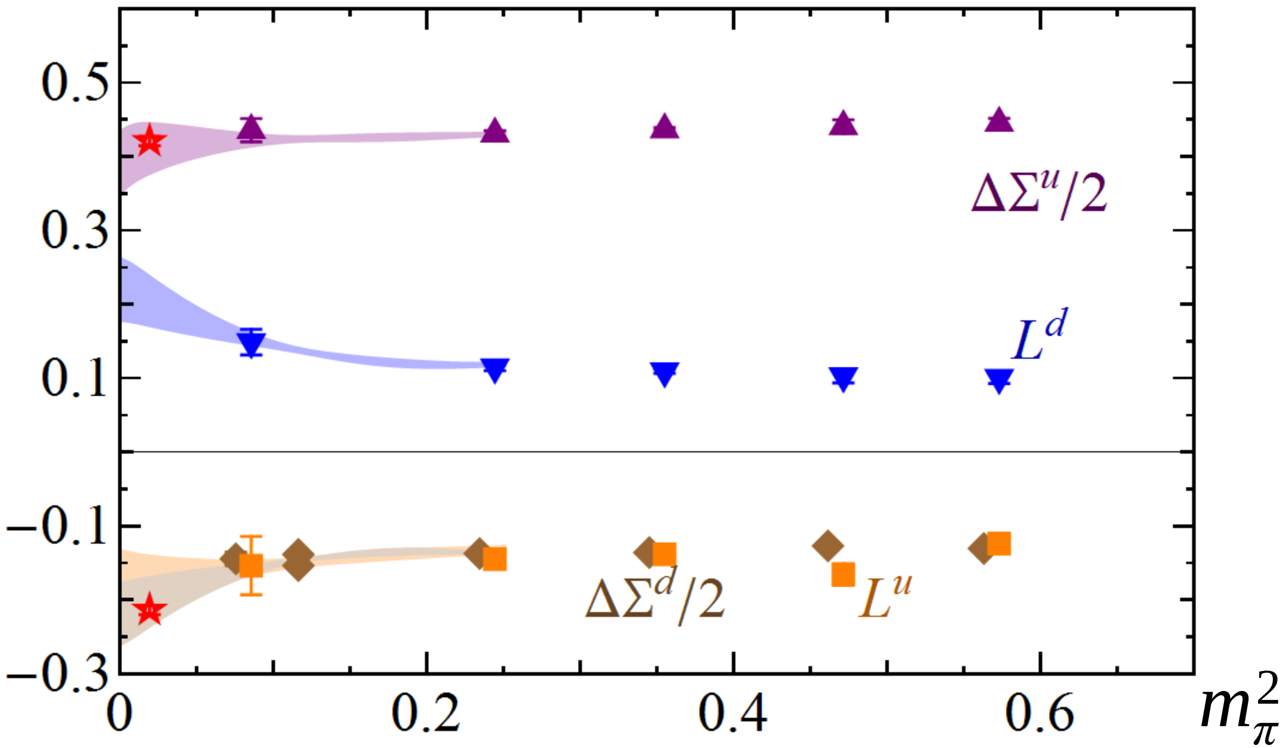
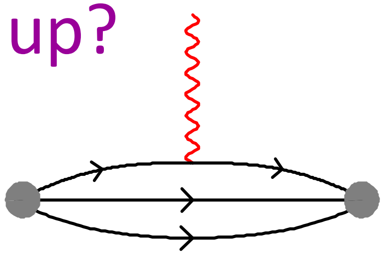
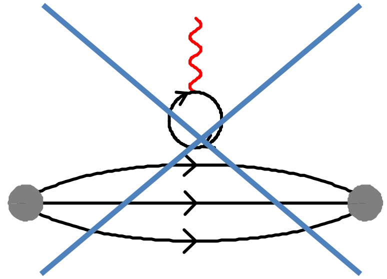




$\Delta\Sigma$: spin

L: orbital angular momentum

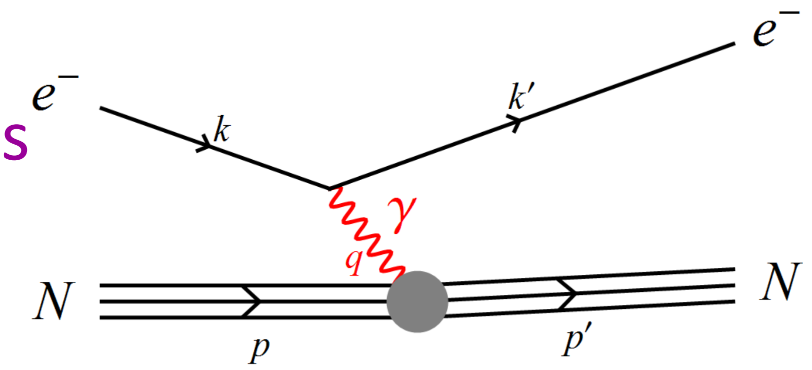
- "Disconnected" contribution
- How much contribution from gluons?




VICTROLA

Electromagnetic Form Factors

- Experimentally studied through elastic scattering process



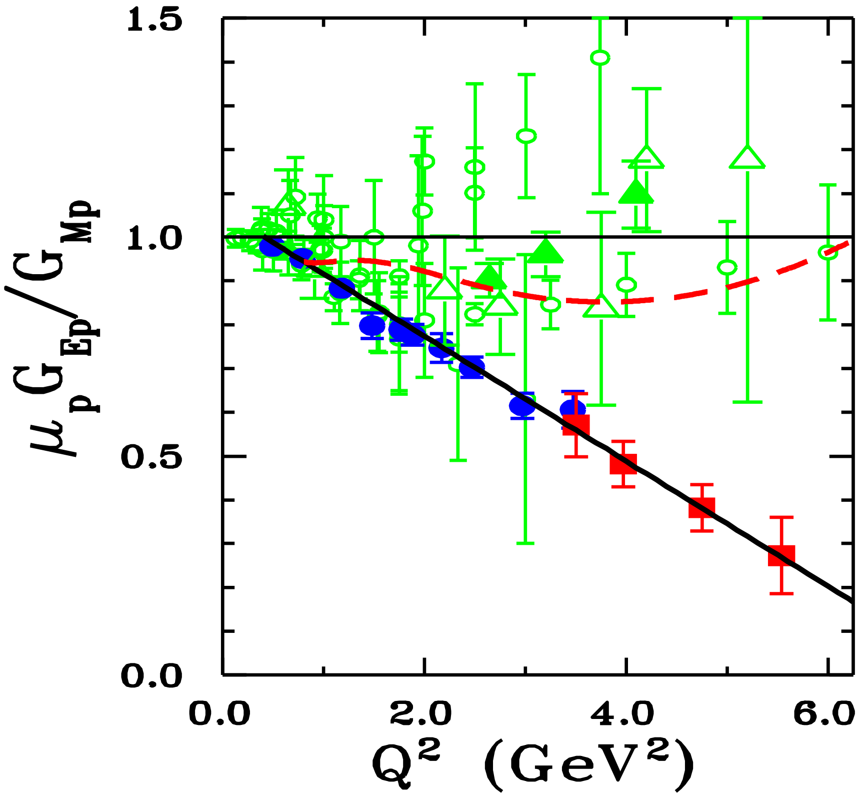
- Two definitions
  - Dirac and Pauli form factors F_1, F_2

$$\langle N | V_\mu | N \rangle (q) = \overline{u}_N(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m} \right] u_N(p)^{\kappa_n}$$

  - Sachs form factors G_E, G_M

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

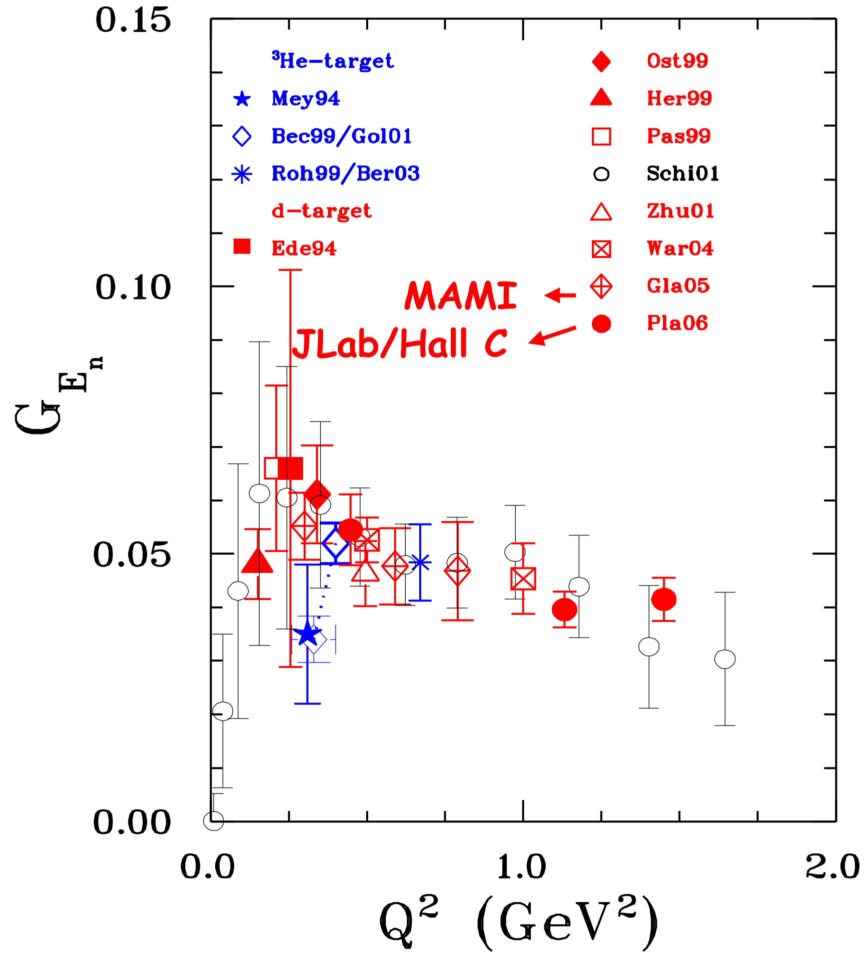
EM Form Factors: Experiment



▲ : Rosenbluth data (SLAC, JLab)

● Pun05 } JLab Hall A
■ Gay02 } recoil pol. data

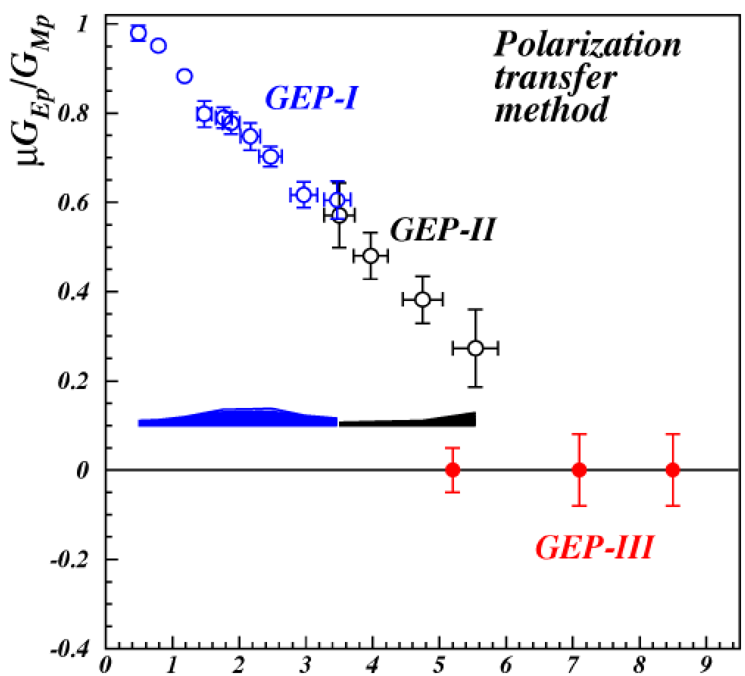
new JLab Hall C recoil pol. exp't extension up to $Q^2 \approx 8.5$ GeV^2

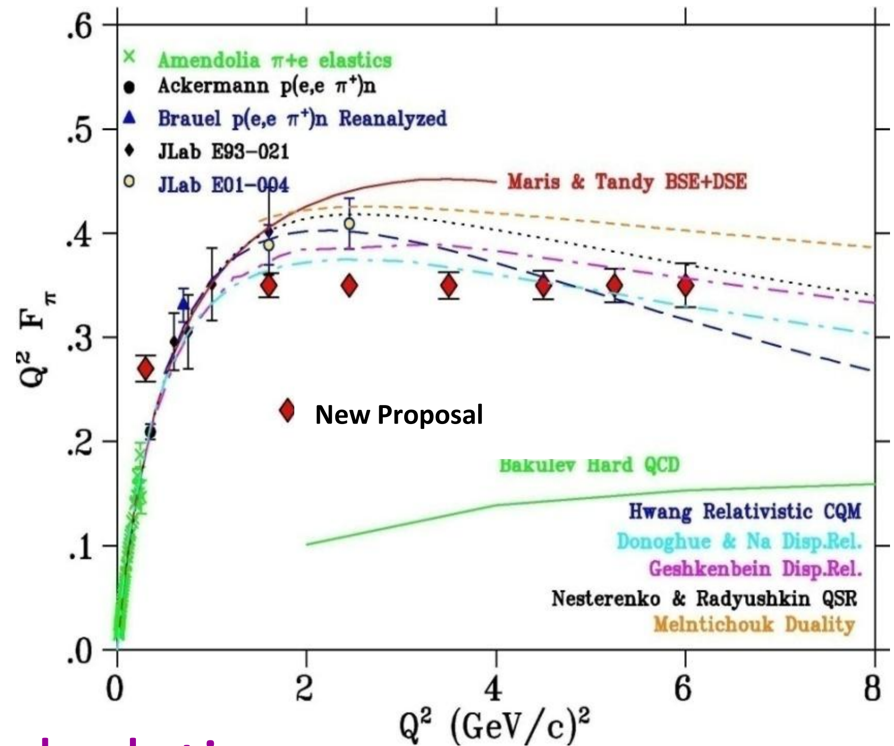


new JLab/Hall A double-pol. exp't extension to $Q^2 \approx 3.5$ GeV^2 completed

Higher-Q^2 Form Factors

- Higher-Q^2 data will help us to understand hadrons and challenge QCD-based models

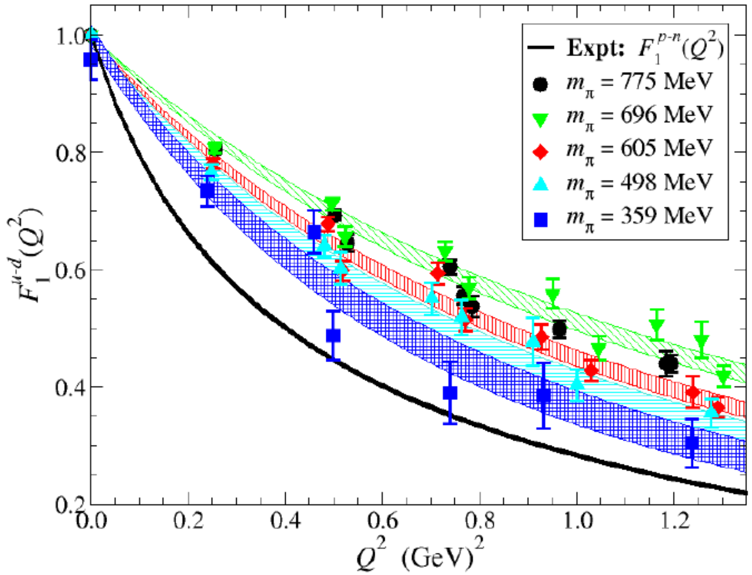


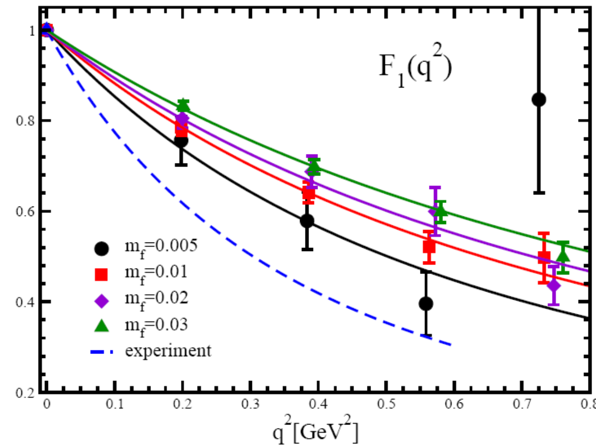


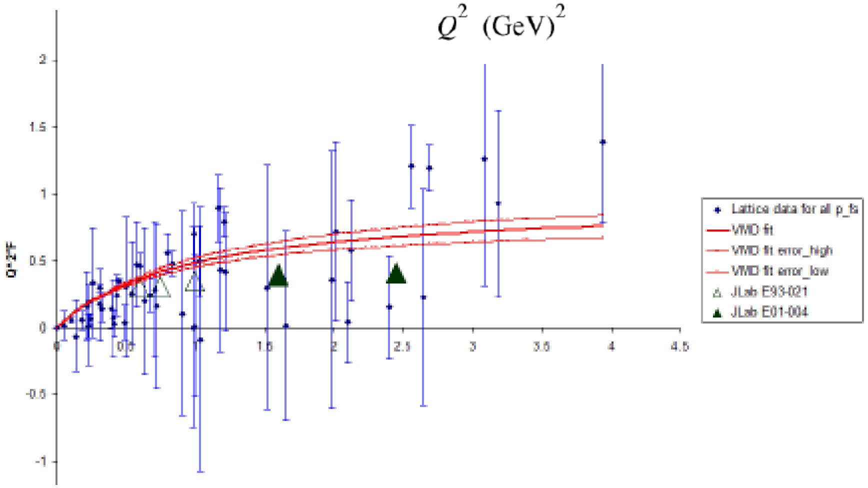
- Challenge for lattice-QCD calculations
  - Typical Q^2 range for nucleon form factors is < 3.0 GeV^2
  - Higher-Q^2 calculations suffer from poor noise-to-signal ratios

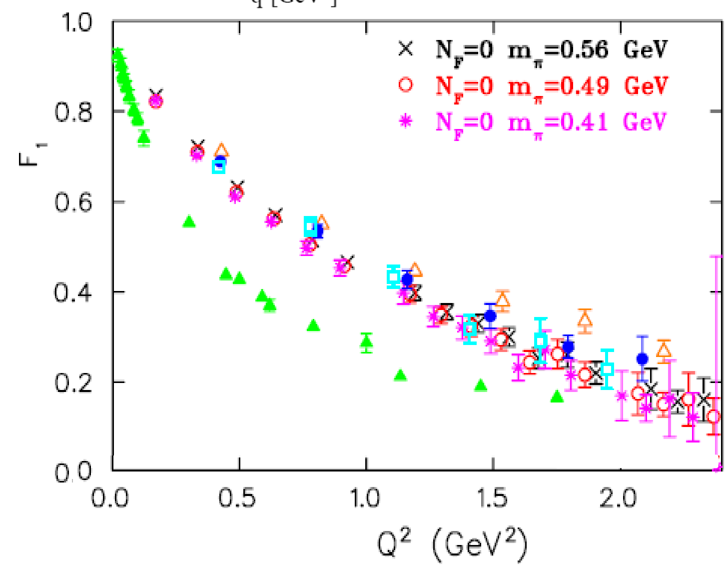
Conventional Calculation

Higher-Q^2 calculations suffer from poor noise-to-signal ratios





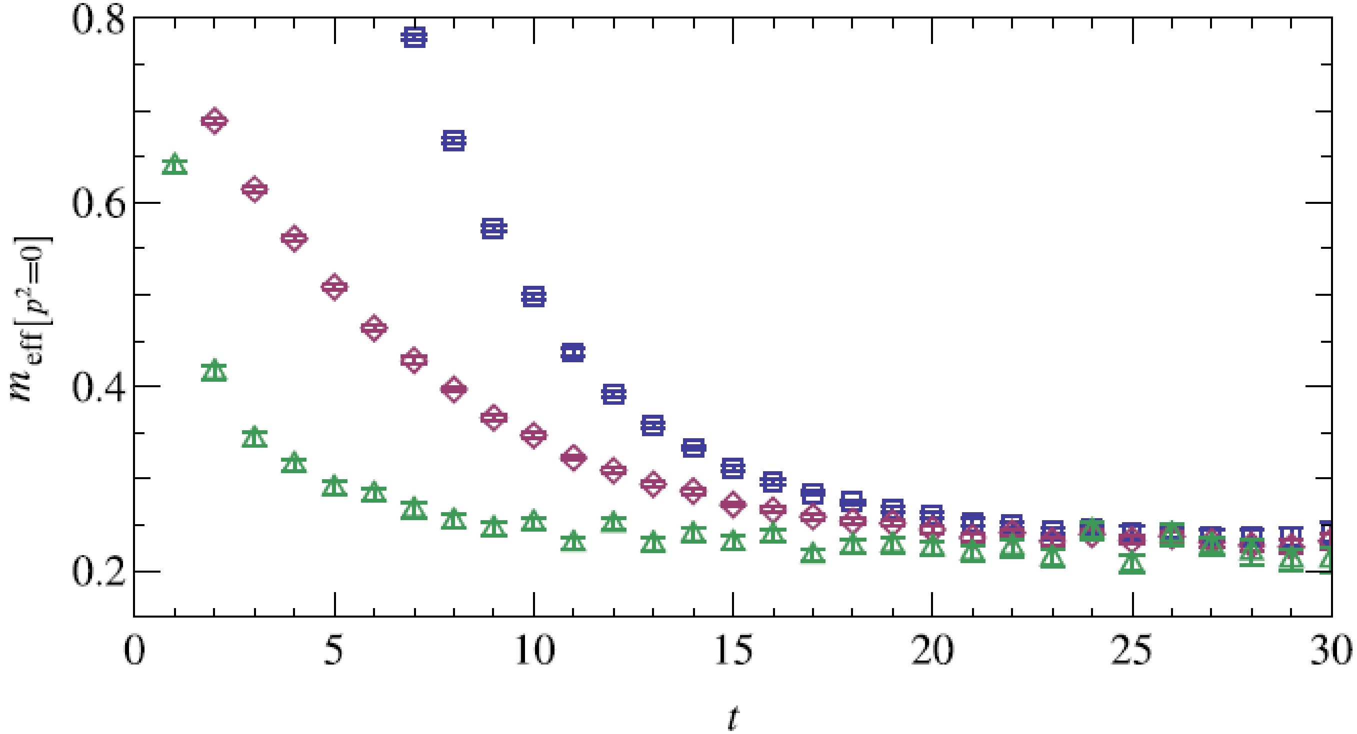




Lattice High-Q^2 Form Factors

§ Problem: traditional approach

simplify to one-state problem



Lattice High-Q^2 Form Factors

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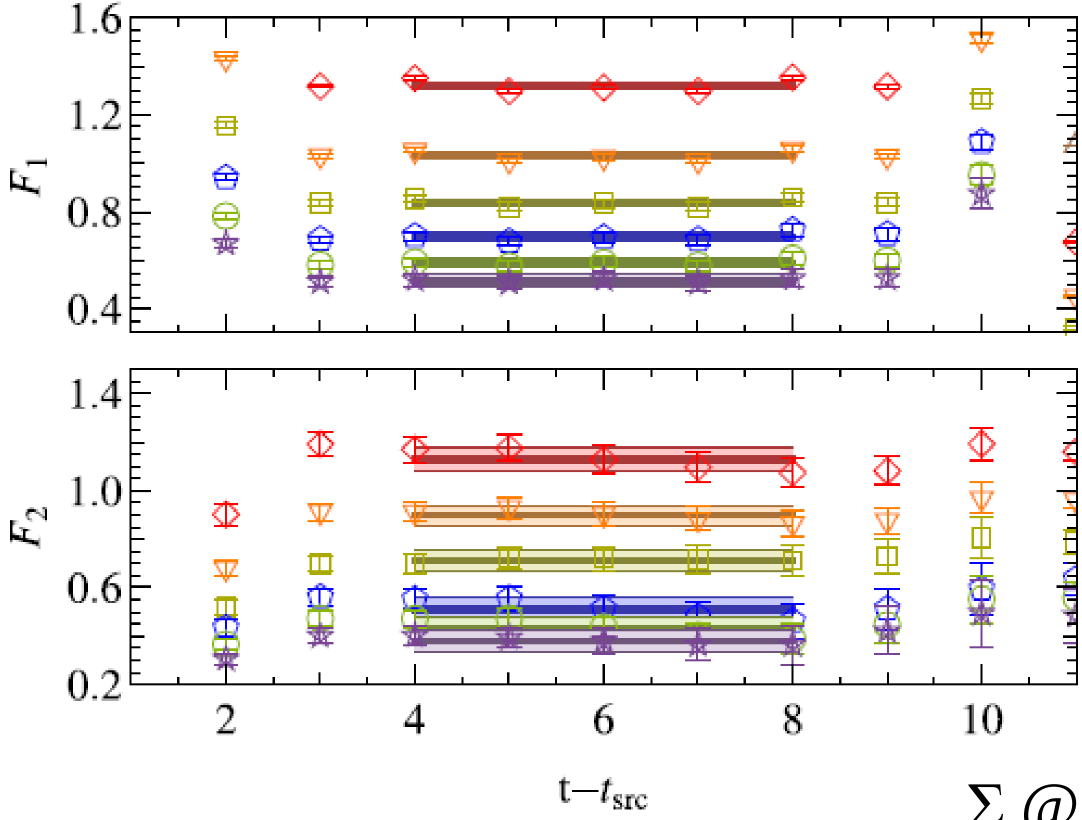
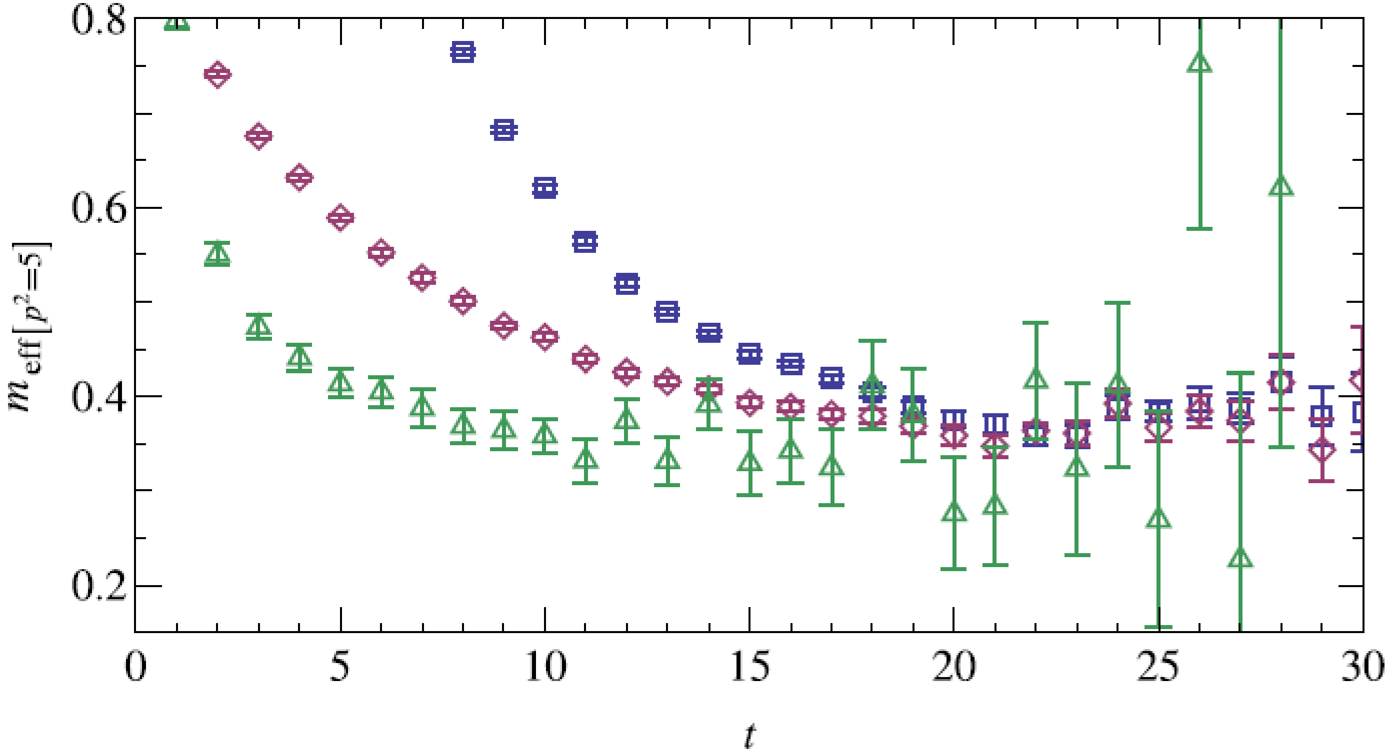


Σ @ 600 MeV pion

Lattice High-Q² Form Factors

- Problem: traditional approach



- Solution: confront excited states directly and allow operators to couple to both ground and excited states

Form Factors

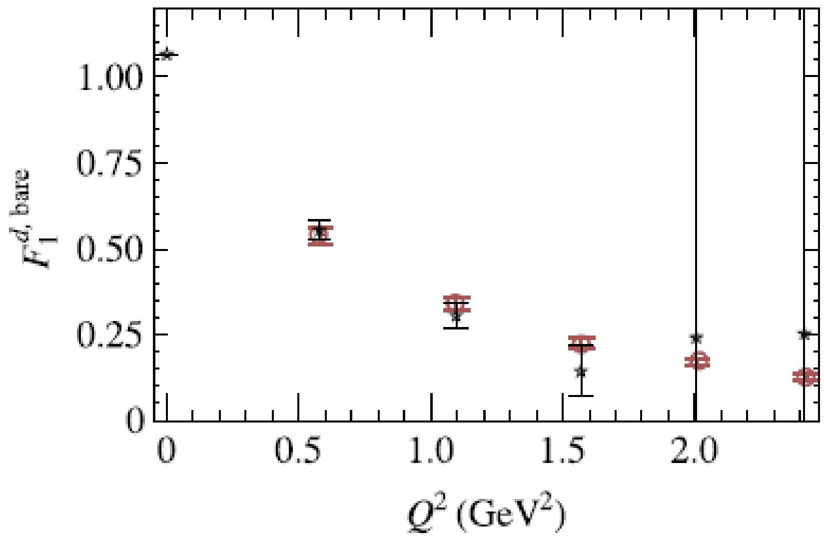
- The form factors are buried in the amplitudes

$$\Gamma^{(3),T}_{\mu,AB}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) = a^3\sum_n\sum_{n'}\frac{1}{Z_j}\frac{Z_{n',B}(p_f)Z_{n,A}(p_i)}{4E'_n(\vec{p}_f)E_n(\vec{p}_i)}e^{-(t_f-t)E'_n(\vec{p}_f)}e^{-(t-t_i)E_n(\vec{p}_i)} \times\sum_{s,s'}T_{\alpha\beta}u_{n'}(\vec{p}_f,s')_\beta\langle N_{n'}(\vec{p}_f,s')\left|j_\mu(0)\right|N_n(\vec{p}_i,s)\rangle\overline{u}_n(\vec{p}_i,s)_\alpha$$

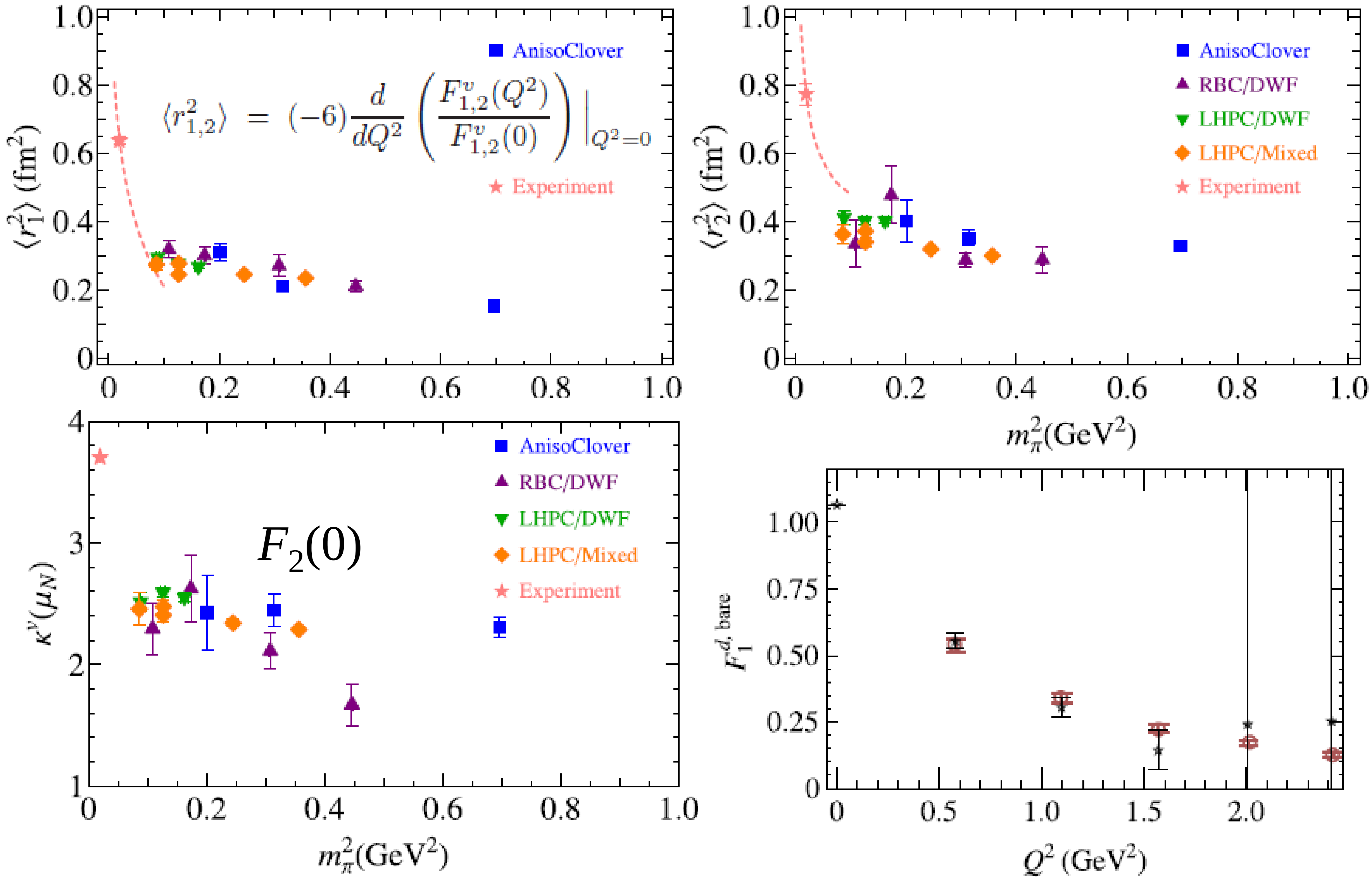
- *n* = *n′* = 0 gives us nucleon

  Matrix Element $\langle N\left|V_\mu\right|N\rangle(q)$

  and solve linear equations for form factors

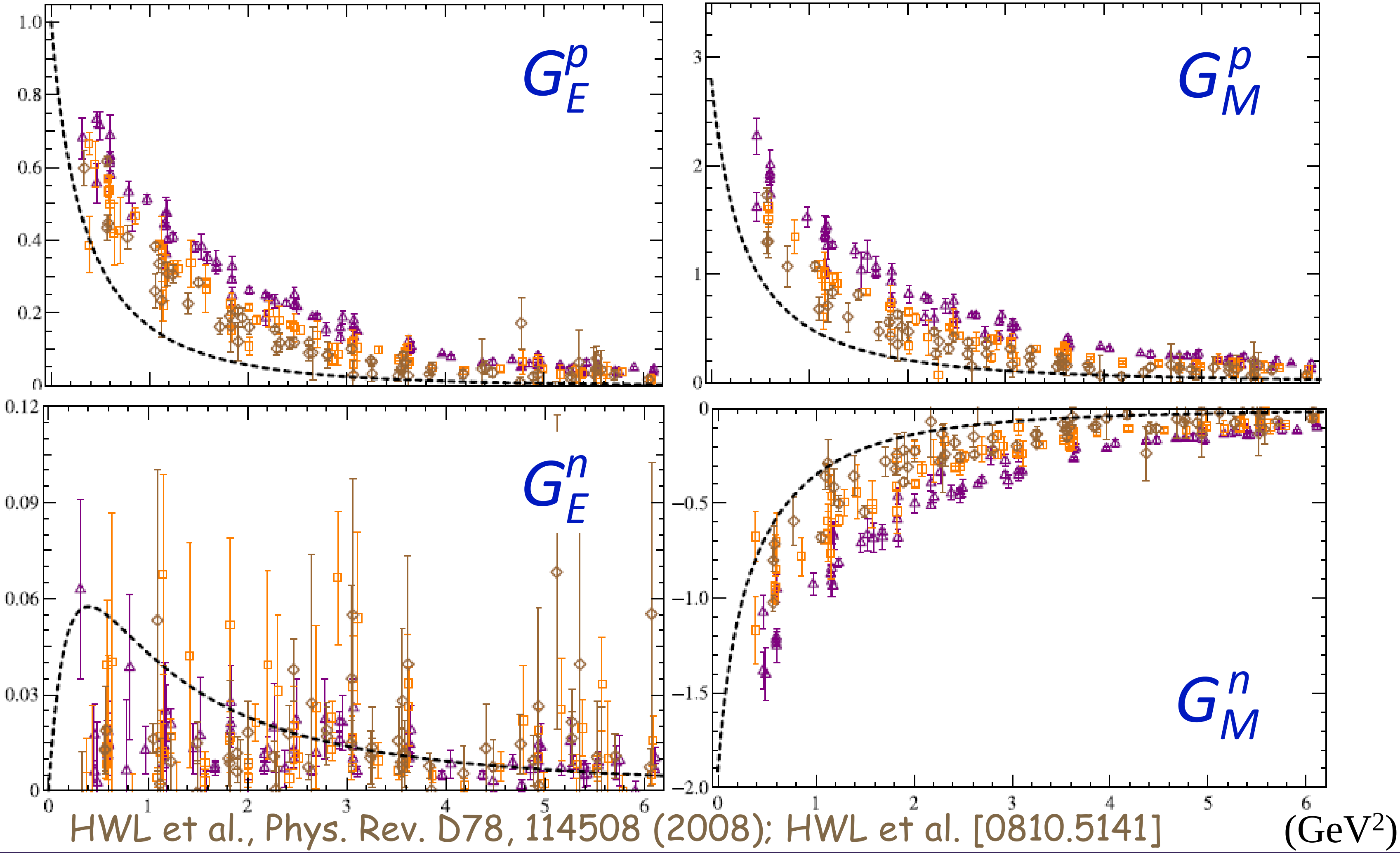


Form Factors



Higher-Q^2 Nucleon Form Factors

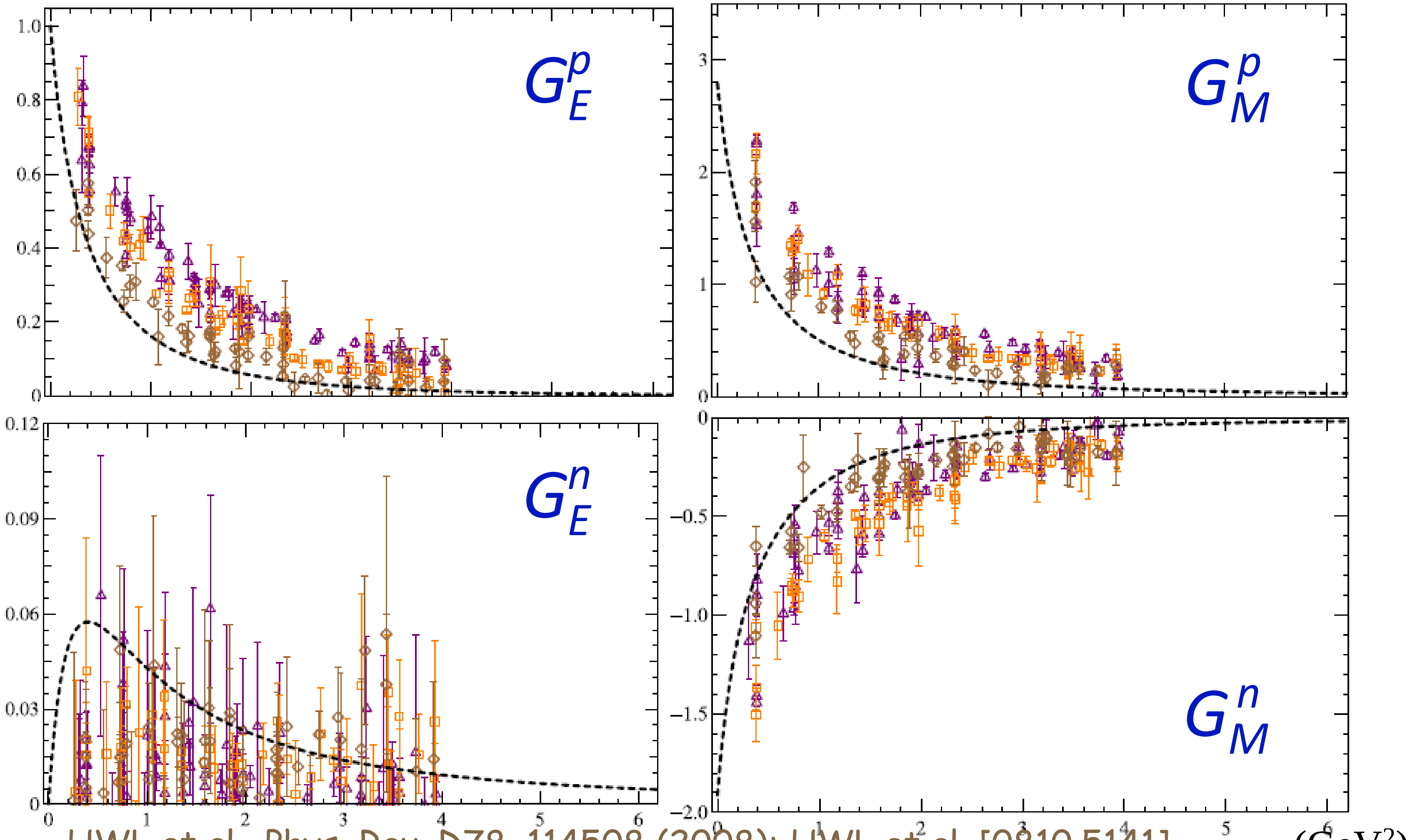
- $N_f = 0$ anisotropic lattices, $M_\pi \approx$ **480**, **720**, **1080** MeV



HWL et al., Phys. Rev. D78, 114508 (2008); HWL et al. [0810.5141]

Higher-Q^2 Nucleon Form Factors

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV



HWL et al., Phys. Rev. D78, 114508 (2008); HWL et al. [0810.5141]

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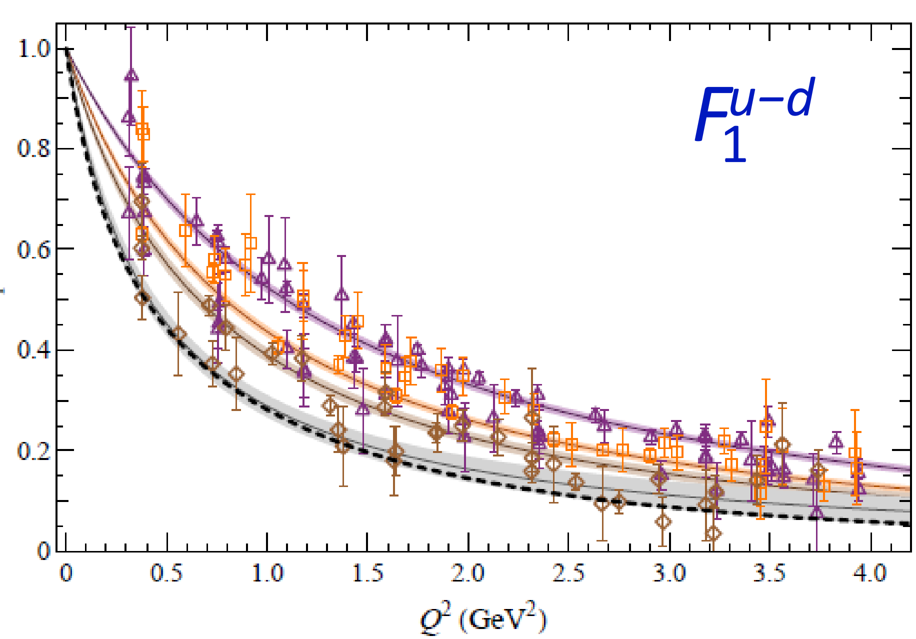
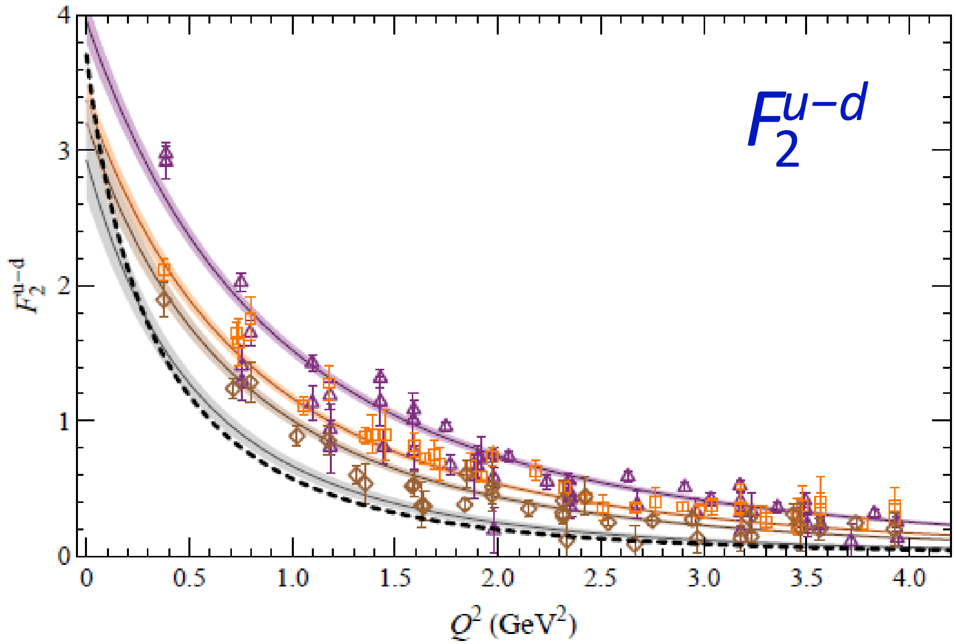




$$F_1 = \frac{a_0 + \sum_{i=1}^{k-2} a_i \tau^i}{1 + \sum_{i=1}^{k} b_i \tau^i}$$

$$\left(\frac{F_2}{\kappa}\right) = \frac{1 + \sum_{i=1}^{k-3} a_i \tau^i}{1 + \sum_{i=1}^{k} b_i \tau^i}.$$

$$\tau = \frac{Q^2}{4m_N^2}$$

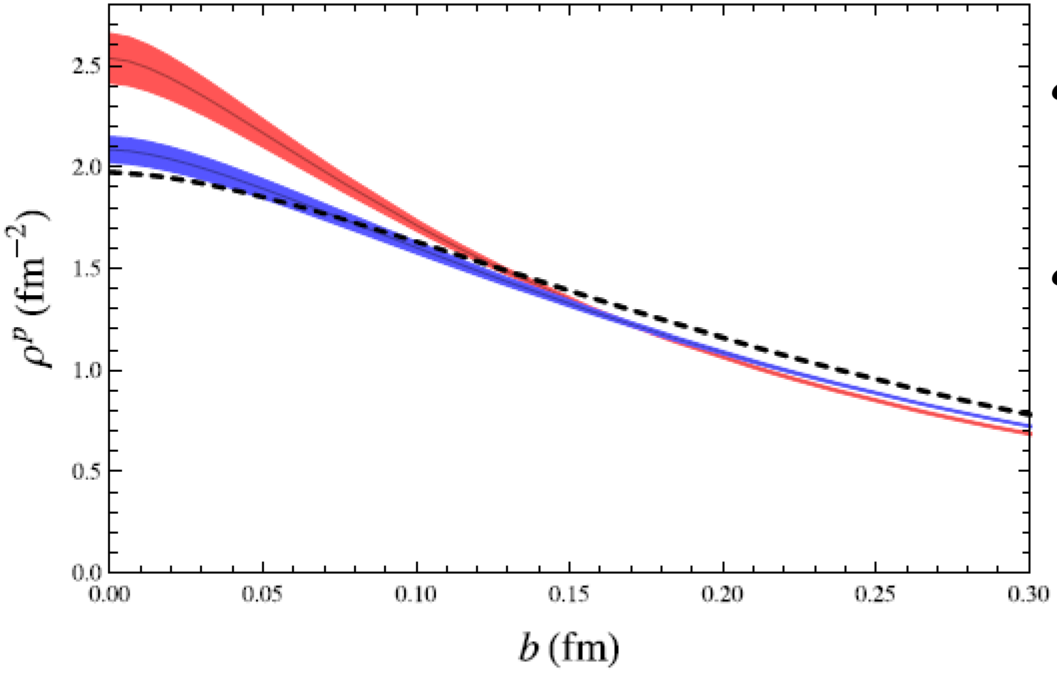
HWL et al., arXiv: 1005.0799

How High is High Enough?

§ For example, how does high-Q^2 affect charge density?

$$\rho(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$

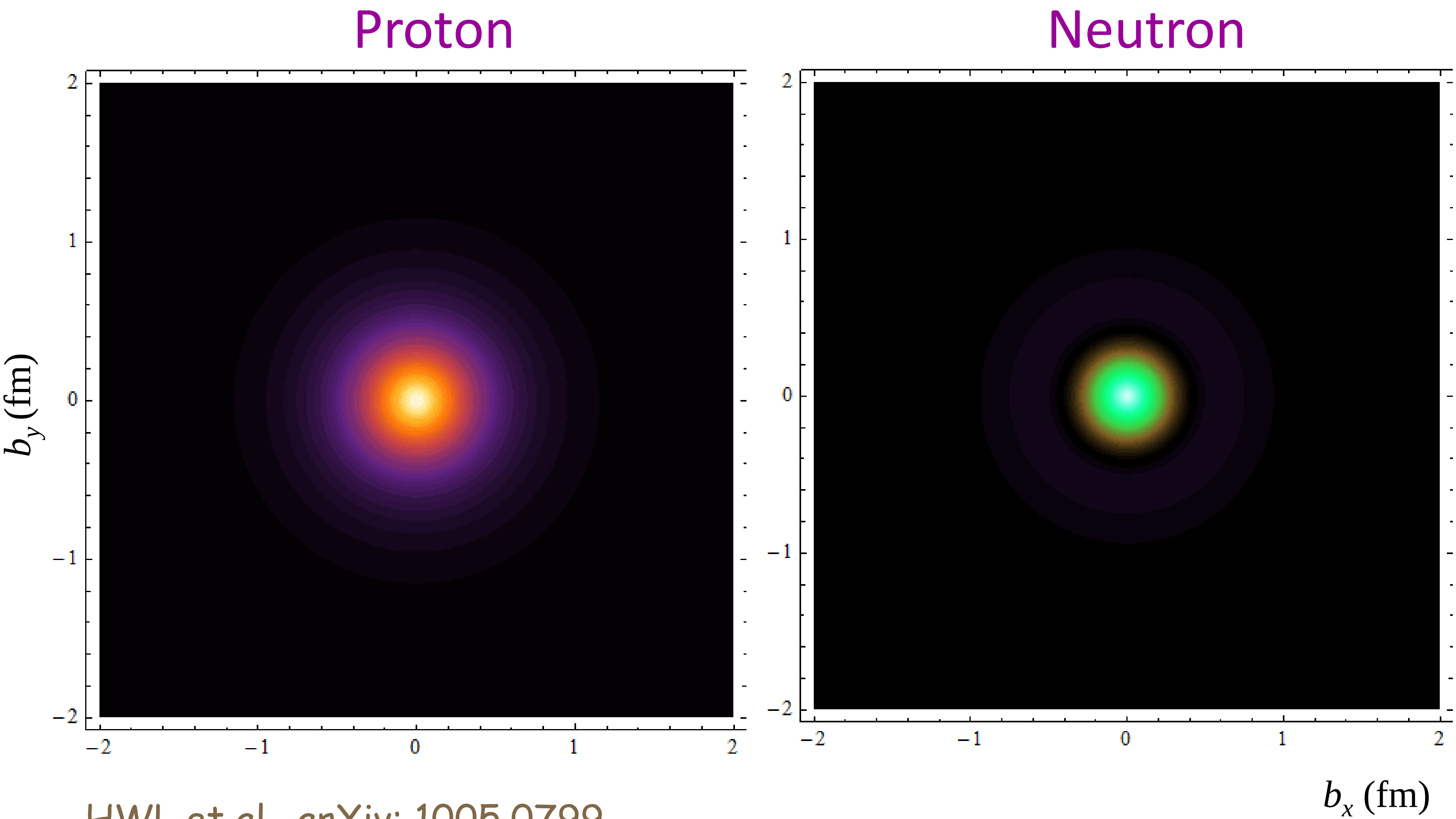
G. A. Miller, arXiv: 1002.0355



- Red band uses lattice data ≤ 2.0 GeV^2
- Blue band uses lattice data ≤ 4.0 GeV^2

HWL et al., arXiv: 1005.0799

Transverse Charge Density

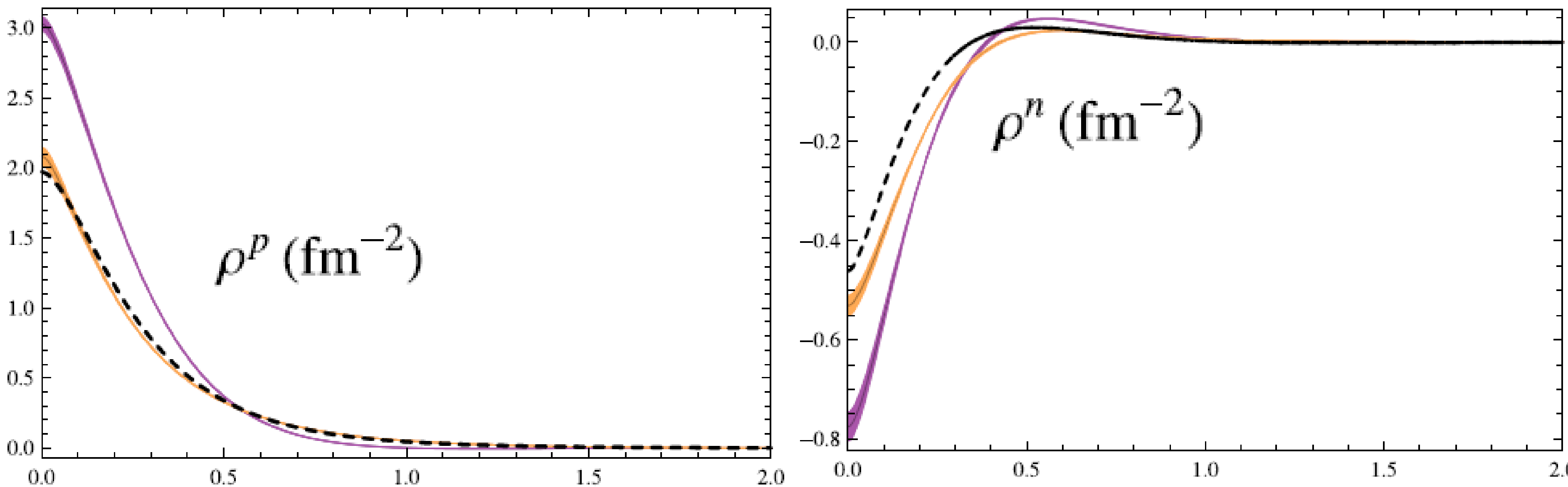


HWL et al., arXiv: 1005.0799

Transverse Charge Density

Proton

Neutron

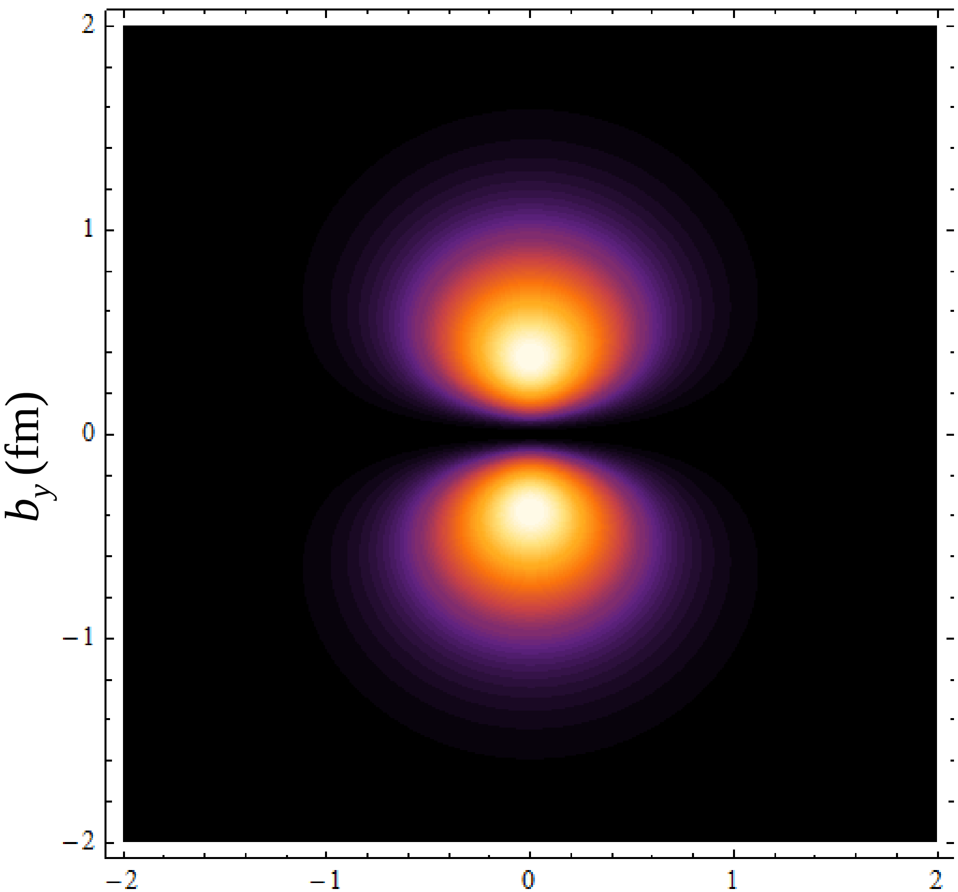


b (fm)

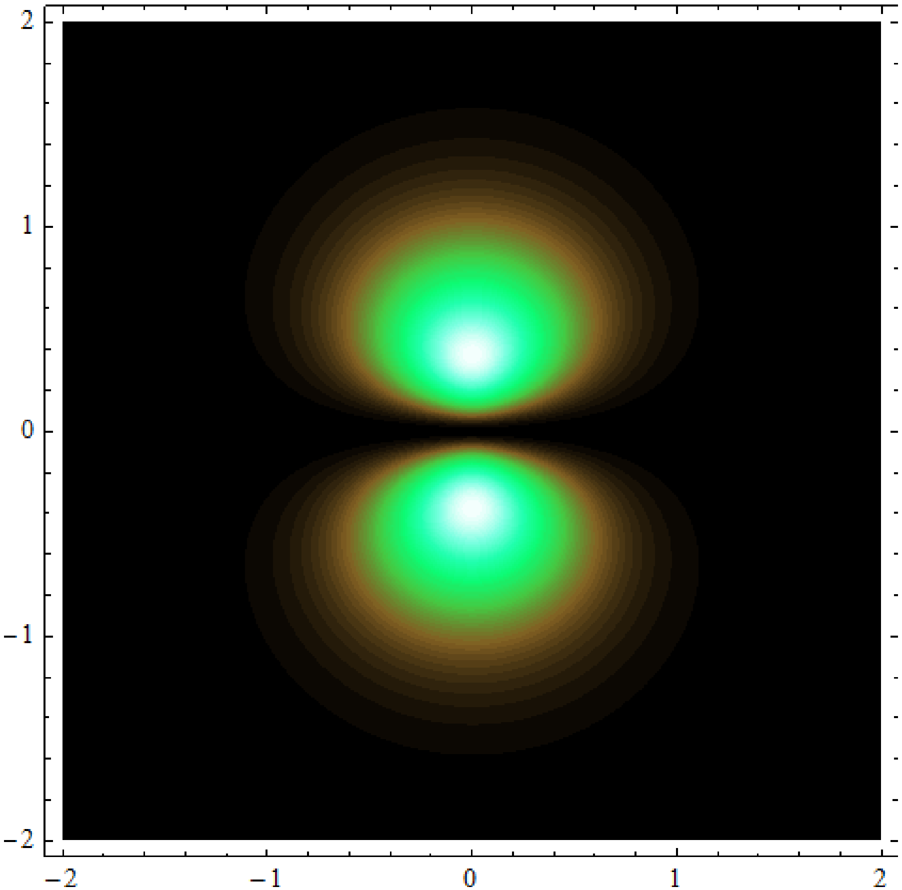
HWL et al., arXiv: 1005.0799

Transverse Magnetization Density

Proton



Neutron

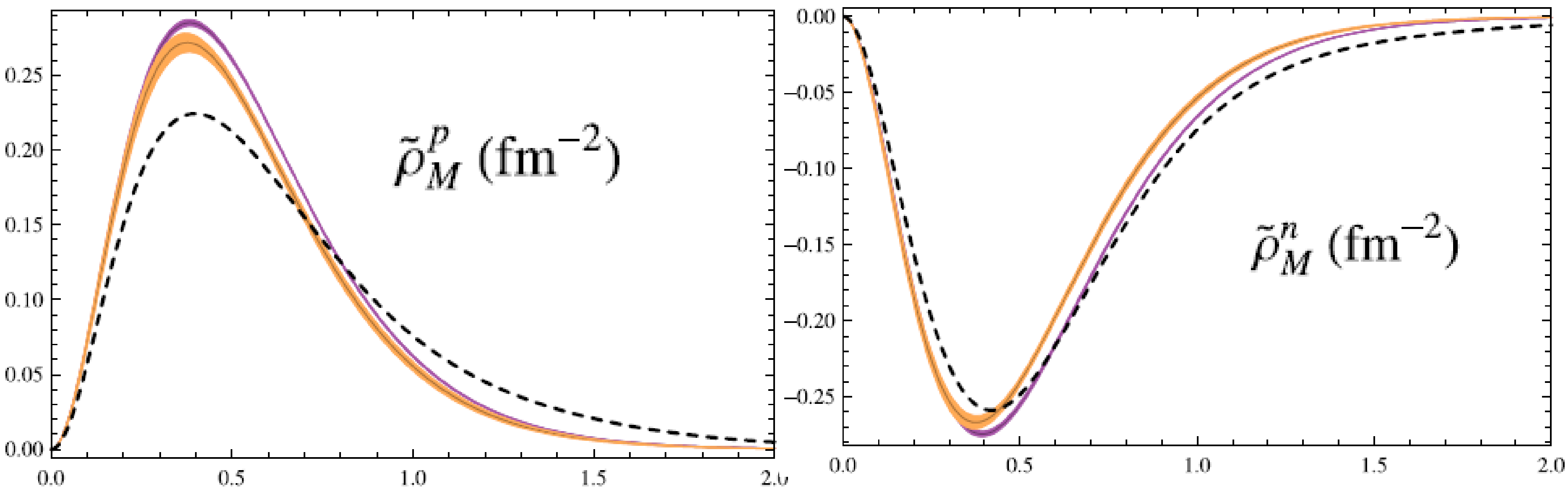


HWL et al., arXiv: 1005.0799

$$b \sin^2 \phi \int_0^\infty \frac{Q^2\, dQ}{2\pi} J_1(bQ) F_2(Q^2)$$

Transverse Magnetization Density

Proton   Neutron



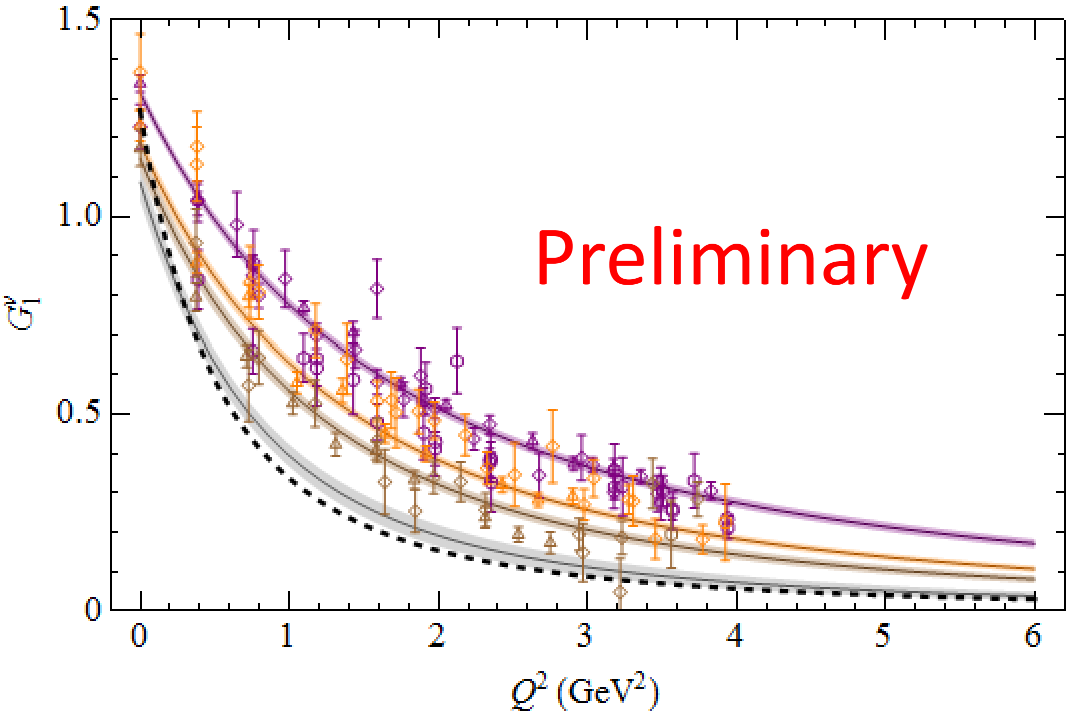
b (fm)

HWL et al., arXiv: 1005.0799 $b \sin^2\phi \int_0^\infty \frac{Q^2\, dQ}{2\pi} J_1(bQ) F_2(Q^2)$

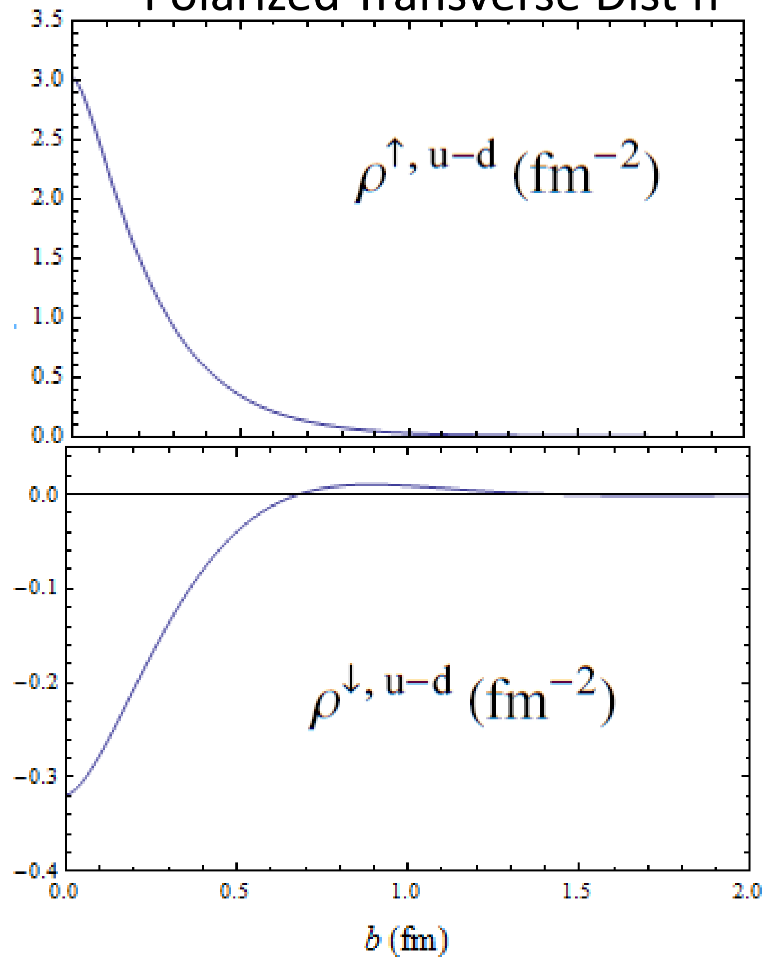
Nucleon Axial Form Factors

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV

$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right] u_B(p)$$

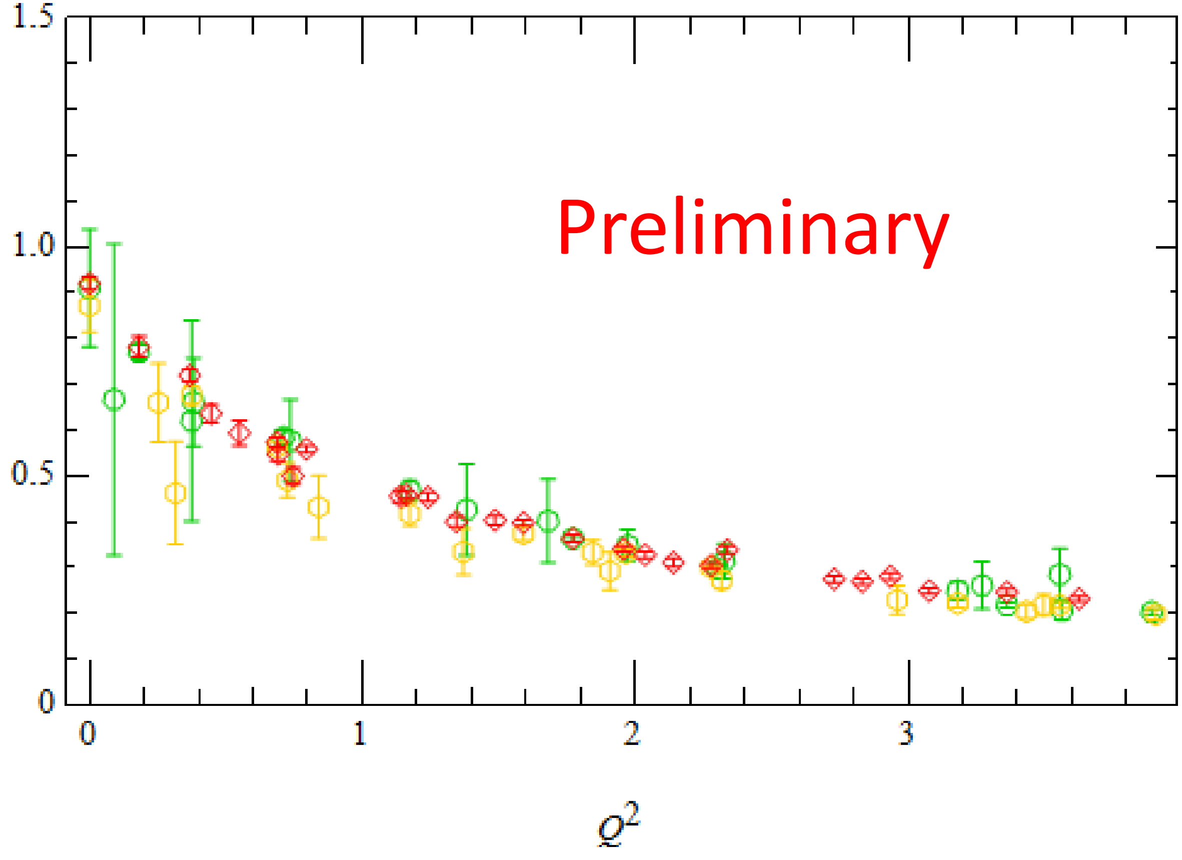


Polarized Transverse Dist'n



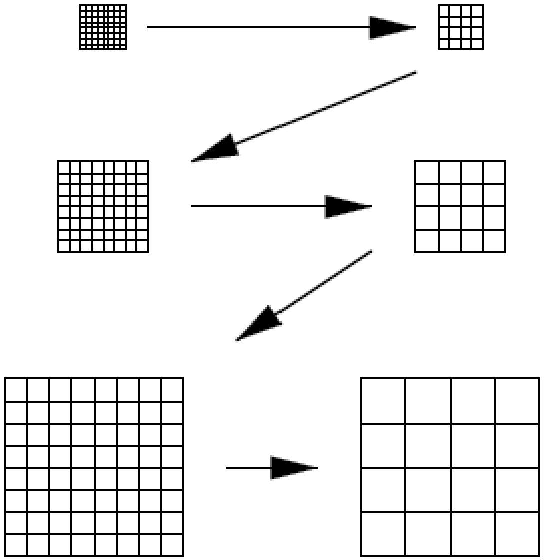
Pion Form Factors

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **875** MeV



Miscellany

- § Disconnected contribution O(10^{-2}) for EM form factor
  - Small for most of the form factors but could be significant for neutron electric form factor
- § To get larger momentum, we use O(*ap*) ≈ 1
  - Rome was not built in a day…
  - Methodology for improving a traditional lattice calculation
- § Possible future improvement
  - Step-scaling through multiple lattice spacings and volumes
  - Higher momentum transfer



Summary

Exciting era using LQCD for studying hadron structure

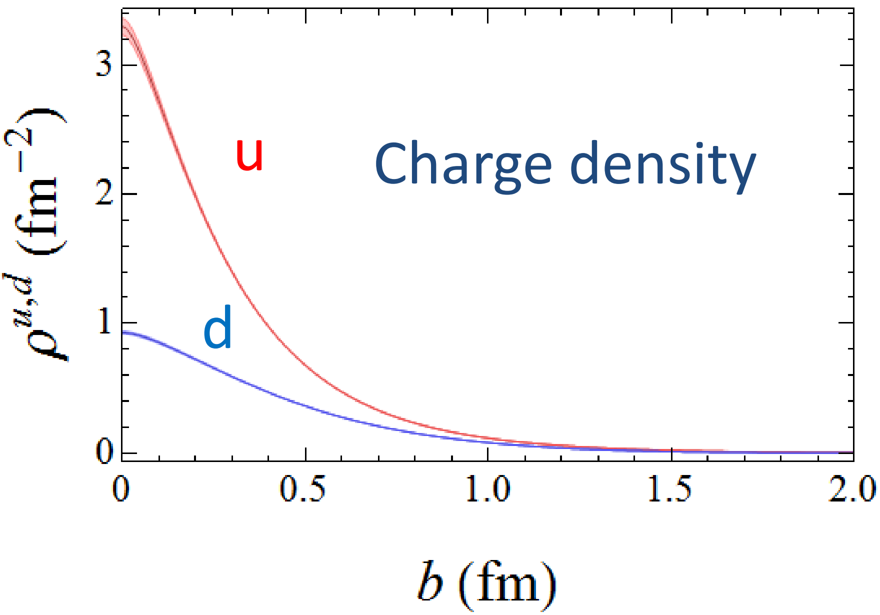
- Improvement
  - Huge leaps due to increasing computational resources world-wide and improved algorithms
- Universality
  - Different lattice actions/groups with independent calculations provide consistency checks: so far so good…
- Confidence
  - Reproducing well measured experimental values gives us confidence for predicting quantities that haven't/couldn't be measured by experiment
- Variety
  - There are many different aspects of hadron structure; only presented a few examples

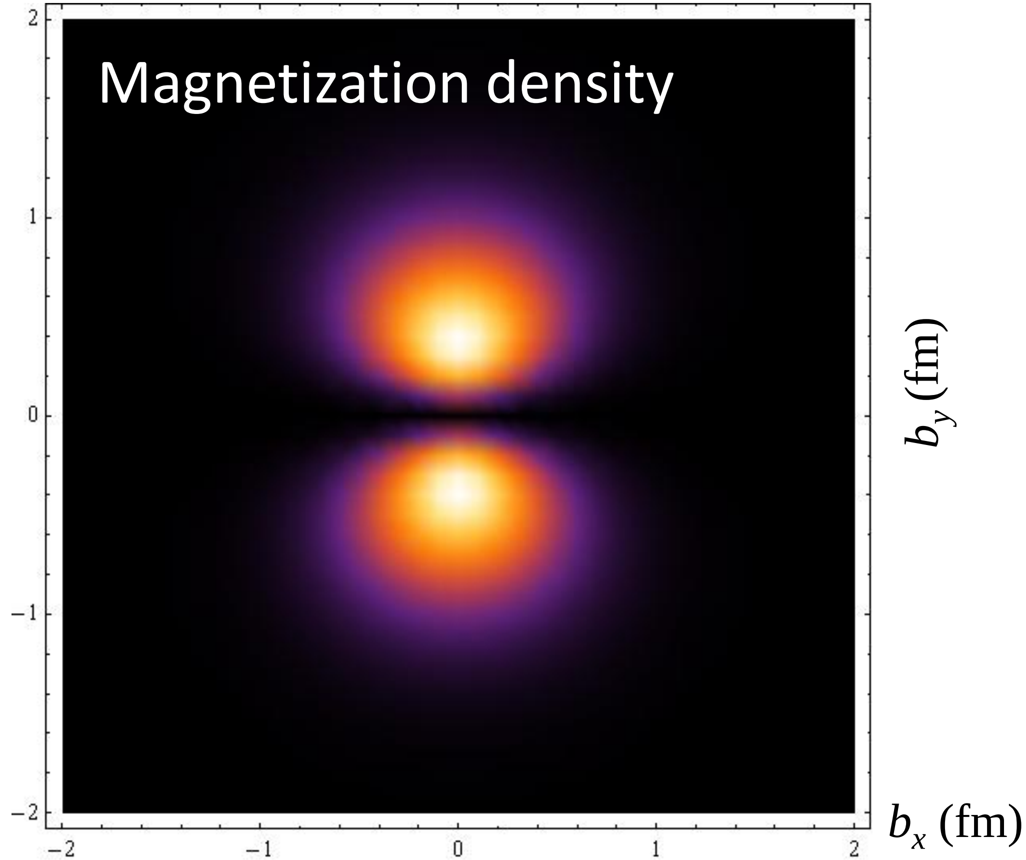
Backup Slides

Proton Transverse Density

§ We know the proton is composed of up and down quarks

$$\rho(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) F_1(Q^2)$$



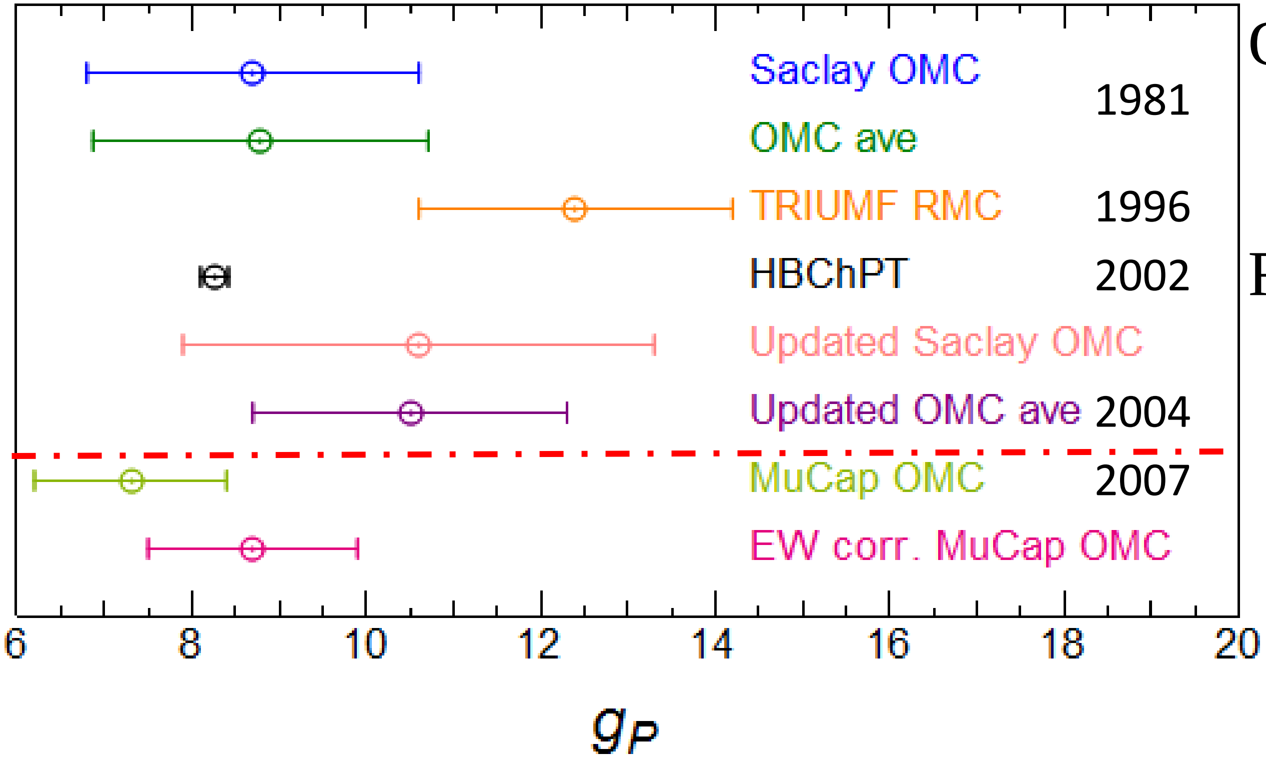


$$b \sin^2\phi \int_0^\infty \frac{Q^2\,dQ}{2\pi} J_1(bQ) F_2(Q^2)$$

HWL et al., arXiv: 1005.0799

g_P

§ g_P induced pseudoscalar coupling constant



Ordinary muon capture

$$\mu^- + p \longrightarrow \nu_\mu + n$$

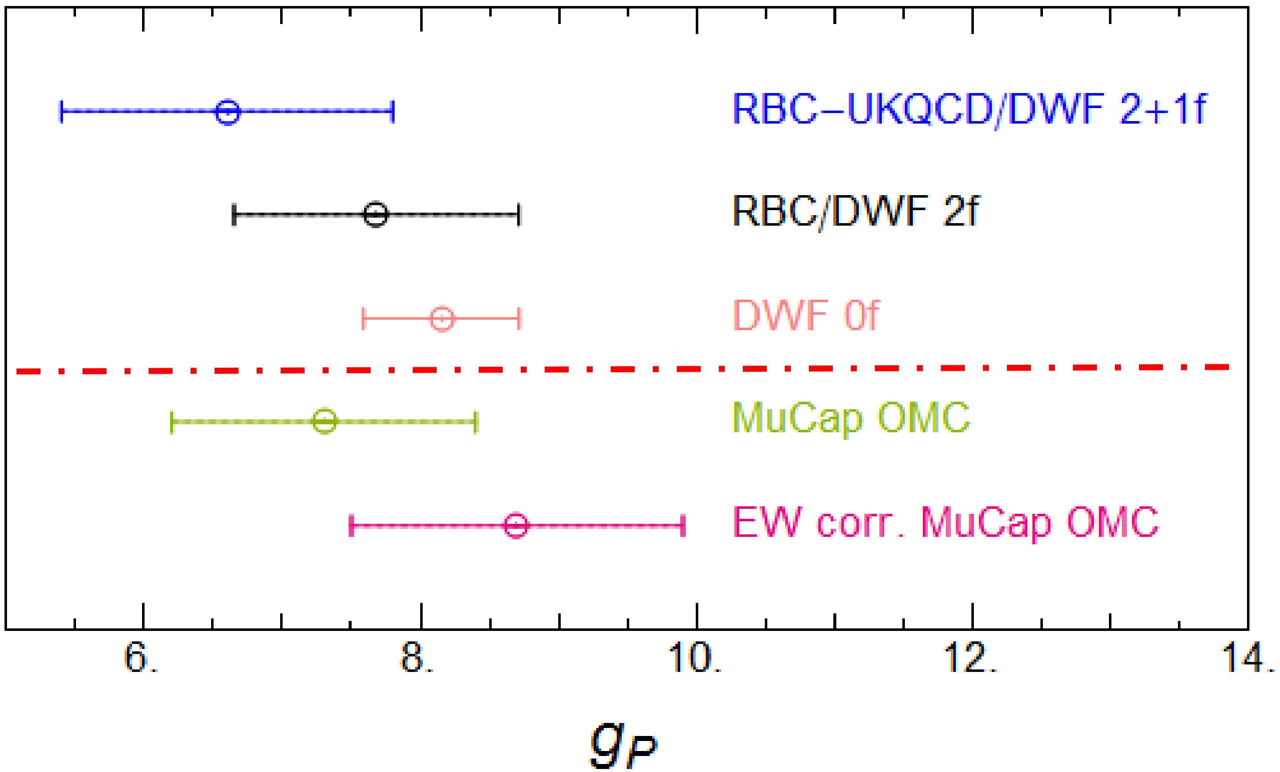
Radiative muon capture

$$\mu^- + p \longrightarrow \gamma + \nu_\mu + n$$

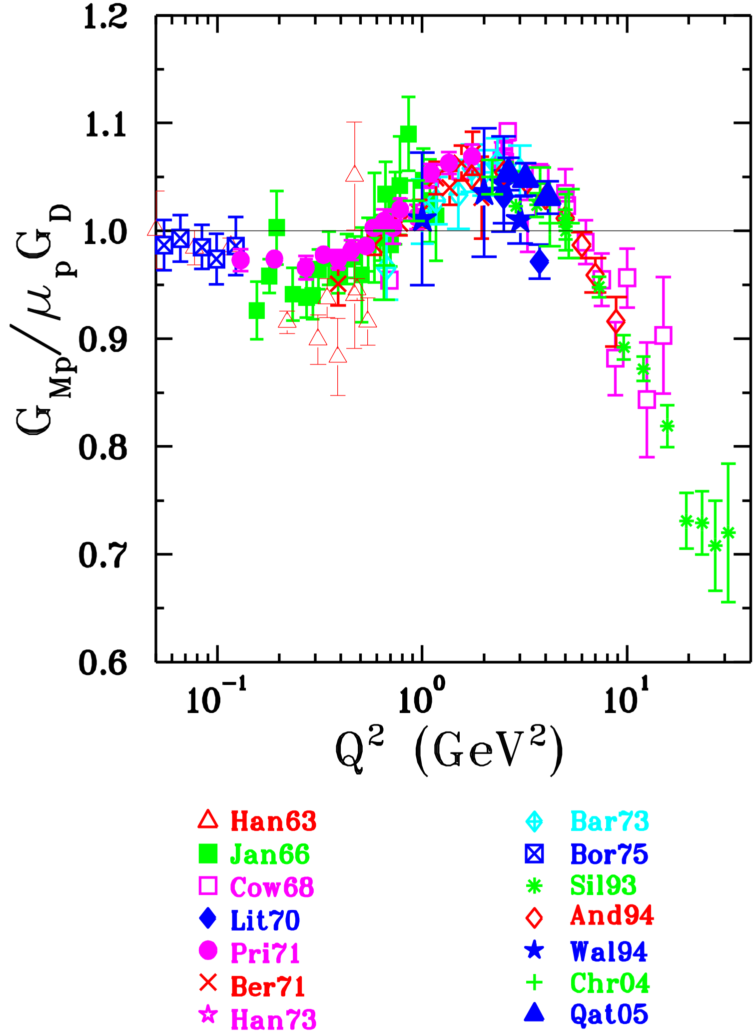
g_P

- g_P induced pseudoscalar coupling constant

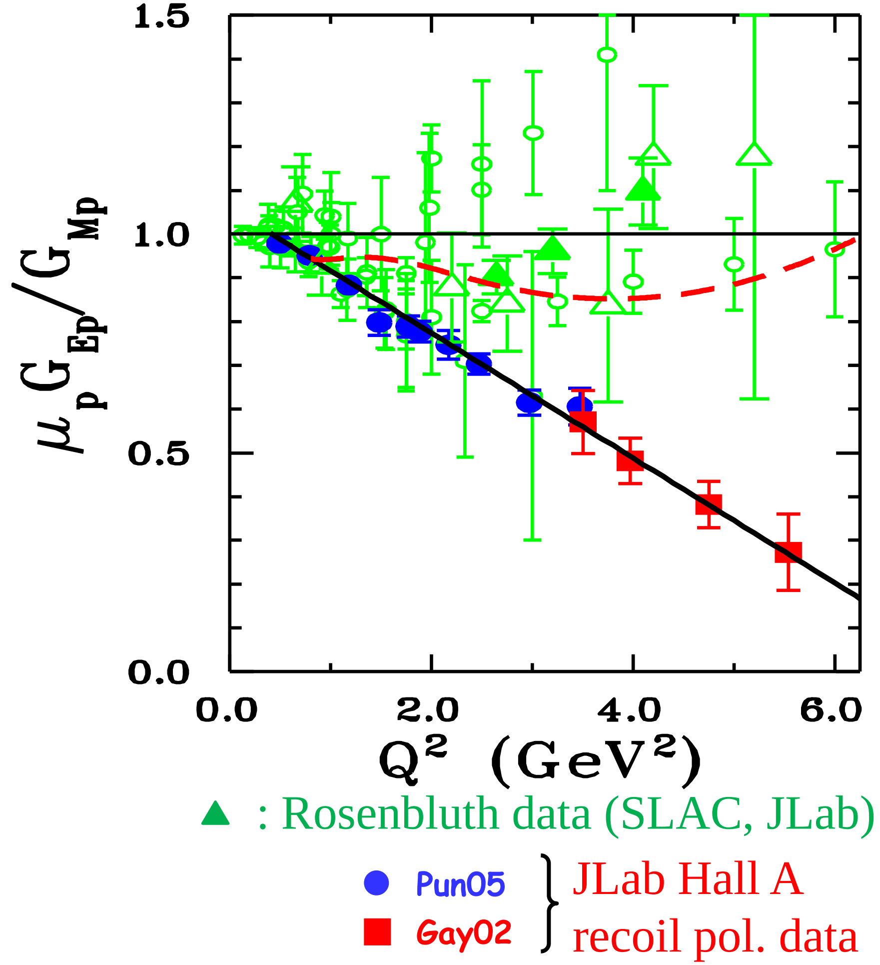
$$g_P = m_\mu G_P(0.88\, m_\mu^2) / 2\, m_N$$



Proton EM Form Factors: Exp't

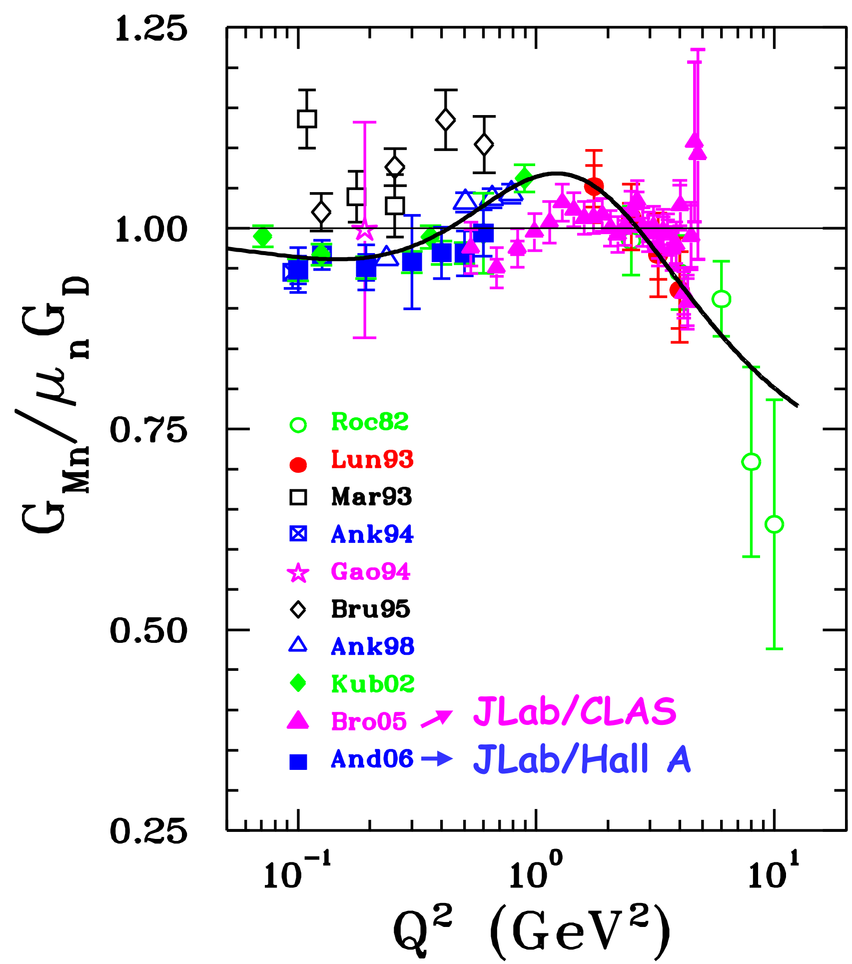


new MAMI/A1 data up to $Q^2 \approx 0.7$ GeV^2

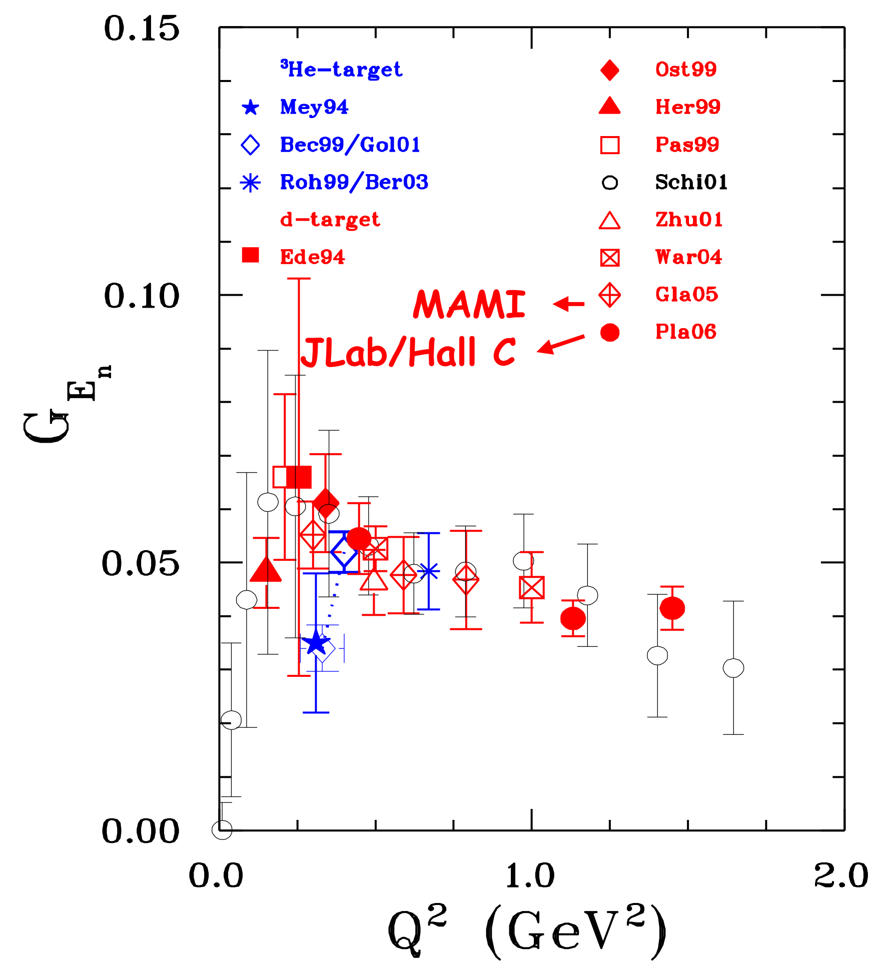


new JLab Hall C recoil pol. exp't extension up to $Q^2 \approx 8.5$ GeV^2

Neutron EM Form Factors: Exp't



new MIT-Bates (BLAST) data for both *p* and *n* at low Q^2



new JLab/Hall A double-pol. exp't extension to $Q^2 \approx 3.5$ GeV^2 completed

Polarized Transverse Distributions

- $N_f = 2+1$ anisotropic lattices, $M_\pi \approx$ **450**, **580**, **875** MeV

$$q^\uparrow(\vec{b}_\perp) = \int_{-1}^{1} dx\, q^\uparrow(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{1}{2}\left[A_{10}^q(-\vec{\Delta}_\perp^2) + \tilde{A}_{10}^q(-\vec{\Delta}_\perp^2)\right]$$

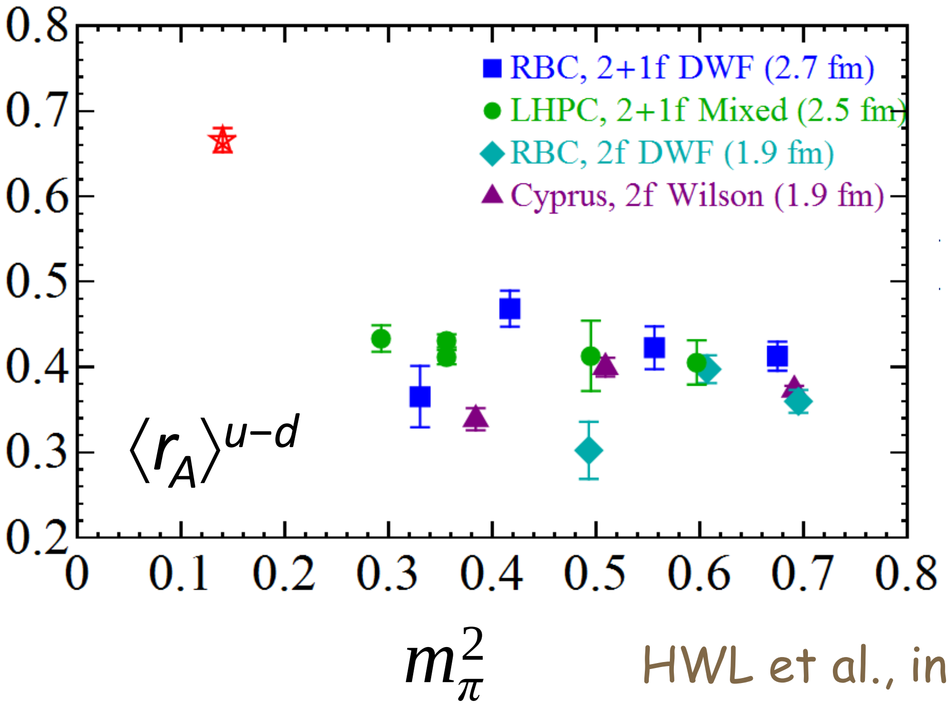
$$q^\downarrow(\vec{b}_\perp) = \int_{-1}^{1} dx\, q^\downarrow(x, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \Delta_\perp} \frac{1}{2}\left[A_{10}^q(-\vec{\Delta}_\perp^2) - \tilde{A}_{10}^q(-\vec{\Delta}_\perp^2)\right]$$

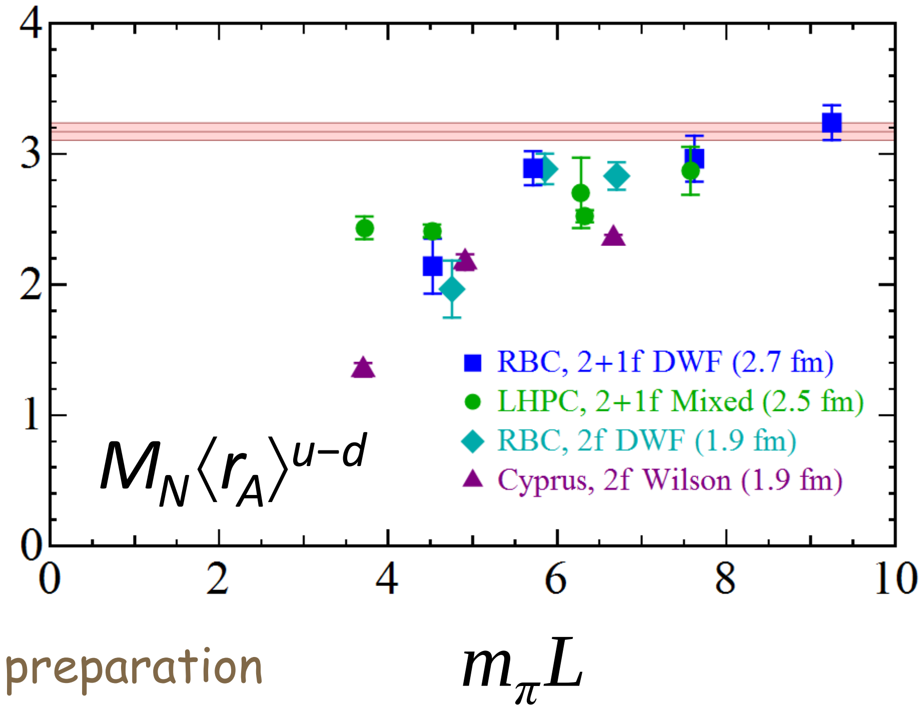
Nucleon Axial Radii

§ Axial RMS $\langle r_A \rangle^{u-d}$

$$\tau = \frac{Q^2}{4m_N^2} \qquad 4(m_N^{\rm lat})^2\langle r_{1,2}^2\rangle = (-6)\frac{d}{d\tau}\left(\frac{F_{1,2}^v(\tau)}{F_{1,2}^v(0)}\right)\Big|_{\tau=0}$$

G_A





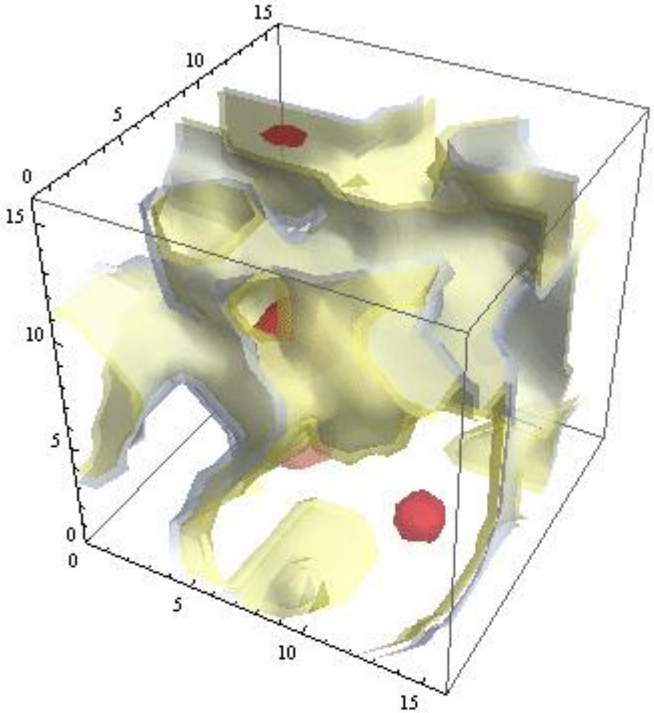
HWL et al., in preparation

Difficulties at Low Energy

§ Even just the vacuum of QCD is complicated

Classical

QCD



Lattice Inputs for LECs

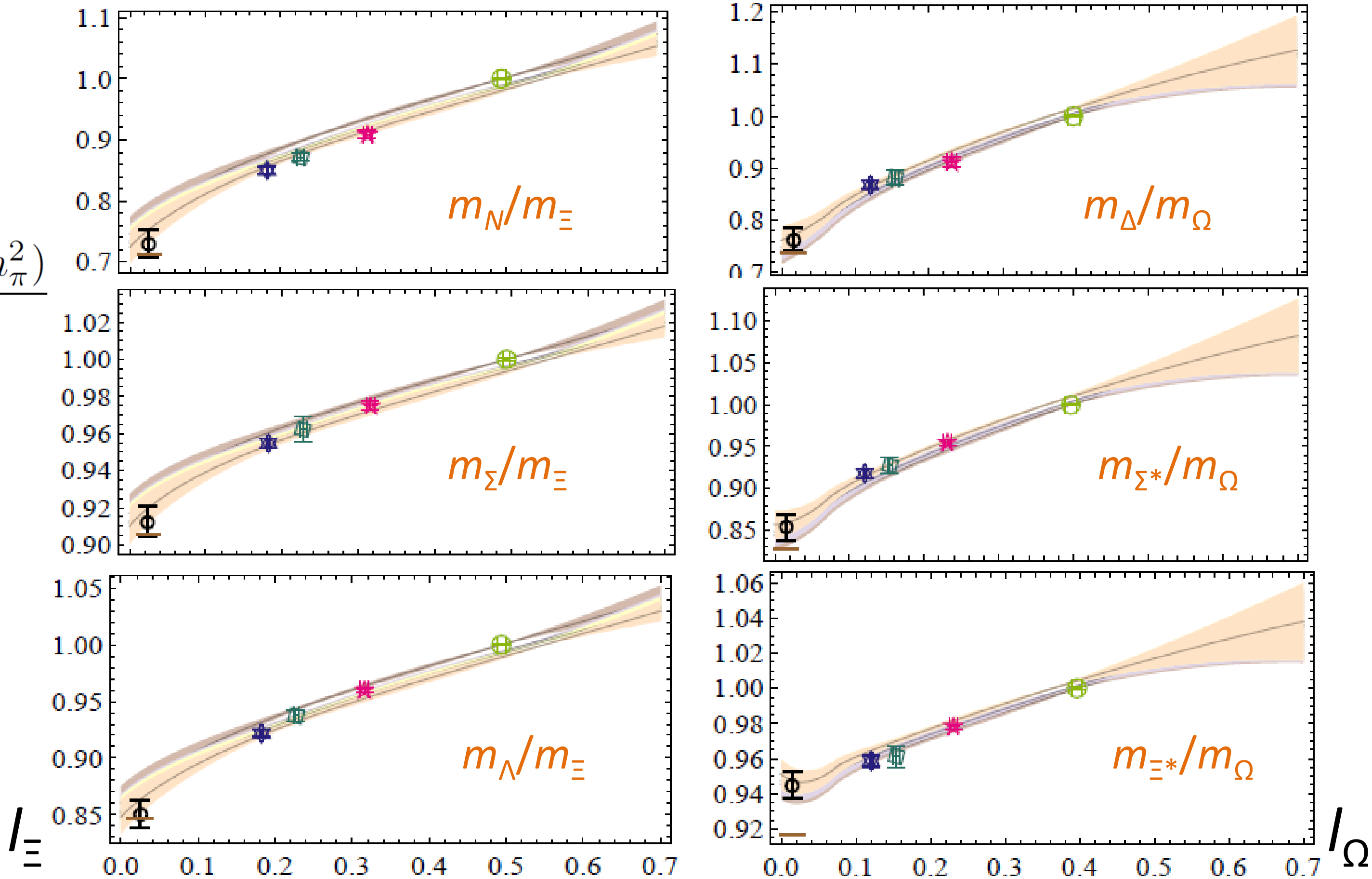
Dynamical Anisotropic Lattices

§ Baryon mass-ratio extrapolation with modified NLO HBXPT

- Dimensionless quantities to avoid lattice-spacing ambiguities
- Mass ratios with finite-volume corrections

$$l_\Xi = \frac{9m_\pi^2}{4m_\Xi^2},$$
$$s_\Xi = \frac{9(2m_K^2 - m_\pi^2)}{4m_\Xi^2}$$

HWL et al.,
in preparation



Coupling Constant

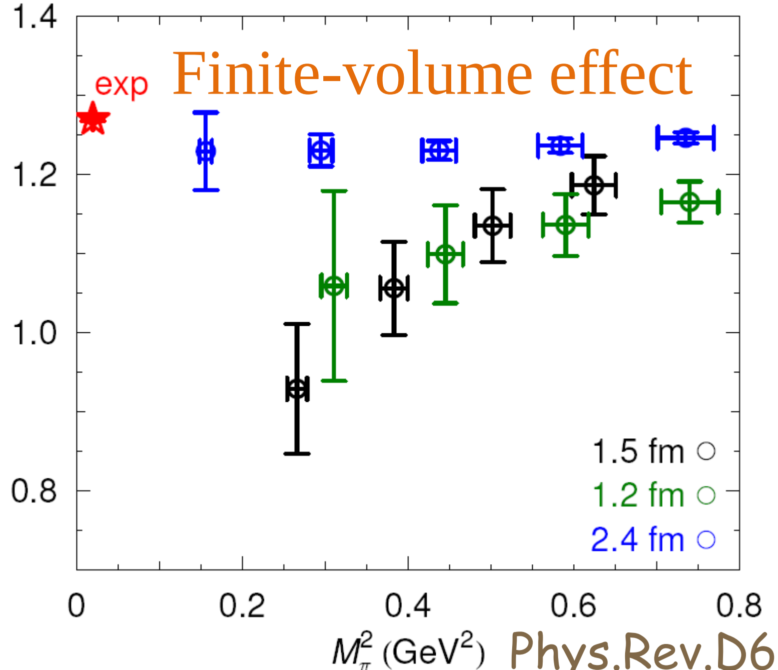
Axial Charge Coupling

- Axial-vector–current matrix element

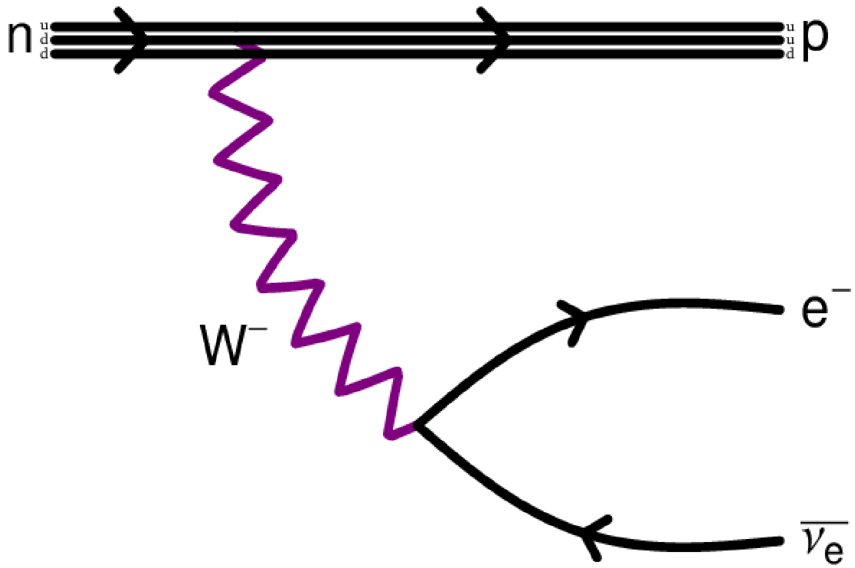
$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\nu\frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

and axial charge coupling $g_A = G_A^{u-d}(Q^2{=}0)$

- Well measured experimentally from neutron beta decay

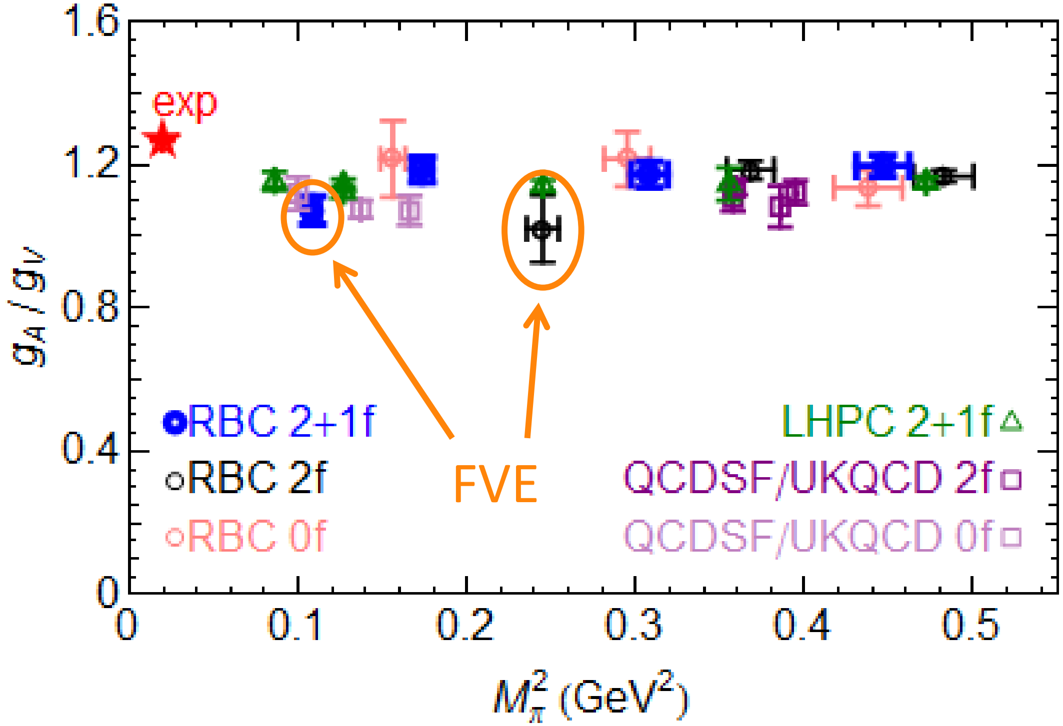


Phys.Rev.D68:054509 (2003)

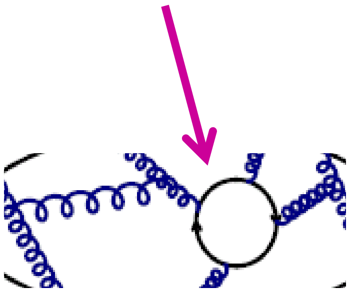


Axial Charge Coupling

§ World data: statistical error-bars only



2+1f : u/d + s
2f : u/d
0f : Ø



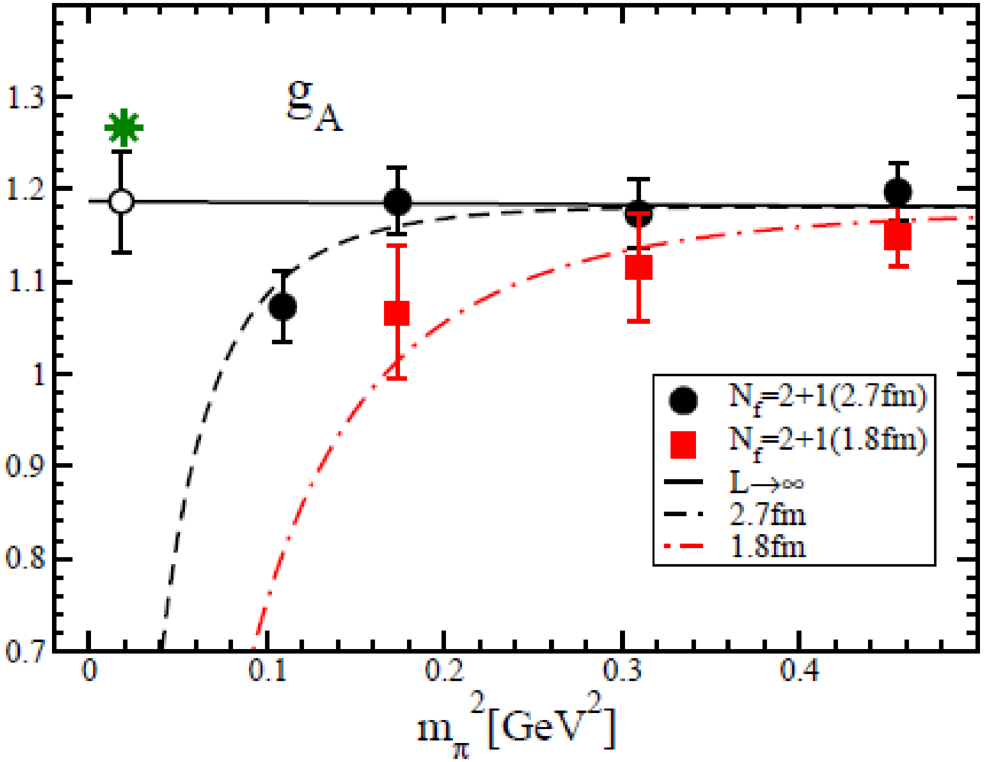
HWL et al., Phys. Rev. D78, 014505 (2008) and Phys. Rev. Lett. 100:171602 (2008);
K. Orginos et al., Phys.Rev.D73:094507 (2005);
D. Dolgov et al., Phys. Rev. D66, 034506 (2002);
M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;
D. Renner et al., PoS(LAT2006)121

Axial Charge Coupling

§ Finite-volume effects

$$A + B\,m_\pi^2 + C\,f_V(m_\pi L)$$

$$f_V \sim e^{-m_\pi L}$$



Systematics:

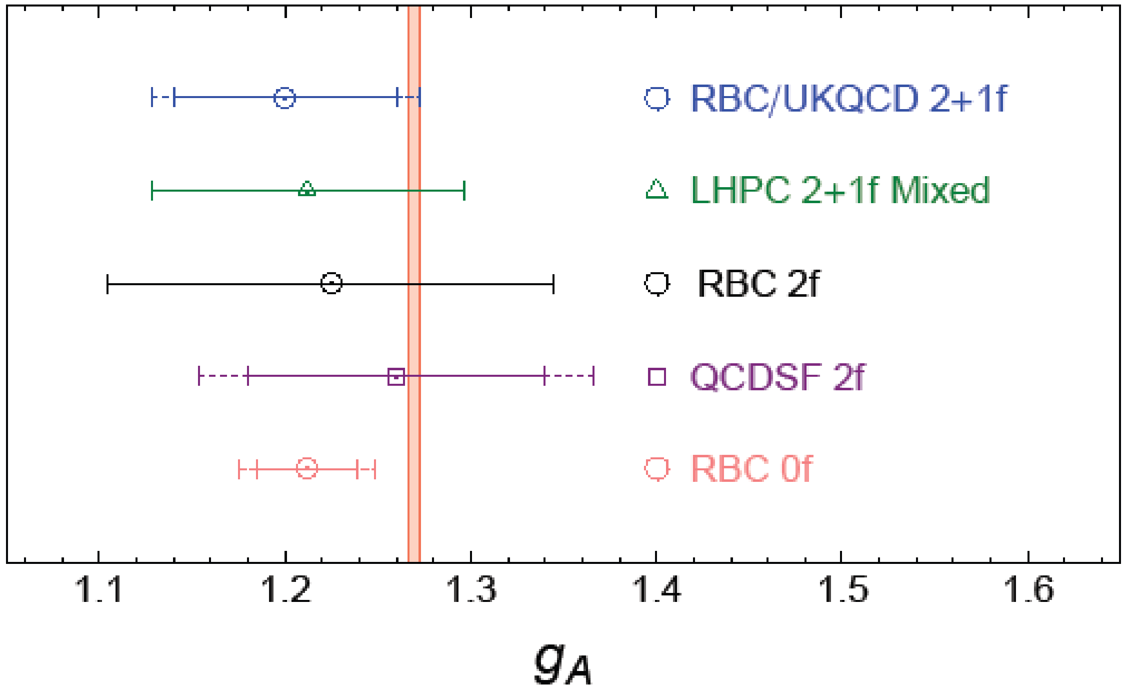
$$f_V \sim (m_\pi L)^{-3}$$

$$f_V \sim m_\pi^2\, e^{-m_\pi L}(m_\pi L)^{-0.5}$$

RBC/UKQCD, Phys. Rev. Lett. 100:171602 (2008)

Axial Charge Coupling

§ Comparison of lattice calculations



HWL et al., Phys. Rev. D78, 014505 (2008) and Phys. Rev. Lett. 100:171602 (2008);
K. Orginos et al., Phys.Rev.D73:094507 (2005); LHPC, arXiv:1001.3620[hep-lat]
D. Dolgov et al., Phys. Rev. D66, 034506 (2002);
M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;

πNN Coupling

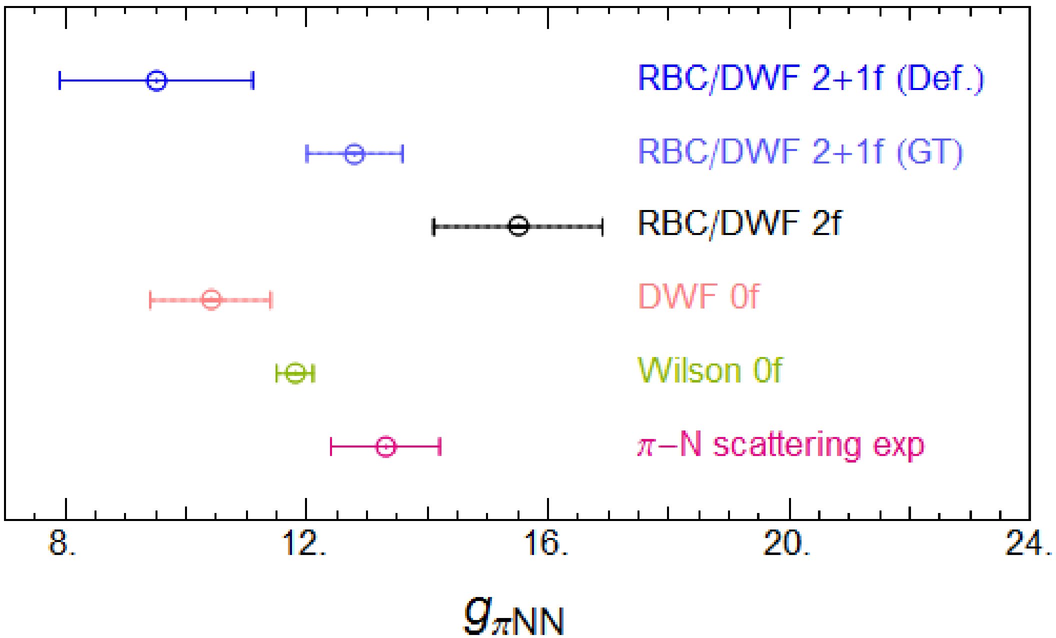
§ $g_{\pi NN}$ via 2 approaches

- Pion-pole domination

$$g_{\pi NN} \approx [(Q^2 + m_\pi{}^2)\, m_N\, G_P(Q^2) / f_\pi] \text{ at } Q^2 = -m_\pi{}^2$$

- Goldberger-Treiman Relation

$$g_{\pi NN} \approx m_N\, G_A(Q^2{=}0) / f_\pi$$



$g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Has applications such as hyperon scattering, non-leptonic decays, …
- Cannot be determined by experiment
- Existing theoretical predictions:
  - Chiral perturbation theory

$$0.35 \le g_{\Sigma\Sigma} \le 0.55 \qquad 0.18 \le -g_{\Xi\Xi} \le 0.36$$

M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - Large-N_c

$$0.30 \le g_{\Sigma\Sigma} \le 0.36 \qquad 0.26 \le -g_{\Xi\Xi} \le 0.30$$

R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

- Loose bounds on the values
- Lattice QCD can provide substantial improvement

$g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

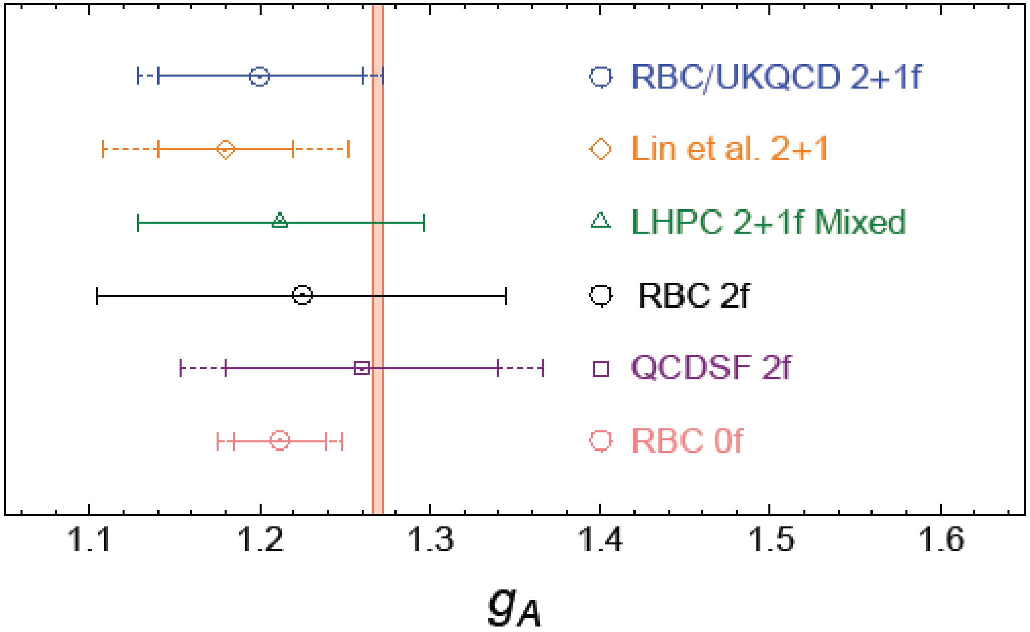
- Pion mass: 350–750 MeV HWL and K. Orginos, Phys.Rev.D79:034507,2009
- First lattice calculation of these quantities; mixed-action full-QCD

	m010	m020	m030	m040	m050
m_π (MeV)	354.2(8)	493.6(6)	594.2(8)	685.4(19)	754.3(16)
m_π/f_π	2.316(7)	3.035(7)	3.478(8)	3.822(23)	4.136(20)
m_K/f_π	3.951(14)	3.969(10)	4.018(11)	4.060(26)	4.107(21)
confs	612	345	561	320	342
$g_{A,N}$	1.22(8)	1.21(5)	1.195(17)	1.150(17)	1.167(11)
$g_{\Sigma\Sigma}$	0.418(23)	0.450(15)	0.451(7)	0.444(8)	0.453(5)
$g_{\Xi\Xi}$	−0.262(13)	−0.270(10)	−0.269(7)	−0.257(9)	−0.261(7)

- Combine with g_A for study of
  - SU(3) symmetry breaking
  - SU(3) simultaneous fits among three coupling constants
    → D, F, and other low-energy constants

Axial Charge Coupling

§ Comparison of lattice calculations



- SU(3)-constrained fit gives Lin et al. smaller extrapolated statistical error than LHPC
- Lighter m_π, finer a, multiple V essential for precise calculation

HWL et al., Phys. Rev. D78, 014505 (2008) and Phys. Rev. Lett. 100:171602 (2008);
HWL et al., Phys.Rev.D79:034507,2009
K. Orginos et al., Phys.Rev.D73:094507 (2005); LHPC, arXiv:1001.3620[hep-lat]
D. Dolgov et al., Phys. Rev. D66, 034506 (2002);
M. Guertler et al., PoS(LAT2006)107; D. Pleiter et al., PoS(LAT2006)120;

πYY Coupling

§ Goldberger-Treiman Relation

$$g_{\pi NN} \approx m_N G_A(Q^2{=}0)/f_\pi$$

$\pi Y_1 Y_2$ Coupling

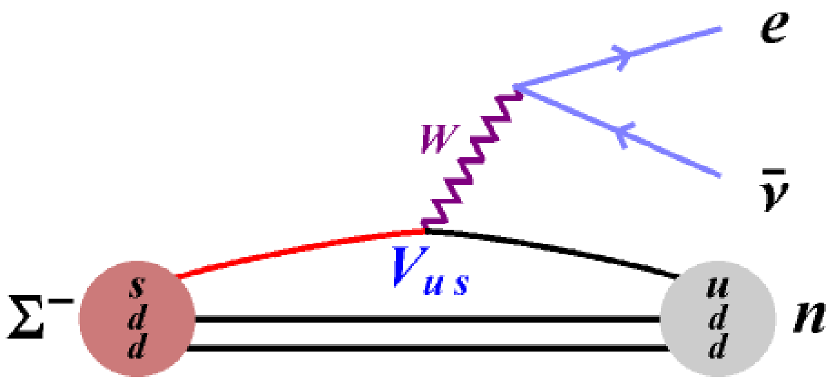
§ Goldberger-Treiman Relation

$$g_{\pi NN} \approx m_N G_A(Q^2{=}0)/f_K$$

- What's the right relation for this?????

$g_{\Xi\Sigma}$ and $g_{\Sigma N}$

§ Matrix element of the hyperon β-decay process $B_1 \to B_2 e^- \overline{\nu}$



$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O_\alpha^{\rm V} + O_\alpha^{\rm A}) u_{B_1} \overline{u}_e \gamma^\alpha (1 + \gamma_5) v_\nu$$

with

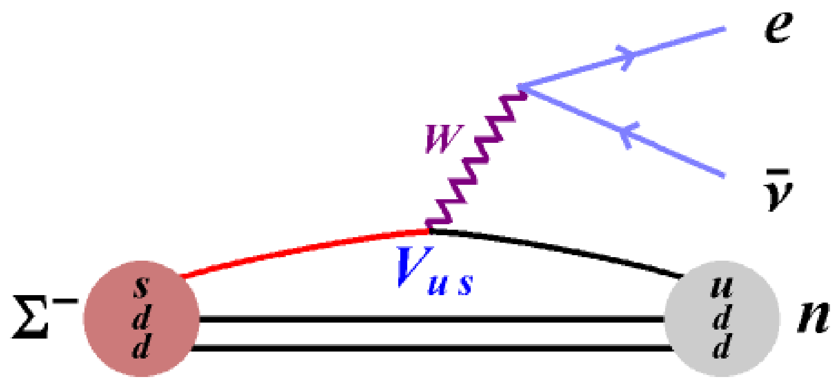
$$O_\alpha^{\rm V} = f_1(q^2)\gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{f_3(q^2)}{M_{B_2}} q_\alpha$$

$$O_\alpha^{\rm A} = \left(g_1(q^2)\gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{g_3(q^2)}{M_{B_2}} q_\alpha \right) \gamma_5$$

SU(3) breaking

$g_{\Xi\Sigma}$ and $g_{\Sigma N}$

§ Matrix element of the hyperon β-decay process



$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O^{\rm V}_\alpha + O^{\rm A}_\alpha) u_{B_1} \overline{u}_e \gamma^\alpha (1+\gamma_5) v_\nu$$

with

$$O^{\rm V}_\alpha = f_1(q^2)\gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_{B_2}}q_\alpha$$

$$O^{\rm A}_\alpha = \left(g_1(q^2)\gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_{B_2}}q_\alpha\right)\boxed{\gamma_5}$$

Hyperon-Decay Experiments

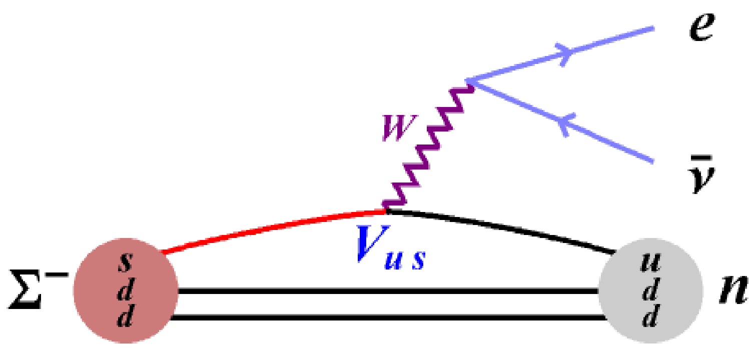
- Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48
- Summary N. Cabibbo et al. 2003

Decay	Rate (μs^{-1})	g_1/f_1
$\Lambda \to p e^- \bar{\nu}$	3.161(58)	0.718(15)
$\Sigma^- \to n e^- \bar{\nu}$	6.88(24)	−0.340(17)
$\Xi^- \to \Lambda e^- \bar{\nu}$	3.44(19)	0.25(5)
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}$	0.876(71)	1.32(+.22/−.18)

- In experiment, only measure linear combination of g_1/f_1 and g_2/f_1
  - usually assume $g_2 \approx 0$
- Systematic from ignoring g_2?
- Better g_1/f_1 from lattice calculations?

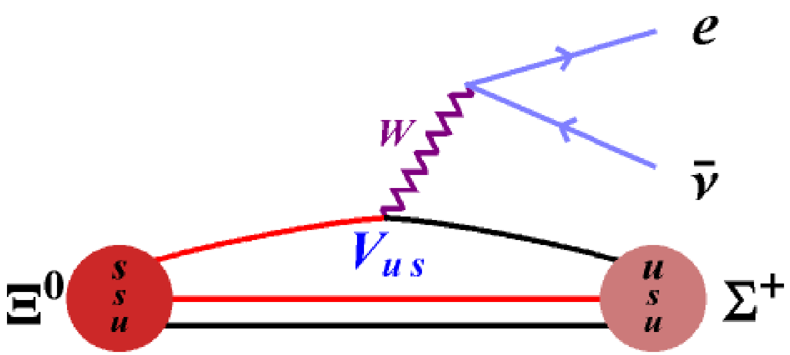
Lattice Studies

§ Two quenched calculations, different channels



- Pion mass > 700 MeV
- $g_1(0) / f_1(0) = -0.287(52)_{\text{stat.}}$
  - Exp't: −0.340(17)
- $g_2(0) / f_1(0) = 0.63(26)_{\text{stat.}}$

Guadagnoli et al., Nucl.Phys.B761:63-91 (2007)



- Pion mass ≈ 540–660 MeV
- $g_1(0) / f_1(0) = 1.248(29)_{\text{stat.}}$
  - Exp't: $1.31(^{21}_{17})$
- $g_2(0) / g_1(0) = 0.68(18)_{\text{stat.}}$

Sasaki et al., 0811.1406[hep-ph]

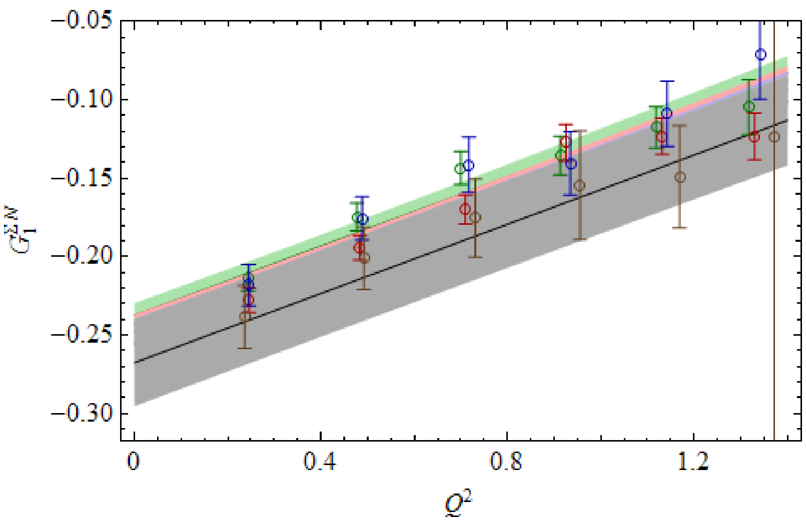
No systematic error estimate from **quenching** effects!

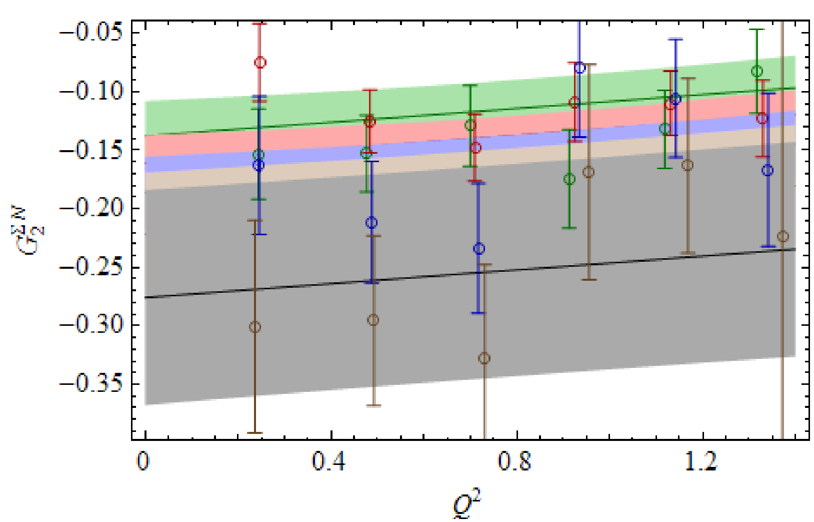
First Dynamical Study

- N_f = 2+1 mixed action, $M_\pi \approx$ 350–700 MeV
- Report on $\Sigma^- \to n$ results for now
- Naïve chiral extrapolation $B_0 + B_1 \cdot (M_K^2 + M_\pi^2) + B_2 \cdot (M_K^2 - M_\pi^2)$

  We got $g_1(0) / f_1(0) = -0.348(37)_{\text{stat}}$

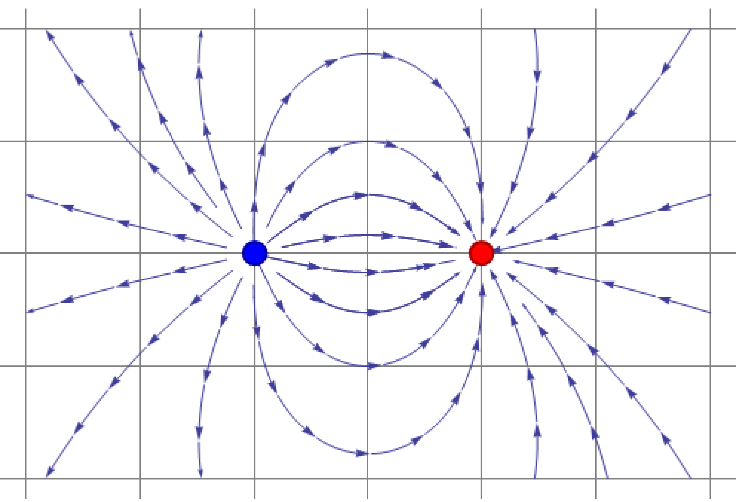
  $g_2(0) / f_1(0) = 0.29(20)_{\text{stat}}$

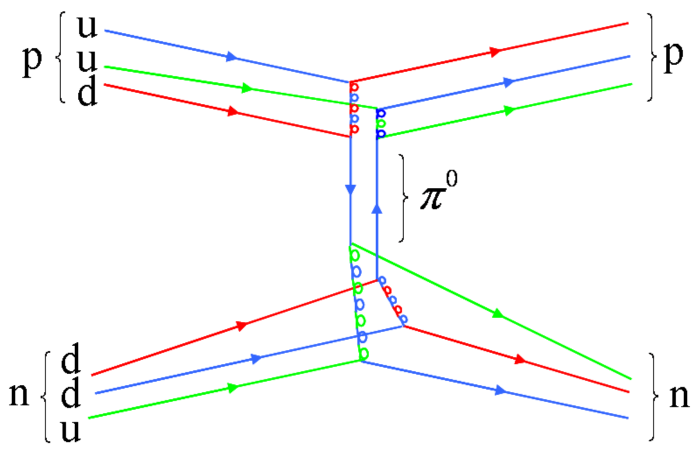


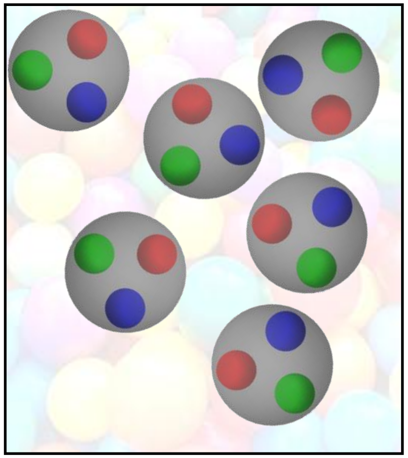


Topics in Nuclear Physics

- What happens when we put hadrons together?







Summary

- Nuclear Physics is not essential for our existence but also for universe we live in
  - Many new experiments are either under construction or in plan
- Lattice QCD is a hard probe for the theory (QCD)
- Helpful in the cases when experimental limitations or challenges
  - Isolated neutrons or pion source (in medium)
  - Unstable particles due to weak decay (hyperons)
  - Predictions of new phenomena to guide experiment
- There is no free lunch
  - The difficulties = The opportunities
  - And some of them have been concurred, as demonstrated in this talk
- Filter for New Physics

Summary

- Nuclear Physics is essential for our existence
  but also for universe we live in
  - Investment in experiment continues (12-GeV, FRIB, RHIC or EIC)
- Lattice QCD is an excellent probe of the theory (QCD)
- Helpful in the cases when experiment is limited
  - Isolated neutrons or pion source (in medium)
  - Unstable particles due to weak decay (hyperons)
  - Predictions of new phenomena to guide experiment
- There is no free lunch
  - **The difficulties = opportunities**
  - And some of them have been conquered, as demonstrated in this talk
- Filter for New Physics