Excited States of the Nucleon in 2+1 flavour QCD

Derek Leinweber
CSSM Lattice Collaboration

Key Collaborators: Selim Mahbub, Waseem Kamleh, Ben Lasscock, Peter Moran, Alan Ó Cais and Tony Williams

Centre for the Subatomic Structure of Matter
University of Adelaide, SA, Australia

Outline

- Two point correlation function:

$$G_{ij}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle .$$

- Inserting completeness

$$\sum_{B,\vec{p'},s} |B,\vec{p'},s\rangle\langle B,\vec{p'},s| = I$$

- Then

$$G_{ij}(t,\vec{p}) = \sum_{B^+} \lambda_{B^+}\bar{\lambda}_{B^+} e^{-E_{B^+}t} \frac{\gamma.p_{B^+} + M_{B^+}}{2E_{B^+}} + \sum_{B^-} \lambda_{B^-}\bar{\lambda}_{B^-} e^{-E_{B^-}t} \frac{\gamma.p_{B^-} - M_{B^-}}{2E_{B^-}}$$

- $\lambda_{B^\pm}$, $\bar{\lambda}_{B^\pm}$ are the couplings of $\chi(0)$ and $\bar{\chi}(0)$ with $|B^\pm\rangle$ defined by

$$\langle\Omega|\chi(0)|B^+,\vec{p},s\rangle = \lambda_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}u_{B^+}(\vec{p},s),$$

$$\langle B^+,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = \bar{\lambda}_{B^+}\sqrt{\frac{M_{B^+}}{E_{B^+}}}\bar{u}_{B^+}(\vec{p},s),$$

and for the negative parity states,

$$\langle\Omega|\chi(0)|B^-,\vec{p},s\rangle = \lambda_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\gamma_5 u_{B^-}(\vec{p},s),$$

$$\langle B^-,\vec{p},s|\bar{\chi}(0)|\Omega\rangle = -\bar{\lambda}_{B^-}\sqrt{\frac{M_{B^-}}{E_{B^-}}}\bar{u}_{B^-}(\vec{p},s)\gamma_5.$$

- At $\vec{p} = 0$

$$G_{ij}^{\pm}(t,\vec{0}) = \mathrm{Tr}_{\mathrm{sp}}[\Gamma_{\pm} G_{ij}(t,\vec{0})] = \sum_{B^{\pm}} \lambda_i^{\pm} \bar{\lambda}_j^{\pm} e^{-M_{B^{\pm}} t}.$$

- Parity projection operator,

$$\Gamma_{\pm} = \frac{1}{2}(1 \pm \gamma_0).$$

- And

$$G_{ij}^{\pm}(t,\vec{0}) \overset{t\to\infty}{=} \lambda_{i0}^{\pm} \bar{\lambda}_{j0}^{\pm} e^{-M_{0^{\pm}} t}.$$

Variational Method

- Consider N interpolating fields, then

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha} \bar{\chi}_i ,$$

$$\phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha} \chi_i ,$$

such that,

$$\langle B_{\beta}, p, s | \bar{\phi}^{\alpha} | \Omega \rangle = \delta_{\alpha\beta} \bar{z}^{\alpha} \bar{u}(\alpha, p, s),$$

$$\langle \Omega | \phi^{\alpha} | B_{\beta}, p, s \rangle = \delta_{\alpha\beta} z^{\alpha} u(\alpha, p, s),$$

- Then a two point correlation function matrix for $\vec{p} = 0$,

$$G^{\pm}_{ij}(t)u^{\alpha}_{j} = (\sum_{\vec{x}} \mathrm{Tr}_{\mathrm{sp}}\{\Gamma_{\pm}\langle\Omega|\chi_i\bar{\chi}_j|\Omega\rangle\})u^{\alpha}_{j}$$
$$= \lambda^{\alpha}_{i}\bar{z}^{\alpha}e^{-m_{\alpha}t}.$$

- There is no sum over α
- t dependence only in the exponential term

- Then one can have a recurrence relation at time $(t + \triangle t)$,

$$G_{ij}(t_0 + \triangle t)u_j^{\alpha} = e^{-m_{\alpha}\triangle t}G_{ij}(t_0)u_j^{\alpha}.$$

- Multiplying by $[G_{ij}(t_0)]^{-1}$ from left,

$$[(G(t_0))^{-1}G(t_0 + \triangle t)]_{ij}u_j^{\alpha} = c^{\alpha}u_i^{\alpha},$$

- where $c^{\alpha} = e^{-m_{\alpha}\triangle t}$ is the eigenvalue.
- Similarly, it can also be solved for the left eigenvalue equation for v^{α} eigenvector,

$$v_i^{\alpha}[G(t_0 + \triangle t)(G(t_0))^{-1}]_{ij} = c^{\alpha}v_j^{\alpha}.$$

- The vectors u_j^α and v_i^α diagonalize the correlation matrix at time t_0 and $t_0 + \triangle t$ making the projected correlation function

$$v_i^\alpha G_{ij}(t) u_j^\beta = \delta^{\alpha\beta} z^\alpha \bar{z}^\beta e^{-m_\alpha t}.$$

- The projected correlator, is then analyzed to obtain masses of different states,

$$v_i^\alpha G_{ij}^{\pm}(t) u_j^\alpha \equiv G_{\pm}^\alpha,$$

- We construct the effective mass

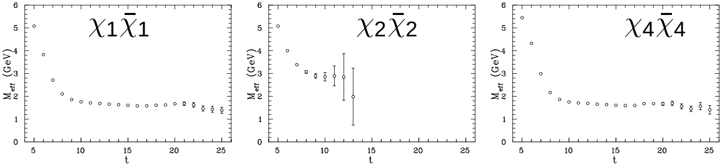
$$M_{\rm eff}^\alpha(t) = \ln\left(\frac{G_{\pm}^\alpha(t,\vec{0})}{G_{\pm}^\alpha(t+1,\vec{0})}\right).$$

Simulation Details

- lattice volume $16^3 \times 32$
- lattice spacing 0.127 fm
- We use FLIC fermion action and quenched QCD
- Analysis is performed for 10 different pion masses: 797, 729, 641, 541, 430, 380, 327, 295, 249, 224 MeV.
- We use a variety of Gaussian smearing sweeps (number of sweeps 1, 3, 7, 12, 16, 26, 35, 48, 65)
- 2×2, 3×3, 4×4, 6×6 and 8×8 correlation matrices are analyzed

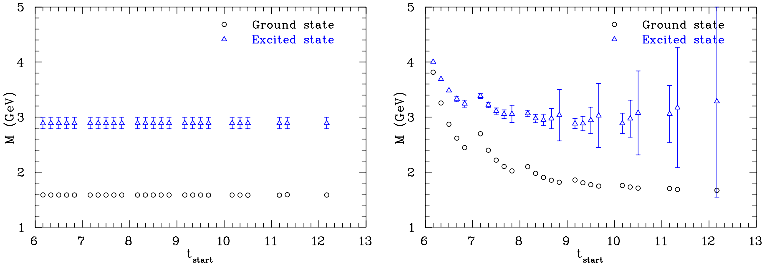
- Interpolators:

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,,$$
$$\chi_4(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\gamma_4\, d^b(x))\, u^c(x).$$



2×2 correlation matrix of $\chi_1\, \chi_2$ for a point source

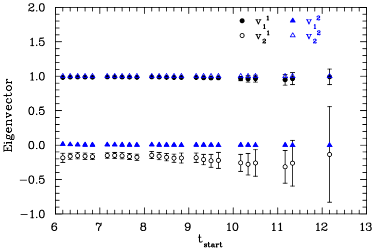
Projected Mass Vs Mass From Eigenvalue



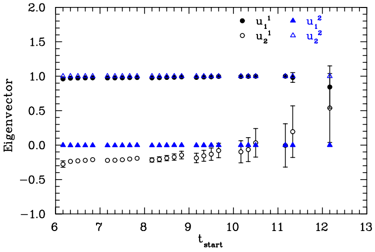
- $t_{\rm start} = t_0$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Eigenvectors - Point Source, for $\chi_1\chi_2$

Left Eigenvectors

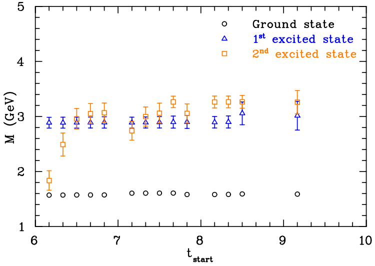


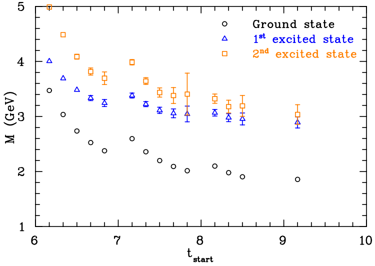
Right Eigenvectors



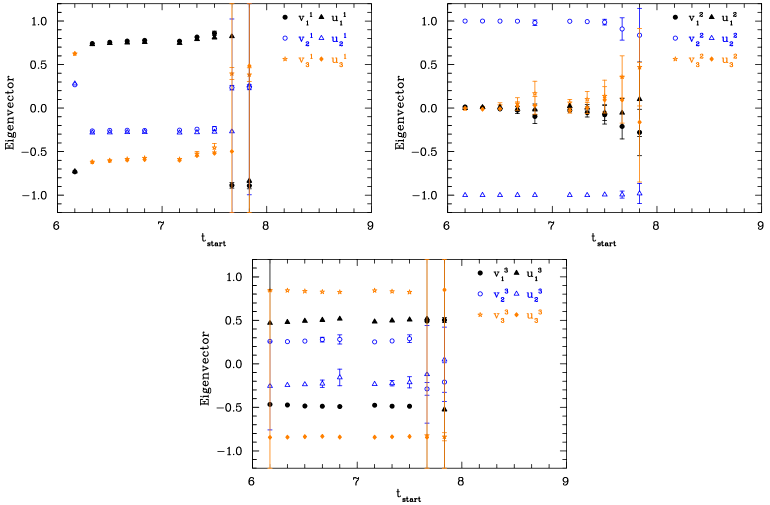
3×3 correlation matrix of $\chi_1\,\chi_2\,\chi_4$ for a point source

Projected Mass Vs Mass From Eigenvalue





Eigenvectors - 3 × 3



Source Smearing

To obtain better overlap with low-lying states, consider source smearing.

$$\psi_i(x,t) = \sum_{x'} F(x,x')\psi_{i-1}(x',t),$$

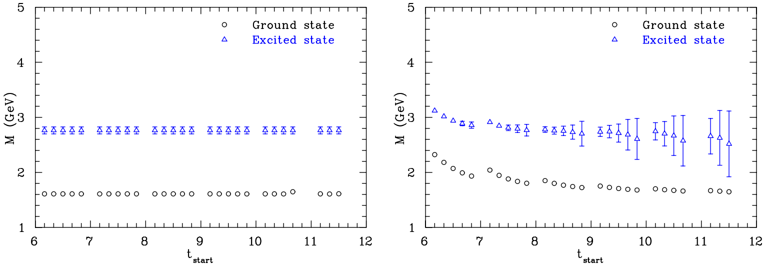
where,

$$F(x,x') = (1-\alpha)\delta_{x,x'} + \frac{\alpha}{6}\sum_{\mu=1}^{3}[U_\mu(x)\delta_{x',x+\hat{\mu}} + U_\mu^\dagger(x-\hat{\mu})\delta_{x',x-\hat{\mu}}],$$

Fixing $\alpha = 0.7$, the procedure is repeated $N_{\rm sm}$ times.

2×2 correlation matrix of $\chi_1\, \chi_2$ for a smeared source

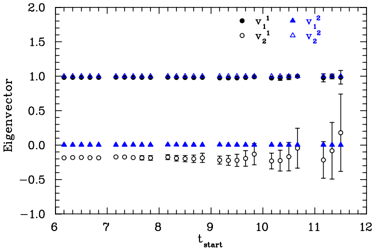
Projected Mass Vs Mass From Eigenvalue



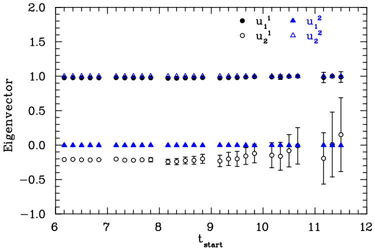
- $t_{\rm start} = t$ is shown in major tick marks
- $\triangle t$ is shown in minor tick marks

Eigenvectors - Smeared source - $\chi_1\ \chi_2$

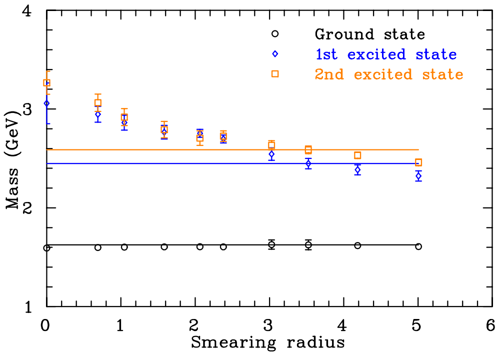
Left Eigenvectors



Right Eigenvectors

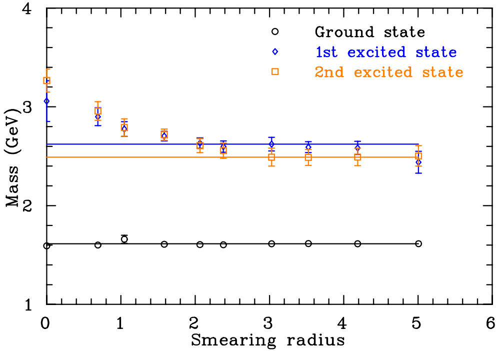


Smeared Source Problem



M. S. Mahbub, *et al.*, Phys. Rev. D **80**, 054507 (2009) [arXiv:0905.3616 [hep-lat]]

Smeared-Smeared Problem



M.S. Mahbub, *et al.*, Phys. Rev. D **80**, 054507 (2009), [arXiv:hep-lat/0905.3616].

Roper Resonance

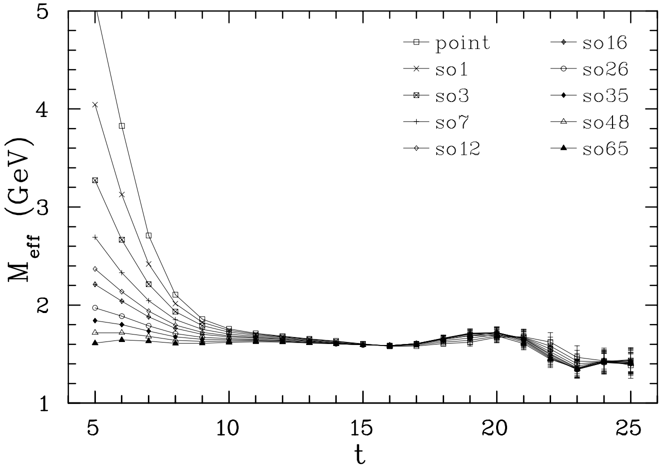
- *Roper resonance* (P_{11}) is the first positive parity excited state of the nucleon
- Observed in 1960's from πN scattering
- The resonance is interesting due to its low mass (1440 MeV) relative to the nearest negative-parity (S_{11}) resonance (1535 MeV).
- In a constituent quark model, the Roper state is $\approx$100 MeV *above* the S_{11} (1535 MeV) state.
- The Roper state appeared very high in all previous lattice simulations using the variational method.

4×4 bases of $\chi_1\bar{\chi}_1$

- Consider smeared–smeared correlation functions
- Variety of smearing sweeps used to form basis interpolators

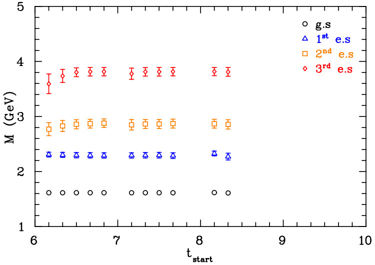
Sweeps →	1	3	7	12	16	26	35	48
Basis No. ↓	Bases							
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

Smeared Source - Point Sink Correlators



4×4 correlation matrix for the 4^{th} basis (3, 12, 26, 35)

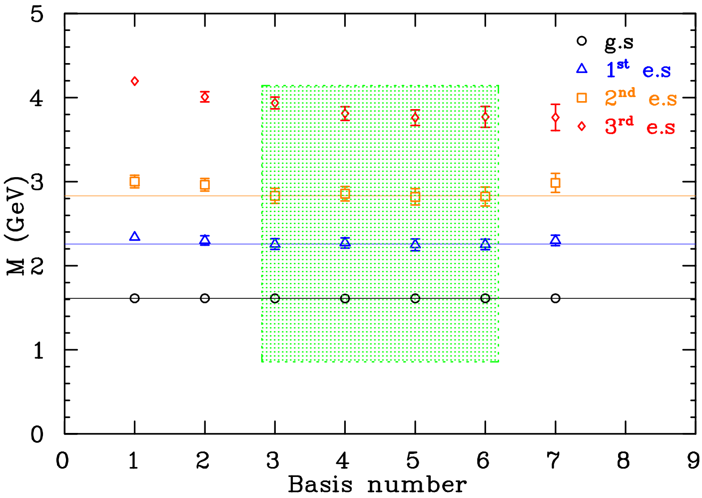
Projected Mass Vs Mass From Eigenvalue

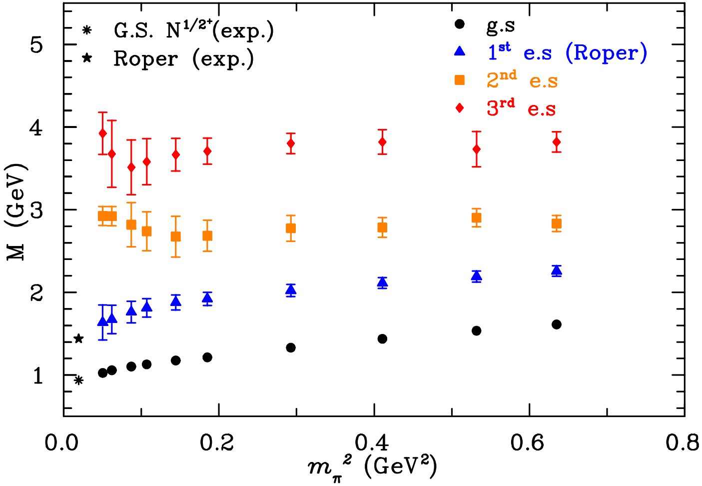


4×4 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$				Bases				
1	1	-	7	-	16	-	35	-
2	-	3	7	-	16	-	35	-
3	1	-	-	12	-	26	-	48
4	-	3	-	12	-	26	35	-
5	-	3	-	12	-	26	-	48
6	-	-	-	12	16	26	35	-
7	-	-	7	-	16	-	35	48

Projected correlator masses from 4 × 4 analysis

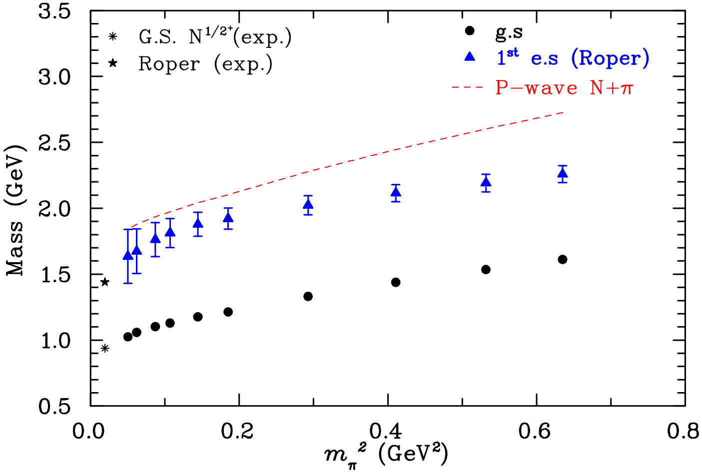



G.S. N^{1/2+}(exp.)
Roper (exp.)
g.s
1st e.s (Roper)
2nd e.s
3rd e.s
M (GeV)
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)

Projected Correlator Fits for the Roper

3^{rd} basis (1,12,26,48)				4^{th} basis (3,12,26,35)				5^{th} basis (3,12,26,48)				6^{th} basis (12,16,26,35)			
t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$	t_1	t_2	aM (Roper)	$\frac{\chi^2}{\mathrm{dof}}$
7	12	1.456(41)	0.58	7	12	1.465(39)	0.63	7	12	1.451(44)	0.51	7	12	1.454(40)	0.57
7	12	1.411(43)	0.55	7	12	1.419(41)	0.62	7	12	1.405(46)	0.48	7	12	1.417(39)	0.60
7	12	1.368(39)	0.54	7	12	1.361(45)	0.60	7	12	1.364(40)	0.53	7	11	1.363(42)	0.68
7	12	1.307(44)	0.57	7	11	1.298(51)	0.60	7	12	1.305(45)	0.57	7	10	1.308(46)	0.54
7	11	1.235(50)	0.43	7	11	1.245(51)	0.57	7	11	1.233(51)	0.37	7	11	1.244(52)	0.38
7	11	1.210(60)	0.42	7	11	1.211(55)	0.58	7	11	1.206(57)	0.38	7	11	1.220(60)	0.49
7	10	1.163(69)	0.60	7	11	1.165(67)	0.56	7	10	1.164(71)	0.53	7	10	1.184(75)	0.56
7	10	1.129(82)	0.61	7	10	1.127(81)	0.84	7	10	1.136(82)	0.58	7	10	1.155(85)	0.54
7	10	1.07(10)	0.56	7	10	1.06(10)	0.95	7	10	1.07(11)	0.68	7	10	1.11(11)	0.63
7	9	1.04(13)	0.85	7	10	1.01(12)	0.97	7	9	1.05(13)	0.79	7	9	1.10(13)	0.70

Roper from 4 × 4 analysis

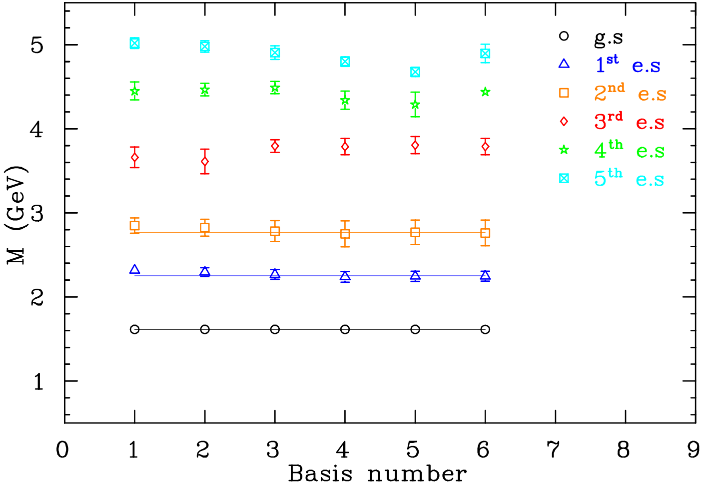


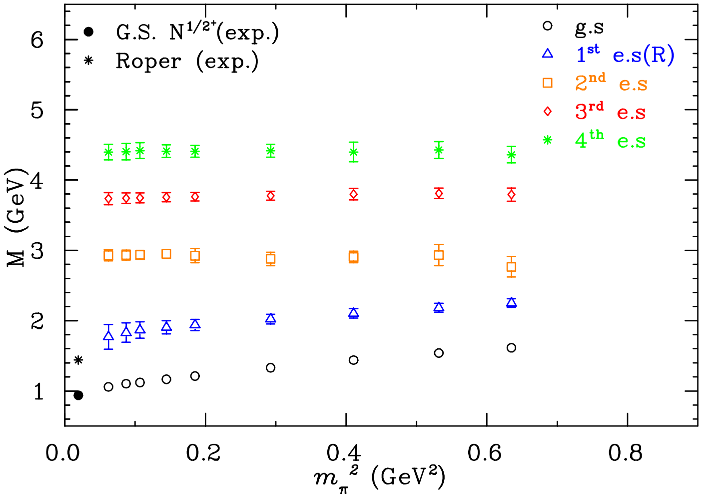
Mahbub *et al.*, Phys. Lett. B **679**, 418 (2009), [arXiv:hep-lat/0906.5433].

6×6 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	3	7	12	16	26	-	-
2	1	3	7	12	16	-	35	-
3	1	3	7	-	16	26	35	-
4	1	3	-	12	16	26	-	48
5	1	-	7	12	16	26	35	-
6	-	3	7	12	16	26	35	-

Projected correlator masses from 6 × 6 analysis

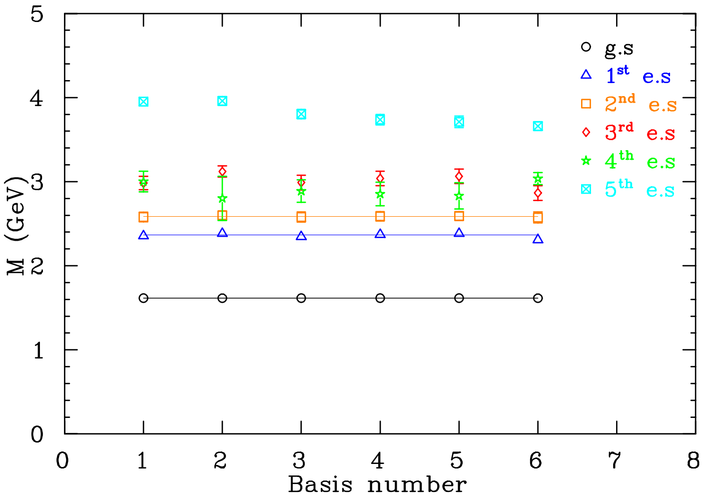


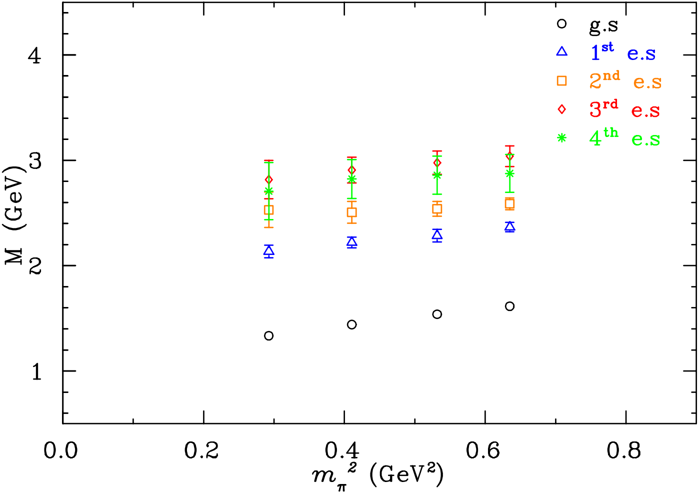

G.S. N1/2+(exp.)
Roper (exp.)
g.s
1st e.s(R)
2nd e.s
3rd e.s
4th e.s
M (GeV)
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
mπ2 (GeV2)

6×6 bases of $\chi_1\,\chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	-	-	16	-	-	48
2	-	3	-	12	-	26	-	-
3	-	3	-	-	16	-	-	48
4	-	-	7	-	16	-	35	-
5	-	-	-	12	16	26	-	-
6	-	-	-	-	16	26	35	-

Projected masses from 6×6 analysis of $\chi_1\,\chi_2$

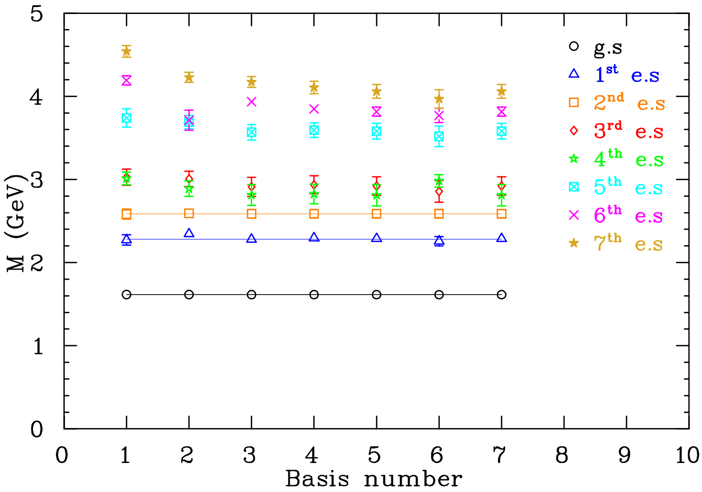


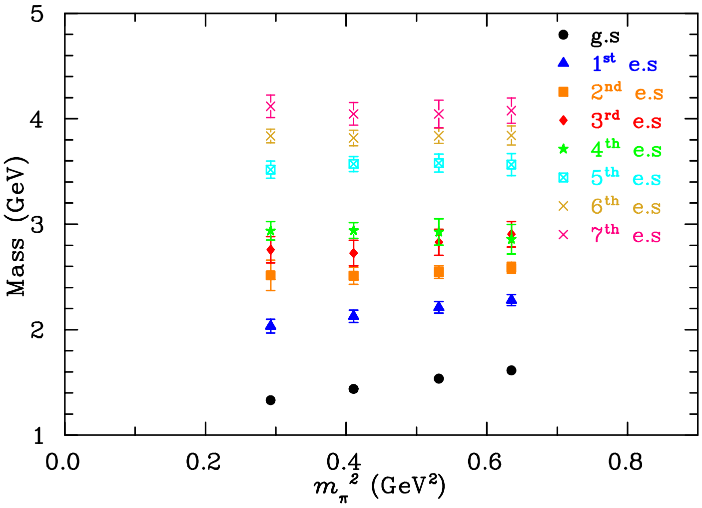

g.s
1st e.s
2nd e.s
3rd e.s
4th e.s
M (GeV)
1
2
3
4
0.0
0.2
0.4
0.6
0.8
m_π^2 (GeV^2)

8×8 bases of $\chi_1\,\chi_2$

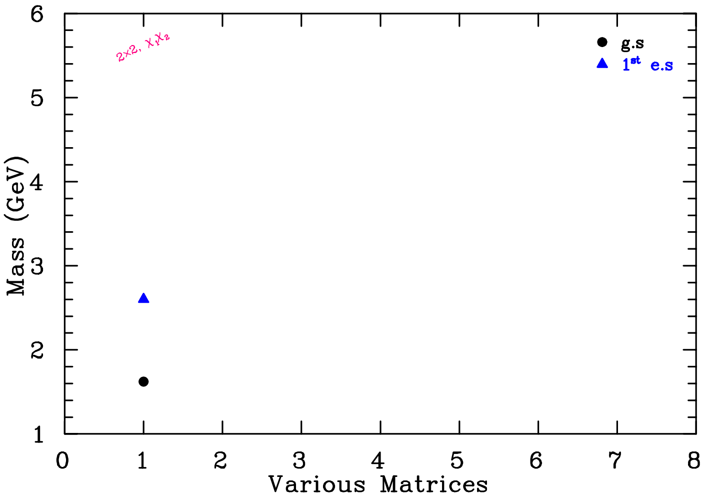
Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$				Bases				
1	1	-	7	-	16	-	35	-
2	-	-	7	12	16	26	-	-
3	-	3	-	12	-	26	-	48
4	-	-	7	12	-	26	35	-
5	-	-	7	-	16	26	35	-
6	-	-	7	-	16	-	35	48
7	-	-	-	12	16	26	35	-

Projected masses from 8 × 8 analysis of $\chi_1\,\chi_2$

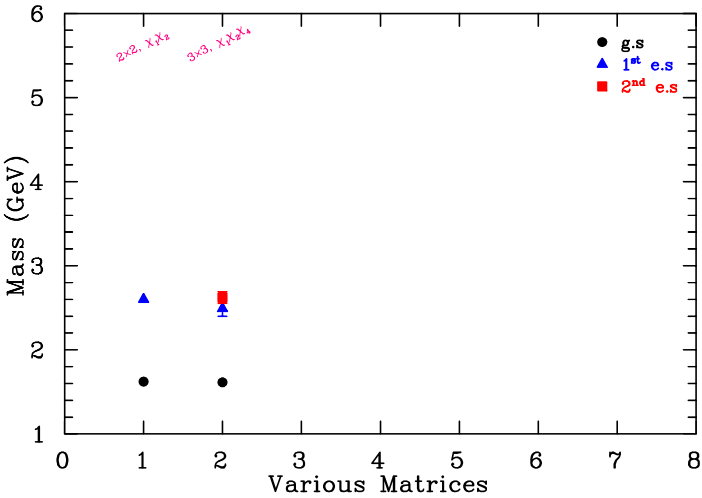



Mass (GeV)
m_π^2 (GeV^2)
g.s
1^{st} e.s
2^{nd} e.s
3^{rd} e.s
4^{th} e.s
5^{th} e.s
6^{th} e.s
7^{th} e.s
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8

Review of excited “states”



Review of excited “states”



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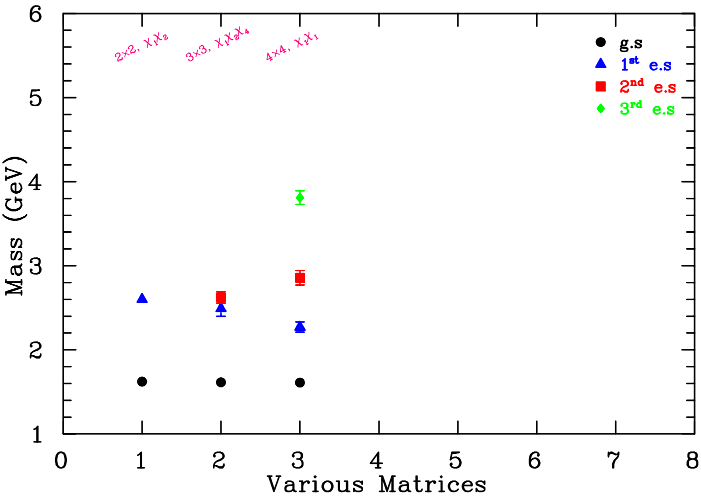
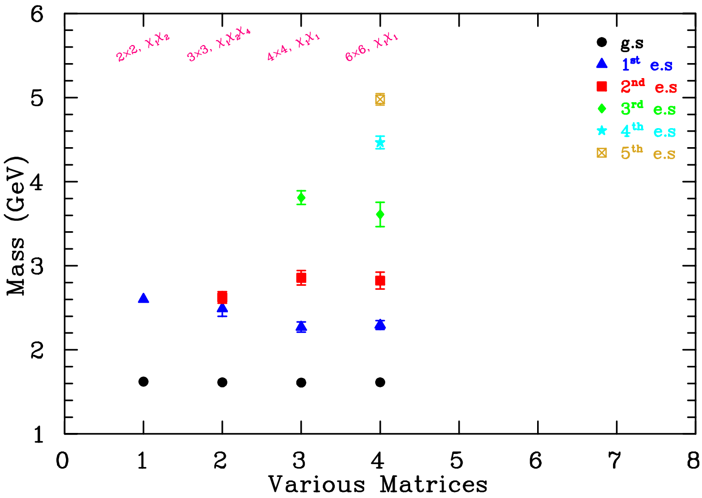
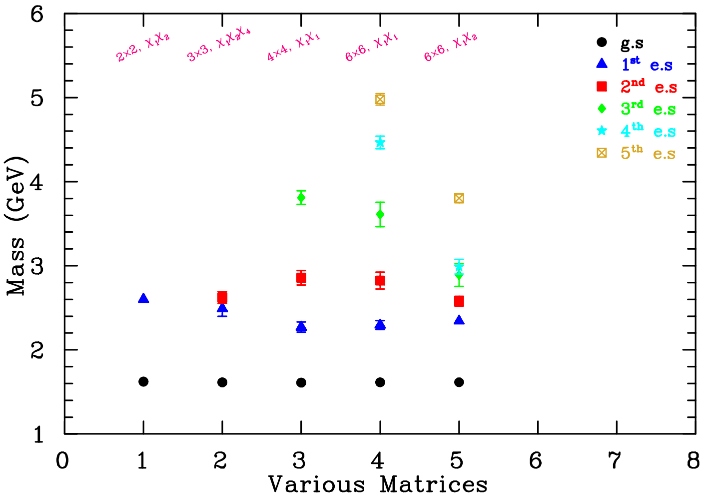


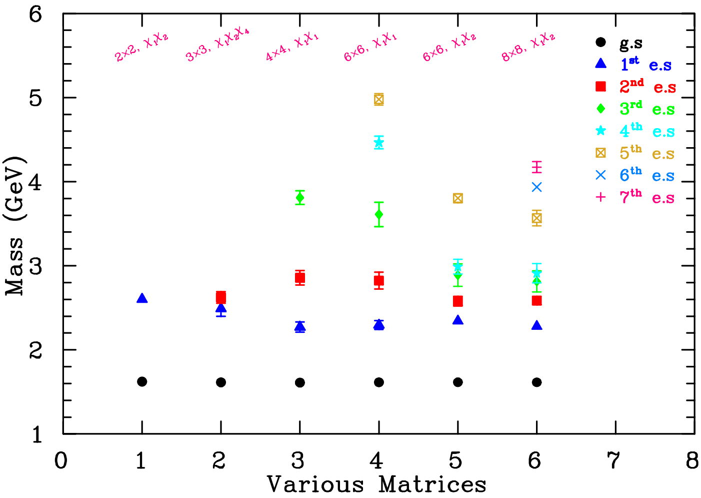
Review of excited “states”



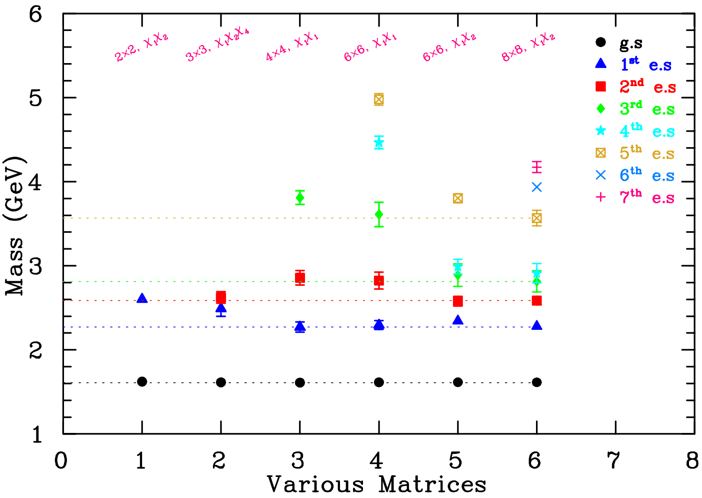
Review of excited “states”



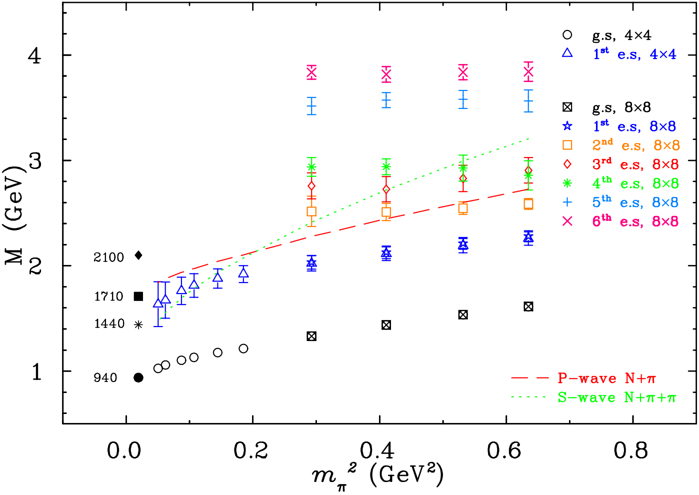
Review of excited "states"



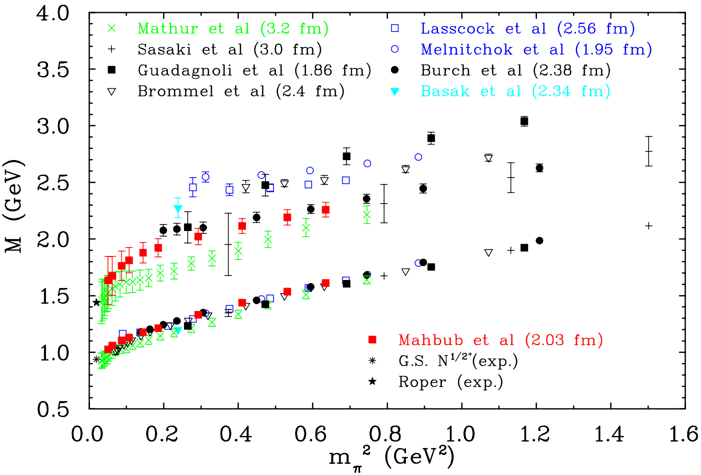
Review of excited “states”



Positive Parity Results



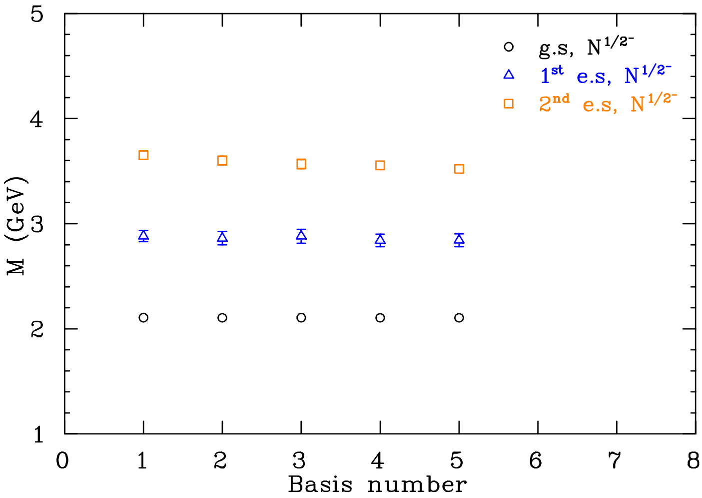
Roper state: Compilation of existing results



3×3 bases of $\chi_1 \bar{\chi}_1$ for $N1/2^-$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	-	-	7	-	16	26	-	-
2	-	-	7	-	16	-	35	-
3	-	-	-	12	16	26	-	-
4	-	-	-	12	-	26	35	-
5	-	-	-	-	16	26	35	-

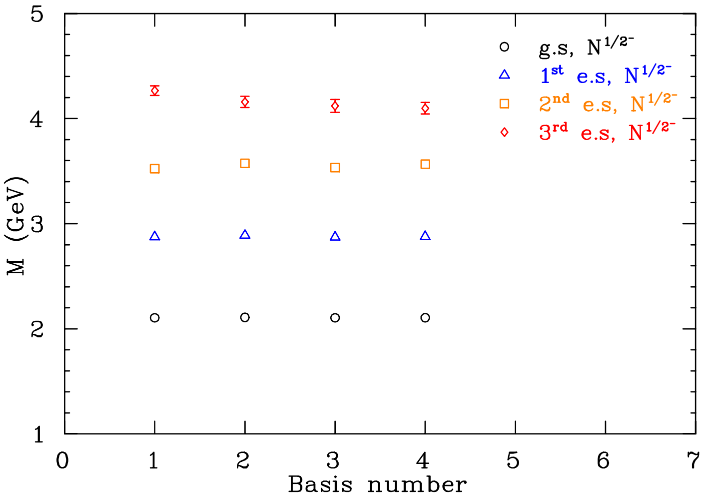
Projected $N1/2^-$ masses from 3×3 bases



4×4 bases of $\chi_1\,\bar{\chi}_1$ for $N1/2^-$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	1	-	-	12	-	26	-	48
2	-	3	-	12	-	26	35	-
3	-	3	-	12	-	26	-	48
4	-	-	-	12	16	26	35	-

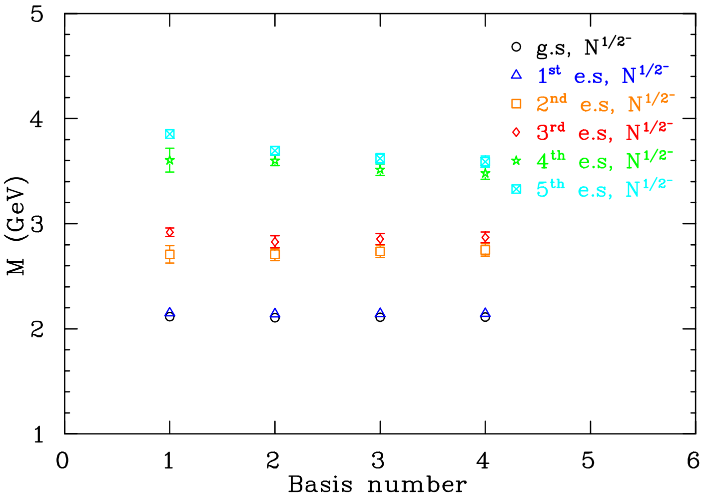
Projected $N1/2^-$ masses from 4×4 bases



6×6 bases of $\chi_1\,\chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	-	3	-	12	-	26	-	-
2	-	3	-	-	16	-	-	48
3	-	-	7	-	16	-	35	-
4	-	-	-	12	16	26	-	-

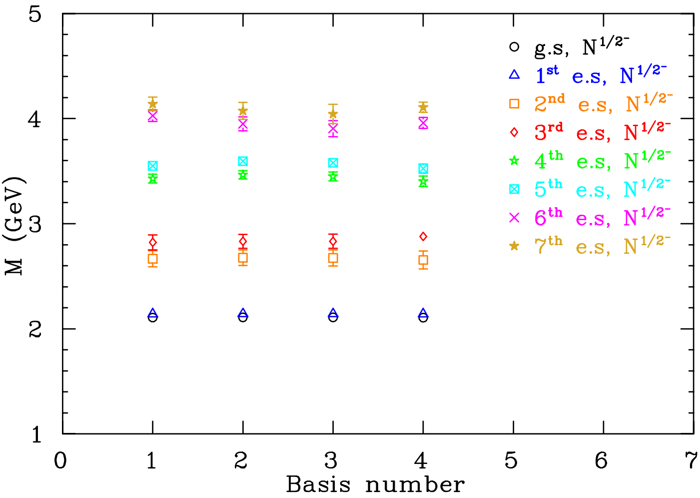
Projected $N1/2^-$ masses from 6×6 bases



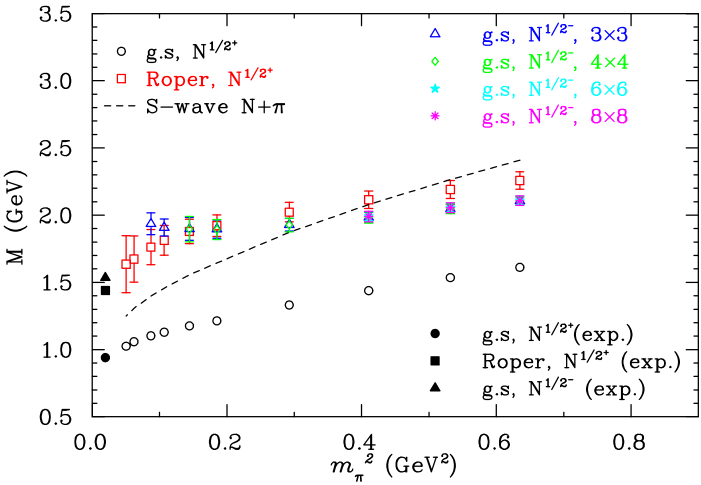
8×8 bases of $\chi_1\,\chi_2$

Sweeps $\rightarrow$	1	3	7	12	16	26	35	48
Basis No. $\downarrow$	Bases							
1	-	3	-	12	-	26	-	48
2	-	-	7	12	-	26	35	-
3	-	-	7	-	16	26	35	-
4	-	-	7	-	16	-	35	48

Projected $N1/2^-$ masses from 8×8 bases

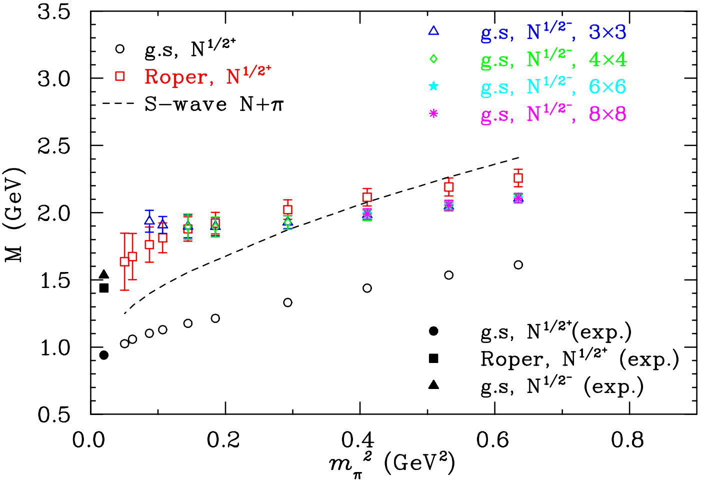


Roper and $N1/2^-$ states



Note: $N1/2^-$ and Roper analyses use same Euclidean times.

Roper and $N1/2^-$ states



Note: $N1/2^-$ and Roper analyses use same Euclidean times.

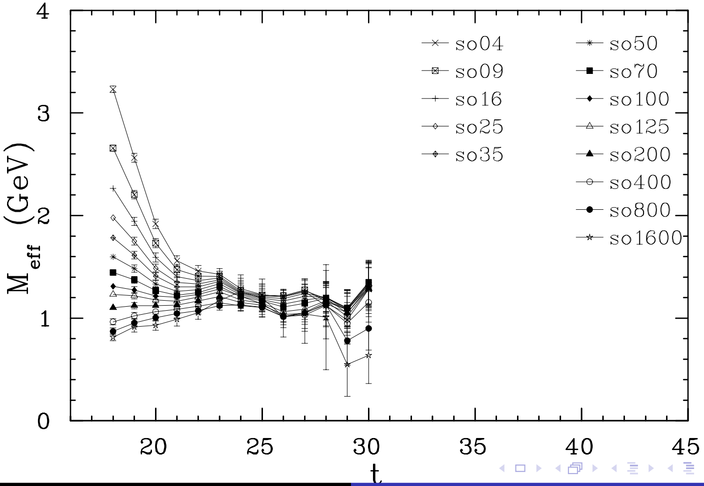
PACS-CS lattice: Simulation details

PACS-CS Collaboration: S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.

- Lattice volume: $32^3 \times 64$
- Non-perturbative $\mathcal{O}(a)$-improved Wilson quark action
- Iwasaki gauge action
- $2+1$ flavour dynamical QCD
- $\beta = 1.9$ providing a =0.0907 fm
- $K_{ud} = \{$0.13700, 0.13727, 0.13754, 0.13770, 0.13781 $\}$
- K_s={ 0.13640, 0.13660 }
- Lightest pion mass is 156 MeV.

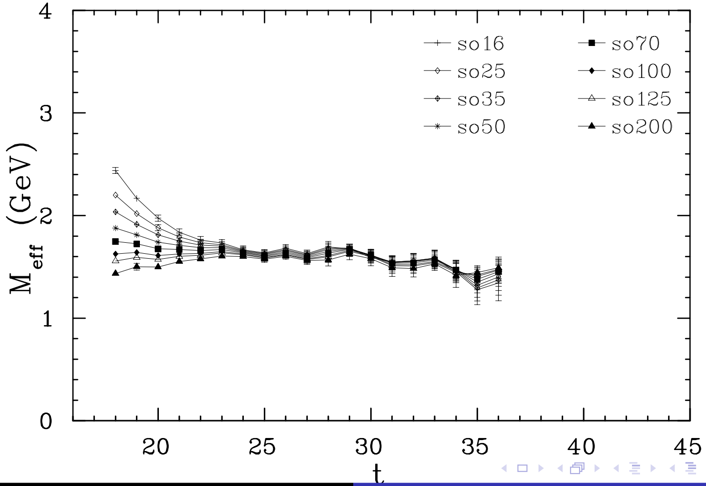
Smeared Source - Point Sink Effective Masses

For second lightest quark : 50 cfgs



Smeared Source - Point Sink Effective Masses

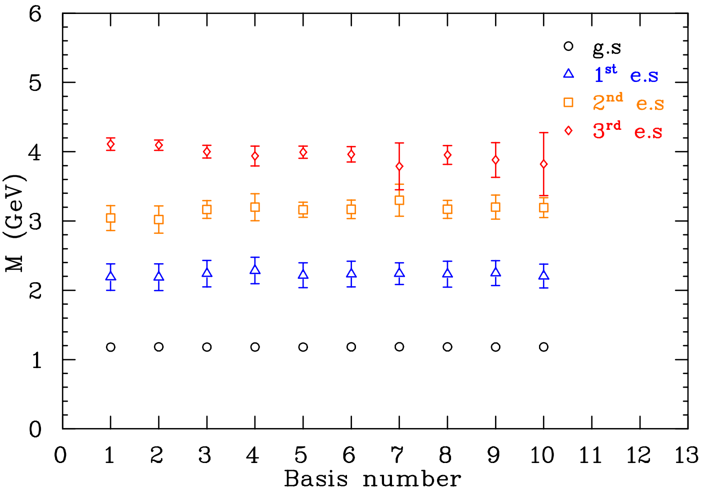
For the heaviest quark: 50 cfgs



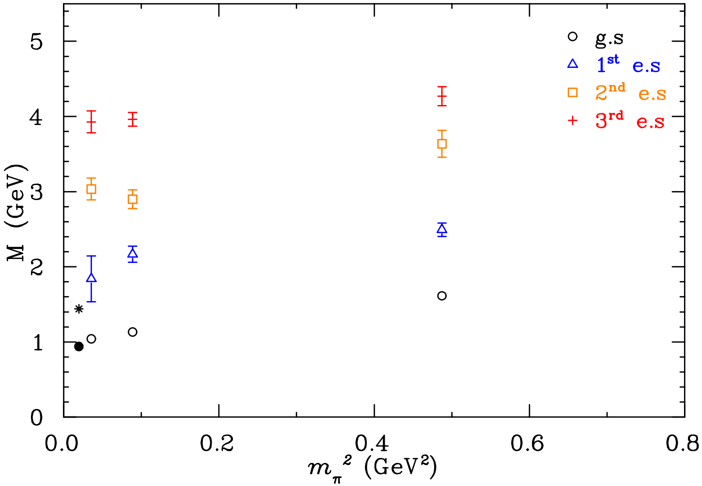
4×4 bases of $\chi_1\bar{\chi}_1$

Sweeps $\rightarrow$	16	25	35	50	70	100	125	200	400	800
Basis No. $\downarrow$						Bases				
1	16	-	35	-	70	100	-	-	-	-
2	16	-	35	-	70	-	125	-	-	-
3	16	-	35	-	-	100	-	200	-	-
4	16	-	35	-	-	100	-	-	400	-
5	16	-	-	50	-	100	125	-	-	-
6	16	-	-	50	-	100	-	200	-	-
7	16	-	-	50	-	-	125	-	-	800
8	-	25	-	50	-	100	-	200	-	-
9	-	25	-	50	-	100	-	-	400	-
10	-	-	35	-	70	-	125	-	400	-

For all 4×4 bases: $K_{ud} = 0.137700$

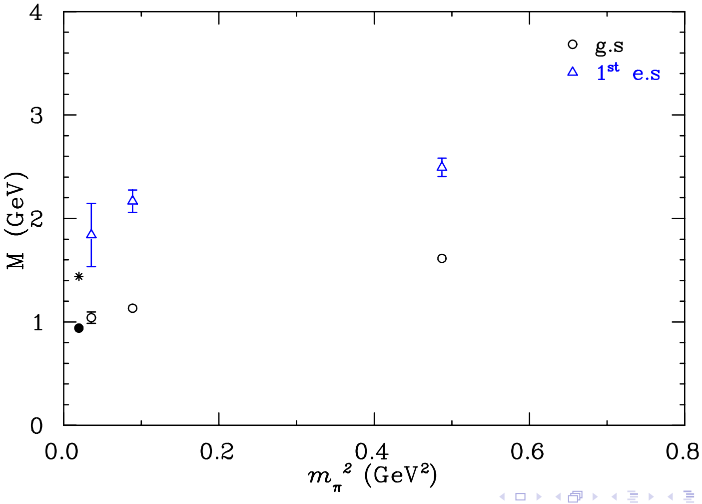


Even Parity Nucleon Spectrum in full QCD

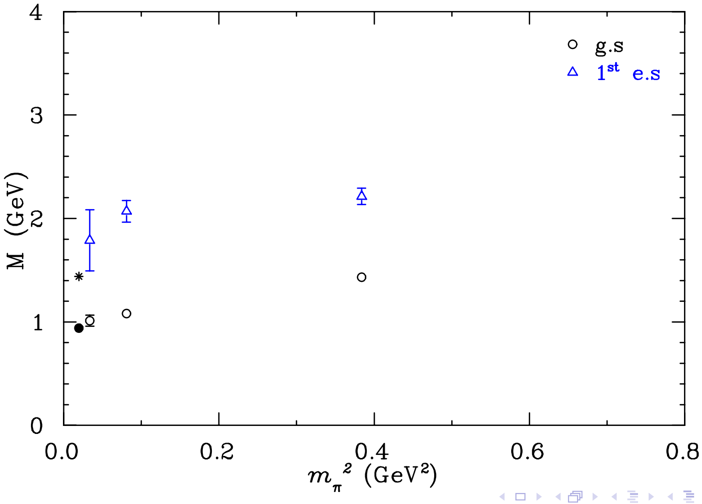


Heaviest = 50 cfgs, second lightest = 200 cfgs, lightest = 40 cfgs.

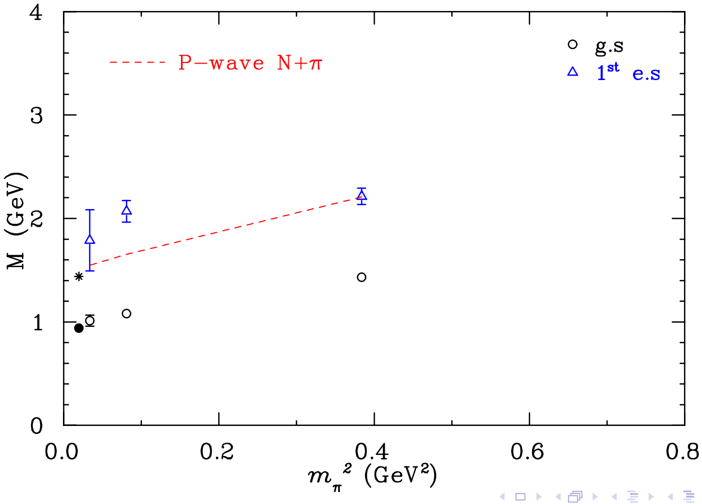
Ground and Roper states (fixed lattice spacing)



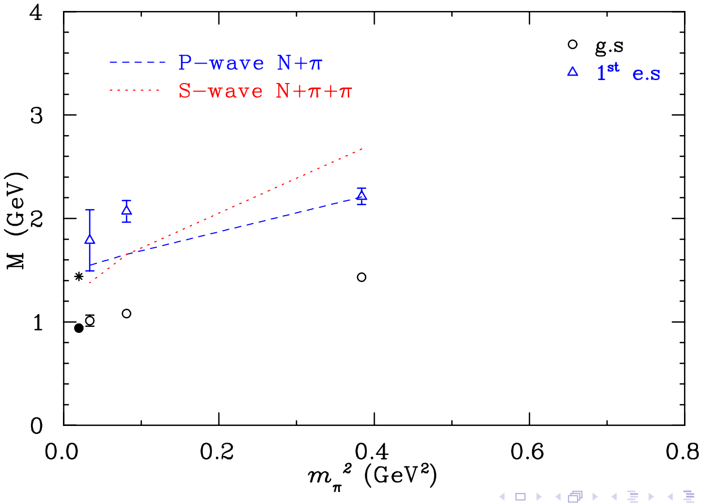
Ground and Roper states (Sommer scale sets *a*)



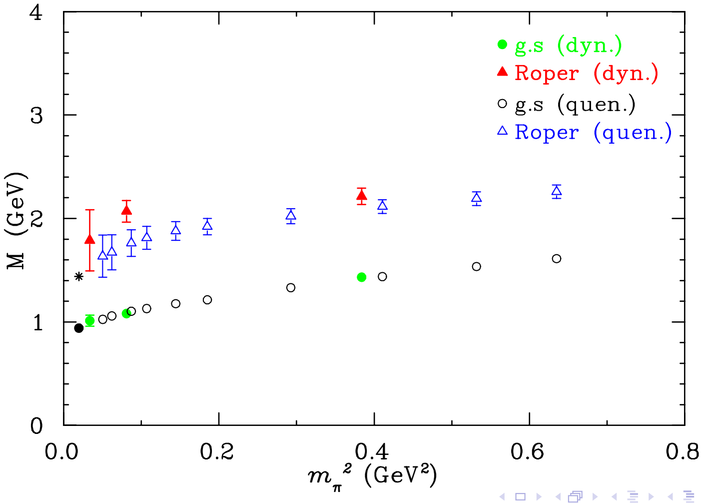
Ground and Roper states (Sommer scale sets *a*)



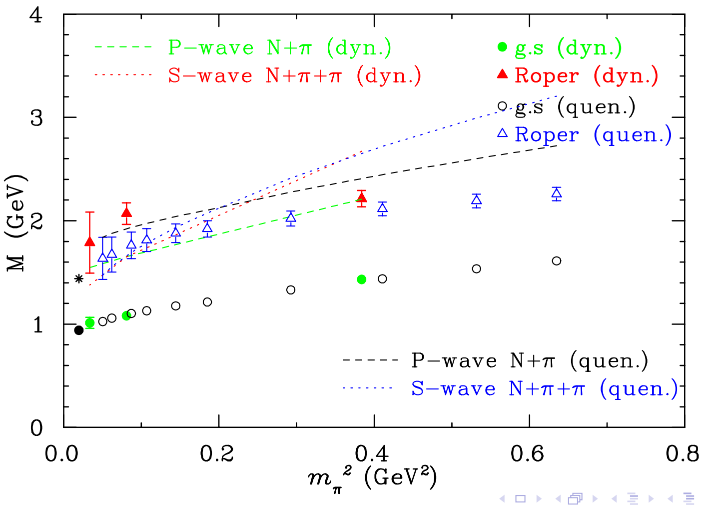
Ground and Roper states (Sommer scale sets *a*)



Quenched Vs Dynamical (Sommer scale)



Quenched Vs Dynamical (Sommer scale)



Summary

- Several levels of smearing have been used to construct correlation matrices.
- Various dimensions of correlation matrices have been analyzed.
- We observed a source-smearing dependence of the excited state masses when using early correlation matrix techniques with fixed source smearing.
- Using several levels of source and sink smearings to construct correlation matrices, a low-lying Roper state has been identified for the first time.
- Several 4×4, 6×6, 8×8 matrices were considered to demonstrate the independence of the eigenstate energies from the basis interpolators.

Summary continued...

- We have shown how apparent eigenstates can be resolved into actual eigenstates states through the consideration of larger correlation matrices and introducing additional quark spin-flavor relationships (via χ_2).
- A level crossing between the Roper and $N1/2^-$ states has been observed for the first time using the variational approach.
- The Roper results in quenched and dynamical QCD reveal significant differences in the approach to the physical point.

Future Plans

- Complete all quark masses at 200 configs (400 at lightest mass).
- Extend to partially quenched QCD and quenched QCD.
- Commence chiral effective field theory analysis of chiral curvature.
- Extend to a comprehensive analysis of all baryons of interest.
- Complete determination of excited-state electromagnetic properties.