Electromagnetic response of nuclei with the LIT method

Outline

- Lorentz integral transform (LIT) method
- Simple Example: deuteron photodisintegration
- Comparison to Lanczos Response
- Applications: $^{3/4}$He(e,e'), ^{7}Li(γ)

Motivation of LIT method

Aim: calculation of reactions involving A-body systems in the continuum

Well known: calculation of A-body continuum state tremendously more difficult than A-body bound state calculation

??? is it possible to calculate continuum observables without explicit knowledge of the corresponding continuum wave function ???

YES, via the LIT method!
Continuum state problem → bound-state like problem

LIT method: V. Efros, W.L., G. Orlandini, PLB 338, 130 (1994),
review: V. Efros, W.L., G. Orlandini, N. Barnea, J. of Phys. G 34, R459 (2007)

LIT for Inclusive Reactions

Cross section described by response functions R(ω)

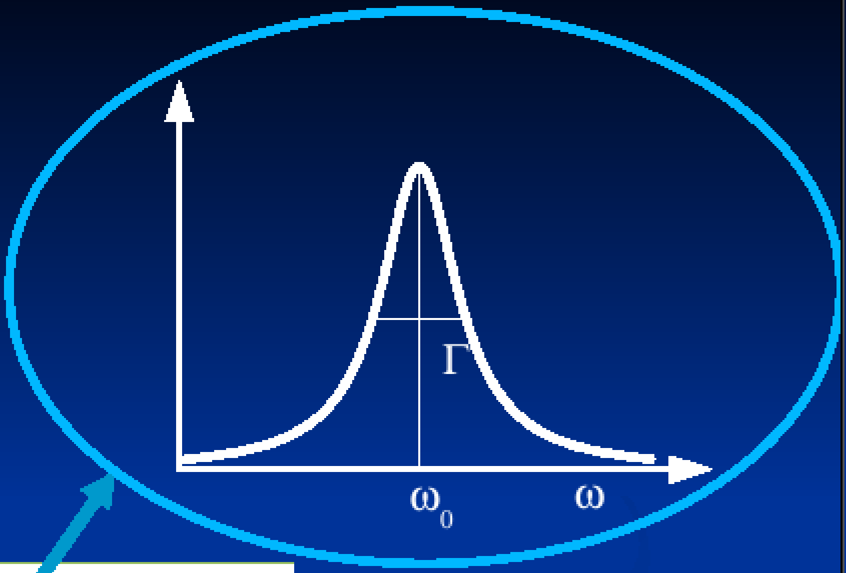
$$R(\omega) = \sum_n |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)$$

steps:

1. Solve for many ω_0 and fixed Γ

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = \Theta\,|0\rangle$$

2. Calculate



$$\left\langle \tilde{\Psi} \middle| \tilde{\Psi} \right\rangle = \int R(\omega) L(\omega, \omega_0, \Gamma) d\omega$$

for a Theorem based on closure

3. Invert transform

$$\int_{E^-_{th}}^{\infty} d\omega \, \frac{R(\omega)}{(\omega-\omega_0)^2 + \Gamma^2} = \int_{E^-_{th}}^{\infty} d\omega \, \frac{R(\omega)}{(\omega-\omega_0-i\Gamma)\,(\omega-\omega_0+i\Gamma)}$$

$$= \int_{E^-_{th}}^{\infty} d\omega \, \frac{\int dn \;\; <0|\Theta^\dagger|n><n|\Theta|0> \;\; \delta(\omega - E_n - E_0)}{(\omega-\omega_0-i\Gamma)\,(\omega-\omega_0+i\Gamma)}$$

$$= \int dn \;\; <0|\Theta^\dagger\,(E_n-E_0-\omega_0-i\Gamma)^{-1}\,|n><n|\,(E_n-E_0-\omega_0+i\Gamma)^{-1}\,\Theta\,|0>$$

H H

$$= \;\; <0|\Theta^\dagger\,(H-E_0-\omega_0-i\Gamma)^{-1}\;(H-E_0-\omega_0+i\Gamma)^{-1}\,\Theta\,|0>$$

$= \; <\tilde{\psi}|\tilde{\psi}>$ with $(H-E_0-\omega_0+i\Gamma)|\tilde{\psi}> = \; \Theta\,|0>$

LIT for Exclusive Reactions

General form of final state wave function for a given channel

$$|\Psi(E)\rangle = |\Phi(E)\rangle + (E - H + i\,\eta)^{-1} V |\Phi(E)\rangle$$

$|\Phi(E)\rangle$ is "channel function" (with proper antisymmetrization),
in general fragment bound states times their free relative motion,
V is the sum of potentials between particles belonging to different fragments

Transition matrix element T_{fi}:

$$T_{fi} = \langle\Psi(E)|\,\Theta\,|0\rangle$$

$$= \langle\Phi(E)\,|\,\Theta\,|0\rangle + \langle\Phi(E)\,|\,V\,(E - H + i\,\eta)^{-1}\,\Theta\,|0\rangle$$

trivial part: T_{Born}

non trivial part: T_{FSI}

Spectral representation for non trivial part

$$\langle \Phi(E) \,|\, V\, (E - H + i\,\eta)^{-1}\, \Theta\, |0\rangle \;=\; \Sigma_n\, (E-E_n)\; F_{fi}(E,E_n)$$

$$+ \int_{E\_th}^{\infty} (E-E' + i\,\eta)^{-1}\, F_{fi}(E,E')\; dE'$$

$$F_{fi}(E,E') = \sum\!\!\!\!\!\!\int d\gamma\, \langle \Phi(E) \,|\, V | \Psi_\gamma \rangle \langle \Psi_\gamma | \,\Theta\, |0\rangle\; \delta(E-E')$$

$F_{fi}(E,E')$ has same form as the inclusive response function $R(\omega)$ therefore we can apply the same formalism, however, here left and right hand side are not identical, hence two LIT equations are obtained

$$(H - \sigma_R + i\sigma_I\,)\, \tilde{\Psi}_1 = \Theta\,|0\rangle \;,\quad (H - \sigma_R + i\sigma_I\,)\, \tilde{\Psi}_2 = V|\Phi(E)\rangle$$

LIT: $\langle \tilde{\Psi}_1 | \tilde{\Psi}_2 \rangle$

1) Calculate LIT for many values of σ_R for fixed σ_I

2) Invert LIT $\Longrightarrow$ $F_{fi}(E,E')$

3) Calculate T_{FSI}

$$T_{FSI}(E) = -i\,\pi\,F_{fi}(E,E) + \mathcal{P}\int_{E\_th}^{\infty} (E-E')^{-1}\,F_{fi}(E,E')\,dE'$$

LIT - Inversion

Standard LIT inversion method

Take the following ansatz for the response function $R(\omega)$ (or $F_{fi}(E,E')$)

$$R(\omega') = \sum_{m=1}^{M_{max}} c_m \, \chi_m(\omega',\alpha_i)$$

with $\omega'=\omega-\omega_{th}$, given set of functions χ_m , and unknown coefficients c_m

Define: $$\tilde{\chi}_m(\sigma_R,\sigma_I,\alpha_i) = \int_0^\infty d\omega' \, \frac{\chi_m(\omega',\alpha_i)}{(\omega'-\sigma_R)^2+\sigma_I^2}$$

Take calculated LIT $L(\sigma_R,\sigma_I) = \langle\tilde{\psi}|\tilde{\psi}\rangle$ for many σ_R and fixed σ_I

and expand in set $\tilde{\chi}_m$: $L(\sigma_R,\sigma_I) = \sum_{m=1}^{M_{max}} c_m \, \tilde{\chi}_m(\omega',\alpha_i)$

Determine c_m via best fit

Increase M_{max} up to the point that stable result is obtained for $R(\omega)$. Even further increase of M_{max} might lead to oscillations in $R(\omega)$

As basis set χ_m we normally use

$$\chi_m(\omega',\alpha_i) = (\omega')^{\alpha_1} \exp(-\alpha_2 \omega'/m)$$

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

The $\tilde{\Psi}$ solution is unique and has ***bound state like*** asymptotic behavior

→ one can apply ***bound state methods***

Our method for calculation of bound states

Hyperspherical **H**armonics Expansions (HH): **CHH** and **EIHH**

CHH: Additional two-body correlation functions are introduced
EIHH: Effective Interaction is constructed via Lee-Suzuki transformation

EIHH: N. Barnea, W.L., G.Orlandini, PRC 61, 054001 (2000)

AB INITIO BOUND STATE CALCULATIONS

BE of ^{4}He (exp. 28.296 MeV)

TABLES

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

from H.Kamada et al. (18 authors 7 groups) PRC 64 (2001) 044001

LIT - Example

As example we consider deuteron photodisintegration calculating the total photoabsorption cross section in unretarded dipole approximation

unretarded dipole approximation $\Longrightarrow$ $\Theta = \sum_{i=1}^{2} z_i \frac{1+\tau_{i,z}}{2}$,

z_i , $\tau_{i,z}$: 3rd componts of position and isospin coordinates

$$\Longrightarrow \quad \sigma_\gamma(\omega) = 4\pi^2 \alpha R(\omega) \quad \text{with} \quad R(\omega) = \sum\!\!\!\!\!\!\int_f |\langle f|\Theta|0\rangle|^2 \delta(\omega - E_{np} - E_d)$$

with $|0\rangle$ and E_d deuteron bound-state wave function and energy

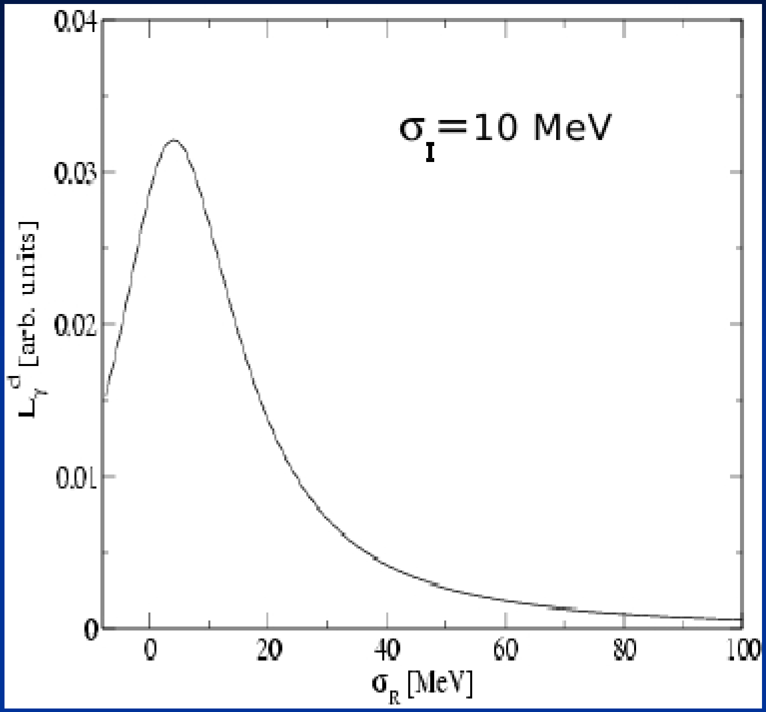
$|f\rangle$ and E_{np} wave function and kinetic energy of final np-pair

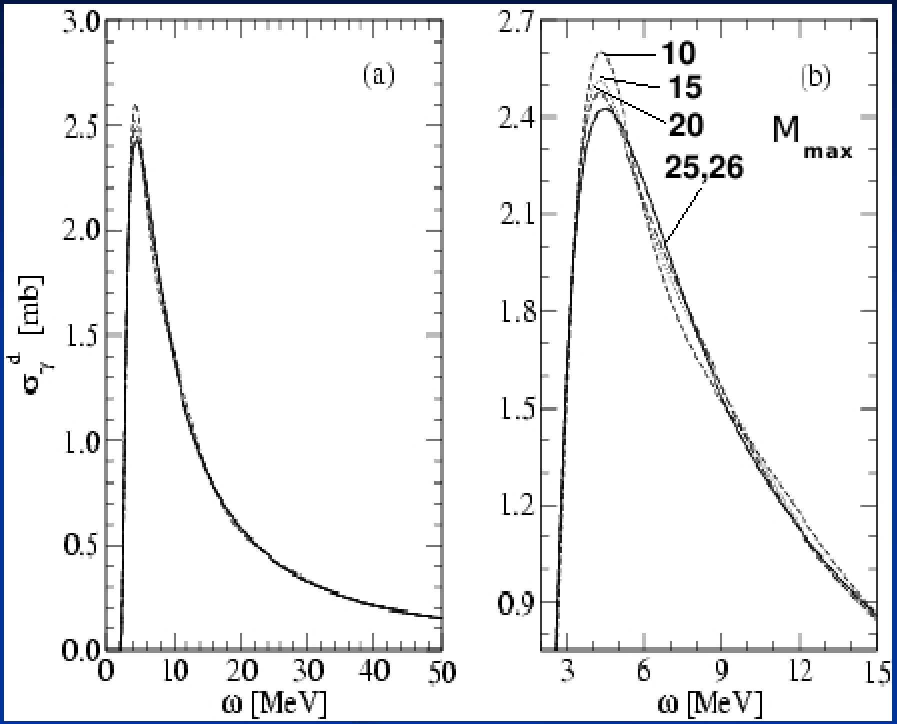
Hamiltonian is given by $H = \frac{p^2}{M} + V_{1,2}$

p relative momentum
M nucleon mass
$V_{1,2}$ NN potential

In the following we will use different models for $V_{1,2}$

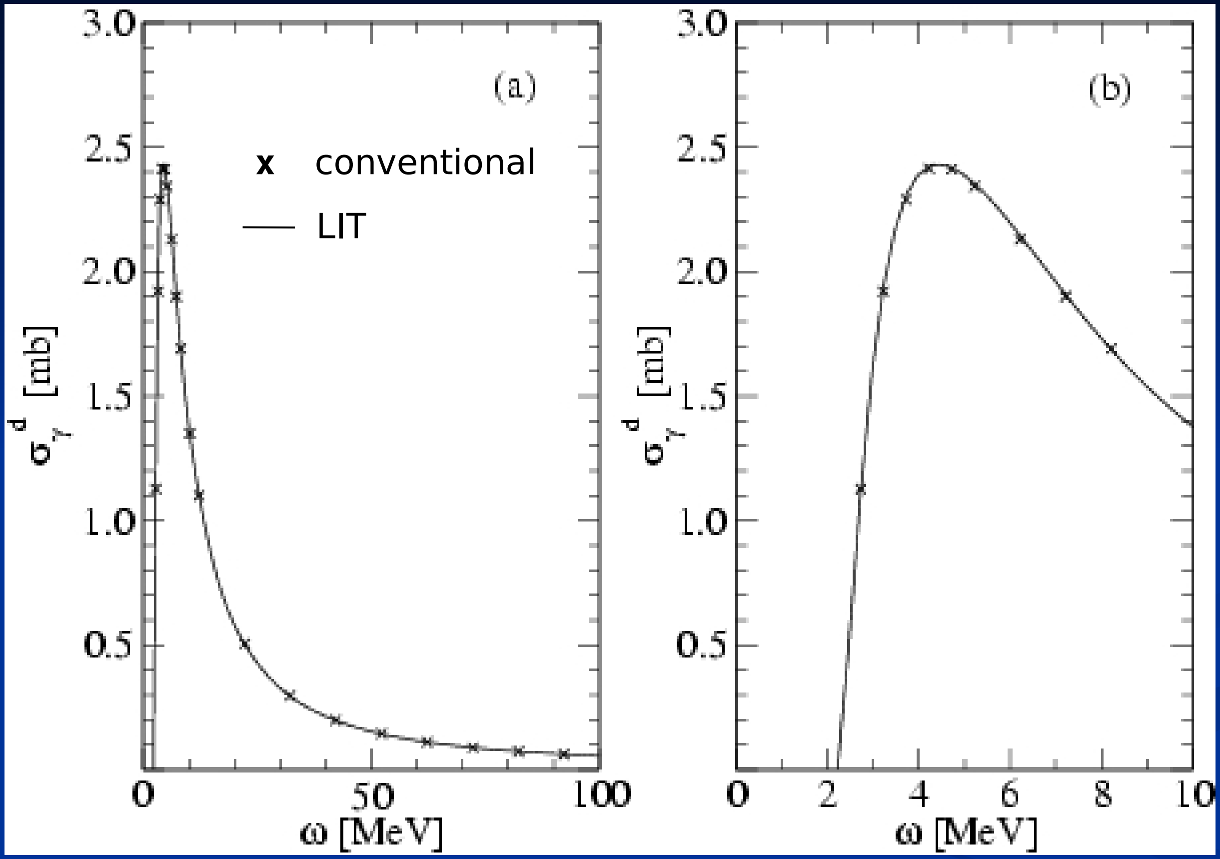
(i) Argonne V14 potential





LIT

$\sigma_\gamma(\omega)$ from inversion with various M_{max}



$\sigma_\gamma(\omega)$ from inversion with various M_{max} = 25

and result from conventional calculation with explicit np continuum wave functions

(ii) JISP-6 potential

Potential is defined via matrix elements for harmonic oscillator (HO) basis: <n'| V | n> up n=n'=4 (n=0,1,2,...; HO quantum number)

Also deuteron wave function and $\tilde{\Psi}$ are expanded on HO basis

Slow convergence for E_d

N_{max} in expansion of deuteron wave function	E_d [MeV]
10	2.057
20	2.195
50	2.2236
100	2.224555
150	2.224574

Potential is interesting because HO and hyperspherical harmonic expansions are used for A>2; one can also consider the so-called Lanczos response

Lanczos response

Since the Lorentzian function is a representation of the δ-function one could think of calculating R(ω) as the limit of $L(\omega,\sigma_R,\sigma_I)$ for $\sigma_I \longrightarrow 0$.
The extrapolation would give

$$R(\omega) = \sum_\nu^N r_\nu \, \delta(\omega - \epsilon_\nu^N)$$

Lanczos response: δ-function is replaced by Lorentzian with small σ_I

$$R(\omega) = \sum_\nu^N r'_\nu \, L(\omega, \epsilon_\nu^N, \sigma)$$

Lanczos technique is used, e.g., for diagonalization of Hamiltonian matrix (dimension: M×M) in a bound-state calculation.
Very efficient: total diagonalization is avoided instead only $N \ll M$ Lanczos steps are needed. They lead to N energy eigenvalues , which are very good approximations of the lower energy eigenvalues of H, especially for $\nu \ll N$.
Lanczos technique is also applicable to solve LIT equation.

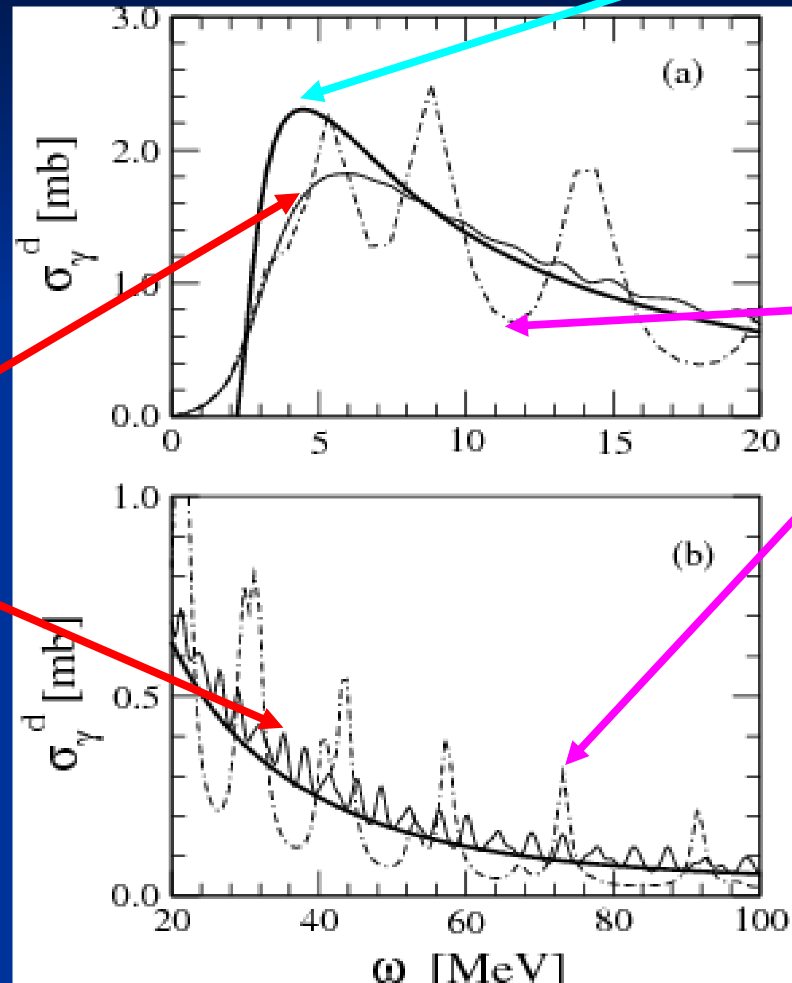
$\sigma_\gamma(\omega)$ from inversion and Lanczos response

"true"

$\sigma_I = 1$ MeV

N_{ho}=150

N_{ho}=2400



$\sigma_\gamma(\omega)$ from inversion and Lanczos response

"true"

$\Gamma = 0.5$ MeV

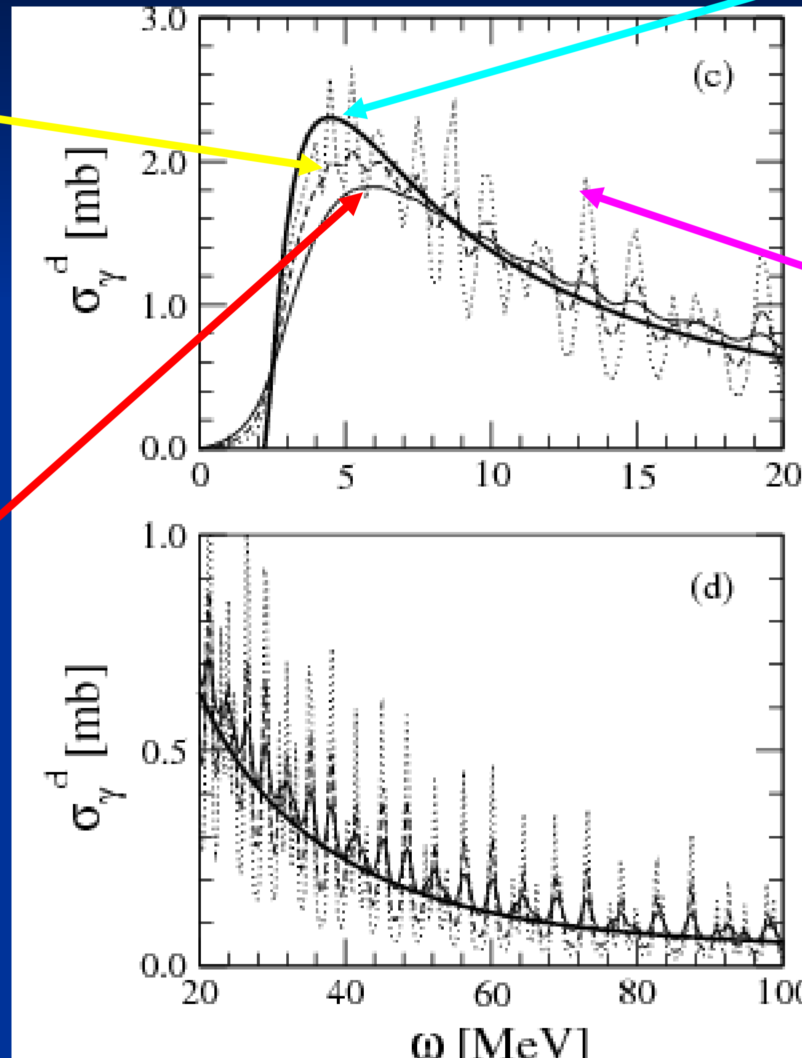
HO basis:

fixed

$N_{HO} = 2400$

$\Gamma = 1$ MeV

$\Gamma = 0.25$ MeV



LIT Application

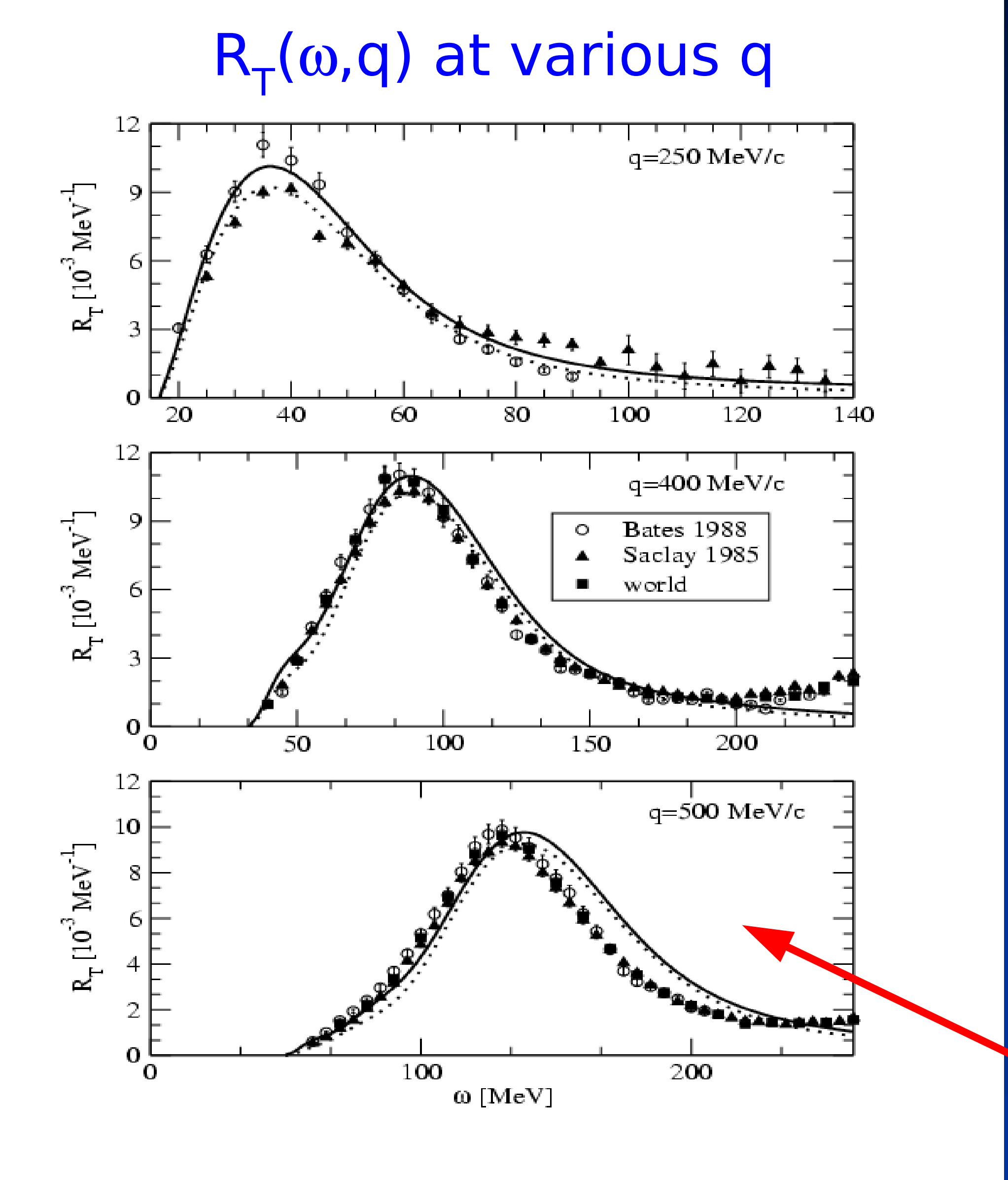
Inclusive Electrodisintegration of Three-Body Nuclei

Nuclear Interaction: AV18 NN potential + Urbana-IX NN force

Various kinematical regions:

- Quasielastic peak region (relativistic effects)
- Breakup threshold region at $q \leq 500$ MeV/c (MEC)
- Breakup threshold region at $q=860$ MeV/c (rel. effects, MEC, Δ)

Motivation



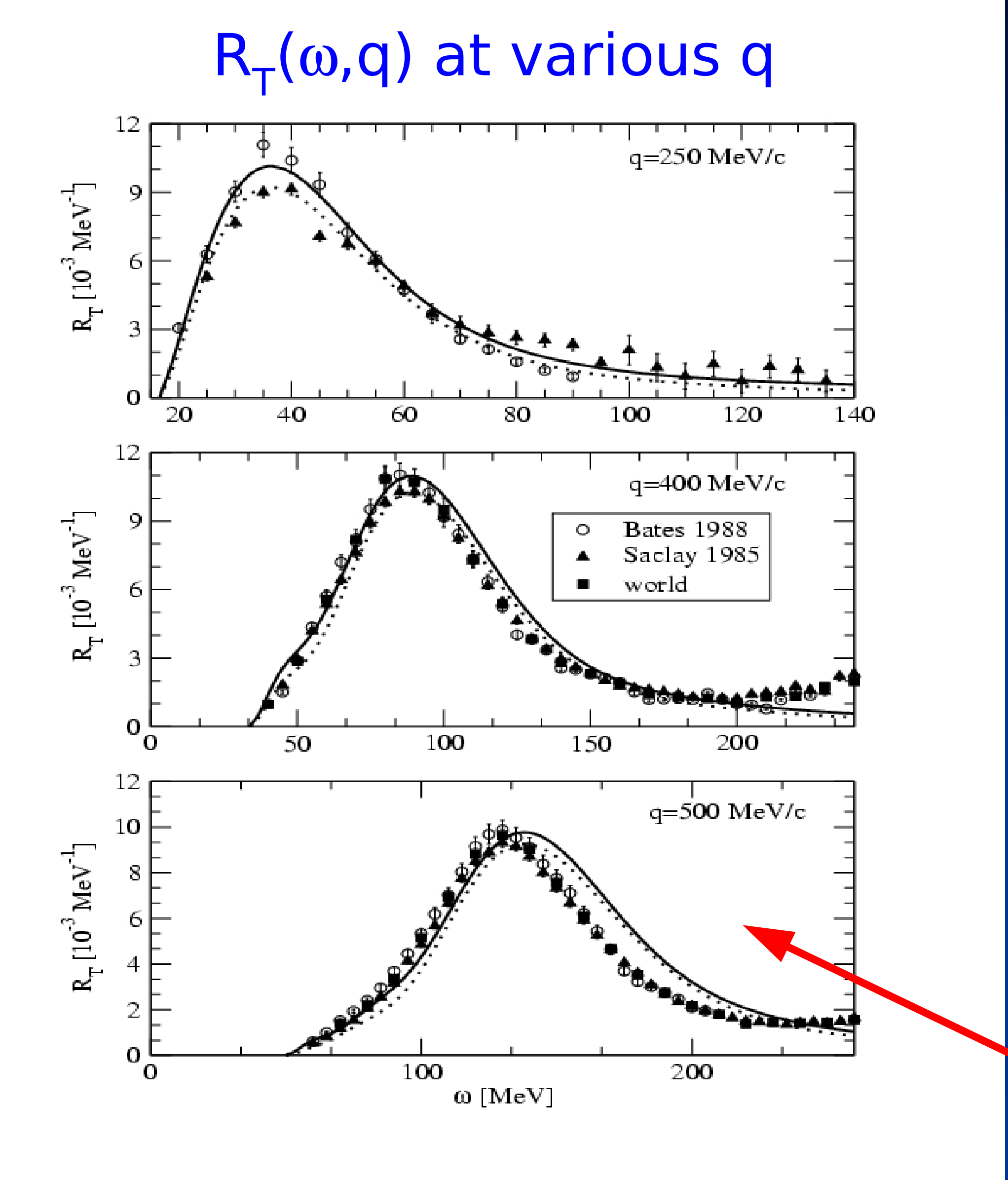
Potential: BonnRA +TM'

one-body current: dashed
one+two-body current: full

(S. Della Monaca et al., PRC 77, 044007 (2008))

Bad agreement between theory and experiment because of non considered relativistic effects

Motivation



Potential: BonnRA +TM'

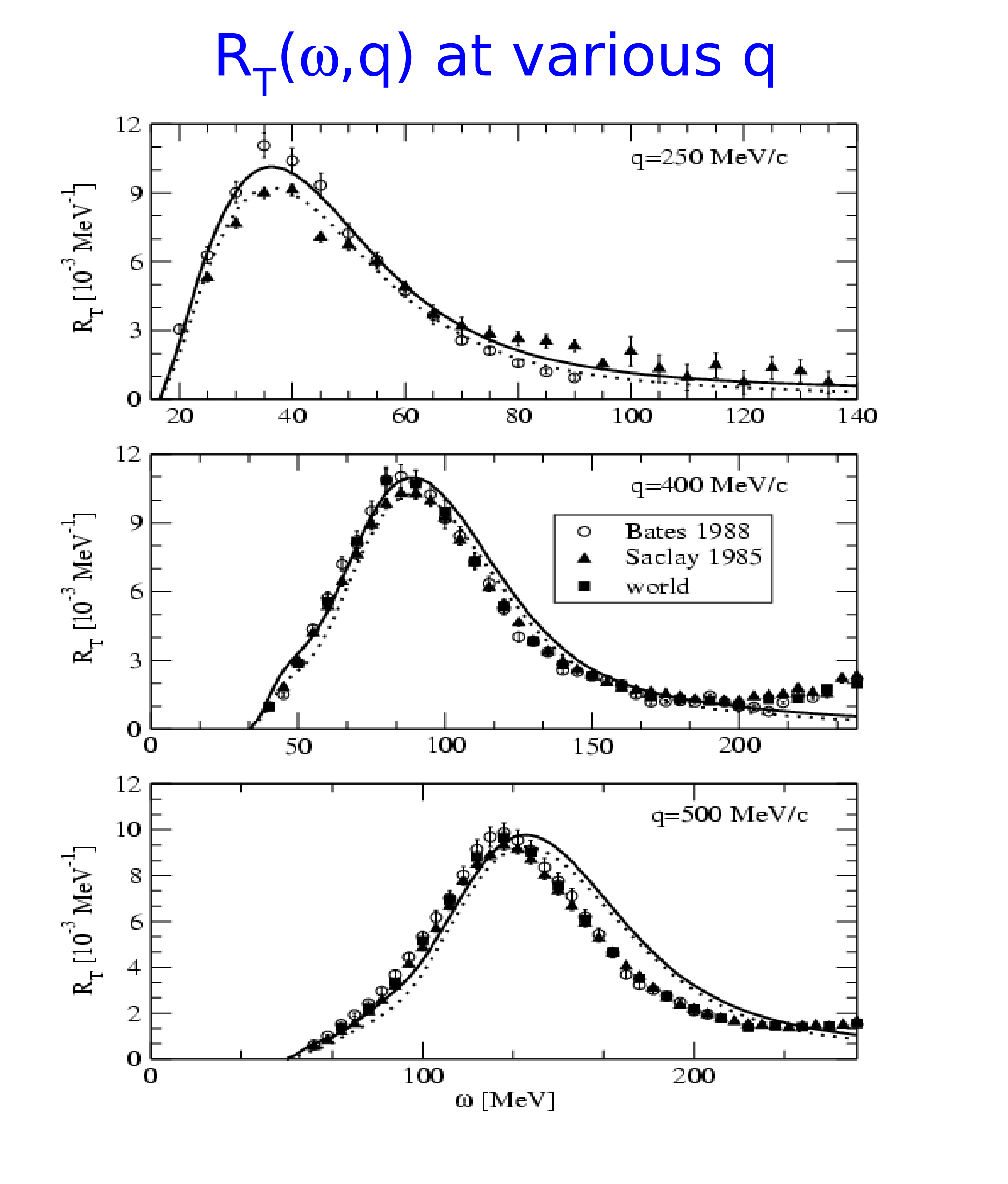
one-body current: dashed
one+two-body current: full

Quasi-elastic kinematics (q=500 MeV/c),
Kinetic energy of outgoing nucleon:

non-rel. : $T = q^2/2m = 133$ MeV
rel.: $T = (m^2+q^2)^{1/2} - m = 125$ MeV

Bad agreement between theory and experiment because of non considered relativistic effects

Motivation



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rel.: $T= (m^2+q^2)^{1/2} - m = 125$ MeV

Active Nucleon Breit (ANB) frame (nucleus moves with -Aq/2) is better
Efros et al., PRC 72, 011002 (2005)

The current operator **J**

$\mathbf{J} = \mathbf{J}^{(1)} + \mathbf{J}^{(2)}$

$\mathbf{J}^{(1)} = \mathbf{J}^{(1)}(q,\omega,P_T) = \mathbf{J}_{spin} + \mathbf{J}_p + \mathbf{J}_q + (\omega/M)\, \mathbf{J}_\omega$

for instance spin current

$\mathbf{J}_{spin} = \exp(i\mathbf{q}\cdot\mathbf{r})\; i\, \boldsymbol{\sigma}\times\mathbf{q}/2M\, [G_M(1\text{-}q^2/8M^2) - G_E\, \kappa^2 q^2/8M^2]$

with $\kappa=1+2P_T/Aq$

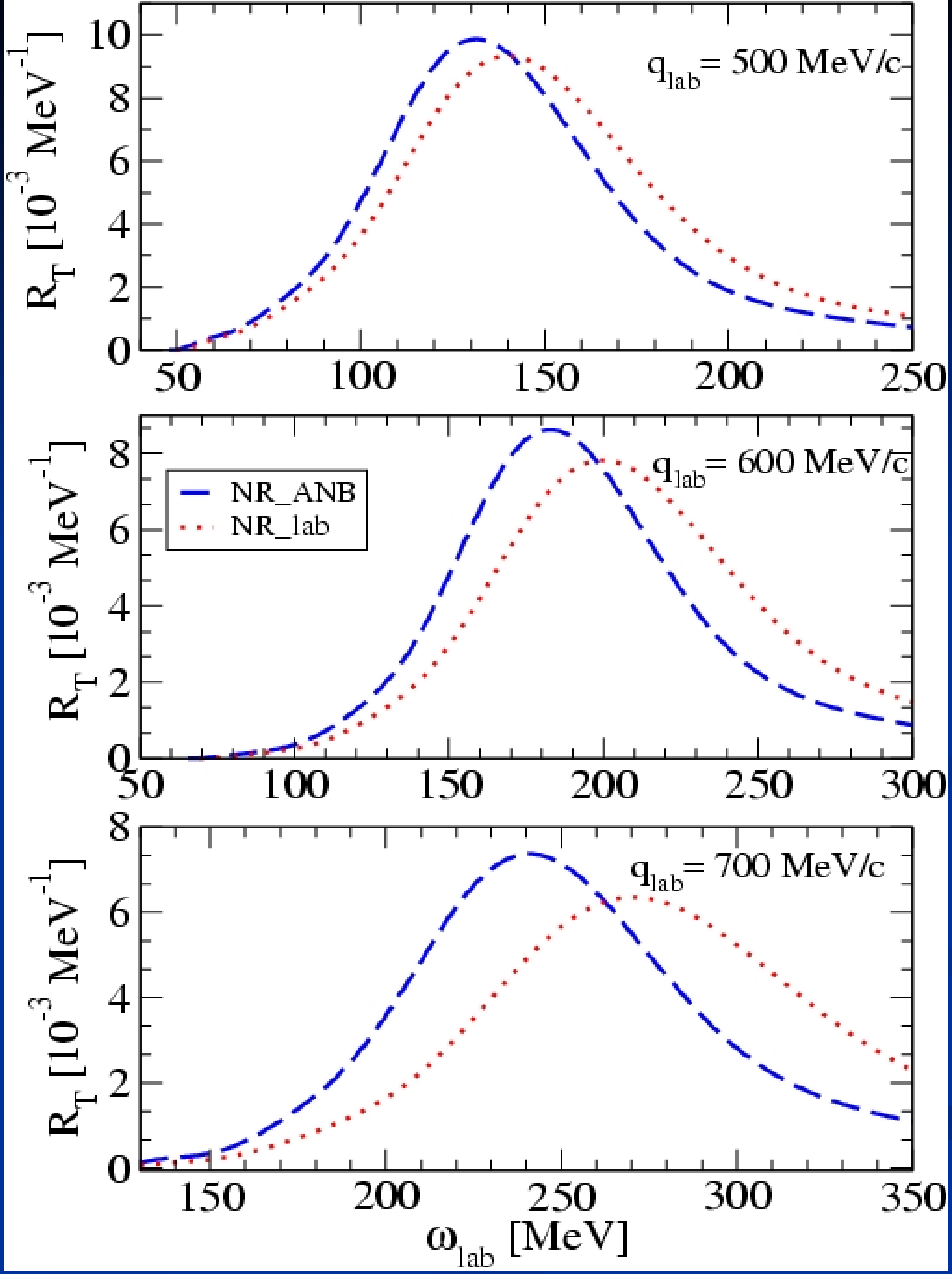
Transformation from ANB frame to LAB frame

$R_T^{LAB}(\omega^{LAB},q^{LAB}) = R_T^{ANB}(\omega^{ANB},q^{ANB})\;\; E_T^{ANB}/M_T$

Efros et al., PRC 81, 034001 (2010)

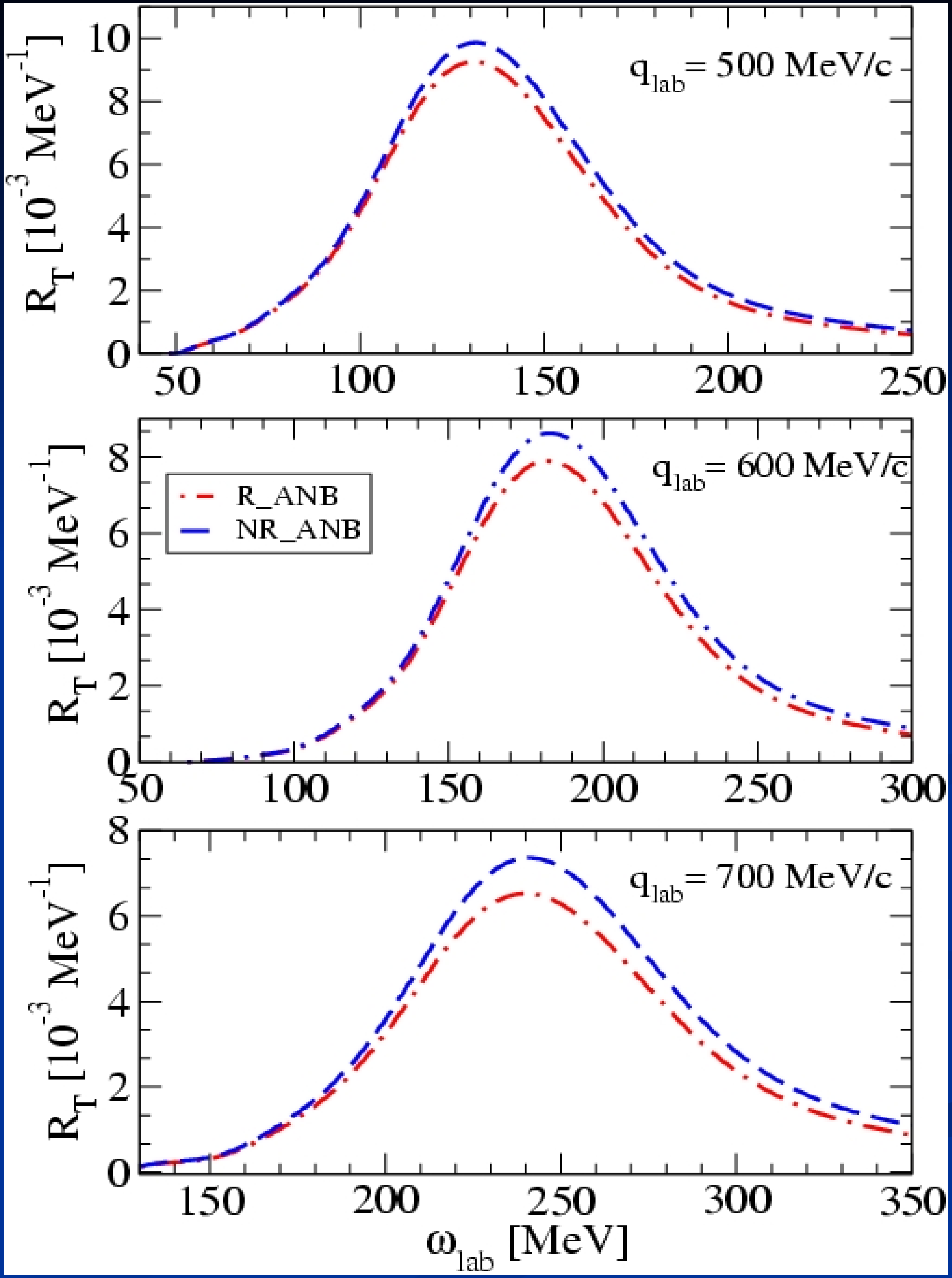
Results

- Comparison of ANB and LAB calculation: strong shift of peak to lower energies! (8.7, 16.7, 29.3 MeV at q=500, 600, 700 MeV/c)



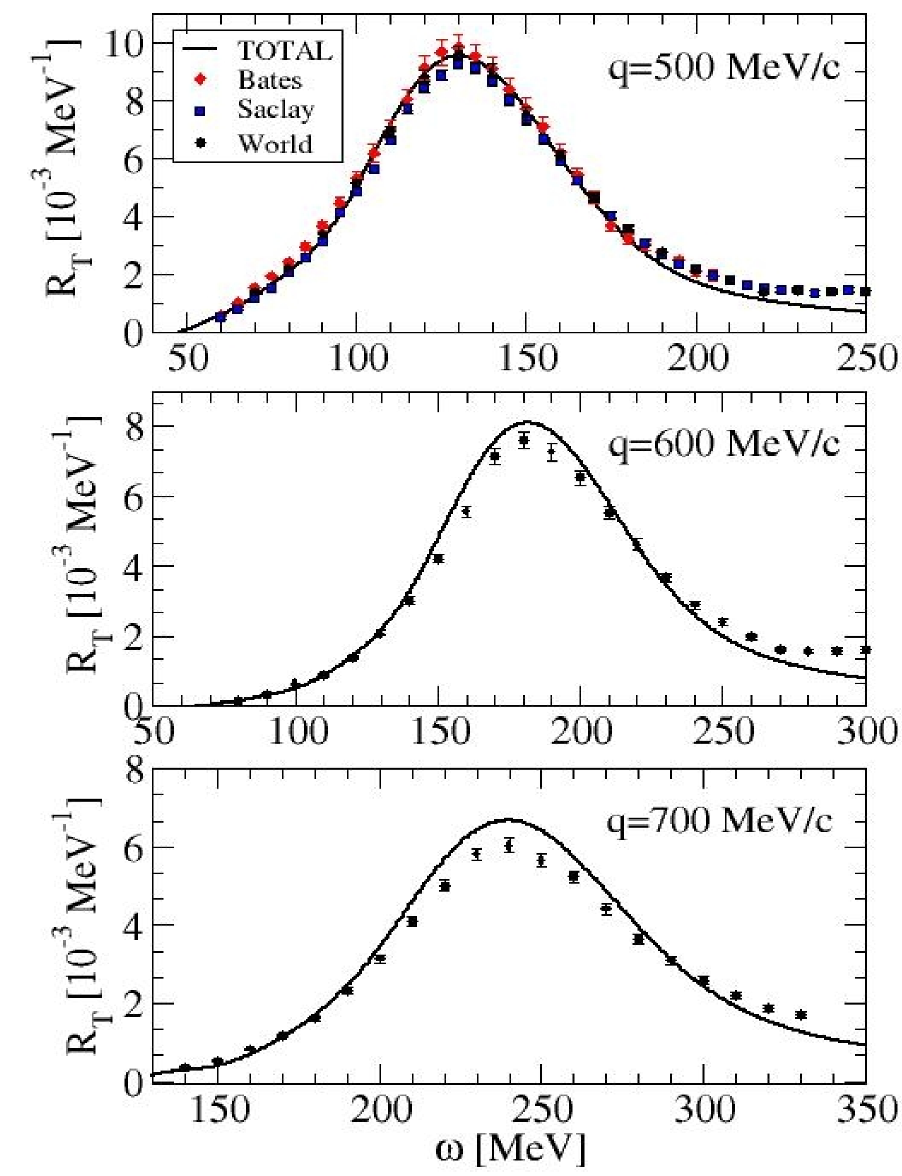
Results

- Rel. contribution: reduction of peak height (6.2%, 8.5%, 11.3 % at q=500, 600, 700 MeV/c)



Results

Comparison with experiment

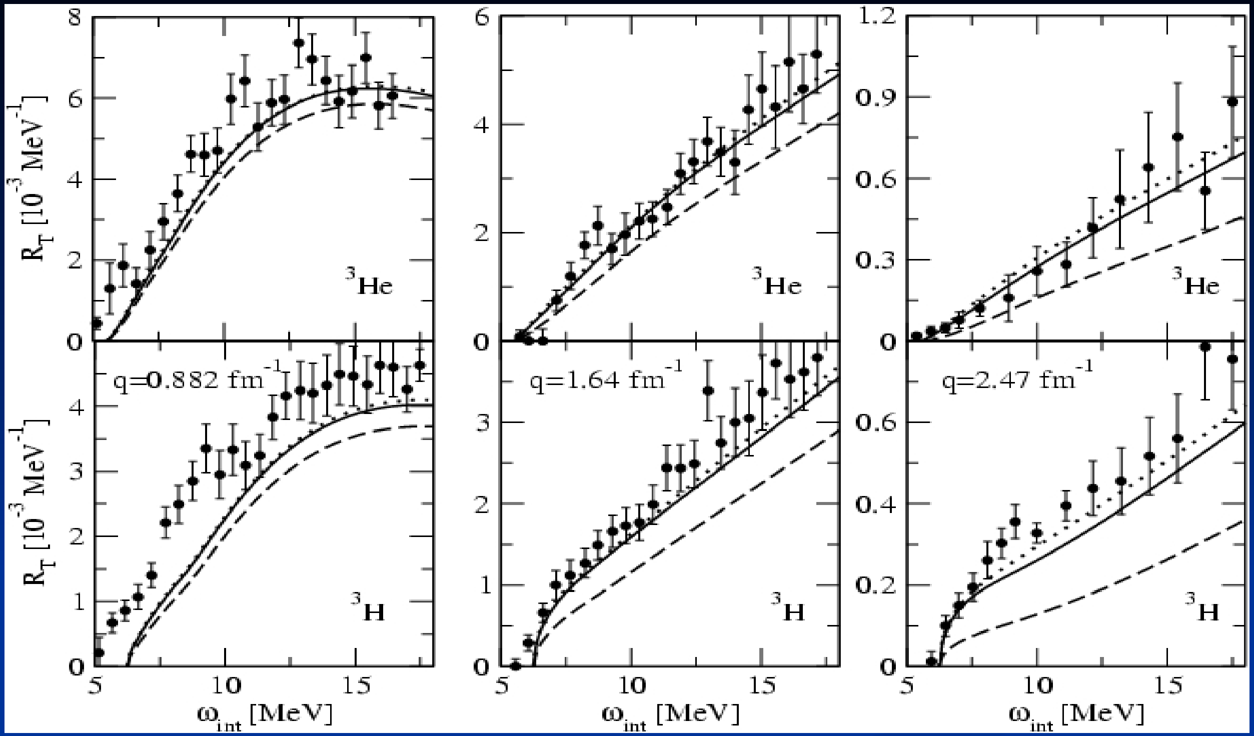


NR: dashed

NR+MEC: dotted

Rel.+MEC: full

Exp.: Retzlaff et al.

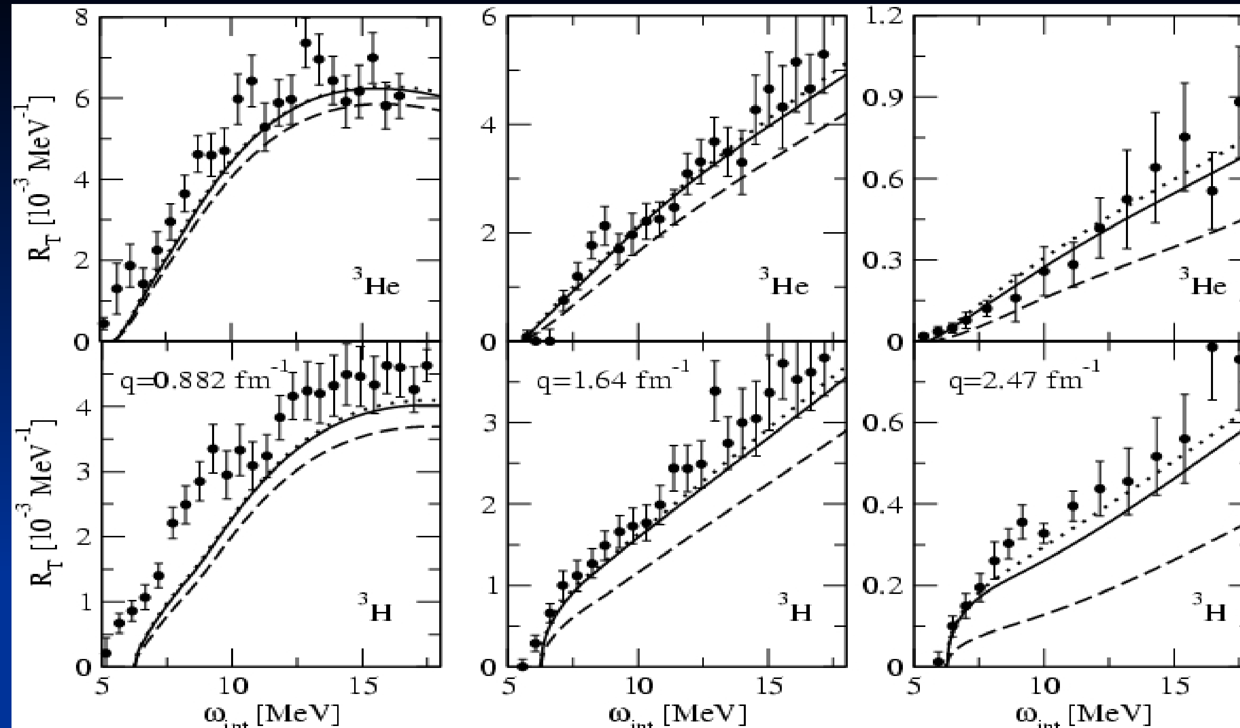


q = 174 MeV/c   q = 324 MeV/c   q = 487 MeV/c

R_T close to break-up threshold

(Few-Body Syst., online first; arXiv:0906.0663)

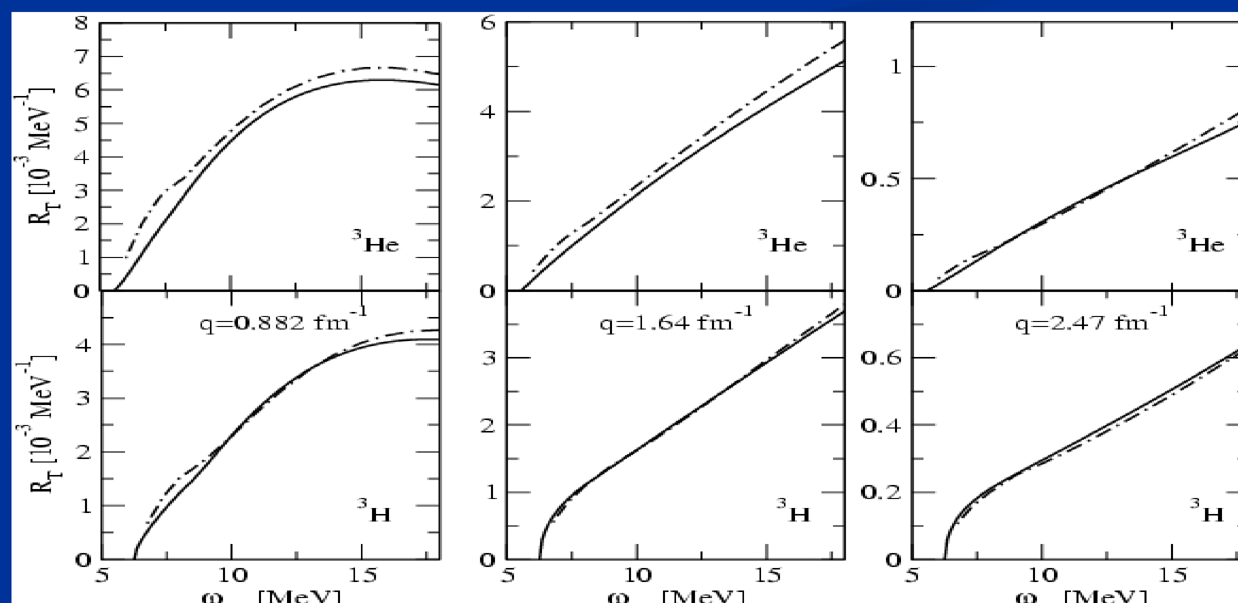
NR+MEC: dotted
Rel.+MEC: full



Faddeev calculation

Golak et al.: dash-dotted

our NR+MEC calc.: full



Incorporation of Δ-Resonance

LIT coupled channel calculation

$$|\tilde{\Psi}\rangle = |\tilde{\Psi}^N\rangle + |\tilde{\Psi}^\Delta\rangle$$

$$(T_N + V_{NN} - \sigma)|\tilde{\Psi}^N\rangle = -V_{NN,N\Delta}\,|\tilde{\Psi}^\Delta\rangle + O_{N\to N}|\Psi_0^N\rangle + O_{\Delta\to N}|\Psi_0^\Delta\rangle$$

$$(\delta m + T_\Delta + V_{ND} - \sigma)|\tilde{\Psi}^\Delta\rangle = -V_{N\Delta,NN}|\tilde{\Psi}^N\rangle + O_{N\to\Delta}|\Psi_0^N\rangle + O_{\Delta\to\Delta}|\Psi_0^\Delta\rangle$$

$V_{NN,N\Delta}$ and $V_{N\Delta,NN}$ transition potentials between NNN and NNΔ spaces

O_k various diagonal (NN, ΔΔ) and transition (NΔ, ΔN) current operators

Incorporation of Δ-Resonance

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$$(T_N + V_{NN} - \sigma)|\tilde{\Psi}^N\rangle = -V_{NN,N\Delta}\,|\tilde{\Psi}^\Delta\rangle + O_{N\to N}|\Psi_0^N\rangle + O_{\Delta\to N}|\Psi_0^\Delta\rangle$$

Solve for ψ^Δ $$(\delta m + T_\Delta + V_{ND} - \sigma)|\tilde{\Psi}^\Delta\rangle = -V_{N\Delta,NN}|\tilde{\Psi}^N\rangle + O_{N\to\Delta}|\Psi_0^N\rangle + O_{\Delta\to\Delta}|\Psi_0^\Delta\rangle$$

$V_{NN,N\Delta}$ and $V_{N\Delta,NN}$ transition potentials between NNN and NNΔ spaces

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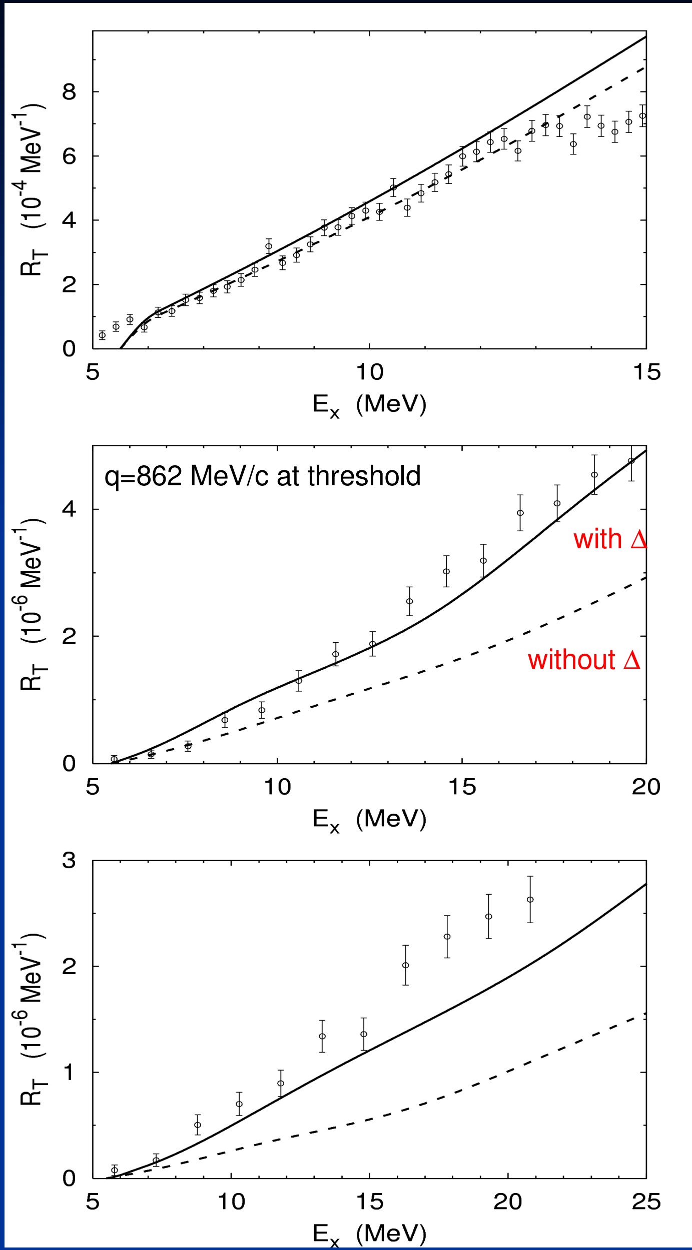
$$(\delta m + T_\Delta + V_{ND} - \sigma)|\tilde\Psi^\Delta\rangle = -V_{N\Delta,NN}|\tilde\Psi^N\rangle + O_{N\to\Delta}|\Psi_0^N\rangle + O_{\Delta\to\Delta}|\Psi_0^\Delta\rangle$$

$V_{NN,N\Delta}$ and $V_{N\Delta,NN}$ transition potentials between NNN and NNΔ spaces

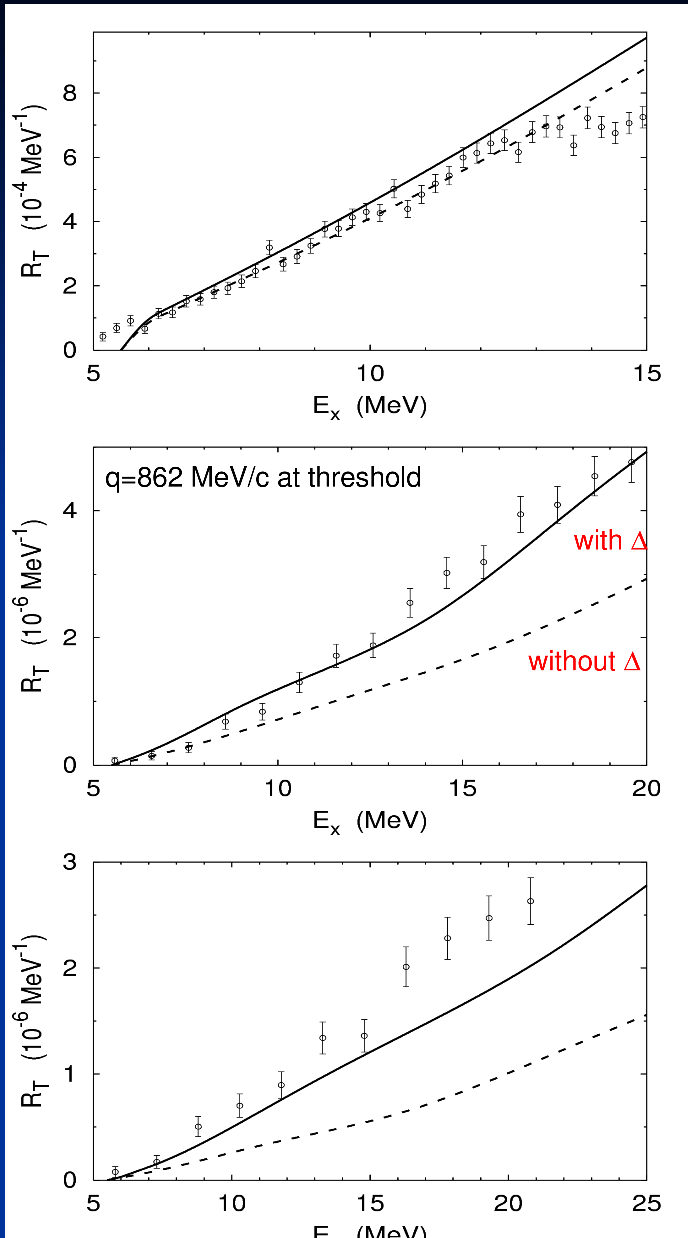
O_k various diagonal (NN, ΔΔ) and transition (NΔ, ΔN) current operators

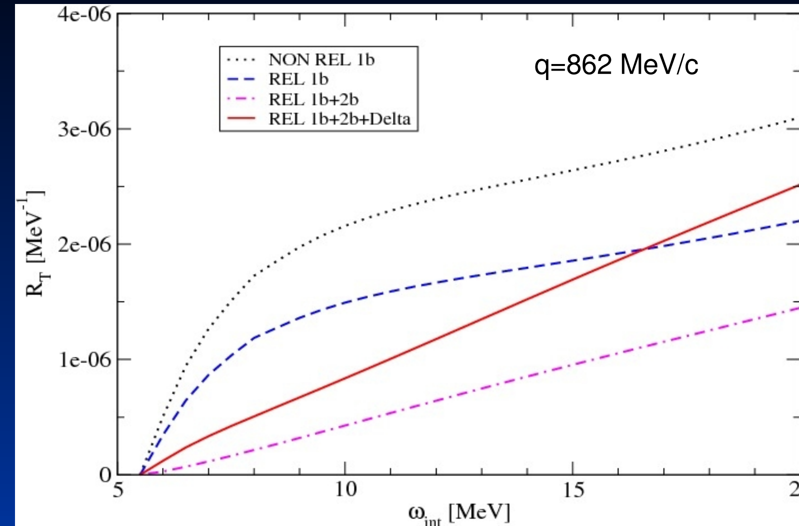
New first equation:

$$(H_N^{\rm new} - \sigma)|\tilde\Psi^N\rangle = -V_{NN,N\Delta}(H_\Delta - \sigma)^{-1}\left(O_{N\to\Delta}|\Psi_0^N\rangle + O_{\Delta\to\Delta}|\Psi_0^\Delta\rangle\right)$$
$$+O_{N\to N}|\Psi_0^N\rangle + O_{\Delta\to N}|\Psi_0^\Delta\rangle$$

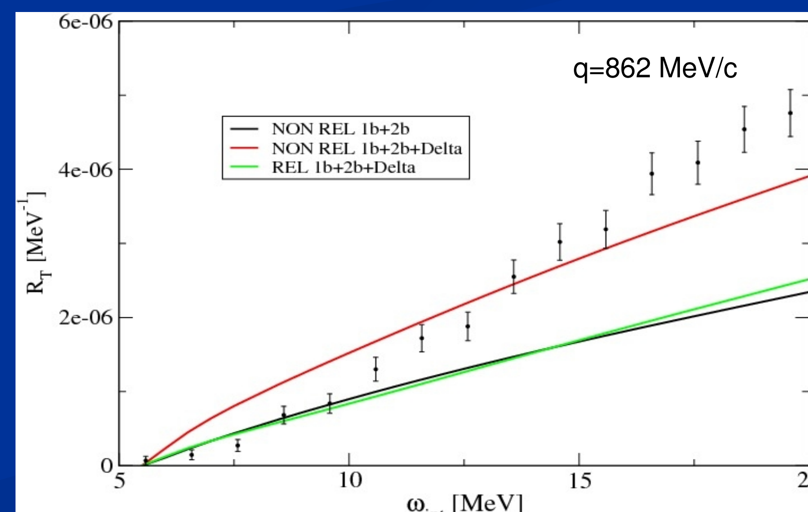


Calculation with CD-Bonn and CD-Bonn-Δ
A. Deltuva, L.P. Yuan, J. Adam, P.U.Sauer





L.P. Yuan, V. Efros, WL, G. Orlandini, E. Tomusiak

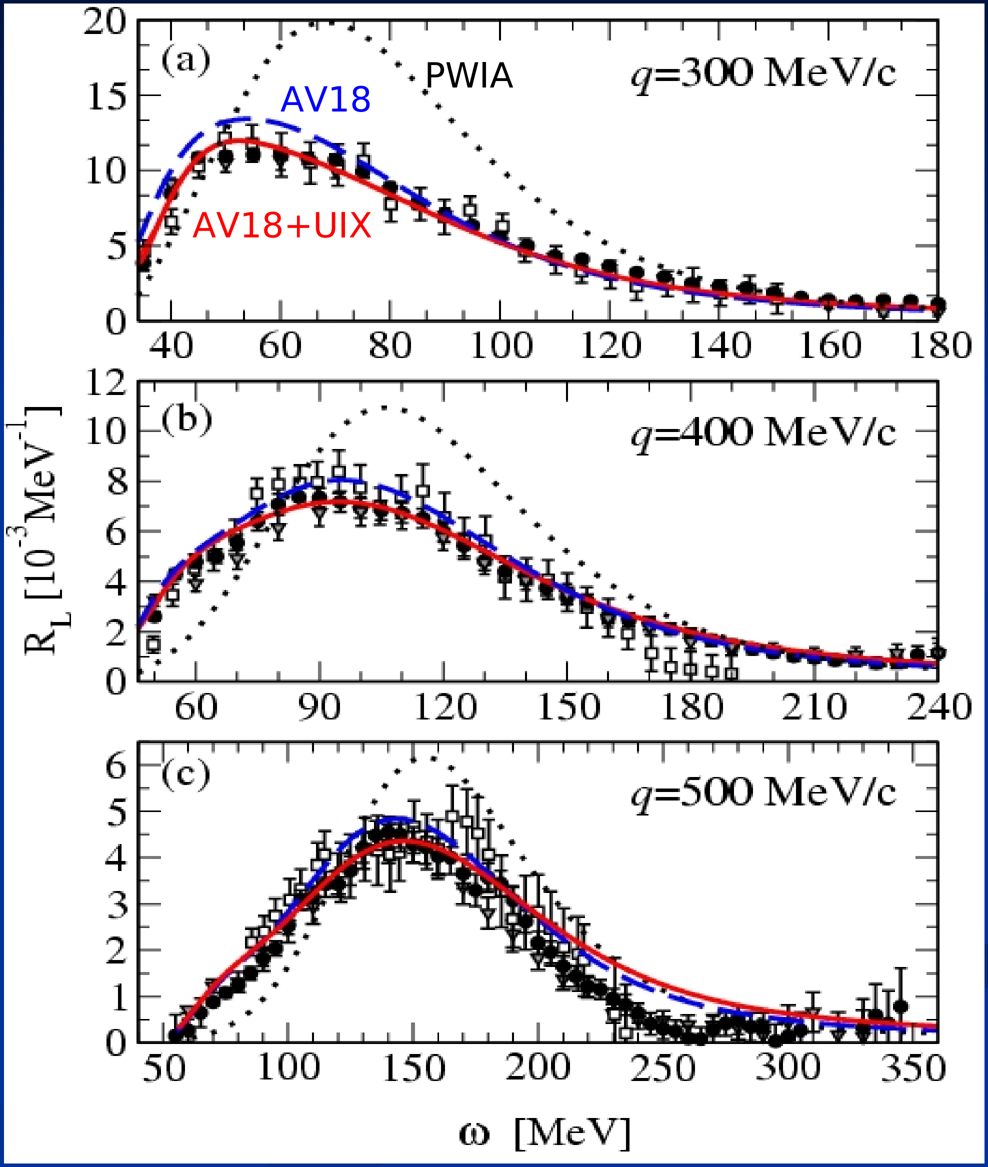


Inclusive Electrodisintegration of ^4He

Nuclear Interaction: AV18 NN potential + Urbana-IX NNN force
AV18 NN potential + TM' NNN force

S. Bacca, N. Barnea, W.L., G.Orlandini, PRL 102, 162501 (2009);
PRC 80, 064001 (2009)

^{4}He (e,e') Longitudinal Response



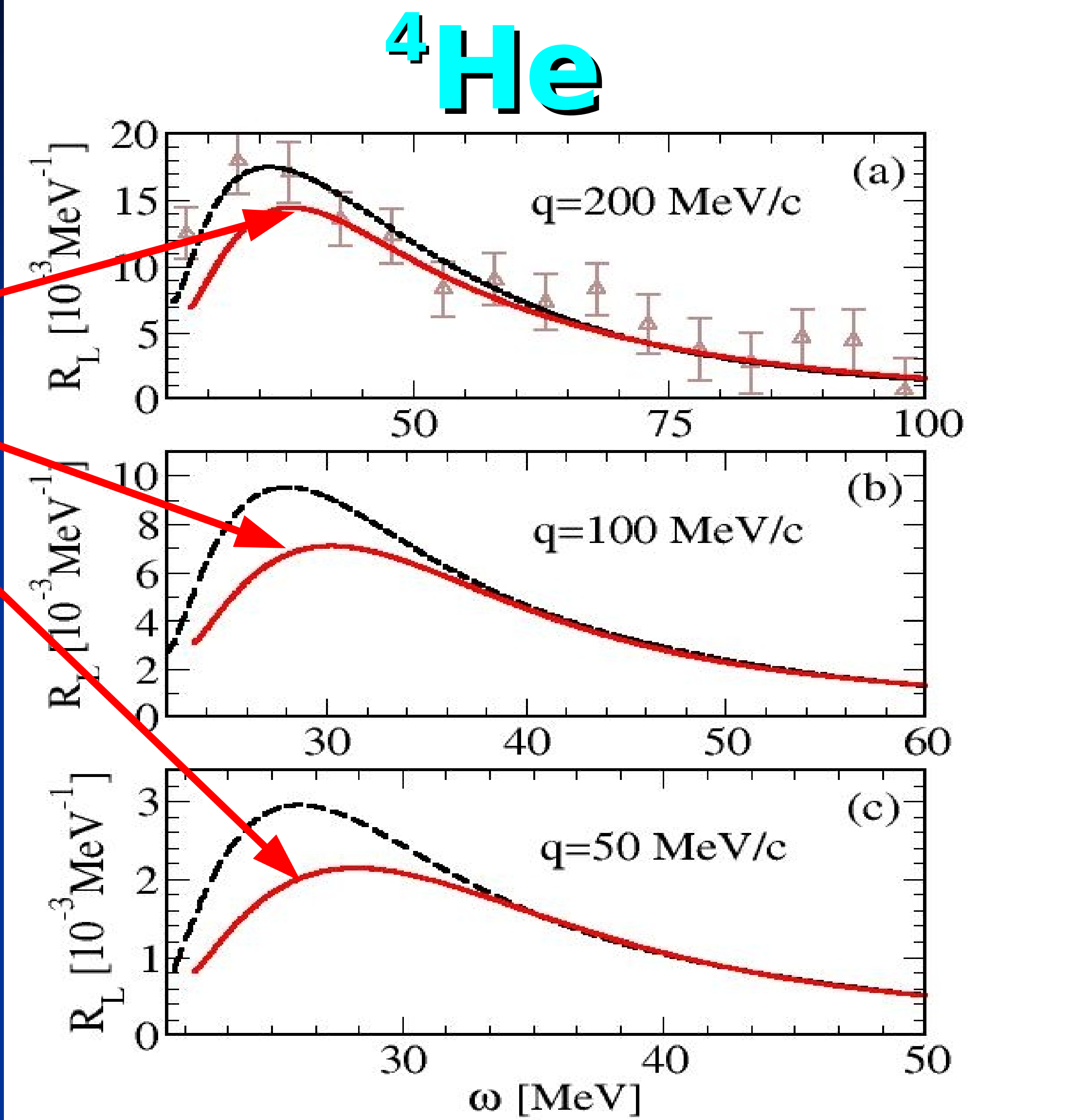
SMALL EFFECT OF 3-BODY FORCE AT HIGH q

Exp.: Saclay
Bates
world data

(e,e') Longitudinal Response

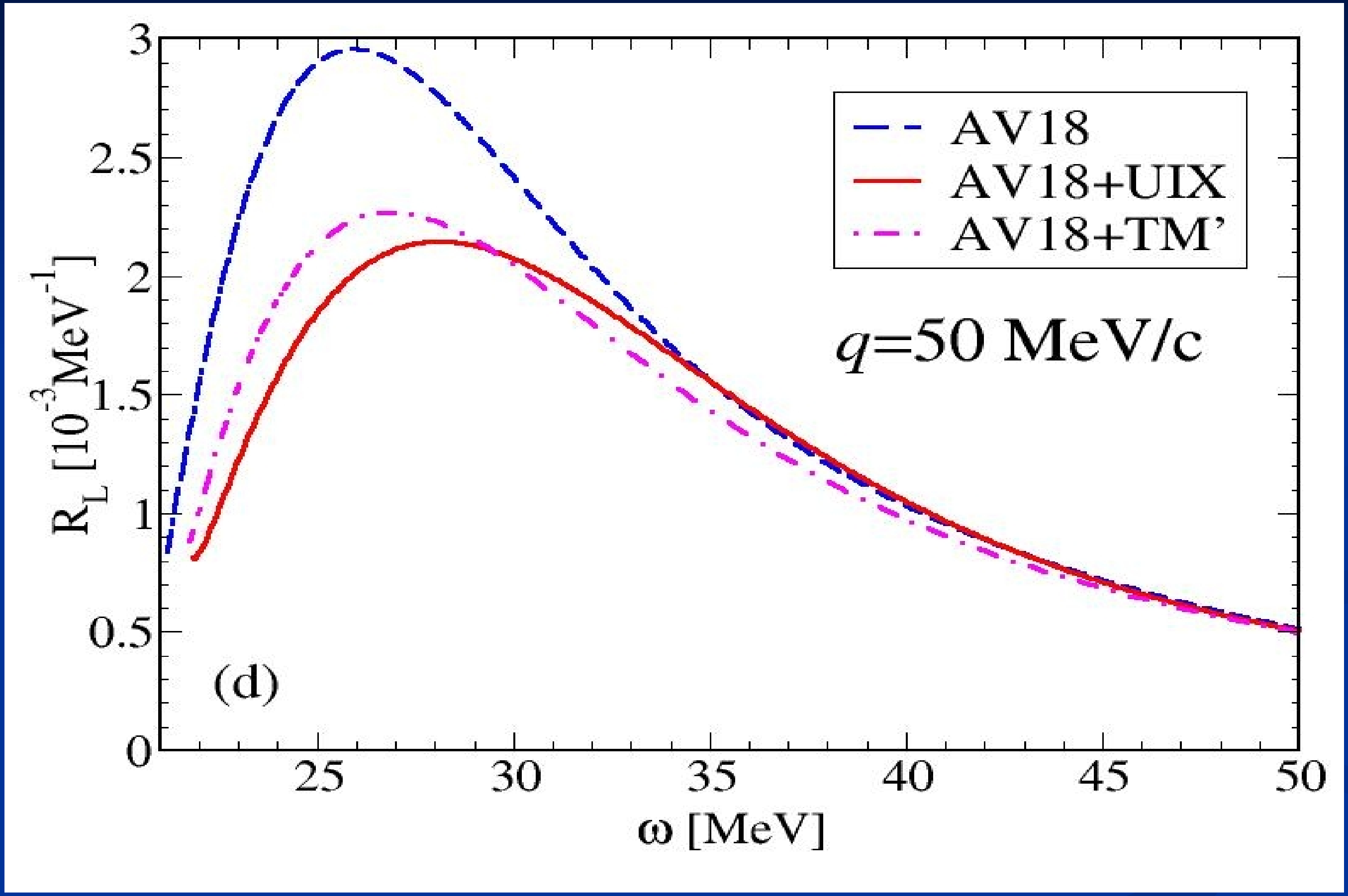
SURPRISE:
LARGE EFFECT OF
3-BODY FORCE
AT LOW q

NO DATA
AT LOWER q !!!
(wait for Mainz results)

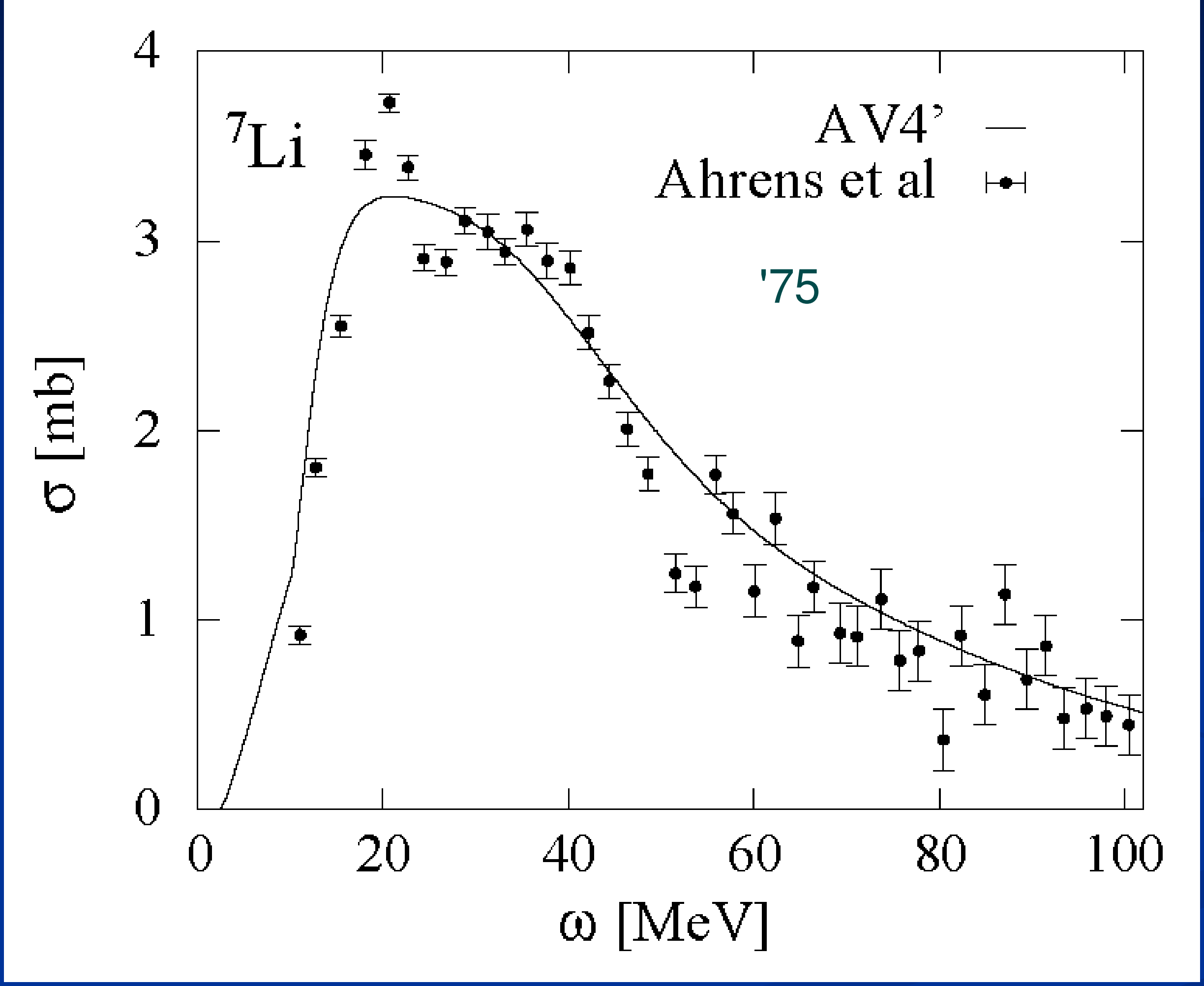


S.Bacca et al., PRL 102 (2009) 162501

Dependence on different 3-nucleon forces



7-Body total photodisintegration



S.Bacca et al.
PLB 603(2004) 159

Conclusion

LIT approach opens up the possibility to
calculate continuum observables with bound-state methods

- Method is benchmarked with conventional calculations for two- and three-body systems
- Various applications for nuclei with A=3-7

Outlook: Search for other integral transform which can be used in GFMC calculations (G. Orlandini, W.L., V.Efros, N.Barnea, J. of Phys. G in print)