One-Pion exchange currents in chiral EFT using the method of unitary transformation

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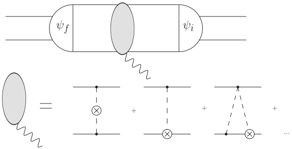
Motivation

- Successful derivation of nuclear potentials using method of unitary transformation & chiral perturbation theory in recent years[Epelbaum et al. '98]
- Consistent derivation of electromagnetic-current J^μ



- $\vec{\nabla} \cdot \vec{J} = -i\,[H, \rho]$
- Treat em-interaction as perturbation
- Convolute between wave-functions.

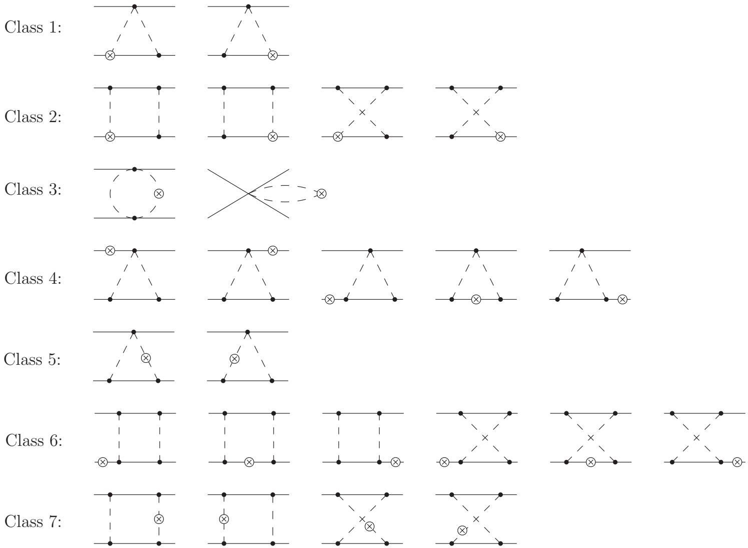
- Define effective current with unitary transformation

$$\eta V_{\mathrm{eff}} \eta = \eta U'^{\dagger} \eta U^{\dagger} (H - H_0) U \eta U' \eta, \quad U = \begin{pmatrix} \eta \left(1 + A^\dagger A\right)^{-\frac{1}{2}} & -A^\dagger \left(1 + AA^\dagger\right)^{-\frac{1}{2}} \\ A \left(1 + A^\dagger A\right)^{-\frac{1}{2}} & \lambda \left(1 + AA^\dagger\right)^{-\frac{1}{2}} \end{pmatrix}$$

$$\eta J^\mu_{\mathrm{eff}} \eta = \eta U'^{\dagger} \eta U^{\dagger} J^\mu U \eta U' \eta,$$

with projectors η (λ) on the purely nucleonic (rest) subspace.

Two-Pion exchange currents



Two-Pion exchange currents in configuration-space

$$
\begin{aligned}
\vec{J}_{c1}(\vec{r}_{10},\vec{r}_{20}) &= e\,\frac{g_A^2 M_\pi^7}{128\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}\,[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c2}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{g_A^4 M_\pi^7}{256\pi^3 F_\pi^4}\left(3\nabla_{10}^2-8\right)\left[\vec{\nabla}_{10}\,[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\delta(\vec{x}_{20})\,\frac{K_0(2x_{10})}{x_{10}}\\
&\quad + e\,\frac{g_A^4 M_\pi^7}{32\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}\times\vec{\sigma}_1\right]\tau_2^3\,\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c3}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{M_\pi^7}{512\pi^4 F_\pi^4}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,(\vec{\nabla}_{10}-\vec{\nabla}_{20})\,\frac{K_2(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})(x_{10}+x_{20}+x_{12})} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c5}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{g_A^2 M_\pi^7}{256\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\left[[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} - 2\tau_1^3\vec{\sigma}_2\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{20}\right]\right]\\
&\quad \times\,\frac{K_1(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c7}(\vec{r}_{10},\vec{r}_{20}) &= e\,\frac{g_A^4 M_\pi^7}{512\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\left[[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} + 4\tau_2^3\vec{\sigma}_1\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{10}\right]\right.\\
&\quad \left.\times\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20}\right]\frac{x_{10}+x_{20}+x_{12}}{x_{10}\,x_{20}\,x_{12}}\,K_0(x_{10}+x_{20}+x_{12}) + (1\leftrightarrow 2)\\
\vec{J}_4(\vec{r}_{10},\vec{r}_{20}) &= \vec{J}_{c6}(\vec{r}_{10},\vec{r}_{20}) = 0\,,
\end{aligned}
$$

with $\vec{r}_{1/2/0}$ the positions of the first/second nucleon/the photon, and $\vec{x}_{10} = M_\pi\,(\vec{r}_1-\vec{r}_0)$, $\vec{x}_{20} = M_\pi\,(\vec{r}_2-\vec{r}_0)$, $\vec{x}_{12} = M_\pi\,(\vec{r}_1-\vec{r}_2)$ and $\vec{\nabla}_{ij} \equiv \partial/\partial x_{ij}$ and $x_{ij} \equiv |\vec{x}_{ij}|$.
All derivatives have to be evaluated as if the variables were independent.

Two-Pion exchange currents in configuration-space Ctd.

$$\rho_{c1}(\vec{r}_{10}, \vec{r}_{20}) = \rho_{c2}(\vec{r}_{10}, \vec{r}_{20}) = \rho_{c3}(\vec{r}_{10}, \vec{r}_{20}) = 0\,,$$

$$\rho_{c4}(\vec{r}_{10}, \vec{r}_{20}) = e\,\frac{g_A^2\, M_\pi^7}{256\pi^2 F_\pi^4}\,\tau_1^3\,\delta(\vec{x}_{20})\left(\nabla_{10}^2 - 2\right)\frac{e^{-2x_{10}}}{x_{10}^2} + (1 \leftrightarrow 2)\,,$$

$$\rho_{c5}(\vec{r}_{10}, \vec{r}_{20}) = -\,e\,\frac{g_A^2\, M_\pi^7}{256\pi^2 F_\pi^4}\,\tau_2^3\,\delta(\vec{x}_{20})\left(\nabla_{10}^2 - 2\right)\frac{e^{-2x_{10}}}{x_{10}^2} + (1 \leftrightarrow 2)\,,$$

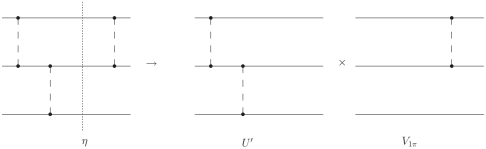
$$\rho_{c6}(\vec{r}_{10}, \vec{r}_{20}) = -\,e\,\frac{g_A^4\, M_\pi^7}{256\pi^2 F_\pi^4}\,\delta(\vec{x}_{20})\left[\tau_1^3\left(2\nabla_{10}^2 - 4\right) + \tau_2^3\,\vec{\sigma}_1\cdot\vec{\nabla}_{10}\,\vec{\sigma}_2\cdot\vec{\nabla}_{10} - \tau_2^3\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\frac{e^{-2x_{10}}}{x_{10}^2}$$
$$-\,e\,\frac{g_A^4\, M_\pi^7}{128\pi^2 F_\pi^4}\,\delta(\vec{x}_{20})\,\tau_1^3\left(3\nabla_{10}^2 - 11\right)\frac{e^{-2x_{10}}}{x_{10}} + (1 \leftrightarrow 2)\,,$$

$$\rho_{c7}(\vec{r}_{10}, \vec{r}_{20}) = -\,e\,\frac{g_A^4\, M_\pi^7}{512\pi^3 F_\pi^4}\Big[(\tau_1^3 + \tau_2^3)\Big(\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} + \vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{10}\times\vec{\sigma}_1\right]\vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{20}\times\vec{\sigma}_2\right]\Big)$$
$$+\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\,\vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{20}\times\vec{\sigma}_2\right]\Big]\frac{e^{-x_{10}}}{x_{10}}\,\frac{e^{-x_{20}}}{x_{20}}\,\frac{e^{-x_{12}}}{x_{12}} + (1 \leftrightarrow 2)\,.$$

- Results also available in momentum-space, expressed in standard loop-function $L(q)$, $A(q)$ and three-point functions.
- Can be easily treated numerically.
- Continuity-equation is fulfilled $\rightarrow$ Current is consistent with potential obtained within the method of unitary transformation

Additional unitary transformations

- Okubo-parameterization only gives a minimal unitary transformation.
- We are free to add additional transformations.
- Consider the 3N-force diagram:

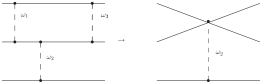


- Contribution from additional transformation!
- Write $U' = e^S = \mathbb{1} + S$, with $S^\dagger = -S$ and $S = \eta S \eta$.

- For this example Epelbaum '07

$$
\begin{aligned}
S &= \alpha_1 S_1 + \alpha_2 S_2 \,, \\
S_1 &= \eta \left[H^1_{21} \frac{\lambda^1}{E_\pi} H^1_{21} \eta H^1_{21} \frac{\lambda^1}{E_\pi^3} H^1_{21} - H^1_{21} \frac{\lambda^1}{E_\pi^3} H^1_{21} \eta H^1_{21} \frac{\lambda^1}{E_\pi} H^1_{21} \right] \eta \,, \\
S_2 &= \eta \left[H^1_{21} \frac{\lambda^1}{E_\pi} H^1_{21} \frac{\lambda^2}{E_\pi} H^1_{21} \frac{\lambda^1}{E_\pi^2} H^1_{21} - H^1_{21} \frac{\lambda^1}{E_\pi^2} H^1_{21} \frac{\lambda^1}{E_\pi} H^1_{21} \frac{\lambda^1}{E_\pi} H^1_{21} \right] \eta \,.
\end{aligned}
$$

- Renormalizability of the potential

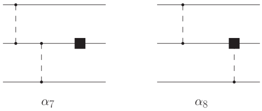


- Divergencies have to absorbed by right diagram.
- Depends only on $1/\omega_2^2$.

- Can fix $\alpha_1 = -1/2$ and $\alpha_2 = 1/4$.

Additional transformations Ctd.

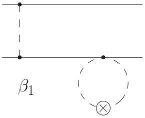
- Further additional transformations (not fixed by renormalizability).

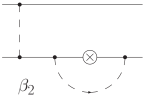


$$S_7 = \alpha_7 \eta \left[H^2_{20} \eta H^1_{21} \frac{\lambda^1}{E^2_\pi} H^1_{21} - H^1_{21} \frac{\lambda^1}{E^2_\pi} H^1_{21} \eta H^2_{20} \right] ,$$

$$S_8 = \alpha_8 \eta \left[H^3_{21} \frac{\lambda^1}{E^3_\pi} H^1_{21} - H^1_{21} \frac{\lambda^1}{E^3_\pi} H^3_{21} \right] \eta .$$

- Where $H^2_{20} = 1/2 m_N \vec{p}^2$, and H^3_{21} is the $1/m_N$ correction to H^1_{21}.
- Additional transformations for the em-current





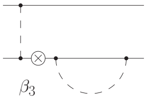
$$U'_{\rm em} = e^{S'} , \quad \text{with} \quad S'(\mathcal{A}) \to 0 \quad \text{for} \quad \mathcal{A} \to 0 ,$$

$$U'_{\rm em} \qquad \text{s.t. transformed Hamiltonian is block-diagonal}$$

$$S'_1 = \beta_1 \eta \left[J^{-1}_{02} \frac{\lambda^2}{E^2_\pi} H^2_{22} - H^2_{22} \frac{\lambda^2}{E^2_\pi} J^{-1}_{02} \right] \eta ,$$

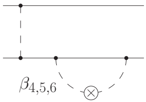
$$S'_2 = \beta_2 \eta \left[H^1_{21} \frac{\lambda^1}{E^2_\pi} J^{-1}_{20} \frac{\lambda^1}{E_\pi} H^1_{21} - H^1_{21} \frac{\lambda^1}{E_\pi} J^{-1}_{20} \frac{\lambda^1}{E^2_\pi} H^1_{21} \right] \eta ,$$

Additional Contributions for the em current Ctd.



$$S_3' = \beta_3 \eta \left[J_{20}^{-1} \eta H_{21}^1 \frac{\lambda^1}{E_\pi^3} H_{21}^1 - H_{21}^1 \frac{\lambda^1}{E_\pi^3} H_{21}^1 \eta J_{20}^{-1} \right] \eta \,,$$

$\longrightarrow$ contribution vanishes in all considered cases!

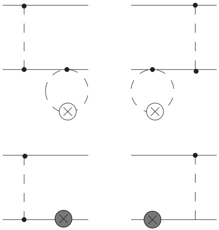


$$S_4 = -\frac{\beta_4}{2} \eta \left[J_{02}^{-1} \frac{\lambda^2}{E_\pi^2} H_{21}^1 \frac{\lambda^1}{E_\pi} H_{21}^1 - H_{21}^1 \frac{\lambda^1}{E_\pi} H_{21}^1 \frac{\lambda^2}{E_\pi^2} J_{02}^{-1} \right] \eta \,,$$

$$S_5 = -\frac{\beta_5}{2} \eta \left[J_{02}^{-1} \frac{\lambda^2}{E_\pi} H_{21}^1 \frac{\lambda^1}{E_\pi^2} H_{21}^1 - H_{21}^1 \frac{\lambda^1}{E_\pi^2} H_{21}^1 \frac{\lambda^2}{E_\pi} J_\pi \right] \eta \,,$$

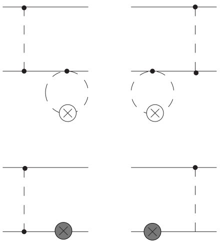
$$S_6 = -\beta_6 \eta \left[H_{21}^1 \frac{\lambda^1}{E_\pi} J_{02}^{-1} \frac{\lambda^1}{E_\pi^2} H_{21}^1 - H_{21}^1 \frac{\lambda^1}{E_\pi^2} J_{02}^{-1} \frac{\lambda^1}{E_\pi} H_{21}^1 \right] \eta \,,$$

- Constraints by renormalizability!



- These diagrams receives contributions from S_1'.
- The divergent part of the diagrams has to be absorbed into LECs.
- The divergent part (β-functions) of the LECs is already known Gasser et al. '02.
- Nontrivial since β-functions computed in different formalism.

Determination of βs

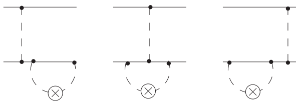


Contributes to:

$$\vec{J}_{c5} = e\,\frac{g_A^2\, i}{16F_\pi^4}(\beta_1 - 1)\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2 + M_\pi^2} \times \int \frac{d^3l}{(2\pi)^3}\vec{l}\,\frac{\omega_+ - \omega_-}{\omega_+\omega_-(\omega_+ + \omega_-)^2} + (1 \leftrightarrow 2)$$

$$\text{with } \omega_\pm^2 = \left(\vec{l} \pm \vec{k}\right)^2 + 4M_\pi^2\,, \qquad \vec{k} = \text{Photon-momentum}\,.$$

- Contributions from LECs vanishes in this case.
- Have to choose $\beta_1 = 1$ to guaranty renormalizability.



Contributes to:

$$\vec{J}_{c7,1} = e\,\frac{i\,g_A^4}{8F_\pi^4}\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2 + M_\pi^2}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3 \int \frac{d^3l}{(2\pi)^3}\vec{l}\,\vec{l}\cdot\left[\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1]\right] \times \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{\omega_+^3\omega_-^3(\omega_+ + \omega_-)} - e\,\frac{i\,g_A^4}{8F_\pi^4}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2 + M_\pi^2}(\beta - 1) \times \vec{q}_2\cdot\vec{\sigma}_1 \int \frac{d^3l}{(2\pi)^3}\vec{l}\left(l^2 - k^2\right)(\vec{\sigma}_1\cdot\vec{q}_2)\,\frac{2\omega_+ + \omega_-}{\omega_+^3\omega_-(\omega_+ + \omega_-)^2} + (1 \leftrightarrow 2)\,,$$

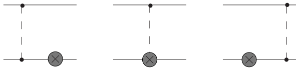
Where, for brevity, we set $\beta_4 = \beta_5 = \beta_6 = \beta$.

Determination of βs Ctd.

Divergent part reads (in dim. reg.)

$$\vec{J}_{\rm c7div} = -e\,\frac{i\,g_A^4}{32F_\pi^4\,\pi^2}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1]\right]\frac{1}{d-4}$$

$$+e\,\frac{5i\,g_A^4}{192F_\pi^4\,\pi^2}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}(\beta-1)\vec{k}\vec{q}_2\cdot\vec{\sigma}_1\,\frac{1}{d-4}+(1\leftrightarrow 2)\,,$$



- Left and right diagram do not contribute.
- The diagram in the middle contributes among other things the following LECs

$$\vec{J} \;=\; -2e\frac{i\,g_A}{F_\pi^2}\left(d_8\tau_2^3+d_9\,(\vec{\tau}_1\cdot\vec{\tau}_2)\right)\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{q}_2\times\vec{k}\right]-e\frac{i\,g_A}{4F_\pi^2}\,(2d_{21}+d_{22})\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1]\right]$$

$$+\,e\frac{i\,g_A\,d_{22}}{4F_\pi^2}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\,\left(\vec{\sigma}_2k^2-\vec{q}_2\vec{\sigma}_2\cdot\vec{k}\right)+(1\leftrightarrow 2)$$

with the β-functions

$$d_i \;=\; d_i^r(\mu)+\frac{\beta_i}{F_\pi^2}L\,,\quad l_i=l_i^r(\mu)+\gamma_i L$$

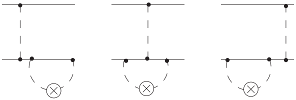
$$\beta_8 \;=\; \beta_9=\beta_{18}=\beta_{22}=0,\quad \beta_{16}=\frac{1}{2}g_A+g_A^3,\quad \beta_{21}=-g_A^3,\qquad \gamma_4=2,\quad \gamma_6=-\frac{1}{3},$$

$$L \;=\; \frac{\mu^{d-4}}{16\pi^2}\left[\frac{1}{d-4}-\frac{1}{2}\left(\Gamma'(1)+1+\log(4\pi)\right)\right].$$

We have to choose $\beta=1$!

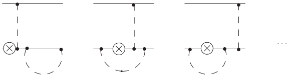
Determination of βs Ctd.

These diagrams have also a contribution to the charge density



For $\beta = 1$

$$\rho_{c7} = -e \frac{g_A^4}{8F_\pi^4} \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \tau_2^3 \int d^3 l \frac{1}{\omega_+^2 \omega_-^2} \left[\vec{\sigma}_1 \cdot \vec{l}\, \vec{l} \cdot \vec{q}_2 - \vec{\sigma}_1 \cdot \vec{k}\, \vec{k} \cdot \vec{\sigma}_2 \right] + (1 \leftrightarrow 2)\,.$$



- Omitted most of the diagrams for brevity (in the figure).
- S_2' and S_3' could potentially contribute, but do not.
- Diagrams are not divergent.

Contribution to the charge density:

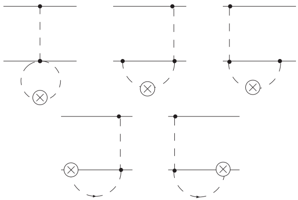
$$\rho_{c6} = e \frac{g_A^4}{4F_\pi^4} \tau_2^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \vec{\sigma}_1 \cdot \vec{q}_2 \frac{1}{3} \int \frac{d^3 l}{(2\pi)^3} \frac{l^2}{\omega_l^4} + (1 \leftrightarrow 2).$$

Partial Summary

By choosing $\beta_1 = \beta_4 = \beta_5 = \beta_6 = 1$ we can get rid of divergencies. β_2 and β_3 remain undetermined.

We still have to check that all divergencies cancel!

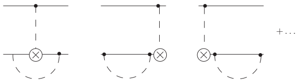
Additional loop contributions



- These diagrams cancel exactly!

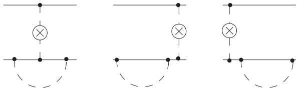
Class 1:

$$\vec{J}_{c1} \quad = \quad -e\,\frac{i\,g_A^2}{16F_\pi^4}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,\frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2}\,\vec{\sigma}_1 \int \frac{d^3 l}{(2\pi)^3}\,\frac{1}{\omega_l^3} + (1 \leftrightarrow 2)\,.$$



Class 2, (Omitted some non-vanishing diagrams in the figure):

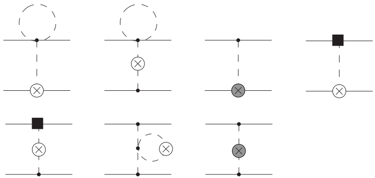
$$\vec{J}_{c2} \quad = \quad e\,\frac{i\,g_A^2}{2F_\pi^4}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,\frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2}\,\vec{\sigma}_1 \int \frac{d^3 l}{(2\pi)^3}\,\frac{1}{\omega_l^3} + (1 \leftrightarrow 2)\,.$$



Class 7:

$$\vec{J}_{c7,2} \quad = \quad e\,\frac{i\,g_A^4}{4F_\pi^4}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,\frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2}\,\frac{\vec{\sigma}_1 \cdot \vec{q}_1}{q_1^2 + M_\pi^2}\,(\vec{q}_2 - \vec{q}_1)\,\vec{\sigma}_1 \times \int \frac{d^3 l}{(2\pi)^3}\,\frac{1}{\omega_l^3} + (1 \leftrightarrow 2)\,.$$

Additional loop contributions Ctd



- However, all of the previous currents are canceled by these diagrams
- plus the contributions from δZ_π and $\delta(g_A/F_\pi)$.
- LECs d_{16} and l_4 disappear, only remaining term

$$
\begin{aligned}
\vec{J} \quad = \quad & e\,\frac{g_A^2\, i}{4F_\pi^2}\,[\vec\tau_1\times\vec\tau_2]^3 \Bigg\{ \frac{d_{18}}{g_A}\left[\frac{\vec\sigma_2\cdot\vec q_2}{q_2^2+M_\pi^2}\,\frac{\vec\sigma_1\cdot\vec q_1}{q_1^2+M_\pi^2}\,(\vec q_1-\vec q_2) - \frac{\vec\sigma_2\cdot\vec q_2}{q_2^2+M_\pi^2}\,\vec\sigma_1 + \frac{\vec\sigma_1\cdot\vec q_1}{q_1^2+M_\pi^2}\,\vec\sigma_2\right] \\
& - l_6\,\vec\sigma_1\cdot\vec q_1\,\vec\sigma_2\cdot\vec q_2\,(\vec q_1+\vec q_2)\left(\frac{1}{q_2^2+M_\pi^2}-\frac{1}{q_1^2+M_\pi^2}\right)+\frac{1}{2}\left(2k^2 l_6-\int\frac{d^3l}{(2\pi)^3}\,\frac{1}{\omega_l}\right)\,\frac{\vec\sigma_2\cdot\vec q_2}{q_2^2+M_\pi^2}\,\frac{\vec\sigma_1\cdot\vec q_1}{q_1^2+M_\pi^2}\,(\\
& - \frac{1}{4}\,\frac{\vec\sigma_2\cdot\vec q_2}{q_2^2+M_\pi^2}\,\frac{\vec\sigma_1\cdot\vec q_1}{q_1^2+M_\pi^2}\int\frac{d^3l}{(2\pi)^3}\,\vec l\;\vec l\cdot(\vec q_1-\vec q_2)\,\frac{1}{\omega_+\omega_-(\omega_++\omega_-)}\Bigg\} + (1\leftrightarrow 2)\,.
\end{aligned}
$$

The remaining divergence can be absorbed in l_6 with the known β-function!

Summary so far

Current density from LECs

$$\vec{J}_{\rm LEC} = -2e\frac{g_A\, i}{F_\pi^2}\left(d_8\tau_2^3 + d_9\left(\vec{\tau}_1\cdot\vec{\tau}_2\right)\right)\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{q}_2\times\vec{k}\right] - e\frac{g_A\, i}{4F_\pi^2}\left[\vec{\tau}_1\times\vec{\tau}_2\right]^3\Bigg\{\left(2\bar{d}_{21}+d_{22}\right)$$
$$\times\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{k}\times\left[\vec{q}_2\times\vec{\sigma}_1\right]\right] + d_{22}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left(\vec{\sigma}_2 k^2 - \vec{q}_2\vec{\sigma}_2\cdot\vec{k}\right)$$
$$+ d_{18}\left[\frac{1}{2}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\left(\vec{q}_1-\vec{q}_2\right) - \frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\vec{\sigma}_1\right]$$
$$- g_A\,\bar{l}_6\,\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_2\left(\vec{q}_1+\vec{q}_2\right)\frac{1}{q_2^2+M_\pi^2} + \frac{1}{2}k^2 g_A\,\bar{l}_6\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\left(\vec{q}_1-\vec{q}_2\right)\Bigg\} + (1\leftrightarrow 2)$$

Loop contribution to current density

$$\vec{J}_{\rm Loop} = -e\frac{i\,g_A^4}{64F_\pi^4\,\pi^2}\left[\vec{\tau}_1\times\vec{\tau}_2\right]^3\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{k}\times\left[\vec{q}_2\times\vec{\sigma}_1\right]\right]\left(2L(k)-1\right)$$
$$+ e\frac{i\,g_A^4}{1536F_\pi^4\,\pi^2}\left[\vec{\tau}_1\times\vec{\tau}_2\right]^3\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\left(\vec{q}_1-\vec{q}_2\right)\left[(4M_\pi^2+k^2)L(k) - \frac{1}{6}(5k^2+24M_\pi^2)\right]$$
$$+ e\frac{i\,g_A^4}{768F_\pi^4\,\pi^2 k^2}\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_2\cdot\vec{q}_2\left(\vec{q}_1+\vec{q}_2\right)\frac{1}{q_2^2+M_\pi^2}\left((4M_\pi^2+k^2)L(k) - \frac{1}{6}(5k^2+24M_\pi^2)\right) + (1\leftrightarrow 2).$$

Summary so far Ctd.

Loop contribution to the charge density

$$\rho_{\rm Loop} = e\frac{g_A^4}{64F_\pi^4\,\pi}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\Big[\tau_2^3\vec{\sigma}_1\cdot\vec{k}\,\vec{q}_2\cdot\vec{k}\left(2A(k)+\frac{M_\pi-(4M_\pi^2+k^2)A(k)}{k^2}\right) + \vec{q}_2\cdot\vec{\sigma}_1\big(M_\pi+(4M_\pi^2+k^2)A(k)-2M_\pi\big)\Big]+(1\leftrightarrow 2)\,,$$

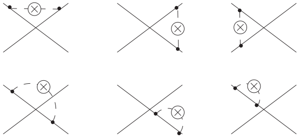
$$A(k) = \frac{1}{2k}\arctan\left(\frac{k}{2M_\pi}\right), \qquad L(k)=\frac{1}{2}\frac{4M_\pi^2+k^2}{k}\log\left(\frac{\sqrt{4M_\pi^2+k^2}+k}{\sqrt{4M_\pi^2+k^2}-k}\right).$$

One-Pion Exchange current

- All divergencies can be canceled by additional unitary transformations
- or by LECs with predetermined β-functions
- Contributions from LECs d_8, d_9, d_{18}, d_{21}, d_{22} and l_6
- Continuity equation is fulfilled.

One-Pion exchange with LO contact potential

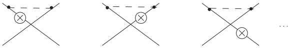
Diagrams with C_S and C_T



$$\vec{J} = e\frac{g_A^2\, i}{16F_\pi^2\,\pi^2} C_T\, \left(\tau_1^3 - \tau_2^3\right) \left[(\vec{\sigma}_1 - \vec{\sigma}_2)\times\vec{k}\right] (2L(k) - 1)\,,$$

$$\rho = -e\frac{g_A^2}{32F_\pi^4\,\pi} C_T\, \left(\tau_1^3 + \tau_2^3\right) \Big[\vec{\sigma}_1\cdot\vec{\sigma}_2 (3M_\pi + (3k^2 + 4M_\pi^2)A(k)) + \vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}\left(\frac{M_\pi - (4M_\pi^2 + k^2)A(k)}{k^2} + 2A(k)\right)\Big]\,.$$

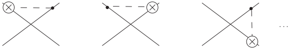
- Contributions from additional unitary transformations cancel!
- Divergent part can be absorbed in contact currents.



Many non-vanishing diagrams not shown here.

$$\rho = e\frac{g_A^2}{8F_\pi^4\,\pi} C_T\, \left(\tau_1^3 + \tau_2^3\right) \vec{\sigma}_1\cdot\vec{\sigma}_2 M_\pi\,.$$

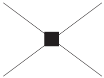
Only dependent on C_T.



$\longrightarrow$ These diagrams vanish.

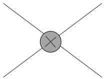
Contact currents

- Seven contact contributions to the potential.



$$
\begin{aligned}
V_{\text{contact}} &= C_1 q^2 + C_2 k'^2 + \left(C_3 q^2 + C_4 k'^2\right)(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + iC_5 \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{2} \cdot \left[\vec{k}\,' \times \vec{q}\right] \\
&\quad + C_6 (\vec{q} \cdot \vec{\sigma}_1)(\vec{q} \cdot \vec{\sigma}_2) + C_7 \left(\vec{k}\,' \cdot \vec{\sigma}_1\right)\left(\vec{k}\,' \cdot \vec{\sigma}_2\right), \\
\vec{k}\,' &= \frac{\vec{p} + \vec{p}\,'}{2}, \qquad \vec{q} = \vec{p}\,' - \vec{p}\,.
\end{aligned}
$$

- Via a gauge transformation and a Fierz-reshuffling, we obtain



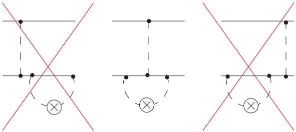
$$
\begin{aligned}
\vec{J}_{\text{contact}} &= +i\,(C_2 + 3C_4 + C_7)\,\frac{e}{16}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,(\vec{q}_1 - \vec{q}_2) \\
&\quad - i\,(-C_2 + C_4 + C_7)\,\frac{e}{16}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3\,(\vec{q}_1 - \vec{q}_2)\,\vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
&\quad - C_5\, i\,\frac{e}{16}\,(\tau_1^3 - \tau_2^3)\,[(\vec{\sigma}_1 + \vec{\sigma}_2) \times (\vec{q}_1 - \vec{q}_2)] \\
&\quad + iC_7 \frac{e}{16}\,[\vec{\tau}_1 \times \vec{\tau}_2]^3 \left[\vec{\sigma}_1 \quad \vec{\sigma}_2 \cdot (\vec{q}_1 - \vec{q}_2) + \vec{\sigma}_2 \quad \vec{\sigma}_1 \cdot (\vec{q}_1 - \vec{q}_2)\right].
\end{aligned}
$$

Plus two contact currants that cannot be obtained from gauge transformations

$$
\vec{J}_{\text{contact}} = -e\,i\,\tilde{C}_1 \left[(\vec{\sigma}_1 + \vec{\sigma}_2) \times \vec{k}\right] - e\,i\,\tilde{C}_2(\tau_1^3 - \tau_2^3)\left[(\vec{\sigma}_1 - \vec{\sigma}_2) \times \vec{k}\right].
$$

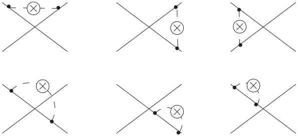
- We checked, that the divergencies of the two-loop current can be absorbed into the contact currents with the same renormalization of the C_i as in the case of the potential.
- $\tilde{C}_1$ can contribute to elastic *ed*-scattering, contribution to *d*-magnetic moment.
- $\tilde{C}_2$ from *d*-breakup reaction at threshold.

Comparison with Pastore et al.



- Pastore et al. (2009) do not take into account these two diagrams.
- In our formalism, however, the contribution from the middle diagram is exactly canceled from the left and right diagrams.
- $\Rightarrow$ Different isospin structure of the loop contributions!

- They do not comment on the LECs l_6 and d_{18}.
- The LEC d_{22} only appears in the combination $2d_{21} + d_{22}$.

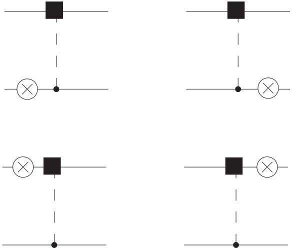


- Loops with C_S and C_T depend on C_S.
- This is similar to the situation with the potential.
- Again a different isospin structure!

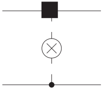
- They work with a different contact Lagrangian, however their results agree with ours.

$1/m_N$-corrections

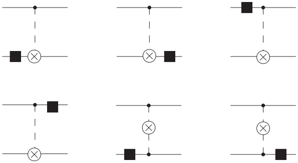
With $\vec{p}_{1/2}(\ \vec{p}^{\,\prime}_{1/2})$ the momentum of the incoming (outgoing) nucleon 1/2.



$$\rho = e\frac{g_A^2}{16F_\pi^2 m_N}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}(1-2\alpha_8)\vec{\sigma}_1\cdot\vec{k}\left[\tau_2^3+\vec{\tau}_1\cdot\vec{\tau}_2\right]$$
$$+\,e\frac{g_A^2\, i}{16F_\pi^2 m_N}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{\tau}_1\times\vec{\tau}_2\right]^3\left[\vec{\sigma}_2\cdot\left(\vec{p}_1+\vec{p}^{\,\prime}_1+\vec{p}_2+\vec{p}^{\,\prime}_2\right)\right.$$
$$\left.+\,2\alpha_8\vec{\sigma}_2\cdot\left(\vec{p}_1+\vec{p}^{\,\prime}_1-\vec{p}_2-\vec{p}^{\,\prime}_2\right)\right]+(1\leftrightarrow 2)$$



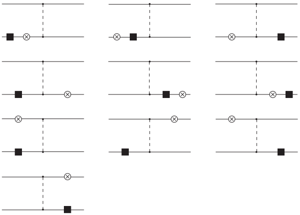
$$\rho = -e\frac{g_A^2\, i}{8F_\pi^2 m_N}\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\left[\vec{\tau}_1\times\vec{\tau}_2\right]^3\vec{\sigma}_2\cdot\left(\vec{p}_1+\vec{p}^{\,\prime}_1\right)+(1\leftrightarrow 2)$$



These diagrams with one insertion

of $\vec{p}^{\,2}/2m_N$ vanish!

$1/m_N$-corrections Ctd.

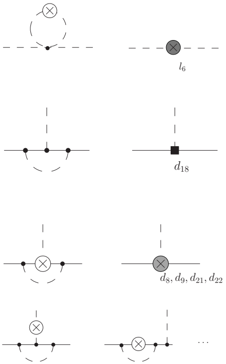


$$
\begin{aligned}
\rho \;=\; & -e\,\frac{g_A^2}{16F_\pi^2\,m_N}(1-2\alpha_7)(\tau_1^3+\vec{\tau}_1\cdot\vec{\tau}_2)\,(\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_2\cdot\vec{q}_2)\,\frac{\vec{q}_1\cdot\vec{k}}{(q_1^2+M_\pi^2)^2} \\
& + e\,\frac{g_A^2\,i}{16F_\pi^2\,m_N}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,(\vec{\sigma}_1\cdot\vec{q}_1\;\vec{\sigma}_2\cdot\vec{q}_2)\,\frac{1}{(q_1^2+M_\pi^2)^2} \\
& \times\Big[(1+2\alpha_7)\vec{q}_1\left(\vec{p}_1+\vec{p}_1{}'+\vec{p}_2+\vec{p}_2{}'\right) \\
& +(1-2\alpha_7)\vec{q}_1\left(\vec{p}_1+\vec{p}_1{}'+\vec{p}_2+\vec{p}_2{}'\right) \\
& -4\beta_2\left(\vec{q}_1\cdot\left(\vec{p}_1+\vec{p}_1{}'\right)+\vec{q}_2\cdot\left(\vec{p}_2+\vec{p}_2{}'\right)\right)\Big]+(1\leftrightarrow 2)\,.
\end{aligned}
$$

- $1/m_N$-corrections depend on three parameters α_7, α_8 and β_2.
- Give only rise to charge density terms.
- In Epelbaum, Glöckle, Meißner, '04 the convention $\alpha_7 = 1/4$ and $\alpha_8 = 0$ has been adopted.

Determining the LECs

- In NLO one-pion exchange current 6 LECs appear:



- l_6 is related to pion vector form factor $\longrightarrow$ well known.
- d_{18} is related to Goldberger-Treiman discrepancy $g_{\pi N}/m_N = g_A/F_\pi(1 - 2M_\pi^2/g_A d_{18}) \longrightarrow$ relatively well known.
- d_8, d_9, d_{21} and d_{22} are related to pion-photoproduction on one nucleon, poorly known.
- Calculate full pion-photoproduction amplitude and fix these constants to data!

- Pastore et. al estimate

$$d_9 = g_{\rho\pi\gamma} g_{\rho NN} \frac{F_\pi}{4 m_\rho^3} \qquad d_8 = g_{\omega\pi\gamma} g_{\omega NN} \frac{F_\pi}{4 m_\omega^3}$$

Conclusion and outlook

Conclusion

- We derived the full NLO em-current including two-pion exchange, one-pion exchange and contact terms.
- An explicit check of renormalizability of the one- and two-pion exchange contributions was performed.
- Expressions are given in momentum-space in terms of loop-functions $L(q)$, $A(q)$ and three-point functions.
- We analytically carried out the Fourier-transform to arrive at very compact expressions in configuration-space.
- The current fulfills the continuity-equation, i.e. is consistent with the potential.
- The two-pion exchange current corresponds to the result of Pastore et al.
- The one-loop current is different.

Outlook

- Calculation of pion-photoproduction off nucleons to determine LECs.
- Calculation of *ed*-scattering observables.
- Inclusion of Δ-degrees of freedom.
- Going to the sub-leading loop-order.