Nuclear Few-Body Physics with a "Pionless" Effective Field Theory

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1. JK, H. W. Grießhammer, D. Shukla, H. M. Hofmann, arXiv:0903.5538 [nucl-th]. Accepted by *EPJ* **A**.
2. JK, H. W. Grießhammer, D. Shukla, H. M. Hofmann, arXiv:0909.5606 [nucl-th]. Proceedings of Chiral Dynamics, Bern '09.
3. JK: *Bound state calculations of 3H and 4He with realistic three-nucleon interactions*, Diploma thesis.

(I) Universality in few-body-systems

- 2^{nd} ^{4}He bound state
  see H.-W. Hammer, L. Platter
  Eur. Phys. J. A **32**, 2007, 113-120.
- unbound ^{5}He
- peculiar Borromean & Halo structures in ^{6,8}He

(II) Alternate universes

- $f: \quad m_\pi \mapsto \mathrm{B}_{A\to ?}(m_\pi)$
  see *e.g.*, E. Epelbaum, H.-W. H., U.-G. M.
  Rev. Mod. Phys. **81**, 2009, 1773-1825.
- conditions for bound multi-neutron states
  see J.P. Kneller, G.C. McLaughlin
  Phys. Rev. D **70**, 2004, 043512.

(III) Low-energy nuclear reactions

- parity violating effects, *e.g.* in $N-\alpha$ scattering
  see D.R. Phillips, M.R. Schindler, R.P. Springer
  Nucl. Phys. A **822**, 2009, 1-19.
- BBN input crosssections
  e.g., ^{2}H, ^{7}Li abundance: d(d,n)^{3}He
  (error $\lesssim 2.0\%$)
  see K.M. Nollett and S. Burles
  Phys. Rev. D **61**, 2000, 123505.
- EFT$_{\not\pi}$ as fundamental interaction of cluster EFTs
  see *e.g.*, C.A. Bertulani, H.-W. Hammer, U. v Kolck,
  Nucl. Phys. A **712**, 2002, 37.

(Refined) Resonating Group Method

see e.g. H. M. Hofmann, proceedings of Models and Methods in Few-Body Physics, 1986.

- provides solution to the N-body stationary Schrödinger equation
- $E \gtrless 0$ *i.e.* scattering & bound state observables accessible
- applicable to bosonic & fermionic systems
- wave function approximated in Gaussian expansion, accurate up to r_{max}
- comparison with other numerical techniques necessary

The Effective Field Theory "without pions"

see e.g. P.F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 2002
L. Platter, FB Syst. **46**, 2009, 139-171.

- systematic approach to a system, parameterized by well separated scales
- universality $\propto$ renormalization scheme dependence
- error estimates
- interaction not over-parameterized
- significance of QCD parameters for nuclear properties

The **R**esonating **G**roup **M**ethod I

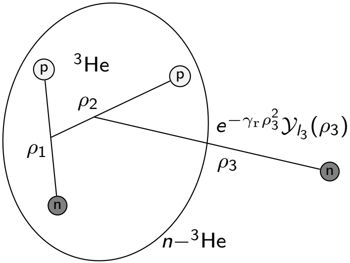
$$\hat{\mathrm{H}}\Psi = E\Psi$$

$$\Psi_l = \mathcal{A}\left\{\sum\nolimits_{\mathrm{k}} \phi_{\mathrm{ch}}^{k}\phi_{\mathrm{rel}}^{lk}\right\}$$

boundary condition

$\overset{r\to\infty}{\curvearrowright}$ Coulomb wave function

inspired by cluster decomposition



$$\langle\vec{r}|(n-p)\rangle = \sum_{a,d}\left\{c_a\left[|S=1\rangle e^{-\beta_a r^2}\mathcal{Y}_0(\vec{r})\right]^{J=1} + c_d\left[|S=1\rangle e^{-\beta_d r^2}\mathcal{Y}_2(\vec{r})\right]^{J=1}\right\}$$

Ritz variation ⇒ bound states

Kohn-Hulthén variation ⇒ S-matrix

- versatile method
  bound-, scattering-, capture-, E&M-, *etc.* calculations,
  no principal limitations on A, (anti) symmertrized states,
  (non-) local interactions, full treatment of Coulomb interaction
- analytic evaluation of many-body matrix elements
- implementation is not "communicative"
  ↷ perfect scaling allows efficient,
  large scale **parallel** computations

John Wheeler's idea:

[...] It was as if, at a party, all the tall people clustered together at one moment, with all the short people in another cluster; then at the next moment [...] four groups formed, consisting of guests from the north, east, west, and south parts of the city; and so on, [...]

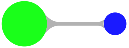
The **R**esonating **G**roup **M**ethod II

H. M. Hofmann

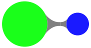
Proceedings of Models and Methods in Few-Body Physics, 1986.

Ritz variation: $\delta\left(\langle\psi|\hat{\mathrm{H}}-E|\psi\rangle\right)=0$

Gaussian parameterization of radial functions $\Rightarrow$ analytic evaluation of integrals



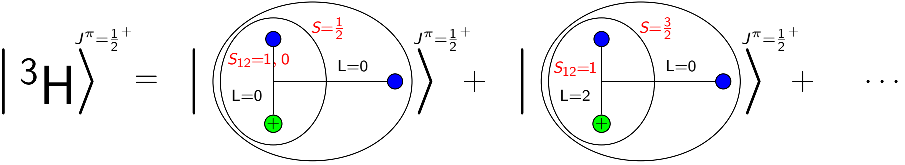
width parameters



$$\psi^{J^\pi}_{\mathrm{BS}}\left(\vec{\rho}_m,\vec{s}_m\right)=\mathcal{A}\left\{\sum_{d,i,j} c_{dij}\left[\left[\prod_{k=1}^{N-1} e^{-\gamma_{dk}\vec{\rho}_k^{\,2}}\mathcal{Y}_{l_{ki}}\left(\vec{\rho}_k\right)\right]^{L_i}\otimes\Xi^{S_j}\right]^{J}\cdot\Upsilon\right\}$$

antisymmetrizer

Jacobi coordinates

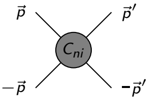


[...] "That's the idea, but I need a name for it. How about '<u>resonating group structure</u>'?"

So that was it. A little cumbersome, but I couldn't think of a better name.

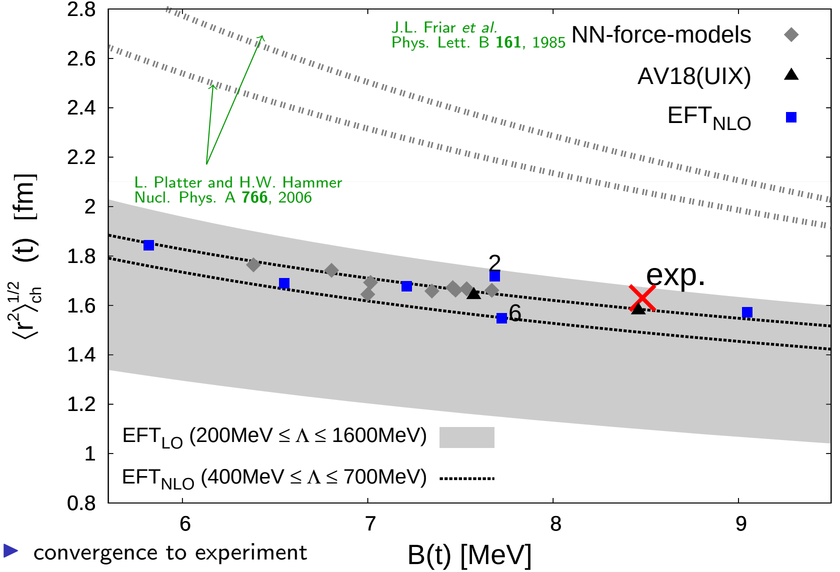
EFT($\not\pi$) NN potential & LEC determination

$$
\begin{aligned}
V_{\not\pi}^{\mathrm{NLO}}(\vec{r}) \;=\;& \overbrace{l_0(r)\left(A_1 + A_2\vec{\sigma}_1\cdot\vec{\sigma}_2\right)}^{\boxed{\text{LO-NN}}} + \left(A_3 + A_4\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2},\vec{\nabla}^2\right\} \;\boxed{\text{NLO}} \;+ \\
& l_0(r)\left(A_5 + A_6\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\vec{r}^2 + l_0(r)A_7\vec{L}\cdot\vec{S} + l_0(r)A_8\left[\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r} - \frac{1}{3}\vec{r}^2\vec{\sigma}_1\cdot\vec{\sigma}_2\right] \\
& -A_9\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2},\left[\left[\partial^r\otimes\partial^s\right]^2\otimes\left[\sigma_1^p\otimes\sigma_2^q\right]^2\right]^{00}\right\} + \overbrace{l_0(r_{12})\,l_0(r_{23})\,A_{3NF}\,\vec{\tau}_1\cdot\vec{\tau}_2}^{\boxed{\text{LO-3NF}}} \;\boxed{\text{NLO}^*}
\end{aligned}
$$



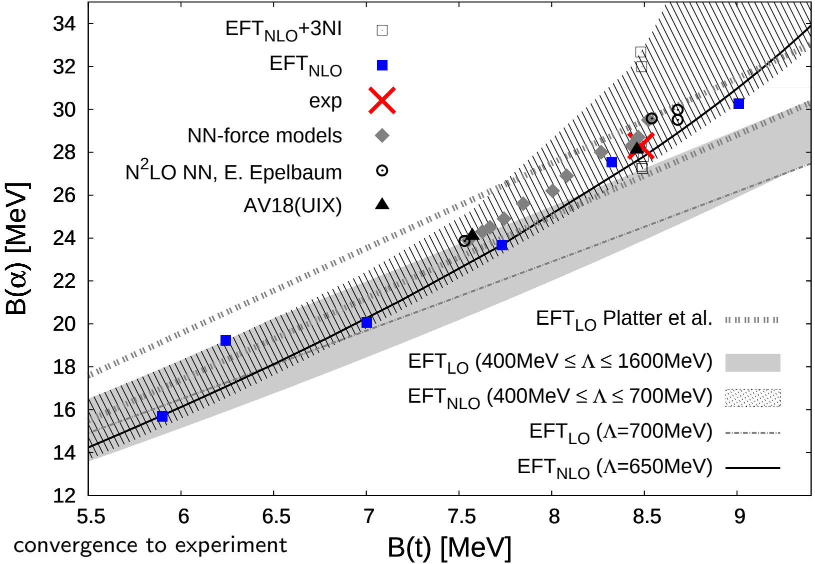
- Gaussian regulator functions $l_0(r,\Lambda) \propto e^{-\frac{\Lambda^2}{4}\vec{r}^2}$
- low-energy-constants depend on cutoff, $A_i = A_i(\Lambda)$

- low-energy input data: $B_d,\ \delta\left({}^1S_0\,,\ {}^3S_1^{1,3}\right),\epsilon_1$ for $E_{\mathrm{cm}} < 1$ MeV
  multi-dimensional fit with a **genetic algorithm**
- two methods to obtain different LEC sets $\leftrightarrow$ different short-range-physics:
  - variation of **cutoff** parameter Λ
  - variation of **low-energy input**

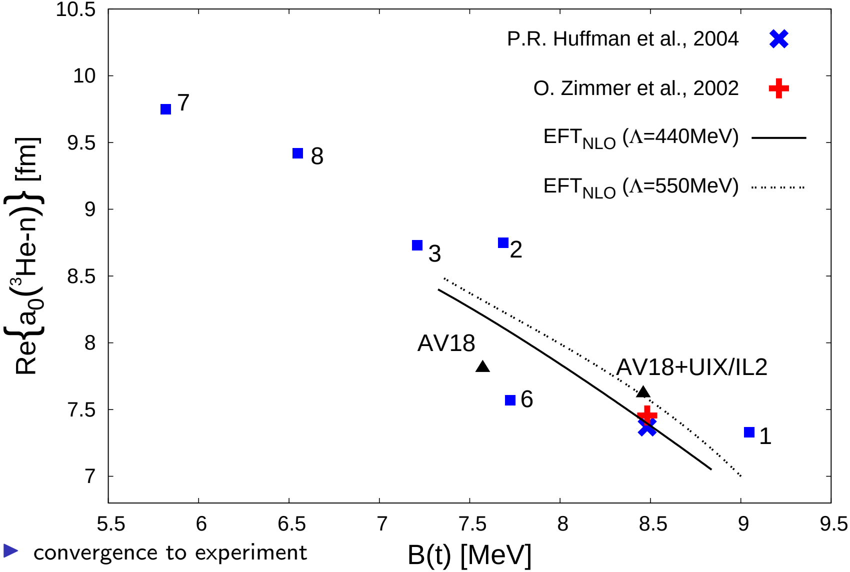


- convergence to experiment
- LO → NLO ⇒ $Q \approx \frac{1}{3}$
- NLO band width ⇒ $Q \approx \frac{1}{3}$

- one three-body parameter needed for a result independent of short distance physics
- EFT$_{\not\pi}$ works at NLO

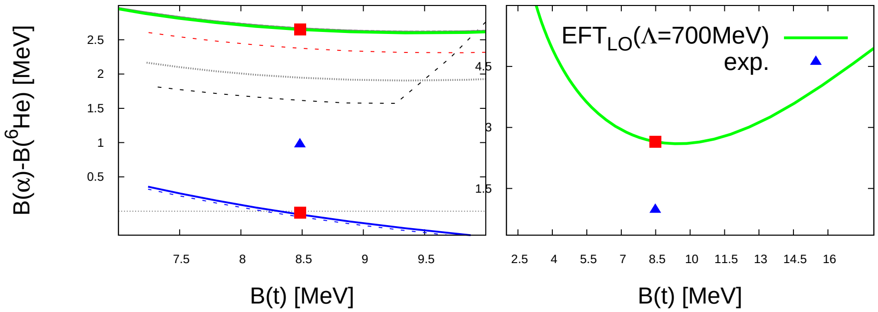


- convergence to experiment
- NLO band width $\Rightarrow \quad Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO
- one three-body parameter needed for a result independent of short distance physics
- EFT$_{\not\pi}$ still works at NLO



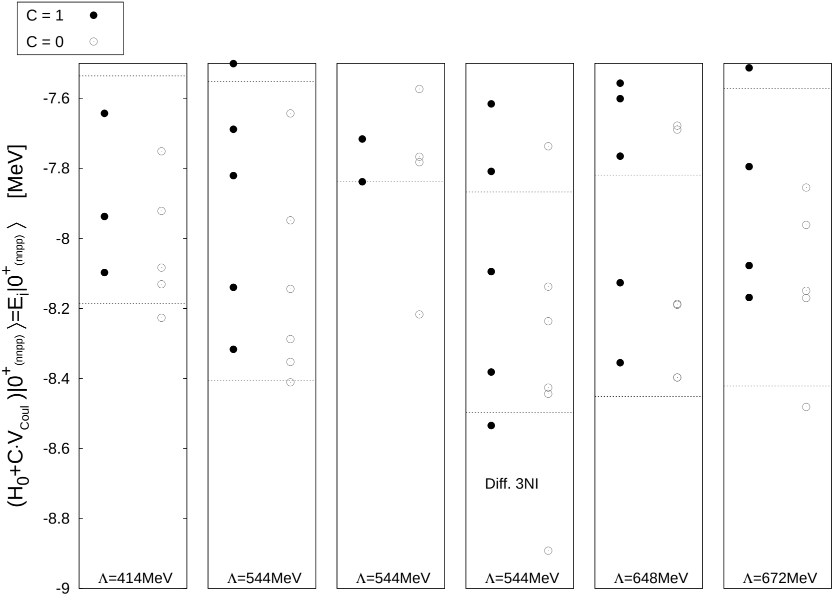
- convergence to experiment
- NLO band width $\Rightarrow \quad Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO

- calculation **not accurate enough** to discriminate between conflicting data
- EFT$_{\not\pi}$ still^2 works at NLO



	EFT$_{\not\pi}$ (700 MeV)	exp.
$\langle \vec{r}^2 \rangle_{ch}^{1/2}$ $\left(^6\mathrm{He}\right)$	3.261 fm	2.054 fm
$\langle \vec{r}^2 \rangle_{m}^{1/2}$ $\left(^6\mathrm{He}\right)$	5.678 fm	2.30 fm

- halo structure at "real" triton
- cutoff & 3NF strength variation exhibit same qualitative effect
- neutron Halo structure still intact at less/more bound "tritons"?


C = 1
C = 0
$(H_0+C\cdot V_{Coul})|0^+_{(nnpp)}\rangle = E_i|0^+_{(nnpp)}\rangle$ [MeV]
-7.6
-7.8
-8
-8.2
-8.4
-8.6
-8.8
-9
Λ=414MeV
Λ=544MeV
Λ=544MeV
Diff. 3NI
Λ=544MeV
Λ=648MeV
Λ=672MeV

Conclusions

- **first** calculations with EFT($\not\pi$) at NLO in $A > 3$ systems
  $\curvearrowright$ evidence for **applicability** the ^{4}He-channel
  - **convergence** from LO to NLO with $Q \approx \frac{1}{3}$
  - new correlation for $a_0\left({}^3\text{He} - n\right)$
  - consistency with high precision models & experiment
  - **four-nucleon** contact interaction **not** necessary at NLO
- $A = 6$ systems accessible with RRGM
- Halo structure of ^{6}He not a universal property of the NN-force?

Outlook

- exotic Fermion systems (Halos, )
- BBN low-energy cross sections
- novel numerical hybrid technique
- PV in ^{4}He-neutron scattering
- univ. in bosonic and atomic systems

$\Longrightarrow$

- understanding of nuclear properties from underlying theory
- criteria for emergent/universal and distinctive properties