EFT Approaches to $\alpha\alpha$ and $N\alpha$ Systems

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Outline

- Motivation
- EFT approach and universality
- halo/cluster EFT
  - ⋆ Coulomb interactions
  - ⋆ $\alpha\alpha$ scattering
  - ⋆ $N\alpha$ scattering
- Summary and outlook

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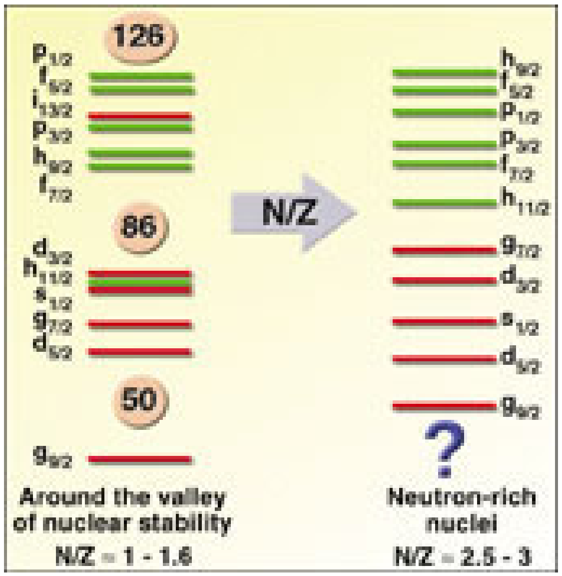
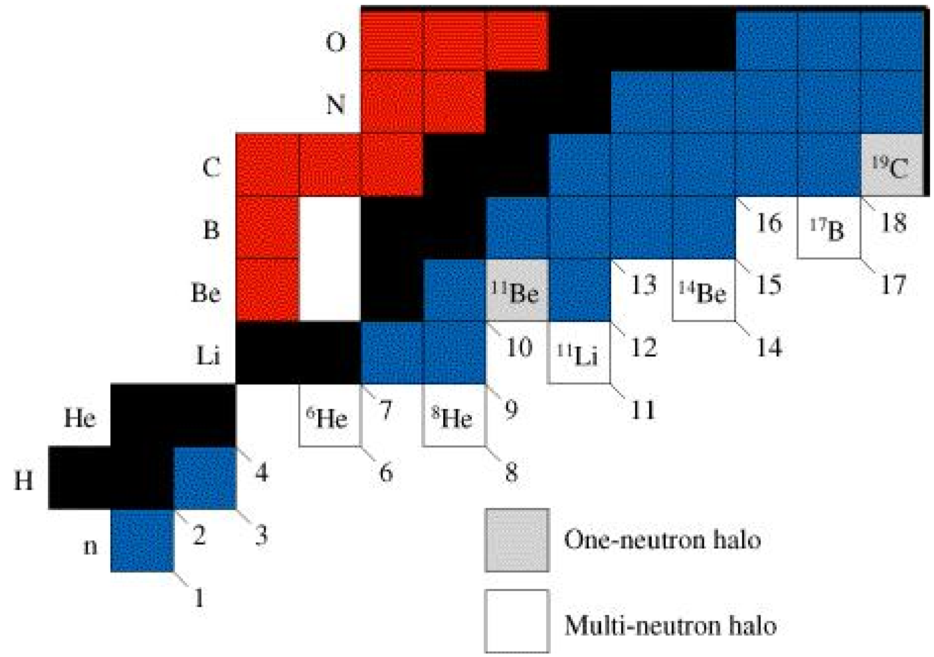


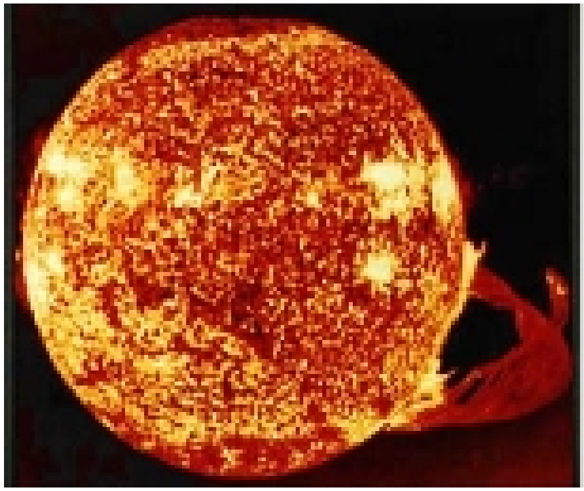
http://fy.chalmers.se/subatom/halo/halo.html



http://www.ornl.gov/info/ornlreview/v34_2_01/search.htm

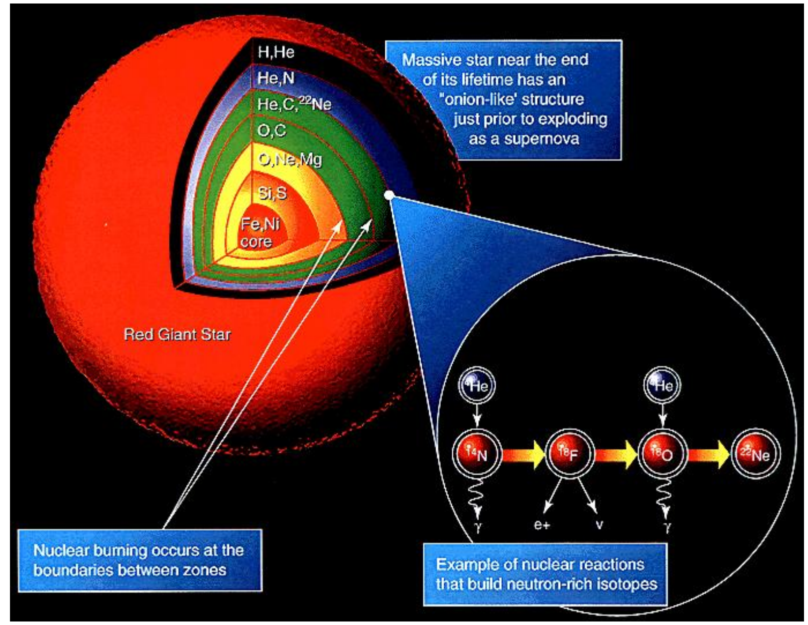
- $N/Z \gg 1$: challenge for shell model
- few nucleon systems: formation of halo systems

⋆ ^{11}Be , ^{19}C, ^{11}Li , ^{6}He , ^{14}Be, ^{8}He, ^{8}B, ^{17}Ne, ...



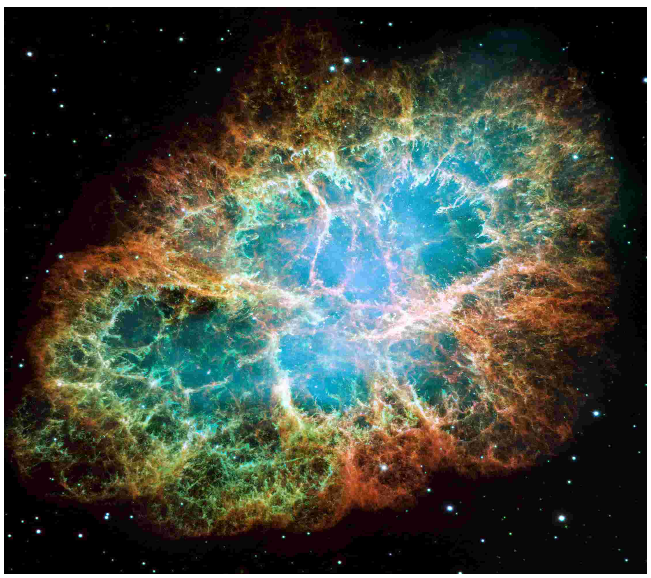
$\mathrm{p} + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$

solar neutrinos



${}^{8}\mathrm{Be} + \alpha \rightarrow {}^{12}\mathrm{C} + \gamma$

red giants



${}^{9}\mathrm{Be}(\frac{1}{2}^{+}) + \alpha \rightarrow {}^{12}\mathrm{C} + \mathrm{n}$

core-colapse supernovae

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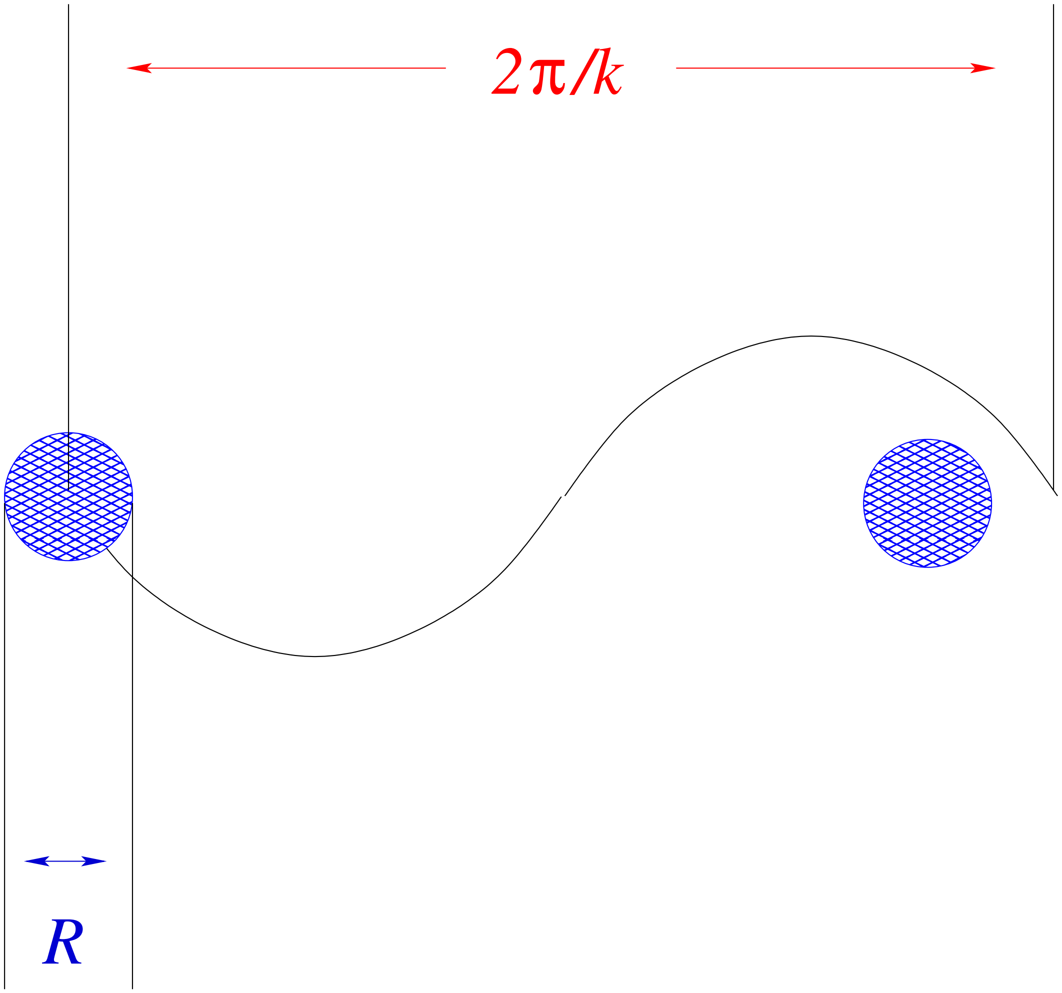
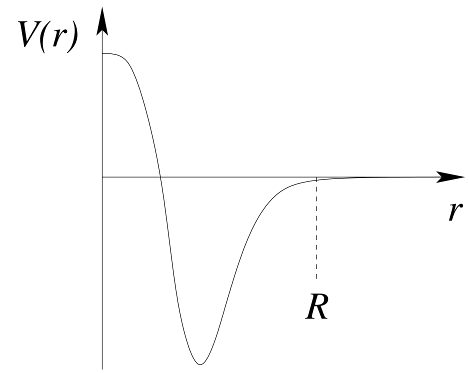
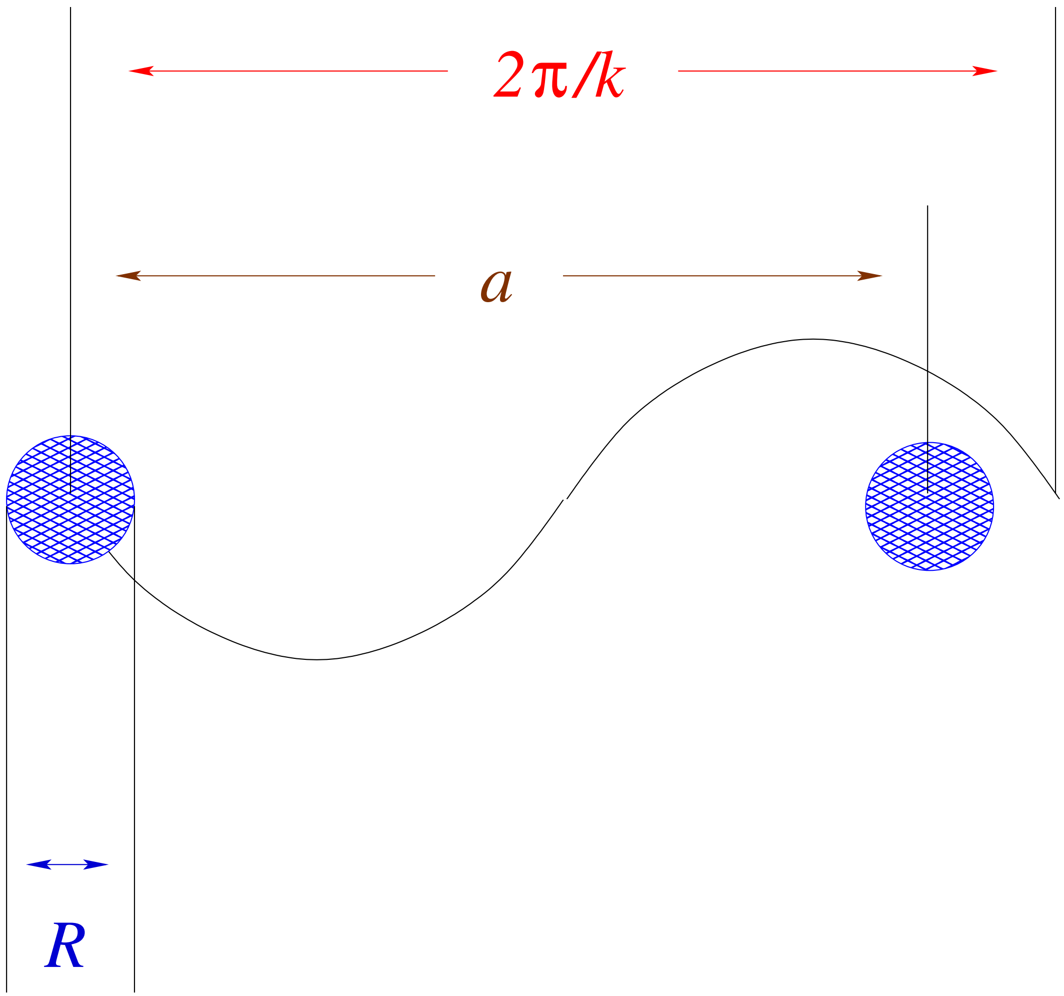
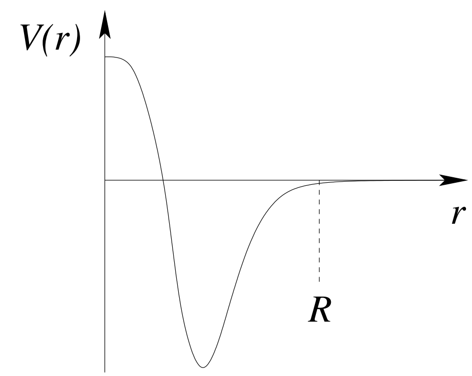
naturalness: where NDA works





strongly interacting systems: fine-tuning

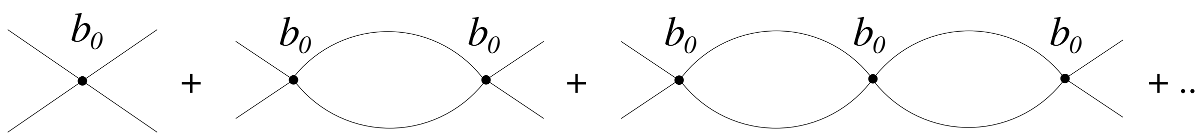




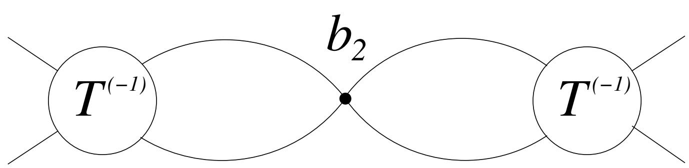
Effective Field Theory

$k, 1/a \sim M_{lo}, \qquad 1/R \sim M_{hi}$

$$\mathcal{L} = \phi^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m} \right] \phi - b_0 \, (\phi\phi)^\dagger (\phi\phi) + \frac{b_2}{8} \left[(\phi\phi)^\dagger \phi (\overleftrightarrow{\boldsymbol{\nabla}})^2 \phi + \text{H.c.} \right] + \cdots$$
$$- c_0 \, (\phi\phi\phi)^\dagger (\phi\phi\phi) + \cdots ,$$

$iT^{(-1)} =$ 

$$= -i \frac{b_0(\Lambda)}{1 + (\Lambda + ik)\frac{m}{4\pi} b_0(\Lambda)}$$

$iT^{(0)} =$ 

$$= -i \frac{b_2(\Lambda)}{\left[1 + (\Lambda + ik)\frac{m}{4\pi} b_0(\Lambda)\right]^2}$$

Pros

- symmetries
- simplicity
- able to handle non-local interactions
- W/EM interactions
- 3-4B extensions
- controlled and systematic low-energy expansion

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Cons

- controlled and systematic low-energy expansion
- complexity for $>$4B (Kirscher *et al.*)
- ...

Universality in two-body systems

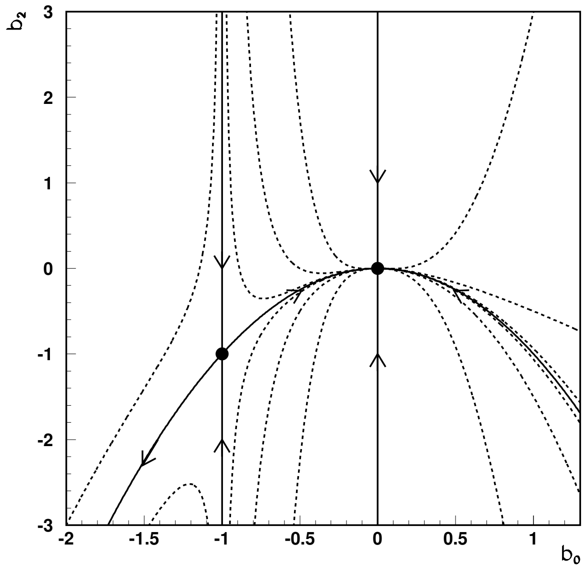
a is the only relevant scale at LO

$$f(\theta) = \frac{1}{-1/a - ik}, \quad E_{B,V} = \frac{1}{ma^2}, \quad \frac{d\sigma}{d\Omega} = \frac{4a^2}{1 + a^2k^2} \tag{1}$$

- $a \longrightarrow \infty$: scale-invariant system
  $\Rightarrow$ BS at threshold, $d\sigma/d\Omega$ saturates the UB
- RG analysis: non-trivial IR fixed point

  (Birse *et al.*, Phys. Lett. B 464, 169)
- close analogy to critical phenomena

  (liquid-gas phase transition, ferromagnets)



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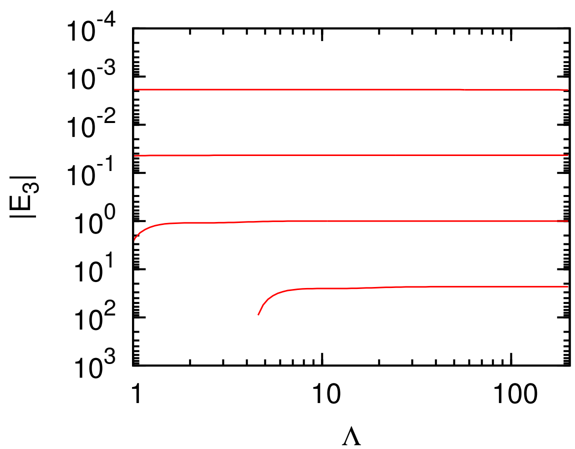
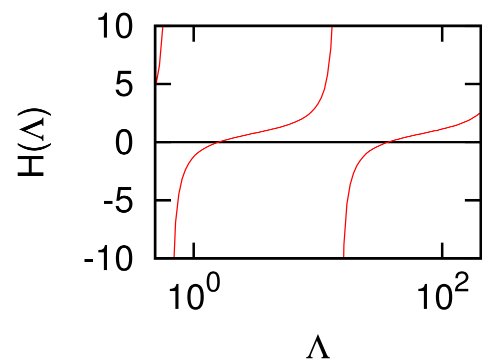
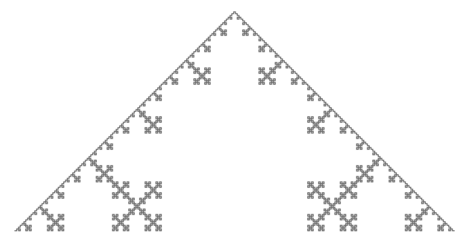
Universality in three-body systems (Braaten and Hammer, Phys. Rept. 428, 259)





(Hammer and RH, Eur. J. Phys. A 37, 193)

- renormalization requires c_0 at LO
  $\Rightarrow$ limit cycle
- $E^{(n)}/E^{(n+1)} \longrightarrow$ const.
  ($\sim$ 515 for bosons)



halo/cluster EFT: separation of scales

- excitation of each cluster $\sqrt{m_c E_c^*} \sim M_{hi}$ $(\gtrsim m_\pi)$
- binding of the valence nucleons (clusters) $\sim M_{lo} \ll M_{hi}$
- extension of the core—treated in *perturbation theory*
- power-counting: modified to account for other effects (resonance/Coulomb)
- expansion around the resonance: rearrangement of the perturbative series, improved convergence
- Coulomb interactions

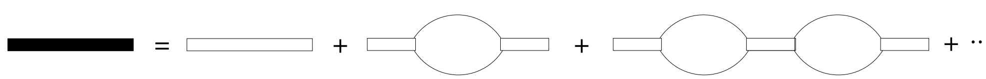
halo/cluster EFT: $k \ll m_\pi, \sqrt{m_c E_c^*} \sim M_{hi}$

Physical quantities: $k, 1/a_0 \sim M_{lo}, \qquad r_0 \sim M_{hi}^{-1}, \mathcal{P} \sim M_{hi}^{-3}, \ldots$

$$T_l = -\frac{2\pi}{\mu} \frac{k^{2l}(2l+1)}{k^{2l+1}(\cot\delta_l - i)} P_l(\cos\theta)$$

$$k^{2l+1}\cot\delta_l \approx -1/a_l + \frac{r_l}{2}k^2 + \frac{\mathcal{P}_l}{4}k^4 + \cdots$$

$$\mathcal{L} = \phi^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{4\mu} \right] \phi + \sigma\, d^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{8\mu} - \Delta \right] d + g \left[d^\dagger \phi\phi + (\phi\phi)^\dagger d \right] + \cdots,$$

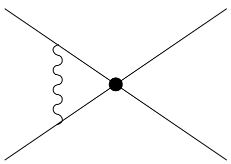


$$\Delta \sim M_{lo} \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{-\Delta + i\epsilon} \sim \frac{1}{M_{lo}} \qquad (NN)$$

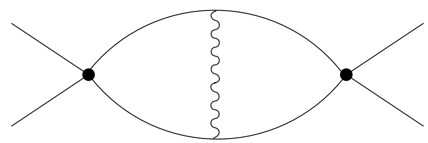
$$\Delta \sim M_{lo}^2/\mu \quad \rightarrow \quad iD_d^{(0)} = \frac{i\sigma}{q_0 - \boldsymbol{q}^2/8\mu - \Delta + i\epsilon} \sim \frac{\mu}{M_{lo}^2} \qquad (\alpha\alpha)$$

Coulomb photons dominant at very low energies

$$k_C = Z_1 Z_2 \alpha_{em} \mu$$



$$\sim \frac{k_C}{k} \equiv \eta \,,$$



$$\sim k_C m \left[\frac{1}{2} \ln \frac{\Lambda}{2k} + \cdots \right]$$

For $\alpha\alpha$: $k_C = Z_\alpha^2 \alpha_{em} m_\alpha / 2 \sim M_{hi}$

- non-perturbative Coulomb (Kong and Ravndal, NPA 665, 137)

Coulomb wave functions: $|\boldsymbol{k}\rangle \to |\chi_k^{(\pm)}\rangle$

$$T \to T_C + T_{CS}$$

$$\eta = Z^2 \alpha_{em} \mu / k = k_C / k$$

$$\sigma_l = \arg \Gamma(1 + l + i\eta)\,, \qquad C_\eta^{(0)\,2} = e^{-\pi\eta} \Gamma(1 + i\eta) \Gamma(1 - i\eta)$$

$$\langle \boldsymbol{r} | \chi_k^{(\pm)} \rangle \equiv \chi_k^{(\pm)}(\boldsymbol{r}) = e^{-\frac{\eta\pi}{2}}\, \Gamma(1 \pm i\eta)\, M(\mp i\eta, 1; \pm ikr - i\boldsymbol{k}\cdot\boldsymbol{r})\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$f_l \sim \sin(kr - \frac{l\pi}{2} - \eta \ln 2kr + \sigma_l)\,, \qquad g_l \sim \cos(kr - \frac{l\pi}{2} - \eta \ln 2kr + \sigma_l)$$

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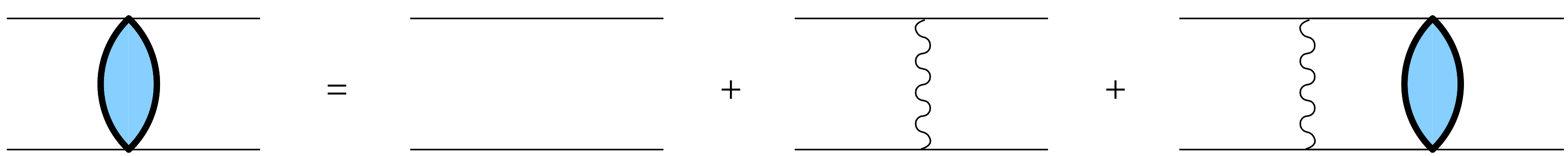
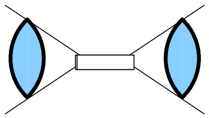
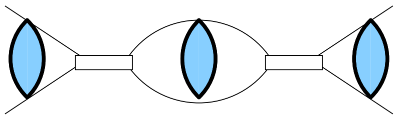
$$\sigma_l = \arg \Gamma(1 + l + i\eta)\,, \qquad C_\eta^{(0)\,2} = e^{-\pi\eta} \Gamma(1 + i\eta) \Gamma(1 - i\eta)$$

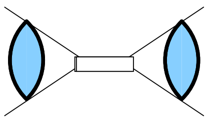
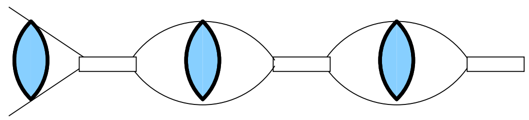
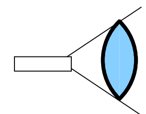
$$G_C^{(\pm)}(E) = \frac{1}{E - \hat{H}_0 - \hat{V}_C \pm i\epsilon} = 2\mu \int \frac{d^3 q}{(2\pi)^3} \frac{|\chi_q^{(\pm)}\rangle\langle\chi_q^{(\pm)}|}{2\mu E - \boldsymbol{q}^2 \pm i\epsilon}$$



$$T_{CS} = \langle \chi_{k'}^{-} | \, \hat{V}_S \, | \chi_k^{+} \rangle + \langle \chi_{k'}^{-} | \, \hat{V}_S \, G_C^{+} \, \hat{V}_S \, | \chi_k^{+} \rangle + \cdots$$

$T_{CS}^{(0)} =$  $= C(E)\, \chi_{k'}^{(-)\,*}(0)\, \chi_k^{(+)}(0) = C(E)\, C_\eta^{(0)\,2}\, e^{2i\sigma_0}\,,$

$T_{CS}^{(1)} =$  $= C(E)\, C_\eta^{(0)\,2}\, e^{2i\sigma_0}\, C(E)\, J_0(E)\,,$

$T_{CS} =$  $+ \cdots +$  ... 

$$= C_\eta^{(0)\,2}\, \frac{C(E)\, e^{2i\sigma_0}}{1 - C(E)\, J_0(E)}\,,$$

$$J_0(E) = 2\mu \int \frac{d^3q}{(2\pi)^3}\, \frac{\chi_q^{(+)}(0)\chi_q^{(+)\,*}(0)}{k^2 - \boldsymbol{q}^2 \pm i\epsilon} = 2\mu \int \frac{d^3q}{(2\pi)^3}\, \frac{2\pi\eta_q}{e^{2\pi\eta_q} - 1}\, \frac{1}{k^2 - q^2 + i\epsilon}$$

$(\eta_q = k_C/q)$

$\alpha\alpha$ scattering

- 0+ resonance (^{8}Be g.s.):

  $E_R^{\mathrm{LAB}} = 184.15 \pm 0.07$ keV , $\Gamma_R^{\mathrm{LAB}} = 11.14 \pm 0.50$ eV

  $M_{lo} \approx \sqrt{\mu E_R^{\mathrm{LAB}}} \sim 20$ MeV, $\quad M_{hi} \sim m_\pi \sim 140$ MeV

- power-counting: $E_{LAB} \leq 3.0$ MeV

- scattering: Afzal *et.al.* (1969)

  ⋆ $E_{LAB} \leq 3.0$ MeV : data from Heydenburg and Temmer (1956)

  ⋆ ERE parameters from Russell *et.al.* (1956), Rasche (1967):

  $a_0 = (-1.65 \pm 0.17) \times 10^3$ fm,

  $r_0 = 1.084 \pm 0.011$ fm $\sim 1/M_{hi}$, $\quad \mathcal{P}_0 = -1.76 \pm 0.22$ fm^3 $\sim 1/M_{hi}^3$

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$$T_{CS} = C_\eta^{(0)\,2} \frac{C(E)\, e^{2i\sigma_0}}{1 - C(E)\, J_0(E)} = -\frac{2\pi}{\mu} \frac{C_\eta^{(0)\,2}\, e^{2i\sigma_0}}{-\frac{1}{a_0} + \frac{r_0}{2} k^2 - i\epsilon + \frac{2\pi}{\mu} J_0(E)}$$

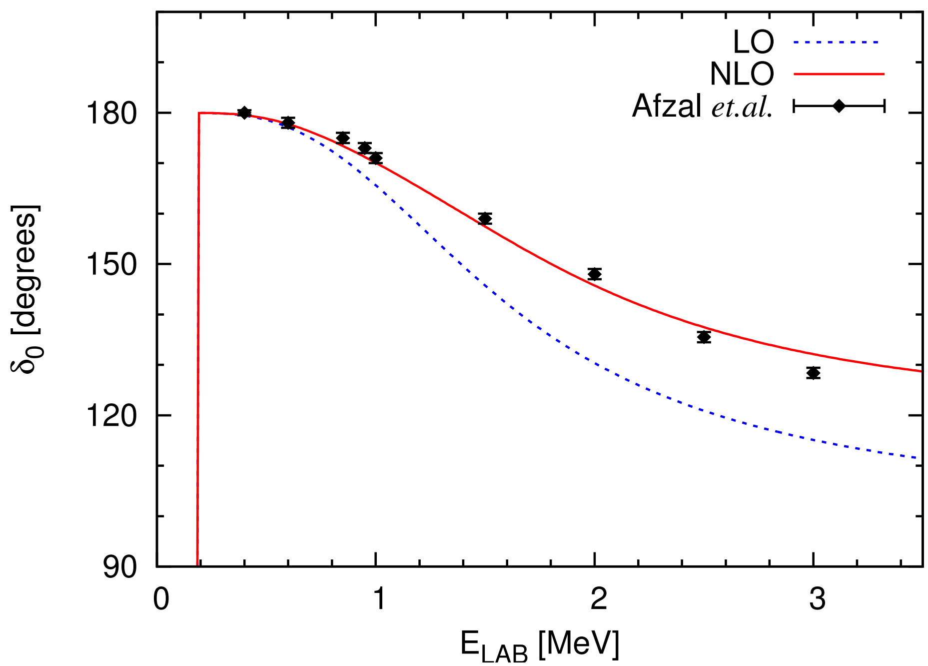
$$= -\frac{2\pi}{\mu} \frac{C_\eta^{(0)\,2}\, e^{2i\sigma_0}}{-\frac{1}{a_0^c} + \frac{r_0}{2} k^2 - \frac{2}{a_B} H(\eta)},$$

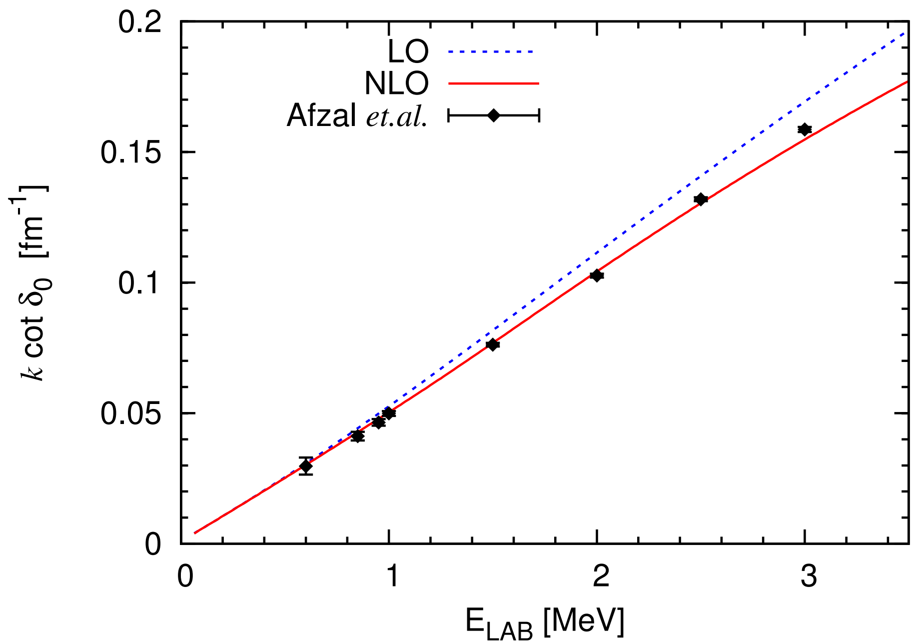
$$a_B = \frac{1}{Z^2 \alpha_{em} \mu} \sim \frac{1}{M_{hi}}$$

$$H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \ln(i\eta) \Rightarrow \begin{cases} \stackrel{\eta \ll 1}{\longrightarrow} \frac{a_B}{2}\, ik \\ \stackrel{\eta \gg 1}{\longrightarrow} \frac{1}{12} (a_B\, k)^2 + \frac{1}{120} (a_B\, k)^4 \end{cases}$$

- without Coulomb: conformal invariance in ^{8}Be , Efimov state in ^{12}C at LO (RH, Hammer, van Kolck, Nucl. Phys. A 809, 171)
- with Coulomb: ^{8}Be and ^{12}C 0+ states remain close to threshold

(RH, Hammer, van Kolck, 2008)





	a_0 (10^3 fm)	r_0 (fm)	$\mathcal{P}_0$ (fm^3)
LO	-1.80	1.083	—
NLO	-1.92 ± 0.09	1.098 ± 0.005	-1.46 ± 0.08
Rasche	-1.65 ± 0.17	1.084 ± 0.011	-1.76 ± 0.22

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fine-tuning puzzle

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}^2}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(natural)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{hi}M_{lo}}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned like } NN\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^2}{\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned to get } E_R\text{)}$$

$$\underbrace{\Delta^{(\mathrm{R})}}_{\frac{M_{lo}^3}{M_{hi}\mu}} = \underbrace{\Delta(\kappa)}_{\frac{M_{hi}^2}{\mu}} - \underbrace{\Delta^{(\mathrm{loops})}}_{\frac{M_{hi}^2}{\mu}} \qquad \text{(fine-tuned to get } \Gamma_R\text{)}$$

$\sim$ factor of 1000!!!

(Oberhummer *et al.*, Science 289, 88; RH, Hammer, van Kolck, 2008)

$p\alpha$ scattering: $S_{1/2}$, $P_{3/2}$, $P_{1/2}$

$$\mathcal{L}_{\mathrm{LO}} = \phi^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2m_\alpha}\right]\phi+N^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2m_N}\right]N$$

$$+\eta_{1+}\,t^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2(m_\alpha+m_N)}-\Delta_{1+}\right]t$$

$$+\frac{g_{1+}}{2}\left\{t^\dagger\vec{S}^\dagger\cdot\left[N\vec{\nabla}\phi-(\vec{\nabla}N)\phi\right]+\text{H.c.}-r\left[t^\dagger\vec{S}^\dagger\cdot\vec{\nabla}(N\phi)+\text{H.c.}\right]\right\}$$

$$\mathcal{L}_{\mathrm{NLO}} = \eta_{0+}\,s^\dagger\left[-\Delta_{0+}\right]s+g_{0+}\left[s^\dagger N\phi+\phi^\dagger N^\dagger s\right]+g'_{1+}\,t^\dagger\left[i\partial_0+\frac{\vec{\nabla}^2}{2(m_\alpha+m_N)}\right]^2 t$$

(Bertulani, Hammer, van Kolck, NPA 712, 37;

Bedaque, Hammer, van Kolck, PLB 569, 159)

P-wave: $k^3 \cot\delta_1 = -1/a_1 + r_1/2\, k^2 + \mathcal{P}_1/4\, k^4 + \cdots$

$$\langle \boldsymbol{p}'|\, \hat{V}_S\, |\boldsymbol{p}\rangle = \frac{\eta_{1+}\, g_{1+}^2\, (S_j S_k^\dagger)_{\beta\alpha}}{q_0 - \boldsymbol{q}^2/2(m_\phi + m_N) - \Delta_{1+} + i\varepsilon}\, p'_j\, p_k$$

Amplitude:

$$T_{CS}^{\rm LO} = -C_\eta^{(1)\,2}\, e^{2i\sigma_1}\, \frac{2\pi}{\mu}\, \frac{k^2\Big(2\cos\theta + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\sin\theta\Big)}{-\frac{1}{a_{1+}} + \frac{r_{1+}}{2}k^2 + i\epsilon + \frac{6\pi}{\mu} J_1(E)}$$

$$= -\frac{2\pi}{\mu}\left[F(k,\theta) + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{n}}\, G(k,\theta)\right]$$

$$J_1 = -\frac{\mu}{3\pi a_B}\left(k^2 + \frac{1}{a_B^2}\right)\left[H(\eta) + \frac{1}{D-4} - \ln\left(\frac{a_B\kappa\sqrt{\pi}}{2}\right) - \frac{4}{3} + \frac{3}{2}C_E + \frac{3\kappa a_B}{4}\right]$$
$$-\frac{4\mu\pi}{3a_B^3}\zeta'(-2) - \frac{\mu\pi\kappa}{12 a_B^2} + \frac{\mu\kappa^2}{4a_B} - \frac{\mu\kappa^3}{12}$$

$p\alpha$ scattering

- $T_{\mathrm{LO}} = T_{\mathrm{LO},S_{1/2}} + T_{\mathrm{LO},P_{3/2}}$
- $T_{\mathrm{NLO}} = T_{\mathrm{NLO},S_{1/2}} + T_{\mathrm{NLO},P_{3/2}}$
- $T_{\mathrm{N^2LO}} = T_{\mathrm{N^2LO},S_{1/2}} + T_{\mathrm{N^2LO},P_{3/2}} + T_{\mathrm{LO},P_{1/2}}$

$$F_C(k,\theta) = -\frac{\eta}{2k}\,\csc^2\theta/2\,\exp\left[i\eta\ln(\csc^2\theta/2) + 2i\sigma_0\right]$$

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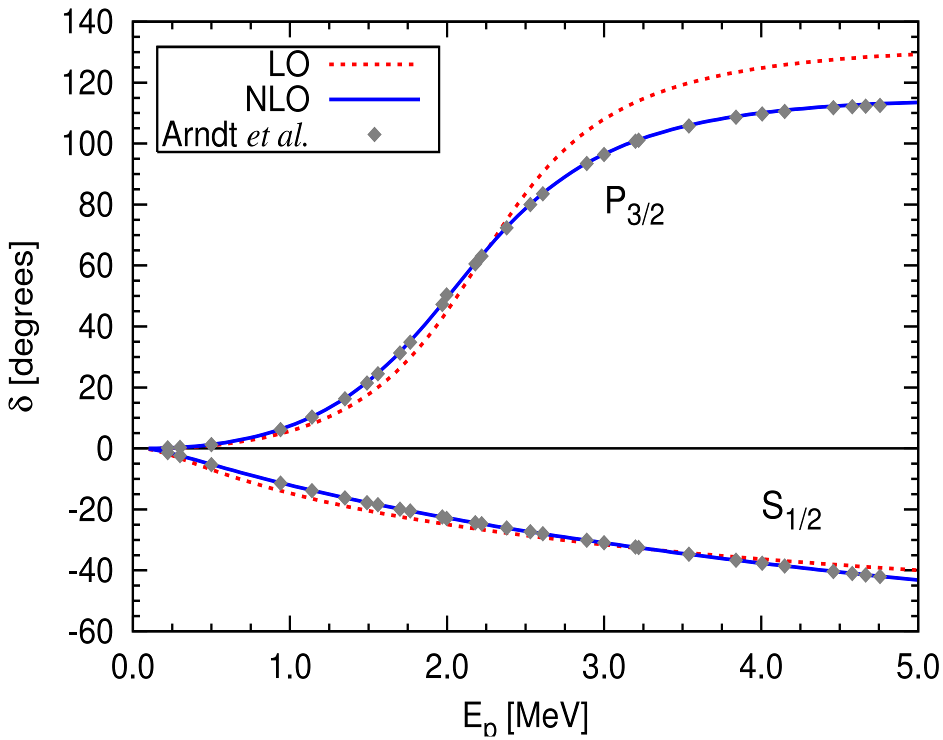
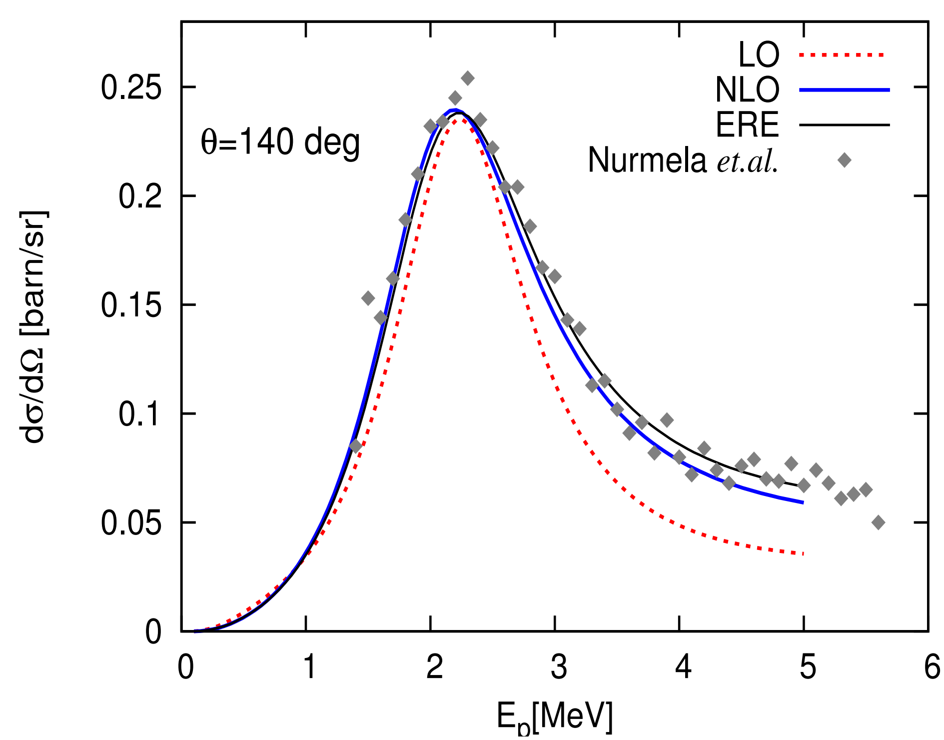
(RH, Bertulani, van Kolck, in preparation)





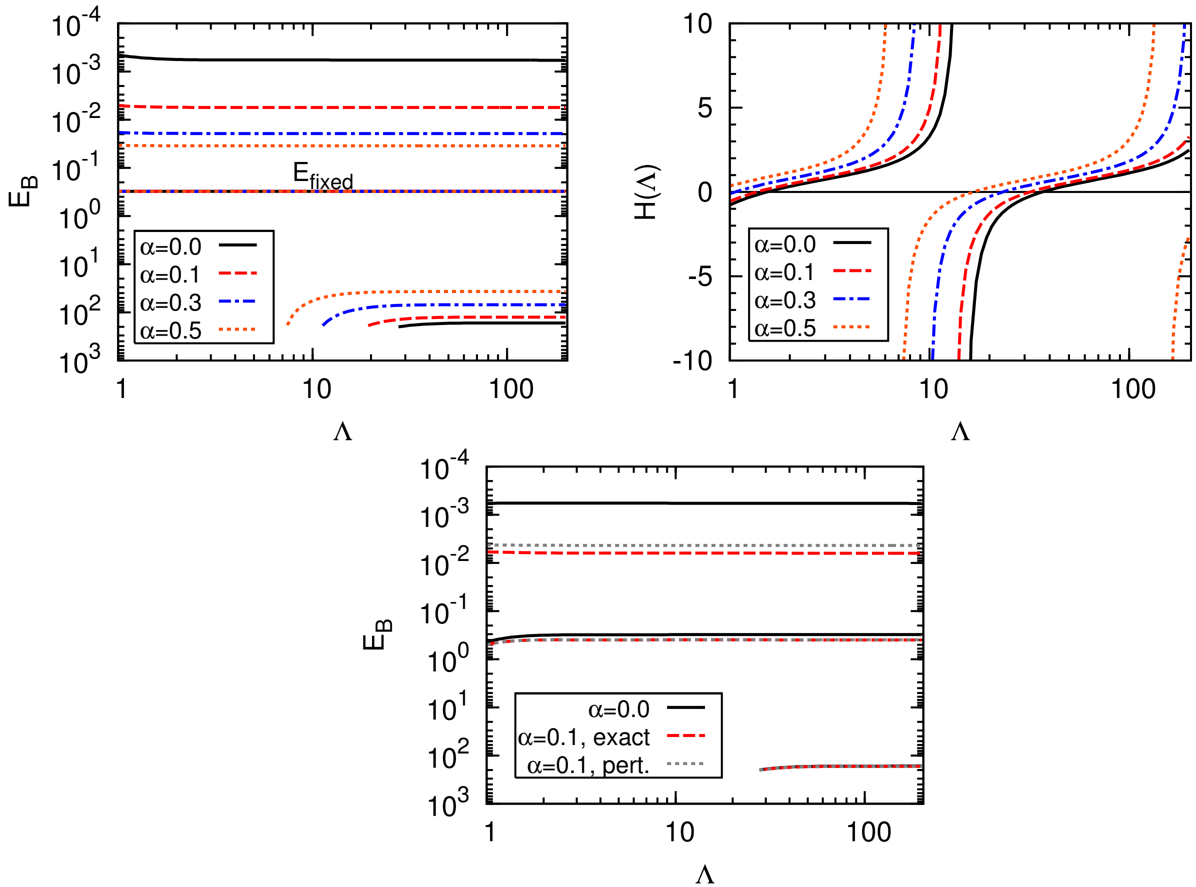
ab-initio: Nollett em et al., PRL 99, 022502 (2007),
Quaglioni & Navrátil, PRL 101, 092501 (2008)

c/r^2+Coulomb: warm-up for 3α

- 3-body problem with large $a \sim$ 1D Schrödinger Eq. with $V(r) = 1/r^2$
- limit cycle for $c < -1/4 \Leftrightarrow$ Efimov spectrum
  (Beane *et al.*, Bawin and Coon, Braaten and Phillips, Long and van Kolck...)
- counterterm: log-periodic function of the cutoff
- Counterterm parameter Λ_*: iteration of quantum corrections
  (dimensional transmutation)
- model to study loss of conformal invariance (Kaplan *et al.*)

$1/r^2$+Coulomb: warm-up for 3α

(Hammer, RH, Eur. J. Phys. A 37, 193)



Summary

- Halo nuclei, cluster systems: opportunities for EFT approach
- universality $\Leftrightarrow$ limit cycles
- $\alpha\alpha$ scattering
  - ⋆ Coulomb turned off $\Rightarrow$ conformal invariance @LO, Efimov spectrum in ^{12}C
  - ⋆ incredible amount of fine-tuning
  - ⋆ LO (parameter-free) works well at very low energies, NLO improves description up to $E_{LAB} \approx 3$ MeV
  - ⋆ extraction of the ERE parameters with improved errorbars
- p-α scattering: good description of the $P_{3/2}$ resonance
- future: 3α, p-^{7}Be, Nradcap (Rupak & RH), Borromean halos, heavier nuclei, ...

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