The Efimov effect in heteronuclear systems

Kerstin Helfrich

Universität Bonn, HISKP and Bethe-Center for Theoretical Physics

in collaboration with H.-W. Hammer and D.S. Petrov

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Outline

Introduction

- We consider ultracold atomic gases.
- Few-body effects are important.

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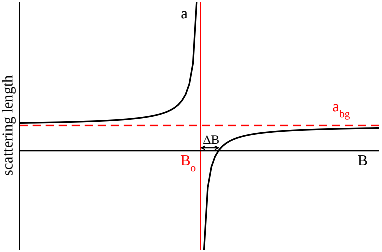

Introduction

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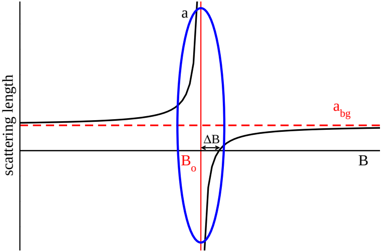
  $\Rightarrow$ universal regime



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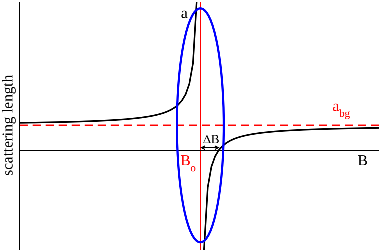


  - Efimov physics plays crucial role.
  - Prediction in 1970 by V. Efimov.

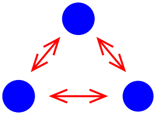
V. Efimov, Phys. Lett. **33B** (1970)

  - First seen in cold atoms in Innsbruck in 2005 with ^{133}Cs atoms.

T. Kraemer *et al.*, Nature **440** (2006)

  - Many more experiments during 2009.
  - Theoretically and experimentally not fully explored yet.

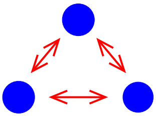
The Efimov effect

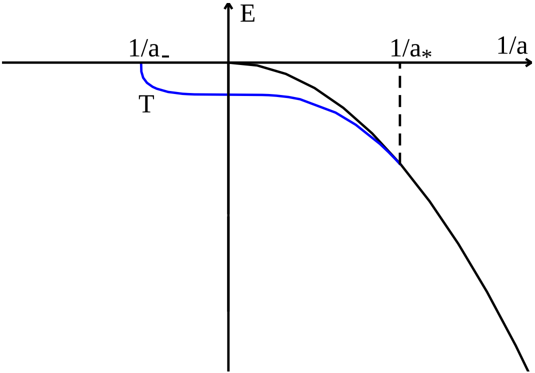


Efimov trimer

The Efimov effect

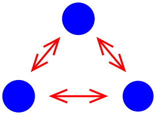
Three-body energy spectrum:





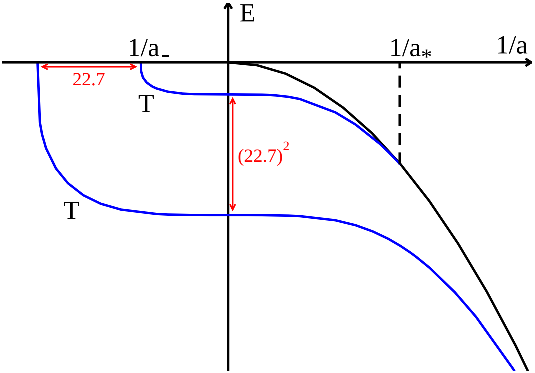
For $a > 0$: shallow dimer with $B_D = \frac{\hbar^2}{ma^2}$.

The Efimov effect



Efimov trimer

Three-body energy spectrum:



For $a > 0$: shallow dimer with $B_D = \frac{\hbar^2}{ma^2}$.

The Efimov effect

- At unitarity:

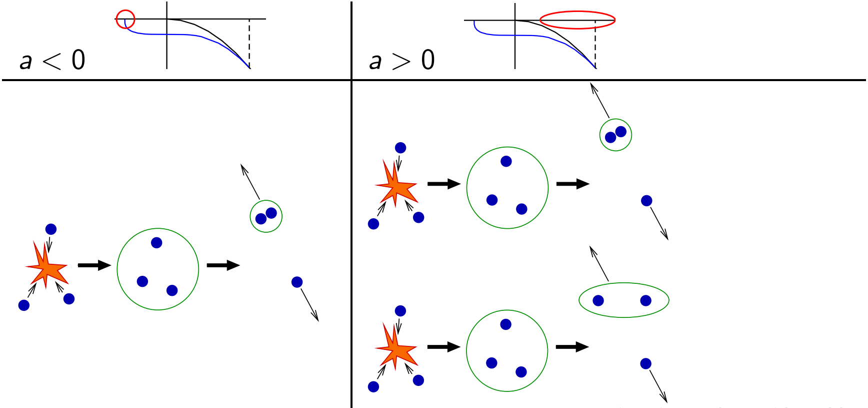
$$B_T^{(n)} = (e^{-2\pi/s_0})^{n-n_*}\hbar^2\kappa_*^2/m$$

with $s_0 = 1.00624\ldots$ for equal bosons.

- Alkali atoms can form deeply bound dimers $\rightarrow$ Efimov trimers are unstable.
- Experimental observation via loss features.

The Efimov effect

Three-body recombination

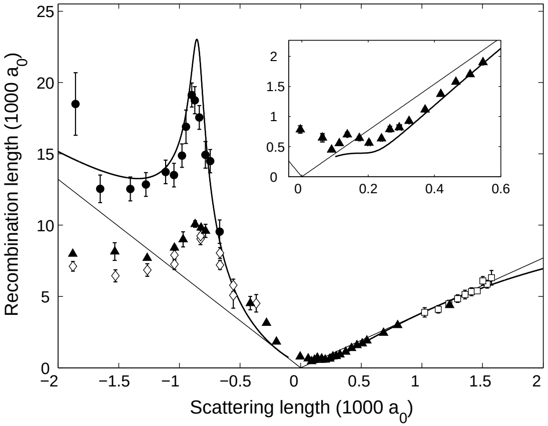


$\longrightarrow$ maxima at a_-

$\longrightarrow$ minima at $a_{0*} = |a_-| \exp(-\pi/(2s_0))$

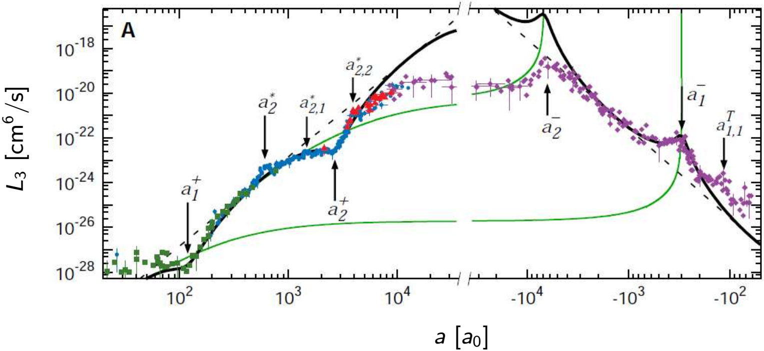
The Efimov effect

Three-body recombination in ^{133}Cs



T. Kraemer *et al.*, Nature **440** (2006)

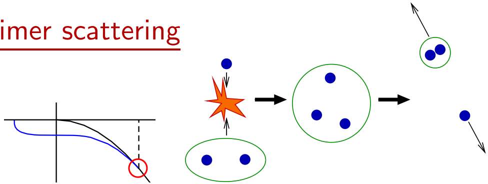
Three-body recombination in ^{7}Li



S.E. Pollack *et al.*, Science **326** (2009)

The Efimov effect

Atom-dimer scattering

$a > 0$  $\rightarrow$ maxima at a_*

The Efimov effect

Atom-dimer scattering

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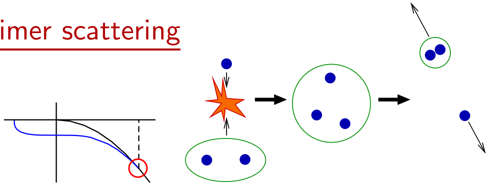
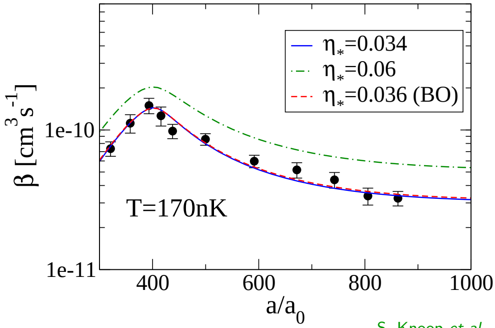


$\rightarrow$ maxima at a_*

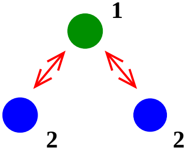
in ^{133}Cs:



S. Knoop *et al.*, Nature Phys. **5** (2009)

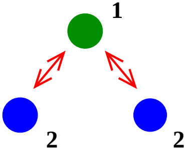
K. Helfrich and H.W. Hammer, EPL **86** (2009)

Heteronuclear systems - EFT



- Only two resonant interactions.
- All observables depend on $m_1/m_2 \equiv \delta$.
- Ongoing experiments e.g. in Florence.

Heteronuclear systems - EFT



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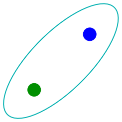
- Use a Lagrangian with auxiliary dimer field d:



$$\mathcal{L} = \psi_1^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_1}\right)\psi_1 + \psi_2^\dagger\left(i\partial_t + \frac{\nabla^2}{2m_2}\right)\psi_2 + g_2 d^\dagger d$$
$$-g_2\left(d^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger d\right) - \frac{g_3}{4} d^\dagger d\psi_2^\dagger\psi_2$$

P.F. Bedaque *et al.*, PRL **82** (1999)

K. Helfrich *et al.*, arXiv:1001.4371

Heteronuclear systems - EFT

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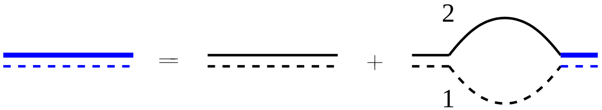
$$-g_2\left(d^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger d\right) - \frac{g_3}{4}d^\dagger d\psi_2^\dagger\psi_2$$

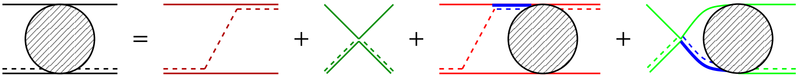
- Derive Feynman rules
- Calculate full dimer propagator



- $D(P_0, \vec{P}) = \frac{2\pi}{g_2^2\mu_{12}}\left[\frac{1}{a} - \sqrt{-2\mu_{12}\left(P_0 - \frac{P^2}{2(m_1+m_2)}\right) - i\epsilon}\right]^{-1}$

Heteronuclear systems - EFT

Solve integral equation for the amplitude $\mathcal{A}_S(p,k;E)$:



$$\mathcal{A}_S(p,k;E) = \frac{2\pi m_1}{a\mu_{12}^2}\left\{\frac{1}{2pk}\ln\left[\frac{p^2+k^2+2pk\frac{\mu_{12}}{m_1}-2\mu_{12}E-i\epsilon}{p^2+k^2-2pk\frac{\mu_{12}}{m_1}-2\mu_{12}E-i\epsilon}\right]+\frac{H(\Lambda)}{\Lambda^2}\right\}$$
$$+\frac{m_1}{\pi\mu_{12}}\int^{\Lambda} dq\, q^2\left\{\frac{1}{2pq}\ln\left[\frac{p^2+q^2+2pq\frac{\mu_{12}}{m_1}-2\mu_{12}E-i\epsilon}{p^2+q^2-2pq\frac{\mu_{12}}{m_1}-2\mu_{12}E-i\epsilon}\right]+\frac{H(\Lambda)}{\Lambda^2}\right\}$$
$$\times\frac{\mathcal{A}_S(q,k;E)}{-1/a+\sqrt{-2\mu\left(E-q^2/(2\mu_{(12)2})\right)-i\epsilon}}$$

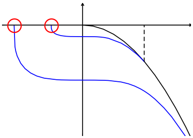
Heteronuclear systems - EFT

Parameters

- Two-body scattering length a
- Complex three-body input $a_* e^{-i\eta_*/s_0}$ with
  - Resonance position a_* (or Λ_*, a_-, a_{0*})
  - Resonance width η_*

Heteronuclear systems - Results

Scaling factor



Heteronuclear systems - Results

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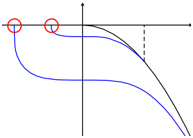
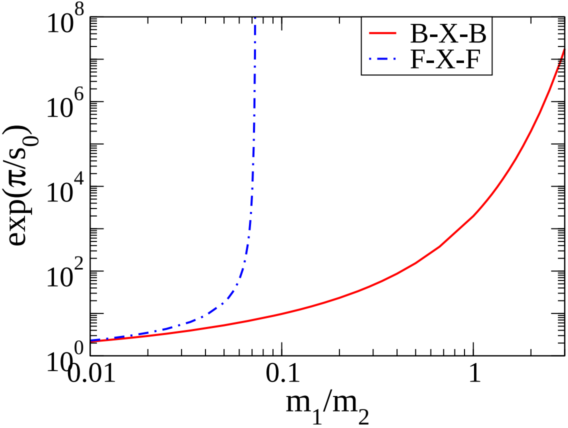
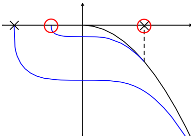


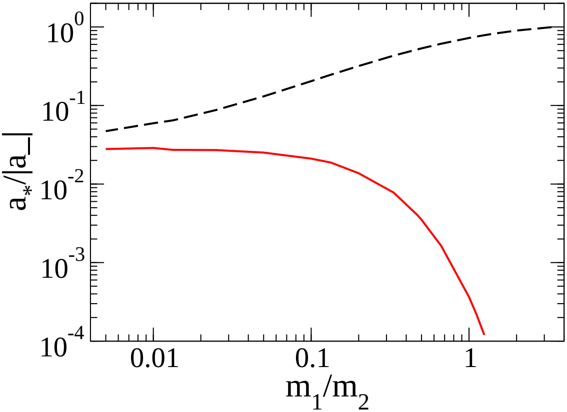


E. Braaten and H.W. Hammer, Phys.Rept. **428** (2006)

Note: for $\delta = 1$, $\exp(\pi/s_0) = 1986.1$

Threshold ratio





Solid line: $a_*^{(n)}/|a_-^{(n)}|$

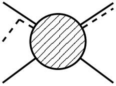
Dashed line: $a_*^{(n+1)}/|a_-^{(n)}| = \exp(\pi/s_0)a_*^{(n)}/|a_-^{(n)}|$

Heteronuclear systems - Results

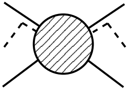
Three-body recombination

Definition of the three-body recombination rate α:

$$\boxed{\frac{d}{dt} n_2 = 2\frac{d}{dt} n_1 = -2\alpha\, n_1 n_2^2}$$

a) Recomb. into shallow dimers:  $\hat{=}\ \mathcal{A}_S\left(0, \sqrt{\frac{\mu_{(12)2}}{\mu}}\frac{1}{a}; 0\right)$

$$\alpha_{shallow} = 4\mu_{(12)2}\sqrt{\frac{\mu_{(12)2}}{\mu}}a^2 \left|\mathcal{A}_S\left(0, \sqrt{\frac{\mu_{(12)2}}{\mu}}\frac{1}{a}; 0\right)\right|^2$$

b) Recomb. into deep dimers:  $\hat{=}\ \mathcal{A}_S(0,0;0)$

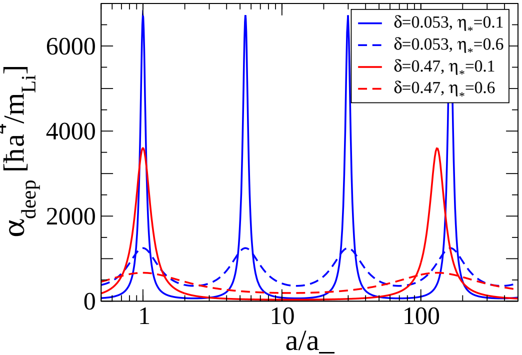
$$\alpha_{total} = \alpha_{deep} + \alpha_{shallow} = \mathrm{Im}\, T_{122\to 122} = 8\pi a^3 \mathrm{Im}\bar{\mathcal{A}}_S(0,0;0)$$

Heteronuclear systems - Results

Three-body recombination for $a < 0$

$$\alpha_{deep} = \frac{C(\delta)}{2} \frac{D \coth(\pi s_0) \sinh(2\eta_*)}{\sinh^2(\eta_*) + \sin^2[s_0 \ln(a/a_-)]} \frac{\hbar a^4}{m_1}$$

with $D = 128\pi^2(4\pi - 3\sqrt{3})$



^{7}Li-^{133}Cs-^{133}Cs

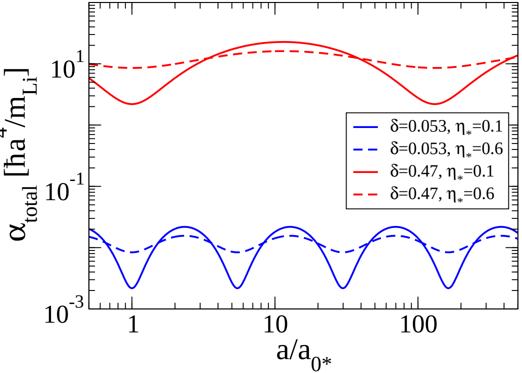
^{41}K-^{87}Rb-^{87}Rb

Heteronuclear systems - Results

Three-body recombination for $a > 0$

$$\alpha_{deep} = C(\delta)\frac{D\coth(\pi s_0)\cosh(\eta_*)\sinh(\eta_*)}{\sinh^2(\pi s_0+\eta_*)+\cos^2[s_0\ln(a/a_{0*})]}\frac{\hbar a^4}{m_1}$$

$$\alpha_{shallow} = C(\delta)\frac{D(\sin^2[s_0\ln(a/a_{0*})]+\sinh^2\eta_*)}{\sinh^2(\pi s_0+\eta_*)+\cos^2[s_0\ln(a/a_{0*})]}\frac{\hbar a^4}{m_1}$$



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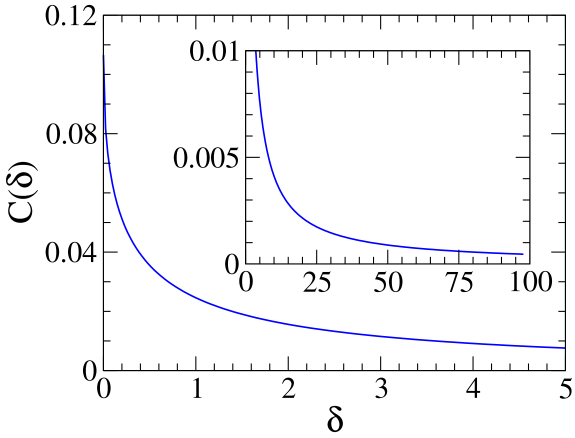
Heteronuclear systems - Results

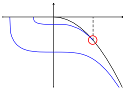
Three-body recombination



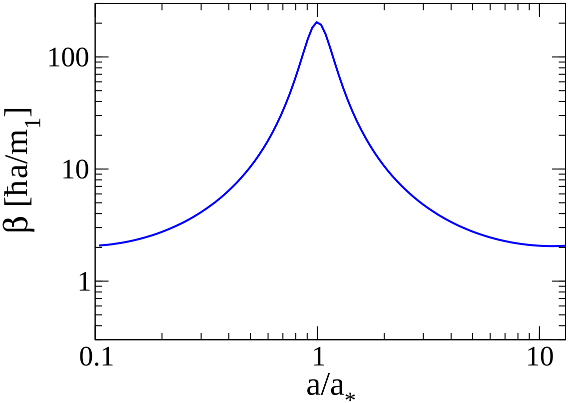
Heteronuclear systems - Results

Atom-dimer scattering

Def: $\boxed{\frac{d}{dt}n_A = \frac{d}{dt}n_D = -\beta n_A n_D}$



$m_1/m_2 = 0.47$
$\eta_* = 0.1$



Heteronuclear systems - Results

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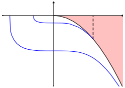

$m_1/m_2 = 0.47$
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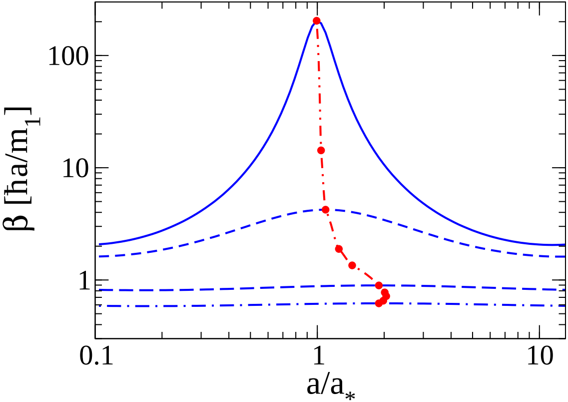


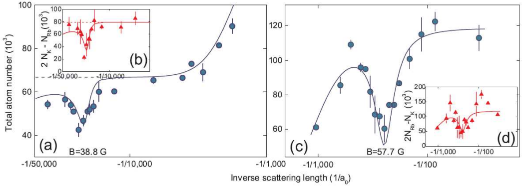
$E/B_D = -1, -0.95, -0.5, 0$

Peak position for $-B_D < E < 0$

Heteronuclear systems - Experiment

In Florence: mixture of ^{41}K and ^{87}Rb, $\exp(\pi/s_0) = 131$

K-K-Rb



K-Rb-Rb

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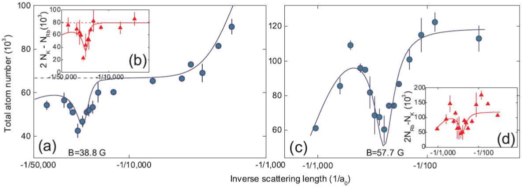
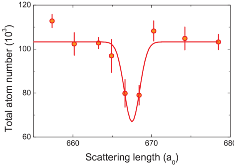


K-Rb-Rb



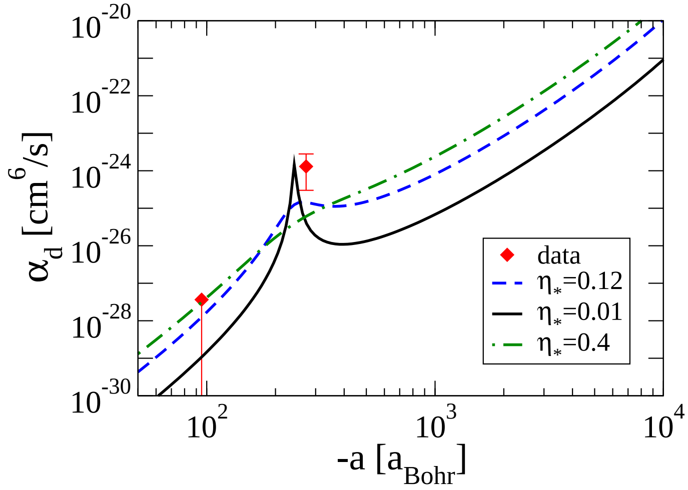
K-Rb-Rb

$a_*/|a_-| = 2.7$ (experiment) $\leftrightarrow$ $a_*/|a_-| = 0.52$ (theory)

G. Barontini *et al.*, PRL **103** (2009)

Heteronuclear systems - Experiment

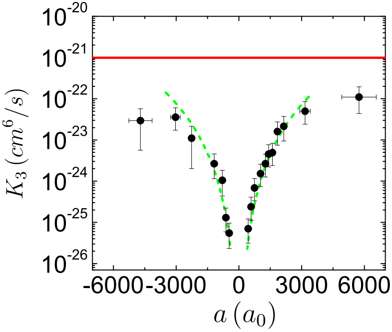
KRbRb resonance at $a = a_- = -246\ a_0$:

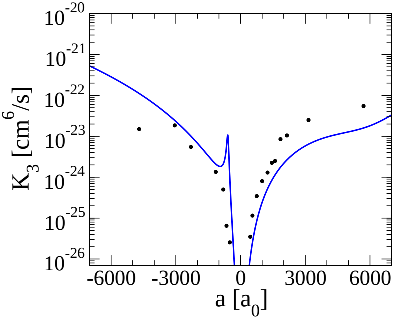


Heteronuclear systems - Experiment

At JILA: mixture of ^{40}K and ^{87}Rb, $\exp(\pi/s_0) = 122.7$

a) Three-body recombination





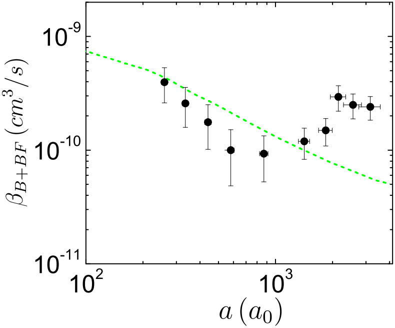
Parameters: $a_* = 300 \pm 100 a_0$, $\eta_* = 0.05 \pm 0.02$

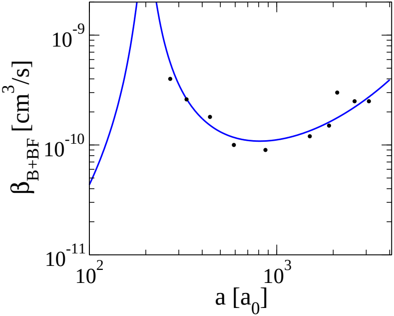
$\Rightarrow$ K_3 peaks at $a = a_- \approx -600 a_0$

J.J. Zirbel *et al.*, PRL **100** (2008)

Heteronuclear systems - Experiment

b) Atom-dimer scattering





Parameters: $a_* = 200 \pm 50 a_0$, $\eta_* = 0.05 \pm 0.02$

Heteronuclear systems - Experiment

Ongoing experiments

- Tübingen: ^{6}Li/^{7}Li and ^{87}Rb, $\exp(\pi/s_0) = 6.8/7.9$
- Heidelberg: ^{7}Li and ^{133}Cs, $\exp(\pi/s_0) = 5.5$
- Heidelberg: ^{6}Li/^{7}Li and ^{23}Na, $\exp(\pi/s_0) = 36.1/48.1$
- Düsseldorf: ^{87}Rb and ^{174}Yb, $\exp(\pi/s_0) = 154$
- probably more...

Heteronuclear systems - Analytical analysis

At $E = 0$, analytical results can be derived in coordinate space.

Resulting integral equation:

$$
\begin{aligned}
f(x) &= -4\pi\Psi_0 \\
&+ \frac{4}{\pi}\int_0^\infty \left[\frac{f(x') - f(x)}{(x^2 - x'^2)^2} + \frac{f(x')}{(x^2 + x'^2)^2 - 4x^2x'^2 \sin^2\phi}\right] x'^2 dx'
\end{aligned}
$$

with

- Ψ_0: wave function of three free atoms
- $f(x)$: atom-dimer wave function

$$\Rightarrow \; C(\delta) = \frac{(1+\delta)^2 \arcsin\left[1/(1+\delta)\right] - \sqrt{\delta(2+\delta)}}{2(4\pi - 3\sqrt{3})}$$

Summary and outlook

- We generalized the effective field theory to heteronuclear systems exhibiting the Efimov effect.
- We calculated theoretical predictions for heteronuclear observables.
- Analytical results for $E = 0$ are available.
- Comparison to Florence experiment is rather inconclusive.

Summary and outlook

- We generalized the effective field theory to heteronuclear systems exhibiting the Efimov effect.
- We calculated theoretical predictions for heteronuclear observables.
- Analytical results for $E = 0$ are available.
- Comparison to Florence experiment is rather inconclusive.
- Comparison to other data from ongoing experiments will be very interesting.
- Possible directions:
  - Generalization to four-bodies
  - Inclusion of effective-range corrections
  - Consideration of trapping geometry
  - Finite temperature calculation
  - Atom-dimer scattering above threshold

Supplement

Renormalization:

$$H(\Lambda) = \frac{\cos[s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0]}{\cos[s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0]}$$

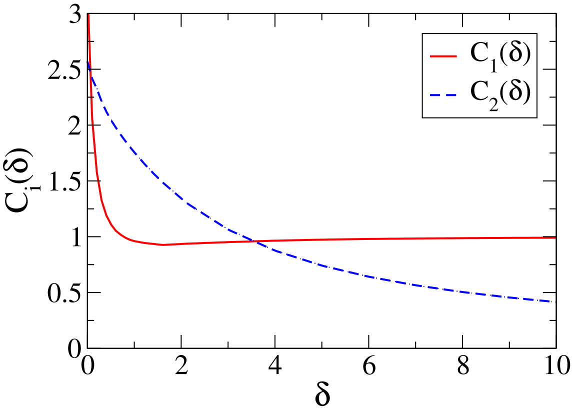
and

$$\Lambda_* \ \propto \ a_*$$

Supplement

Atom-dimer scattering length:

$$a_{AD} = \Big(C_1(\delta) + C_2(\delta) \cot[s_0 \ln(a/a_*)] \Big) a$$



Supplement

Atom-dimer recombination rate:

$$\beta(E=-B_d) = -(4\pi\hbar/\mu_{(12)2})\,\mathrm{Im}a_{AD}$$
$$= 2\pi C_2(\delta)\frac{\delta(\delta+2)}{\delta+1}\frac{\sinh(2\eta_*)}{\sin^2[s_0\ln(a/a_*)]+\sinh^2\eta_*}\frac{\hbar a}{m_1}$$

$$\beta(E=-0) = \pi\frac{[\delta(\delta+2)]^{3/2}}{(\delta+1)^2}$$
$$\times \frac{\sinh(2\pi s_0)\sinh(2\eta_*)}{\sinh^2(\pi s_0+\eta_*)+\cos^2[s_0\ln(a/a_{0*})]}\frac{\hbar a}{m_1}$$