The neutron-neutron scattering length

Anders Gårdestig

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Relevant publications

In collaboration with Daniel Phillips (supported by NSF and DOE):

- A.G. and D.R. Phillips
  Phys. Rev. C **73**, 014002 (2006)
  `arXiv.org/abs/nucl-th/0501049`
- A.G. and D.R. Phillips
  Phys. Rev. Lett. **96**, 232301 (2006)
  `arXiv.org/abs/nucl-th/0603045`
- A.G.
  Phys. Rev. C **74**, 017001 (2006)
  `arXiv.org/abs/nucl-th/0604035`

Review on a_{nn}:
A.G., J. Phys. G: Nucl. Part. Phys. **35**, 053001 (2009).

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Similar, but different! Why?

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QCD Lagrangian almost symmetric under $u \leftrightarrow d$ exchange (Charge Symmetry, CS), $P_{CS} = \exp(i\pi\tau_2/2)$
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Experimental evidence:

n-p mass difference
ρ^0-ω mixing ($e^+e^- \rightarrow \pi^+\pi^-$)
mirror nuclei (e.g. $^3\mathrm{He}$-$^3\mathrm{H}$) binding energy, N-S anomaly
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CSB reviews:

[Miller, Nefkens, and Šlaus, PRt**194**, 1 (1990);
Miller and van Oers, nucl-th/9409013;
Miller, Opper, and Stephenson, ARNPS**56**, 293 (2006), nucl-ex/0602021]

CSB and scattering lengths

$$\text{CSB} \Rightarrow a_{nn}^{\text{str}} \neq a_{pp}^{\text{str}}$$

$a_{nn}^{\text{str}} \neq a_{pp}^{\text{str}} \leftrightarrow$ enhancement factor: $\frac{\Delta a_{\text{CSB}}}{a} = (10-15)\frac{\Delta V_{\text{CSB}}}{V}$

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WHAT TO DO?

Solutions(?)

Direct measurements:

Wild idea #1: Simultaneous underground nuclear explosions

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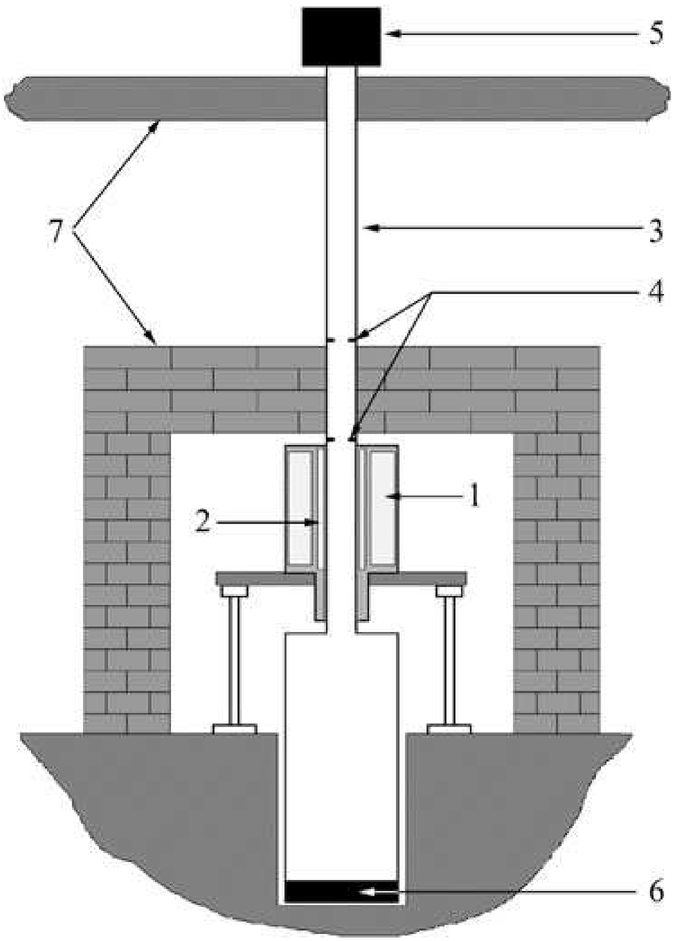
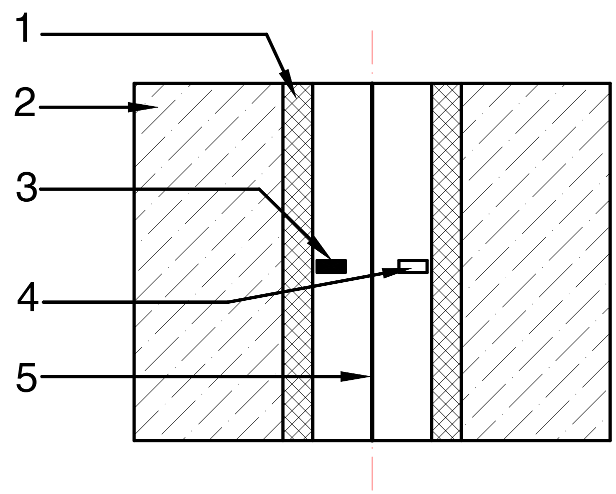
Recent idea: Pulsed reactor YAGUAR in Snezhinsk, Russia





[Furman et al., JPG **28**, 2627 (2002);
Muzichka et al., NPA **789**, 30 (2007)]

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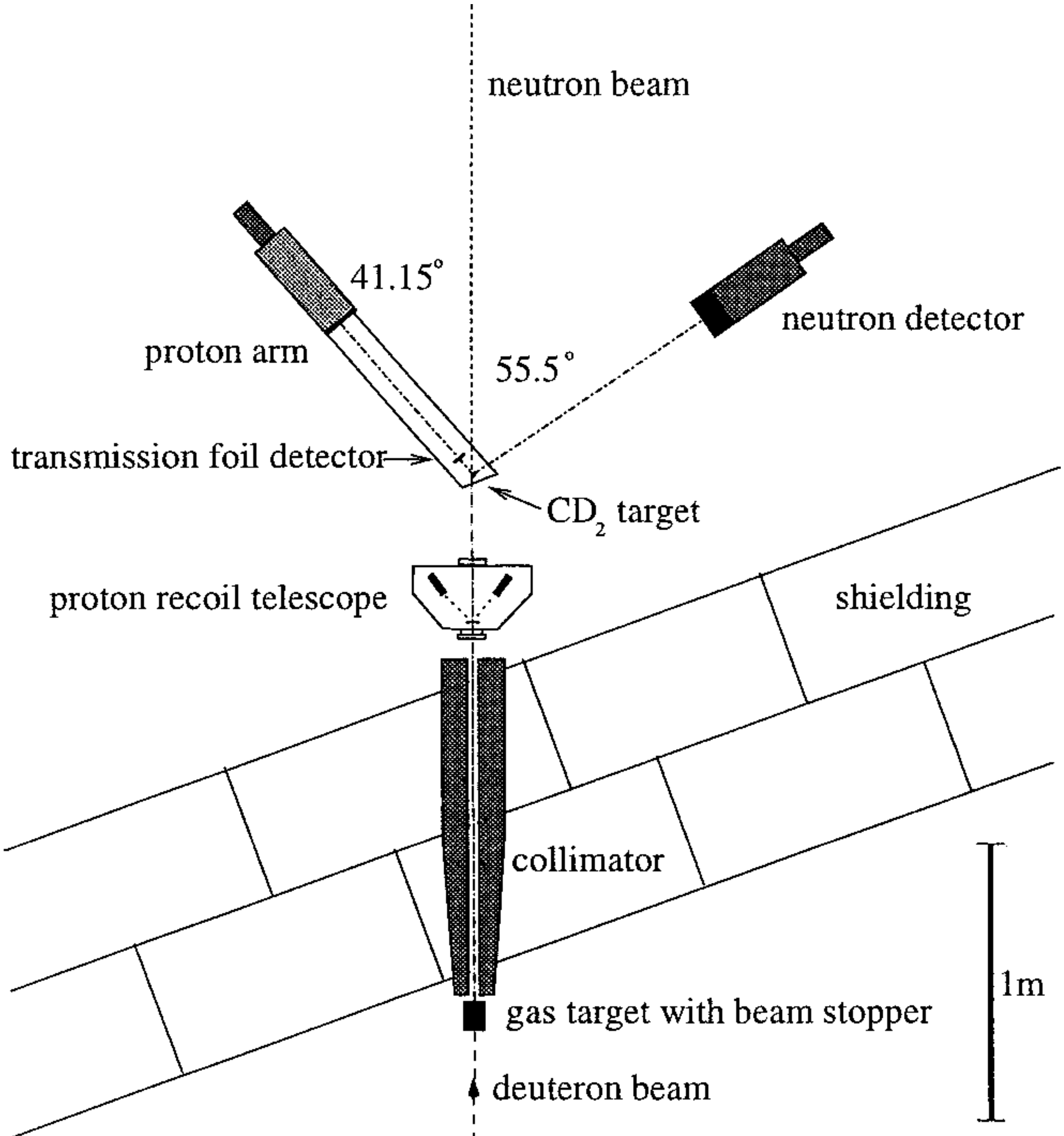
Indirect nn experiments:
Implemented idea: Reactions giving nn with small rel. energy

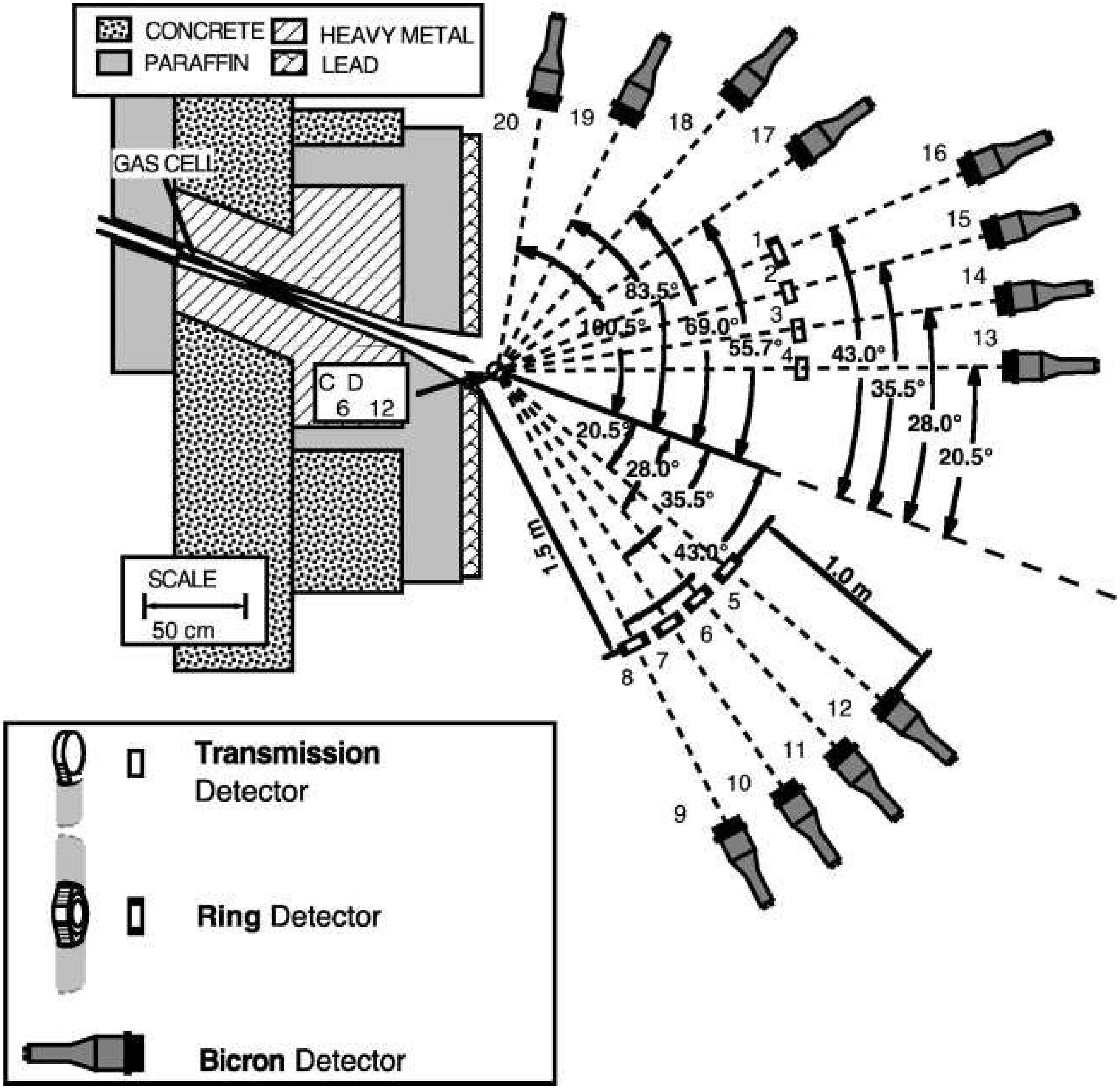
- $nd \to nnp$: 3-body forces needed, expts differ:
  $a_{nn} = -16.1 \pm 0.4$ fm (n, np) [Huhn et al., PRL**85**, 1190 (2000)] and
  $a_{nn} = -16.5 \pm 0.9$ fm (n, p) [von Witsch et al., PRC**74**, 014001 (2006)] vs
  $a_{nn} = -18.7 \pm 0.7$ fm (n, nnp) [González Trotter et al., PRC**73**, 034001 ('06)]
- $\pi^- d \to nn\gamma$: -18.59 ± 0.40 fm $(\pi^-, n\gamma)$ $\Rightarrow$ standard value (PSI and LAMPF) [Machleidt and Slaus, JPG:NPP**27**, R69 (2001)]

Need accurate theoretical input for extraction!

Bonn $nd \to nnp$ set-up



TUNL $nd \to nnp$ set-up



LAMPF set-up.

Stopped pions captured on d in atomic s-wave orbitals

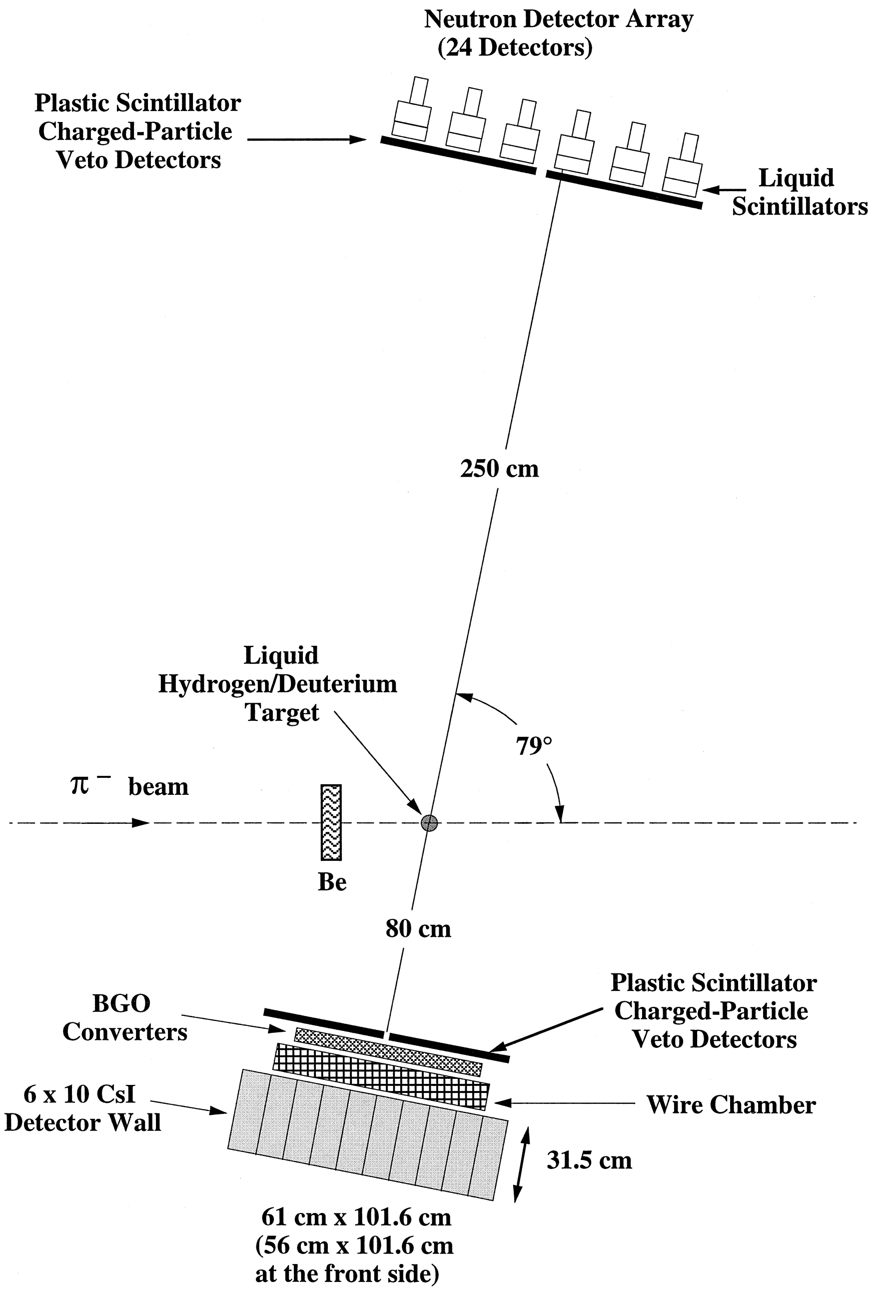
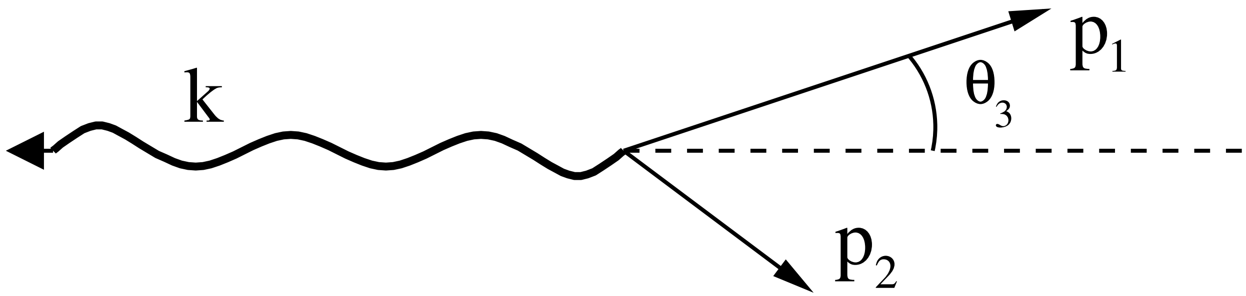


Fig. 1. Schematic of the mid-level cut-away view of the experimental layout.

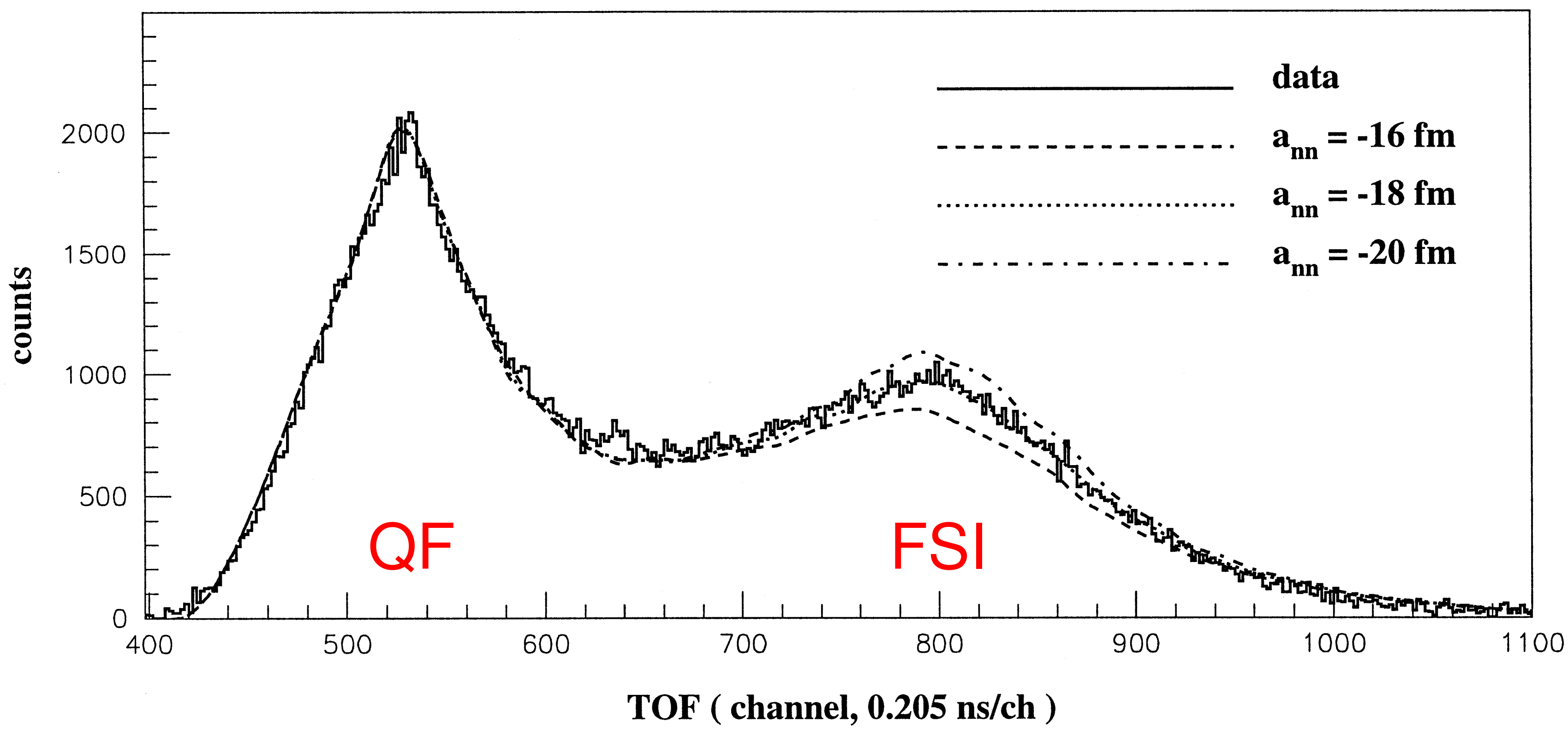
Lab = cm

γ and n detected in coincidence

$\pi^- d \rightarrow nn\gamma$ data (LAMPF).



γ and n_1 detected at $0.05 < \theta_3 < 0.1$ (rad) [Howell et al., PLB**444**, 252 (1998)]



Unnormalized, but shape fitted to give a_{nn}!

Old theory for $\pi^- d \to nn\gamma$

Gibbs, Gibson, and Stephenson (GGS) [PRC**11**, 90 (1975)]:

- $\pi^- p \to \gamma n$, rel corr up to $O(p/M)$
- estimated pion rescattering
- tried different wave functions
- theoretical error (mainly SD): $\Delta a_{nn} = \pm 0.3$ fm
- Only accurate under the FSI peak!

de Téramond *et al.*, [PRC**16**, 1976 (1977);**36**, 691 (1987)]
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Can chiral perturbation theory (χPT) do better?

EFT credo

Advantages of an effective field theory like χPT:

- Consistent amplitudes and wave functions
- Recipe to estimate theoretical error
- Systematic improvement possible
- χPT = low-energy limit of QCD, retains chiral symmetry of QCD

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At low E: expansion in $\alpha_S \sim 1$ not possible. Instead:
Power counting gives hierarchy of amplitudes. Here:

- $Q \sim m_\pi$ small momentum/energy of problem
- $\Lambda_\chi \sim M \sim 4\pi f_\pi \sim$ 1 GeV energy scale where χPT breaks down
- Expand in Q/Λ_χ

χPT for $\pi^- d \to nn\gamma$.

For $\pi^- d \to nn\gamma$ we get

- $O(Q^3)$ = GGS + π loops + 2-body
- $O(Q^3)$ $\pi N \to \gamma N$ fitted to data $\Rightarrow$ no free parameters

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2B $\mathcal{O}(Q^4)$: π exchanges and contact term
1B $\mathcal{O}(Q^4)$: under investigation

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2B $\mathcal{O}(Q^4)$: π exchanges and contact term
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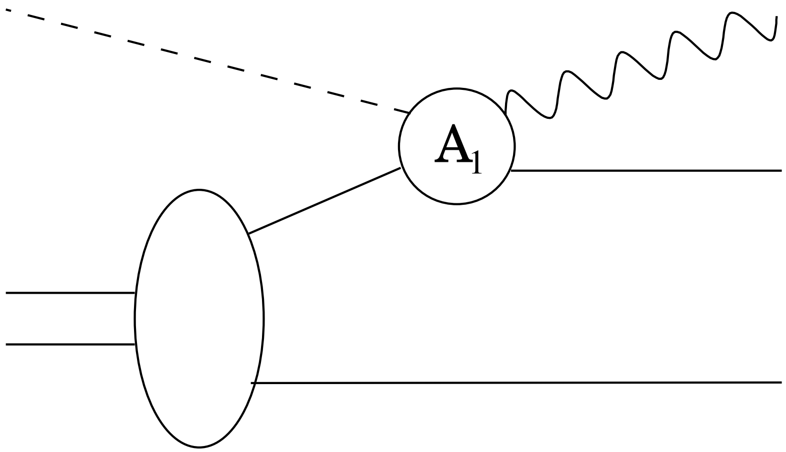
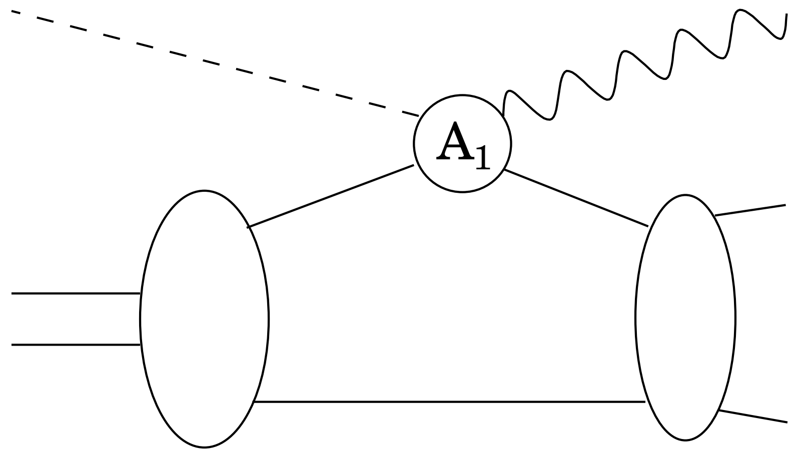
$\Rightarrow$ High precision possible

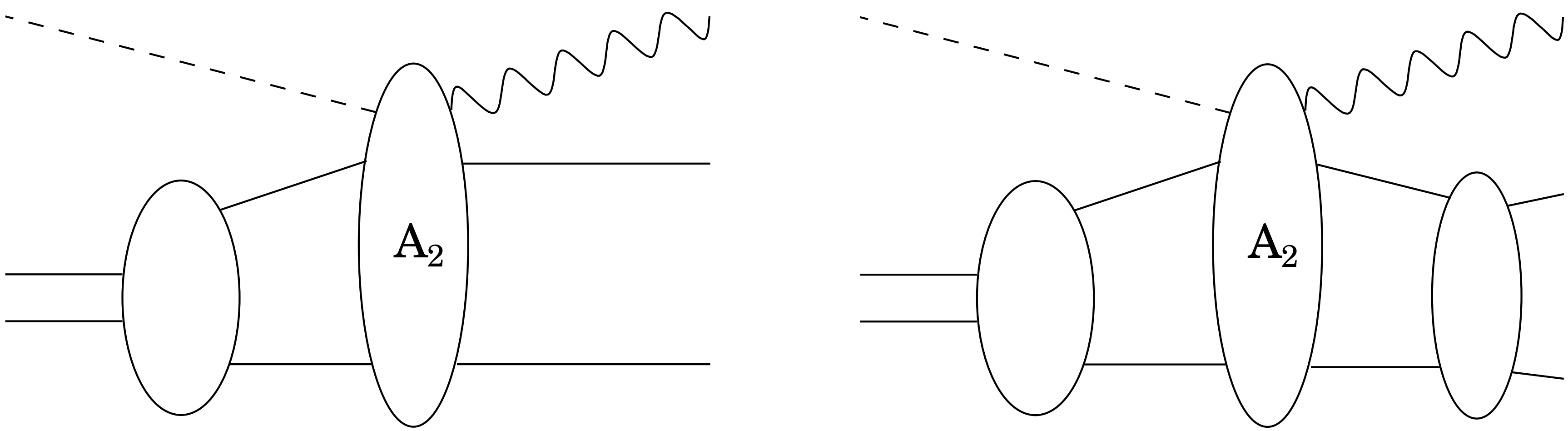
Generic diagrams



Quasifree (QF)



Final State Interaction (FSI)



Two-body effects (2)

$$\Gamma \propto |\mathcal{M}_{\rm QF} + \mathcal{M}_{\rm FSI} + \mathcal{M}_2|^2$$

Chirally inspired wave functions.

Start from asymptotic wave functions
SE integrated in from $r = \infty$ with chiral OPEP and TPEP

[Phillips & Cohen, NPA**668**, 45 (2000)]:

- Coupled integral equations for d ($^3S_1$–$^3D_1$)
- Uncoupled integral equations for nn ($^1S_0$, ^3P_J, $^1D_2$, no $^3F_2$)

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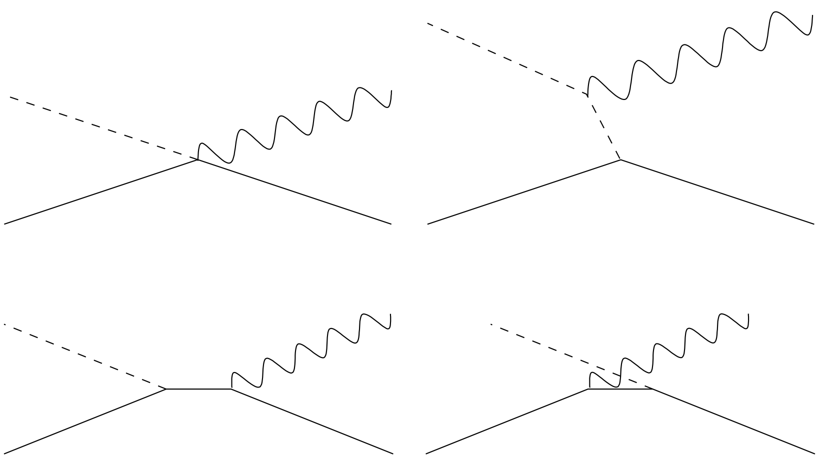
Match with spherical well solution at $r = R =$ 1.4 to 3.0 fm
(Regulates unknown short-distance physics)

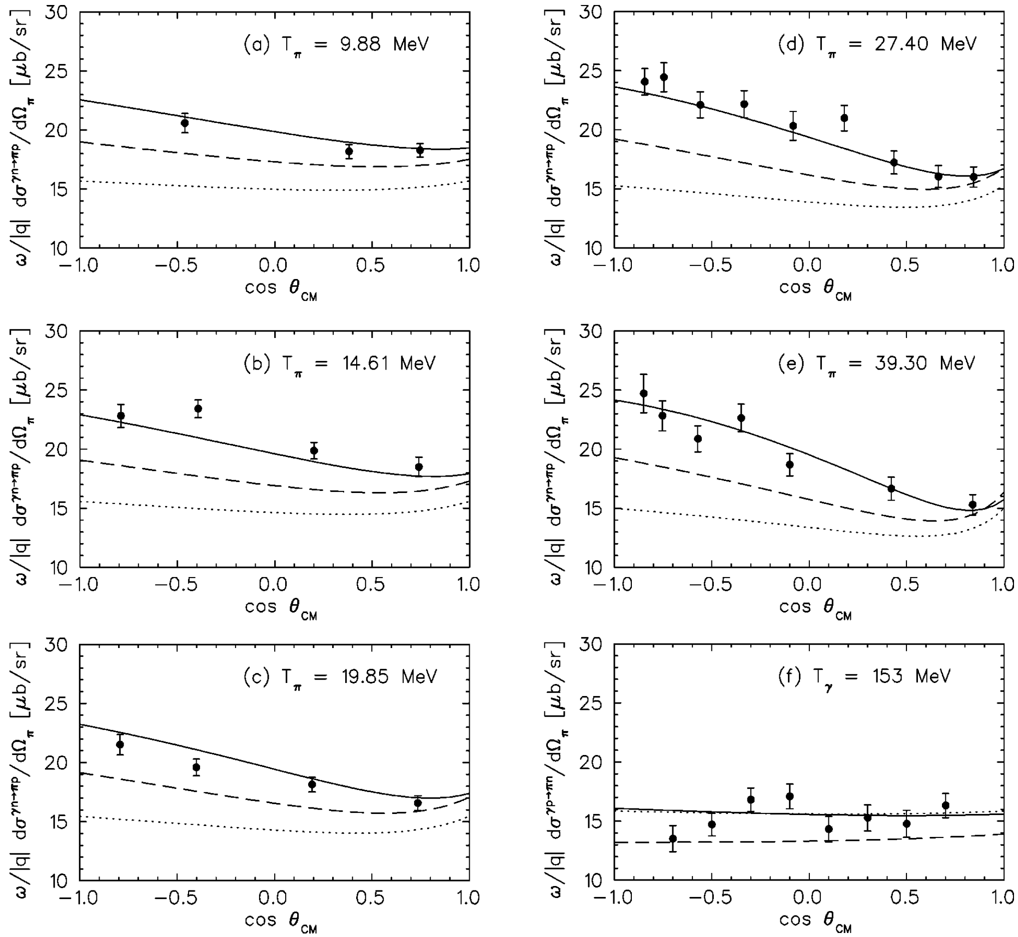
Calc indep of R?

One-body amplitudes.

EFT to $O(Q^3)$

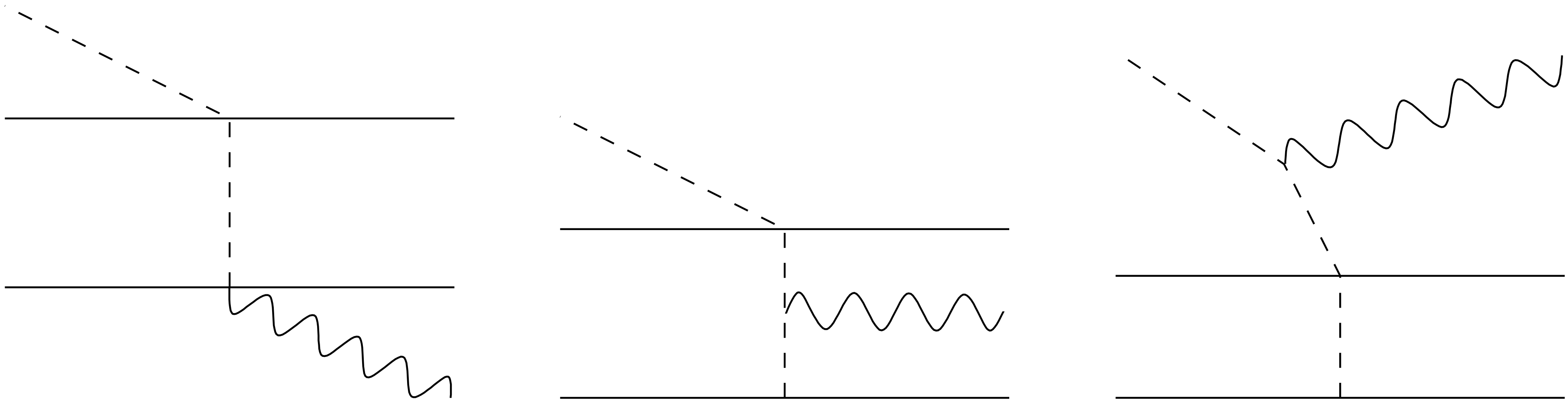
[Fearing et al., PRC**62**, 054006 (2000)]:





+pion loops at $O(Q^3) \Rightarrow \mathcal{A}_1$
all parameters fitted to data

Two-body amplitudes $O(Q^3)$



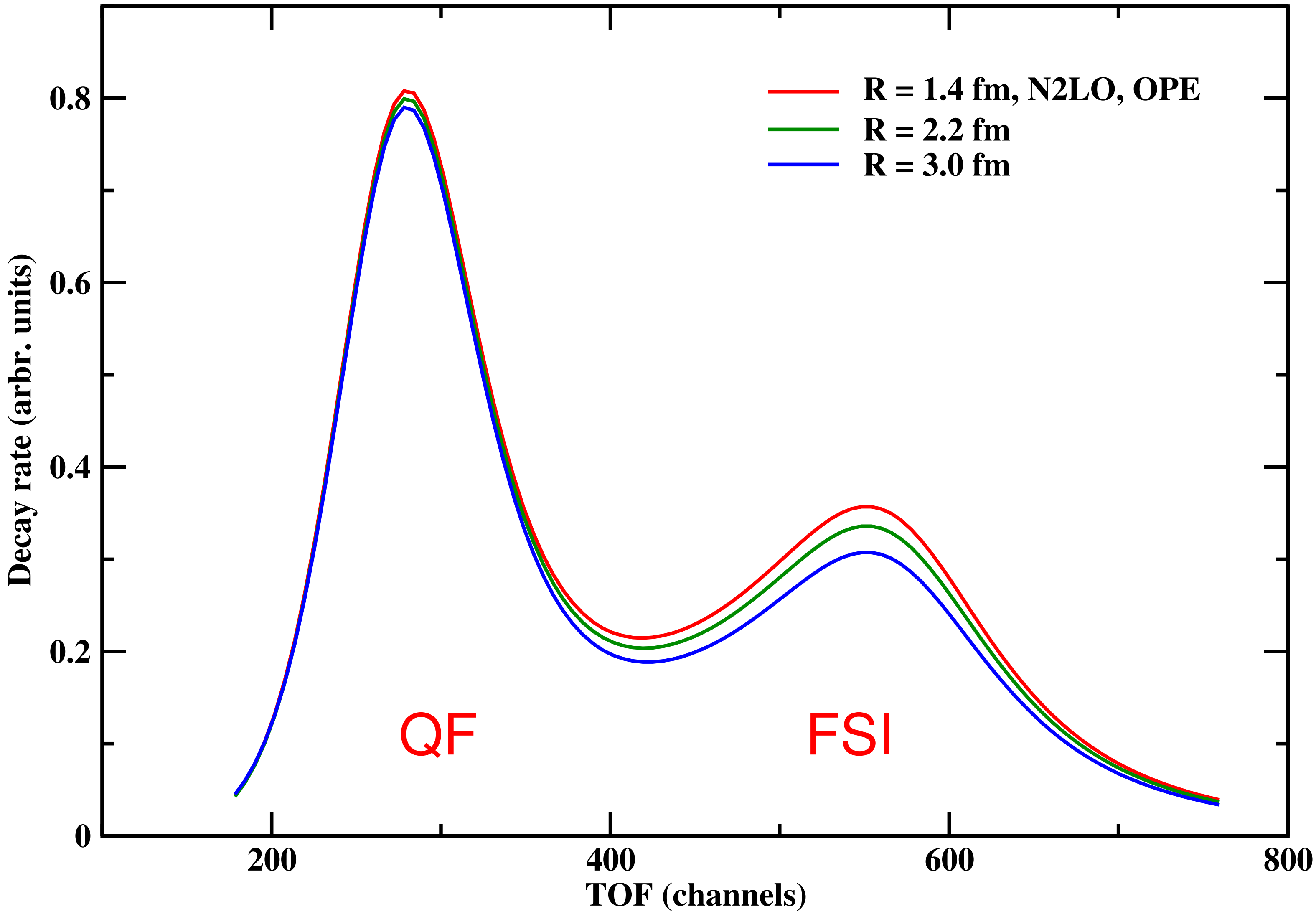
$\Rightarrow \mathcal{A}_2$

First diagram has a Coulomb-like propagator, $1/\vec{q}^{\,2}$

Second diagram has $1/\vec{q}^{\,2}$ and also an off-shell pion prop

Third diagram (2 off-shell props) vanishes in Coulomb gauge

R-dep error at $\mathcal{O}(Q^3)$



Δa_{nn}(theory) $= \pm 0.2$ fm (FSI only)

Δa_{nn}(theory) $= \pm 1$ fm (full spectrum)

Chiral relations

Chiral Lagrangian:

$$\mathcal{L} = N^\dagger(iv\cdot D + g_A S\cdot u)N$$
$$- 2d_1 N^\dagger S\cdot u N N^\dagger N + 2d_2 \epsilon^{abc}\epsilon_{\kappa\lambda\mu\nu}v^\kappa u^{\lambda,a} N^\dagger S^\mu \tau^b N N^\dagger S^\nu \tau^c N \ldots$$

where $f_\pi u_\mu = -\tau^a \partial_\mu \pi^a - \epsilon^{3ba} V_\mu \pi^b \tau^a + f_\pi A_\mu + \mathcal{O}(\pi^3)$

1N (g_A): Goldberger-Treiman and Kroll-Ruderman

$$\frac{g_A}{f_\pi} = \frac{g_{\pi NN}}{M} \qquad |\mathcal{A}_{KR}| = \frac{e g_A}{f_\pi}$$

relate axial coupling to πN coupling and $\gamma\pi N$ coupling

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2N (d_i): Axial isovector coupling to NN ($^3S_1 \leftrightarrow {}^1S_0$)

Connects π (photo)prod to EW reactions

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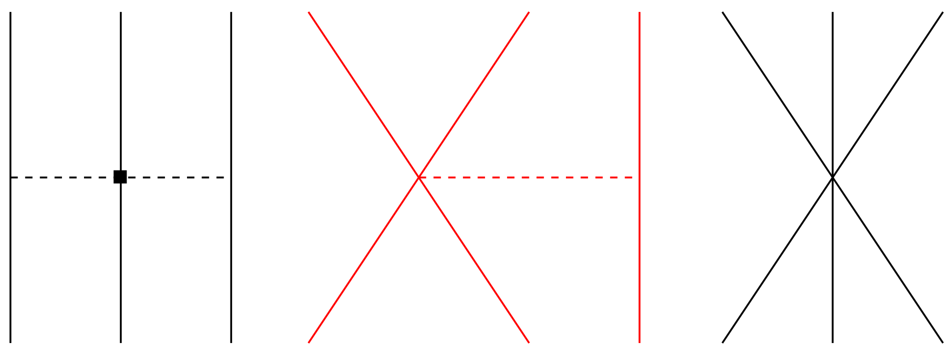
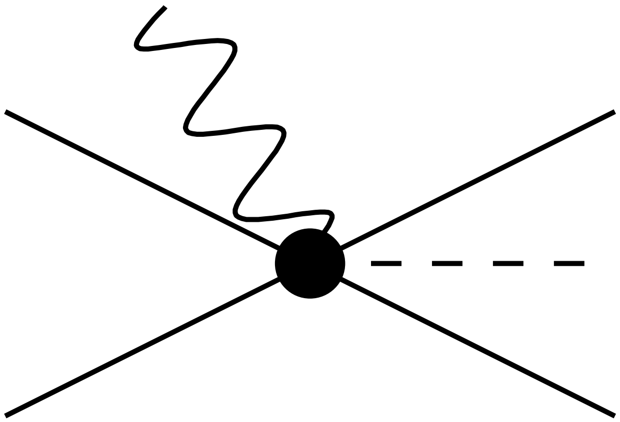
Connects π (photo)prod to EW reactions and chiral 3NF!



$\mathcal{O}(Q^4)$ axial isovector contact term



For ${}^3S_1 \leftrightarrow {}^1S_0$ one single LEC:

$$\hat{d} \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{\hat{c}_3}{3} + \frac{2\hat{c}_4}{3} + \frac{1}{6}$$

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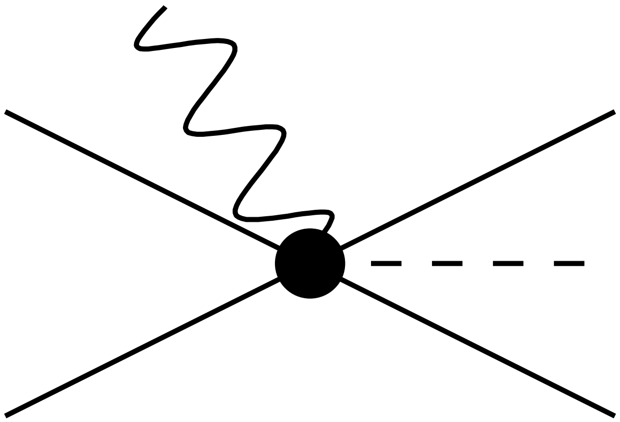

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Relates SD physics of

pp fusion, ${}^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \bar{\nu}_e$ (not EFT): [Schiavilla *et al.*, PRC**58**, 1263 (1998)]

p-wave π prod+3NF: [Hanhart, van Kolck, Miller, PRL**85**, 2905 (2000)]

$\mu^- d \to nn\nu_\mu$: [Ando *et al.*, PLB**533**, 25 (2002)]

$\nu(\bar{\nu})d$ breakup: [Ando *et al.*, PLB**555**, 49 (2003)]

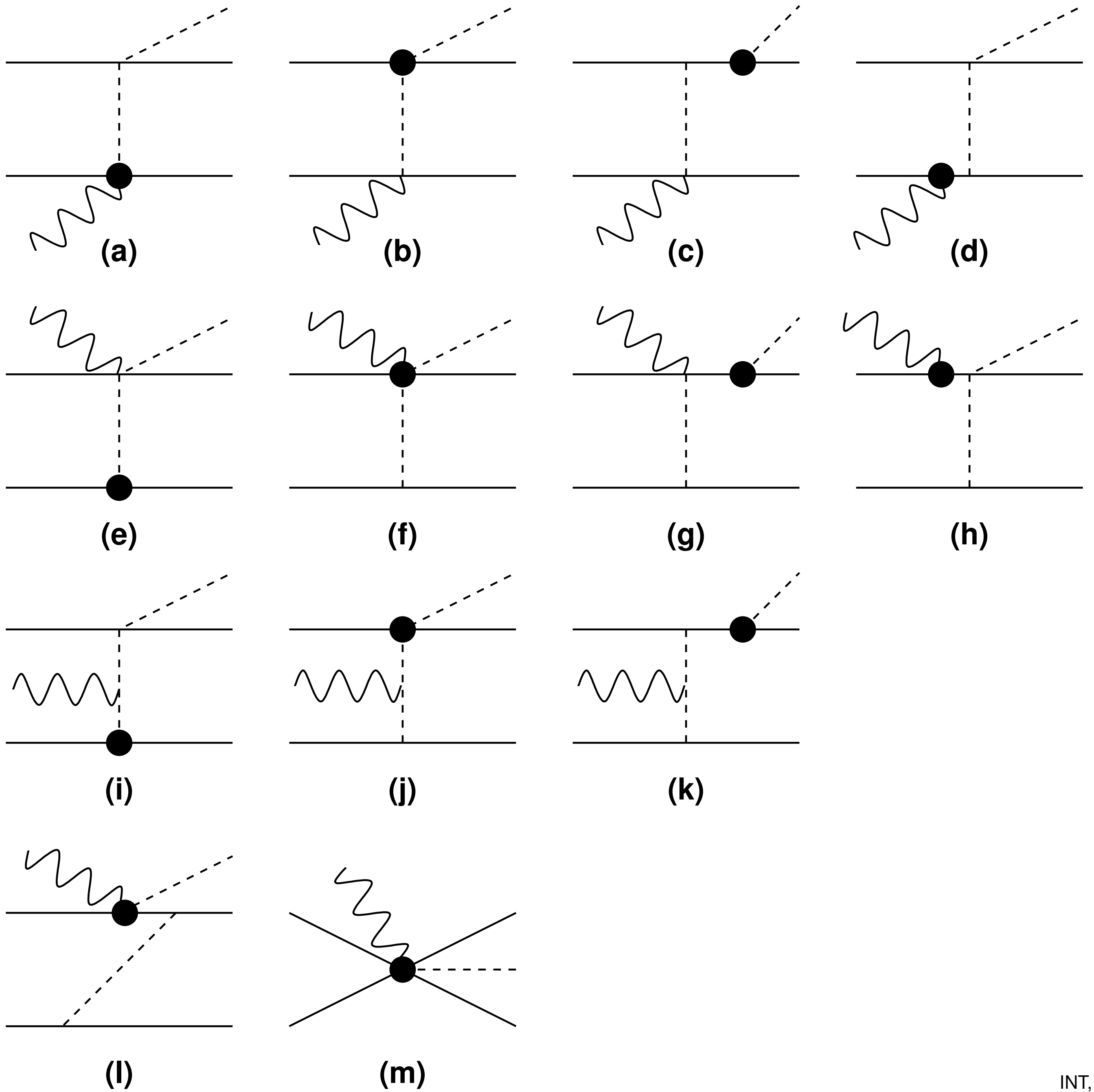
pp fusion, hep, ${}^3\mathrm{H} \to {}^3\mathrm{He}\, e^- \bar{\nu}_e$: [Park *et al.*, PRC**67**, 055206 (2003)]

pp fusion, $\pi^- d \to nn\gamma$, $\gamma d \to nn\pi^+$: [AG+DRP, PRL**96**, 232301 (2006); AG, PRC**74**, 017001 (2006)]

pp fusion, $\nu(\bar{\nu})d$, $\mu^- d \to nn\nu_\mu$: [Butler *et al.*, PLB**520**, 97 (2001);

EFT($\not\pi$): $\hat{d} \leftrightarrow L_{1,A}$ Chen *et al.*, PRC**72**, 061001(R) (2005)]

Two-body amplitudes $O(Q^4)$



Constraining contact term

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Calculated by [Park *et al.*, PRC**67**, 055206 (2003)]
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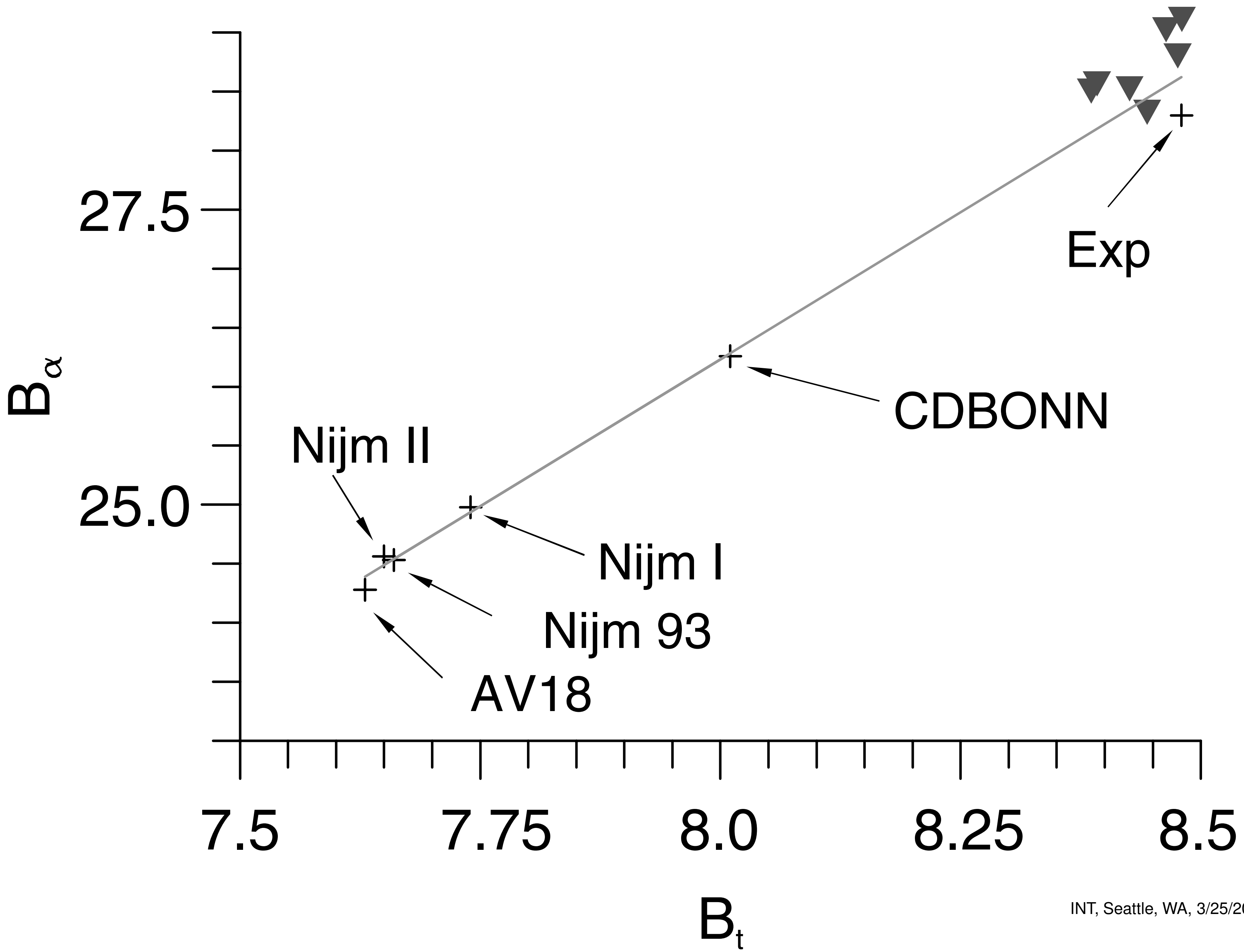
Let's do a numerical experiment!

Remember:

Tjon line: $B(^4\mathrm{He})$ vs $B(^3\mathrm{H})$
Phillips line: $^2a_{nd}$ vs $B(^3\mathrm{H})$

Tjon line [Nogga, Kamada, Glöckle, PRL85, 944 (2000)]



Phillips line [Witała et al., PRC68, 034002 (2003)]

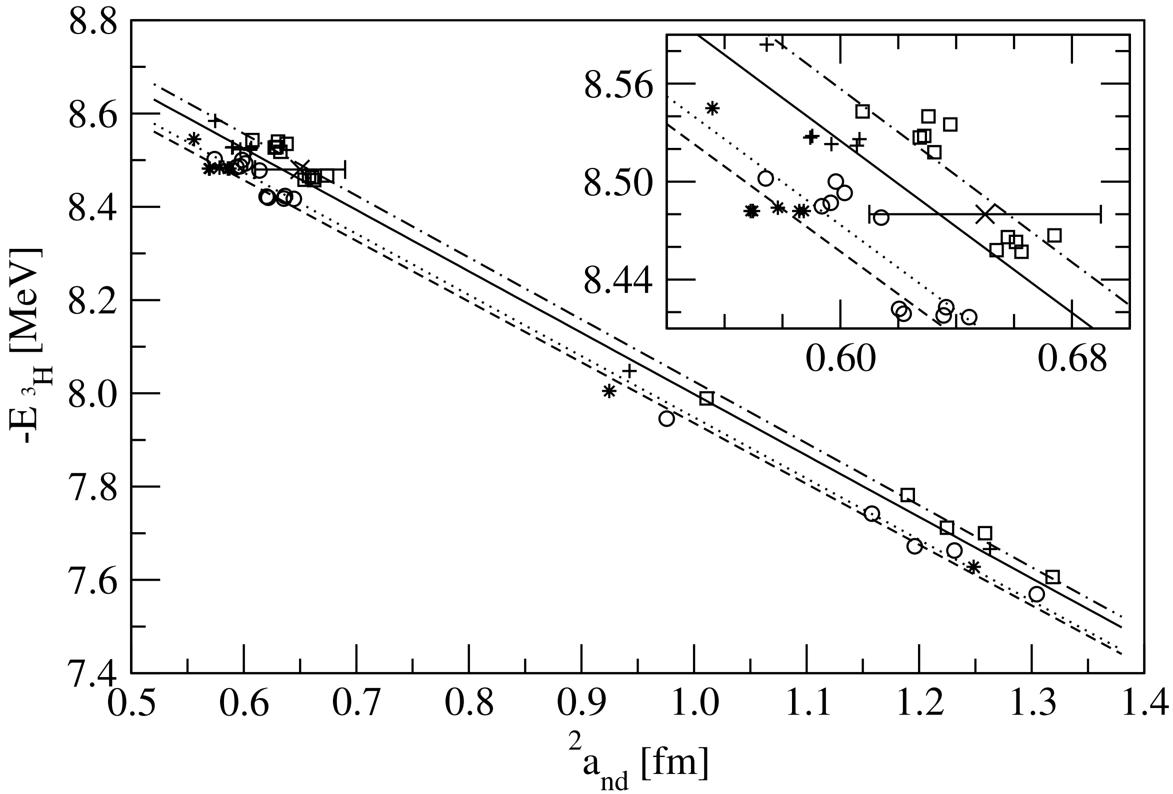
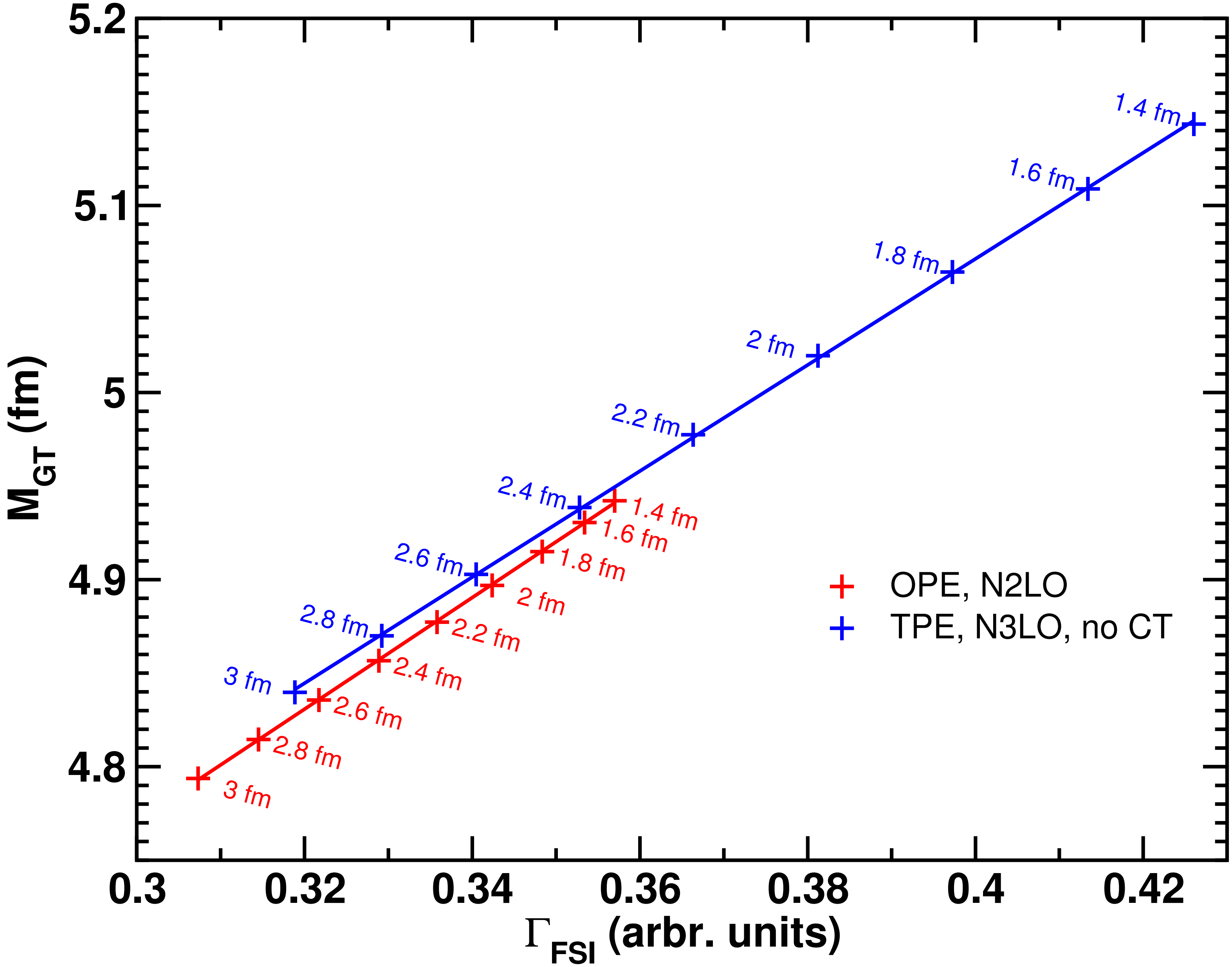


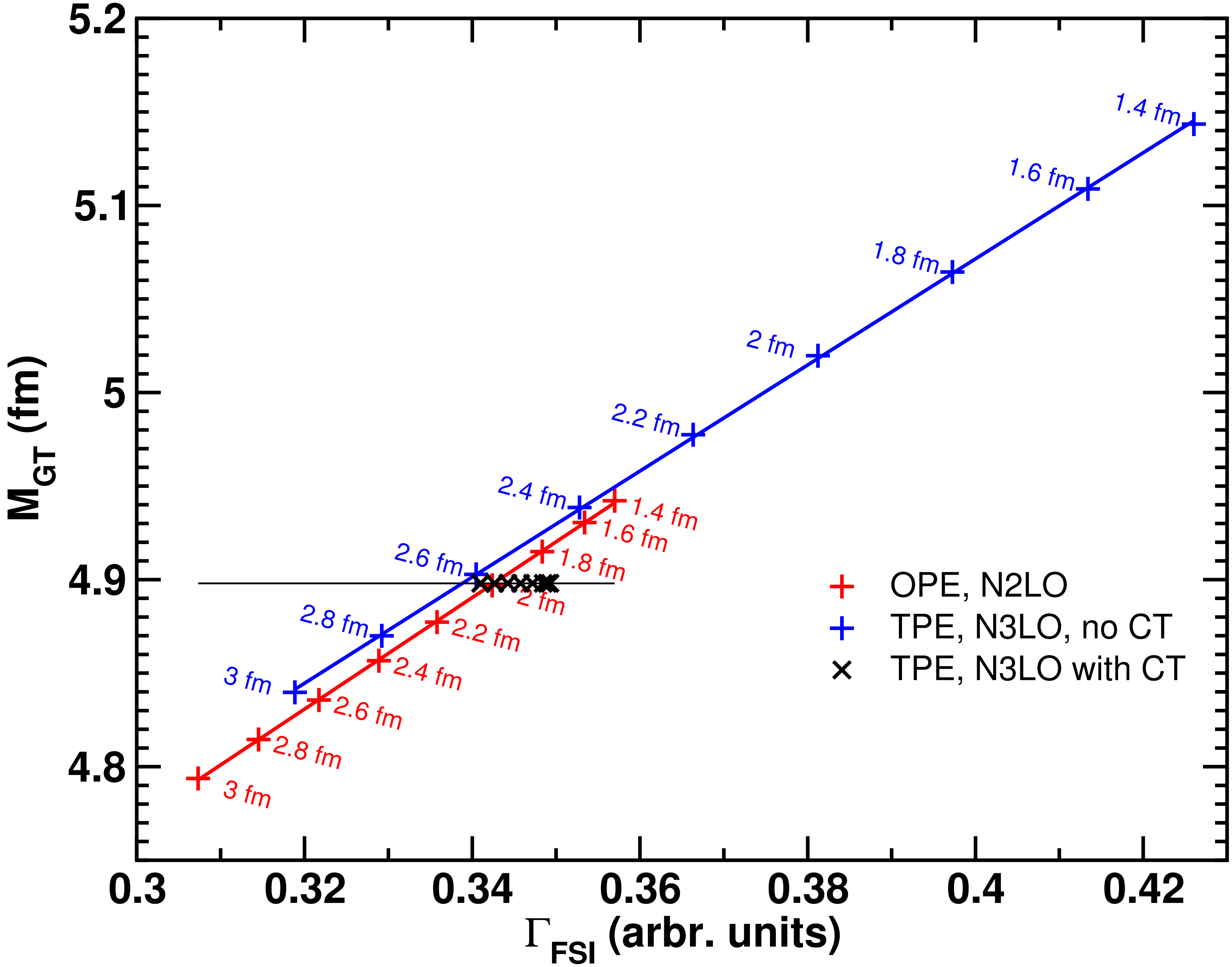
FIG. 4. The results for $^2a_{nd}$ and $E_{^3\mathrm{H}}$ from Table I: *np*-*nn* forces alone (pluses), *np*-*pp* forces alone (squares), and *np*-*nn* and *np*-*pp* forces plus electromagnetic interactions (stars and circles, respectively). The four straight lines (Phillips lines) are χ^2 fits (*np*-*nn*, solid; *np*-*pp*, dashed-dotted; *np*-*nn* with EMI's, dashed; *np*-*pp* with EMI's, dotted). The lines with EMI's miss the experimental error bar for $^2a_{nd}$ [33]. The physically interesting domain around the experimental values is shown in the inset.

Gamow-Teller vs FSI



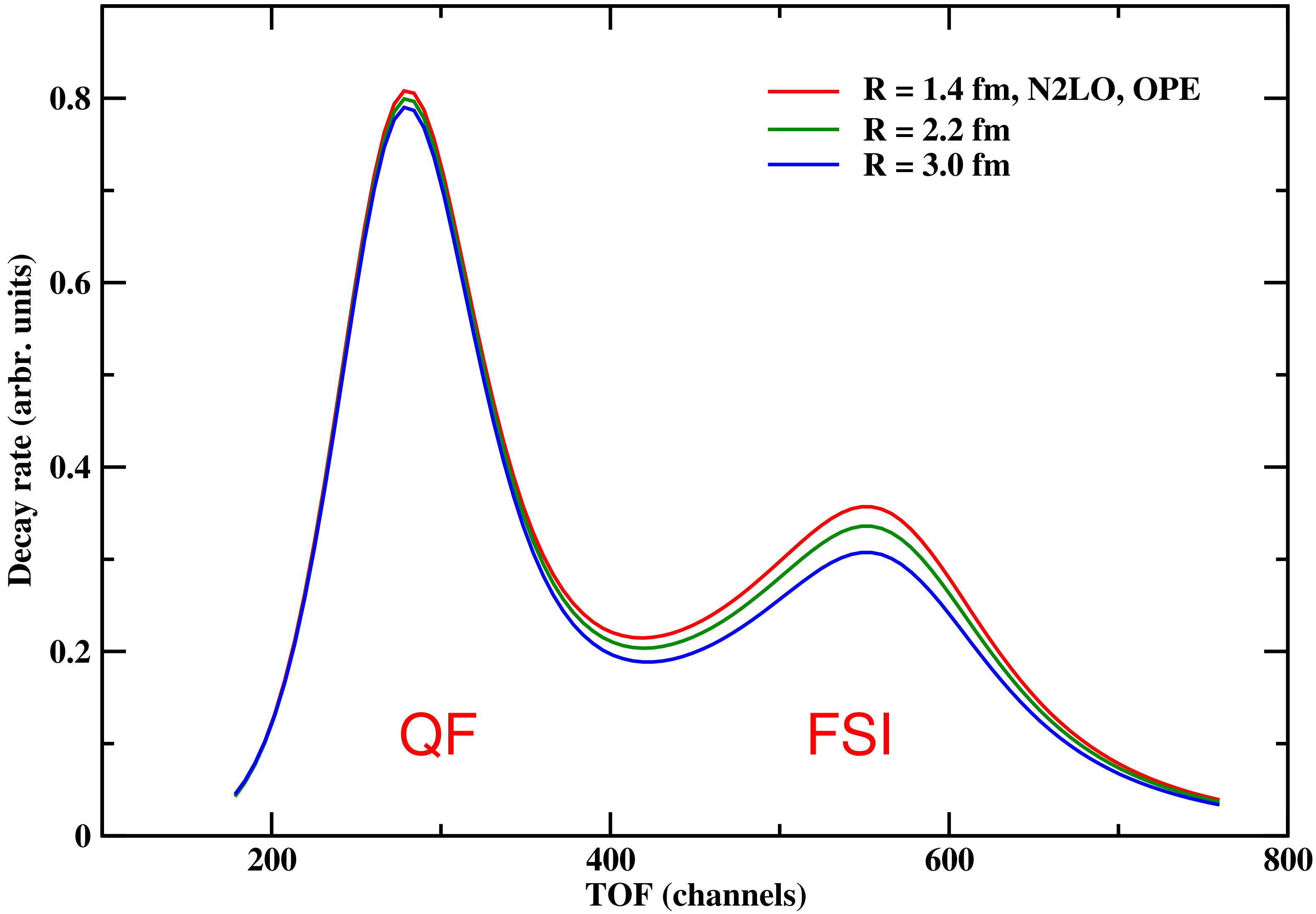
M_{GT} of pp fusion vs FSI peak height

Gårdestig-Phillips line(s)



M_{GT} of pp fusion vs FSI peak height

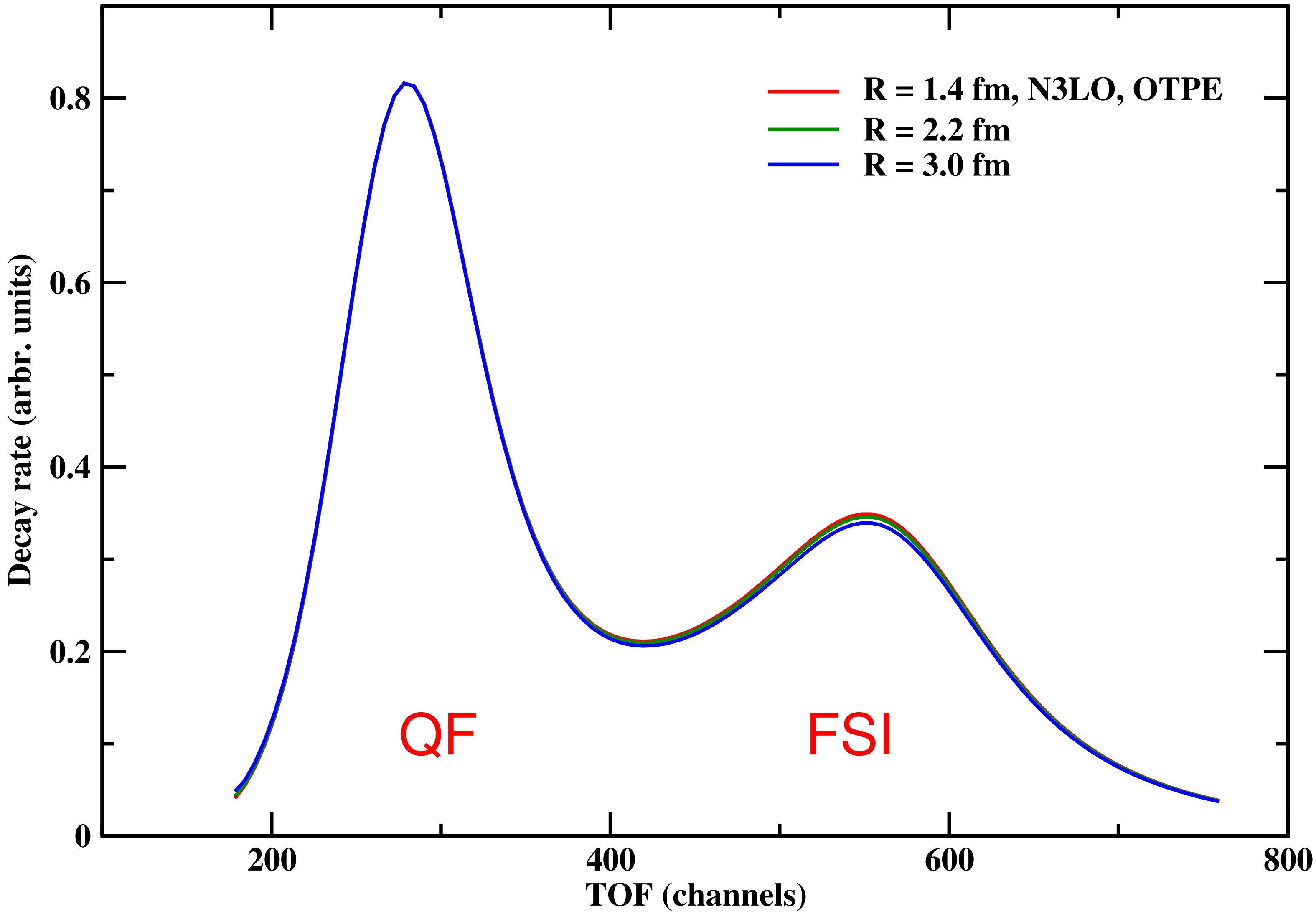
R-dep error at $\mathcal{O}(Q^3)$



$\Delta a_{nn}(\text{theory}) = \pm 0.2$ fm (FSI only)

$\Delta a_{nn}(\text{theory}) = \pm 1$ fm (full spectrum)

R-dep error at $\mathcal{O}(Q^4)$



Δa_{nn}(theory) $= \pm 0.05$ fm (FSI only)

Δa_{nn}(theory) $= \pm 0.3$ fm (full spectrum)

Summary and Conclusions I

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  [AG+DRP, PRL **96**, 232301 (2006)]

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- Better input possible from $\gamma d \to nn\pi^+$ or $\mu^- d \to nn\nu_\mu$?
- Complete the circle:
  $\mu^- d \to nn\nu_\mu$ (1%) at PSI; expt and calc under way!
  $\nu(\bar{\nu})d$ breakup (SNO) with chiral wfs
  $^3\mathrm{H} \to {}^3\mathrm{He} e^- \bar{\nu}_e$ with (r-space) chiral wfs?

Summary and Conclusions II

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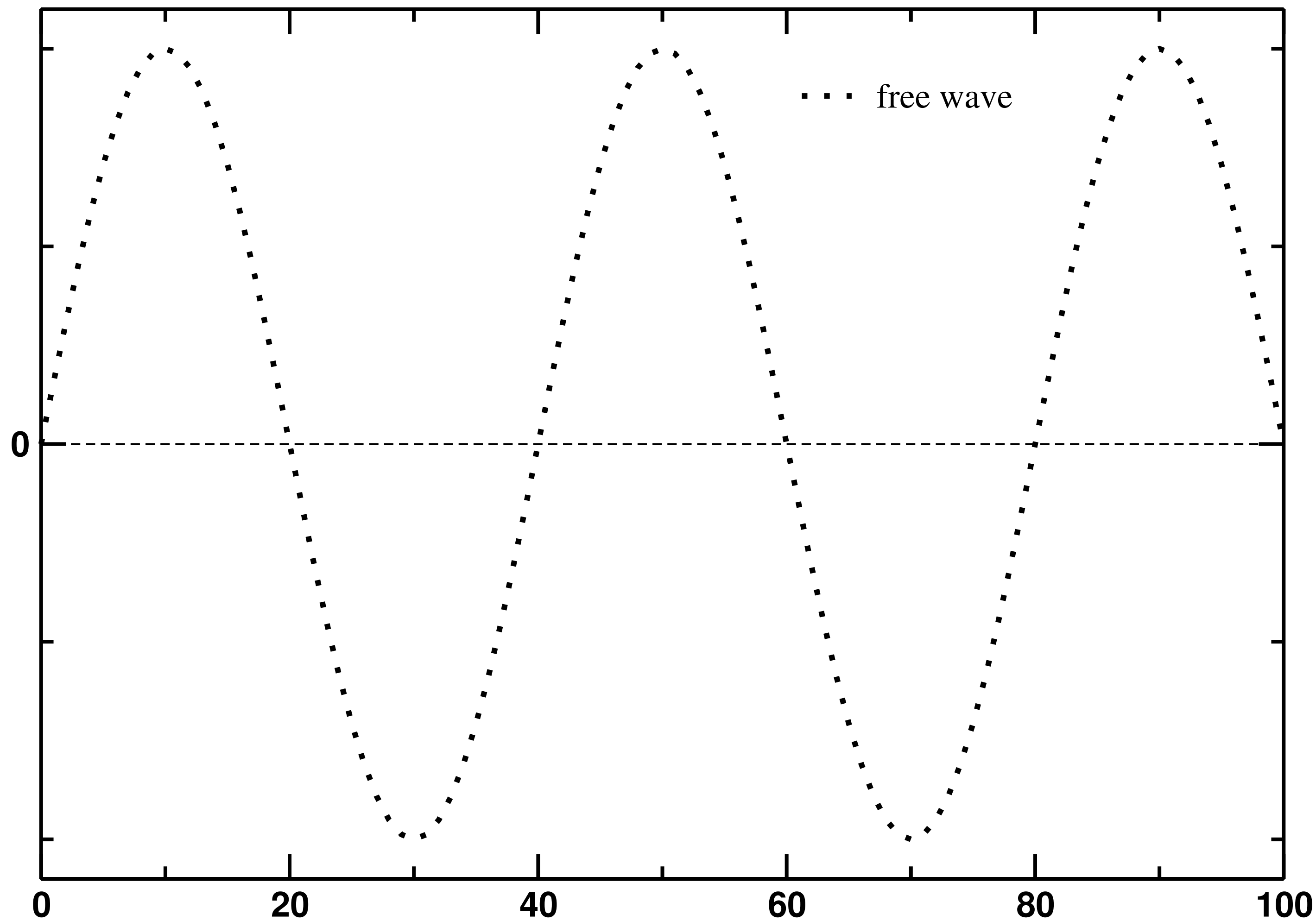
- direct measurement of a_{nn} finally possible?

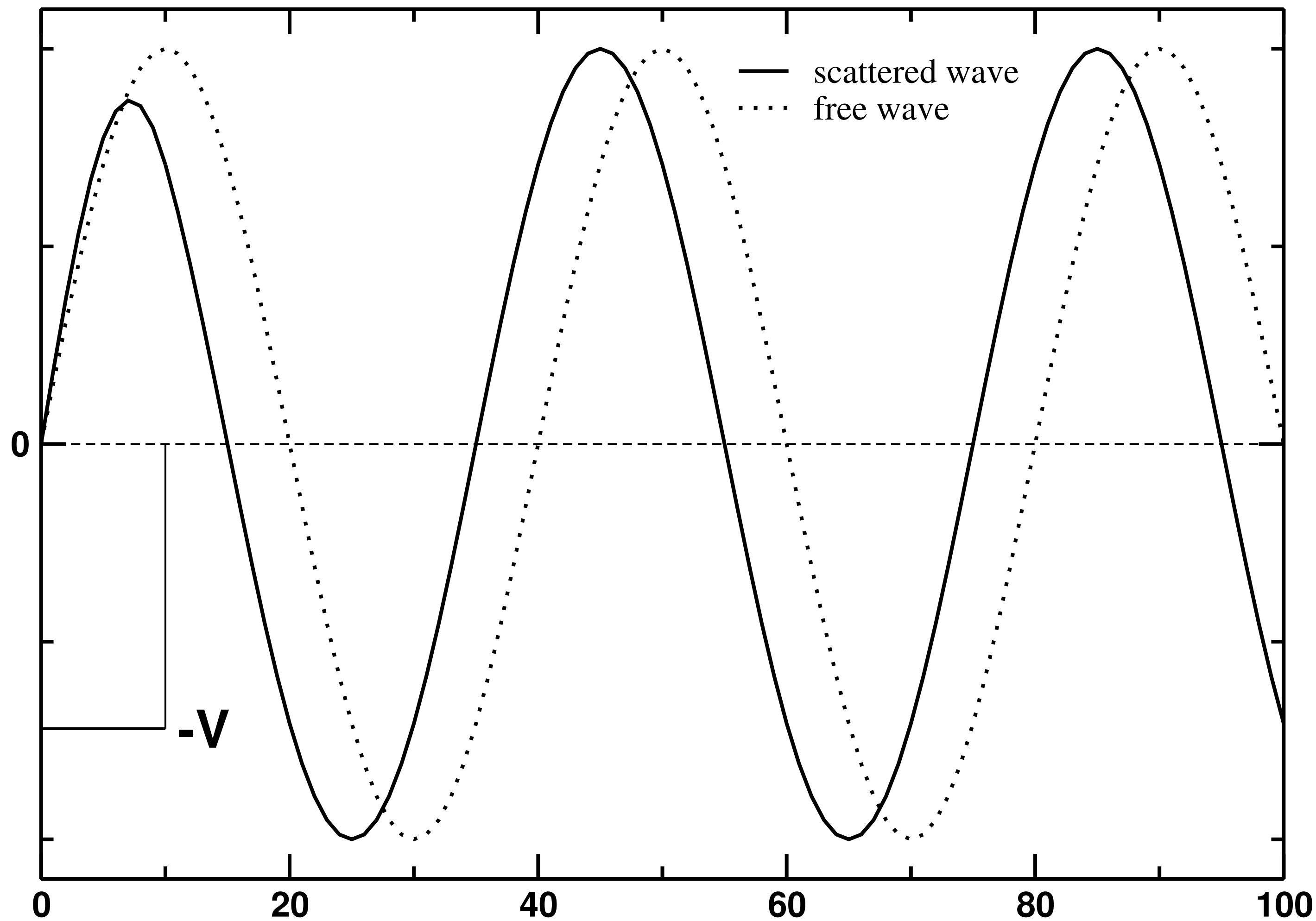
Intro to Phase Shifts

Free wave (radial part of 3D): $u_l \sim \sin(pr - \frac{l\pi}{2})$



Intro to Phase Shifts

Scattered wave: $$u_l \sim \sin(pr - \frac{l\pi}{2} + \delta_l)$$



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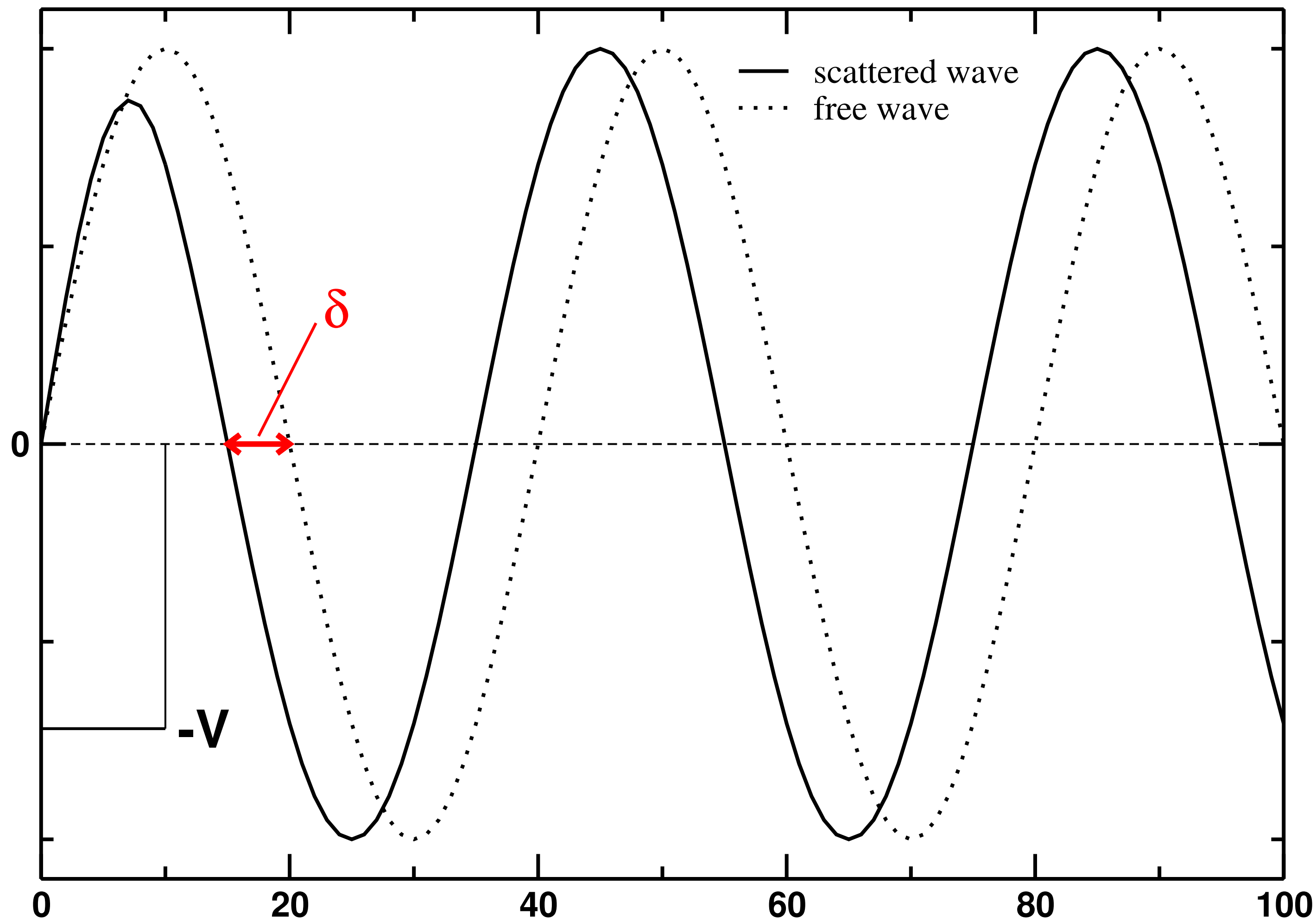


Interaction $V \Rightarrow$ phase shift:

$\delta > 0$ attractive potential

$\delta < 0$ repulsive potential

Definition of Scattering Length

The low energy cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{p^2}\sin^2\delta_0 \stackrel{p\to 0}{\longrightarrow} a^2$$

defining the (S-wave) scattering length

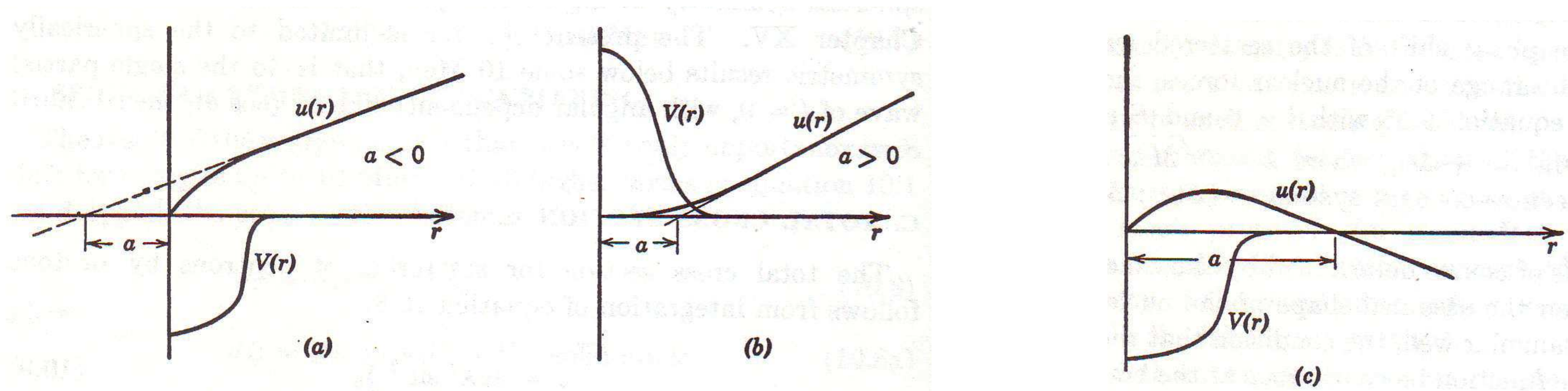
$$a \equiv -\lim_{p\to 0}\frac{\delta_0}{p}$$

where the sign is conventional.

For impenetrable sphere: $a > 0$ and $\sigma = 4\pi a^2$

Scattering length and wfs

At zero energy the asymptotic NN wave function behaves as $1 - r/a$



a) $a < 0$, attractive potential, scattering state

b) $a > 0$, repulsive potential

c) $a > 0$, attractive potential, bound state

CGLN amplitudes

Spin decomposition

$$\begin{aligned}\mathcal{A}_{\mathrm{I}}(\gamma N \to \pi N) &= F_1(E_\pi, x) i\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_\gamma + F_2(E_\pi, x)\boldsymbol{\sigma}\cdot\widehat{\mathbf{q}}\,\boldsymbol{\sigma}\cdot(\widehat{\mathbf{k}}\times\boldsymbol{\epsilon}_\gamma) \\ &+ F_3(E_\pi, x) i\boldsymbol{\sigma}\cdot\widehat{\mathbf{k}}\,\widehat{\mathbf{q}}\cdot\boldsymbol{\epsilon}_\gamma + F_4(E_\pi, x) i\boldsymbol{\sigma}\cdot\widehat{\mathbf{q}}\,\widehat{\mathbf{q}}\cdot\boldsymbol{\epsilon}_\gamma\end{aligned}$$

Isospin

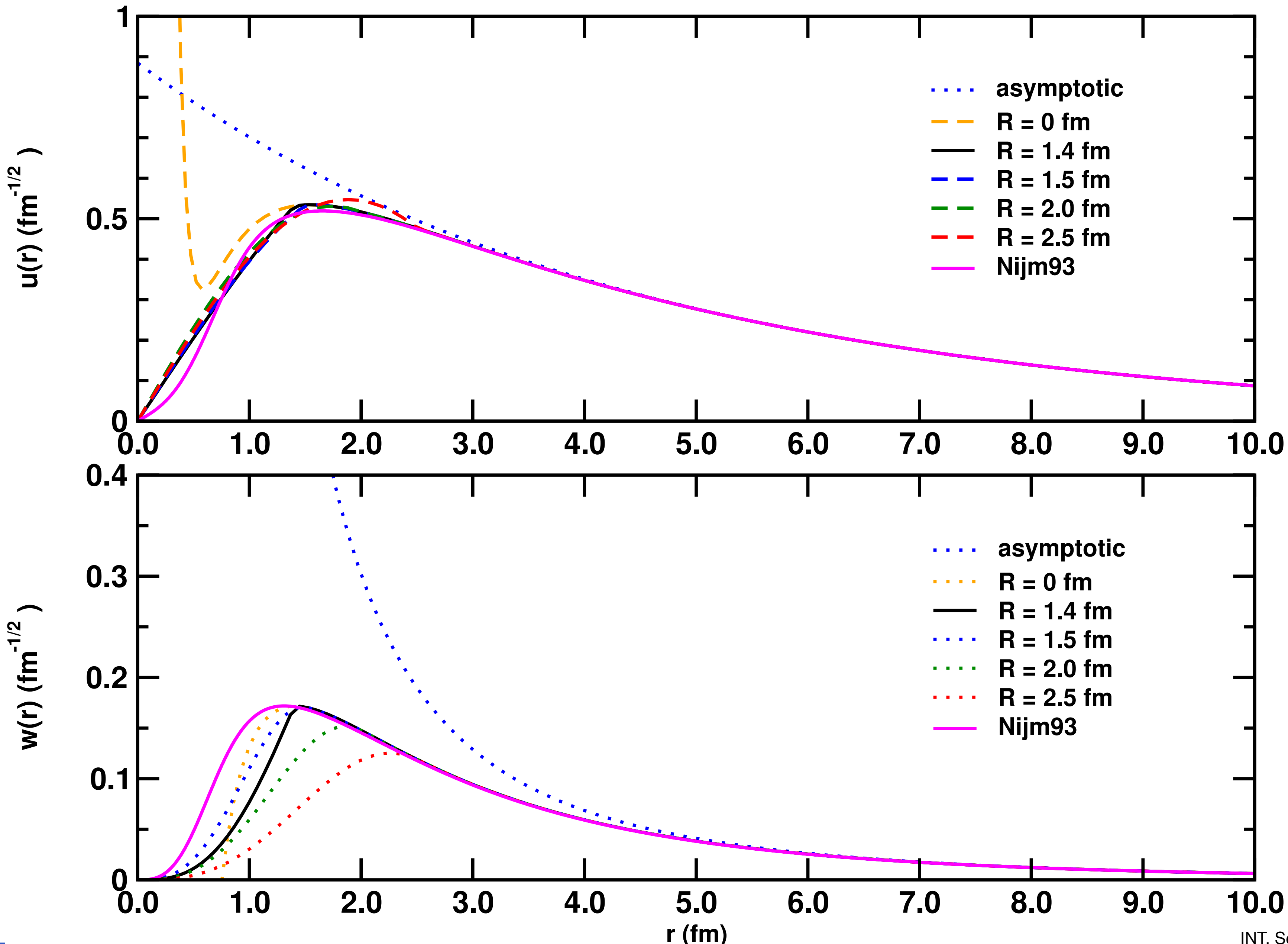
$$F_i^a(E_\pi, x) = F_i^{(-)}(E_\pi, x) i\epsilon^{a3b}\tau^b + F_i^{(0)}(E_\pi, x)\tau^a + F_i^{(+)}(E_\pi, x)\delta^{a3}$$

and for $\gamma n \to \pi^- p$

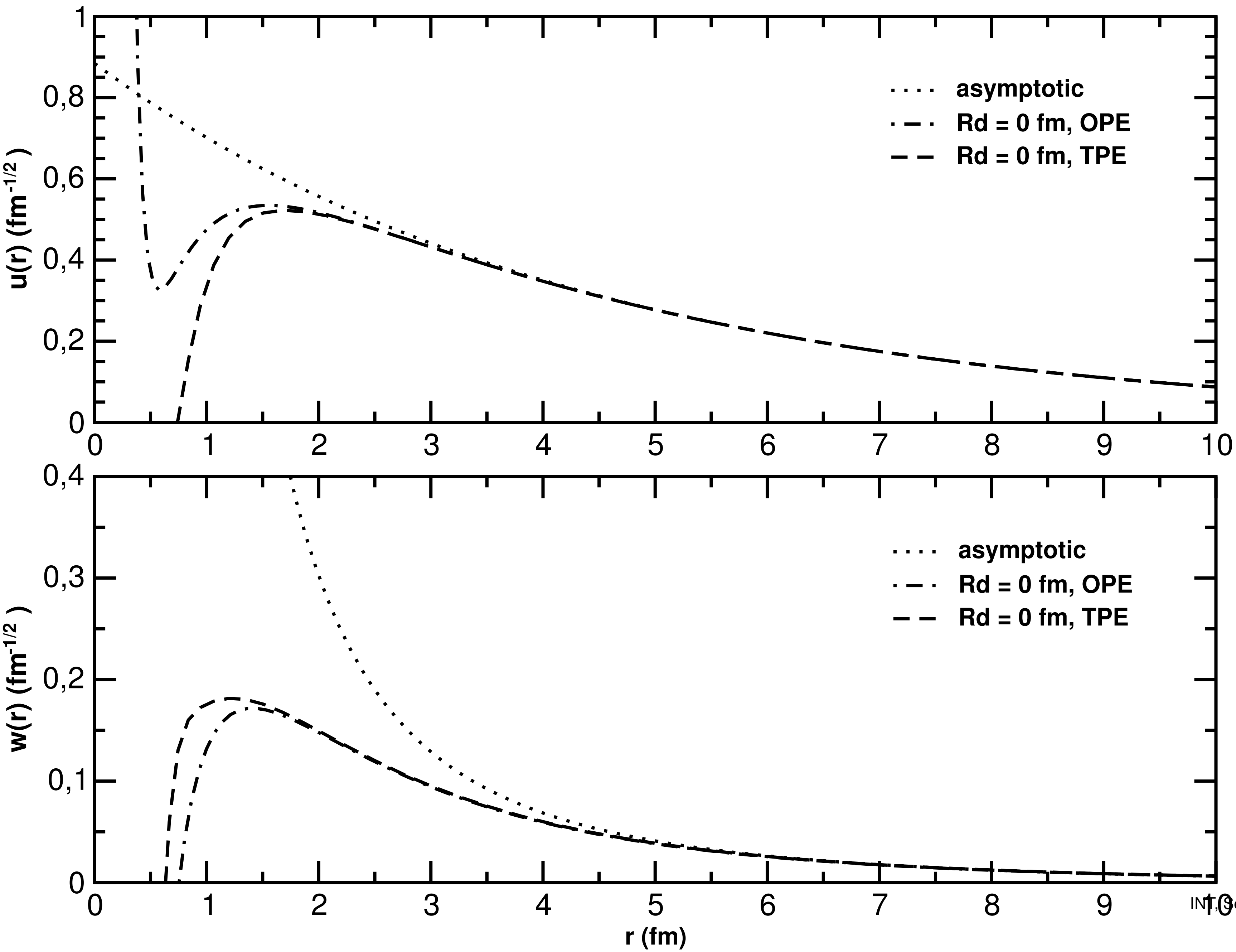
$$F_i(\gamma n \to \pi^- p) = \sqrt{2}[F_i^{(0)} - F_i^{(-)}]$$

$\boldsymbol{q} = \mathbf{0} \Rightarrow$ only F_1, dominated by KR for charged pions

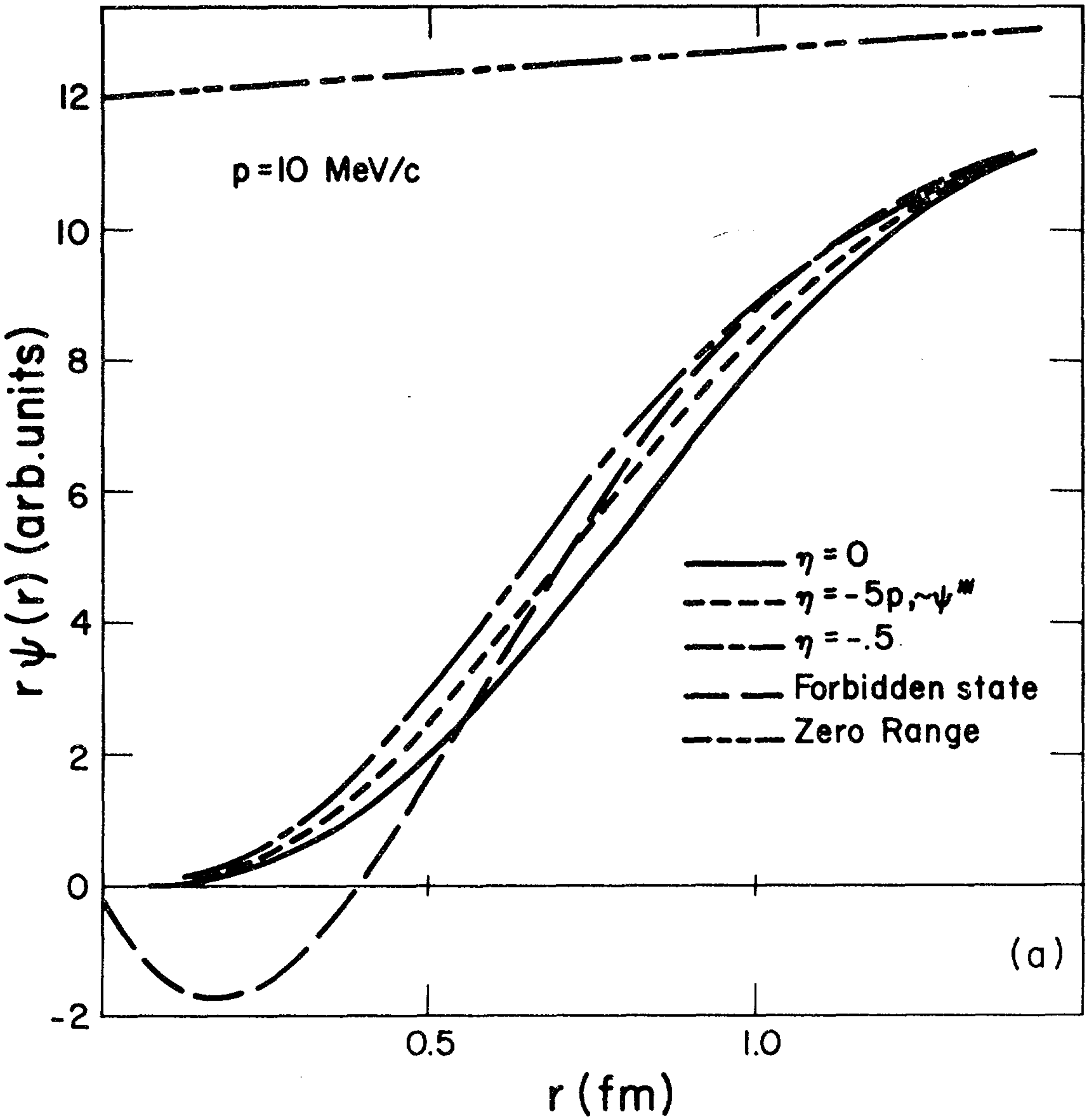
Deuteron wave functions (OPE).



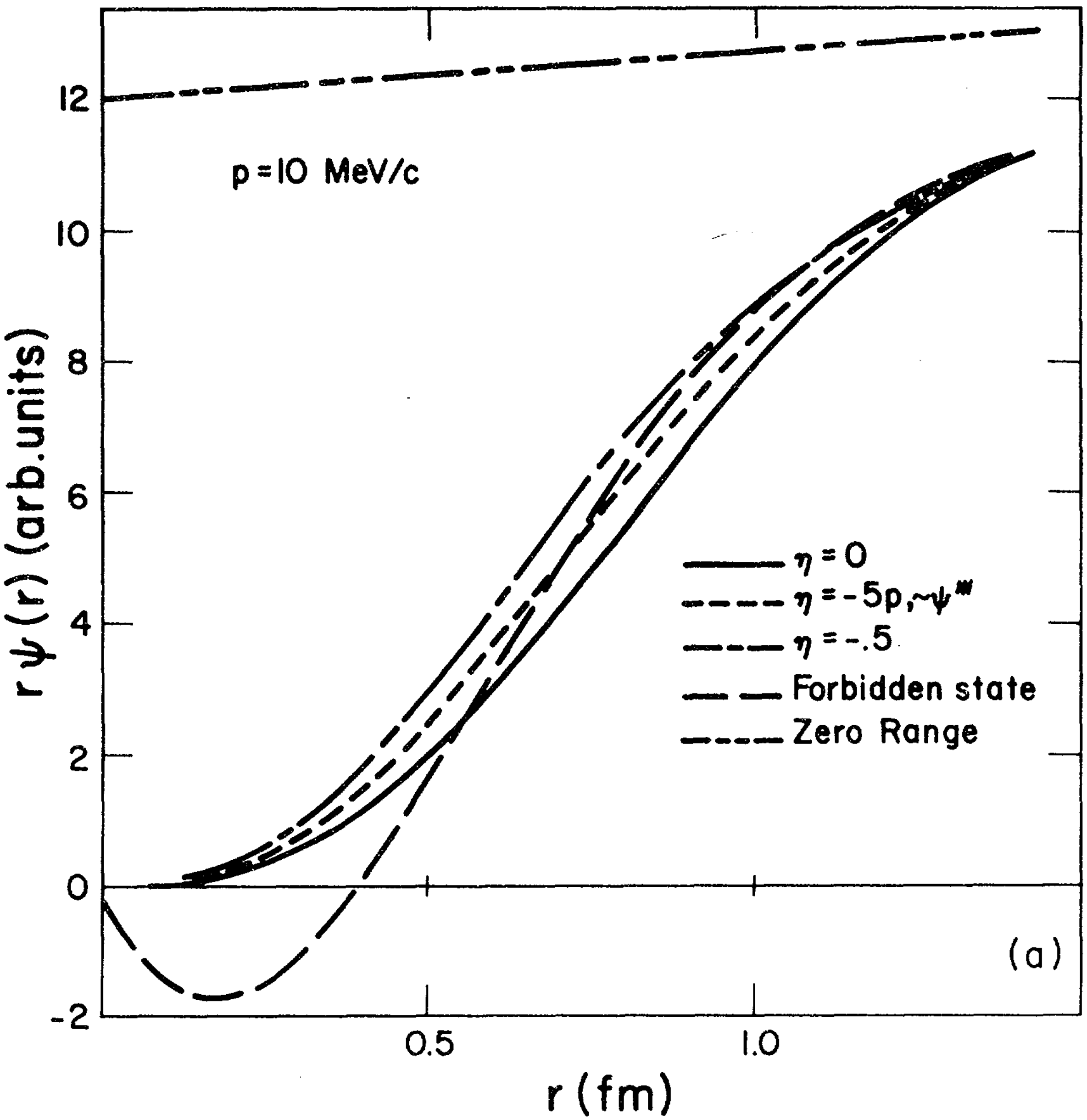
Deuteron wave functions (TPE).



nn scattering wfs, GGS



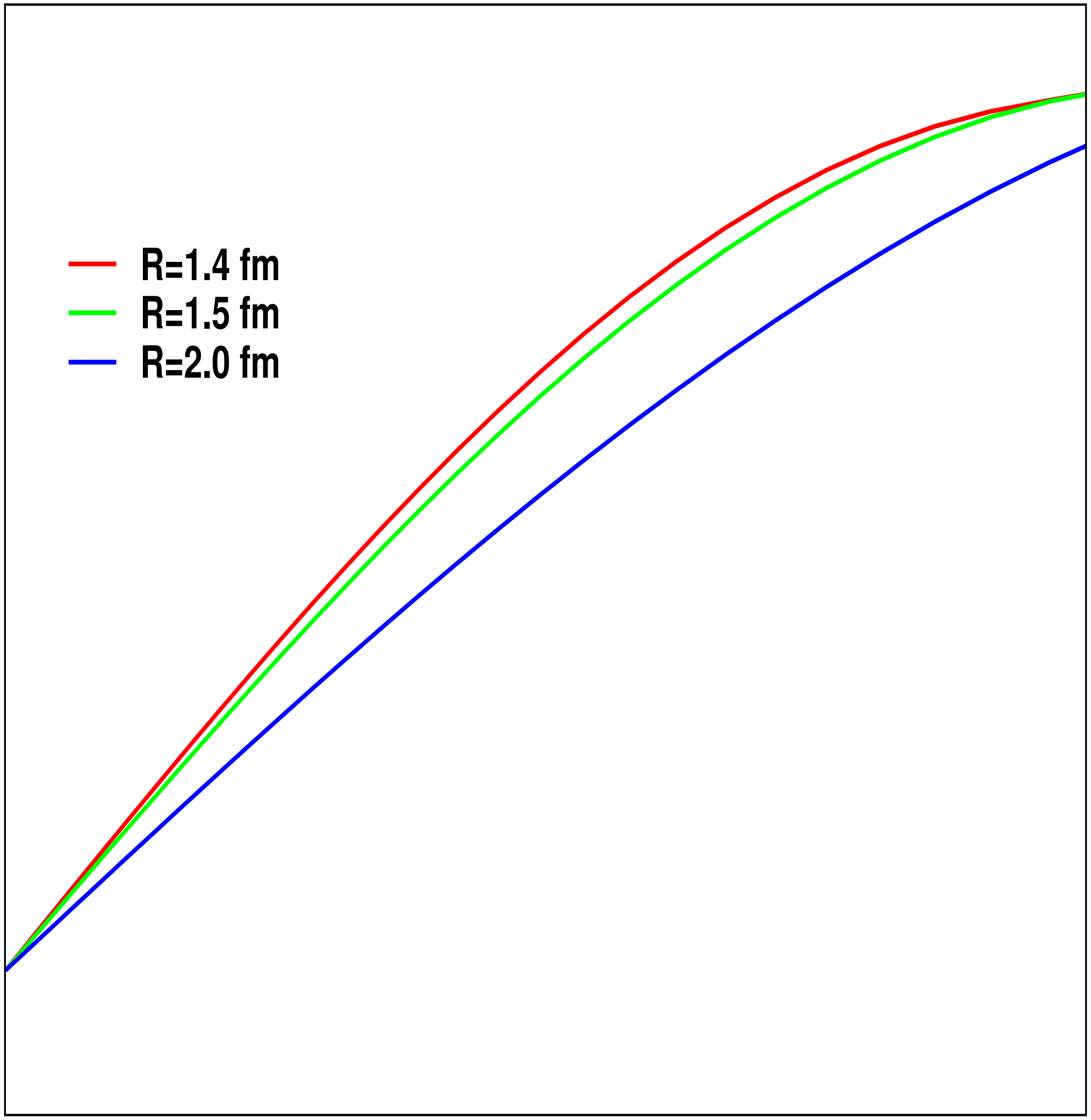
nn scattering wfs, GGS vs GP



nn scattering wfs, GGS

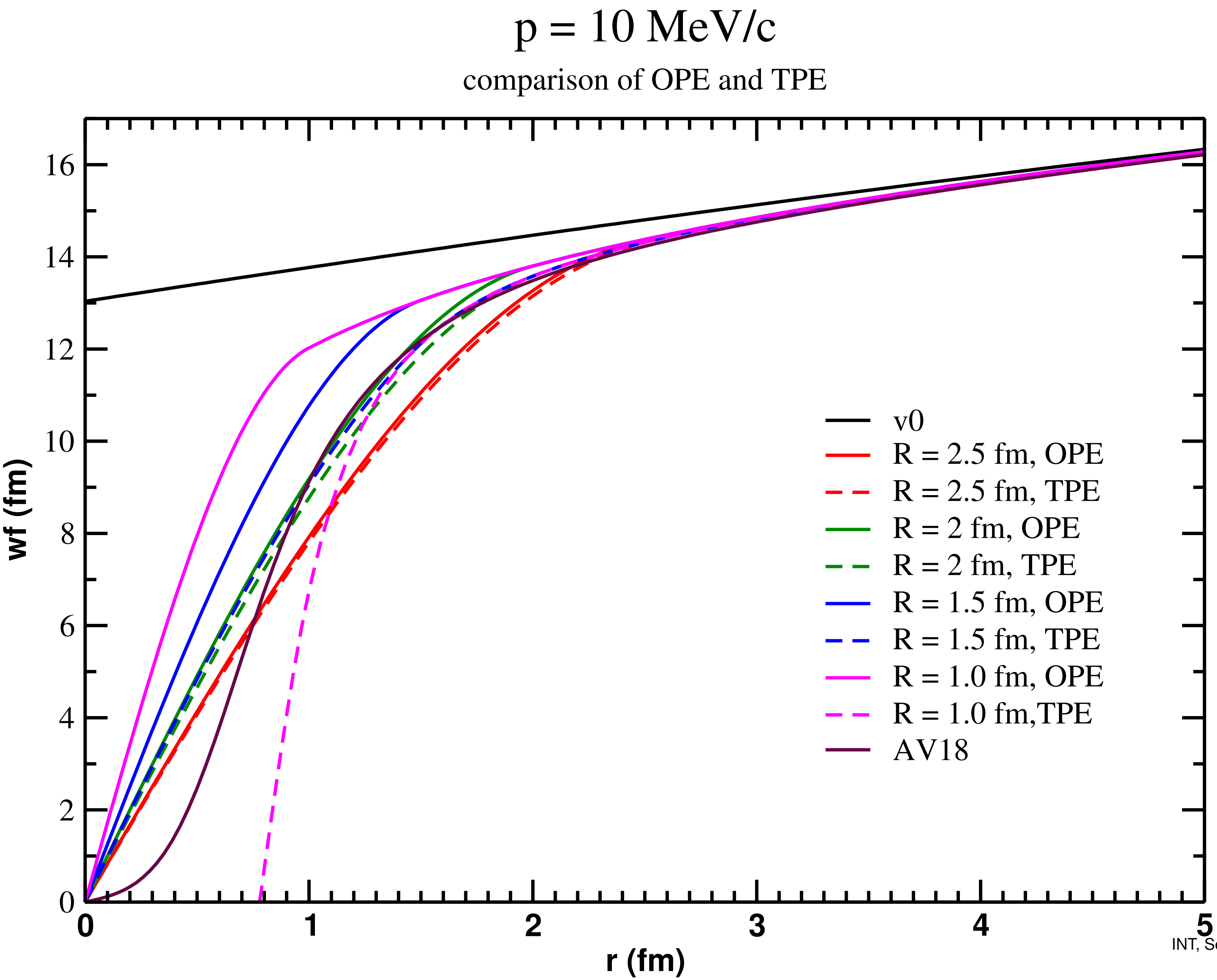
GGS use $P_5(r)$

nn scattering wfs, GGS vs GP



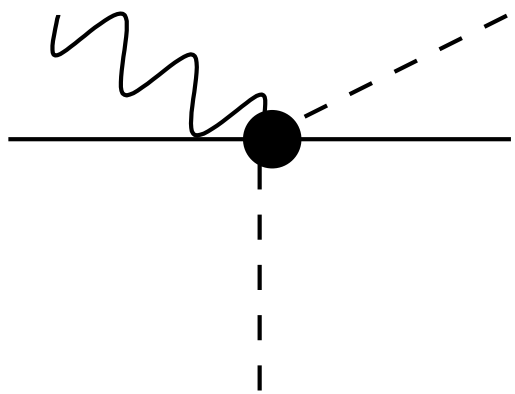
GGS use $P_5(r)$

TPE wfs


p = 10 MeV/c
comparison of OPE and TPE
wf (fm)
r (fm)
0
2
4
6
8
10
12
14
16
0
1
2
3
4
5
v0
R = 2.5 fm, OPE
R = 2.5 fm, TPE
R = 2 fm, OPE
R = 2 fm, TPE
R = 1.5 fm, OPE
R = 1.5 fm, TPE
R = 1.0 fm, OPE
R = 1.0 fm,TPE
AV18

Feynman rule for $\gamma\pi\pi NN$

$\hat{d}$ can only be established if new FR derived:

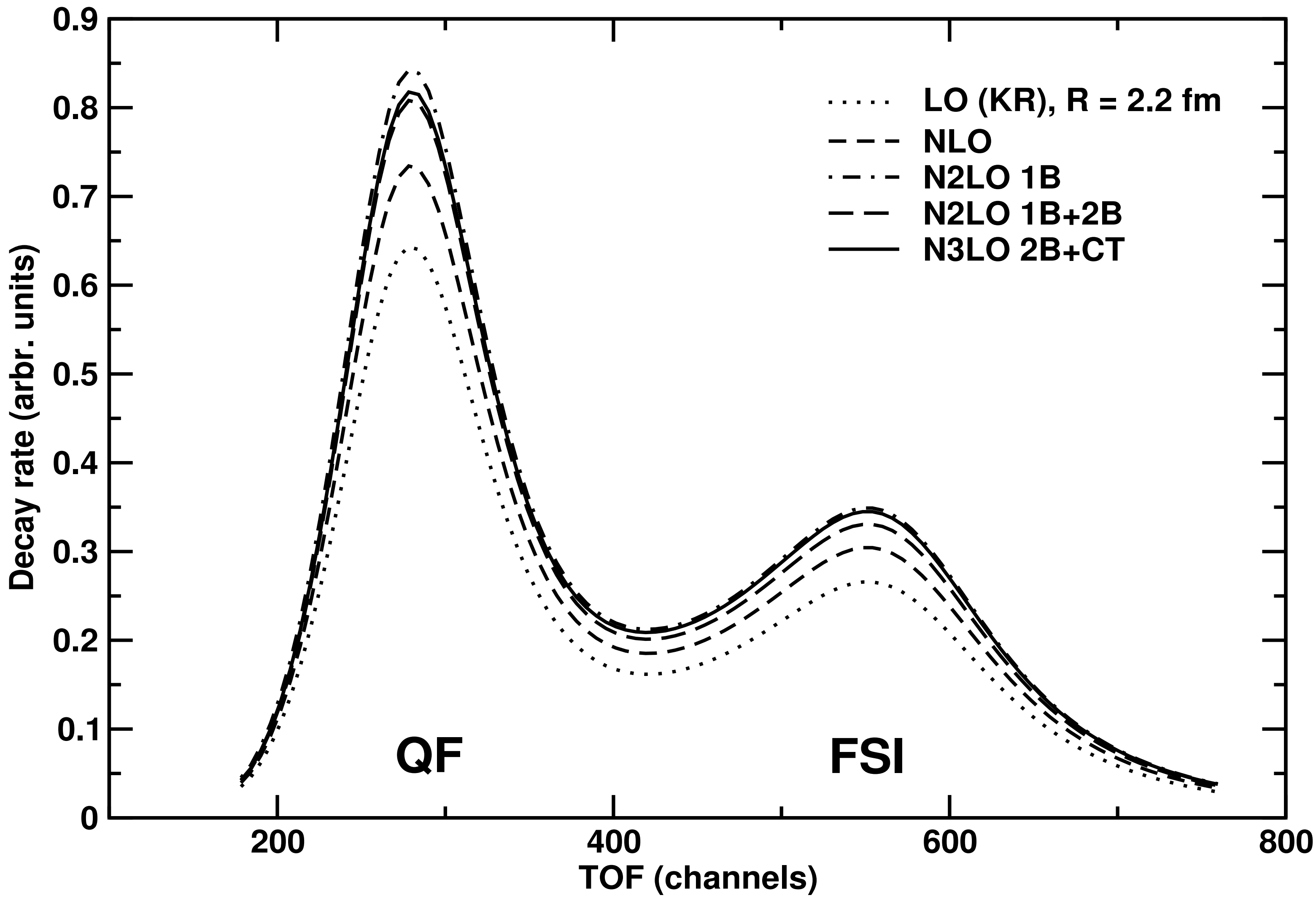


$$\left(c_4 + \frac{1}{4M}\right)\frac{2ie}{f_\pi^2}\left[\left(\delta^{ab}\tau^3 - \delta^{a3}\tau^b\right)[S\cdot q_1, S\cdot\epsilon_\gamma] - \left(\delta^{ab}\tau^3 - \delta^{b3}\tau^a\right)[S\cdot q_2, S\cdot\epsilon_\gamma]\right]$$

Not published before (not in [Bernard, Kaiser, Meißner, IJMPE **4**, 193 (1995)])

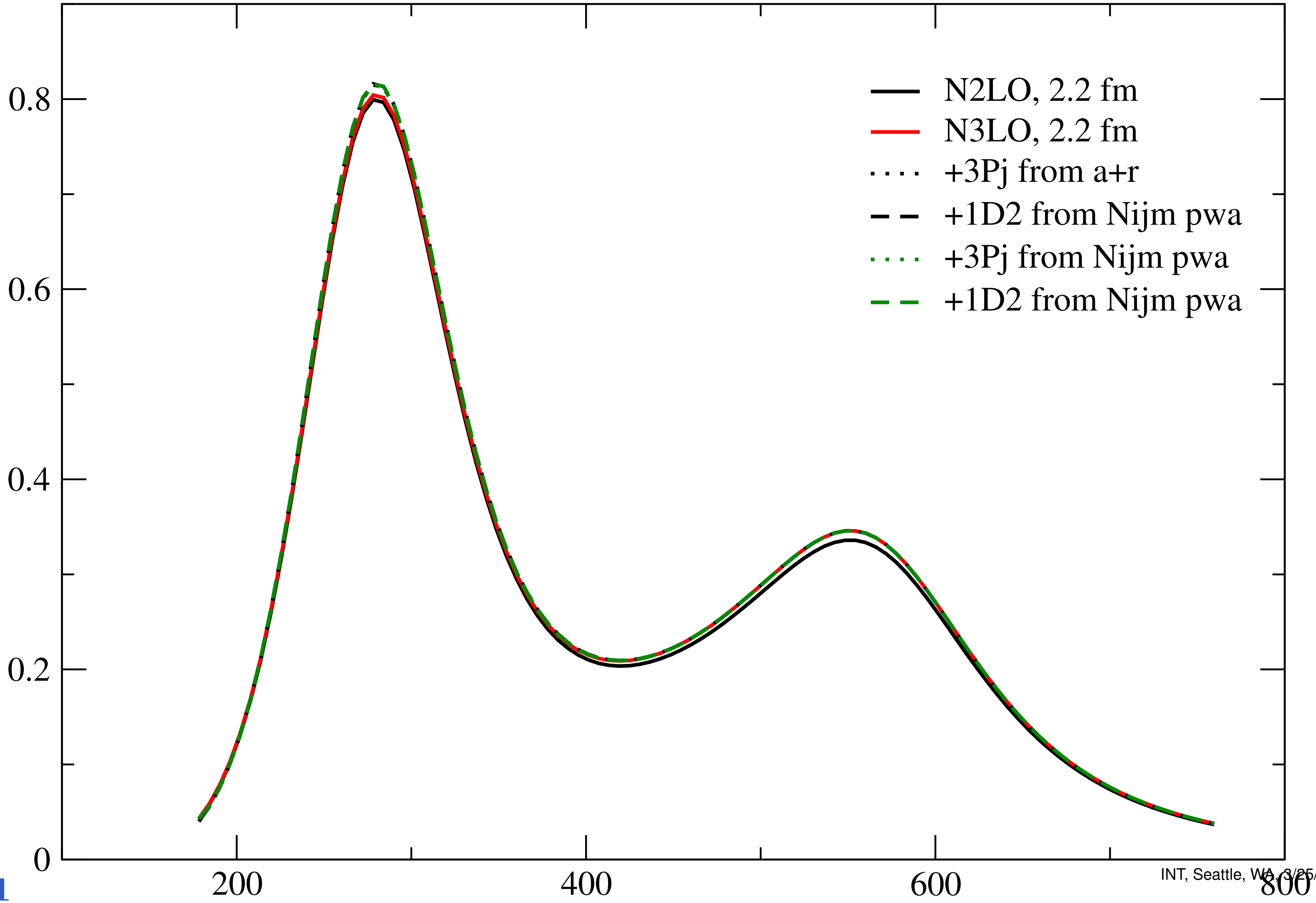
[AG, PRC **74**, 017001 (2006)]

Convergence

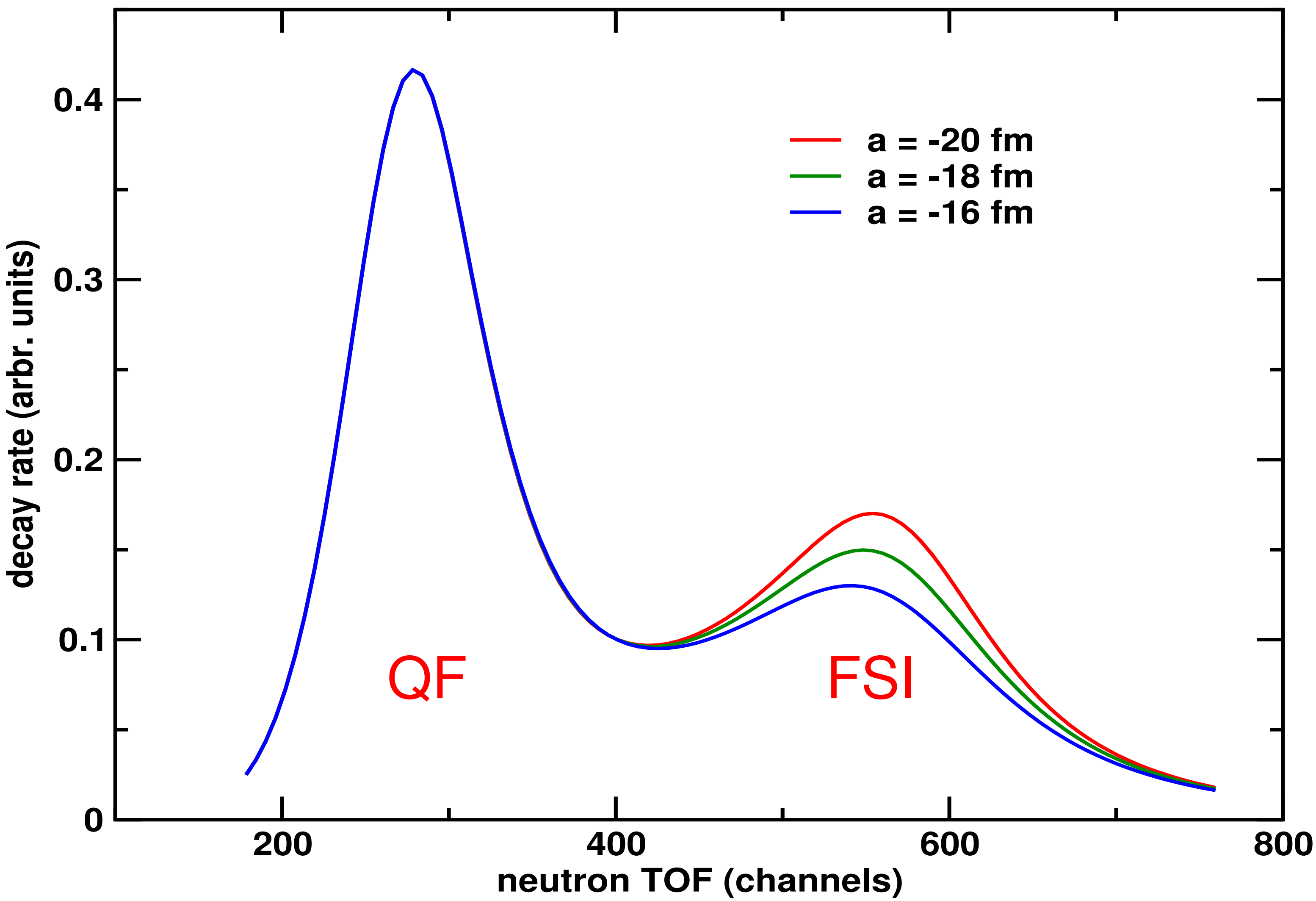


$$\frac{\Gamma_{\mathrm{QF}}}{\Gamma_{\mathrm{FSI}}} = 2.422(1 - 0.0035 + 0.0003 + 0.013 - 0.035)$$

Role of higher partial waves



Sensitivity to a_{nn}



Neutron TOF spectrum at $\theta_3 = 0.075$ rad $\Rightarrow \frac{\Delta a_{nn}}{a_{nn}} = 0.83 \frac{\Delta \Gamma}{\Gamma}$

Boost corrections

Corrections to CGLN

$$\Delta F_1^{(0)}(E_\pi) = \frac{eg_A}{2f_\pi}\frac{-(E_\pi \mathbf{p}_n\cdot\widehat{\mathbf{k}}+E_\pi^2)}{2M^2}(\mu_p+\mu_n)$$

$$\Delta F_1^{(-)}(E_\pi) = \frac{eg_A}{2f_\pi}\frac{E_\pi \mathbf{p}_n\cdot\widehat{\mathbf{k}}+E_\pi^2}{M^2}$$

New spin-momentum structures

$$G^{(0)}(E_\pi) = \frac{eg_A}{2f_\pi}\frac{iE_\pi \mathbf{p}_n\cdot\boldsymbol{\epsilon}_\gamma\boldsymbol{\sigma}\cdot\widehat{\mathbf{k}}}{2M^2}(\mu_p+\mu_n-1)$$

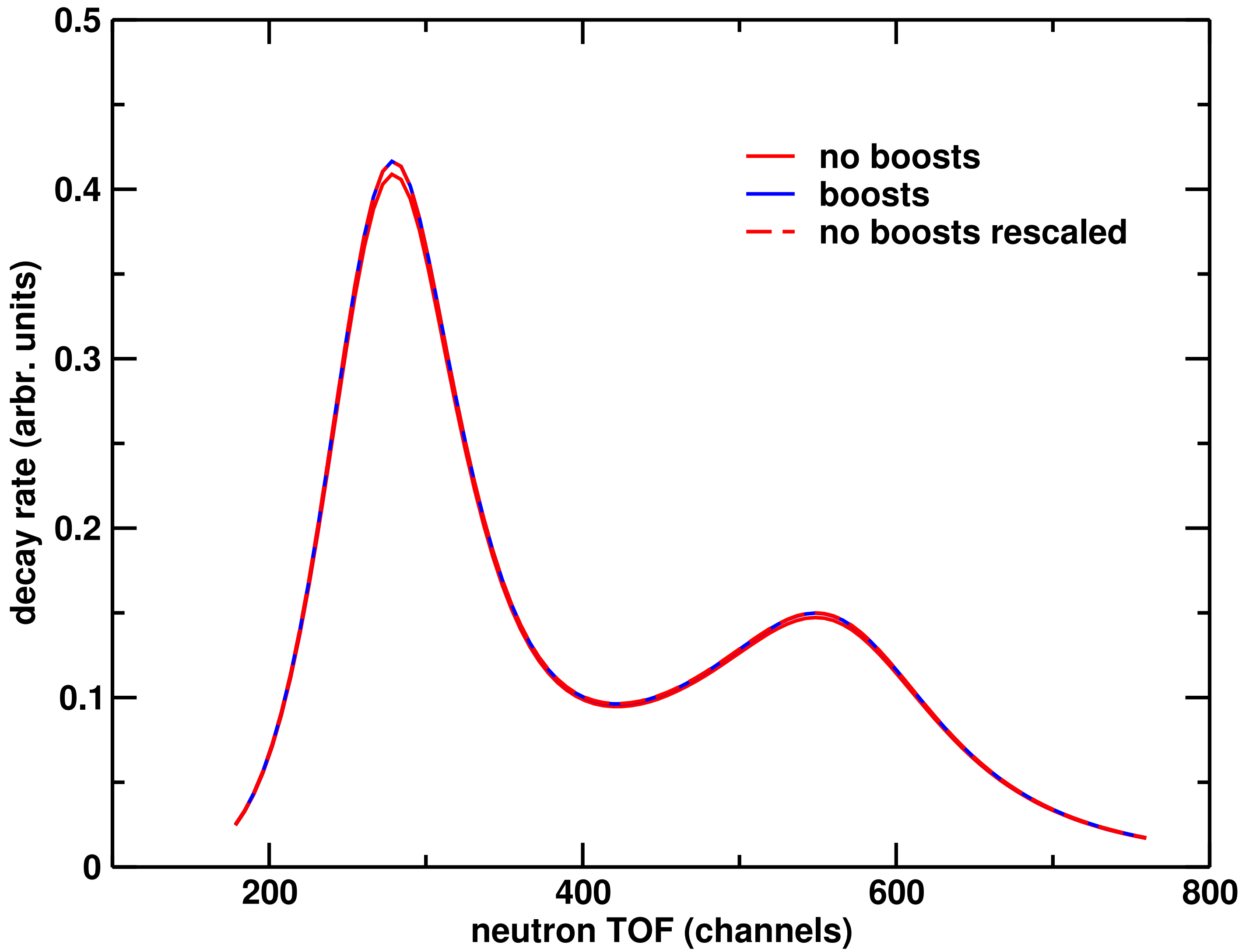
$$G^{(-)}(E_\pi) = \frac{eg_A}{2f_\pi}\left(\frac{E_\pi \mathbf{p}_n\cdot(\widehat{\mathbf{k}}\times\boldsymbol{\epsilon}_\gamma)}{2M^2}(\mu_p-\mu_n+\frac{1}{2})-\frac{i\mathbf{p}_n\cdot\boldsymbol{\epsilon}_\gamma\boldsymbol{\sigma}\cdot(2\mathbf{p}_n+E_\pi\widehat{\mathbf{k}})}{M^2}\right)$$

$\mu_p-\mu_n+\frac{1}{2}=5.2$, but $\mathbf{p}_n\cdot(\widehat{\mathbf{k}}\times\boldsymbol{\epsilon}_\gamma)\approx E_\pi^2\sin\theta_3$ with $\theta_3=0.075$ rad

similarly $\mathbf{p}_n\cdot\boldsymbol{\epsilon}_\gamma\approx E_\pi\sin\theta_3$

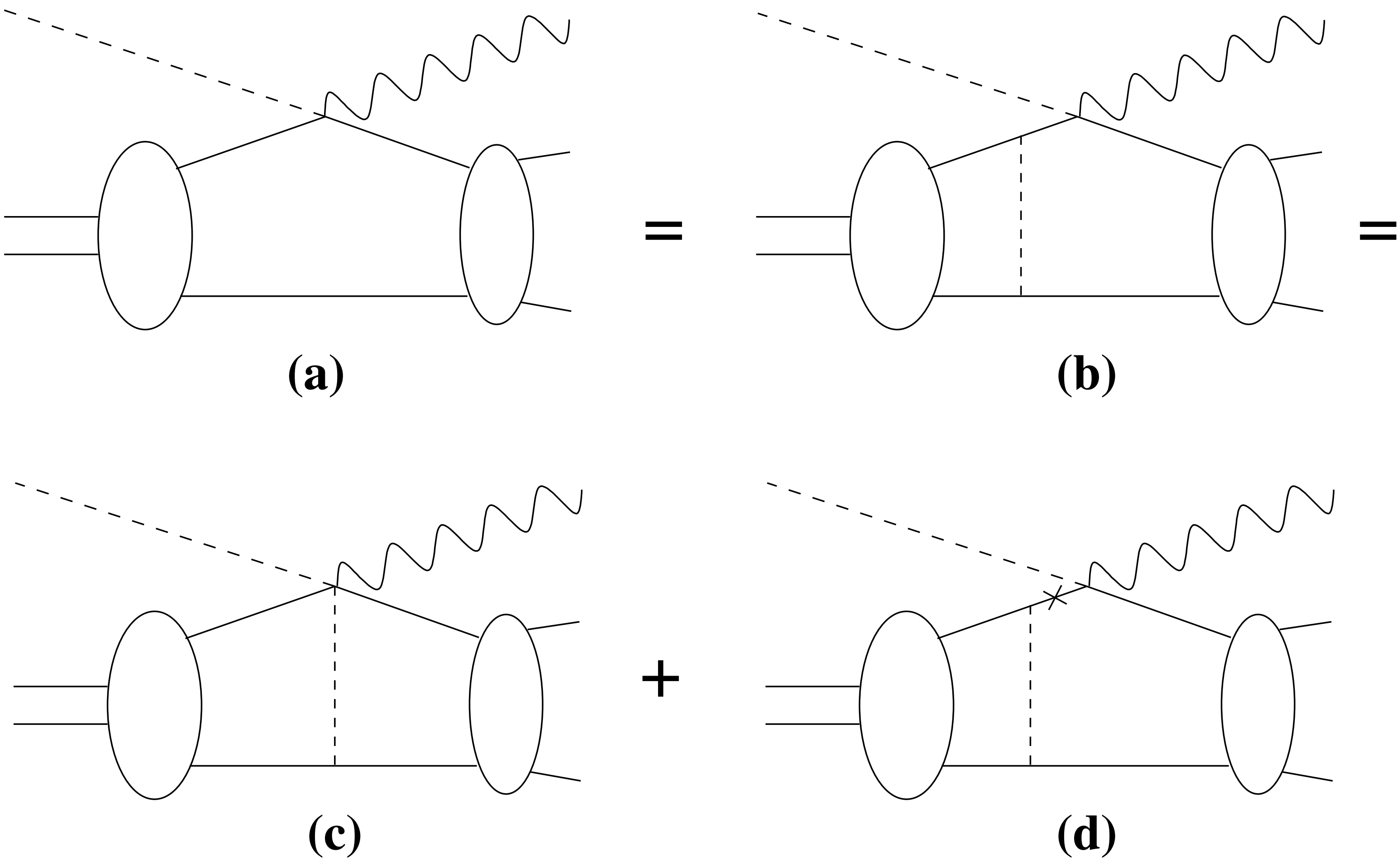
Thus only CGLN corr's important, $O(\mu^2/2M^2)\sim 1\%$

To boost or not to boost?



Both peaks scale the same way $\Rightarrow 0.10\%$ for a_{nn}

‘Off-shellness’

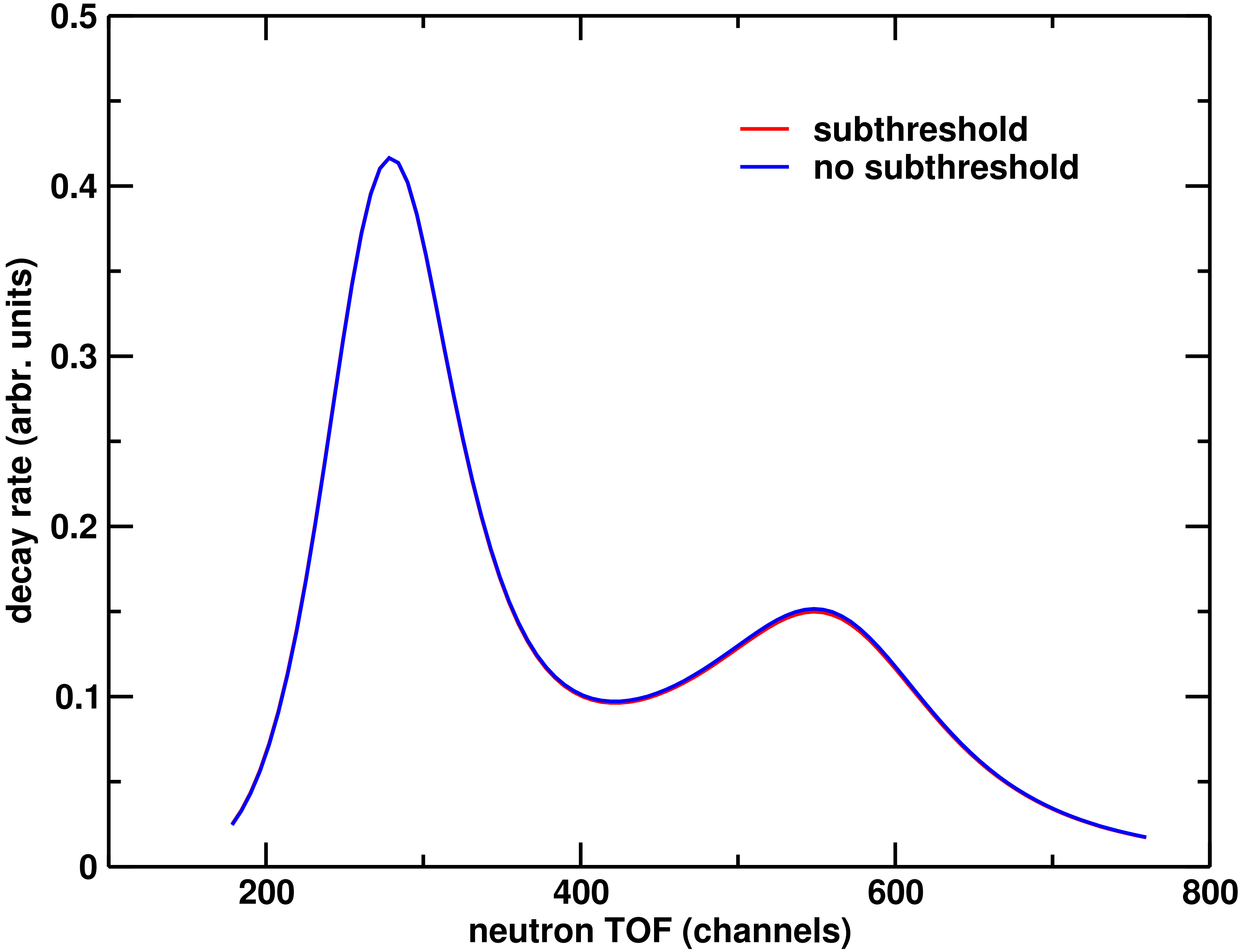


Off-shell nucleon transformed into 2B and on-shell 1B

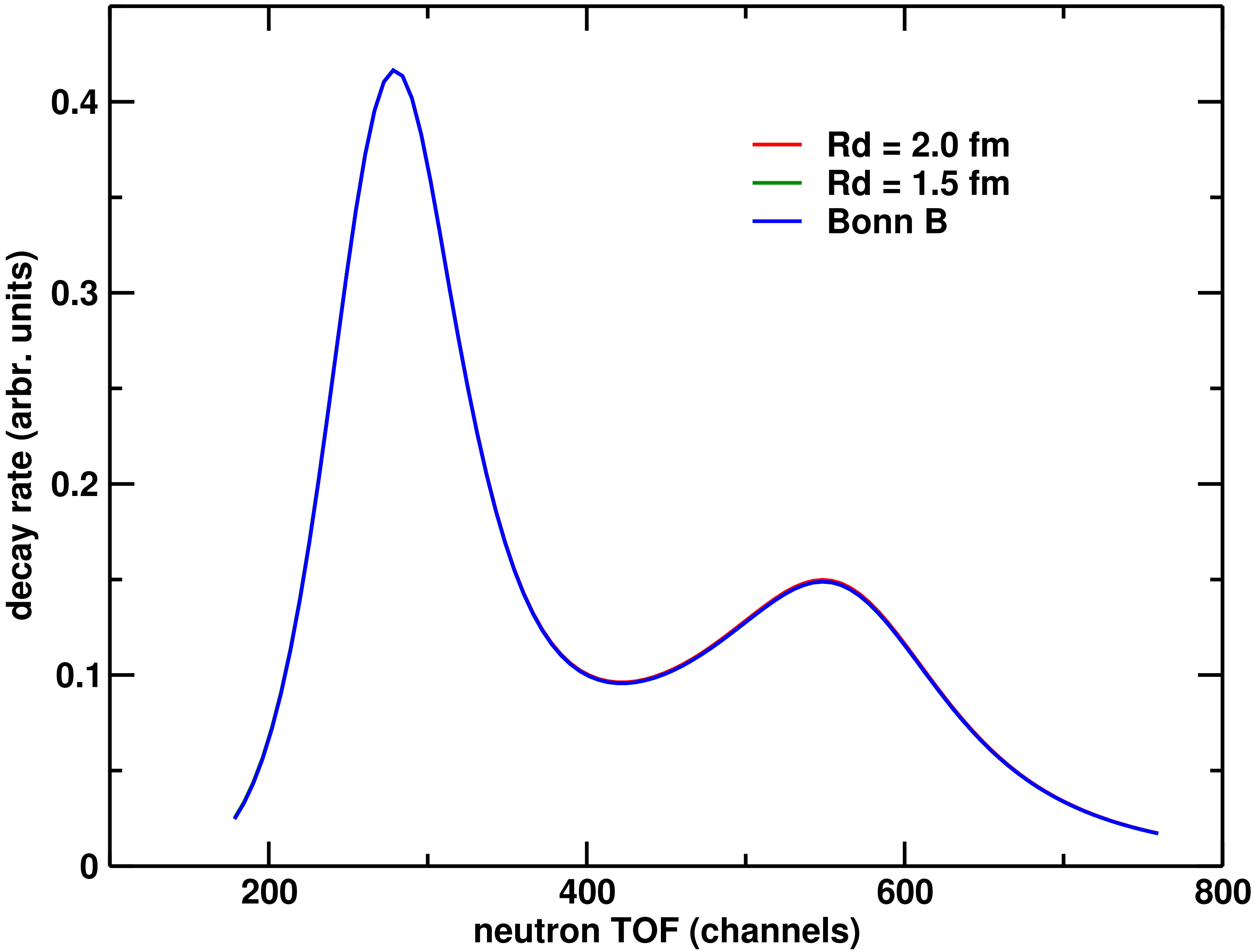
New 2B $O(Q^5) \Leftrightarrow p^2/M^2 \sim \mu^2/M^2 \sim 2\%$ of $O(Q^3)$ 2B

$\Rightarrow \Delta a_{nn} = 0.02$ fm

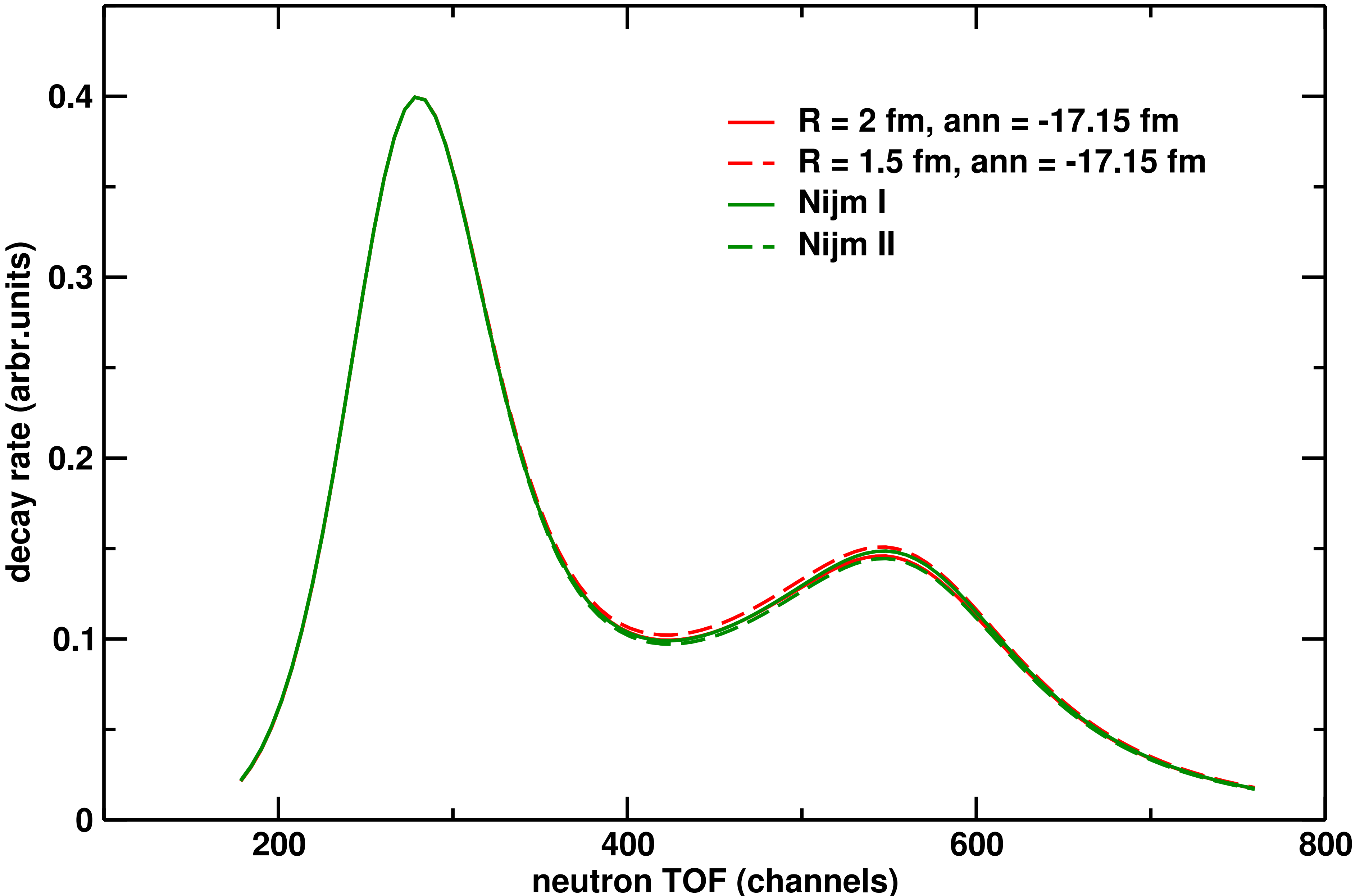
Subthreshold extrapolation



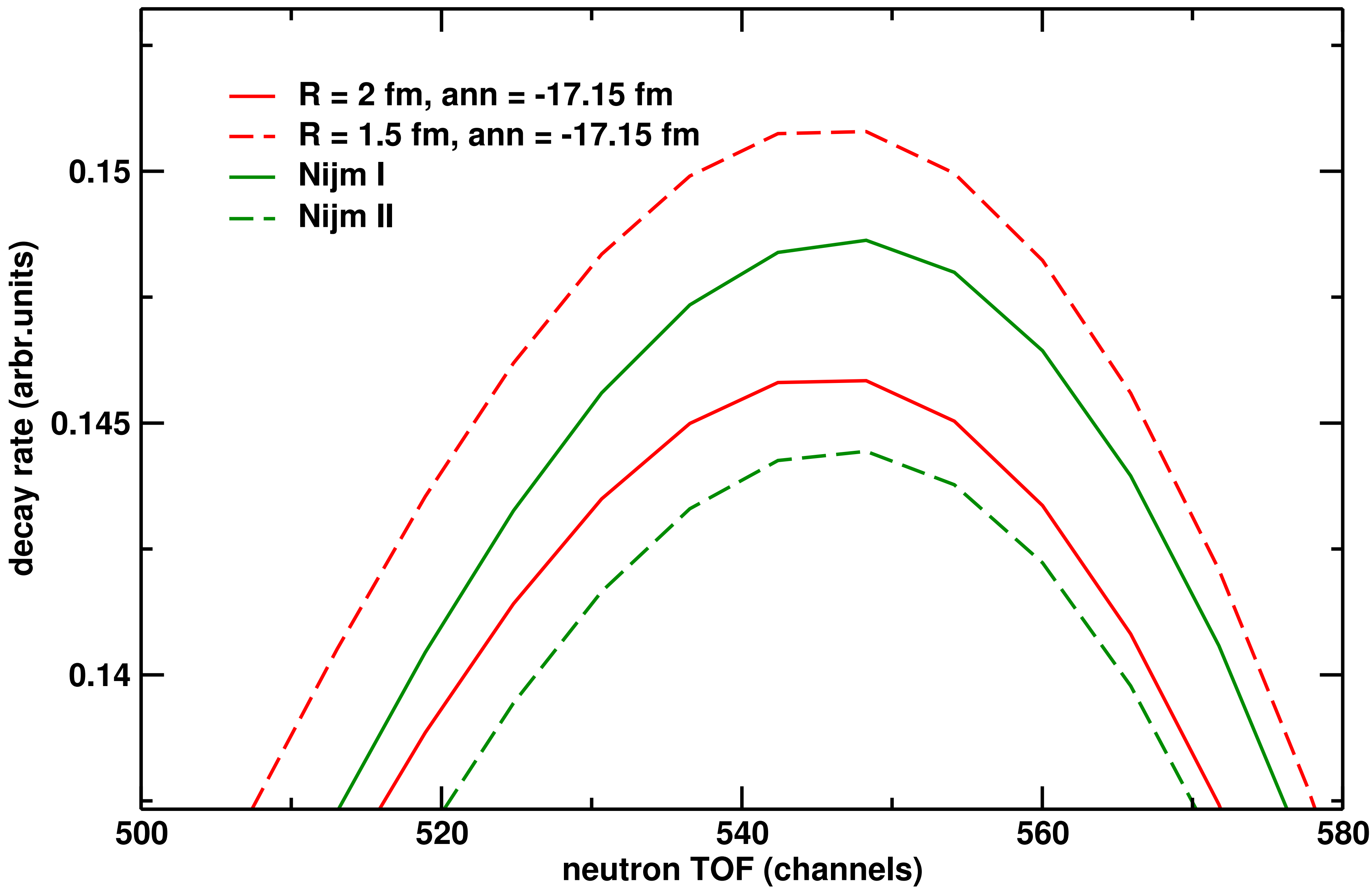
Error from *d* wfs



Error from nn wfs



FSI only



R = 2 fm, ann = -17.15 fm
R = 1.5 fm, ann = -17.15 fm
Nijm I
Nijm II
0.15
0.145
0.14
decay rate (arbr.units)
500
520
540
560
580
neutron TOF (channels)