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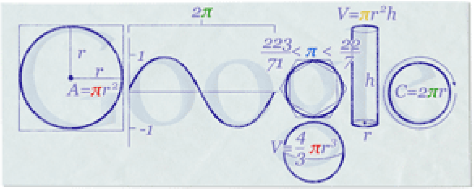
Seattle, 24.03.2010

On the chiral expansion of nπ-exchange: Hunting the powers of π

dedicated to the international Pi Day, 3/14 ☺



- The Why
- 2π-exchange: q-space vs r-space
- Partial resummation of n-pion exchange
- Further examples of the "π-enhancement"
- Summary and conclusions

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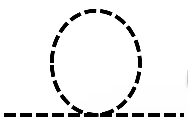
Introduction

ChPT = expansion of observables in powers of $\{q_i/\Lambda_\chi\,,\ M_\pi/\Lambda_\chi\}$

Pion loops are suppressed by $\dfrac{Q^2}{(4\pi F_\pi)^2}$. Here, Q^2/F_π^2 can be understood from dimensional reasons, while the factor $(4\pi)^2$ arises from the angular integration:

$$\int \frac{d^d l}{(2\pi)^d} = \int \frac{d\Omega_d}{(2\pi)^d} \int l^{d-1} dl = \frac{1}{2^{d-1}\,\pi^{d/2}\,\Gamma(d/2)} \int l^{d-1} dl \xrightarrow{d\to 4} \frac{2}{(4\pi)^2} \int l^3 dl$$



$$\Longrightarrow \frac{M_\pi^2}{F_\pi^2} \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - M_\pi^2 + i0^+} \xrightarrow{DR} M_\pi^2 \frac{M_\pi^2}{(16\pi^2 F_\pi^2)} \left[-\frac{2}{\epsilon} - \Gamma'(1) - 1 - \ln(4\pi) + \ln\left(\frac{M_\pi^2}{\mu^2}\right) + \mathcal{O}(\epsilon) \right]$$

194 *A. Manohar, H. Georgi / Chiral quarks*

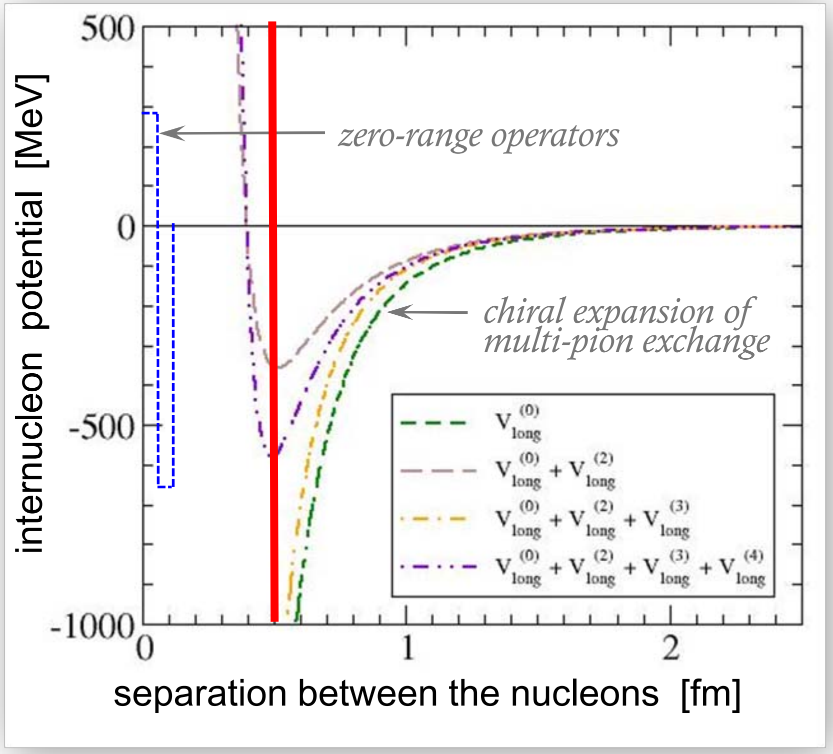
This suggests that we use

$$\Lambda_{\chi\mathrm{SB}} = 4\pi f\,, \qquad (2.26)$$

as an estimate of the chiral symmetry breaking scale and as the dimensionful parameter that suppresses non-renormalizable terms in $\mathfrak{L}$. What we have actually

Manohar, Georgi, Nucl. Phys. B234 (1984) 189

Chiral expansion of the nucleon force



Is the breakdown distance of the order:

$R \sim (4\pi F_\pi)^{-1} \sim 0.2 \ \ \mathrm{fm}$?

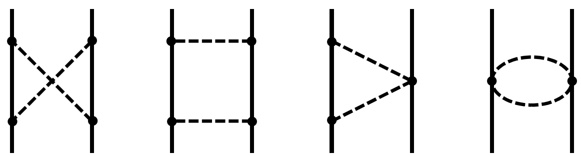
- The (hard) scale entering contact terms is strongly scheme dependent (power counting, cutoff)
- The knowledge of the breakdown scale of the GB exchange is important to organize the expansion in a most efficient way (implications for the power counting → *Birse '09,'10*)

Pion exchange

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa et al. ’03; …

- **Leading order** $$V_{1\pi}^{(0)}(\vec{q}) = -\frac{g_A^2}{4F_\pi^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+M_\pi^2}$$

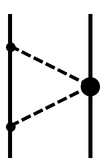
- **Next-to-leading order**



$$\begin{aligned} V_{2\pi}^{(2)}(\vec{q}) \;=\; & -\frac{1}{384\pi^2F_\pi^4}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right]L(q)\\ & -\frac{3g_A^4}{64\pi^2F_\pi^4}\left[\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-q^2\,\vec{\sigma}_1\cdot\vec{\sigma}_2\right]L(q) \end{aligned}$$

where $$L(q)\equiv\frac{\sqrt{4M_\pi^2+q^2}}{q}\,\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{2M_\pi}$$

- **Next-to-next-to-leading order**



$$\begin{aligned} V_{2\pi}^{(3)}(\vec{q}) \;=\; & -\frac{3g_A^2}{16\pi F_\pi^4}\left[2M_\pi^2(2c_1-c_3)-c_3q^2\right](2M_\pi^2+q^2)\,A(q)\\ & -\frac{g_A^2c_4}{32\pi F_\pi^4}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}-q^2\,\vec{\sigma}_1\cdot\vec{\sigma}_2\right](4M_\pi^2+q^2)\,A(q) \end{aligned}$$

where $$A(q)\equiv\frac{1}{2q}\arctan\frac{q}{2M_\pi}$$

Exception or general rule?
What is the breakdown scale of the χ–expansion of the pion exchange?

Pion exchange in coordinate space

$V(\vec{q})$ grows as $q \to \infty$ ⟹ Fourier transform does not exist…

- At finite r, the potential may be obtained from suitably regularized q-space expressions:

$$V_\pi(\vec{r}) = \lim_{\Lambda\to\infty} \int \frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, V_\pi(\vec{q})\, F_{\rm reg}\left(q^2/\Lambda^2\right)$$

— unique result, does not depend on $F_{\rm reg}\left(q^2/\Lambda^2\right)$

- Alternatively, one can use dispersive spectral representation for multiple π-exchange:

$$V_\pi^{\rm cent}(q) = \frac{2}{\pi}\int d\mu\, \mu \frac{\rho(\mu)}{\mu^2+q^2} \quad \text{with} \quad \rho(\mu) = {\rm Im}\left[V_\pi^{\rm cent}(0^+ - i\mu)\right] \quad \Longrightarrow \quad V_\pi^{\rm cent}(r) = \frac{1}{2\pi^2 r}\int d\mu\, \mu e^{-\mu r}\rho(\mu)$$

Local r-space potential: $V(r) = V_C + W_C \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + [V_S + W_S \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T + W_T \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\,(3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)$

- **LO**

$$W_T^{(0)}(r) = \frac{g_A^2 M_\pi^2}{48\pi F_\pi^2}\frac{e^{-x}}{r}\left(1+\frac{3}{x}+\frac{3}{x^2}\right), \quad W_S^{(0)}(r) = \frac{g_A^2 M_\pi^2}{48\pi F_\pi^2}\frac{e^{-x}}{r} \quad \text{with} \quad x \equiv M_\pi r$$

- **NLO**

$$W_C^{(2)}(r) = \frac{M_\pi}{128\pi^3 F_\pi^4}\frac{1}{r^4}\left\{\left[1+2g_A^2(5+2x^2)+g_A^4(23+12x^2)\right]K_1(2x) + x\left[1+10g_A^2-g_A^4(23+4x^2)\right]K_0(2x)\right\},$$

$$V_T^{(2)}(r) = -\frac{g_A^4 M_\pi}{128\pi^3 F_\pi^4}\frac{1}{r^4}\left[12xK_0(2x)+(15+4x^2)K_1(2x)\right], \qquad V_S^{(2)}(r) = \frac{g_A^4 M_\pi}{32\pi^3 F_\pi^4}\frac{1}{r^4}\left[3xK_0(2x)+(3+2x^2)K_1(2x)\right],$$

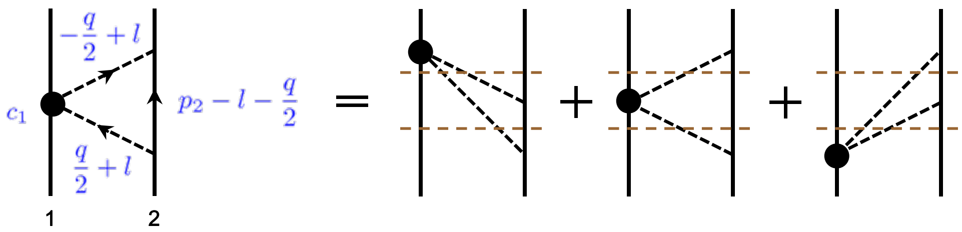
- **NNLO**

$$V_C^{(3)}(r) = \frac{3g_A^2}{32\pi^2 F_\pi^4}\frac{e^{-2x}}{r^6}\left[2c_1x^2(1+x)^2 + c_3(6+12x+10x^2+4x^3+x^4)\right],$$

$$W_T^{(3)}(r) = -\frac{g_A^2}{48\pi^2 F_\pi^4}\frac{e^{-2x}}{r^6}c_4(1+x)(3+3x+x^2), \qquad W_S^{(3)}(r) = \frac{g_A^2}{48\pi^2 F_\pi^4}\frac{e^{-2x}}{r^6}c_4(1+x)(3+3x+2x^2)$$

Pion exchange in coordinate space

Consider the NNLO triangle diagram proportional to c_1 :



$$\frac{1}{2\omega_1\, 2\omega_2}\left[\frac{1}{\omega_1(\omega_1+\omega_2)}+\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_2(\omega_1+\omega_2)}\right]=\frac{1}{2\omega_1^2\omega_2^2}$$

$$V_{2\pi}^{(3)} = 2\left(\frac{g_A}{2F_\pi}\right)^2 4c_1 M_\pi^2 \frac{1}{F_\pi^2}(-i)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_2)\int\frac{d^4l}{(2\pi)^4}\frac{[\vec\sigma_2\cdot(\vec l+\vec q/2)][\vec\sigma_2\cdot(\vec q/2-\vec l)]}{[(l+q/2)^2-M_\pi^2+i\epsilon][(l-q/2)^2-M_\pi^2+i\epsilon][p_2^0-l^0-q^0/2+i\epsilon]}$$

$$= \frac{3g_A^2}{2F_\pi^4}c_1M_\pi^2\int\frac{d^3l}{(2\pi)^3}\frac{\vec l^{\,2}-\vec q^{\,2}}{\omega_+^2\omega_-^2} \quad \text{with} \quad \omega_\pm\equiv\sqrt{(\vec l\pm\vec q)^2+4M_\pi^2}$$

Fourier transformation:

$$V_{2\pi}^{(3)}(\vec r) = \int\frac{d^3q}{(2\pi)^3}\, e^{i\vec q\cdot\vec r}\, V_{2\pi}^{(3)}(\vec q)$$

$$= \frac{3g_A^2}{2F_\pi^4}\, c_1M_\pi^2\int\frac{d^3q}{(2\pi)^3}e^{i\vec q\cdot\vec r}\int\frac{d^3l_1}{(2\pi)^3}\frac{d^3l_2}{(2\pi)^3}\,(2\pi)^3\,\delta(\vec l_1+\vec l_2-\vec q)\,\frac{(-2\vec l_1\cdot\vec l_2)}{[\vec l_1^{\,2}+M_\pi^2][\vec l_2^{\,2}+M_\pi^2]}$$

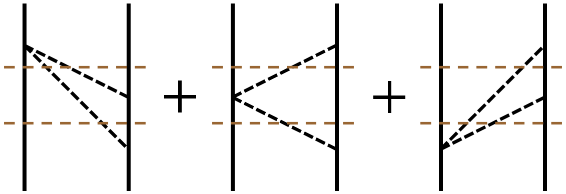
$$= -\frac{3g_A^2}{F_\pi^4}\, c_1M_\pi^2\left[\int\frac{d^3l}{(2\pi)^3}\, e^{i\vec l\cdot\vec r}\frac{\vec l}{\vec l^{\,2}+M_\pi^2}\right]^2$$

$$\longrightarrow -\frac{3g_A^2}{F_\pi^4}\, c_1M_\pi^2\left[-i\vec\nabla\left(\frac{1}{4\pi}\,\frac{e^{-M_\pi r}}{r}\right)\right]^2 = \frac{3g_A^2}{16\pi^2F_\pi^4}\, c_1M_\pi^2\frac{e^{-2x}}{r^4}(1+x)^2$$

only $c_1M_\pi^3/(4\pi F_\pi^2)$ *times suppressed compared to* $V_{1\pi}^{(0)}$ *at* $x\sim 1$

Pion exchange in coordinate space

On the other hand, for the NLO triangle diagram one obtains:



$$\propto \frac{1}{\omega_1\,\omega_2\,(\omega_1+\omega_2)} \Bigg| = \frac{2}{\pi}\int_0^\infty d\beta\,\frac{1}{[\omega_1^2+\beta^2][\omega_2^2+\beta^2]}$$

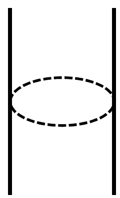
One obtains for the isovector central potential:

$$W_c^{(2)}(q) = -\frac{g_A^2}{4F_\pi^4}\int\frac{d^3l_1}{(2\pi)^3}\,\frac{d^3l_2}{(2\pi)^3}\,(2\pi)^3\delta(\vec{q}-\vec{l}_1-\vec{l}_2)\,\frac{2}{\pi}\int_0^\infty d\beta\,\frac{\vec{l}_1\cdot\vec{l}_2}{[\vec{l}_1^{\,2}+M_\pi^2+\beta^2][\vec{l}_2^{\,2}+M_\pi^2+\beta^2]}$$

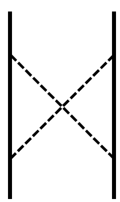
$$\begin{aligned}
W_c^{(2)}(r) &= \frac{g_A^2}{4F_\pi^4}\,\frac{2}{\pi}\int_0^\infty d\beta\left[\vec{\nabla}\,\frac{1}{4\pi r}\,e^{-\sqrt{M_\pi^2+\beta^2}\,r}\right]^2\\
&= \frac{g_A^2}{32\pi^3F_\pi^4}\,\frac{1}{r^4}\int_0^\infty d\beta\left[1+2\sqrt{M_\pi^2+\beta^2}\,r+(M_\pi^2+\beta^2)r^2\right]e^{-2\sqrt{M_\pi^2+\beta^2}\,r}\\
&= \frac{g_A^2}{32\pi^3F_\pi^4}\,\frac{1}{r^4}\left[1-r\frac{\partial}{\partial r}+\frac{1}{4}r^2\frac{\partial^2}{\partial r^2}\right]\underbrace{\int_0^\infty d\beta\,e^{-2\sqrt{M_\pi^2+\beta^2}\,r}}_{M_\pi\,K_1(2M_\pi r)}\\
&= \frac{g_A^2}{64\pi^3F_\pi^4}\,\frac{M_\pi}{r^4}\left[(5+2x^2)K_1(2x)+5xK_0(2x)\right]
\end{aligned}$$

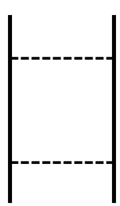
Pion exchange in coordinate space

Similar results are obtained for all other NLO diagrams:



$$\propto \frac{1}{\omega_1\,\omega_2\,(\omega_1+\omega_2)} = \frac{2}{\pi}\int_0^\infty d\beta\,\frac{1}{[\omega_1^2+\beta^2][\omega_2^2+\beta^2]}$$

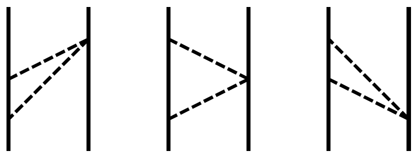




$$\propto\ \pm\frac{\omega_1^2+\omega_1\omega^2+\omega_2^2}{\omega_1^3\omega_2^3(\omega_1+\omega_2)} = \pm\frac{4}{\pi}\int_0^\infty d\beta\left[\frac{1}{[\omega_1^2+\beta^2]^2[\omega_2^2+\beta^2]}+\frac{1}{[\omega_1^2+\beta^2][\omega_2^2+\beta^2]^2}\right]$$

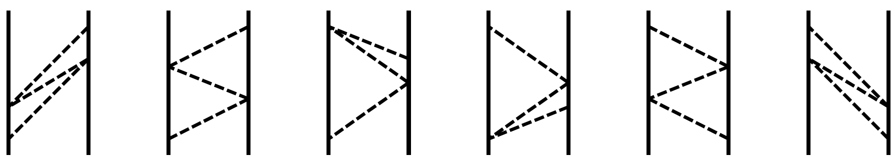
Time-ordered graphs (time-indep. vertices)

- Two pions



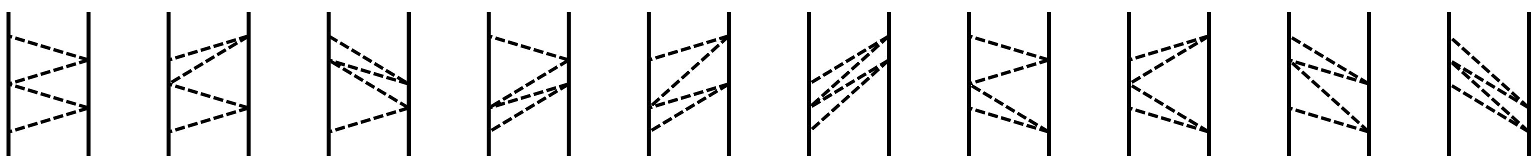
$$\frac{1}{\omega_1(\omega_1+\omega_2)}+\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_2(\omega_1+\omega_2)}=\frac{2}{\omega_1\omega_2}$$

- Three pions



$$-\frac{2}{\omega_1\omega_2\omega_3}-\frac{1}{\omega_1\omega_2(\omega_2+\omega_3)}-\frac{1}{(\omega_1+\omega_2)\omega_2\omega_3}+\frac{1}{\omega_1(\omega_1+\omega_2)(\omega_1+\omega_2+\omega_3)}+\frac{1}{(\omega_1+\omega_2+\omega_3)(\omega_2+\omega_3)\omega_3}$$

- Four pions

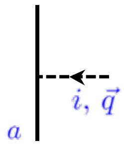


$$\frac{2}{\omega_1\omega_2\omega_3\omega_4}+\frac{1}{\omega_1\omega_2\omega_3(\omega_3+\omega_4)}+\frac{1}{(\omega_1+\omega_2)\omega_2\omega_3\omega_4}+\frac{1}{(\omega_1+\omega_2)\omega_2(\omega_2+\omega_3+\omega_4)\omega_4}$$
$$+\frac{1}{\omega_1(\omega_1+\omega_2+\omega_3)\omega_3(\omega_3+\omega_4)}+\frac{1}{(\omega_1+\omega_2)(\omega_1+\omega_2+\omega_3)(\omega_2+\omega_3+\omega_4)\omega_4}+\frac{1}{\omega_1(\omega_1+\omega_2+\omega_3)(\omega_2+\omega_3+\omega_4)(\omega_3+\omega_4)}$$
$$-\frac{1}{\omega_1\omega_2(\omega_2+\omega_3)(\omega_2+\omega_3+\omega_4)}-\frac{1}{(\omega_1+\omega_2+\omega_3)(\omega_2+\omega_3)\omega_3\omega_4}+\frac{1}{(\omega_1+\omega_2)\omega_2\omega_3(\omega_3+\omega_4)}$$

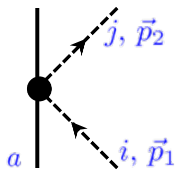
Time-ordered multiple scattering graphs

Consider t-channel multiple-scattering time-ordered diagrams

"Feynman rules":

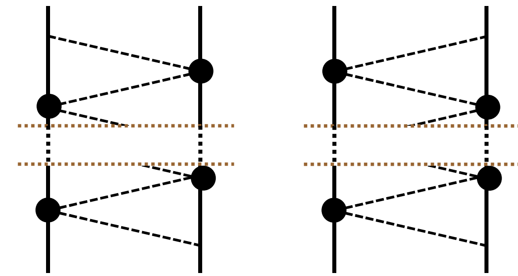


$$i\frac{g_A}{2F_\pi}\tau_a^i\vec{\sigma}_a\cdot\vec{q}\,\frac{1}{\sqrt{2\omega}}$$



$$\frac{2\delta_{ij}}{F_\pi^2}\left(2c_1M_\pi^2 + c_3\vec{p}_1\cdot\vec{p}_2\right)\frac{1}{\sqrt{2\omega_1}\sqrt{2\omega_2}}$$

(Notice: this is not the complete vertex)



Diagrams with an odd number of pion exchanges yield the following potential:

$\frac{1}{4\pi r_1}e^{-M_\pi r_1}$

$$V^{(3n)}_{(n+1)\pi}(\vec{r}) = \frac{g_A^2}{2F_\pi^{2n+2}}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec{\sigma}_1\cdot\vec{\nabla}_{n+1})(\vec{\sigma}_2\cdot\vec{\nabla}_1)\left[2c_1M_\pi^2 + c_3\vec{\nabla}_1\cdot\vec{\nabla}_2\right]\ldots\left[2c_1M_\pi^2 + c_3\vec{\nabla}_n\cdot\vec{\nabla}_{n+1}\right]U(r_1)\ldots U(r_{n+1})\Big|_{r_i=r}$$

$$= \frac{g_A^2}{2(4\pi F_\pi^2)^{n+1}}\frac{e^{-(n+1)x}}{r^{3(n+1)}}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})\left\{\sum_{m=0}^{n-1}\sum_{l=0}^{m}(2c_1x^2)^{n-m}\,c_3^m\,y_n^{m,l}\,X^{2+2(m-l)}(X^2+1)^l\right.$$

$$\left. + \; c_3^n\left[(X^2+1)^{n+1} + X^{n+1}\right]\right\} - \frac{g_A^2}{2(4\pi F_\pi^2)^{n+1}}\frac{e^{-(n+1)x}}{r^{3(n+1)}}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,c_3^n\,X^{n+1}$$

where $X \equiv 1+x$ and $y_n^{m,l}$ are known combinatorial coefficients with the properties:

$$y_n^{m,l} = 0 \text{ for } l > 2m-n, \quad y_n^{0,0} = 1, \quad y_n^{n-1,n-1} = n, \quad \sum_{l,m} y_n^{l,m} = 2^n - 1$$

Time-ordered multiple scattering graphs

Similarly, diagrams with an even number of pion exchanges yield the following potential:

$$V^{(3n)}_{(n+1)\pi}(\vec{r}) = \frac{3g_A^2}{2(4\pi F_\pi^2)^{n+1}} \frac{e^{-(n+1)x}}{r^{3(n+1)}} \left\{ \sum_{m=0}^{n-1} \sum_{l=0}^{m} (2c_1 x^2)^{n-m}\, c_3^m\, y_n^{m,l} \underbrace{X^{2+2(m-l)}(X^2+1)^l}_{X \equiv 1+x} + c_3^n \left[(X^2+1)^{n+1} + 2X^{n+1} \right] \right\}$$

For example:

- **Two pions**

$$V^{(3)}_{2\pi}(r) = \frac{3g_A^2}{2(4\pi F_\pi^2)^2} \frac{e^{-2x}}{r^6} \left\{ 2c_1 x^2 X^2 + c_3 \left[(X^2+1)^2 + 2X^2 \right] \right\}$$

⟵ agrees with the known result

- **Four pions**

$$V^{(9)}_{4\pi}(r) = \frac{3g_A^2}{2(4\pi F_\pi^2)^4} \frac{e^{-4x}}{r^{12}} \left\{ 8c_1^3 x^6 X^2 + 4c_1^2 c_3 x^4 X^2 \left[3X^2+1 \right] + 6c_1 c_3^2 x^2 X^2 (X^2+1)^2 + c_3^3 \left[(X^2+1)^4 + 2X^4 \right] \right\}$$

- **Six pions**

$$\begin{aligned} V^{(15)}_{6\pi}(r) \;=\; & \frac{3g_A^2}{2(4\pi F_\pi^2)^6} \frac{e^{-6x}}{r^{18}} \Big\{ 32c_1^5 x^{10} X^2 \;+\; 16c_1^4 c_3 x^8 X^2 [5X^2+2] \;+\; 8c_1^3 c_3^2 x^6 X^2 [10X^4+12X^2+3] \\ & + \; 8c_1^2 c_3^3 x^4 X^2 [3X^2(X^2+1)^2 + 2(X^2+1)^3] \;+\; 10c_1 c_3^4 x^2 X^2 (X^2+1)^4 + c_3^5 [(X^2+1)^6 + 2X^6] \Big\} \end{aligned}$$

Time-ordered multiple scattering graphs

For either $c_1 = 0$ or $c_3 = 0$ the potential can be easily resummed (geometrical series)

For the isoscalar central potential (even number of pions) one obtains:

- $$V_{\text{resummed}}^{c_3=0}(r) = \frac{3g_A^2 c_1 M_\pi^2}{16\pi^2 F_\pi^4} \frac{e^{-2x}}{r^4} \frac{(1+x)^2}{1 - \frac{4c_1^2 M_\pi^4}{(4\pi F_\pi^2)^2} \frac{e^{-2x}}{r^2}}$$

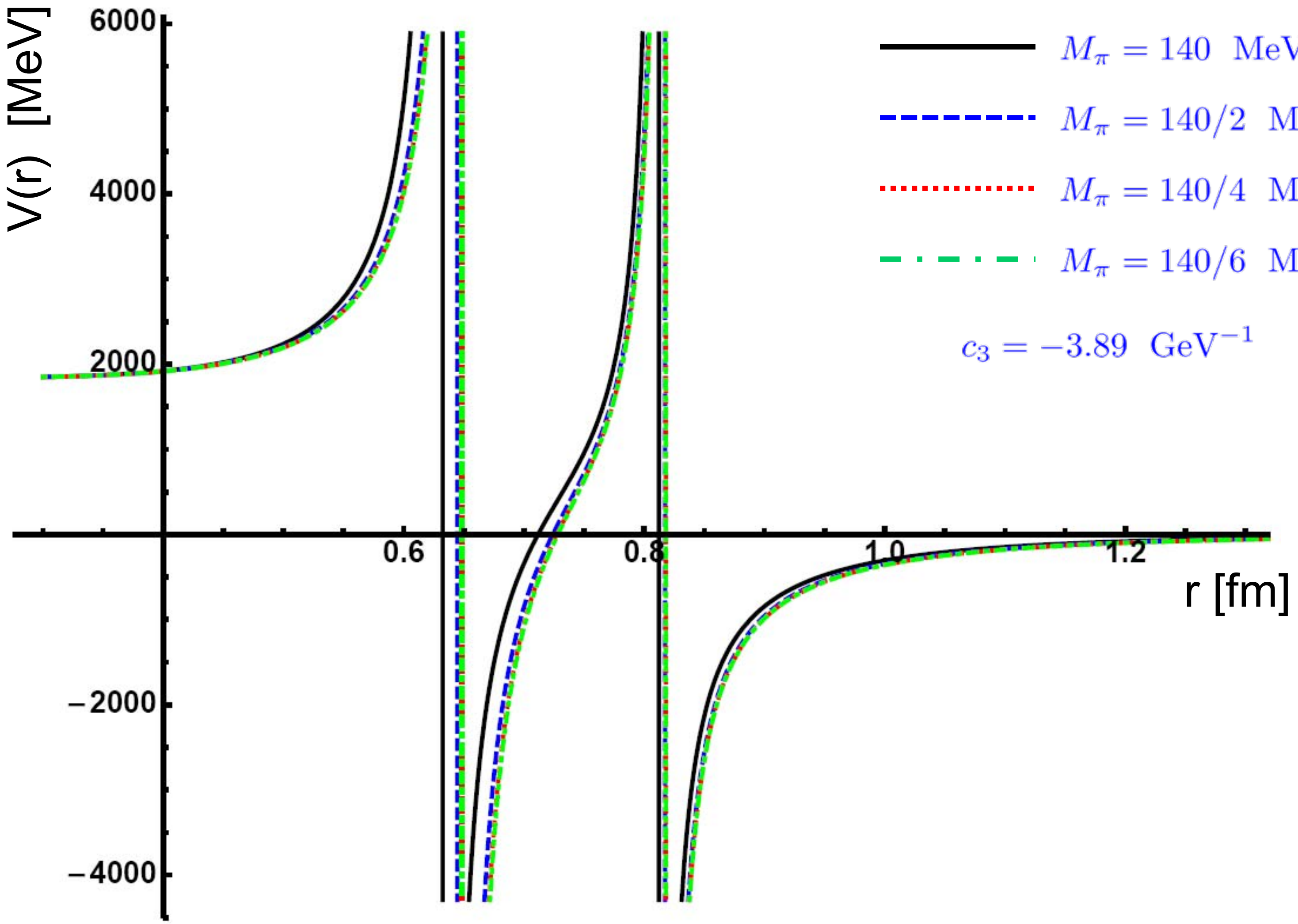
  ⟹ Poles at negative r (irrelevant) and at $r \sim \frac{c_1 M_\pi^2}{2\pi F_\pi^2} \sim 0.05$ fm

- $$V_{\text{resummed}}^{c_1=0}(r) = \frac{3g_A^2 c_3}{32\pi^2 F_\pi^4} \frac{e^{-2x}}{r^6} \left[\frac{(2+2x+x^2)^2}{1 - \frac{c_3^2}{(4\pi F_\pi^2)^2} \frac{e^{-2x}}{r^6}(2+2x+x^2)^2} + \frac{2(1+x)^2}{1 - \frac{c_3^2}{(4\pi F_\pi^2)^2} \frac{e^{-2x}}{r^6}(1+x)^2} \right]$$

  ⟹ Poles at negative r (irrelevant) and at $r \sim 0.6$ fm and $r \sim 0.8$ fm (for $c_3 = -3.89$ GeV^{-1})

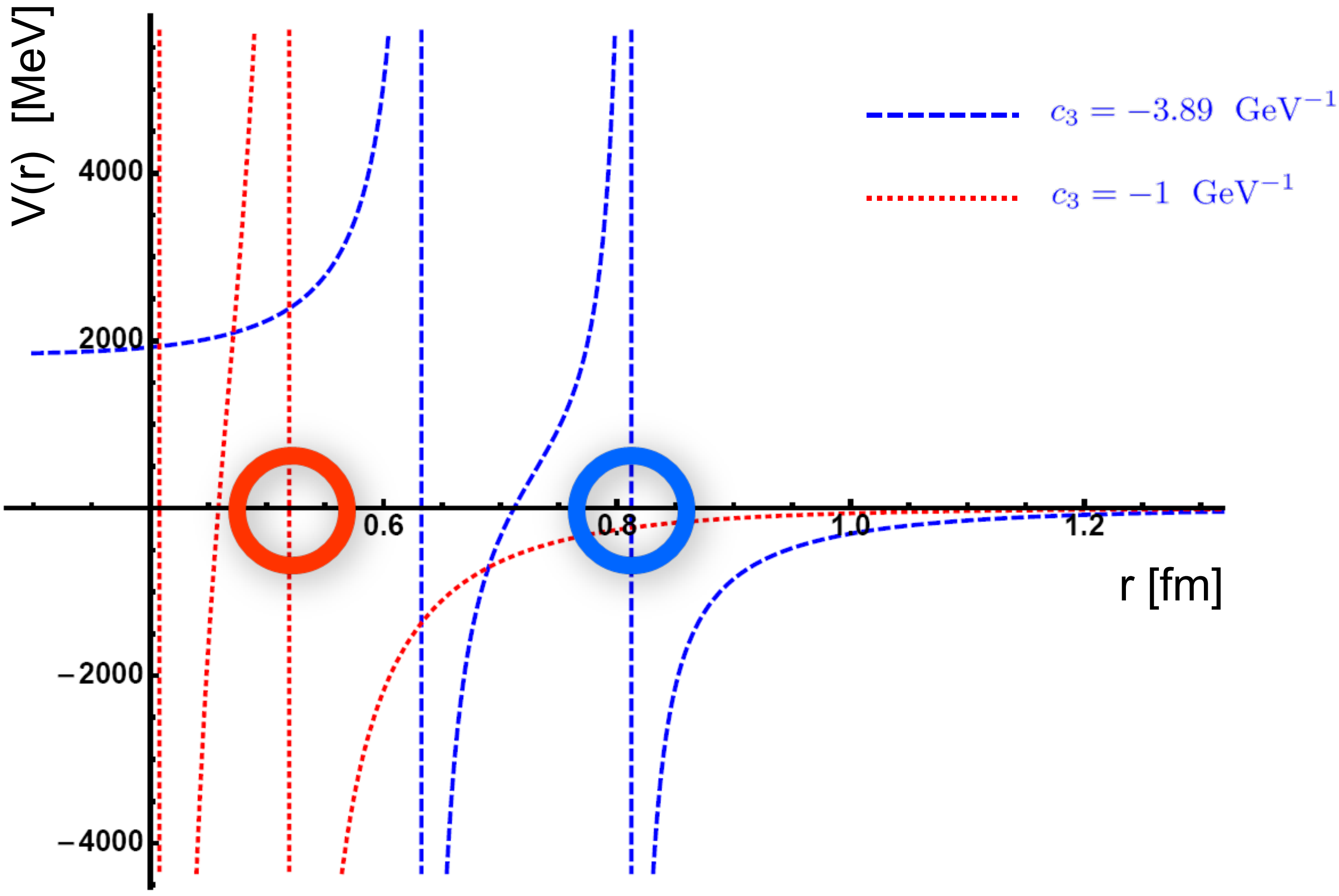
Time-ordered multiple scattering graphs

Resummed c_3-potential: M_π-dependence



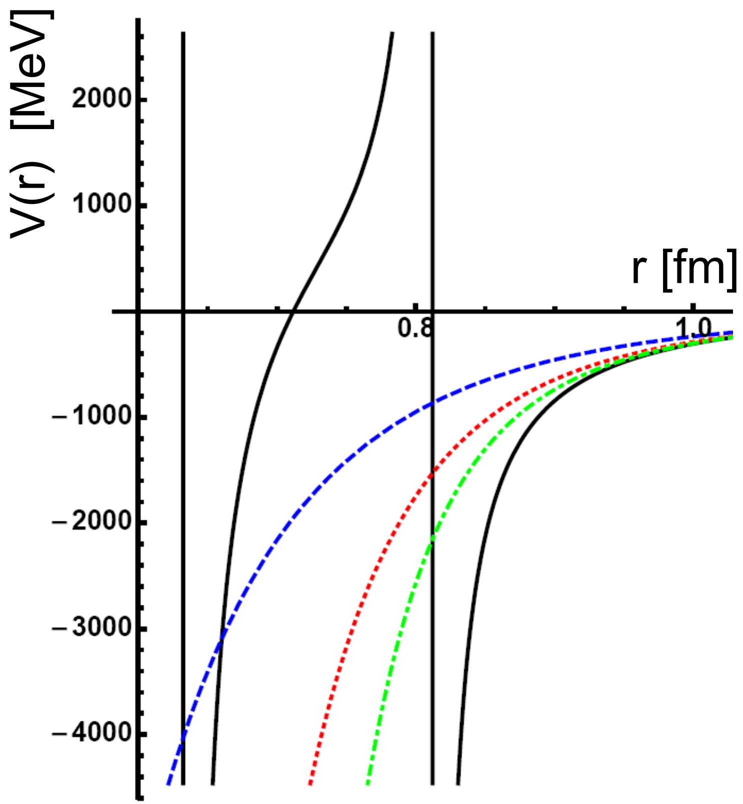
Time-ordered multiple scattering graphs

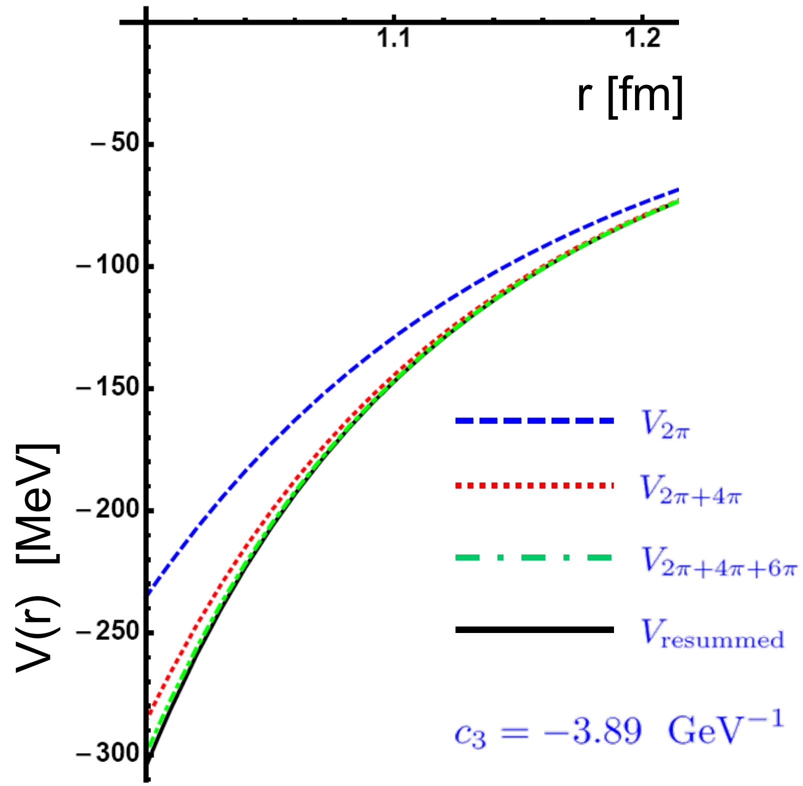
Resummed potential: c_3-dependence



Time-ordered multiple scattering graphs

Convergence of the chiral expansion





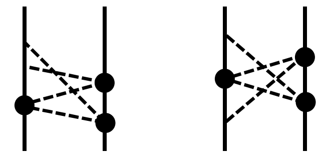
In spite of the breakdown at $r \sim 0.8\ \text{fm}$, reasonably fast convergence for $r > 1\ \text{fm}$

Indeed, at $x = 1$ the expansion goes as $\frac{M_\pi^3 c_3}{4\pi F_\pi^2} e^{-2}$ ⟹ $\Lambda_\chi^{NN} = \left(4\pi F_\pi^2 e^2 c_3^{-1}\right)^{1/3} \sim 600\ \text{MeV}$

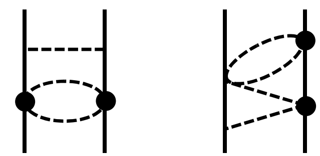
Disclaimer

Are these poles in the potential real ?? Well… Recall that:

- Except for the triangle diagram, I have only considered ***parts*** of the corresponding Feynman diagrams (which are most strongly "π-enhanced"),
- Crossed diagrams may lead to partial cancellations and were not considered (irrelevant for the triangle diagram),



- The part of the c_3-vertex with time derivatives was not considered (it affects the energy denominators),
- There are many more $n\pi$-exchange diagrams $\propto g_A^2 c_i^{n-1}$ that contribute at chiral orders $\mathcal{O}\left(q^{\geq 3(n-3)}\right)$,



- Last but not least, one should be careful with the FT of the resummed potential (***finite range*** contributions due to resummed 0-range terms with increasing # of derivatives).

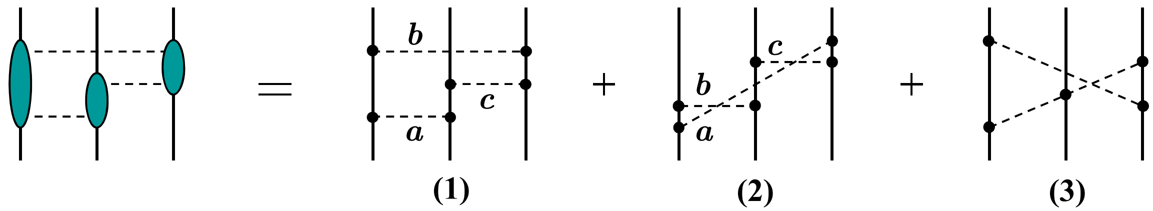
Nevertheless, the results indicate that

- The "potential picture" breaks down at distances $r \sim 0.5 \text{ fm}$ (at best)
- Certain classes of contributions to the GB exchange are enhanced by powers of π. This is not restricted to the considered topology.

Three-nucleon force at N^{3}LO

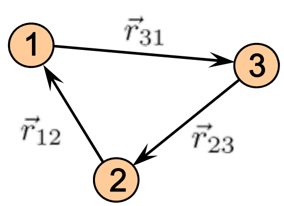
Bernard, E.E., Krebs, Meißner, PRC 77 (2008) 064004

Ring diagrams:



One finds for irreducible contributions: $V_{1,2} \propto \left[\pm \frac{1}{\omega_a^4 \omega_b^2 \omega_c^2} \pm \frac{1}{\omega_a^2 \omega_b^4 \omega_c^2} - \frac{1}{\omega_a^2 \omega_b^2 \omega_c^4} \right], \quad V_3 \propto \frac{1}{\omega_a^2 \omega_b^2 \omega_c^2}$

$$
\begin{aligned}
V_{\rm ring} \;=\; & -\frac{g_A^6 M_\pi^7}{4096\pi^3 F_\pi^6} \Big[-4\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \vec{\nabla}_{23} \times \vec{\nabla}_{12} \cdot \vec{\sigma}_2\, \vec{\nabla}_{23} \times \vec{\nabla}_{31} \cdot \vec{\sigma}_3\, \vec{\nabla}_{31} \cdot \vec{\nabla}_{12} \\
& \quad - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\nabla}_{23} \cdot \vec{\nabla}_{31}\, \vec{\nabla}_{23} \cdot \vec{\nabla}_{12}\, \vec{\nabla}_{31} \cdot \vec{\nabla}_{12} \\
& \quad + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\nabla}_{23} \times \vec{\nabla}_{12} \cdot \vec{\sigma}_2\, \vec{\nabla}_{23} \cdot \vec{\nabla}_{31}\, \vec{\nabla}_{31} \cdot \vec{\nabla}_{12} \\
& \quad + 3\vec{\nabla}_{31} \times \vec{\nabla}_{12} \cdot \vec{\sigma}_1\, \vec{\nabla}_{23} \times \vec{\nabla}_{31} \cdot \vec{\sigma}_3\, \vec{\nabla}_{23} \cdot \vec{\nabla}_{12} \Big] \frac{e^{-x_{23}}}{x_{23}}\, e^{-x_{31}}\, \frac{e^{-x_{12}}}{x_{12}} \\
& + \frac{g_A^4 M_\pi^7}{2048\pi^3 F_\pi^6} \Big[2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \big(\vec{\nabla}_{23} \cdot \vec{\nabla}_{31}\, \vec{\nabla}_{31} \cdot \vec{\nabla}_{12} - \vec{\nabla}_{31} \times \vec{\nabla}_{12} \cdot \vec{\sigma}_1\, \vec{\nabla}_{23} \times \vec{\nabla}_{31} \cdot \vec{\sigma}_3 \big) \\
& \quad + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\nabla}_{31} \times \vec{\nabla}_{12} \cdot \vec{\sigma}_1\, \vec{\nabla}_{23} \cdot \vec{\nabla}_{31} \Big] \frac{e^{-x_{23}}}{x_{23}}\, \frac{e^{-x_{31}}}{x_{31}}\, \frac{e^{-x_{12}}}{x_{12}} \; + \; 5 \text{ permutations}
\end{aligned}
$$

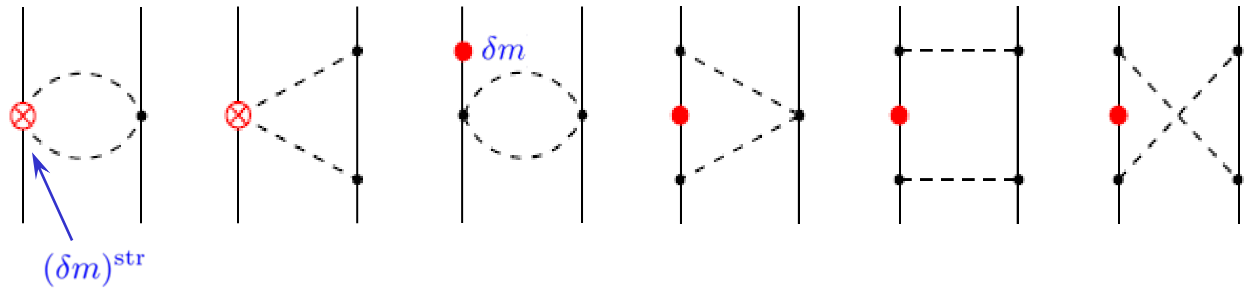


Same sort of enhancement occurs also for the 2π-1π topology

Leading CSB two-pion exchange potential

Niskanen '02; Friar et al. '03, '04; E.E. & Meißner '05

The leading CSB 2π–exchange is governed by the proton-neutron mass difference:



The corresponding energy denominators are $\propto \frac{1}{\omega_a^2 \omega_b^2}$ or $\propto \left[\frac{1}{\omega_a^4 \omega_b^2} + \frac{1}{\omega_a^2 \omega_b^4} \right]$

Decomposition in the momentum space:

$$V_{\rm CSB}^{2\pi} = (\tau_1^3 + \tau_2^3) \left[V_C + V_S \, (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T \, (\vec{\sigma}_1 \cdot \vec{q}\,)(\vec{\sigma}_2 \cdot \vec{q}\,) \right]$$

One finds:

$$V_C = -\frac{g_A^2}{64 \pi F_\pi^4} \left[\frac{2 \, g_A^2 \, \delta m \, M_\pi^3}{4 M_\pi^2 + q^{\,2}} - \left(4 g_A^2 \, \delta m - (\delta m)^{\rm str} \right) (2 M_\pi^2 + q^2) A(q) \right]$$

$$V_T = -\frac{1}{q^2} V_S = \frac{g_A^4 \, \delta m}{32 \pi F_\pi^4} A(q)$$ where $q = |\vec{p}\,' - \vec{p}\,|$ and $A(q) \equiv \frac{1}{2q} \arctan \frac{q}{2 M_\pi}$

Summary & conclusions

- Pion loops are enhanced by powers of π for certain classes of GB exchange diagrams in the few-N sector

  $\Longrightarrow$ perhaps more appropriate to estimate $\Lambda_\chi^{NN} \sim 4\sqrt{\pi}F_\pi$ or even $\Lambda_\chi^{NN} \sim \sqrt{4\pi}F_\pi$

- Don't trust the chiral pion-exchange potential/wave function at distances of the order of $r \sim 0.5 \text{ fm}$ and below (even at $N^{100}LO$)

  $\Longrightarrow$ implications for power counting ?

Open questions

- What happens when Δ is included as an explicit DOF?
- Is it possible to systematically keep track of the π-enhancement?
- If yes, can power counting for long-range terms be adjusted appropriately (to make χ-expansion more efficient)?