INPP @OHIO UNIVERSITY
INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

The NN System in Three Dimensions

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Challenges in 3N Physics

- **Test of nuclear forces in the simplest nuclear environment** (over a large energy range!)
  - Two-body forces
  - Genuine three-body forces

- **Reaction mechanisms**
  - Examples: deuteron breakup, (p,n) charge exchange, exclusive breakup (specific configurations) …
  - Higher Energy: Lorentz vs. Galilean Invariance
    - Check commonly used approximations (e.g. Glauber approach)
  - Application to nuclear reactions

Roadmap for 3N problem without PW

Scalar NN model |

- NN scattering + bound state
- 3N bound state
- 3N bound state + 3NF
- 3N scattering:
- Faddeev Calculation
  - Elastic scattering
  - Below and above break-up
  - Break-up

- **Poincarė Invariant Faddeev Calculations**
  - **Elastic scattering**
  - **Breakup scattering**
  - **up to 2 GeV**

Relativistic Faddeev Calculations

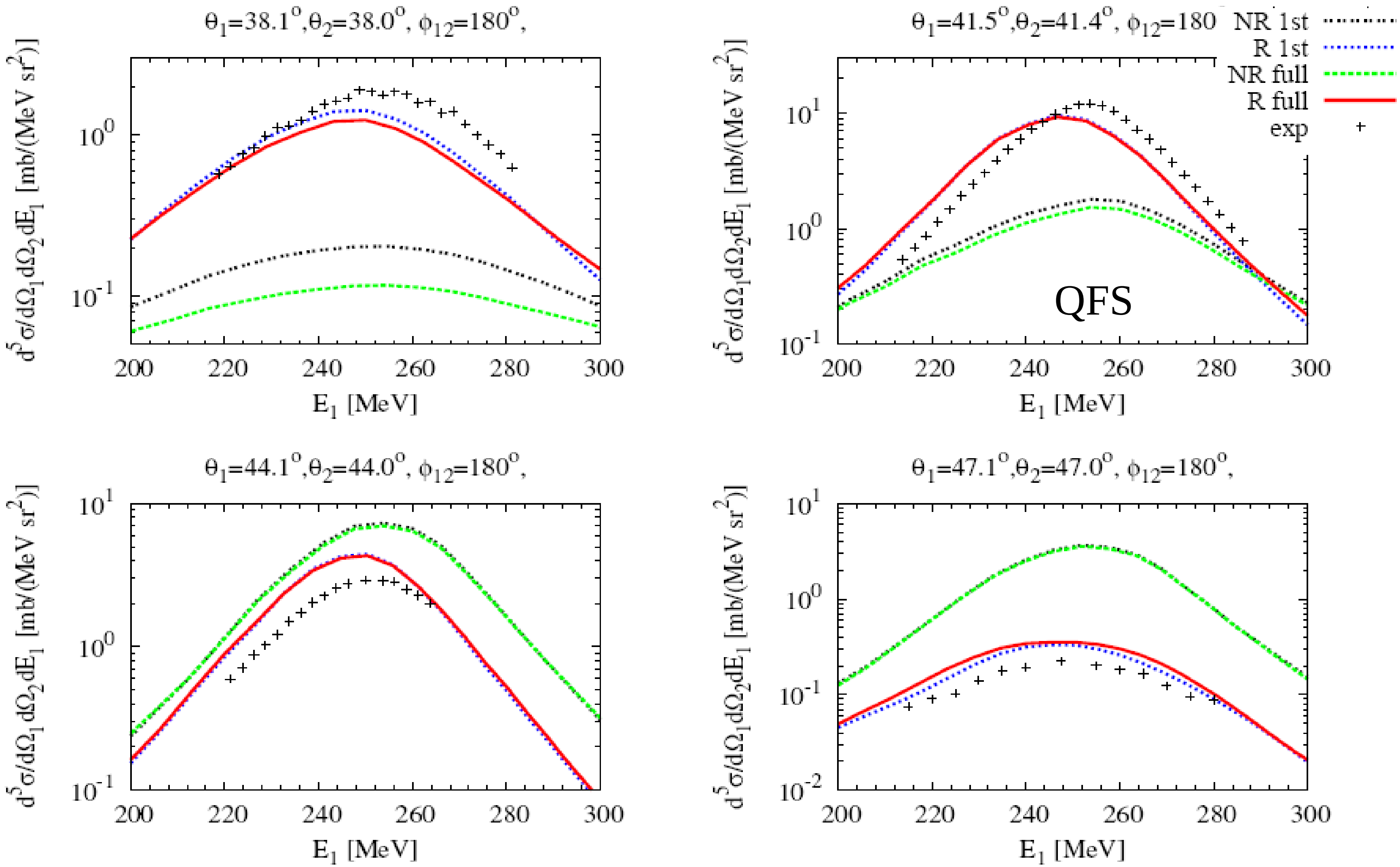
- Context: Poincarė Invariant Quantum Mechanics
  - Poincarė invariance is exact symmetry, realized by a unitary representation of the Poincarė group on a few-particle Hilbert space
  - Instant form
  - Faddeev equations same operator form but different ingredients
- **Kinematics**
  - Lorentz transformations between frames
- **Dynamics**
  - Bakamjian-Thomas Scheme: Mass Operator $M=M_0+V$ replaces Hamiltonian $H=H_0+\nu$
  - Connect Galilean two-body ν with Poincarė two-body v
  - Construct V := $\sqrt{M^2+q^2}-\sqrt{M_0^2+q^2}$

Exclusive Breakup Scattering

(symmetric configuration)

E_{lab} = 508 MeV

(V.Punjabi et al. PRC 38, 2728 (1998))



Roadmap for 3N problem without PW

Scalar NN model | Realistic Systems

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- Atomic Systems
  - L. Tomio et al.

- Spin ½ - Spin 0
  - R. Crespo et al. (Lisbon)

- Spin ½ - Spin ½
  - 3N problem

Roadmap for 3N problem without PW

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- NN scattering + deuteron
  - **Based on Helicity formulation**
  - Potentials AV18 and Bonn-B
- Break-up in first order:
  - (p,n) charge exchange

PhD Thesis
Imam Fachruddin
Bochum 2001

Work along this line continued by group in Teheran: Hadizadeh et al. – for bound states only

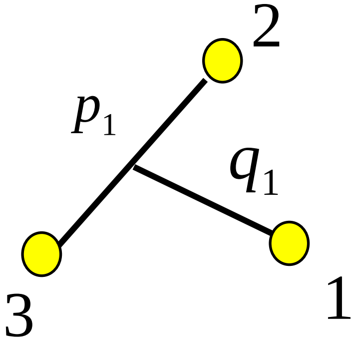
What did we learn from the scalar model?

Technical aspects: Bound State

$$|\psi\rangle = G_0 t\, P|\psi\rangle$$

$$+\, G_0 V_4^{(1)}\, (1+P)|\psi\rangle + G_0 t\, G_0 V_4^{(1)}\, (1+P)|\psi\rangle$$

Faddeev Equation for 2NF only



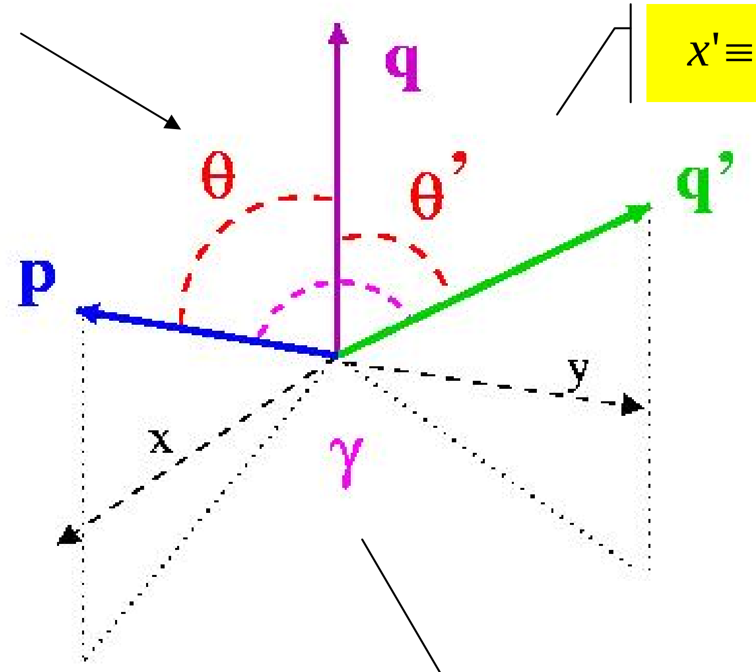
$$\langle \mathbf{pq}|\psi\rangle = \frac{1}{E - \frac{1}{m}p^2 - \frac{3}{4m}q^2}$$

$$\times \int d^3q'\; t_s(\mathbf{p}, \tfrac{1}{2}\mathbf{q} + \mathbf{q}', E - \tfrac{3}{4m}q^2)\; \langle \mathbf{q} + \tfrac{1}{2}\mathbf{q}', \mathbf{q}'|\psi\rangle$$

Variables: q, p, q’

$x \equiv \hat{\mathrm{p}} \cdot \hat{\mathrm{q}} = \cos\theta$

$x' \equiv \hat{\mathrm{q}}' \cdot \hat{\mathrm{q}} = \cos\theta'$



$y \equiv \hat{\mathrm{p}} \cdot \hat{\mathrm{q}}' = \cos\gamma(x, x', \varphi)$

3D Equation for Bound State: Scalar function in 3 variables

$$\psi(p,q,x) = \frac{1}{E - \frac{1}{m}p^2 - \frac{3}{4m}q^2} \int_0^\infty dq' q'^2 \int_{-1}^{1} dx' \int_0^{2\pi} d\varphi'$$

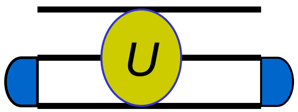
$$\times t_s\left(p, \sqrt{\tfrac{1}{4}q^2 + q'^2 + qq'x'}, \frac{\frac{1}{2}qx + q'y}{|\frac{1}{2}\mathbf{q} + \mathbf{q}'|}; E - \tfrac{3}{4m}q^2\right)$$

$$\times\psi\left(\sqrt{q^2 + \tfrac{1}{4}q'^2 + qq'x'}, q', \frac{qx' + \frac{1}{2}q'}{|\mathbf{q} + \frac{1}{2}\mathbf{q}'|}\right)$$

Three-Body Scattering - General

- Transition operator for elastic scattering

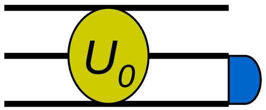
$$U = PG_0^{-1} + PT$$



- Transition operator for breakup scattering

$$U_0 = (1 + P)T$$

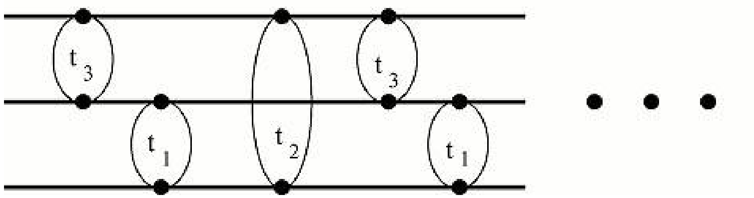
$$T = tP + tG_0PT$$



- Faddeev equation (Multiple Scattering Series)

$$T = tP + tG_0PtP + \cdots$$

1st Order in tP



$t = v + vg_0t$ =: NN t-matrix

$P = P_{12}P_{23} + P_{13}P_{23} \equiv$ Permutation Operator

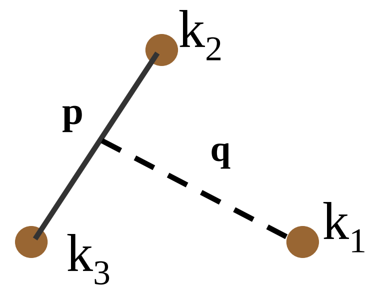
3-Body Transition Amplitude (NR)

$$T\left|\mathrm{q}_0\varphi_d\right\rangle = tP\left|\mathrm{q}_0\varphi_d\right\rangle + tG_0PT\left|\mathrm{q}_0\varphi_d\right\rangle$$

$$\mathrm{p} = \frac{1}{2}\left(\mathrm{k}_2 - \mathrm{k}_3\right)$$

$$\mathrm{q} = \frac{2}{3}\left(\mathrm{k}_1 - \frac{1}{2}\left(\mathrm{k}_2 + \mathrm{k}_3\right)\right)$$

The Faddeev Equation in momentum space by using Jacobi Variables

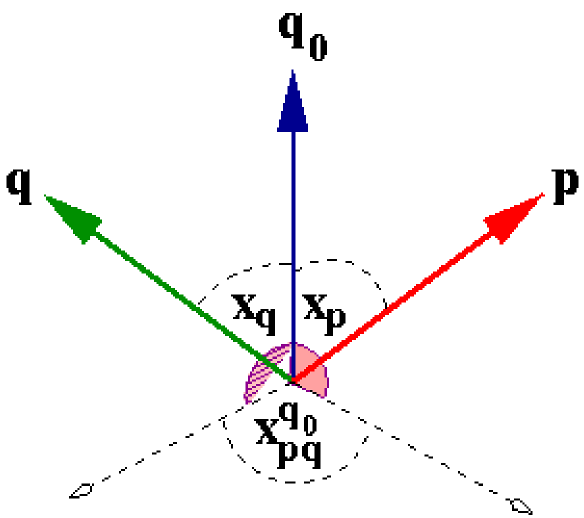


$$\left\langle \mathrm{pq}\left|\hat{T}\right|\mathrm{q}_0\varphi_d\right\rangle = \varphi_d\left(\mathrm{q}+\tfrac{1}{2}\mathrm{q}_0\right)\hat{t}_s\left(\mathrm{p},\tfrac{1}{2}\mathrm{q}+\mathrm{q}_0,E-\tfrac{3}{4m}q^2\right)$$
$$+\int d^3\mathrm{q}''\,\frac{\hat{t}_s\left(\mathrm{p},\tfrac{1}{2}\mathrm{q}+\mathrm{q}'',E-\tfrac{3}{4m}q^2\right)}{E-\tfrac{1}{m}\left(q^2+q''^2+\mathrm{q}\cdot\mathrm{q}''\right)+i\varepsilon}\,\frac{\left\langle \mathrm{q}+\tfrac{1}{2}\mathrm{q}'',\mathrm{q}''\left|\hat{T}\right|\mathrm{q}_0\varphi_d\right\rangle}{E-\tfrac{3}{4m}q''^2-E_d+i\varepsilon}$$

$\hat{t}_s \equiv$ symmetrized 2-body t-matrix

Variables for 3D Calculation

3 distinct vectors in the problem: $\mathbf{q}_0$ $\mathbf{q}$ $\mathbf{p}$



5 independent variables:

$$p = |\mathrm{p}| \; , \; q = |\mathrm{q}|$$

$$x_p = \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}_0 , \; x_q = \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}_0$$

$$x_{pq}^{q_0} = (\widehat{\mathbf{q}_0 \times \mathbf{q}}) \cdot (\widehat{\mathbf{q}_0 \times \mathbf{p}})$$

q system : $\mathbf{z} \parallel \mathbf{q}$

q_0 system : $\mathbf{z} \parallel \mathbf{q}_0$

Variables invariant under rotation:

freedom to choose coordinate system for numerical calculation

3D Integral Equation in 5 Variables

Solved by Padé Summation

$$
\begin{aligned}
&\langle p, x_p, x_{pq}^{q_0}, x_q, q|\hat{T}|q_0\varphi_d\rangle \\
=\ & \varphi_d\left(\sqrt{q^2+\frac{1}{4}q_0^2+qq_0x_q}\right) \\
\times\ & \hat{t}_s\left(p, \sqrt{\frac{1}{4}q^2+q_0^2+qq_0x_q}, \frac{\frac{1}{2}qy_{pq}+q_0x_p}{\sqrt{\frac{1}{4}q^2+q_0^2+qq_0x_q}}; E-\frac{3}{4m}q^2\right) \\
+\ & \int_0^\infty dq''q''^2\int_{-1}^{+1}dx''\int_0^{2\pi}d\varphi''\frac{1}{E-\frac{1}{m}(q^2+qq''x''+q''^2)+i\varepsilon} \\
\times\ & \hat{t}_s\left(p, \sqrt{\frac{1}{4}q^2+q''^2+qq''x''}, \frac{\frac{1}{2}qy_{pq}+q''y_{pq''}}{\sqrt{\frac{1}{4}q^2+q''^2+qq''x''}}; E-\frac{3}{4m}q^2\right) \\
\times\ & \frac{\left\langle\sqrt{q^2+\frac{1}{4}q''^2+qq''x''}, \frac{qx_q+\frac{1}{2}q''y_{q_0q''}}{\sqrt{q^2+\frac{1}{4}q''^2+qq''x''}}, \frac{\frac{qx''+\frac{1}{2}q''}{\sqrt{q^2+\frac{1}{4}q''^2+qq''x''}}-x_{\pi_p}x_{\pi_q}}{\sqrt{1-x_{\pi_p}^2}\sqrt{1-x_{\pi_q}^2}}, y_{q_0q''}, q''|\hat{T}|q_0\varphi_d\right\rangle}{E-\frac{3}{4m}q''^2-E_d+i\varepsilon}
\end{aligned}
$$

Singularities:

Position depends on q, q”, x”

Fixed “deuteron” pole

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General Form of NN interaction

- Space (e.g. momenta)
  - Basis: vector variables

$$\vec{p}\,' - \vec{p}, \quad \vec{p}\,' + \vec{p}, \quad \vec{p}\,' \times \vec{p}$$

- Spin – Operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$

- Isospin - Operators $\vec{\tau}_1$ and $\vec{\tau}_2$

Idea: form scalar functions with the vector variables

handle operators analytically

General Form of NN interaction, cont'd

Allow explicit iso-spin dependence: $\langle t' m_t' \mid V \mid t m_t \rangle = \delta_{tt'} \delta_{m_t m_t'} V^{t m_t}$

Spin momentum operator structure invariant under rotation, parity, time-reversal

$$
\begin{aligned}
w_1(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= 1 \\
w_2(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
w_3(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= i\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{p} \times \mathbf{p}') \\
w_4(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p} \times \mathbf{p}')\ \boldsymbol{\sigma}_2 \cdot (\mathbf{p} \times \mathbf{p}') \\
w_5(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p}' + \mathbf{p})\ \boldsymbol{\sigma}_2 \cdot (\mathbf{p}' + \mathbf{p}) \\
w_6(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p}) &= \boldsymbol{\sigma}_1 \cdot (\mathbf{p}' - \mathbf{p})\ \boldsymbol{\sigma}_2 \cdot (\mathbf{p}' - \mathbf{p})
\end{aligned}
$$

Most general expression for any NN potential:

$$
V^{t m_t} \equiv \sum_{j=1}^{6} v_j^{t m_t}(\mathbf{p}', \mathbf{p})\ w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})
$$

where $v_j^{t m_t}(\mathbf{p}', \mathbf{p})$ is a scalar function of |p'|, |p|, and p'·p

Examples:

χEFT LO potential:

$$V_{LO} = -\frac{1}{(2\pi)^3}\frac{g_A^2}{4F_\pi^2}\frac{\boldsymbol{\sigma}_1\cdot\mathbf{q}\,\boldsymbol{\sigma}_2\cdot\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \frac{C_S}{(2\pi)^3} + \frac{C_T}{(2\pi)^3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,,$$

$$\mathbf{q} = \mathbf{p}' - \mathbf{p}$$

χEFT NLO potential:

$$\begin{aligned}
V_{NLO} \;=\; & -\frac{1}{(2\pi)^3}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{384\pi^2F_\pi^4}\,L^{\tilde{\Lambda}}(q)\left[4m_\pi^2(5g_A^4-4g_A^2-1)+\mathbf{q}^{\,2}(23g_A^4-10g_A^2-1)+\frac{48g_A^4m_\pi^4}{4m_\pi^2+\mathbf{q}^{\,2}}\right] \\
& -\frac{1}{(2\pi)^3}\frac{3g_A^4}{64\pi^2F_\pi^4}\,L^{\tilde{\Lambda}}(q)\left(\boldsymbol{\sigma}_1\cdot\mathbf{q}\,\boldsymbol{\sigma}_2\cdot\mathbf{q}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,\mathbf{q}^{\,2}\right) \\
& +\frac{C_1}{(2\pi)^3}\,\mathbf{q}^{\,2}+\frac{C_2}{(2\pi)^3}\,\mathbf{k}^2+\Big(\frac{C_3}{(2\pi)^3}\,\mathbf{q}^{\,2}+\frac{C_4}{(2\pi)^3}\,\mathbf{k}^2\Big)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
& +\frac{C_5}{(2\pi)^3}\,\frac{i}{2}\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}+\frac{C_6}{(2\pi)^3}\,\mathbf{q}\cdot\boldsymbol{\sigma}_1\,\mathbf{q}\cdot\boldsymbol{\sigma}_2+\frac{C_7}{(2\pi)^3}\,\mathbf{k}\cdot\boldsymbol{\sigma}_1\,\mathbf{k}\cdot\boldsymbol{\sigma}_2\,, \qquad \text{(C2)}
\end{aligned}$$

$$\mathbf{k} \;=\; \tfrac{1}{2}\,(\mathbf{p}'+\mathbf{p}\,)$$

NN t-matrix:

$$t^{tm_t} = V^{tm_t} + V^{tm_t} G_0 t^{tm_t}$$

$$t^{tm_t} \equiv \sum_{j=1}^{6} t_j^{tm_t}(\mathbf{p}', \mathbf{p})\; w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})$$

$$V^{tm_t} \equiv \sum_{j=1}^{6} v_j^{tm_t}(\mathbf{p}', \mathbf{p})\; w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{p}', \mathbf{p})$$

$$\begin{aligned} \sum_{j=1}^{6} t_j^{tm_t}(\mathbf{p}', \mathbf{p})\, \mathrm{w}_j(\mathbf{p}', \mathbf{p}) &= \sum_{j=1}^{6} v_j^{tm_t}(\mathbf{p}', \mathbf{p})\, \mathrm{w}_j(\mathbf{p}', \mathbf{p}) \\ &+ 2\mu \lim_{\epsilon \to 0} \sum_{k,j=1}^{6} \int d\mathbf{p}'' \frac{v_k^{tm_t}(\mathbf{p}', \mathbf{p}'')\, \mathrm{w}_k(\mathbf{p}', \mathbf{p}'')\, t_j^{tm_t}(\mathbf{p}'', \mathbf{p})\, \mathrm{w}_j(\mathbf{p}'', \mathbf{p})}{p^2 + i\epsilon - p''^2} \end{aligned}$$

Project with w_k from the left and perform the trace in NN spin space

$$\sum_j A_{kj}(\mathbf{p}',\mathbf{p})t_j^{tm_t}(\mathbf{p}',\mathbf{p}) = \sum_j A_{kj}(\mathbf{p}',\mathbf{p})v_j^{tm_t}(\mathbf{p}',\mathbf{p}) + \int d^3p'' \sum_{jj'} v_j^{tm_t}(\mathbf{p}',\mathbf{p}'')G_0(p'')\; t_{j'}^{tm_t}(\mathbf{p}'',\mathbf{p})\; B_{kjj'}(\mathbf{p}',\mathbf{p}'',\mathbf{p})$$

All functions are scalar !

$$A_{kj}(\mathbf{p}',\mathbf{p}) \;\equiv\; \mathrm{Tr}\Big(w_k(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})\; w_j(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})\Big)$$

$$B_{kjj'}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) \;\equiv\; \mathrm{Tr}\Big(w_k(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})\; w_j(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p}'')\; w_{j'}(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}'',\mathbf{p})\Big)$$

NN t-matrix consists of 6 coupled eqs of scalar functions

Structure of some of the A_{kj} and $B_{kjj'}$:

$$
\begin{aligned}
A_{26}(\mathbf{p}',\mathbf{p}) &= 4(\mathbf{p}'-\mathbf{p})^2 \\
A_{33}(\mathbf{p}',\mathbf{p}) &= -8(\mathbf{p}\times\mathbf{p}')^2 \\
A_{56}(\mathbf{p}',\mathbf{p}) &= 4(p'^2-p^2)^2
\end{aligned}
$$

14 non-vanishing

$$
\begin{aligned}
B_{261}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= 4(\mathbf{p}'-\mathbf{p}'')^2 \\
B_{612}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= 4(\mathbf{p}'-\mathbf{p})^2 \\
B_{133}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= -8(\mathbf{p}''\times\mathbf{p}')\cdot(\mathbf{p}\times\mathbf{p}'') \\
B_{331}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= -8(\mathbf{p}\times\mathbf{p}')\cdot(\mathbf{p}''\times\mathbf{p}') \\
B_{313}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= -8(\mathbf{p}\times\mathbf{p}')\cdot(\mathbf{p}\times\mathbf{p}'') \\
B_{145}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= 4\{(\mathbf{p}\times\mathbf{p}')\cdot\mathbf{p}''\}^2 \\
B_{155}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= 4\{(\mathbf{p}'+\mathbf{p}'')\cdot(\mathbf{p}''+\mathbf{p})\}^2 \\
B_{551}(\mathbf{p}',\mathbf{p}'',\mathbf{p}) &= 4\{(\mathbf{p}'+\mathbf{p})\cdot(\mathbf{p}'+\mathbf{p}'')\}^2
\end{aligned}
$$

148 non-vanishing

General form of scattering amplitude (Wolfenstein representation)

$$M = a + c(\sigma^{(1)} + \sigma^{(2)})\hat{N} + m(\sigma^{(1)}\hat{N})(\sigma^{(2)}\hat{N}) + (g+h)(\sigma^{(1)}\hat{P})(\sigma^{(2)}\hat{P}) + (g-h)(\sigma^{(1)}\hat{K})(\sigma^{(2)}\hat{K}) .$$

$$\hat{K} \equiv (q' - q)/|q' - q|$$

$$\hat{P} \equiv (q + q')/|q + q'|$$

$$\hat{N} \equiv (q \times q')/|q \times q'| .$$

Solution of our T-matrix equation can be directly mapped to the Wolfenstein amplitudes (initial state needs to be antisymmetrized)

Wolfenstein amplitudes lead directly to observables

(see e.g. N. Hoshizaki, Prog. Theor. Phys. 42, 107 (1968))

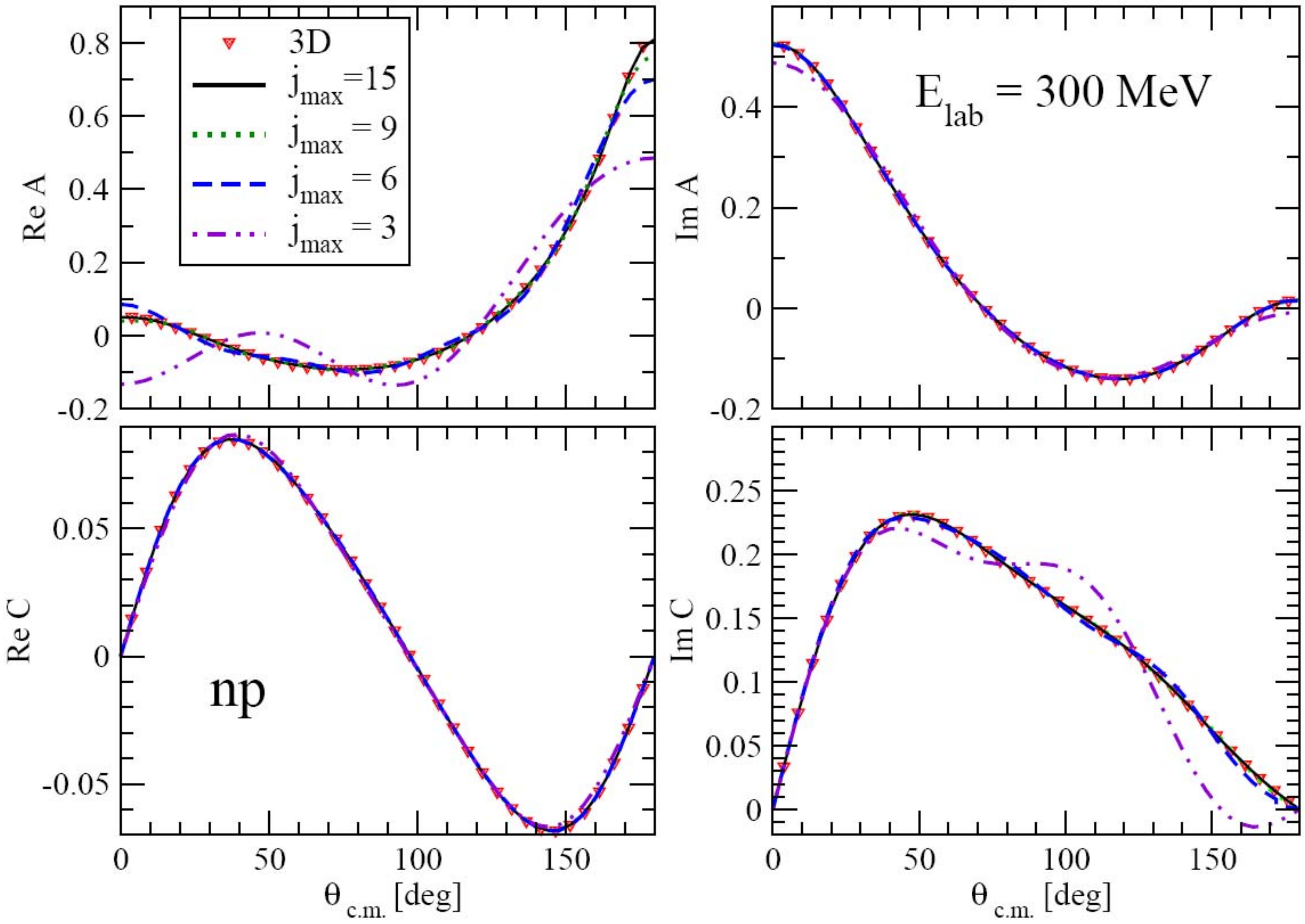
Wolfenstein Amplitudes:

$$
\begin{aligned}
a^{tm_t} &= \frac{1}{4}\,\mathrm{Tr}\,(M) \\
c^{tm_t} &= -i\frac{1}{8}\,\mathrm{Tr}\left(M\;\frac{w_3(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})}{|\mathbf{p}\times\mathbf{p}'|}\right) \\
m^{tm_t} &= \frac{1}{4}\,\mathrm{Tr}\left(M\;\frac{w_4(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})}{|\mathbf{p}\times\mathbf{p}'|^2}\right) \\
(g+h)^{tm_t} &= \frac{1}{4}\,\mathrm{Tr}\left(M\;\frac{w_5(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})}{(\mathbf{p}+\mathbf{p}')^2}\right) \\
(g-h)^{tm_t} &= \frac{1}{4}\,\mathrm{Tr}\left(M\;\frac{w_6(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})}{(\mathbf{p}-\mathbf{p}')^2}\right)
\end{aligned}
$$

with

$$
\begin{aligned}
M^{tm_t}_{m_1'm_2',m_1m_2} &= -\frac{m}{2}(2\pi)^2\sum_{j=1}^{6}\Big[t_j^{tm_t}(\mathbf{p}',\mathbf{p})\;\langle m_1'm_2'|w_j(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',\mathbf{p})|m_1m_2\rangle \\
&+\;(-)^t t_j^{tm_t}(\mathbf{p}',-\mathbf{p})\;\langle m_1'm_2'|w_j(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathbf{p}',-\mathbf{p})|m_2m_1\rangle\Big]
\end{aligned}
$$

np scattering at 300 MeV – Bonn B



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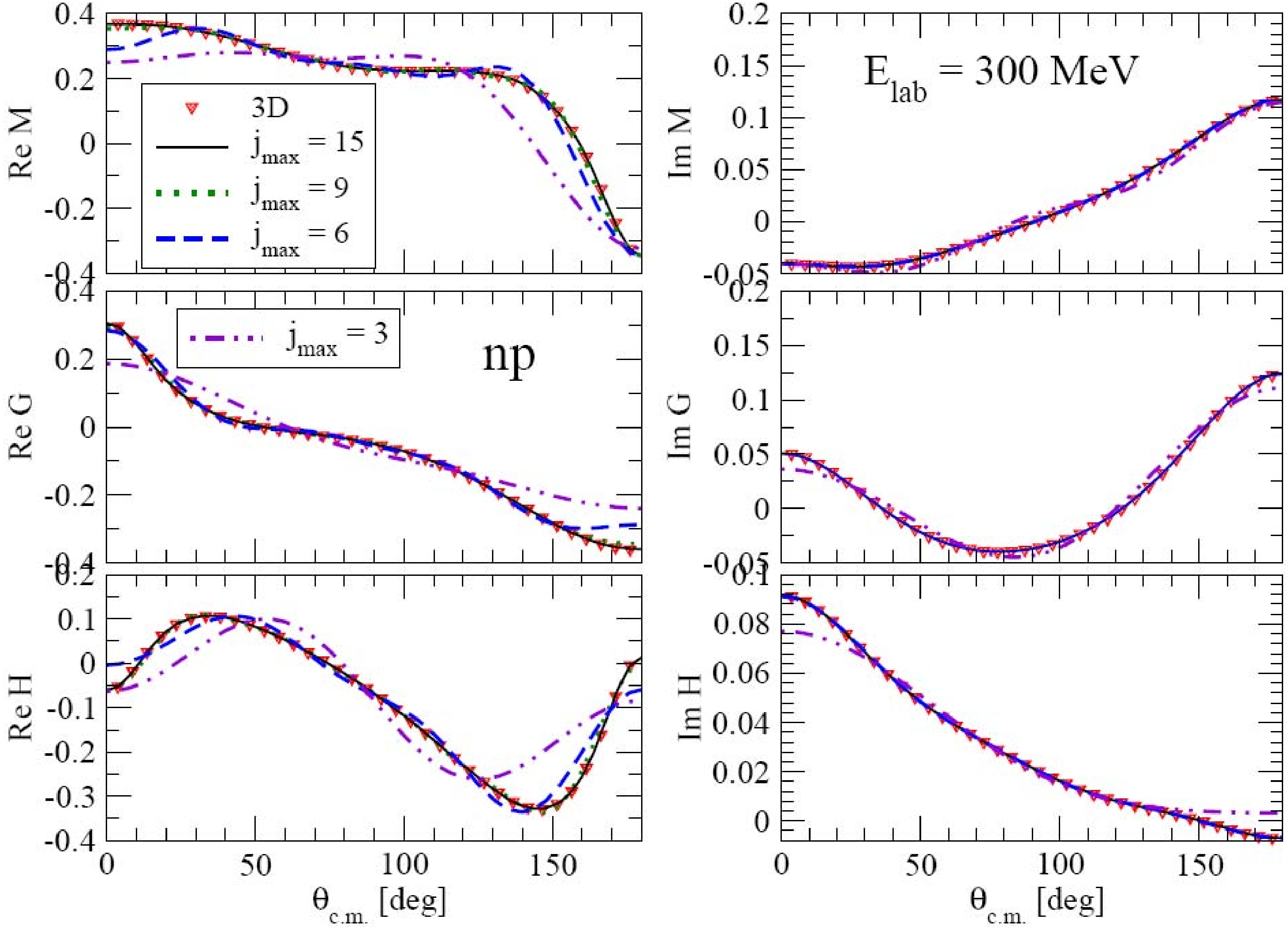
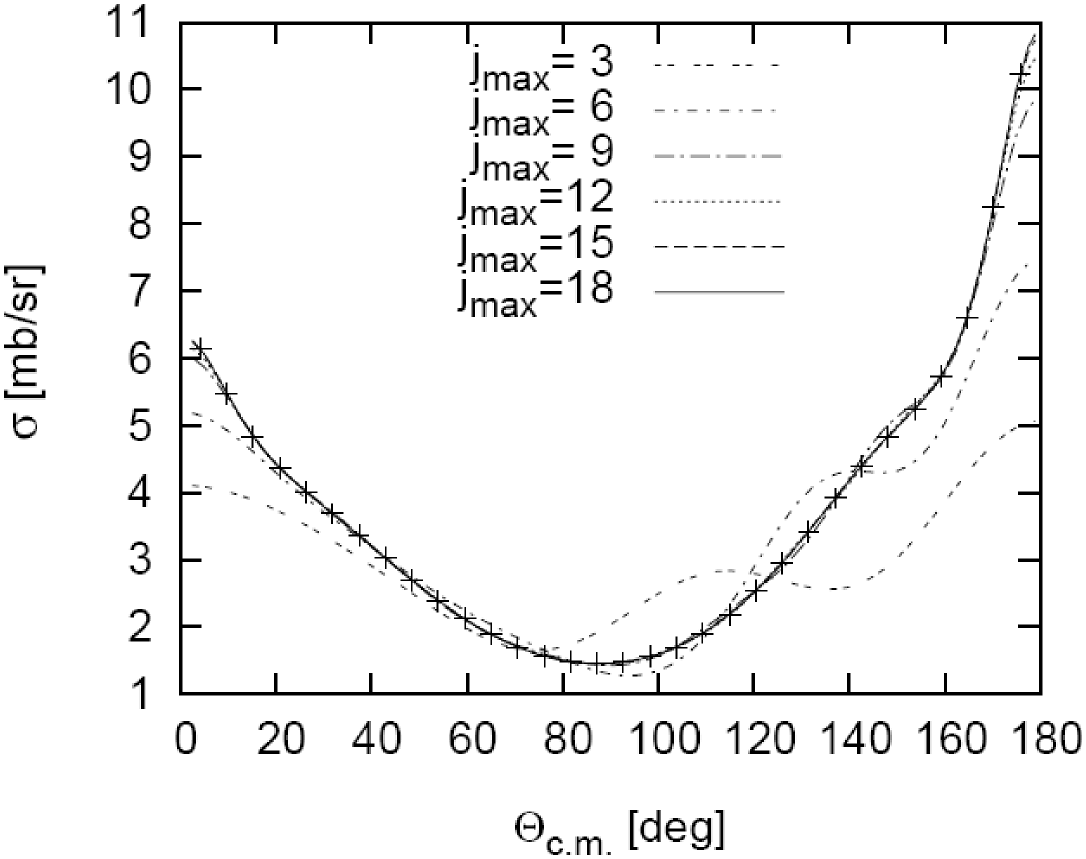


Spin averaged differential cross section:

$$I_0 = \frac{1}{4}$$



Deuteron: t=0, s=1

$$\langle \mathrm{p}|\Psi_{m_d}\rangle = \left[\phi_1(p) + \left(\boldsymbol{\sigma}_1\cdot\mathrm{p}\,\boldsymbol{\sigma}_2\cdot\mathrm{p} - \frac{1}{3}p^2\right)\ \phi_2(p)\right]|1m_d\rangle$$
$$\equiv \sum_{k=1}^{2}\phi_k(p)\ b_k(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathrm{p})|1m_d\rangle,$$

Standard s-wave $\quad \psi_0(p) = \phi_1(p),$

d-wave $\quad \psi_2(p) = \dfrac{4p^2}{3\sqrt{2}}\,\phi_2(p).$

Schrödinger equation: $\quad \Psi_{m_d} = G_0 V^{00}\Psi_{m_d}.$

$$\left[\phi_1(p) + \left(\boldsymbol{\sigma}_1\cdot\mathrm{p}\,\boldsymbol{\sigma}_2\cdot\mathrm{p} - \frac{1}{3}p^2\right)\phi_2(p)\right]|1m_d\rangle =$$
$$\frac{1}{E_d - \frac{p^2}{m}}\int d^3p' \sum_{j=1}^{6} v_j^{00}(\mathrm{p},\mathrm{p}')\ w_j(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2,\mathrm{p},\mathrm{p}')$$
$$\times\ \left[\phi_1(p') + \left(\boldsymbol{\sigma}_1\cdot\mathrm{p}'\,\boldsymbol{\sigma}_2\cdot\mathrm{p}' - \frac{1}{3}p'^2\right)\phi_2(p')\right]|1m_d\rangle,$$

Project from left with $\langle 1 m_d | b_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p})$ **and sum over** m_d

$$\sum_{m_d=-1}^{1} \langle 1 m_d | b_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}) \sum_{k'=1}^{2} \phi_{k'}(p) b_{k'}(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}) | 1 m_d \rangle =$$

$$\frac{1}{E_d - \frac{p^2}{m}} \sum_{m_d=-1}^{1} \int d^3p' \sum_{j=1}^{6} v_j^{00}(\mathrm{p}, \mathrm{p}') w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}, \mathrm{p}') \sum_{k''=1}^{2} \phi_{k''}(p')\, b_{k''}(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}') | 1 m_d \rangle$$

With scalar functions:

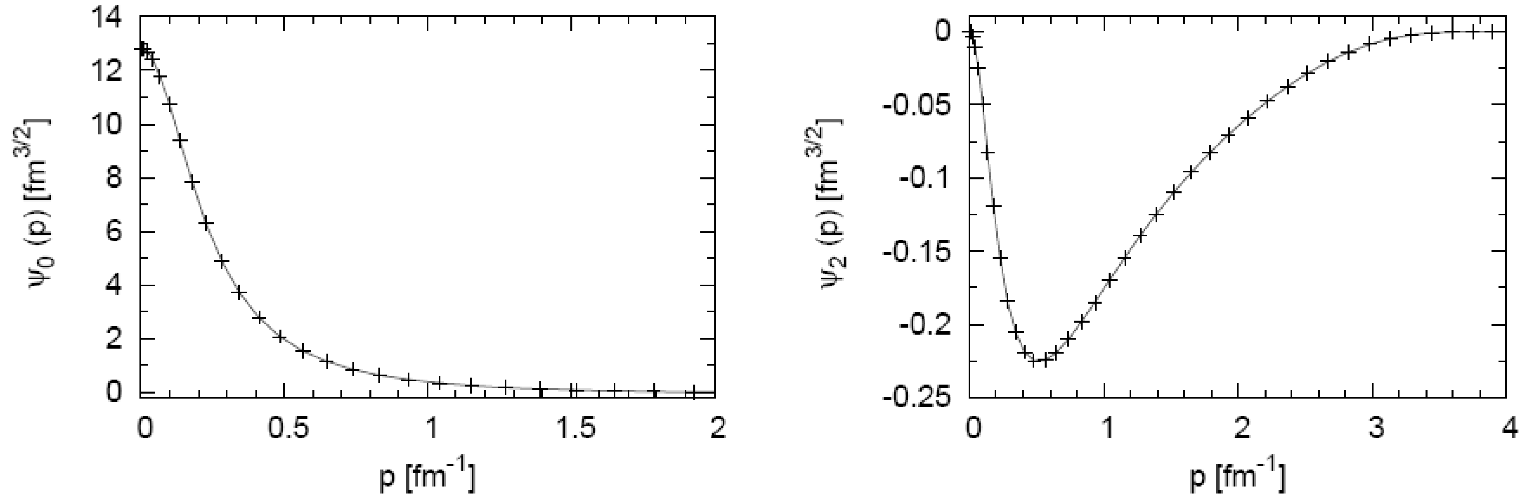
$$A^d_{kk'}(p) \equiv \sum^{1} \langle 1 m_d | b_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}) b_{k'}(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}) | 1 m_d \rangle$$

$$B^d_{kjk''}(\mathrm{p}, \mathrm{p}') \equiv \sum_{m_d=-1}^{1} \langle 1 m_d | b_k(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}) w_j(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}, \mathrm{p}') b_{k''}(\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathrm{p}') | 1 m_d \rangle$$

Deuteron: coupled eq. of 2 scalar functions:

$$\sum_{k'=1}^{2} A^d_{kk'}(p) \phi_{k'}(p) = \frac{1}{E_d - \frac{p^2}{m}} \int d^3p' \sum_{j=1}^{6} v_j^{00}(\mathrm{p}, \mathrm{p}') \sum_{k''=1}^{2} B^d_{kjk''}(\mathrm{p}, \mathrm{p}') \phi_{k''}(p')$$

Example: S- and D-wave of chiral NNLO potential.



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The Two-Nucleon System in Three Dimensions.

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Outlook for the 3N bound state:

Faddeev Equation: $\psi = G_0 t P \psi + (1 + G_0 t) G_0 V^{(1)} (1 + P) \psi$

$P = P_{12} P_{23} + P_{13} P_{23}$

Wave function: $\Psi = (1 + P)\psi.$

3 possible iso-spin states:

$$|\gamma_0\rangle = |\left(0\frac{1}{2}\right)\frac{1}{2}\rangle$$
$$|\gamma_1\rangle = |\left(1\frac{1}{2}\right)\frac{1}{2}\rangle$$
$$|\gamma_2\rangle = |\left(1\frac{1}{2}\right)\frac{3}{2}\rangle.$$

Iso-spin structure of Faddeev component

$$\psi = \sum_\gamma |\gamma\rangle \psi_\gamma \equiv \sum_{tT} |\left(t\frac{1}{2}\right) T\rangle \psi_{tT}$$

Faddeev component $\psi_{tT}(\vec{p},\vec{q}) = \sum_{i=1}^{8} \phi_{tT}^{(i)}(\vec{p},\vec{q})\; O_i|\chi^m\rangle = \sum_{i=1}^{8} \tilde{\phi}_{tT}^{(i)}(\vec{p},\vec{q})|\chi_i\rangle$

$$
\begin{aligned}
O_1 &= 1\\
O_2 &= \vec{\sigma}(23)\cdot\vec{\sigma}_{(1)}\\
O_3 &= \vec{\sigma}_{(1)}\cdot(\hat{\vec{p}}\times\hat{\vec{q}})\\
O_4 &= \vec{\sigma}(23)\cdot\hat{\vec{p}}\times\hat{\vec{q}}\\
O_5 &= \vec{\sigma}(23)\cdot\hat{\vec{q}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{p}}\\
O_6 &= \vec{\sigma}(23)\cdot\hat{\vec{p}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{q}}\\
O_7 &= \vec{\sigma}(23)\cdot\hat{\vec{p}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{p}}\\
O_8 &= \vec{\sigma}(23)\cdot\hat{\vec{q}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{q}}.
\end{aligned}
$$

$$\vec{\sigma}(23) \equiv \tfrac{1}{2}(\vec{\sigma}_{(2)} - \vec{\sigma}_{(3)})$$

$$
\begin{aligned}
\chi_1 &= |\chi^m\rangle\\
\chi_2 &= \frac{1}{\sqrt{3}}\,\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)}|\chi^m\rangle\\
\chi_3 &= \sqrt{\frac{3}{2}}\frac{1}{i}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{p}}\times\hat{\vec{q}}|\chi^m\rangle\\
\chi_4 &= \frac{1}{\sqrt{2}}\Big(i\vec{\sigma}(23)\times(\hat{\vec{p}}\times\hat{\vec{q}}) - \vec{\sigma}_{(1)}\times\vec{\sigma}(23)\cdot(\hat{\vec{p}}\times\hat{\vec{q}})\Big)|\chi^m\rangle\\
\chi_5 &= \frac{1}{i}\Big(\vec{\sigma}(23) - \frac{i}{2}\vec{\sigma}_{(1)}\times\vec{\sigma}(23)\Big)\times(\hat{\vec{p}}\times\hat{\vec{q}})|\chi^m\rangle\\
\chi_6 &= \sqrt{\frac{3}{2}}\Big(\vec{\sigma}(23)\cdot\hat{\vec{p}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{p}} - \frac{1}{3}\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)}\Big)|\chi^m\rangle\\
\chi_7 &= \sqrt{\frac{3}{2}}\Big(\vec{\sigma}(23)\cdot\hat{\vec{q}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{q}} - \frac{1}{3}\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)}\Big)|\chi^m\rangle\\
\chi_8 &= \frac{3}{2}\frac{1}{\sqrt{5}}\Big(\vec{\sigma}(23)\cdot\hat{\vec{q}}\,\vec{\sigma}_{(1)}\cdot\hat{\vec{p}} + \vec{\sigma}(23)\cdot\hat{\vec{p}}\vec{\sigma}_{(1)}\cdot\hat{\vec{q}}\\
&- \frac{2}{3}\hat{\vec{p}}\cdot\hat{\vec{q}}\vec{\sigma}(23)\cdot\vec{\sigma}_{(1)}\Big)|\chi^m\rangle\ .
\end{aligned}
$$

E. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942).

I. Fachruddin, W. Glöckle, Ch. Elster and A. Nogga, Phys. Rev. C **69**, 064002 (2004)