Low-energy four-boson scattering

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Four-boson scattering

- scattering equations
- momentum-space techniques
- first results

Scattering: wave function vs transition operator

- Schrödinger equation

$$(H_0 + v)|\psi\rangle = E|\psi\rangle$$

+ impose asymptotic boundary conditions explicitly

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wave function $|\psi\rangle = |\mathbf{k}\rangle + G_0 v|\psi\rangle$

$$G_0 = (E + i0 - H_0)^{-1}$$

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wave function $$|\psi\rangle = |\mathbf{k}\rangle + G_0 v|\psi\rangle$$

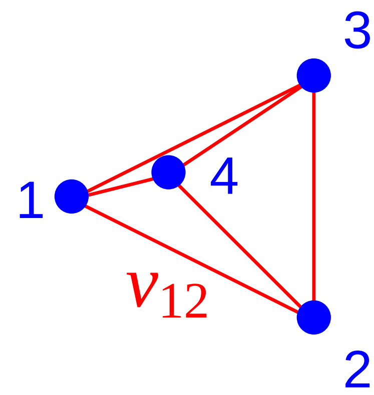
$$G_0 = (E + i0 - H_0)^{-1}$$

transition matrix $$T|\mathbf{k}\rangle = v|\psi\rangle$$

$$T = v + vG_0T$$

$$|\psi\rangle = |\mathbf{k}\rangle + G_0T|\mathbf{k}\rangle$$

Four identical bosons



Hamiltonian $H_0 + \sum_{i>j} v_{ij}$

- Wave function:
  Faddeev-Yakubovsky equations
- Transition operators:
  Alt-Grassberger-Sandhas equations
- Permutation symmetry:
  18 $\to$ 2 FY components: 3+1 and 2+2 partitions

Symmetrized AGS equations

$$\mathcal{U}_{11} = (G_0 T G_0)^{-1} P_{34} + P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{11} + U_2 G_0 T G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 T G_0)^{-1} (1 + P_{34}) + (1 + P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} + P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{12} + U_2 G_0 T G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 + P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{12}$$

$$T = v + v G_0 T$$

$$U_j = P_j G_0^{-1} + P_j T G_0 U_j$$

$$3 + 1: \quad P_1 = P = P_{12} P_{23} + P_{13} P_{23}$$

$$2 + 2: \quad P_2 = \tilde{P} = P_{13} P_{24}$$

scattering amplitude $\quad \mathcal{T}_{fi} = S_{fi} \langle \phi_f | \mathcal{U}_{fi} | \phi_i \rangle$

$$|\phi_j\rangle = G_0 T P_j |\phi_j\rangle$$

basis states symmetric in pair (12) only

Wave function components

Scattering:

$$|\psi_{j,i}\rangle = \delta_{ji}|\phi_i\rangle + G_0 T G_0 U_j G_0 T G_0 \mathcal{U}_{ji}|\phi_i\rangle$$

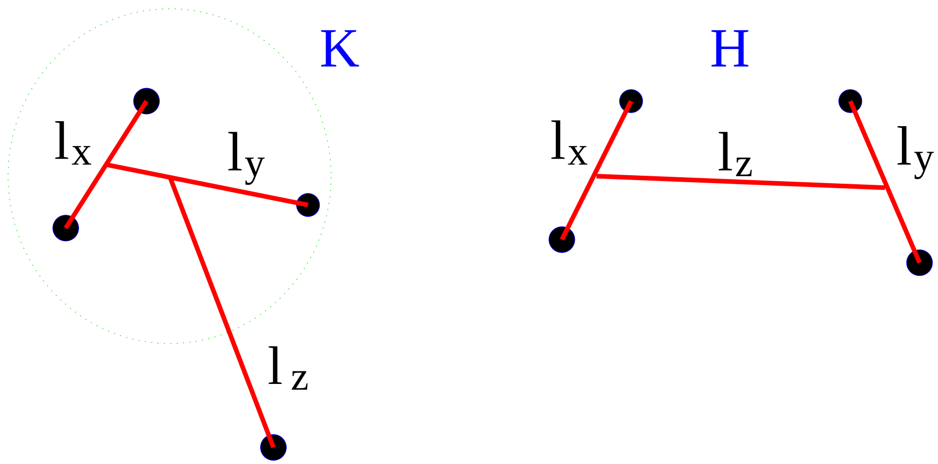
Bound state:

$$|\psi_1\rangle = G_0 T G_0 U_1 (P_{34}|\psi_1\rangle + |\psi_2\rangle)$$

$$|\psi_2\rangle = G_0 T G_0 U_2 (1 + P_{34})|\psi_1\rangle$$

Solution of AGS equations

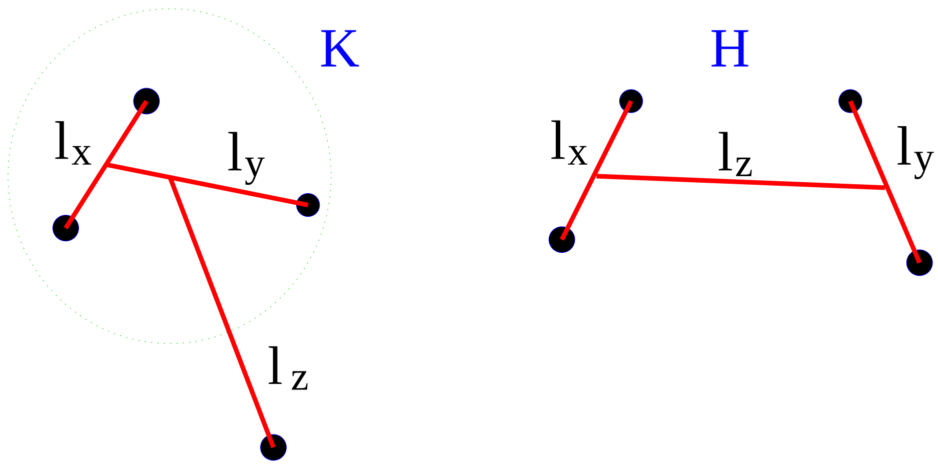
$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} + P_{34} U_1 G_0 T G_0\, \mathcal{U}_{12} + U_2 G_0 T G_0\, \mathcal{U}_{22}$$



- momentum-space partial-wave basis
  $|k_x k_y k_z [(l_x l_y) K l_z] J\rangle$ & $|h_x h_y h_z [(l_x l_y) H l_z] J\rangle$
- set of coupled integral equations in 3 variables:
  $\langle k_x k_y k_z [(l_x l_y) K l_z] J | \mathcal{U}_{1i} | \phi_i \rangle$
  $\langle h_x h_y h_z [(l_x l_y) H l_z] J | \mathcal{U}_{2i} | \phi_i \rangle$

Solution of AGS equations

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} + P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{12} + U_2 G_0 T G_0 \, \mathcal{U}_{22}$$



- bound state poles:
  $U_j(E_j \to -B_j) \to P_j G_0^{-1} |\phi_j\rangle S_{jj}(E_j + i0 + B_j)^{-1} \langle \phi_j | G_0^{-1} P_j$
- numerical methods:
  pole subtraction
  Gaussian integration
  spline interpolation
  (double) Padé summation

Interaction model

- Realistic interactions very repulsive at short distances
- Cutoff: ^{4}He trimer BE and atom-dimer scattering in agreement with r-space results by R. Lazauskas
- Separable potentials in present four-boson calculations

AGS equations with separable potential

$$v = |\mathbf{g}\rangle\lambda\langle\mathbf{g}|$$

$$T = |\mathbf{g}\rangle\tau\langle\mathbf{g}|$$

$$\mathcal{U}_{12} = (G_0\, T\, G_0)^{-1} + P_{34}U_1G_0|\mathbf{g}\rangle\tau\langle\mathbf{g}|G_0\,\mathcal{U}_{12}$$
$$+ U_2G_0|\mathbf{g}\rangle\tau\langle\mathbf{g}|G_0\,\mathcal{U}_{22}$$

- set of coupled integral equations in 2 variables:
  $\langle g_n k_y k_z[(l_x l_y)K l_z]J|G_0\,\mathcal{U}_{1i}|\phi_i\rangle$
  $\langle g_n h_y h_z[(l_x l_y)H l_z]J|G_0\,\mathcal{U}_{2i}|\phi_i\rangle$
- low rank & few partial waves:
  direct solution of discretized 2-variable equations (matrix inversion) feasible

^{4}He **atoms**

S-wave rank 1 separable potential

$$\langle p'|v|p\rangle = g(p')\,\lambda\,g(p)$$

$$g(p) = \exp(-p^2/\Lambda^2)$$

$$\Lambda = 0.4\ \text{Å}^{-1}$$

$$a_0 = 104.0\ \text{Å}$$

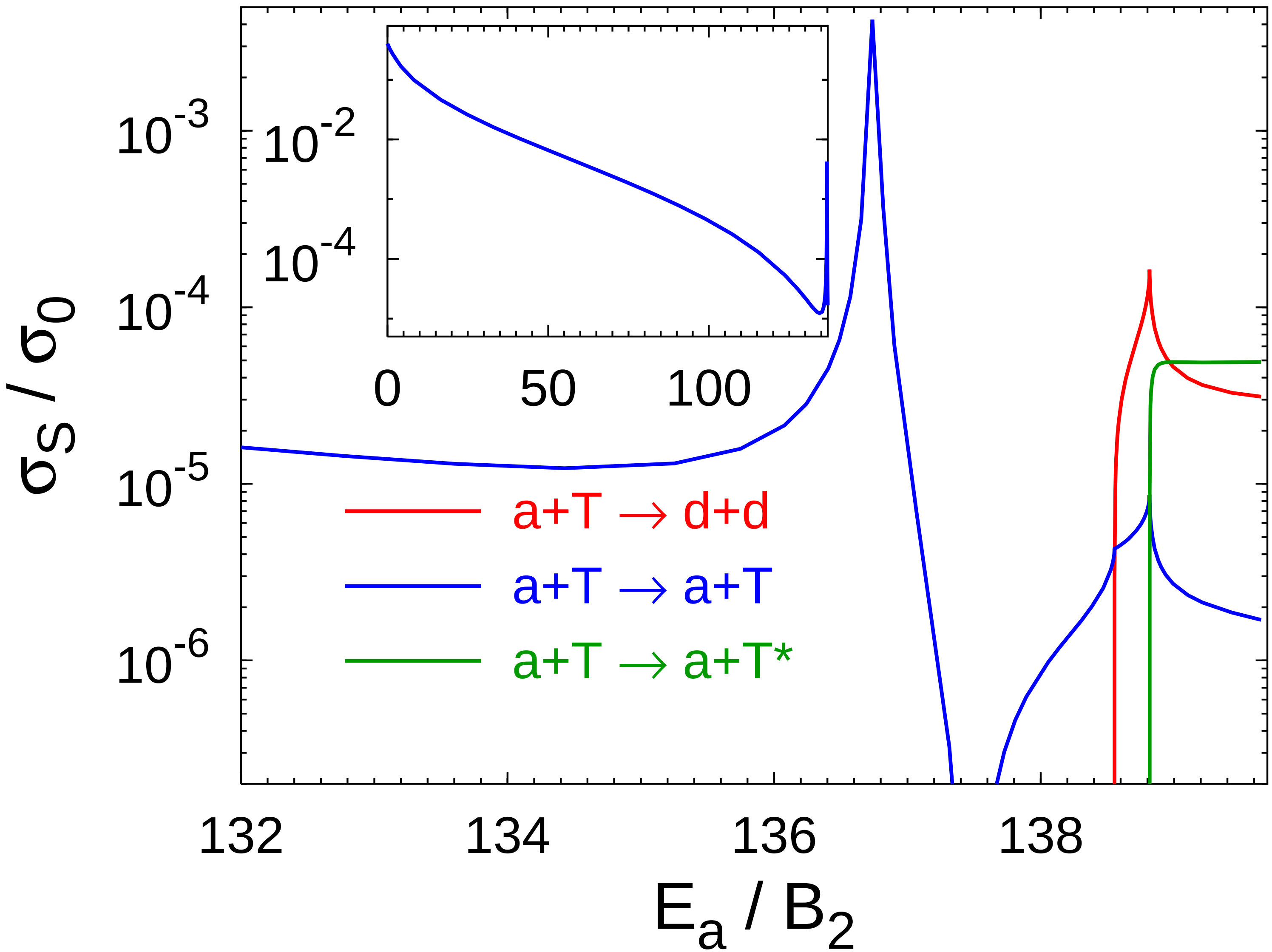
$$B_2 = 1.2126\ \text{mK}$$

$$B_3 = 128.427\ \text{mK}$$

$$B_{3*} = 2.1855\ \text{mK}$$

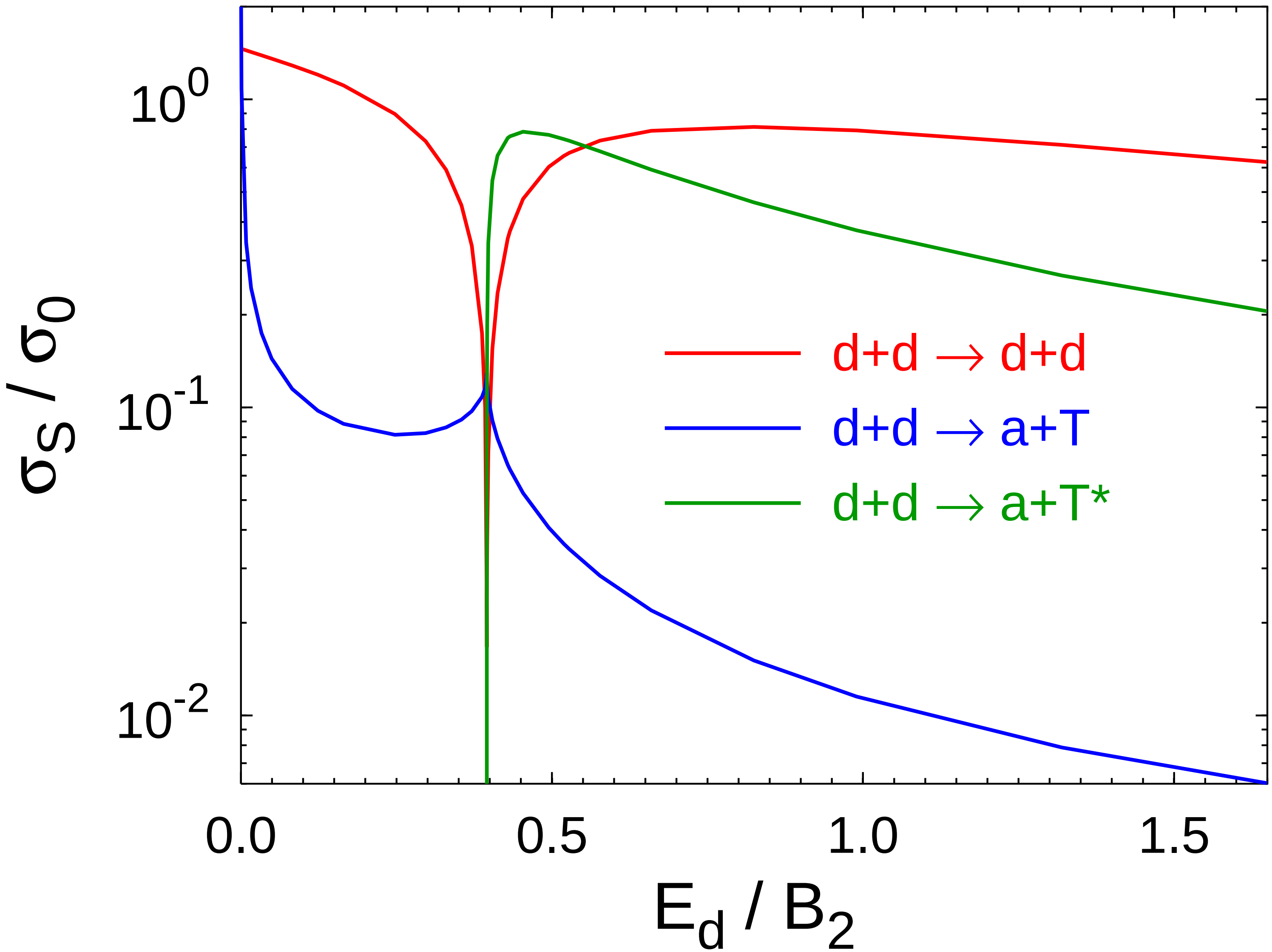
$l_x = 0$ but $l_y, l_z \leq 2$ in 4-boson calculations!

Atom-trimer S-wave scattering

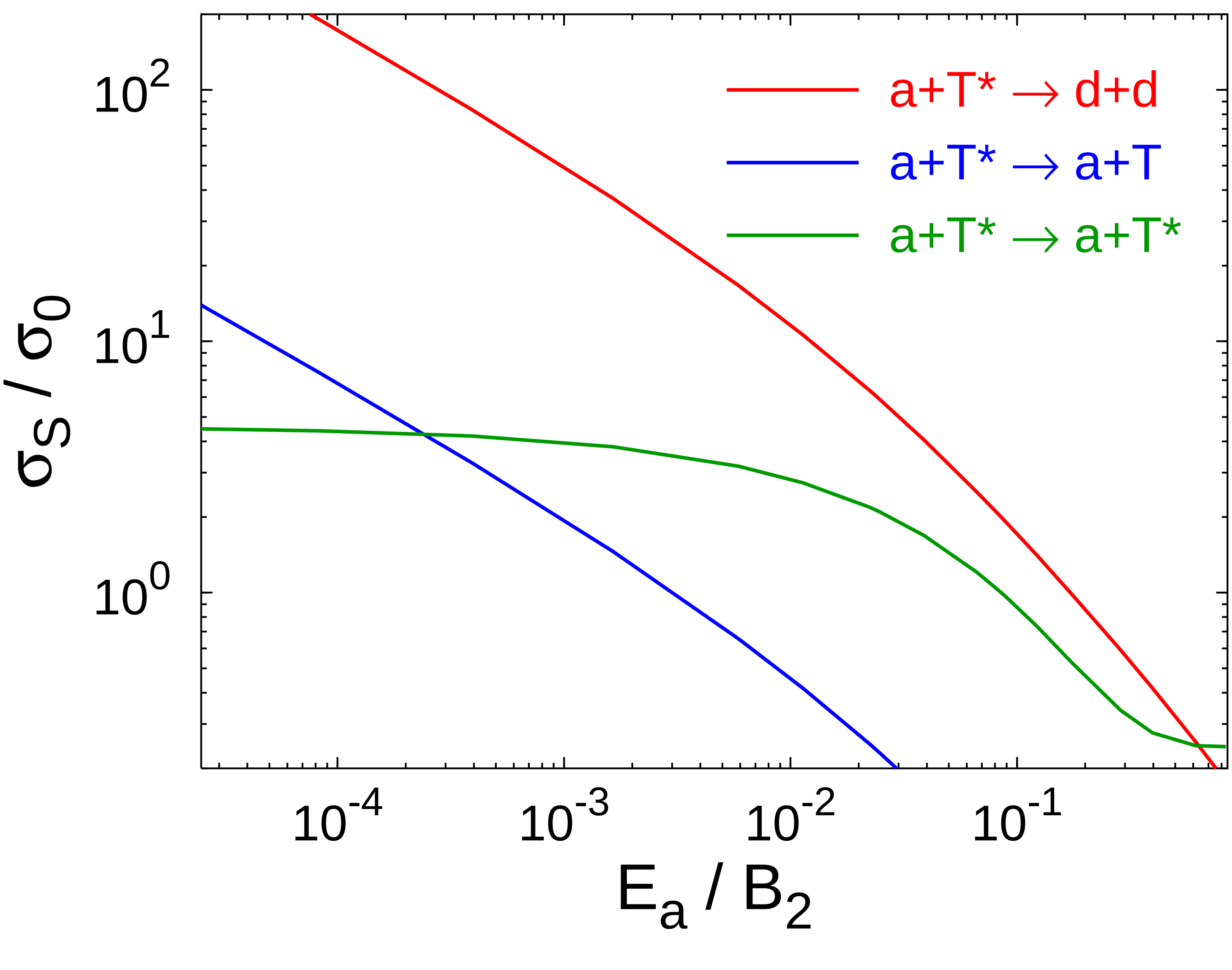


$\sigma_0 = 4\pi a_0^2$

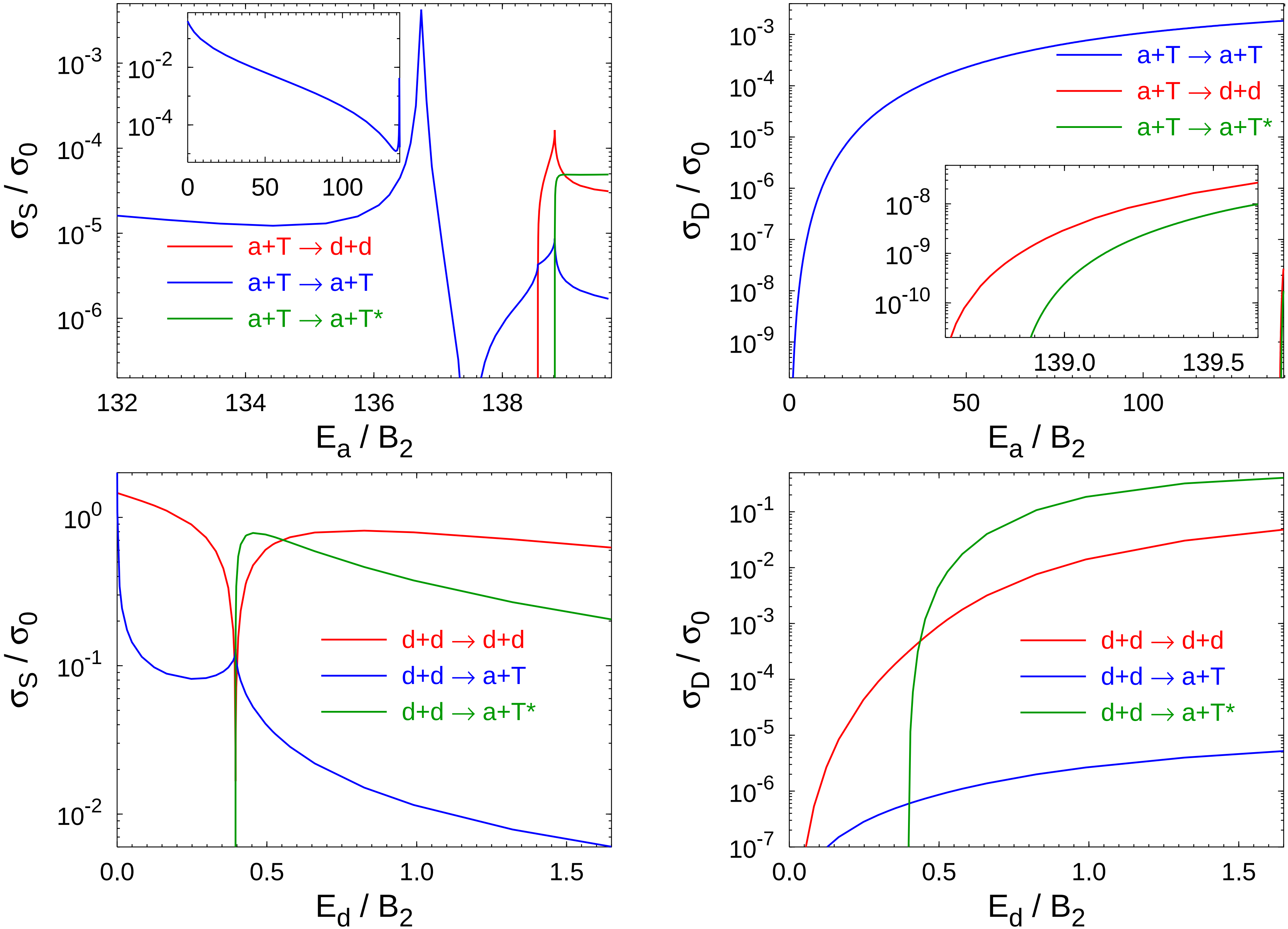
Dimer-dimer S-wave scattering



Atom-trimer (excited) S-wave scattering



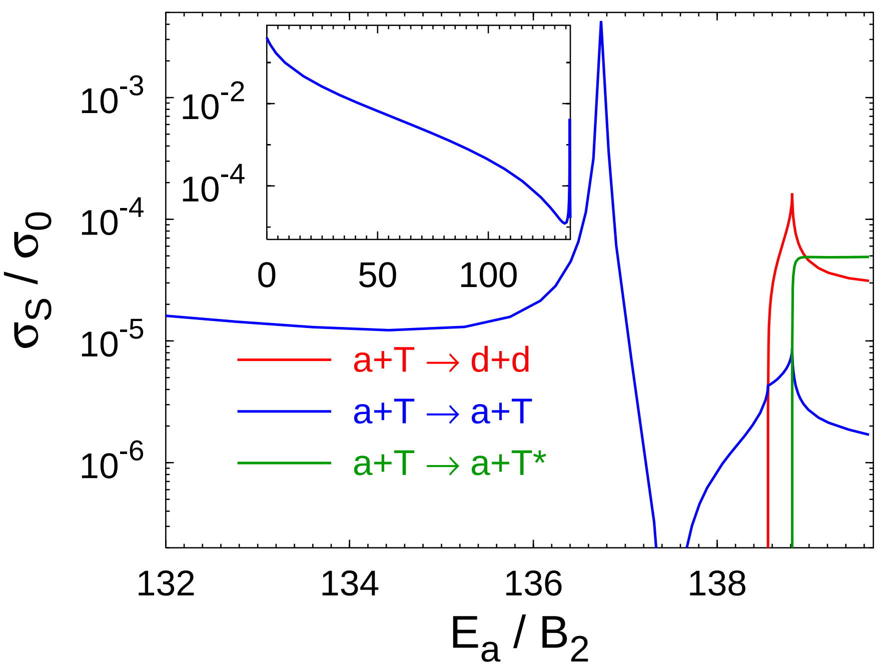
S-wave vs D-wave scattering

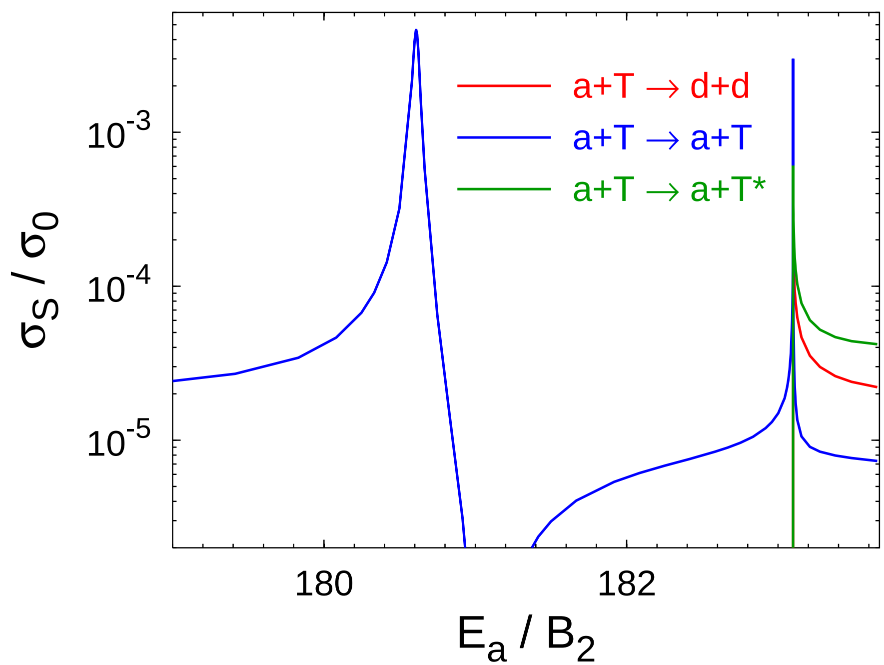


Modified potential: $B_{3*} \approx 2B_2$

$\Lambda = 0.4$ Å^{-1}
$a_0 = 104.0$ Å
$B_2 = 1.2126$ mK
$B_3 = 128.427$ mK
$B_{3*} = 2.1855$ mK

$\Lambda = 0.46817$ Å^{-1}
$a_0 = 104.0$ Å
$B_2 = 1.1984$ mK
$B_3 = 166.960$ mK
$B_{3*} = 2.3967$ mK $\approx 2B_2$





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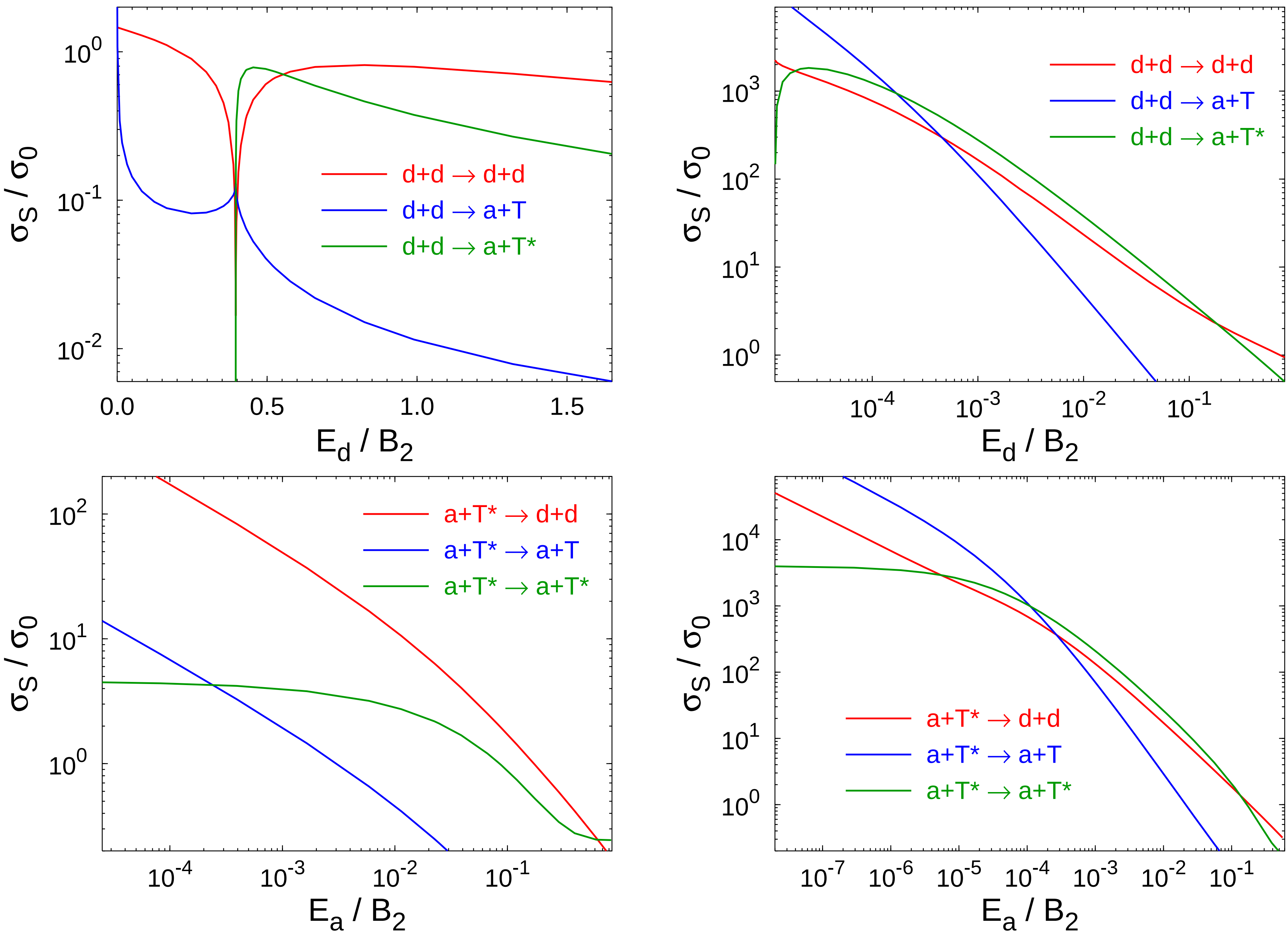
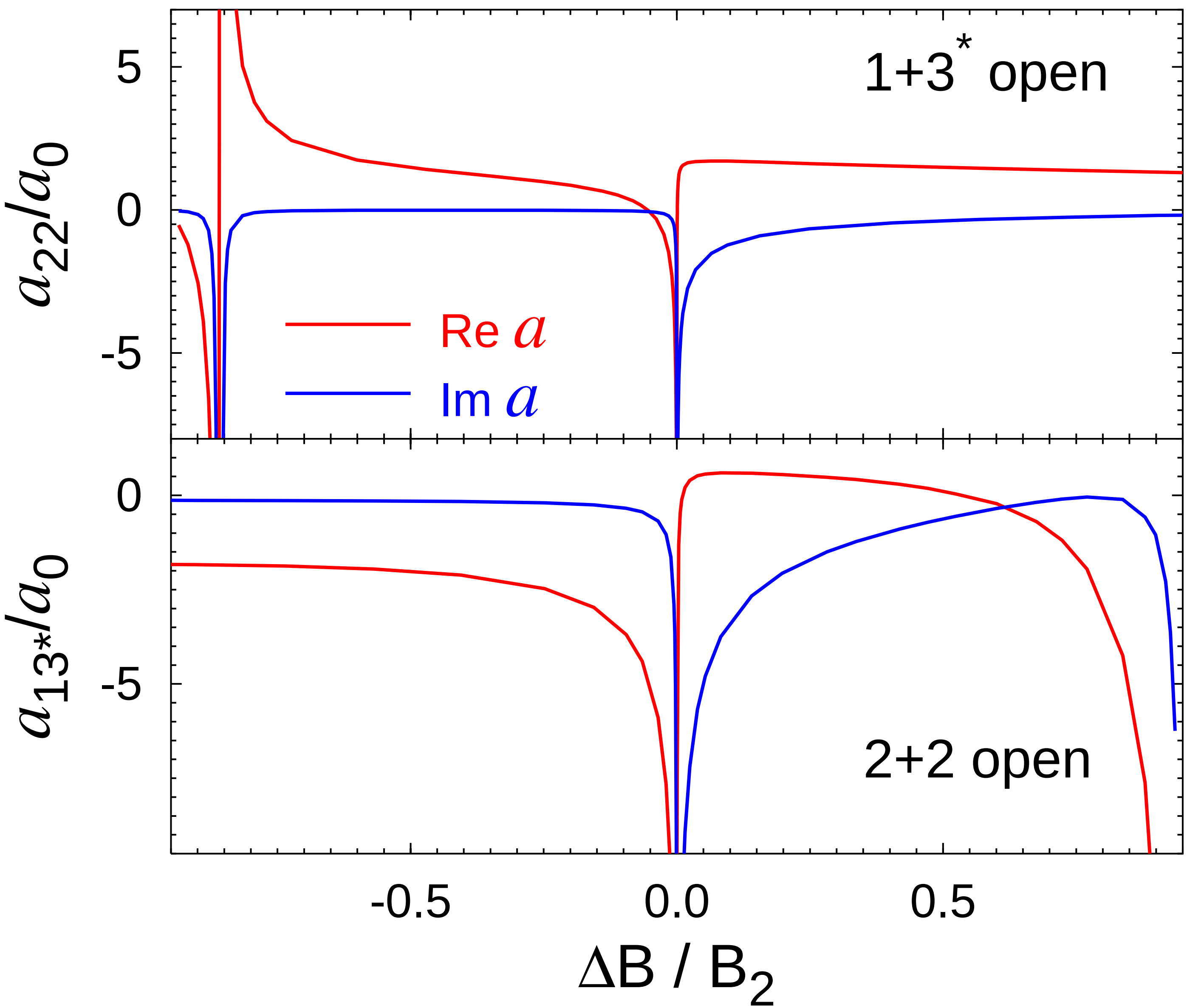
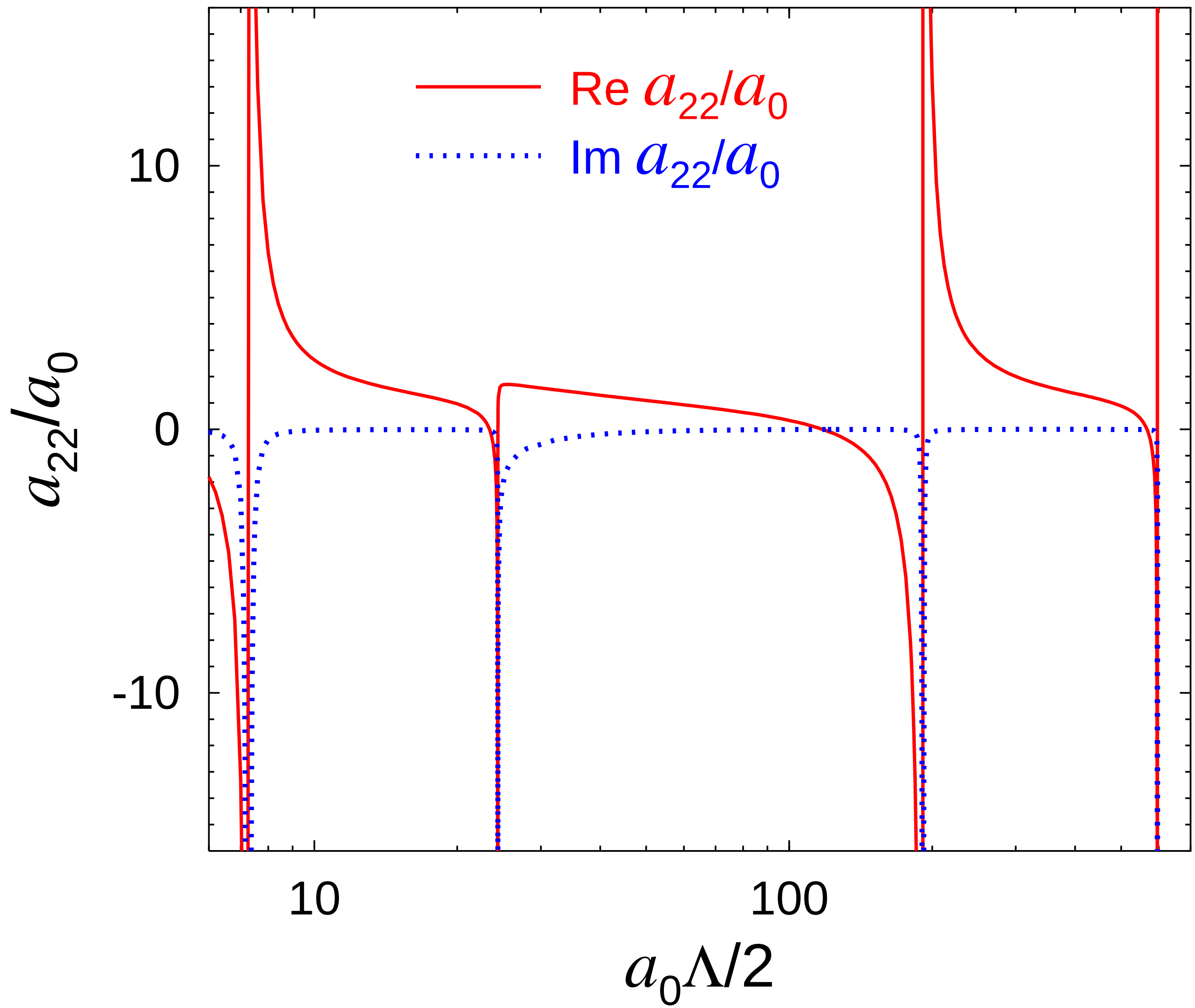


Scattering length vs threshold difference ΔB



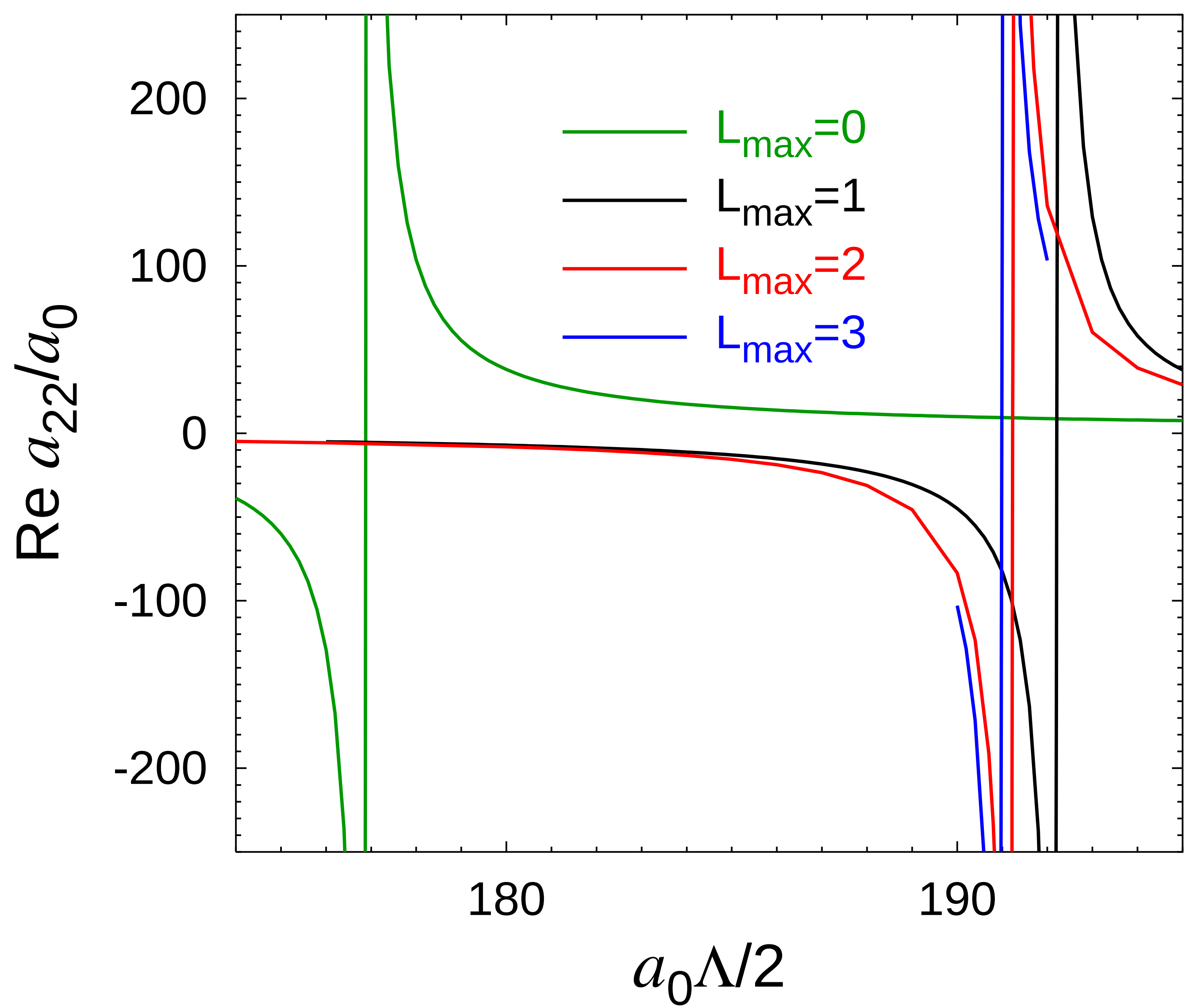
$a_0 = 104.0$ Å, $\quad 0.1$ Å$^{-1} < \Lambda < 0.8$ Å^{-1}

Dimer-dimer vs atom-atom scattering length



$\Lambda = 0.4$ Å^{-1}

2+2 scattering length: convergence with l_y & $l_z \leq L_{max}$



Breakdown of universality

In collaboration with R. Lazauskas (CNRS Strasbourg)

series of rank 2 separable potentials

$$\langle p'|v|p\rangle = \sum_{i,j=1}^{2} g_i(p')\,\lambda_{ij}\,g_j(p)$$

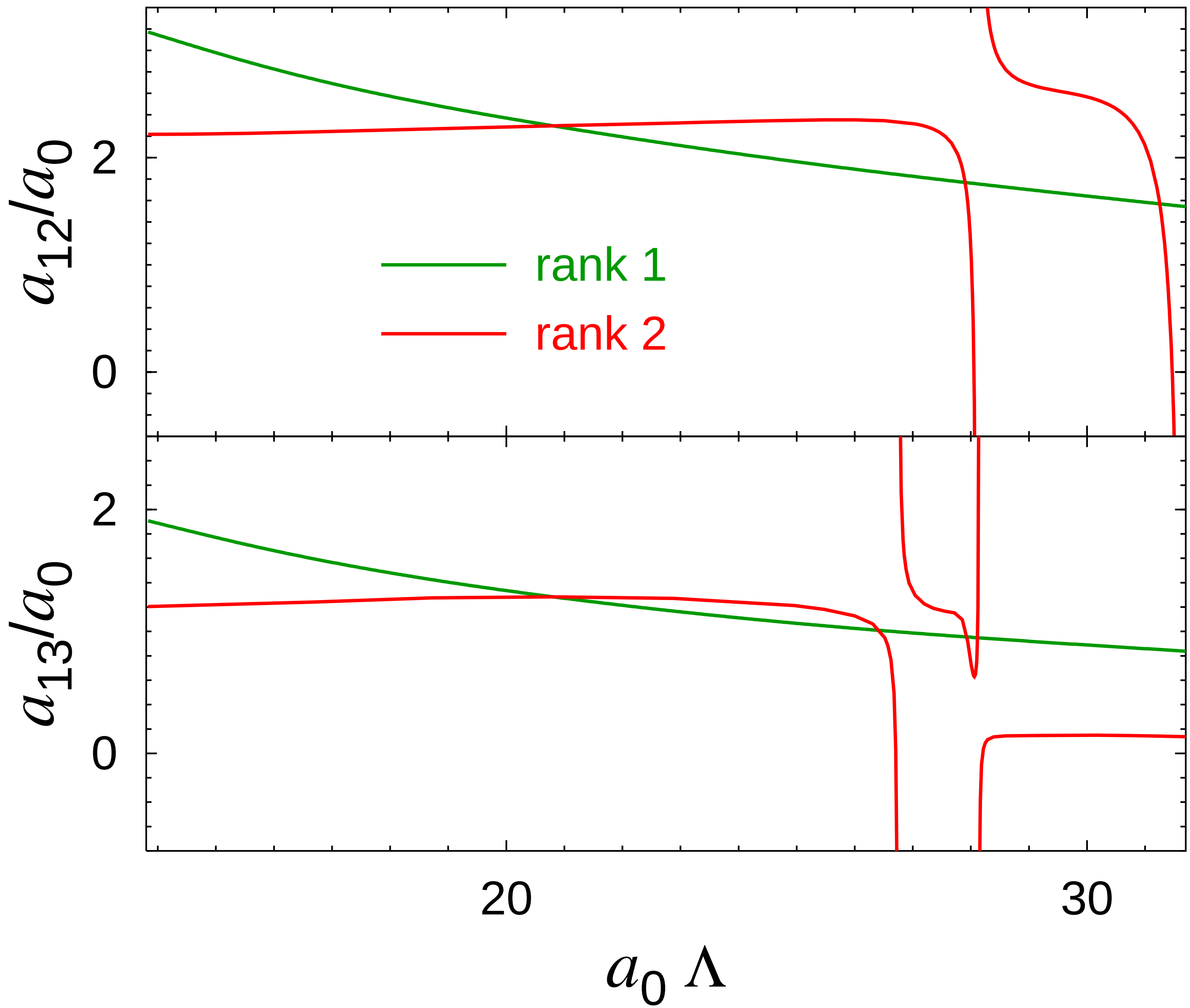
$$g_j(p) = \exp(-p^2/\Lambda_j^2) \text{ with } \Lambda_j = j\Lambda$$

phase equivalent (up to $50B_2$) to

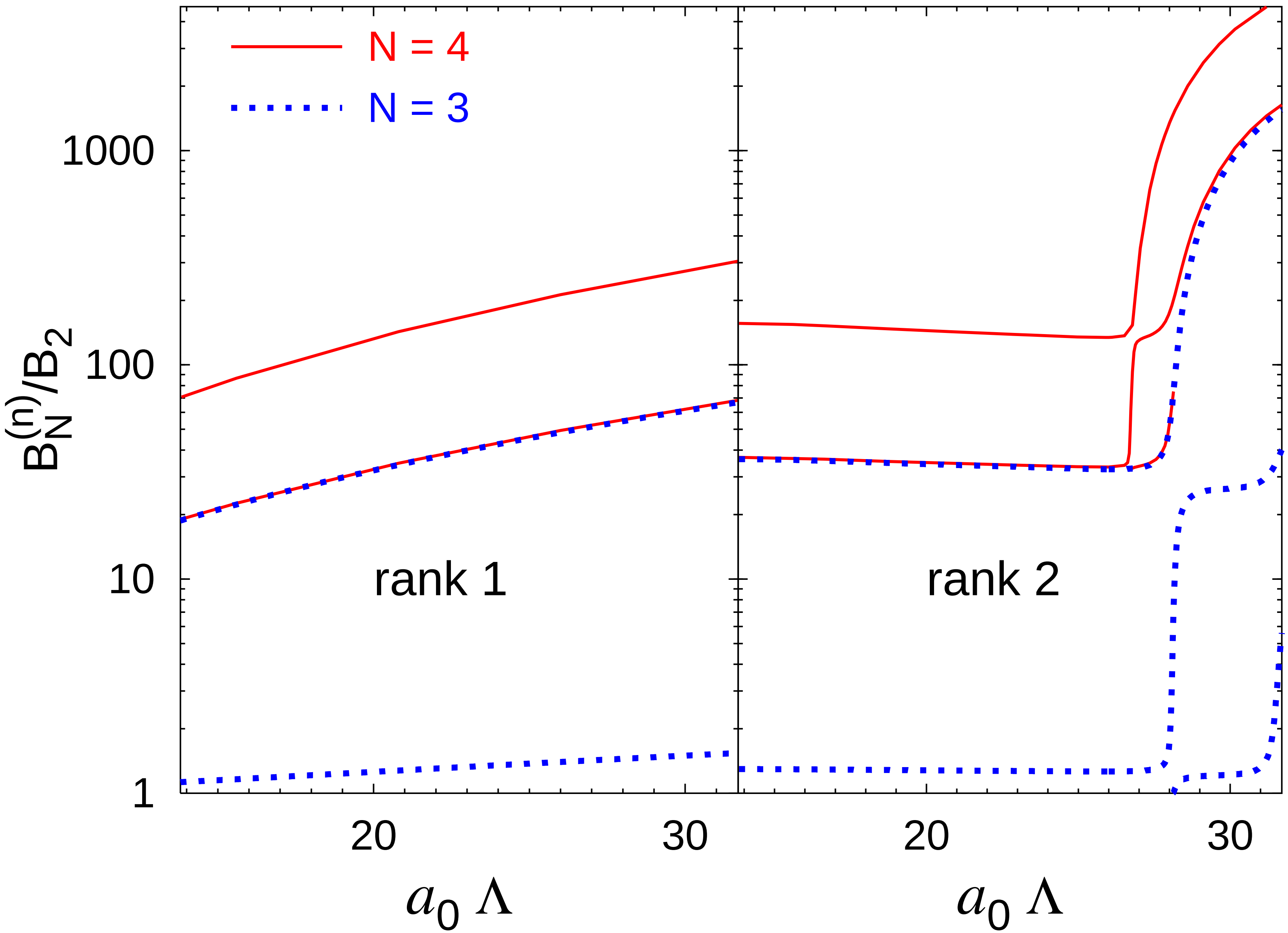
$\Lambda = 0.2$ Å^{-1} rank 1 potential:

$a_0 = 104.0$ Å, $r_0 = 15.0$ Å, $B_2 = 1.318$ mK

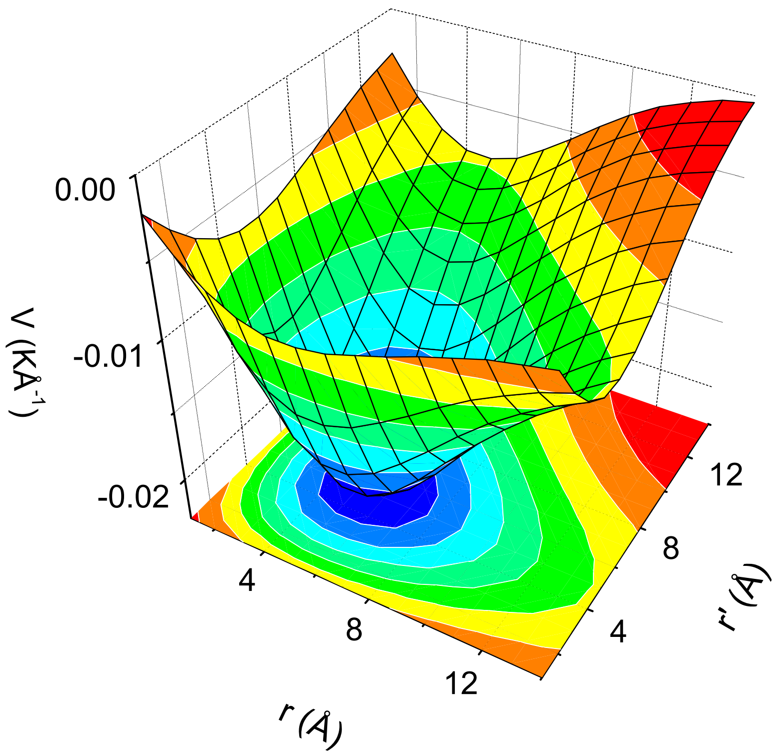
Breakdown of universality: 1+2 and 1+3 scattering lengths

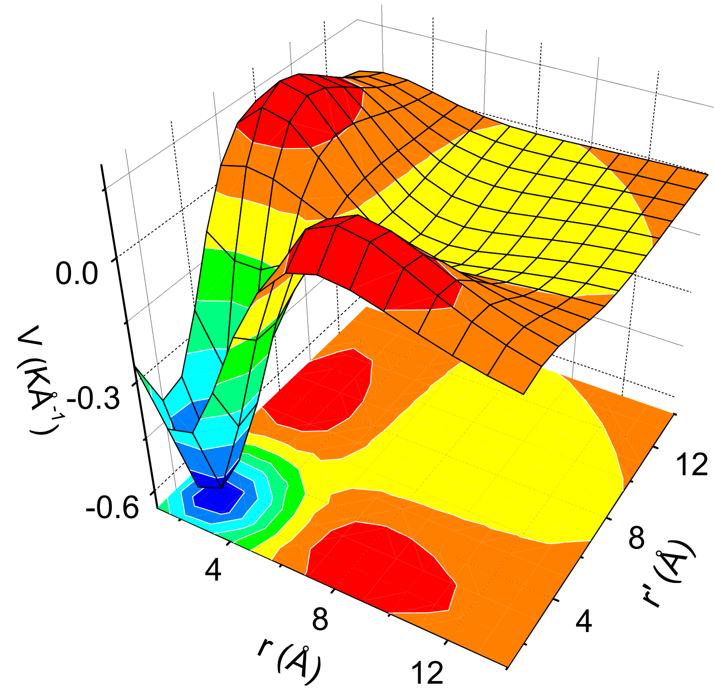


Breakdown of universality: few-boson binding energies



Breakdown of universality: $v(r',r)r'r$

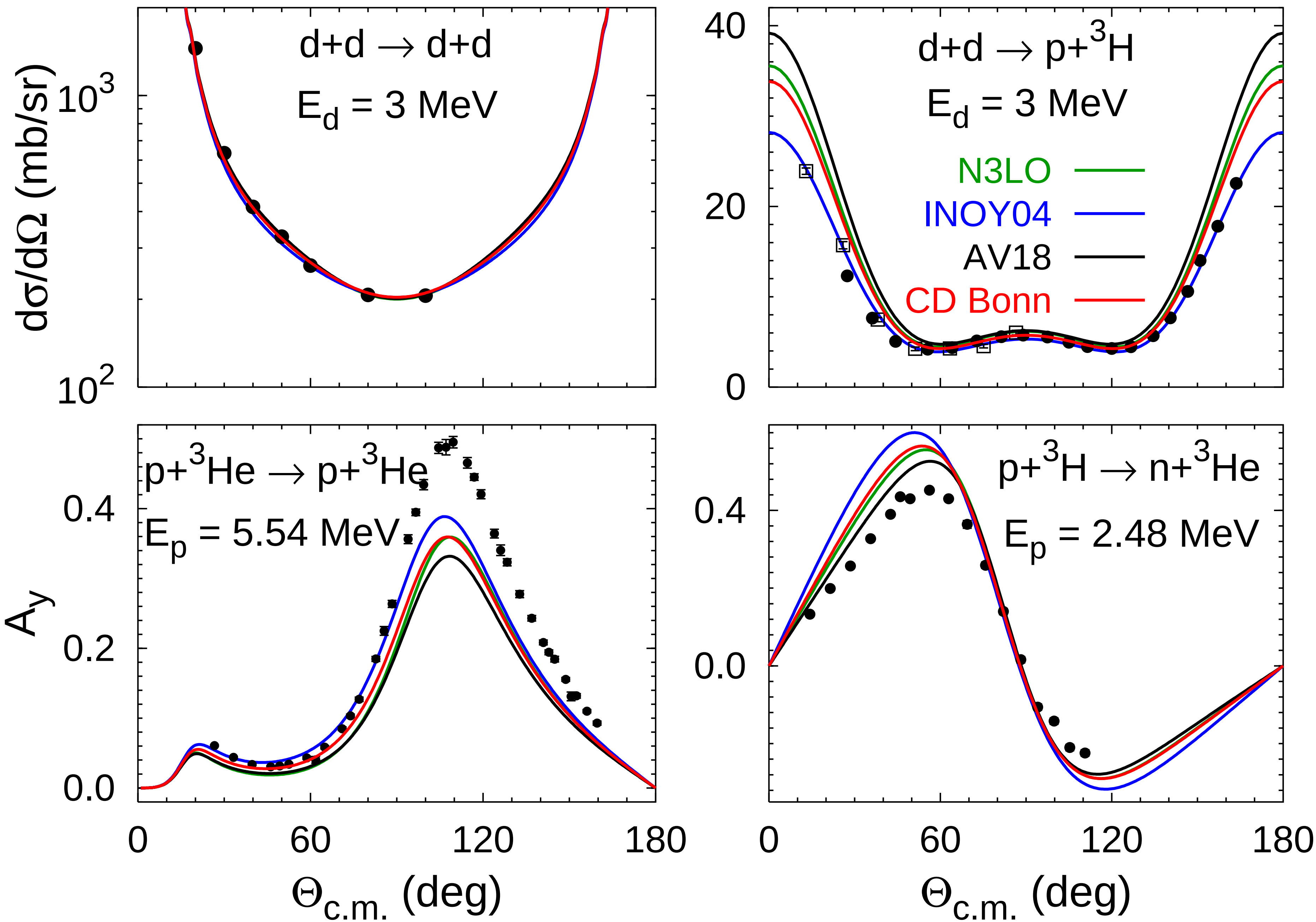




universal

non-universal

Fermionic system: 4N reactions



Summary

- four-particle AGS scattering equations in momentum space
- S-wave separable potentials
- low-energy elastic, inelastic, and transfer reactions

Summary

- four-particle AGS scattering equations in momentum space
- S-wave separable potentials
- low-energy elastic, inelastic, and transfer reactions
- still a lot to do (too much to be listed here)