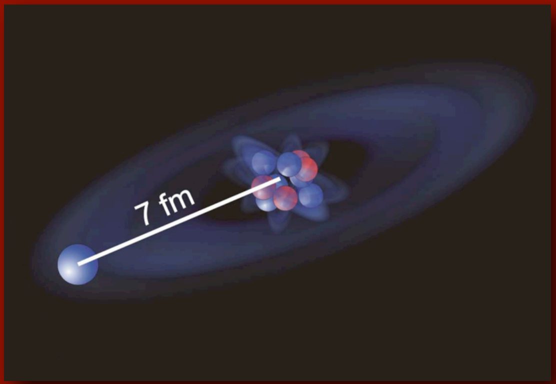
TRIUMF

Canada's National Laboratory for Particle and Nuclear Physics
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Halo Nuclei from Low-momentum Interactions

in collaboration with:
Nir Barnea, Gaute Hagen, Thomas Papenbrock and Achim Schwenk

Sonia Bacca | Theory Group | TRIUMF



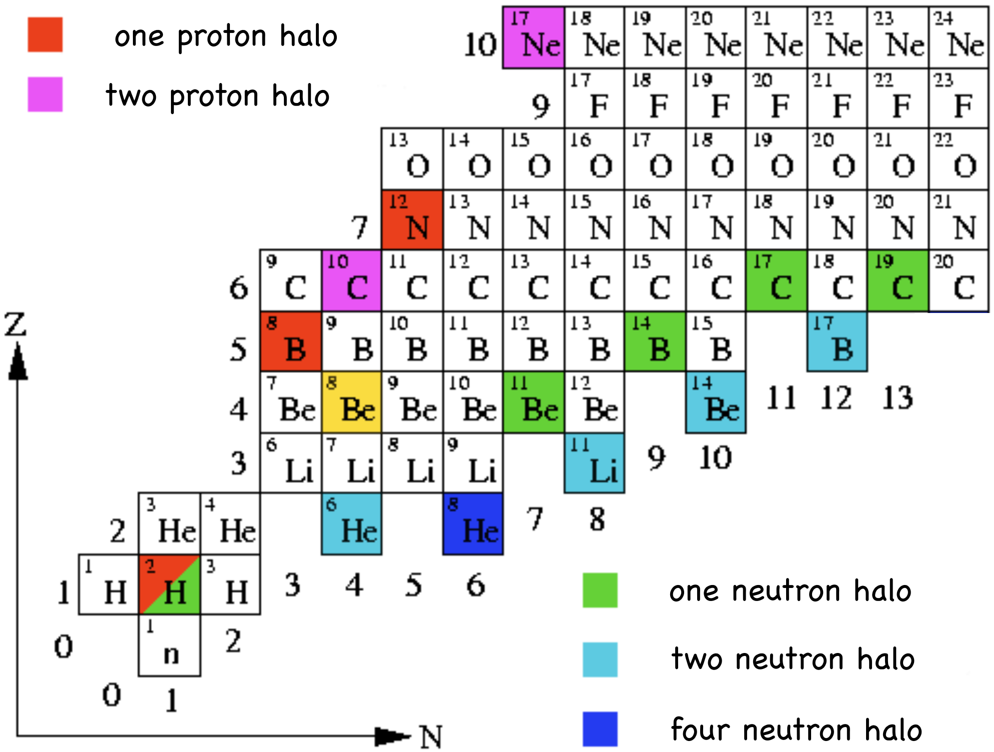
Nuclear Halo

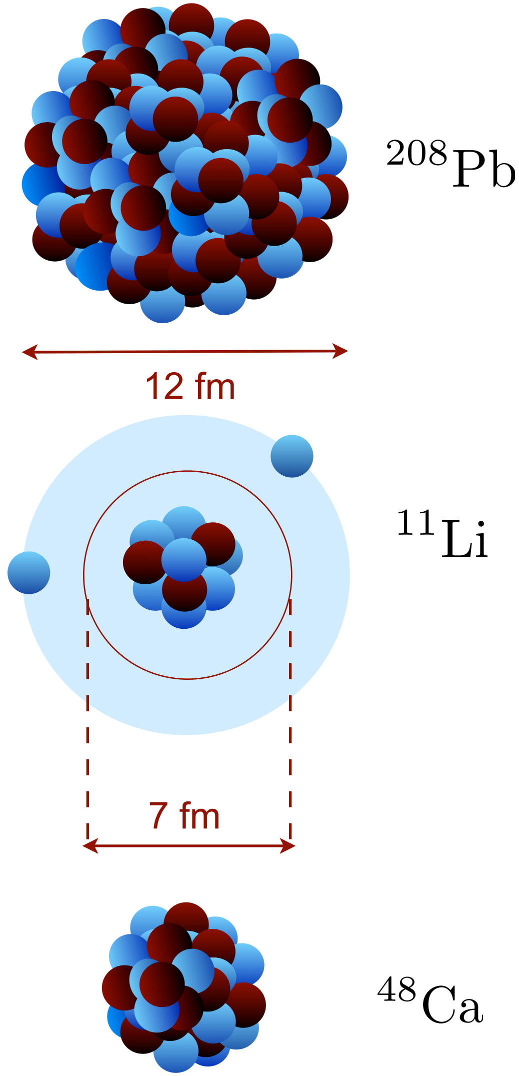
Moon Halo

Outline

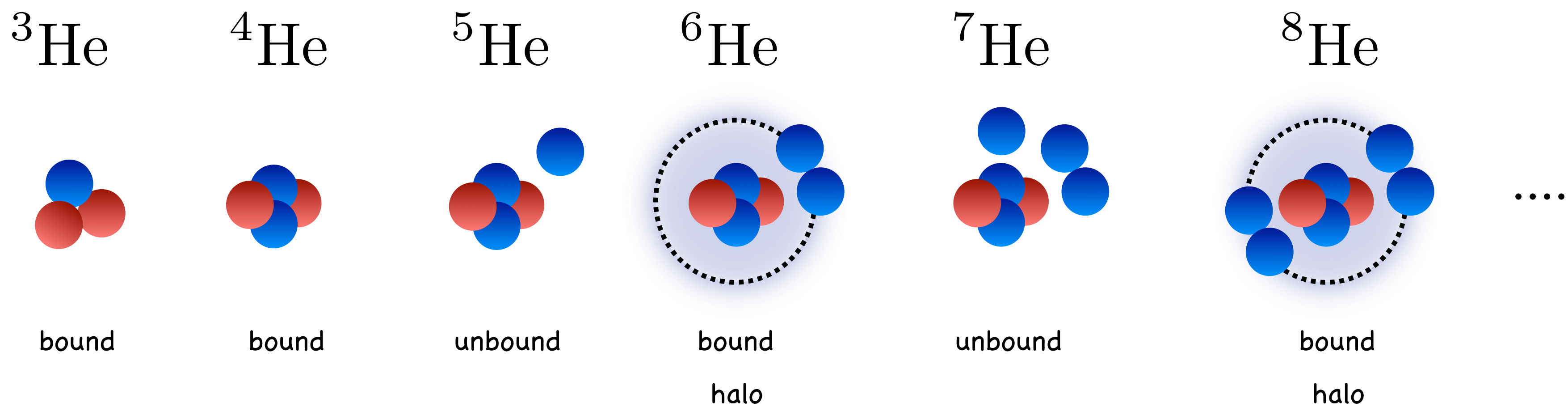
- Why are halo nuclei interesting?
- Brief summary on experimental advances
- Overview of different theoretical approaches
- Our approach:
  - Use hyper-spherical harmonics for ^{6}He
  - Use coupled cluster theory for ^{8}He
- Results for binding energy and radii
- Using the EIHH to improve radii
- Summary and Outlook

Halo Nuclei


one proton halo
two proton halo
one neutron halo
two neutron halo
four neutron halo
Z
N

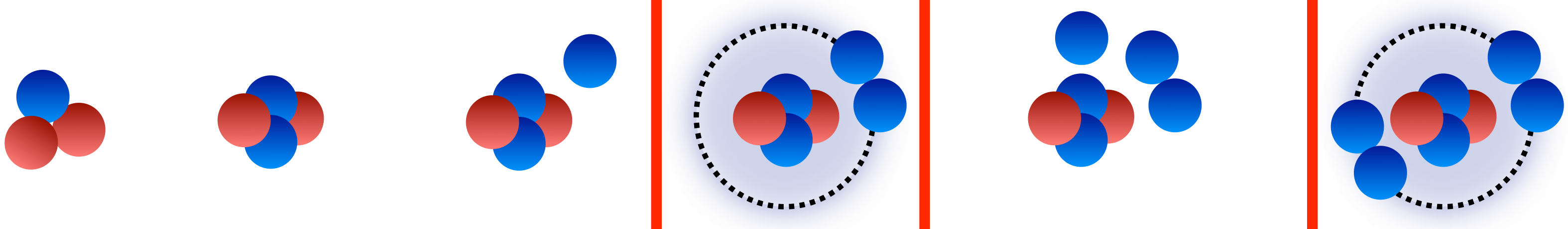

^{208}Pb
12 fm
^{11}Li
7 fm
^{48}Ca

The helium isotope chain



The helium isotope chain

$^{3}\mathrm{He}$	$^{4}\mathrm{He}$	$^{5}\mathrm{He}$	$^{6}\mathrm{He}$	$^{7}\mathrm{He}$	$^{8}\mathrm{He}$	....



bound	bound	unbound	bound halo	unbound	bound halo

Borromean system

lives 806 ms

Most exotic nucleus
"on earth"

$$\frac{N}{Z} = 3$$

lives 108 ms

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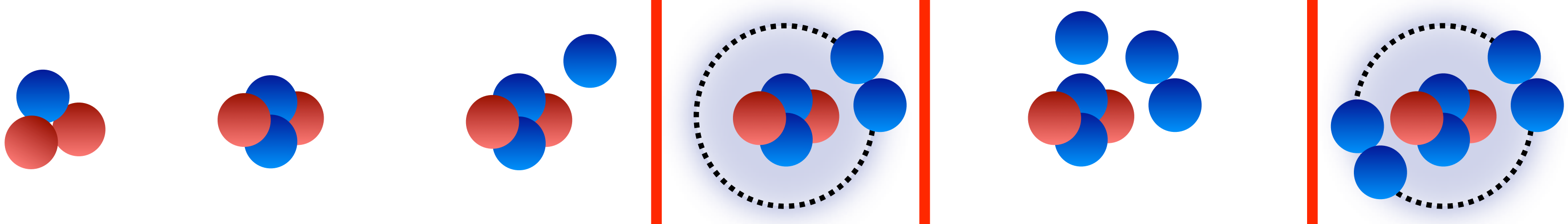

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bound	bound	unbound	bound halo	unbound	bound halo

$^{6}\mathrm{He}$: Borromean system

lives 806 ms

$^{8}\mathrm{He}$: Most exotic nucleus "on earth"

$$\frac{N}{Z} = 3$$

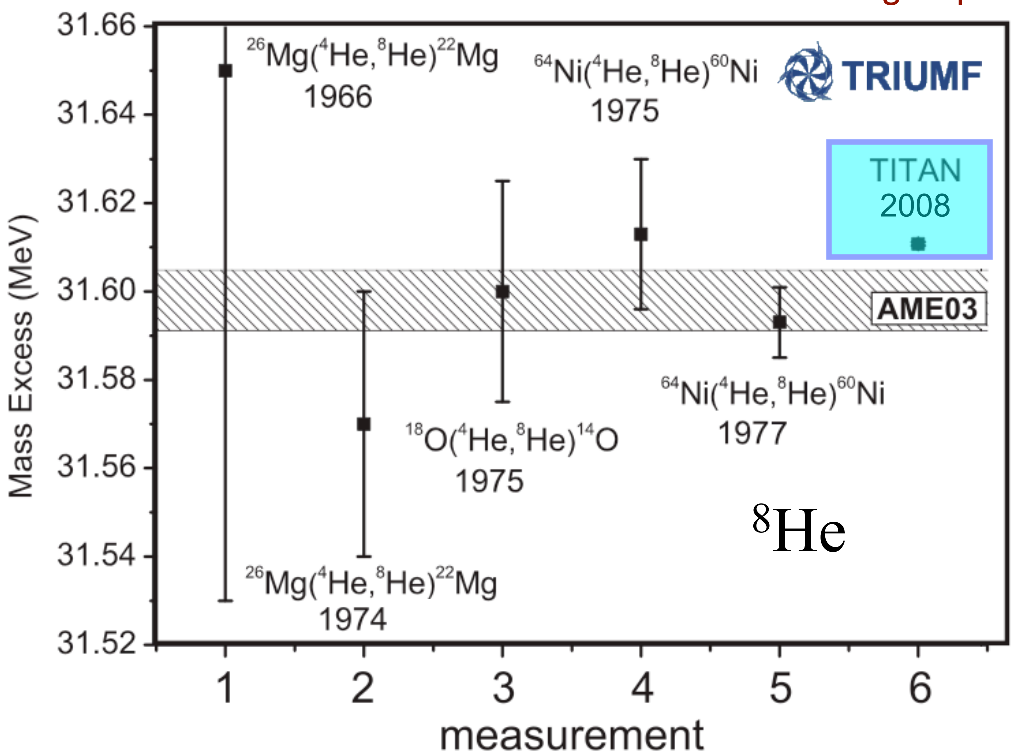
lives 108 ms

Even if they are exotic short lived nuclei, they can be investigated experimentally. From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region

Halo Nuclei - Experiment

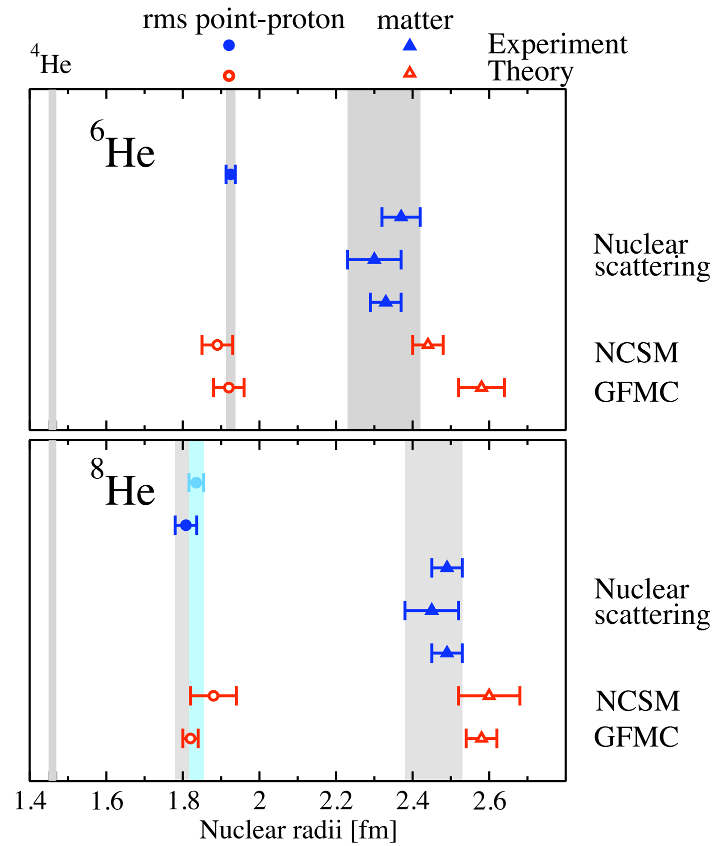
New Era of Precision Measurements for masses and radii

Mass measurement of ^{8}He with the Penning trap



TRIUMF, Ryjkov et al. PRL 101, 012501 (2008)

Measurement of charge radii via isotope shift



ARGONNE, Wang et al. PRL 93, 142501 (2004)
GANIL, Mueller et al. PRL 99, 252501 (2007)

$$\delta\nu_{AA'} = \delta\nu^{mass}_{A,A'} + K\delta\langle r^2_{\rm ch}\rangle_{AA'}$$

$$\langle r_p^2\rangle = \langle r_{ch}^2\rangle - \langle R_p^2\rangle - \frac{3}{4M_p^2} - \frac{N}{Z}\langle R_n^2\rangle$$

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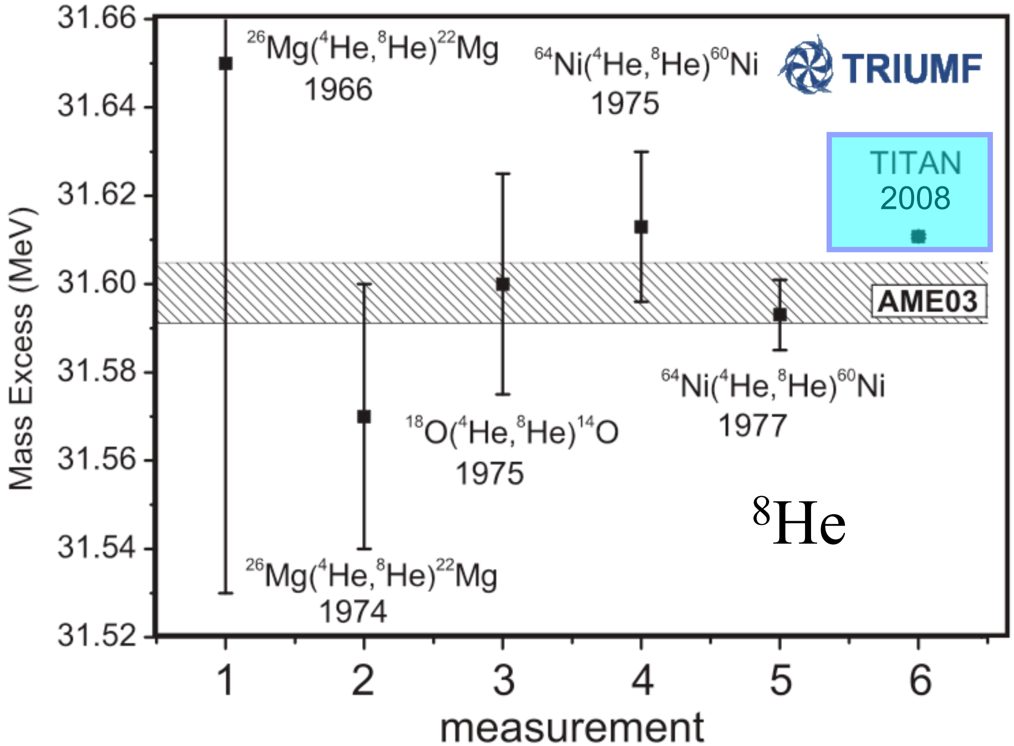

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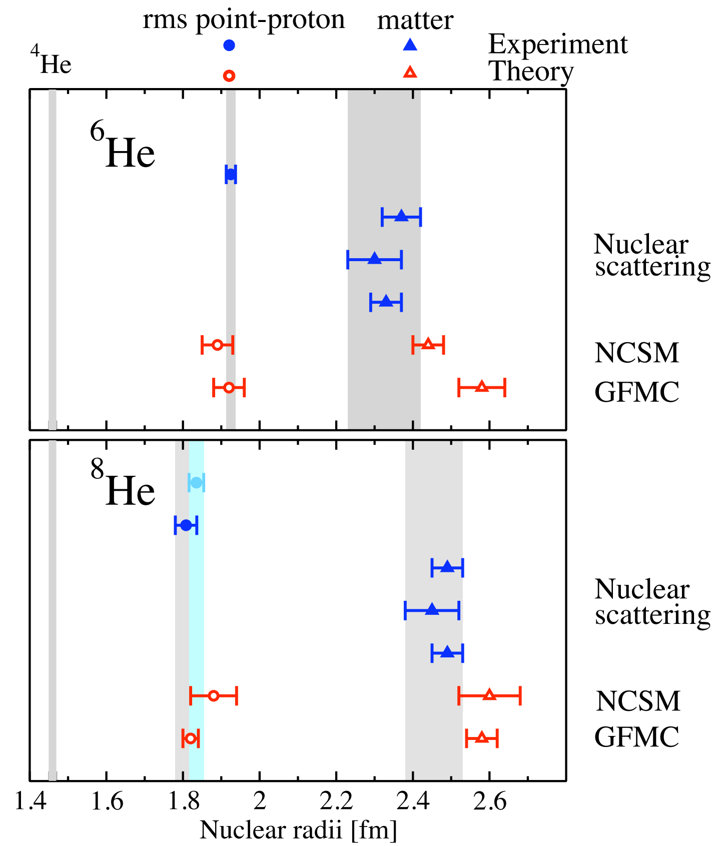

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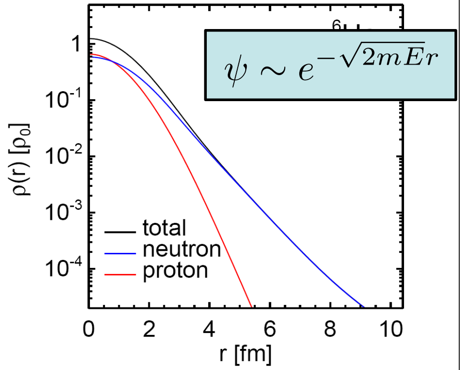
Masses and radii of helium isotopes are important challenges for theory!

Halo Nuclei -Theory

Why are halo nuclei a challenge to theory?

- It is difficult to describe the long extended wave function
- They test nuclear forces at the extremes, where less is known



$$\psi \sim e^{-\sqrt{2mE}r}$$

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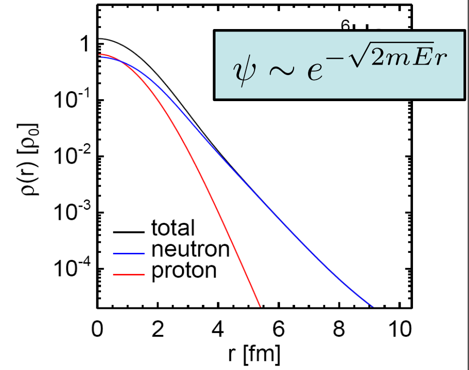
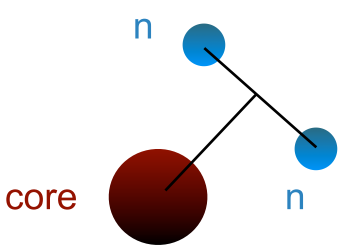


Cluster models:



3-body models with phenomenological interactions

^{6}He, ^{11}Li - borromean systems

can do reactions, Faddeev calculations

but difficult to add core polarizations

Efros, Fedorov, Garrido, Hagino, Bertulani, ...

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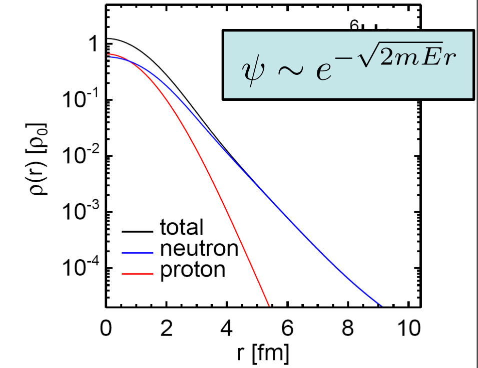

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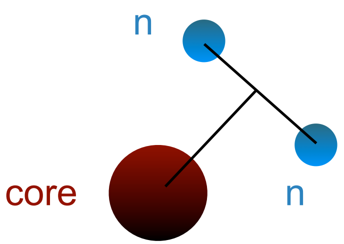

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New: Revived by halo EFT

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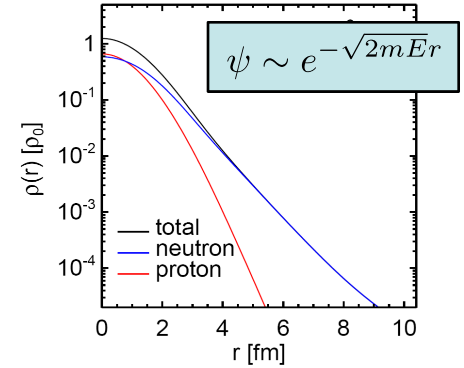

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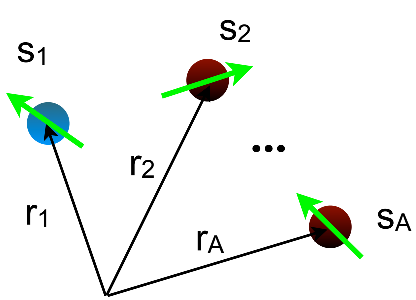
- They test nuclear forces at the extremes, where less is known

Ab-initio calculations:

treat the nucleus as an A-body problem



full antisymmetrization of the w.f.

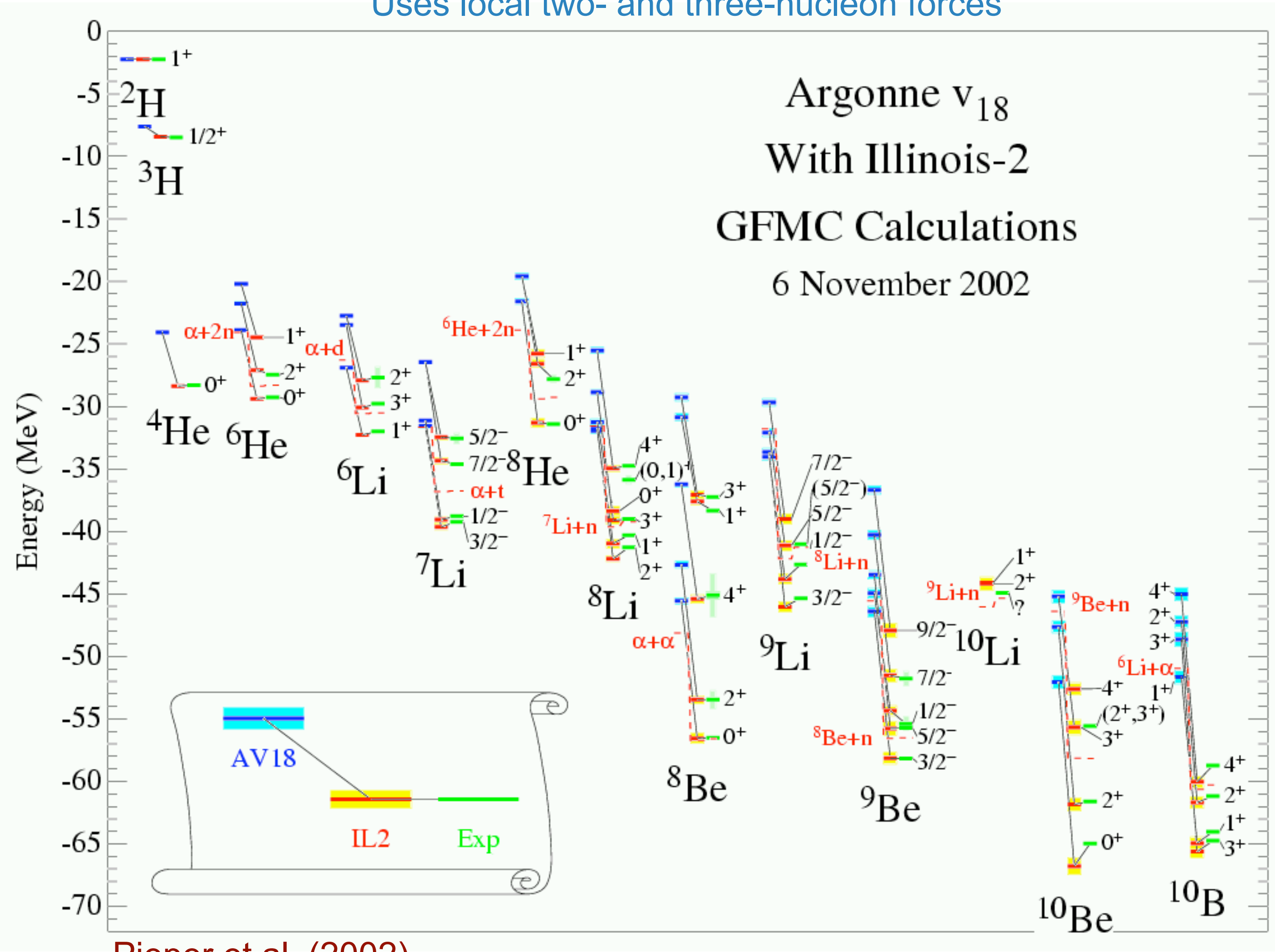
use modern Hamiltonians to predict halo properties

$$H = T + V_{NN} + V_{3N} + \ldots$$

Methods: GFMC, NCSM, CC, HH

Ab-initio Calculations

GFMC Quantum Monte Carlo Method,
Uses local two- and three-nucleon forces → short range phenomenology



Pieper et al. (2002)

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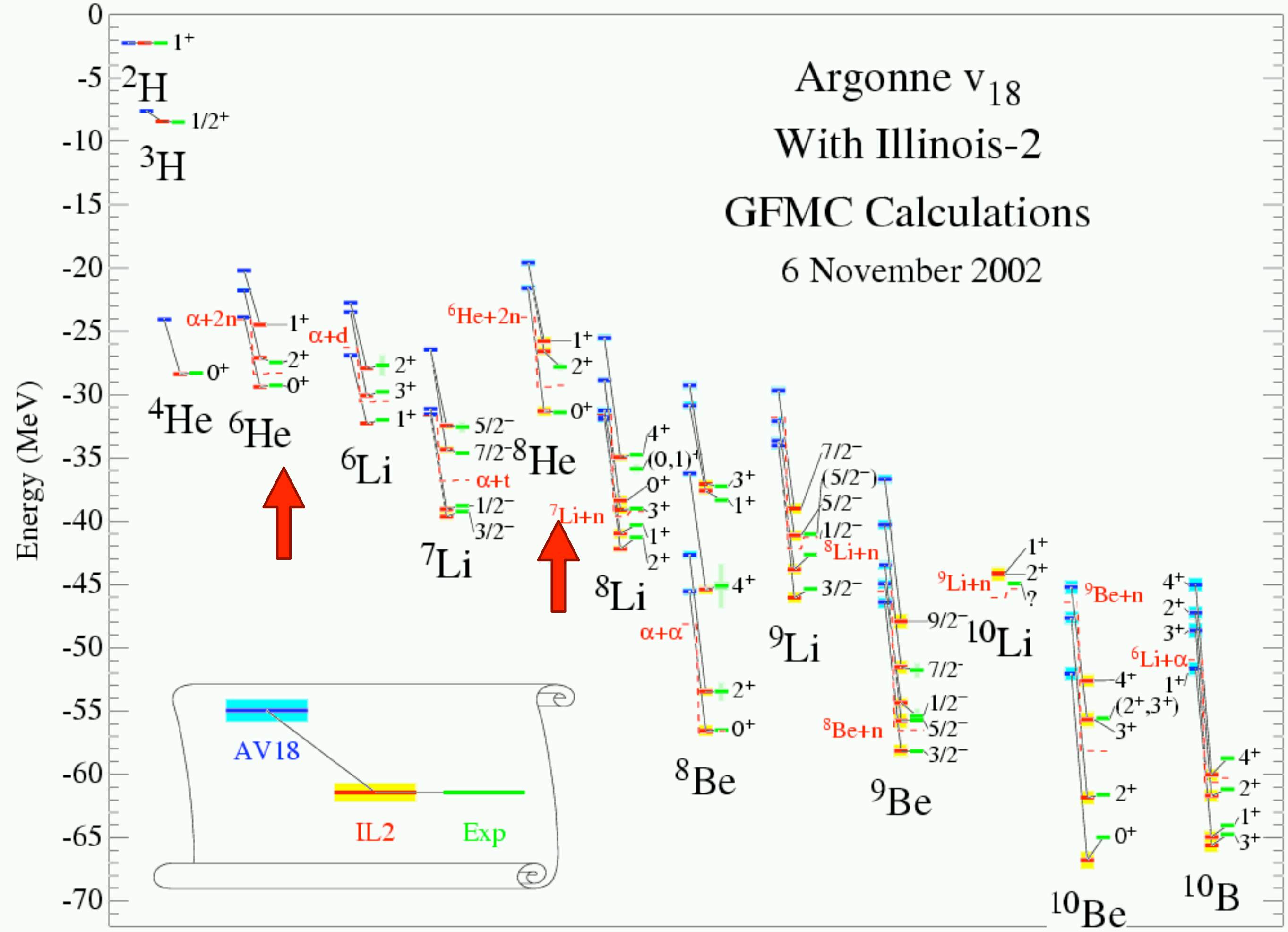

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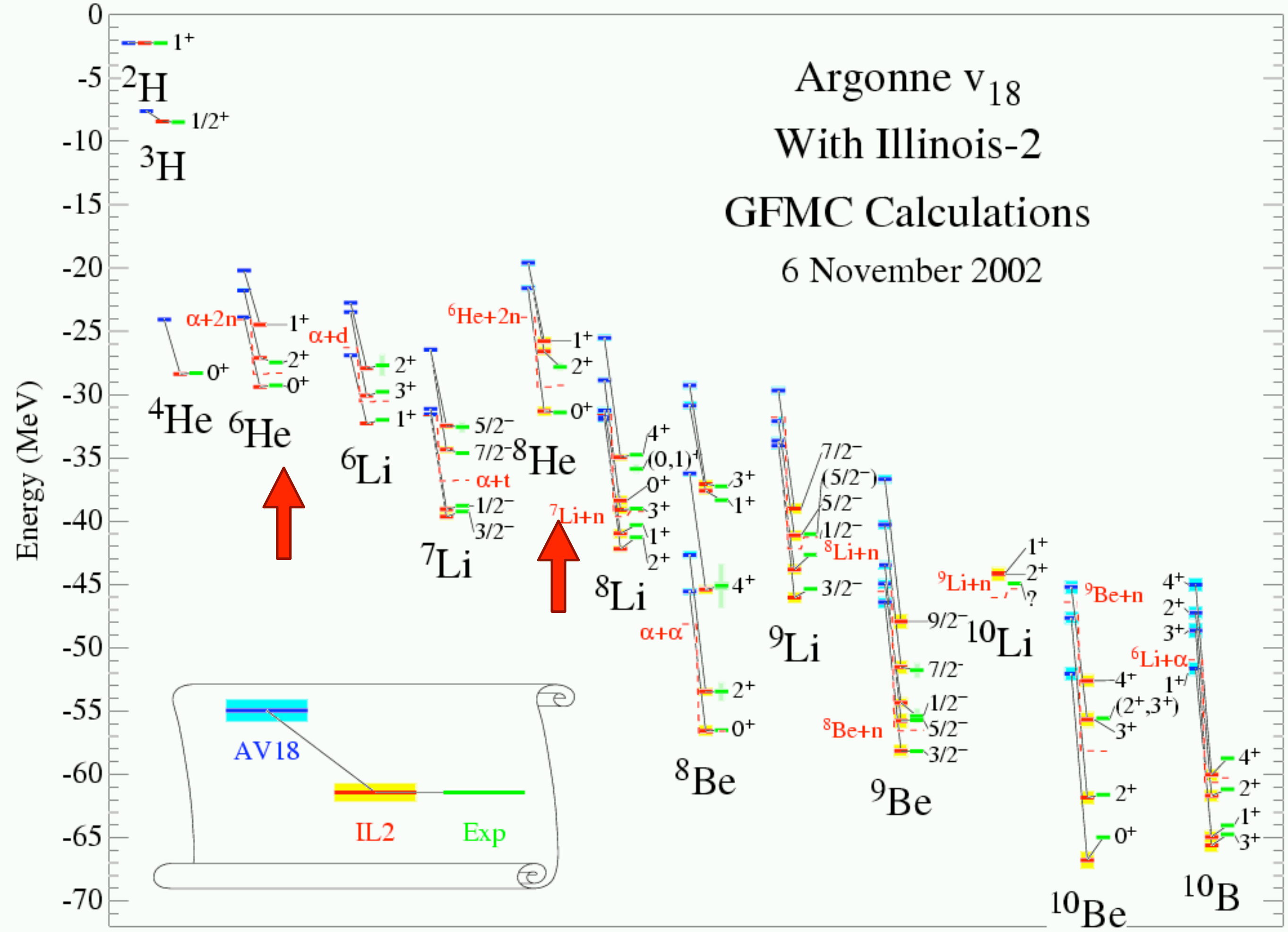

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AV18

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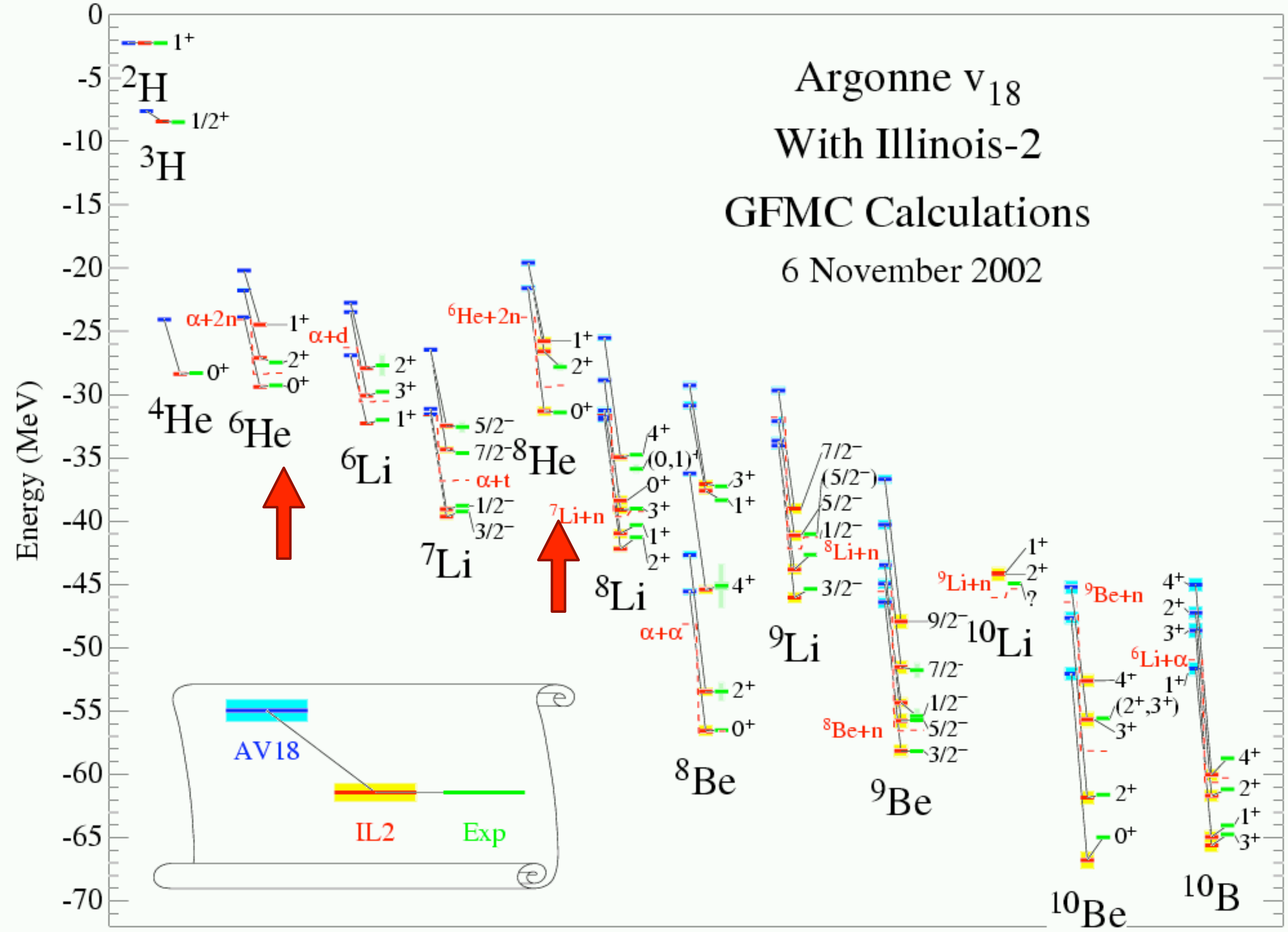
Ab-initio Calculations

GFMC

Quantum Monte Carlo Method,
Uses local two- and three-nucleon forces ➡ short range phenomenology



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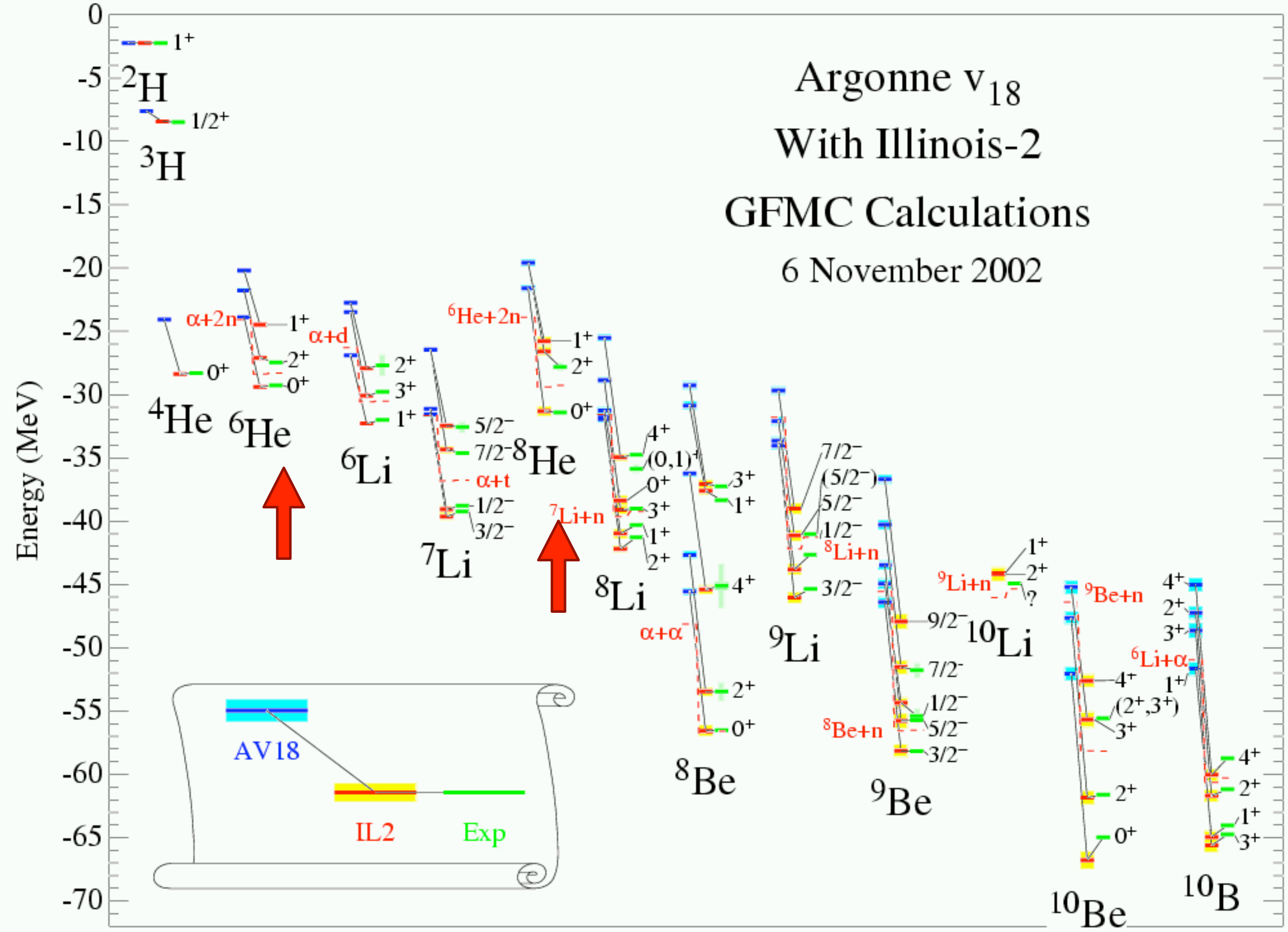
IL2

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$

Ab-initio Calculations

GFMC Quantum Monte Carlo Method, Uses local two- and three-nucleon forces → short range phenomenology



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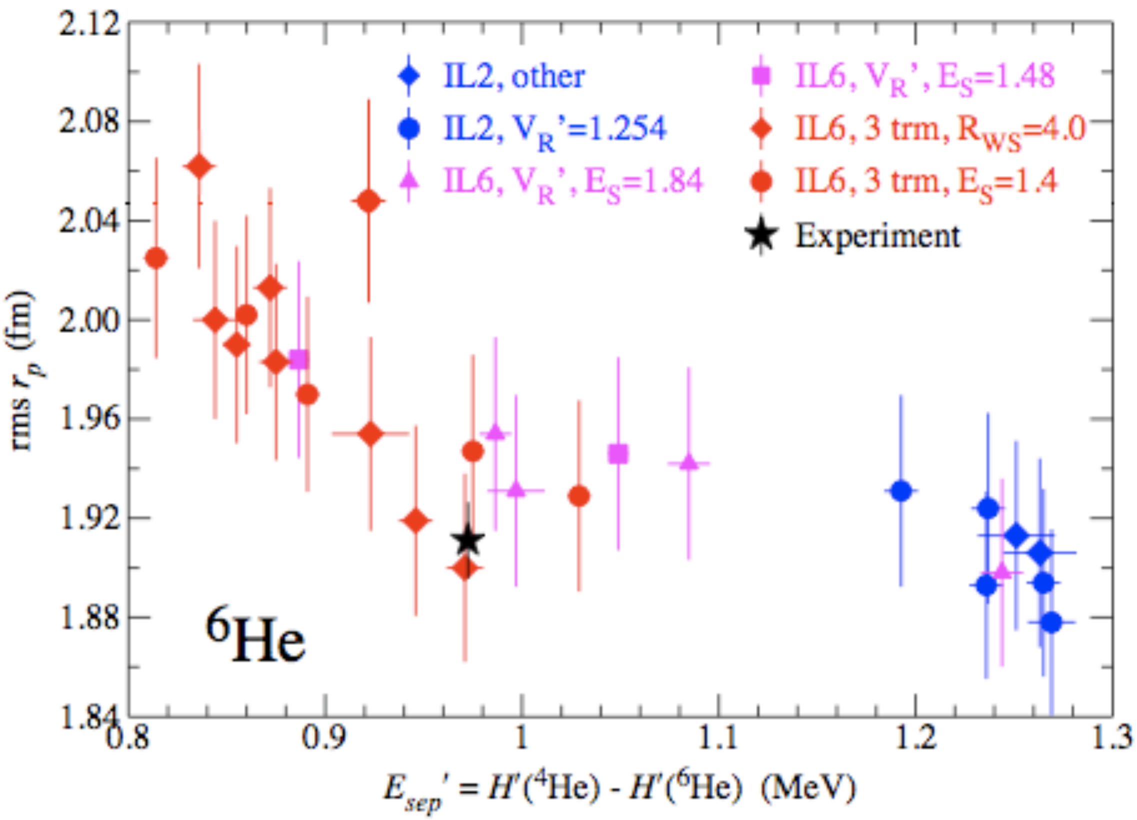
$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi,R} + V_{ijk}^{R}$$

N.B.: parameters of the IL2 force are obtained from a fit of 17 states of A<9 including the binding energy of ^{6}He and ^{8}He

Nota Bene

- GFMC estimation of the proton radius -



S.C. Pieper, arXiv:0711.1500, proceedings of Enrico Fermi School

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Ab-initio Calculations

NCSM

Diagonalization Method using Harmonic Oscillator Basis $\psi_{nl}(r) \sim e^{-\nu r^2} L_n^{l+1/2}(2\nu r^2) \quad \nu = m\omega/2\hbar$

Can use non-local two- and ~~three-nucleon~~ forces

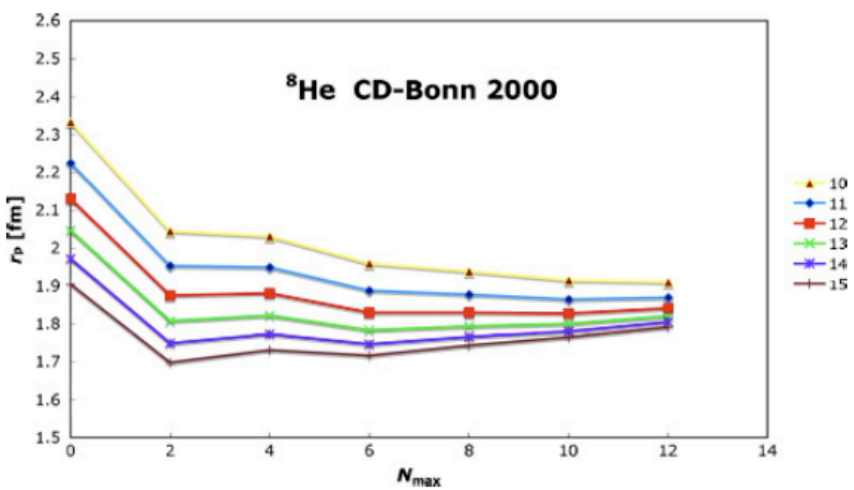
so far not for halo nuclei in large spaces

Navratil and Ormand, PRC 68, 034305 (2003),

^{6}He AV8'+TM $6\hbar\omega$

Helium Isotopes

Caurier and Navratil, PRC 73, 021302(R) (2006)



CD-Bonn → meson exchange theory

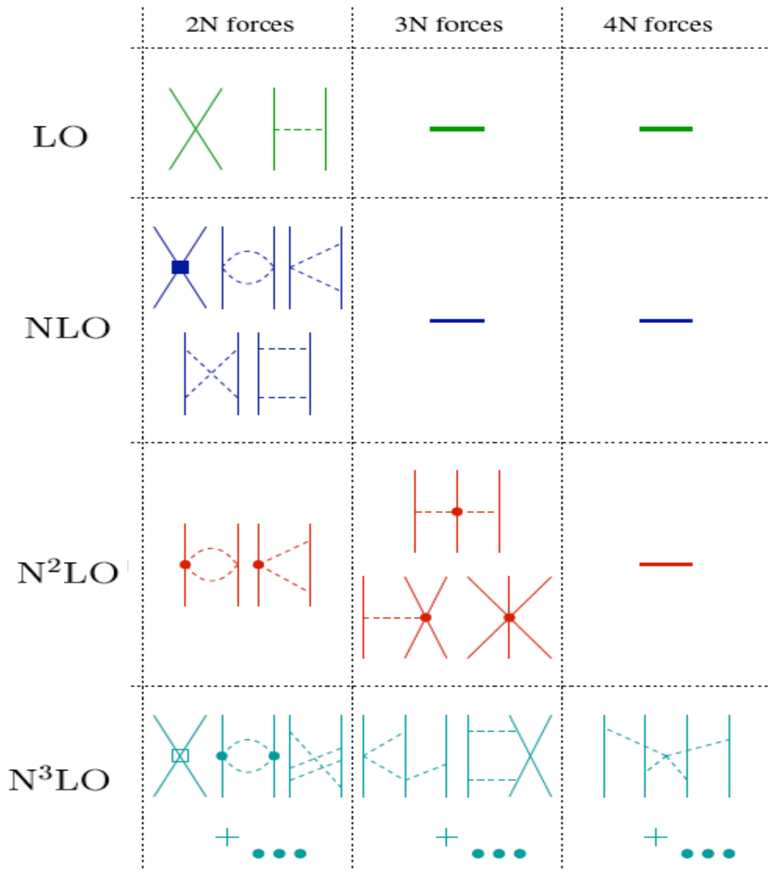
E_B [MeV]	Expt.	CD-Bonn 2000
^{4}He	28.296	26.16 (6)
^{6}He	29.269	26.9 (3)
^{8}He	31.408 (7)	26.0 (4)

NN only with effective interaction (Lee-Suzuki)

Slow convergence and HO parameter dependence in radius

What we aim at

An ab-initio calculation of helium halo nuclei from chiral effective field theory potentials



Ideally we want:

- To use methods that enable to incorporate the correct asymptotic of the w.f. for loosely bound systems
- To obtain convergent calculations, with no dependence on the model space parameters
- To systematically study the cutoff (in)dependence of predicted observables

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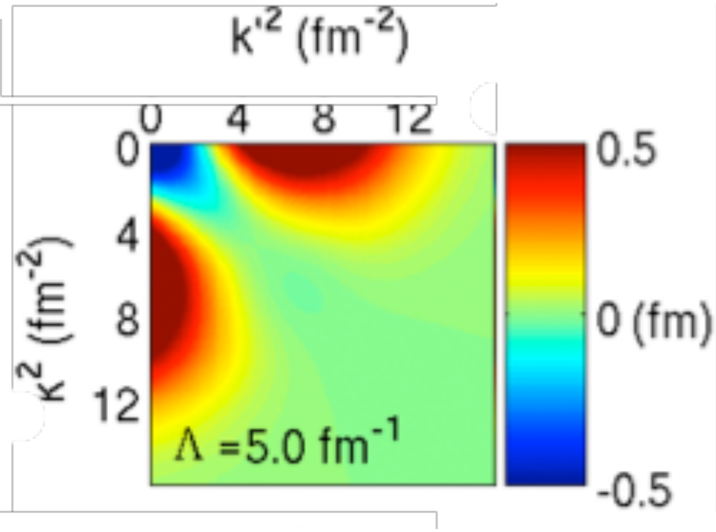
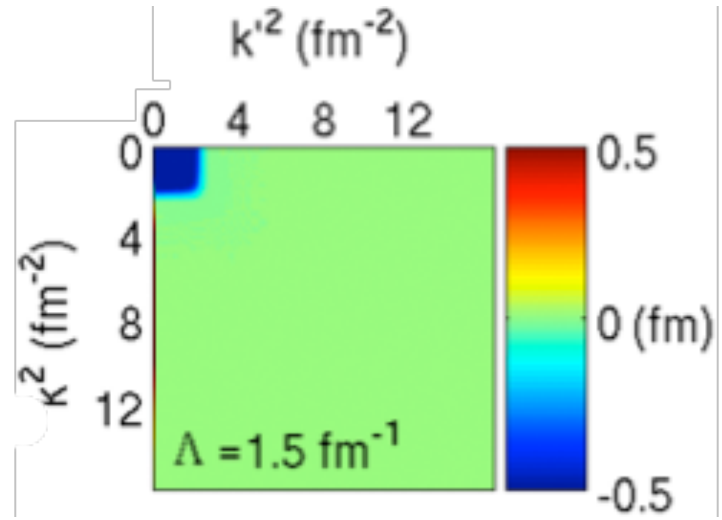
To facilitate convergence we use low-momentum interactions

Low momentum interactions

Effective field theory potentials and low-momentum evolution $V_{\rm low\ k}$

evolve to lower resolution (cutoffs) by integrating out
high-momenta Bogner, Kuo, Schwenk (2003)
smooth cutoff Bogner, Furnstahl, Ramanan, Schwenk (2007)





$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$

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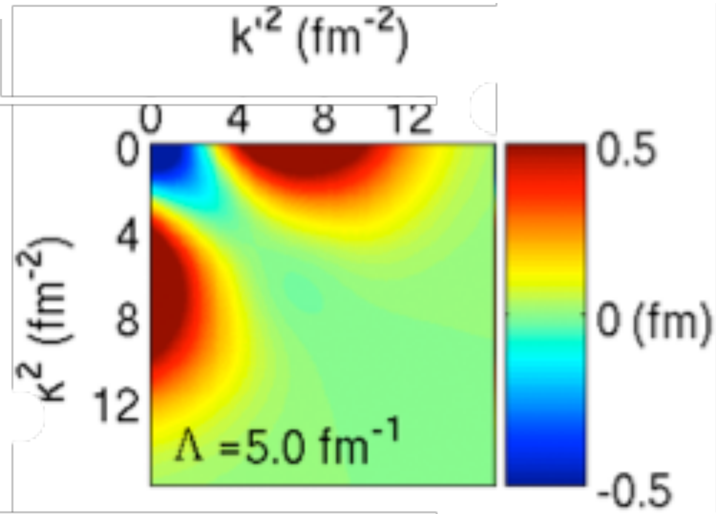

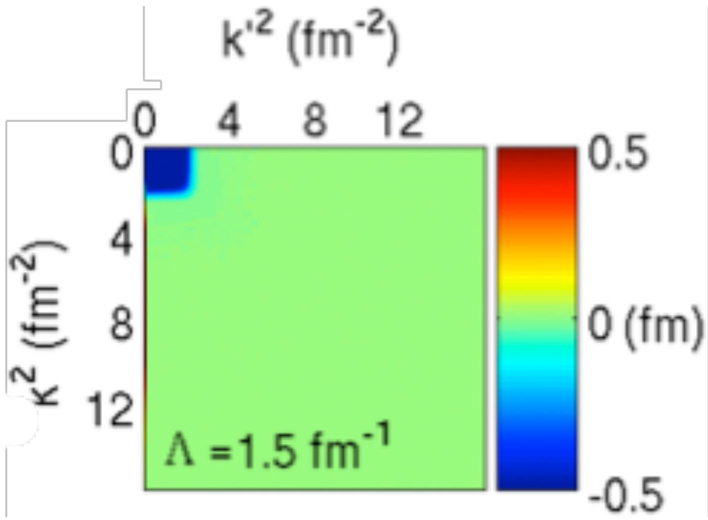

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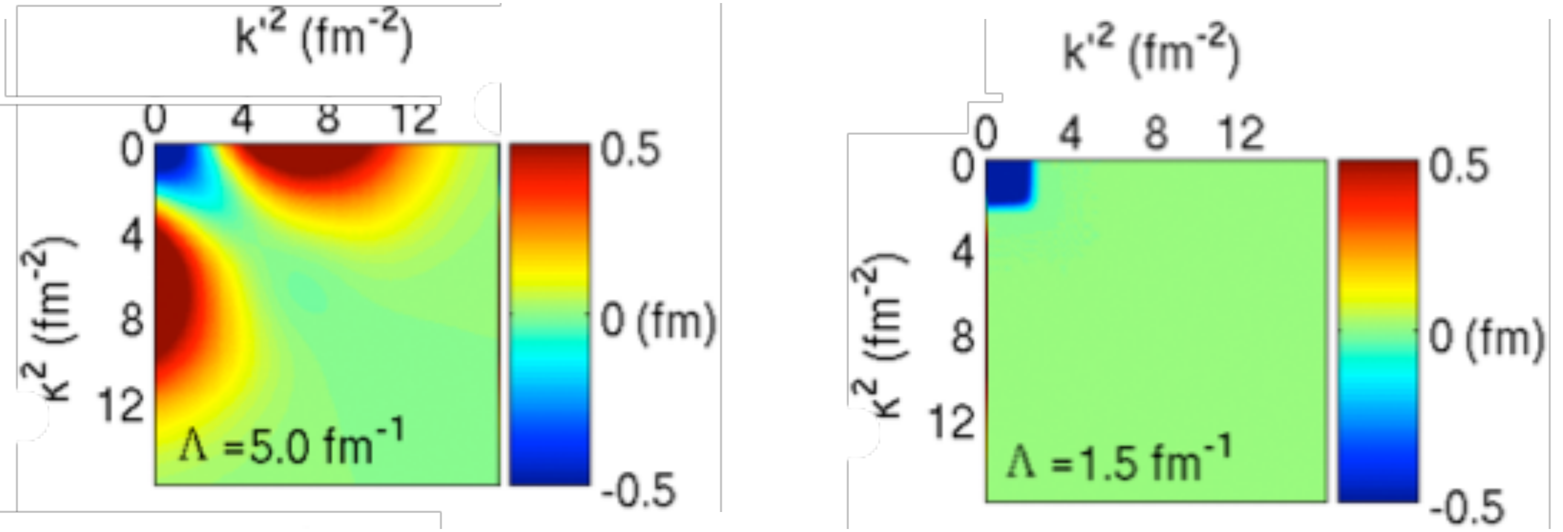

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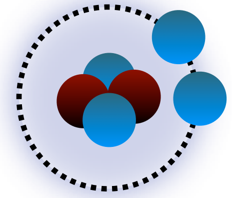
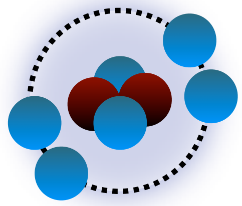
Variation of the cutoff provides a tool to estimate the effect of short range 3N forces

Our Approach

- Hyper-spherical Harmonics Expansion for ^6He



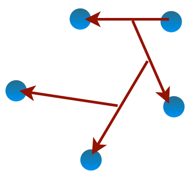
- Cluster Cluster Theory for ^8He



Hyper-spherical Harmonics

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle$$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

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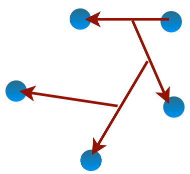

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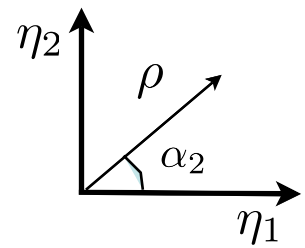

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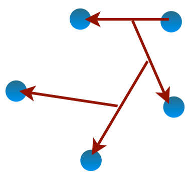

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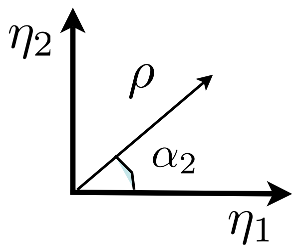
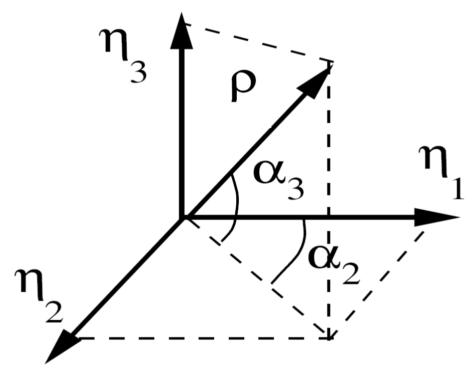


A=4

$$\begin{cases} \vec{\eta}_1 = \{\eta_1, \theta_1, \phi_1\} \\ \vec{\eta}_2 = \{\eta_2, \theta_2, \phi_2\} \\ \vec{\eta}_3 = \{\eta_3, \theta_3, \phi_3\} \end{cases} \qquad \begin{cases} \rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2} \\ \sin\alpha_2 = \frac{\eta_2}{\rho} \\ \sin\alpha_3 = \frac{\eta_3}{\rho} \end{cases}$$



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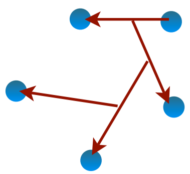

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$$H(\rho,\Omega) = T_\rho + \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n(\tfrac{\rho}{b}) [\mathcal{Y}_{[K]}^{\mu}(\Omega)\chi_{ST}^{\bar{\mu}}]_{JT}^a$$

Asymptotic $e^{-a\rho}$ $\rho \to \infty$

Model space truncation $K \le K_{max}$, Matrix Diagonalization $\langle\psi|H_{(2)}|\psi\rangle = \frac{A(A-1)}{2}\langle\psi|H_{(A,A-1)}|\psi\rangle$

Can use non-local interactions

Most applications in few-body; challenge in A>4 Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

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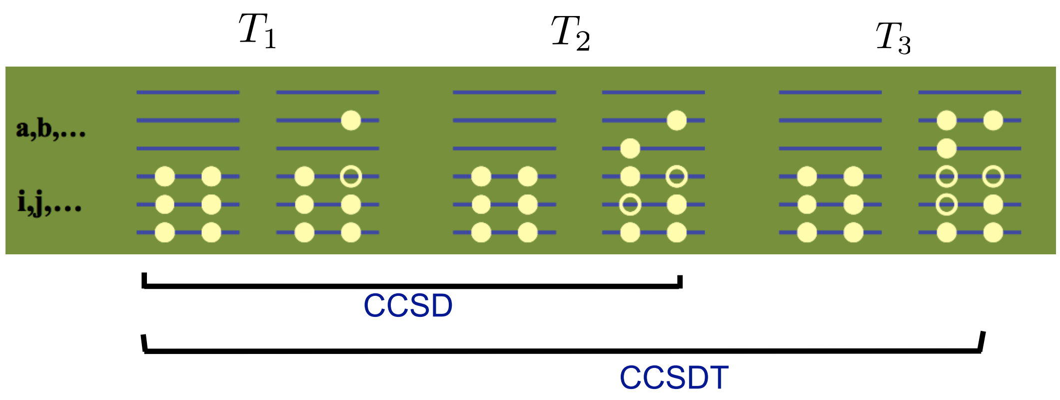
Coupled Cluster Theory

$$|\psi(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle = e^T|\phi(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle \qquad T=\sum T_{(A)}$$

→ reference SD



CCSD Equations

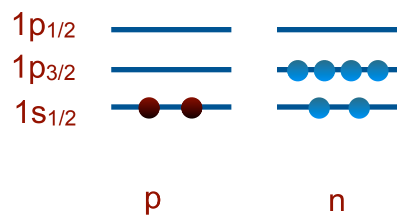
$$E = \langle\phi|e^{-T}He^{T}|\phi\rangle$$

$$0 = \langle\phi_i^a|e^{-T}He^{T}|\phi\rangle$$

$$0 = \langle\phi_{ij}^{ab}|e^{-T}He^{T}|\phi\rangle$$

Asymptotic $\phi_i \sim e^{-k_i r_i} \quad r \to \infty$

Use it for ^{8}He, closed shell nucleus



Model space truncation $N \leq N_{max}$

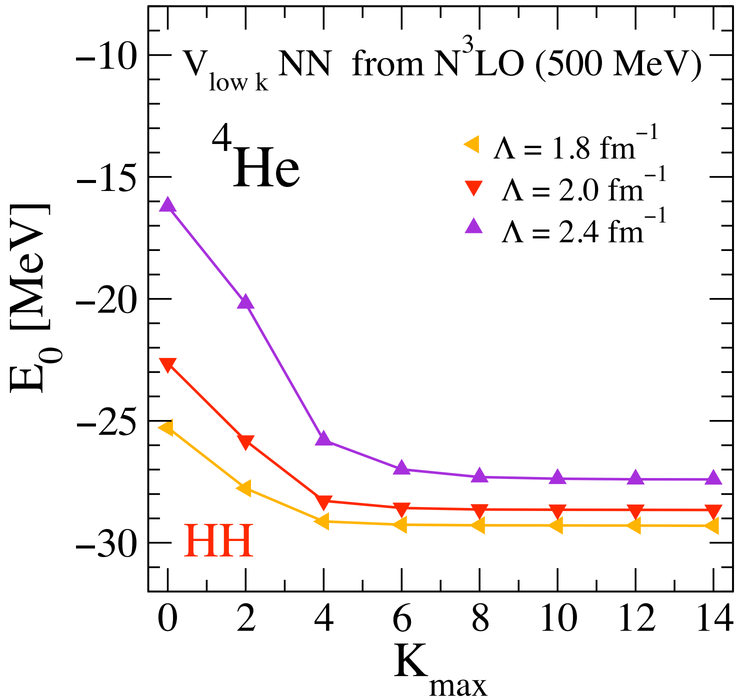
Can use non-local interactions

Applicable to medium-mass nuclei

Results for binding energies

Benchmark on ^{4}He

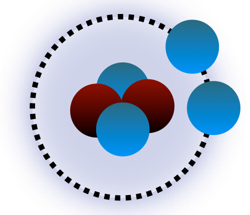
HH-CC-FY

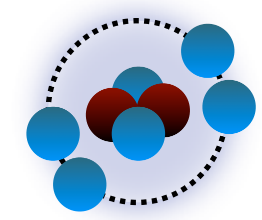


Method	$\Lambda = 2.0$ fm^{-1}	$E_0(^4\text{He})$ [MeV]
Faddeev-Yakubovsky (FY)		-28.65(5)
Hyperspherical harmonics (HH)		-28.65(2)
CCSD level coupled-cluster theory (CC)		-28.44
Lambda-CCSD(T) (CC with triples corrections)		-28.63

E^{exp}=-28.296 MeV

Helium Halo Nuclei

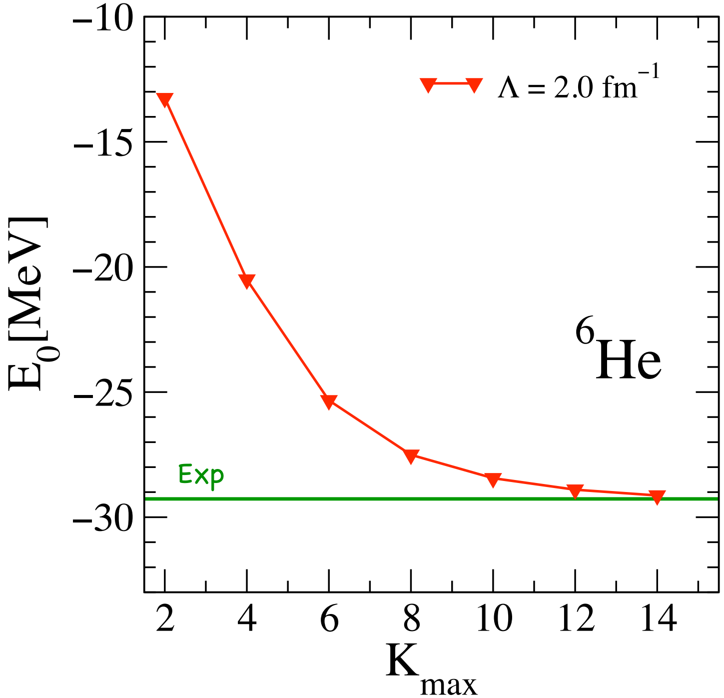


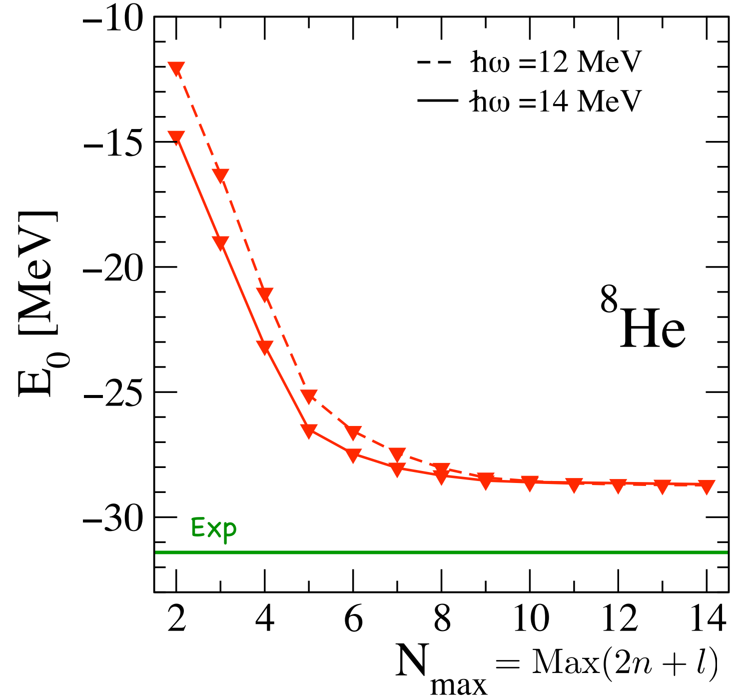


HH

$V_{low\,k}$ NN from N^3LO

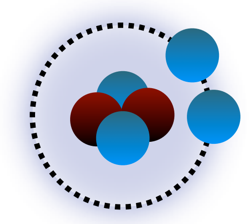
CCSD

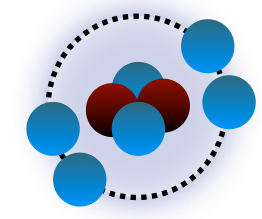




S.Bacca et al., Eur. Phys. J. A 42, 553 (2009)

Helium Halo Nuclei

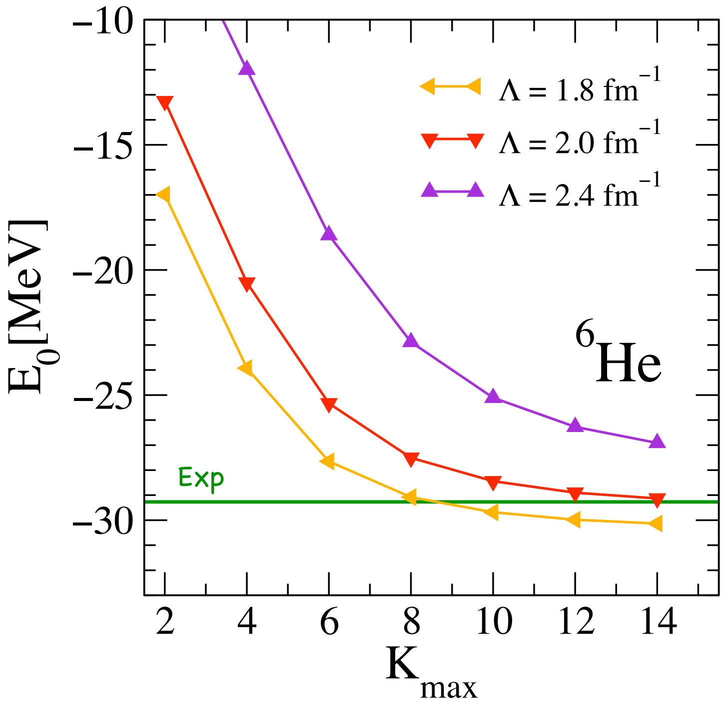


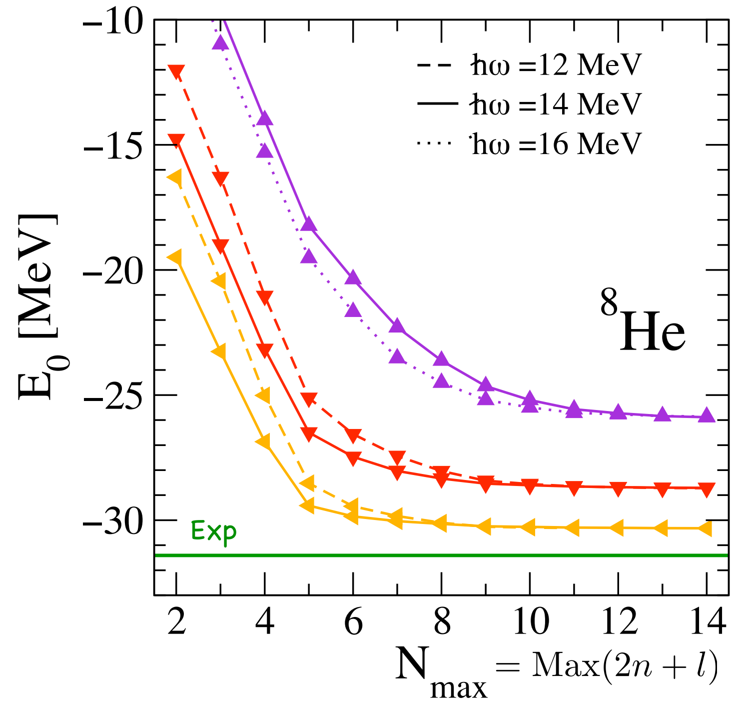


HH

$V_{\text{low k}}$ NN from N^3LO

CCSD

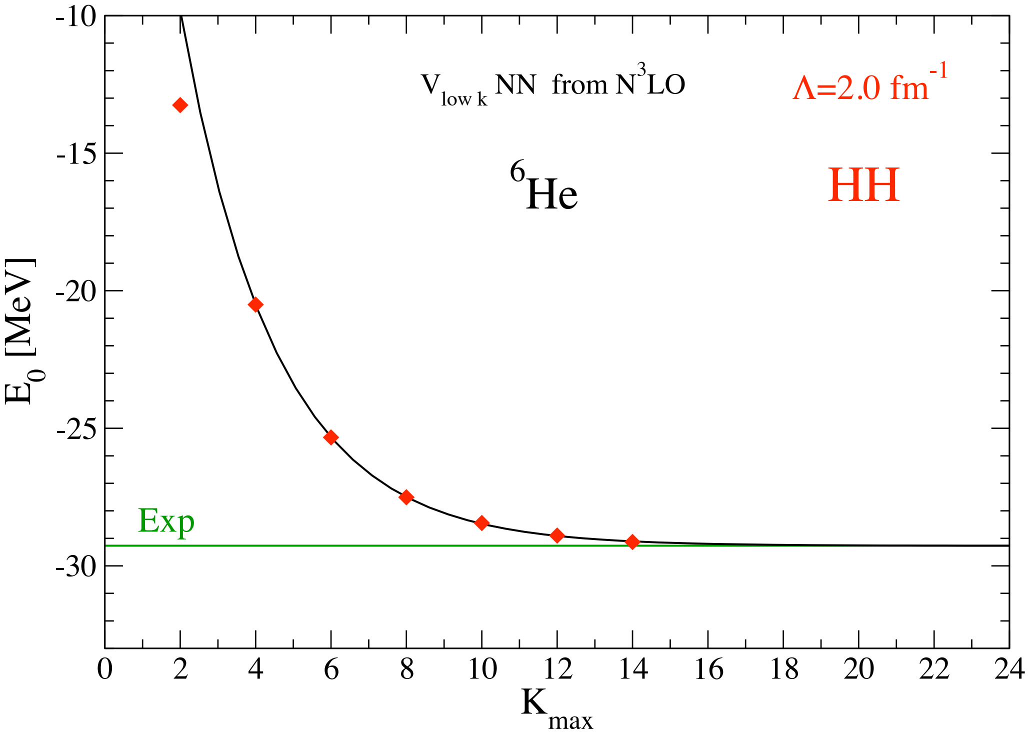




S.Bacca et al., Eur. Phys. J. A 42, 553 (2009)

Binding Energy ^{6}He

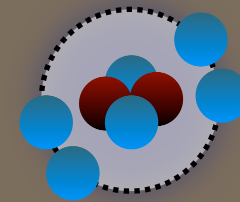
- Extrapolation -



$$E(K_{max}) = E^{\infty} + Ae^{-BK_{max}}$$

Λ	$E(K_{max} = 14)$	E^{∞}
1.8	-30.13	-30.28(3)
2.0	-29.13	-29.35(13)
2.4	-26.91	-27.62(19)

Binding Energy ^{8}He



- CC Theory: Add Triples Correction -

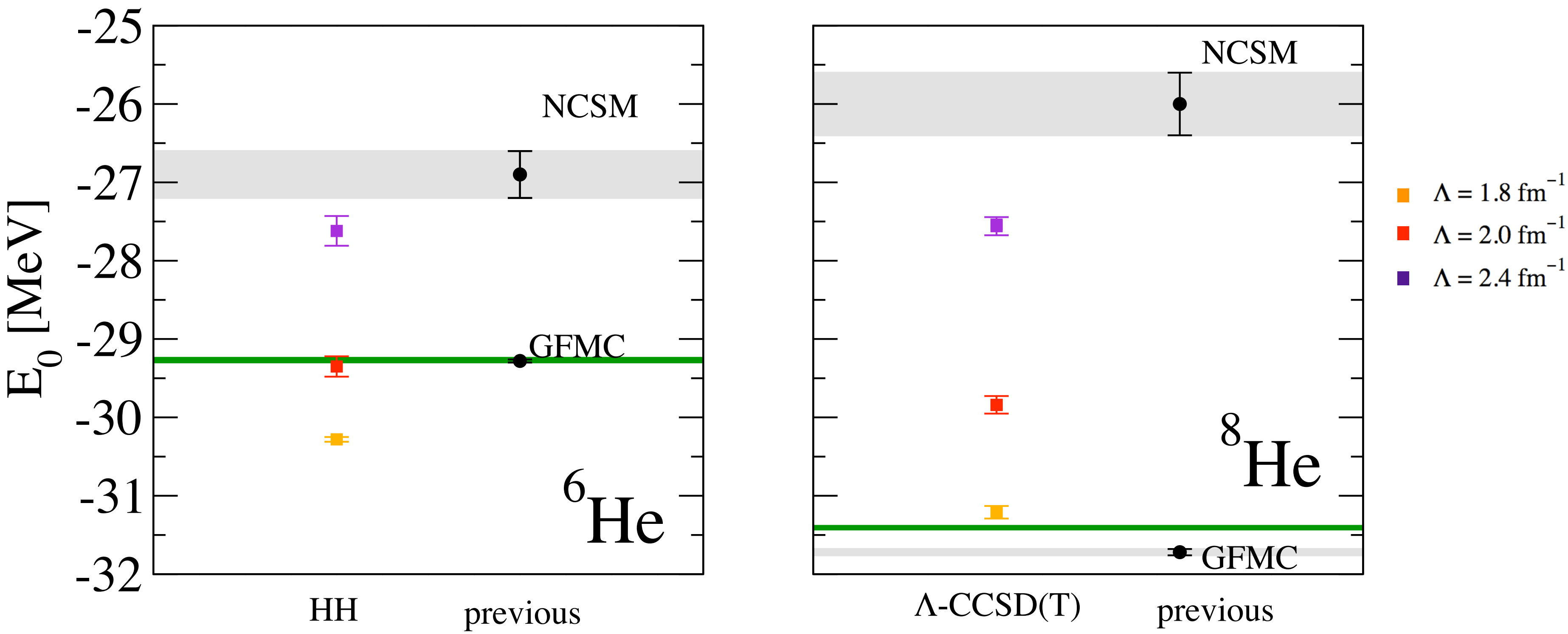
Hilbert space: 15 major shell

Values in MeV

Λ	E[CCSD]	E[Lambda-CCSD(T)]	Δ
1.8	-30.33	-31.21	0.88
2.0	-28.72	-29.84	1.12
2.4	-25.88	-27.54	1.66

- Triples corrections are larger for larger cutoff
- Their relative effect goes from 3 to 6%

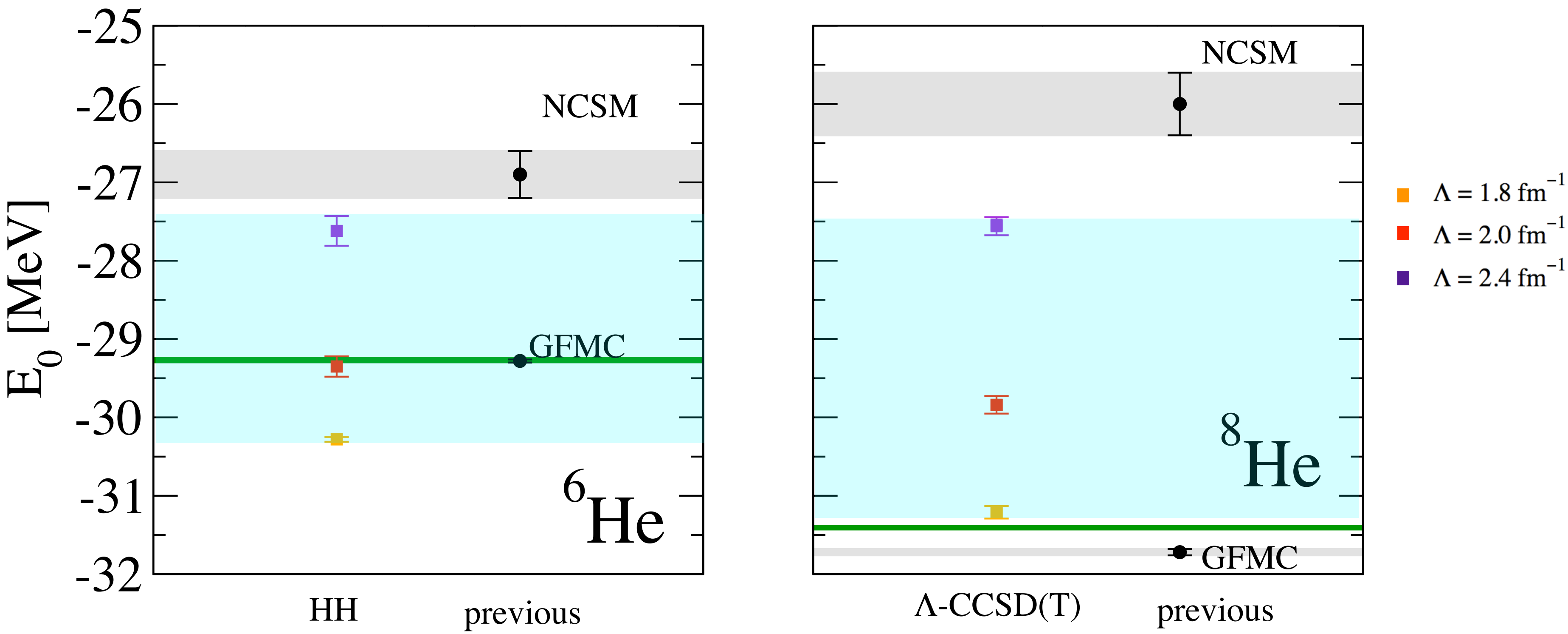
Binding Energy Summary



Experimental data

TITAN ISAC-TRIUMF

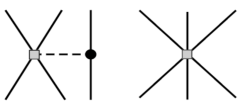
Binding Energy Summary



Experimental data

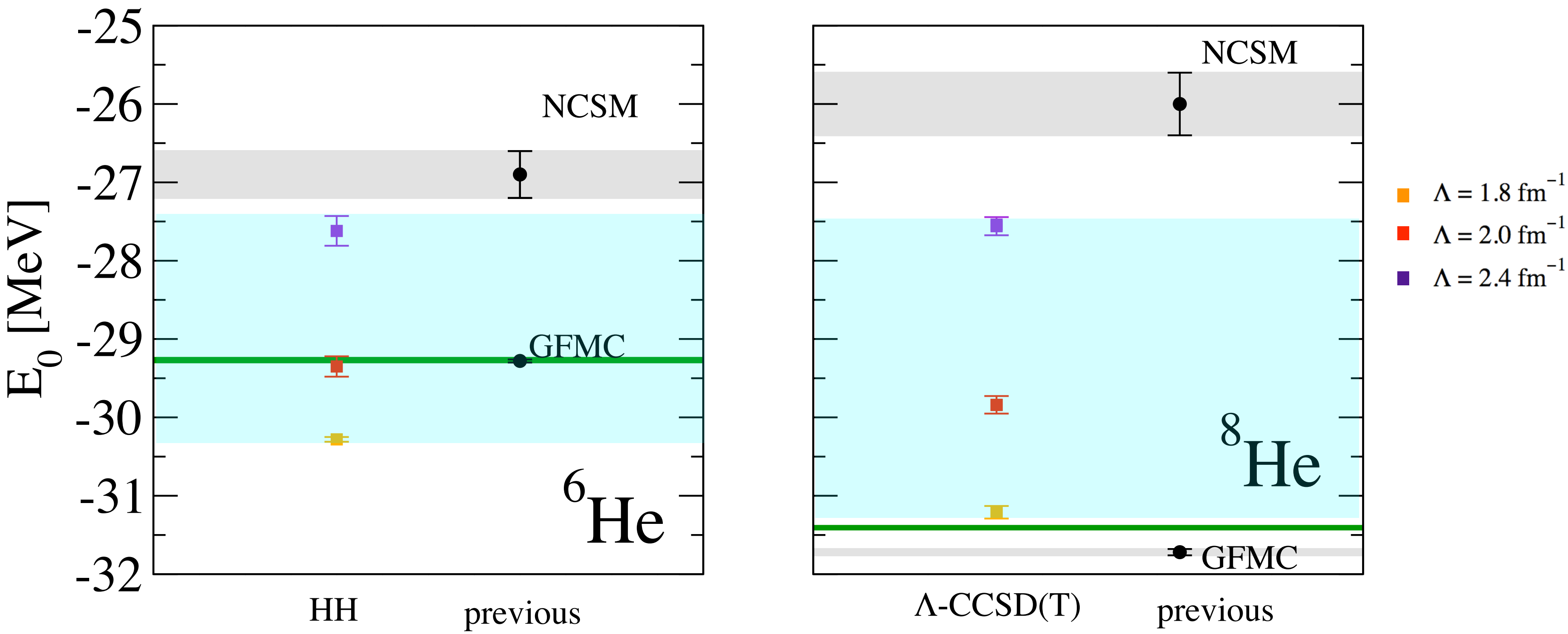
— TITAN ISAC-TRIUMF

Our estimated error in neglected



TRIUMF

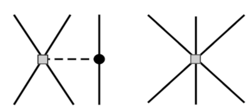
Binding Energy Summary



Experimental data

TITAN ISAC-TRIUMF

Our estimated error in neglected



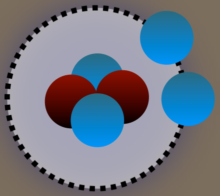
- For cutoff 2.0 fm^{-1} ^{4}He and ^{6}He are close to experiment, but ^{8}He is under-bound
- Low momentum 3NF are overall repulsive in s-shell nuclei and nuclear matter, but two-pion exchange c_i are attractive in ^{4}He and could provide further attractive spin-orbit (LS) contributions for the halo neutrons

Results for radii

TRIUMF

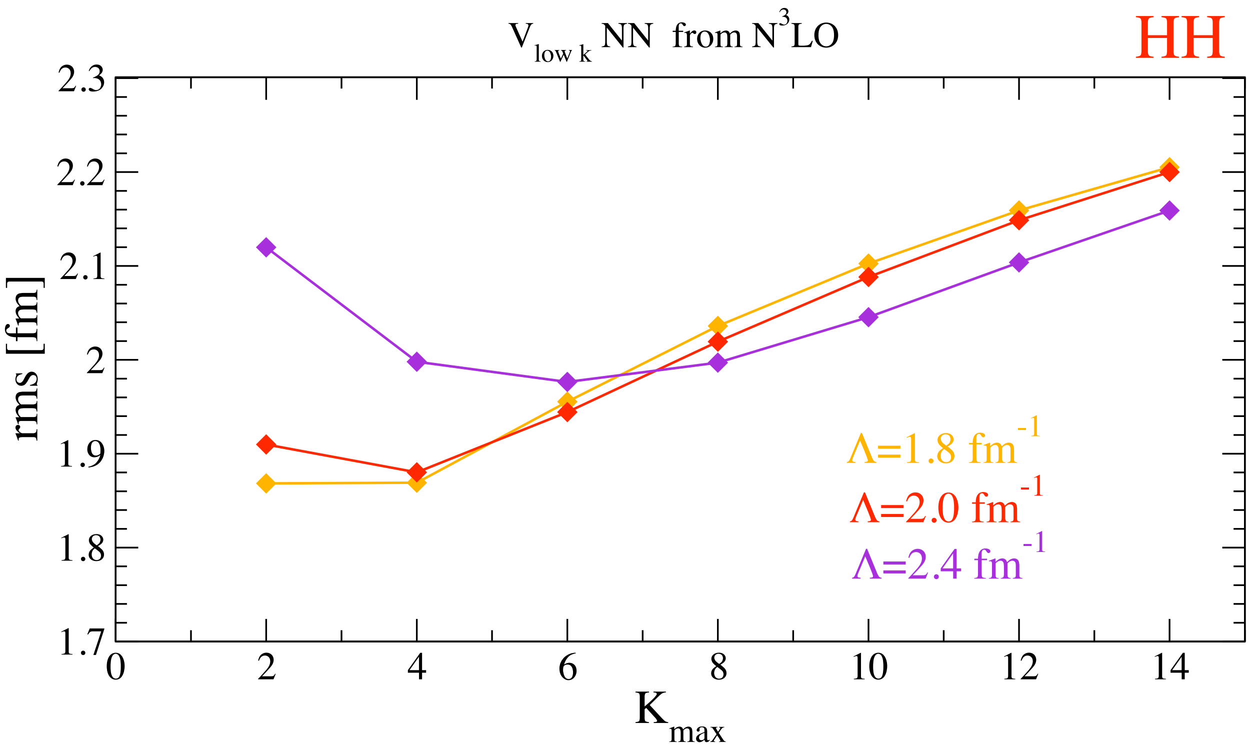
Radii for ^{6}He

- Matter radius -



$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2 \rangle}$$

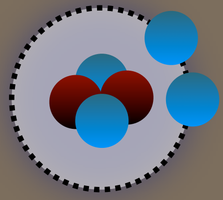
- Convergence in HH expansion is slow!



TRIUMF

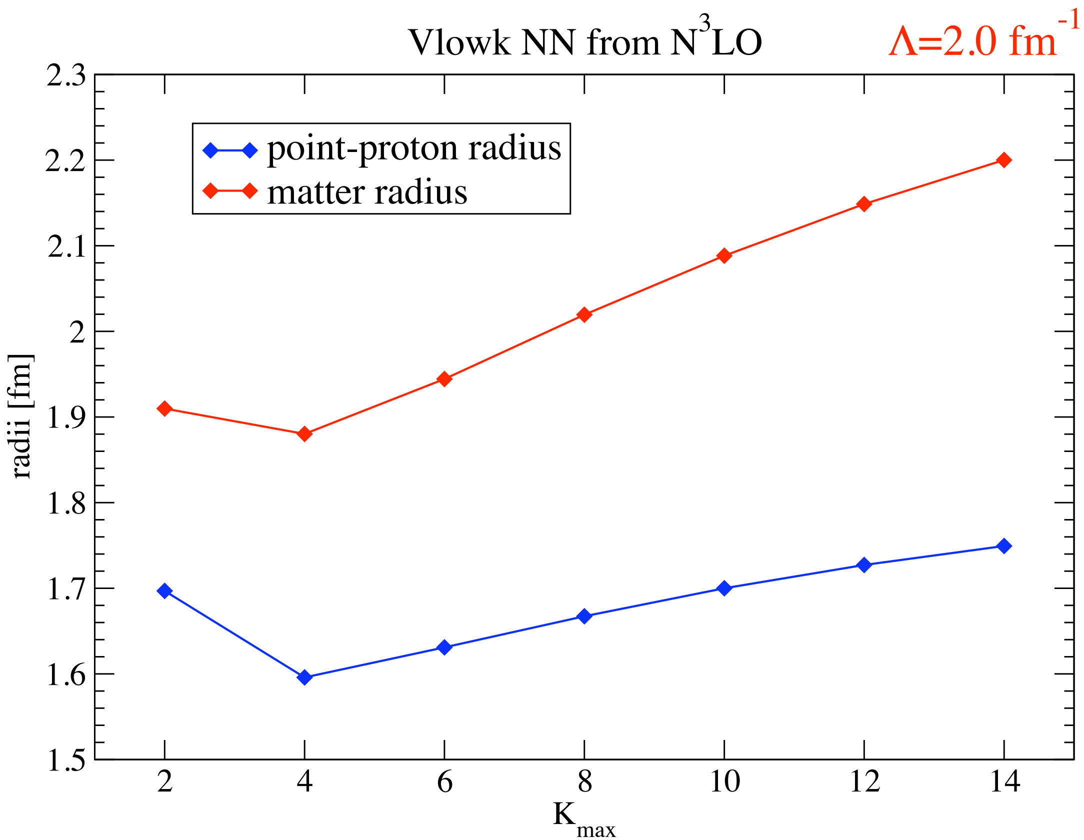
Radii for ^{6}He

- Matter and proton radius -



$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2\rangle}$$

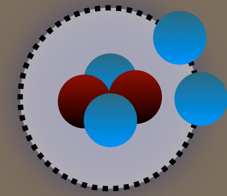
$$r_p^2 = \frac{1}{ZA}\sum_{i<j}(r_i - r_j)^2(q_i + q_j) - \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 \longrightarrow r_p = \sqrt{r_p^2}$$



TRIUMF

Radii for ^{6}He

- Matter and proton radius -



$$r^2 = \frac{1}{A^2}\sum_{i<j}(r_i - r_j)^2 = \frac{1}{A}\rho^2 \longrightarrow \text{rms radius} = \sqrt{\langle r^2 \rangle}$$

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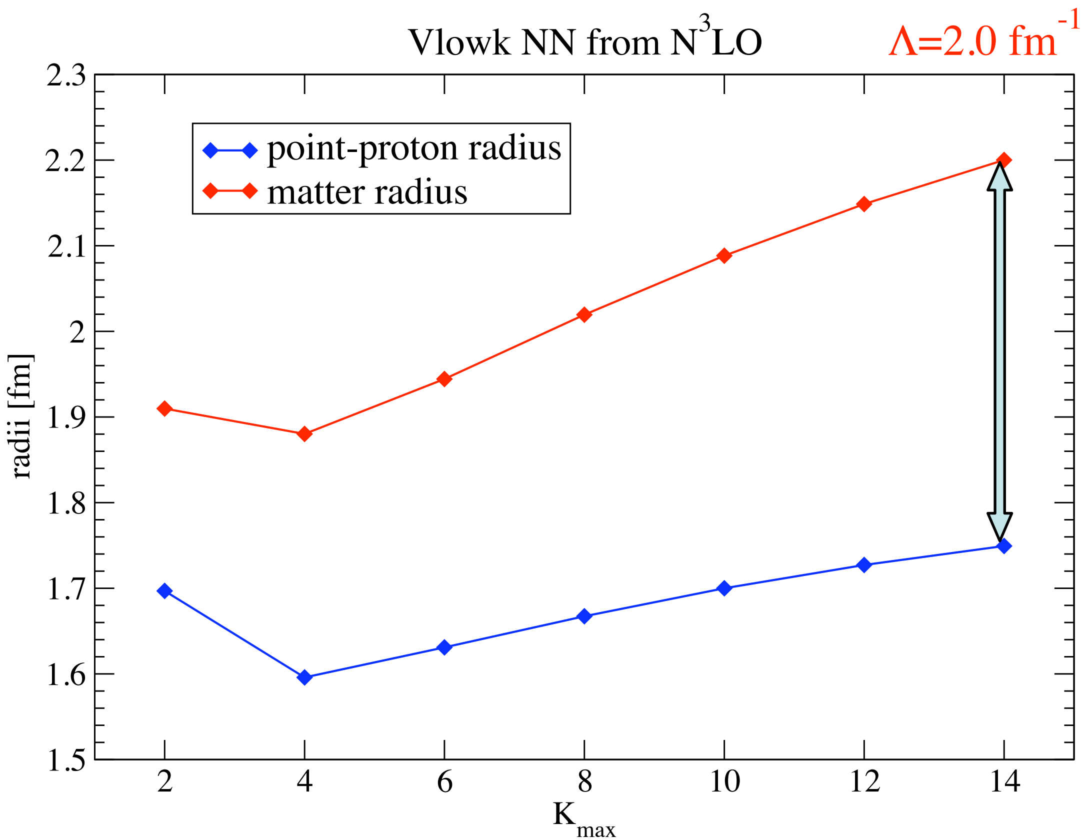
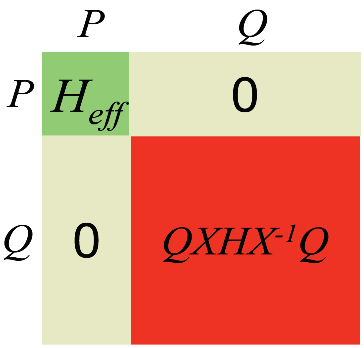


- Proton radii converge better and are smaller than matter radii ⇨ halo structure
- Convergence is pretty slow
  can we improve it?

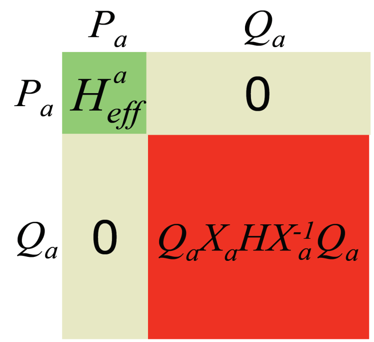
Introducing EIHH



Introduce X

$$H \rightarrow H_{eff} = PXHX^{-1}P$$

H_{eff} is an A-body operator



$a < A (a = 2)$

Solve an a-body problem

Find $H^a_{eff} \rightarrow v^{[a]eff}_{ij}$

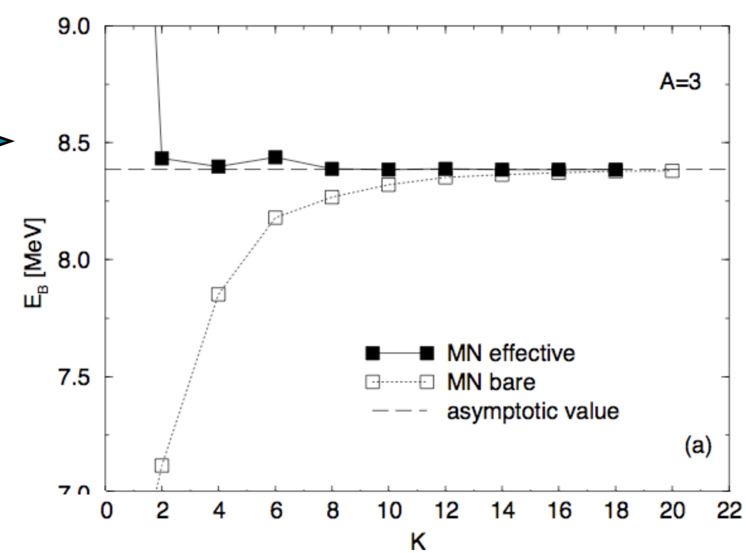
Use it into the A-body problem

$$V = \sum_{ij} v^{[2]eff}_{ij}$$

Increase the model space in the A-body problem

- For local potentials with a hard core

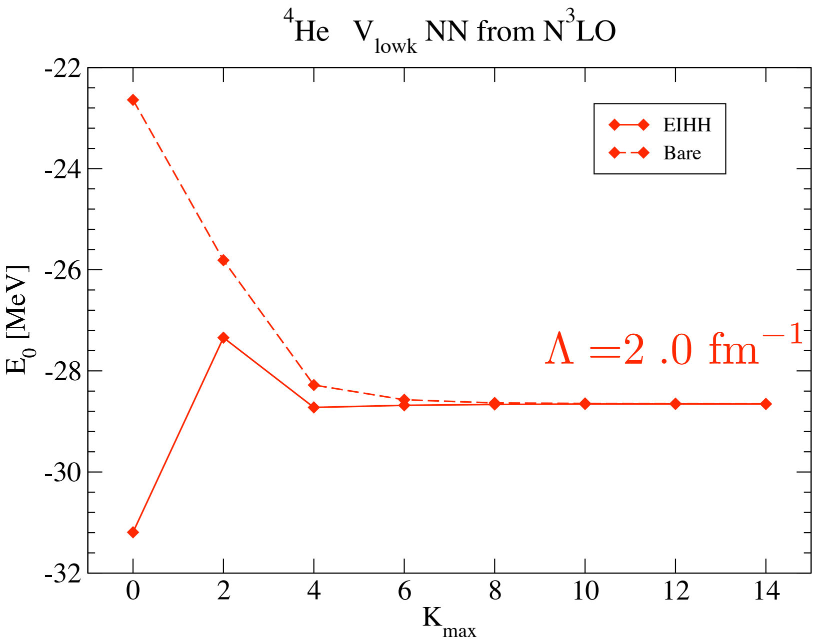
  Barnea et al. PRC 61 (2000) 054001

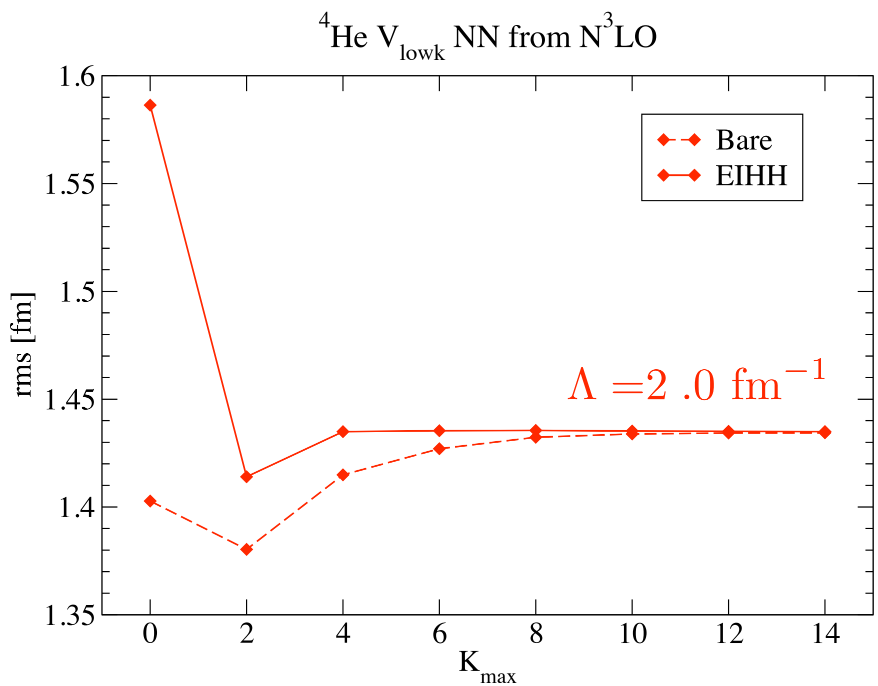


- Can be extended to nonlocal forces

  Barnea et al., in preparation

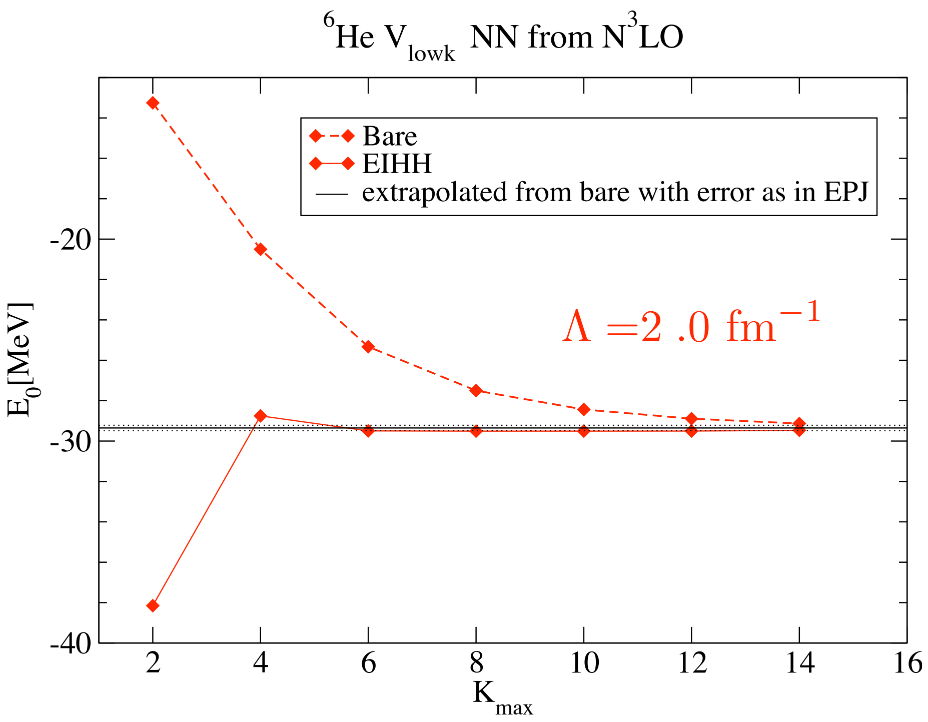
EIHH on ^{4}He

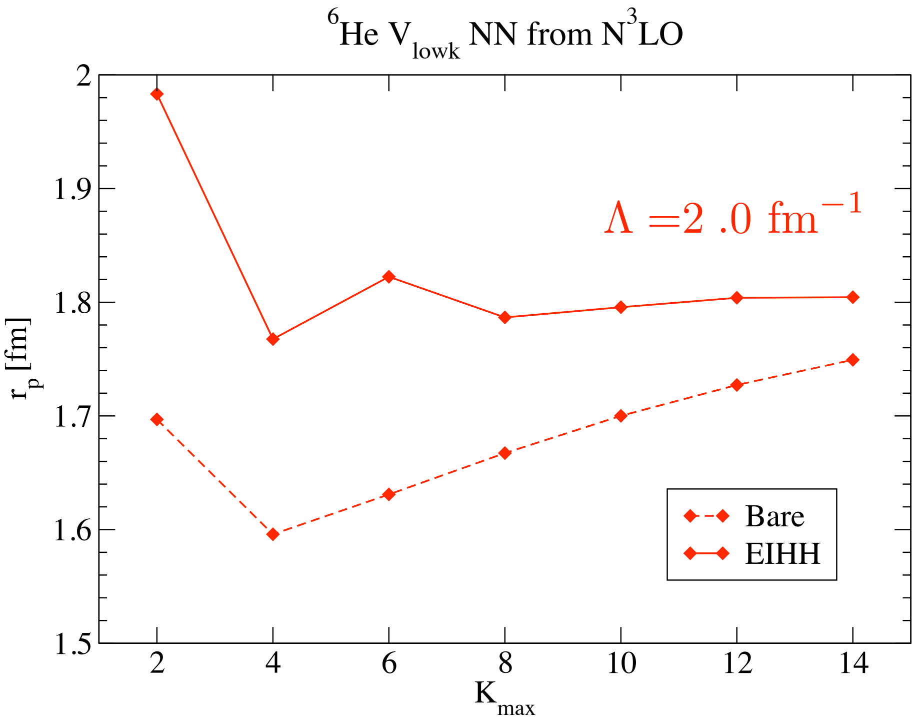




- Convergence is sensibly accelerated both for energy and radius

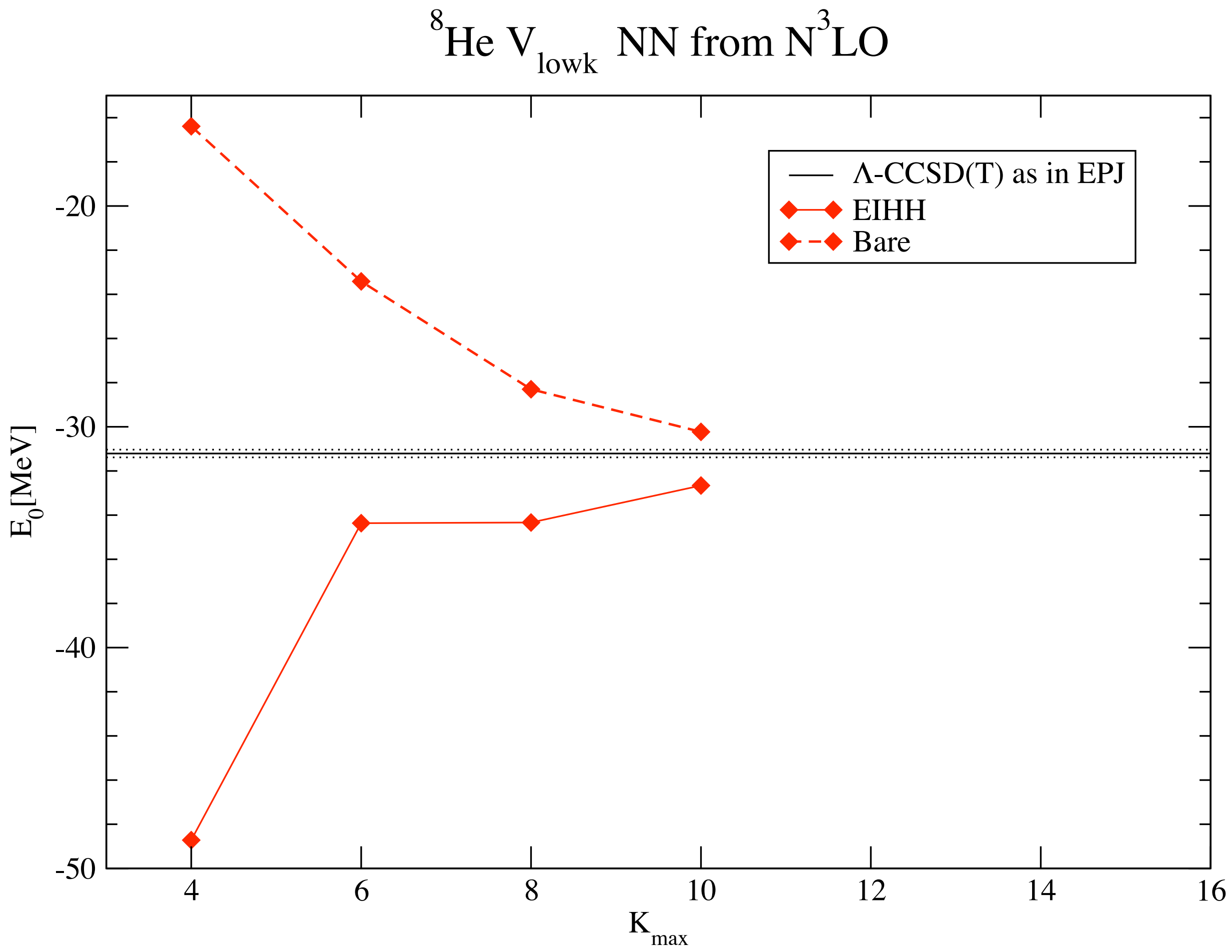
EIHH on ^{6}He





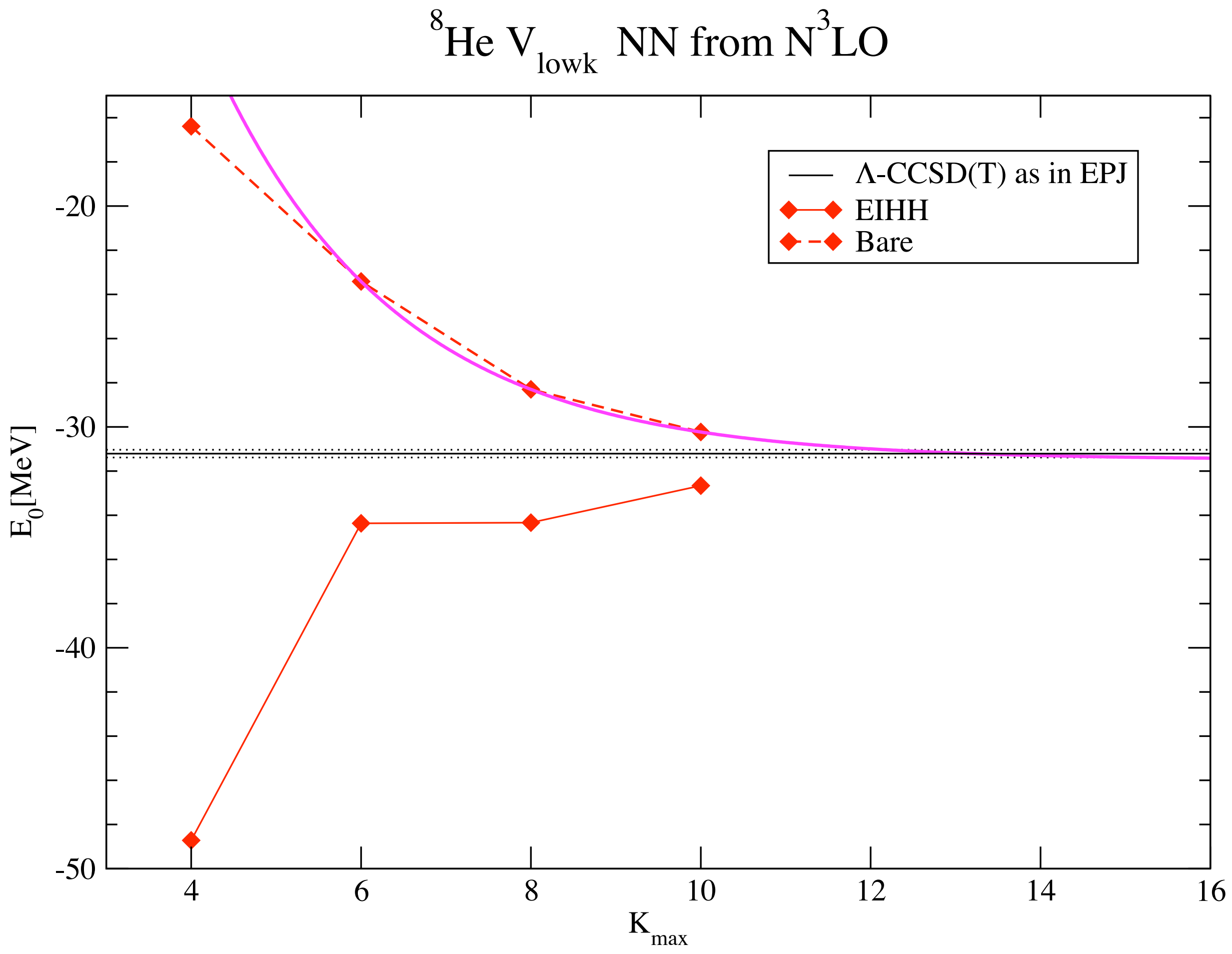
- Great improvement in convergence for both for energy and radius

^{8}He with HH



- Difference between Bare and EIHH is about 2.4 MeV
- EIHH seems less effective than for ^{6}He

^{8}He with HH



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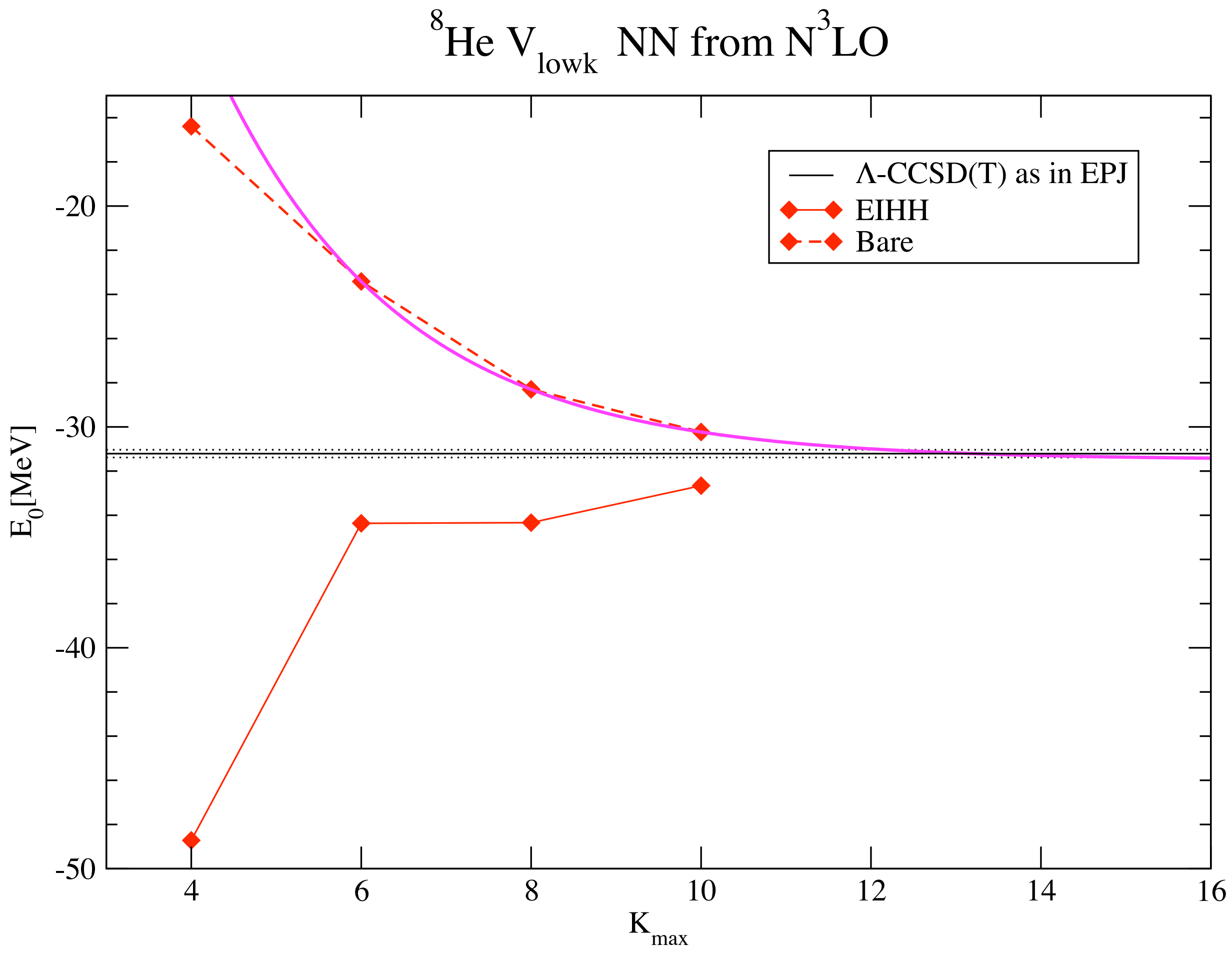
^{8}He with HH



- Difference between Bare and EIHH is about 2.4 MeV
- EIHH seems less effective than for ^{6}He
- Extrapolating Bare results get

$$E_\infty = -31.49\text{MeV}$$

$$E_{\Lambda-\text{CCSD(T)}} = -31.21\text{MeV}$$

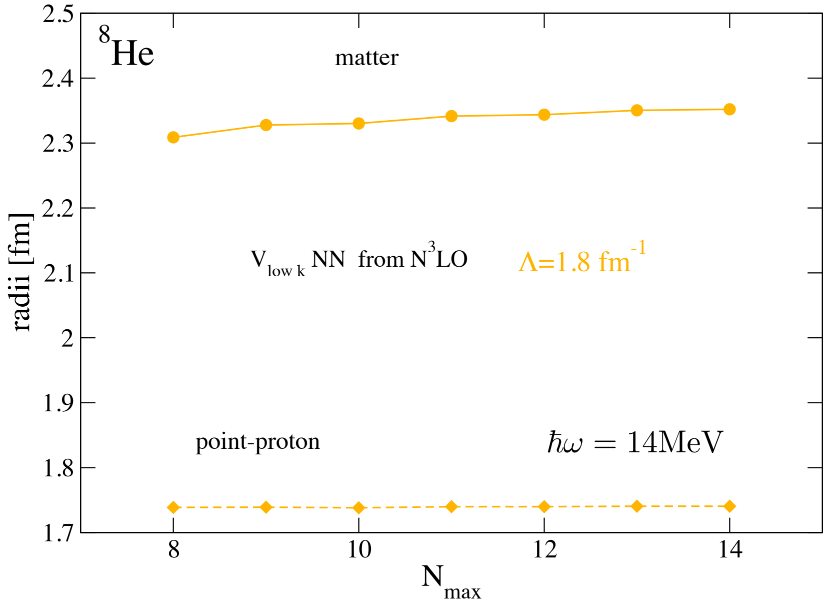
Radii for ^{8}He

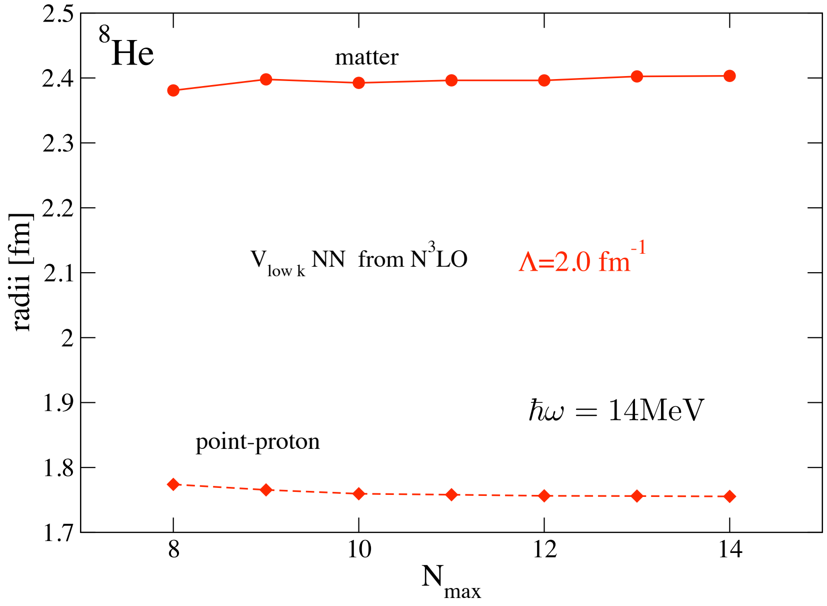
$$r^2 = \frac{1}{A}\sum_i^A r_i^2$$

$\sqrt{\langle r^2 \rangle}$ matter radius

$$r_p^2 = \frac{1}{Z}\sum_i^A r_i^2 \left(\frac{1+\tau_i^3}{2}\right)$$

$\sqrt{\langle r_p^2 \rangle}$ point-proton radius





- Large Hilbert space ⇨ small model space dependence
- Point-proton radius is smaller than matter radius ⇨ halo structure
- Operators are not translational invariant

Matter radii Summary

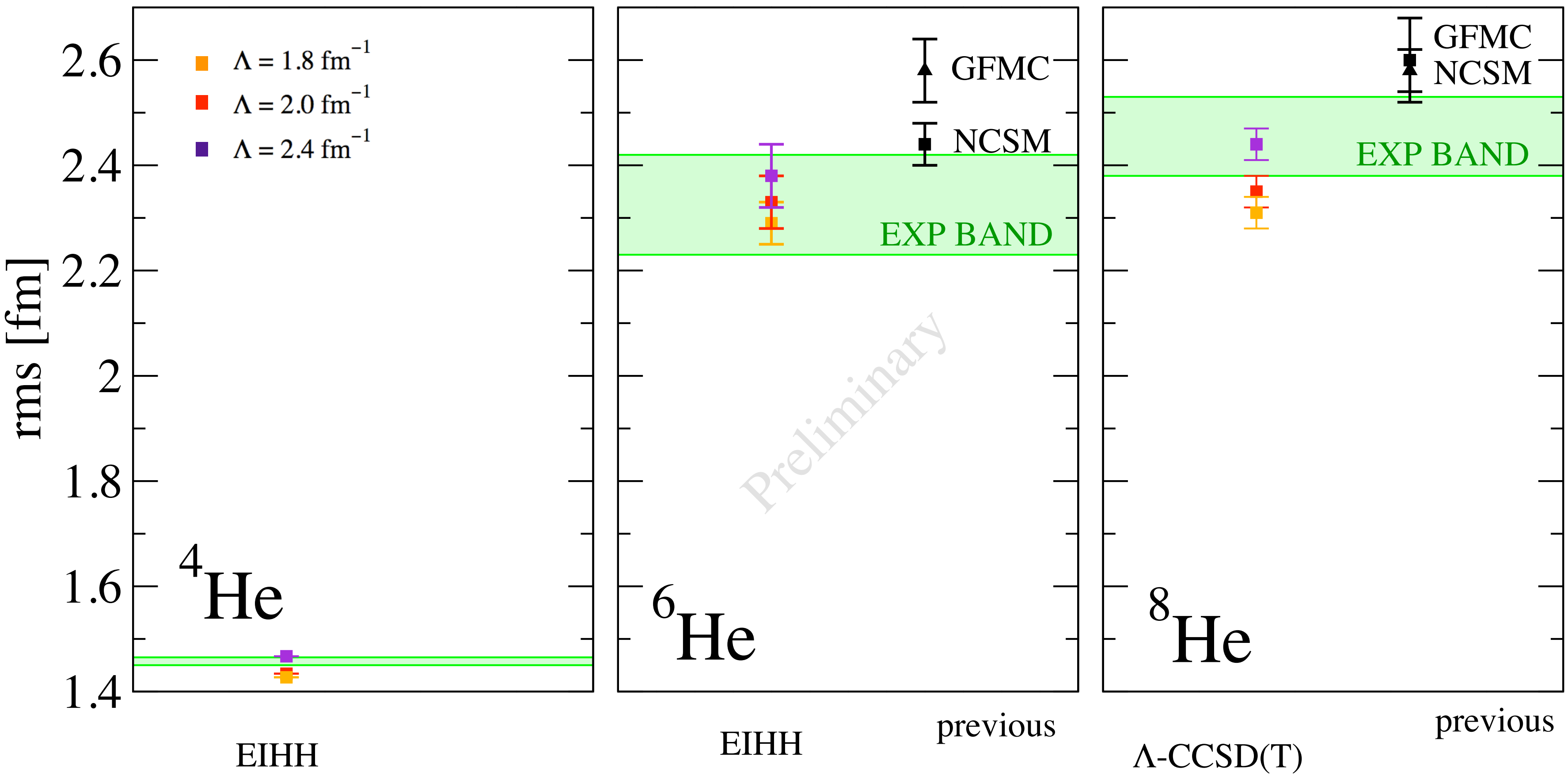
- Benchmark on ^{4}He, CC using translational invariant operators

$$\Lambda = 2.0 \mathrm{fm}^{-1} \qquad \mathrm{HH} \quad 1.434 \quad \mathrm{fm} \qquad \Lambda - \mathrm{CCSD(T)} \quad 1.429 \quad \mathrm{fm}$$

Matter radii Summary

- Benchmark on ^{4}He, CC using translational invariant operators

$\Lambda = 2.0 \mathrm{fm}^{-1}$ HH 1.434 fm Λ – CCSD(T) 1.429 fm



Matter radii Summary

- Benchmark on ^{4}He, CC using translational invariant operators

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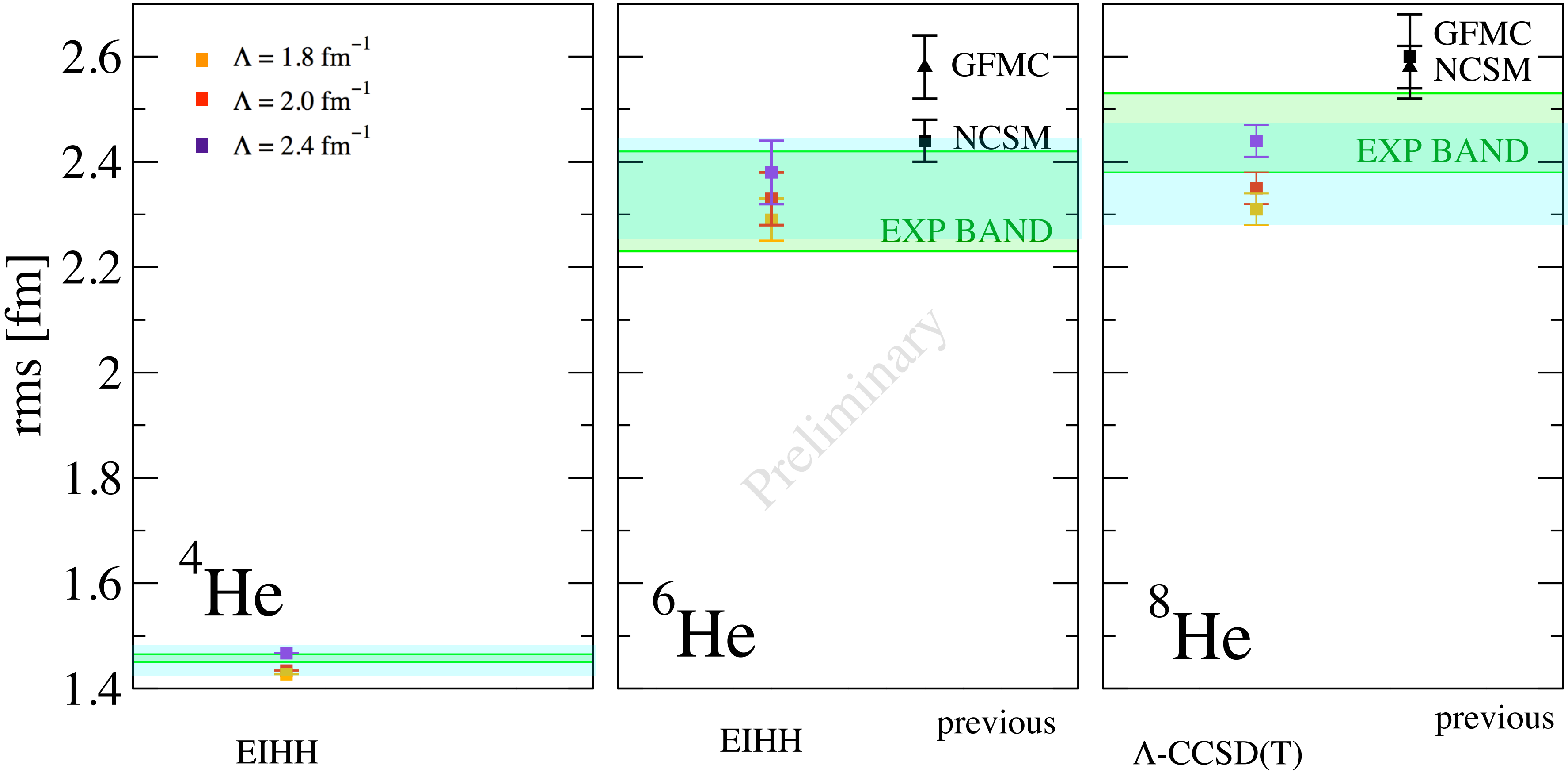
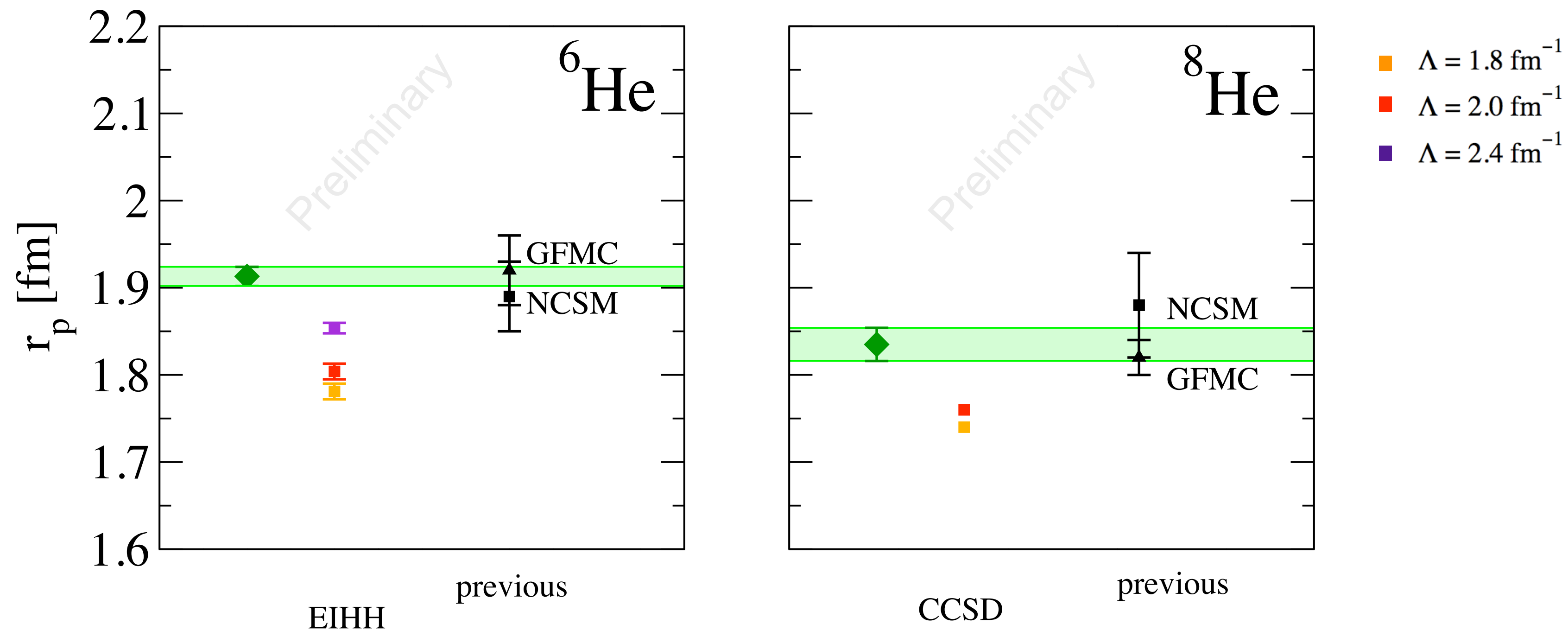


Proton radii Summary

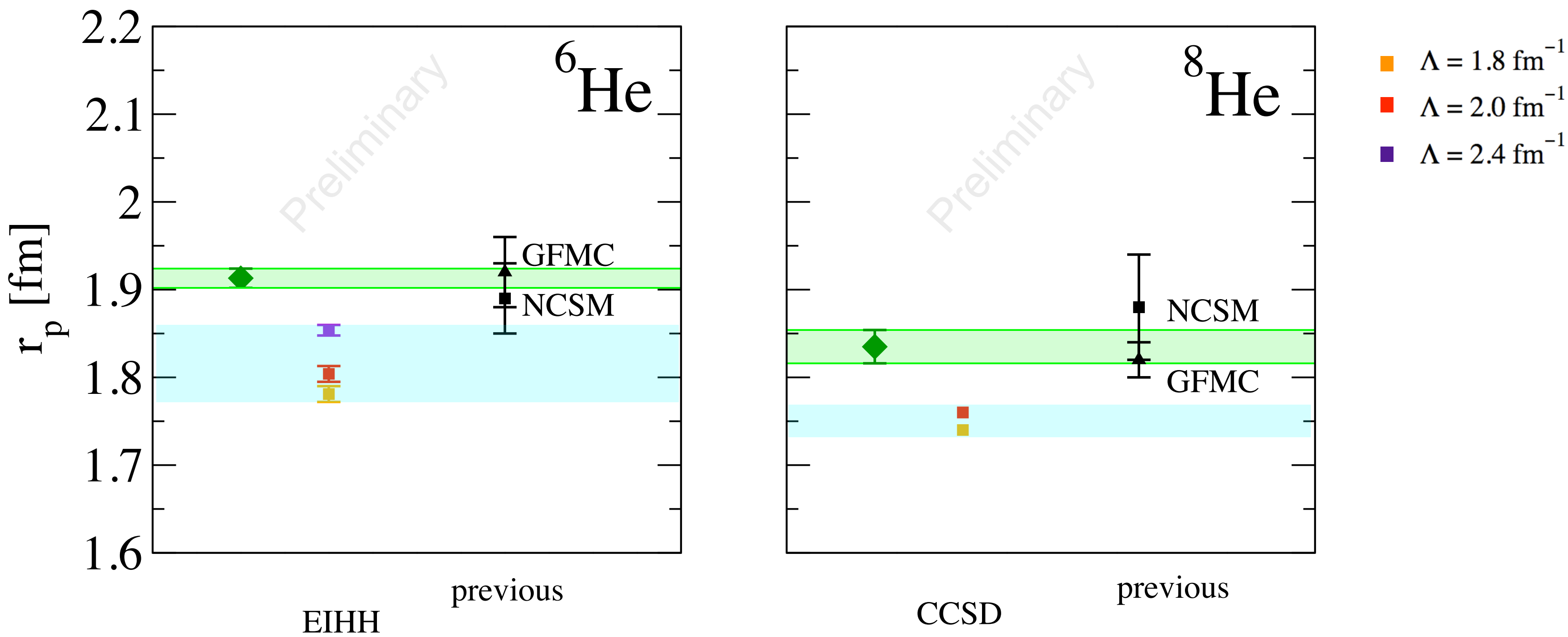


◆ from laser spectroscopy

using TITAN ISAC-TRIUMF

binding energy as input

Proton radii Summary

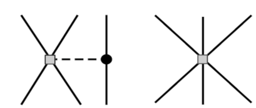


◆ from laser spectroscopy

using TITAN ISAC-TRIUMF

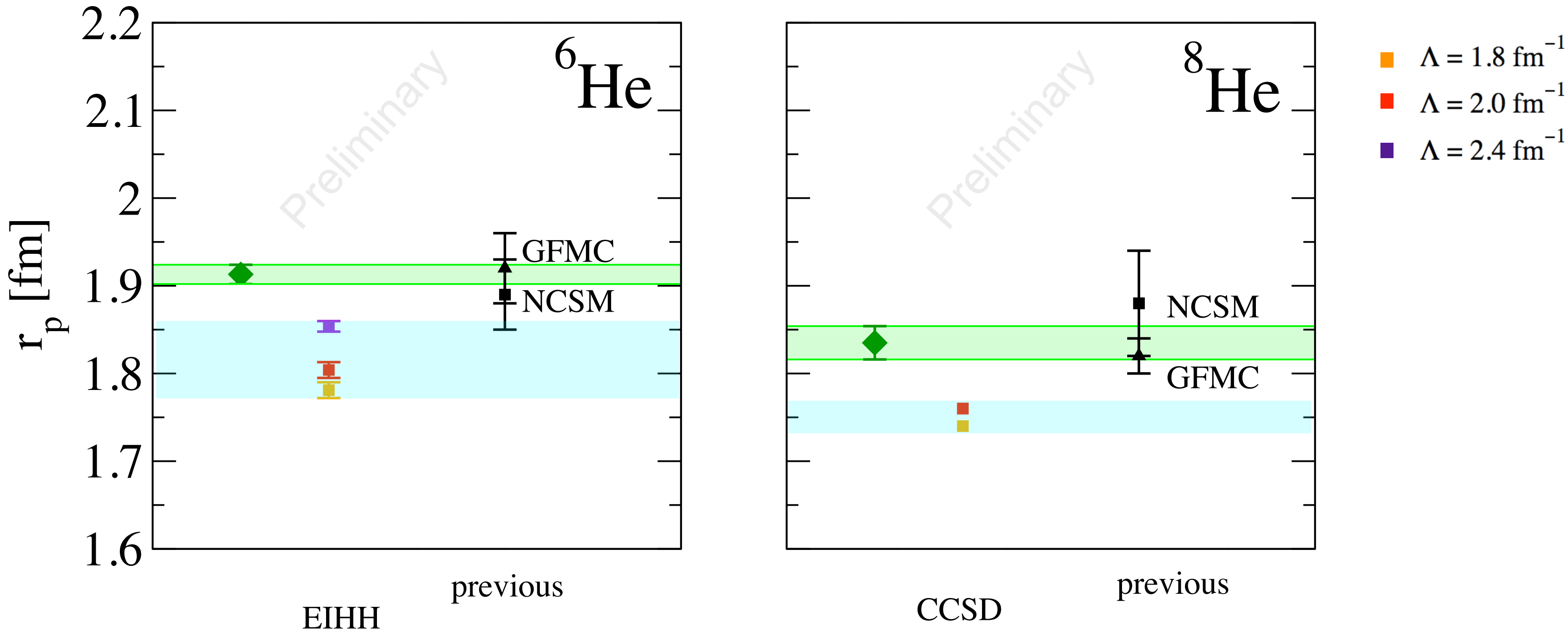
binding energy as input

Our estimated error in neglected



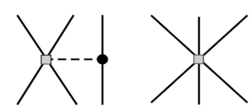
is in percentage consistent with the results for binding energy

Proton radii Summary



◆ from laser spectroscopy using TITAN ISAC-TRIUMF binding energy as input

Our estimated error in neglected



is in percentage consistent with the results for binding energy

- The fact that for some "choice" of the NN force one gets correct radii and wrong energies (or vice-versa) shows that halo nuclei provide important tests of the different aspects of nuclear forces, which includes 3NF

- We provide improved description of helium halo nuclei from evolved EFT interactions with the correct asymptotic in the wave function

- We estimate the effect of short range three-nucleon forces on binding energies and radii by varying the cutoff of the evolved interaction

- Our matter radii agree with experiment whereas our point-proton radii under-predict experiment

Future:

- Include three-nucleon forces

- Extend coupled cluster theory calculations to heavier neutron rich nuclei, e.g. lithium ➡ $^{11}\mathrm{Li}$ or oxygen isotope chain