Role of Symmetries in SRG Transformations of NN Interactions

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Thanks also to:

J. P. Vary (ISU), R. Furnstahl (OSU),

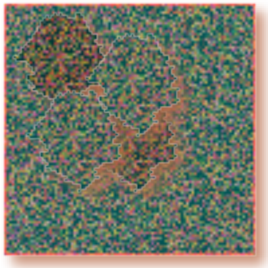
R. Roth (TU Darmstadt), S. Bogner (MSU)

Similarity Renormalization Group and V_{NN}

Bare interaction

$(V_{NN}, V_{NNN}, \ldots)$

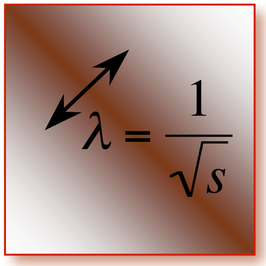
$H_{s=0}$



Unitary transformations

Renormalized interaction

H_s



$$\boxed{H_s = e^{s\eta} H_{s=0} e^{-s\eta}}$$

$s \ldots$ *'flow parameter'*

'flow equation'

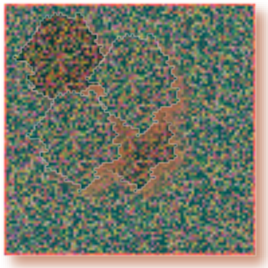
$$\frac{dH_s}{ds} = [\eta, H_s] \qquad \eta = [O, H_s]$$

SRG and V_{NN}: SU(3) Symmetry

Bare interaction

$(V_{NN}, V_{NNN}, \ldots)$

$H_{s=0}$



$$\boxed{O \equiv C_2^{su(3)} \quad \eta_s = \left[C_2^{su(3)}, H_s\right]}$$

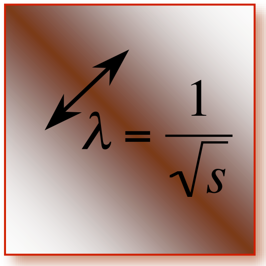
Unitary

transformation

Renormalized

interaction

H_s



flow equation

$$\frac{dH_s}{ds} = \left[\left[\underbrace{C_2^{su(3)}}, H_s\right], H_s\right]$$

Second-order invariant operator of SU(3)

(*diagonal* in SU(3) basis)

$$\underbrace{6C_2^{su(3)} = Q.Q + 3L^2}_{\textit{single shell}}$$

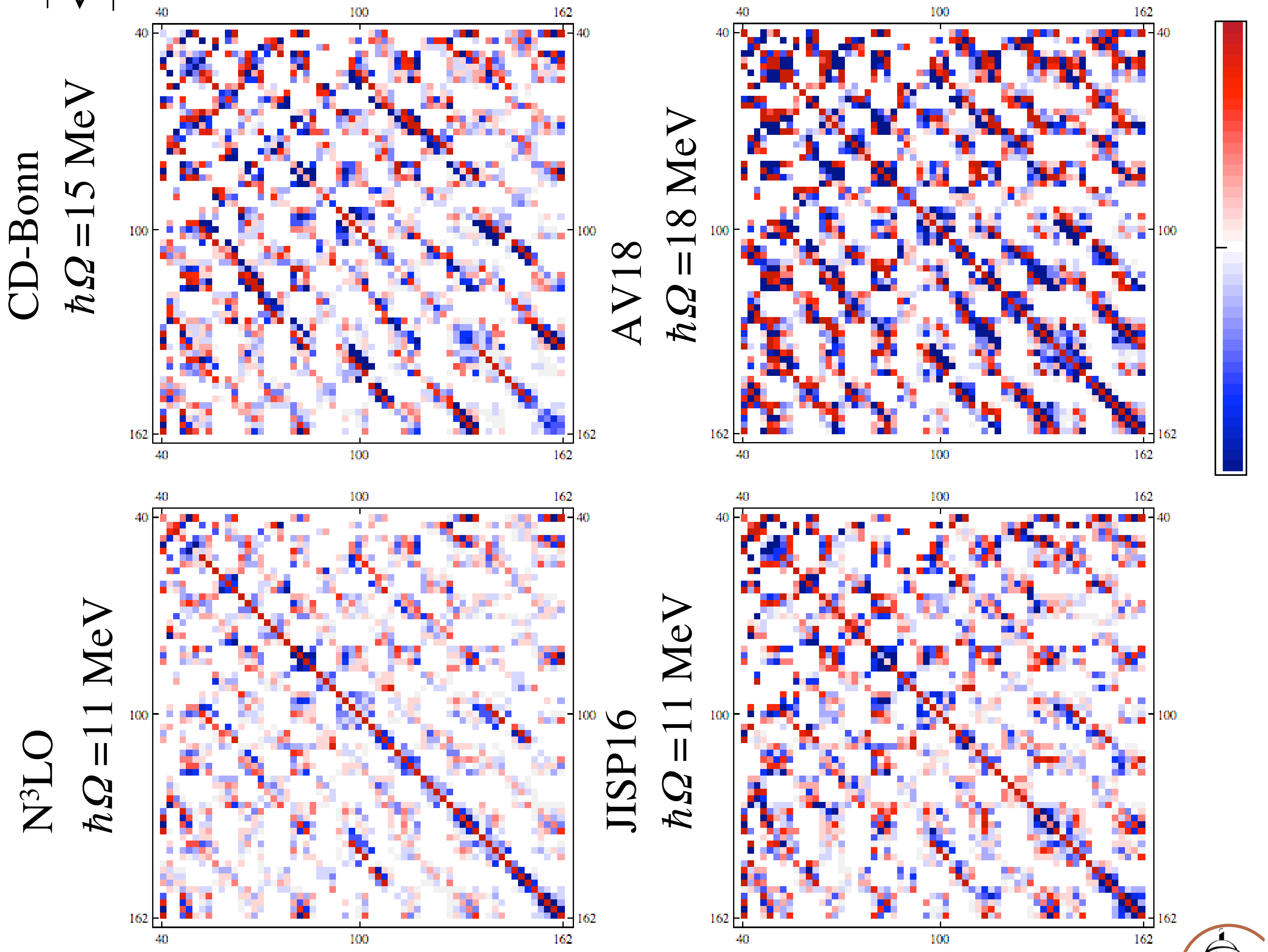
SU(3) basis states: $\frac{1}{\mathrm{N}}\left[a^{\dagger}_{(n_1\,0)st} \times a^{\dagger}_{(n_2\,0)st}\right]^{(\lambda\mu)S,TT_0}_{\kappa L;JM} |0\rangle$

SU(3)-SRG with $C^{(2)}$:

- ➢a very effective scheme for renormalization of realistic interactions in HO basis
- ➢ addresses the SRG-induced many-body forces

Simple Illustration (8-shell Basis Space)

- Bare *NN*+ Relative Kinetic Energy
- Restricted Basis Space (8 shells)
- Shown: *J*=1, *T*=0
- Decouples higher-lying from lower-lying modes



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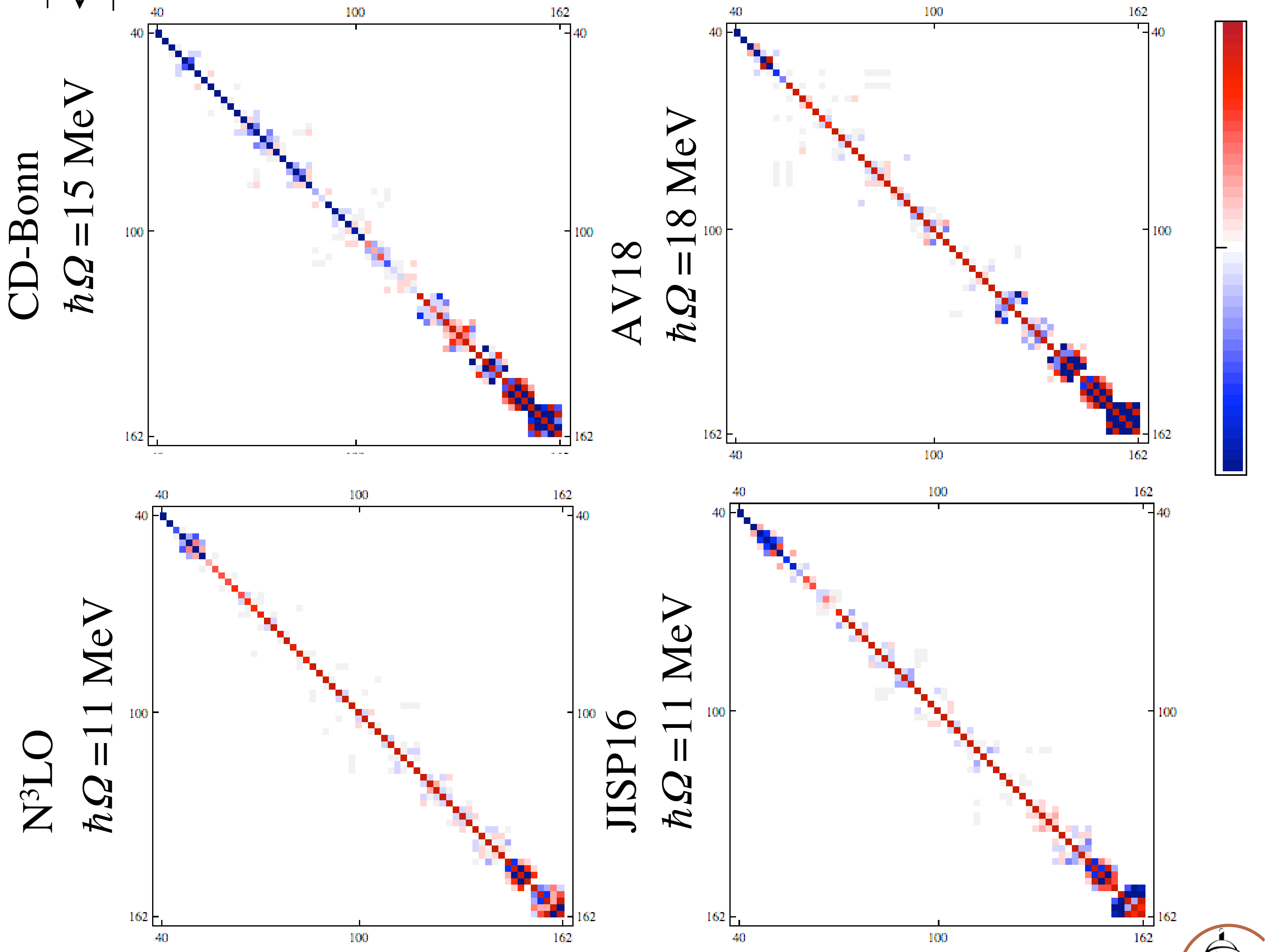


Significance of Symplectic and SU(3) Symmetry

microscopic

collective

For A particles:

$x_i,\ p_i \longleftrightarrow L^{(11)}_{1,M}\quad C^{(11)}_{2,M}\quad N^{(00)}\quad A^{(20)}_{L,M}\quad B^{(02)}_{L,M}$

Sp(3,R) ⊃ SU(3) ⊃ SO(3)

Elliott Model (single shell)

Angular Momentum .... $\vec{x}\times\vec{p}$ **SO(3)**

Quadrupole Moment .... $x_i x_j$ **SU(3)**

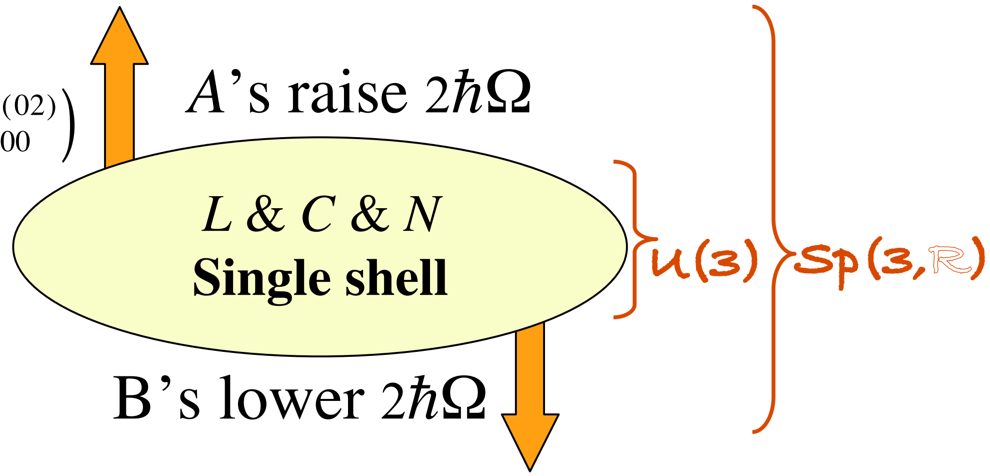
Number Operator ...... essentially HO Hamiltonian

[Hamiltonian $H_0 = (p^2+x^2)/2$]

Multi-shell Coupling ...

[Monopole, $L = 0$ & Quadrupole, $L = 2$]

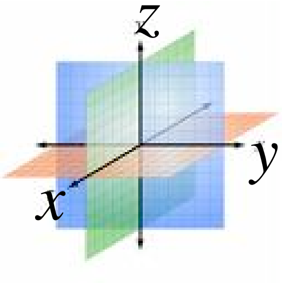
- Higher-lying and core excitations
- Kinetic energy: $\frac{H^{(00)}_{00}}{2} - \frac{\sqrt{6}}{4}\left(A^{(20)}_{00} + B^{(02)}_{00}\right)$



G. Rosensteel and D.J. Rowe, Phys. Rev. Lett. 38 (1977) 10

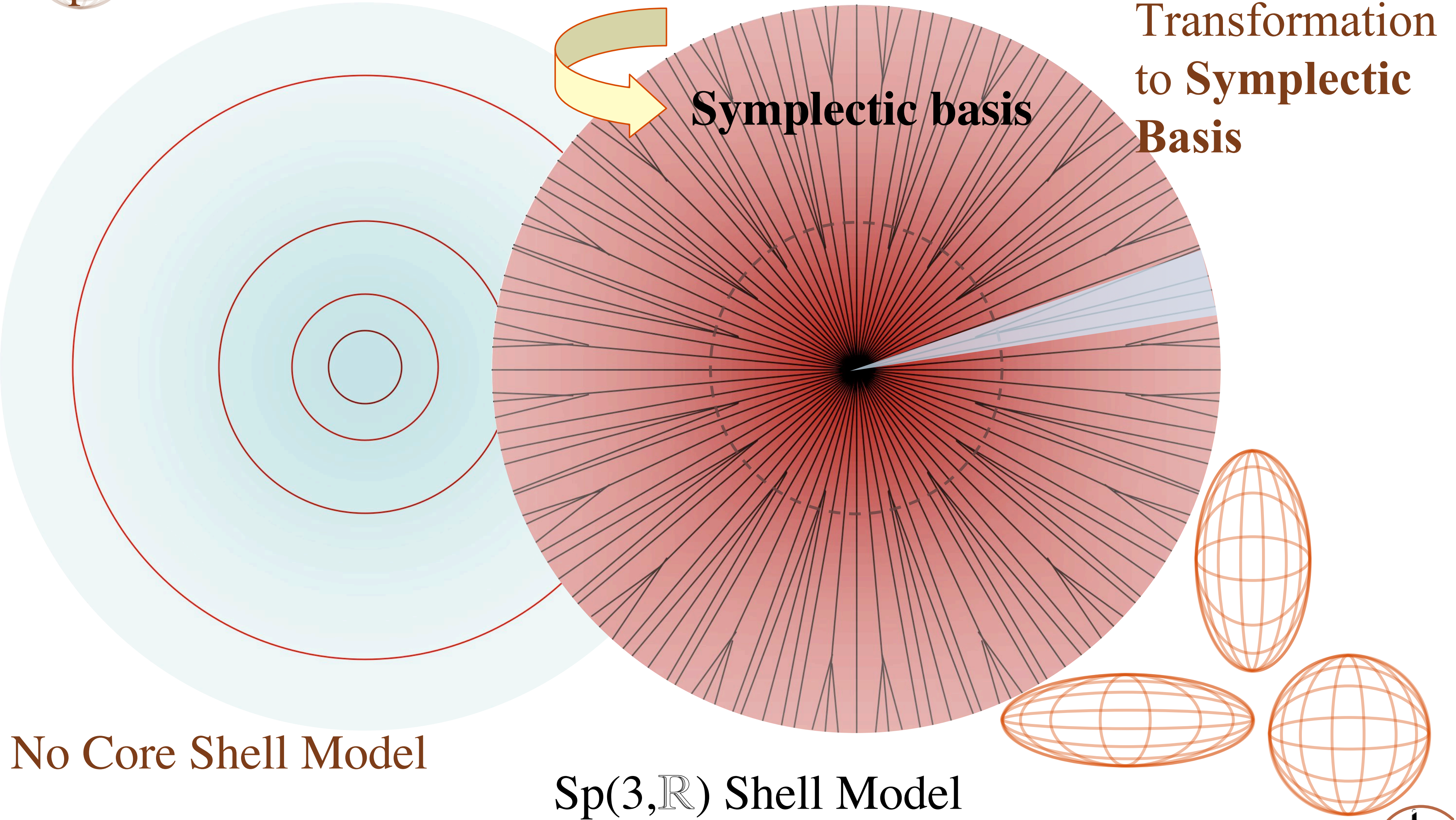
Symplectic Sp(3,ℝ) ⊃ SU(3)

Mars' Deimos



Intrinsic nucleon dynamics - reflected in shape deformation, irrelevant of the orientation in space

Significance of Symplectic Basis



Spherical harmonic oscillator basis

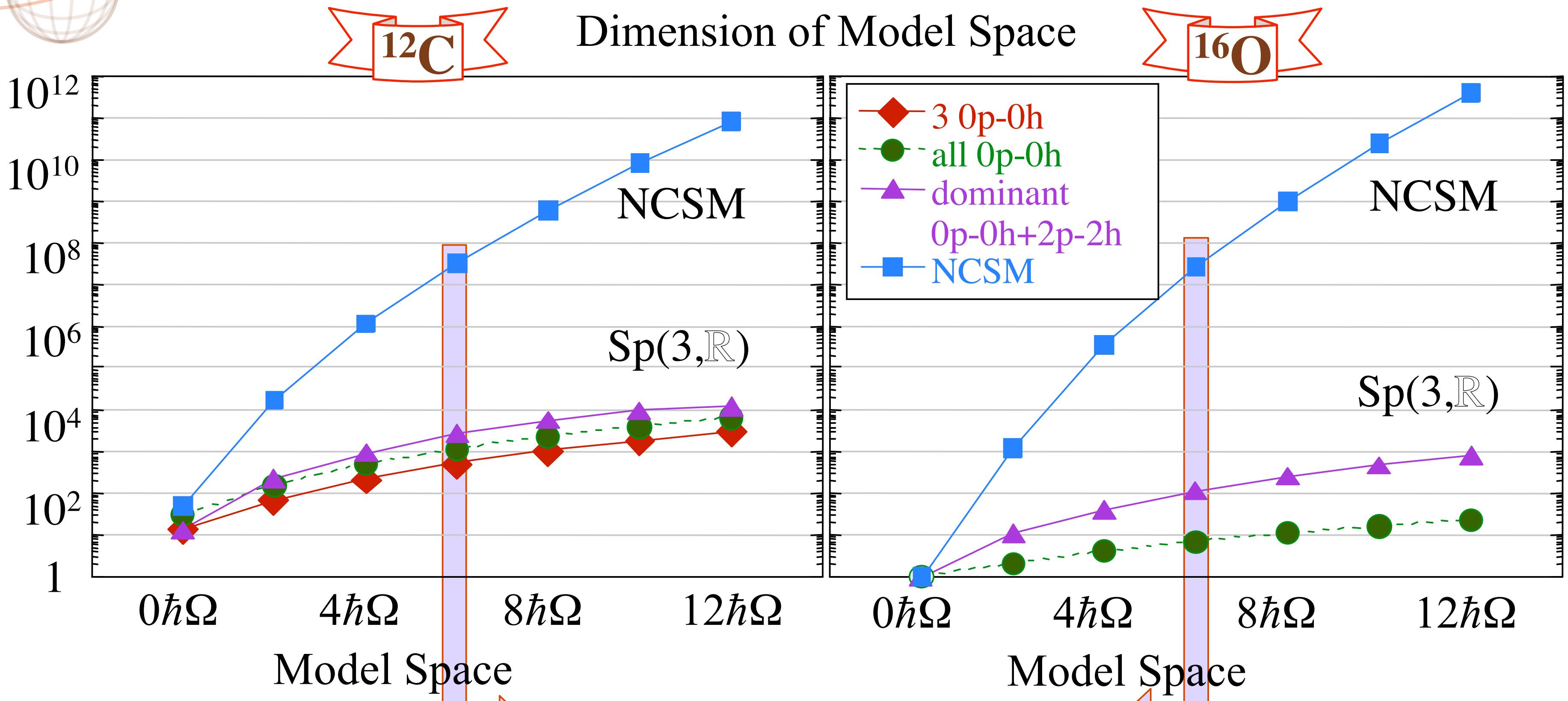
Symplectic basis

Unitary Transformation to **Symplectic Basis**

No Core Shell Model

Sp(3,$\mathbb{R}$) Shell Model

Symplectic Basis: Major Reductions in Model Space



Reproducing NCSM results

Dimension of model space

0.009% for ^{12}C

0.0004% for ^{16}O

T. Dytrych, KDS,
C. Bahri, J.P. Draayer, J.P. Vary,
Phys. Rev. Lett. 98 (2007) 162503

Choice of the SRG-generator: Is it vital?

Bare interaction

$(V_{NN}, V_{NNN}, \ldots)$

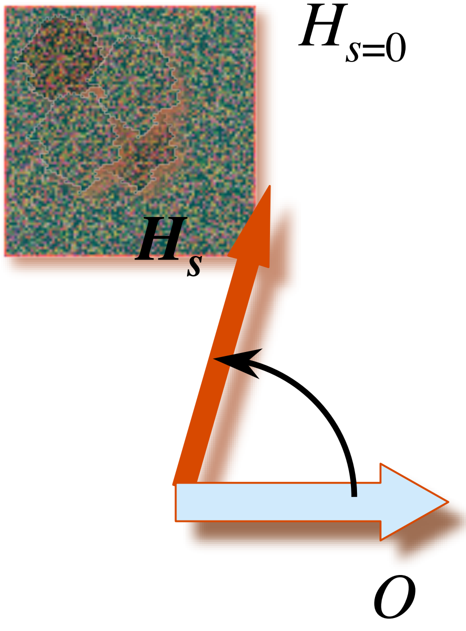
$H_{s=0}$

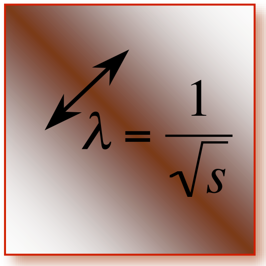
$$\boxed{H_s = e^{s\eta} H_{s=0} e^{-s\eta}}$$

Unitary transformations

Renormalized interaction

H_s





Spectral Distribution Theory

$$\zeta^{\alpha}_{H,H'} = \frac{\langle (H^{\dagger} - \langle H^{\dagger}\rangle^{\alpha})(H' - \langle H'\rangle^{\alpha})\rangle^{\alpha}}{\sigma_H \sigma_{H'}}$$

$$(\sigma^{\alpha}_{H})^2 = \langle (H - \langle H\rangle^{\alpha})^2\rangle^{\alpha}$$

$$\cos\theta = \frac{\vec{v}\cdot\vec{v}'}{|\vec{v}||\vec{v}'|}$$

$$\boxed{\frac{dH_s}{ds} = [\eta, H_s] \qquad \eta = [O, H_s]}$$

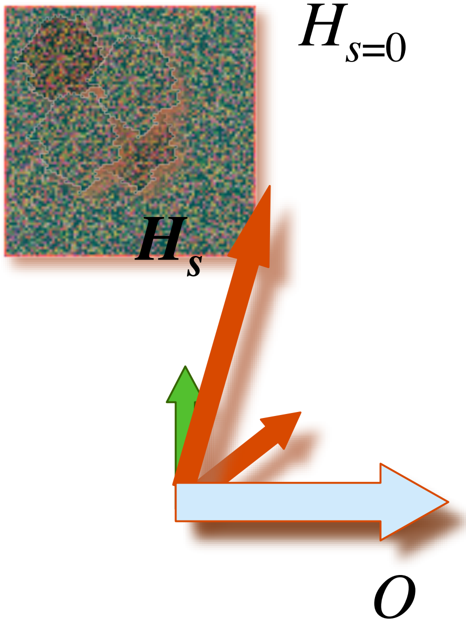
Choice of the SRG-generator: Is it vital?

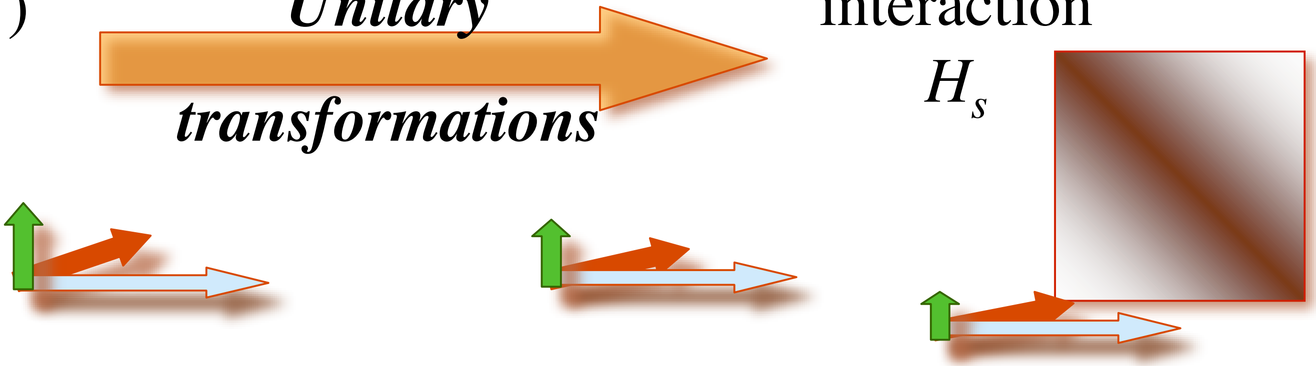
$$H_s = e^{s\eta} H_{s=0} e^{-s\eta}$$

Bare interaction
$(V_{NN}, V_{NNN}, \ldots)$
$H_{s=0}$

Unitary transformations

Renormalized interaction
H_s





For any fixed O:

$$\sigma_H \sigma_O \frac{d}{ds} \zeta_{H,O} = \sigma_\eta^2 = \left\langle [H,O][O,H] \right\rangle \geq 0$$

Correlation between O and H increases until $[O,H]=0$

$[X,O]=0$

$$H(s \to \infty) = \alpha(s) O + \beta(s) \{X\}^n$$

$$\frac{dH_s}{ds} = [\eta, H_s] \qquad \eta = [O, H_s]$$

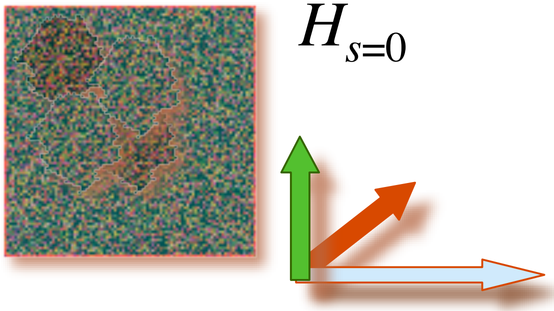
Choice of the SRG-generator: Is it vital?

$$\boxed{H_s = e^{s\eta} H_{s=0} e^{-s\eta}}$$

Bare interaction
(V_{NN}, V_{NNN}, …)
$H_{s=0}$

Unitary transformations

Renormalized interaction
H_s



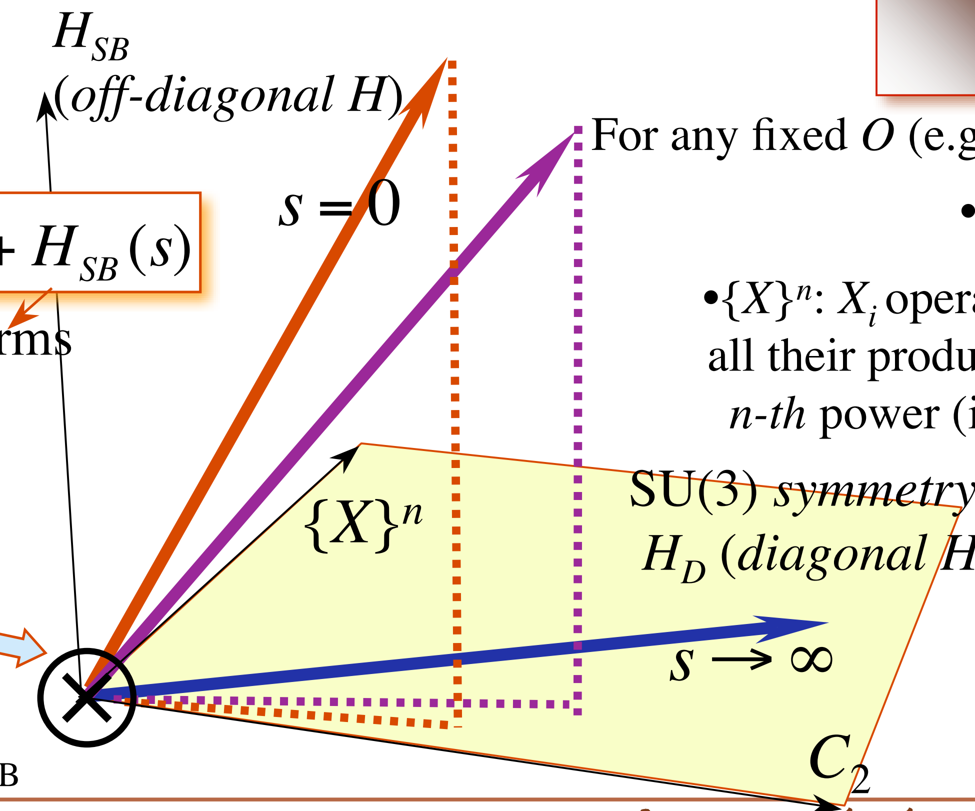
$$\boxed{H(s) = \alpha(s) C_2^{su(3)} + \beta(s)\{X\}^n + H_{SB}(s)}$$

Symmetry breaking terms

$$\boxed{H(s) = H_D(s) + H_{SB}(s)}$$

$$\boxed{\eta = \left[C_2^{su(3)}, H_s\right]}$$

Perpendicular to C_2,$\{X\}^n$,H_{SB}

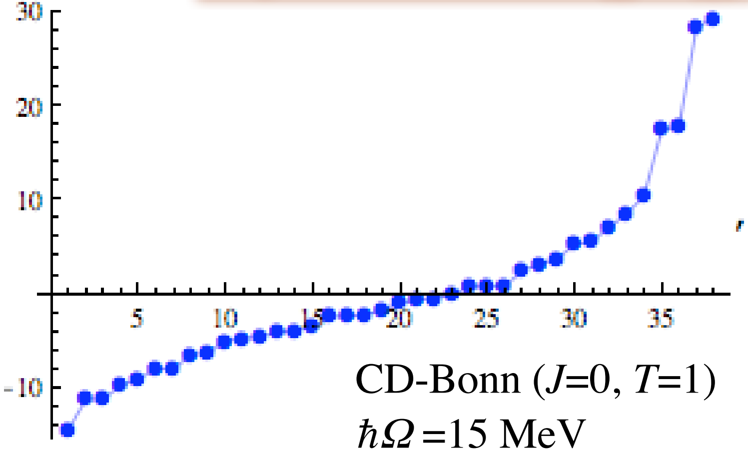


For any fixed O (e.g, O=C_2):

- $[X_i,C]=0$
- $\{X\}^n$: X_i operators and all their products up to *n-th* power (incl. 2-b)

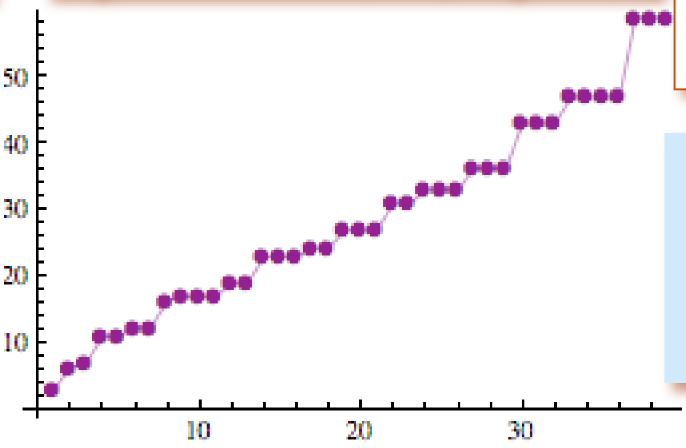
"Minimizing" the Many-body forces

Eigenvalues of $H=T_{\mathrm{rel}}+V_{NN}$



CD-Bonn (J=0, T=1)
$\hbar\Omega$ =15 MeV

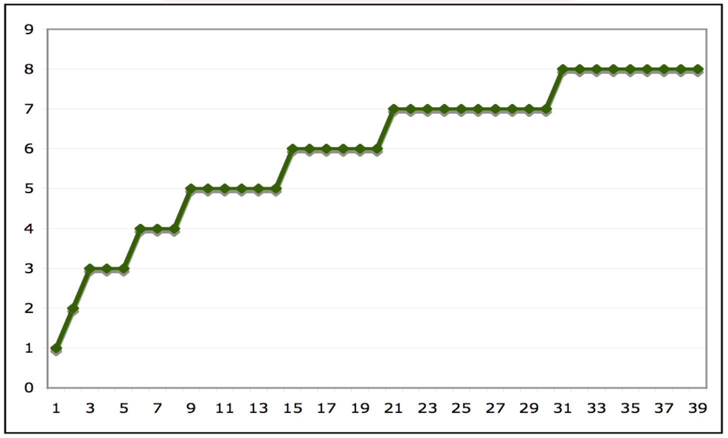
Eigenvalues of $C^{(2)}$ (diag. MEs)



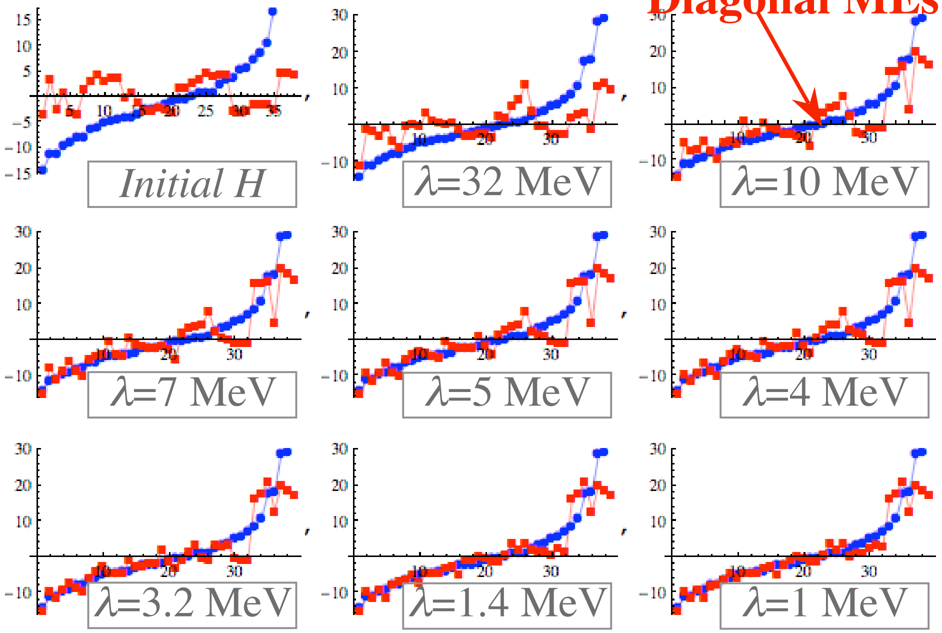
$$6C^{(2)}_{su(3)} = \underbrace{Q.Q + 3L^2}_{\textit{single shell}}$$

Similar energy spectra imply close correlation

For comparison:
HO Hamiltonian



Diagonal MEs of *H*



Eigenvalues of various NN and for different JT: close similarity to C_2

AV18 $\hbar\Omega=18$ MeV

N^{3}LO $\hbar\Omega=11$ MeV

JISP16 $\hbar\Omega=11$ MeV

CD-Bonn $\hbar\Omega=15$ MeV

$C^{(2)}$

Eigenval. of Trel

J=0, T=0

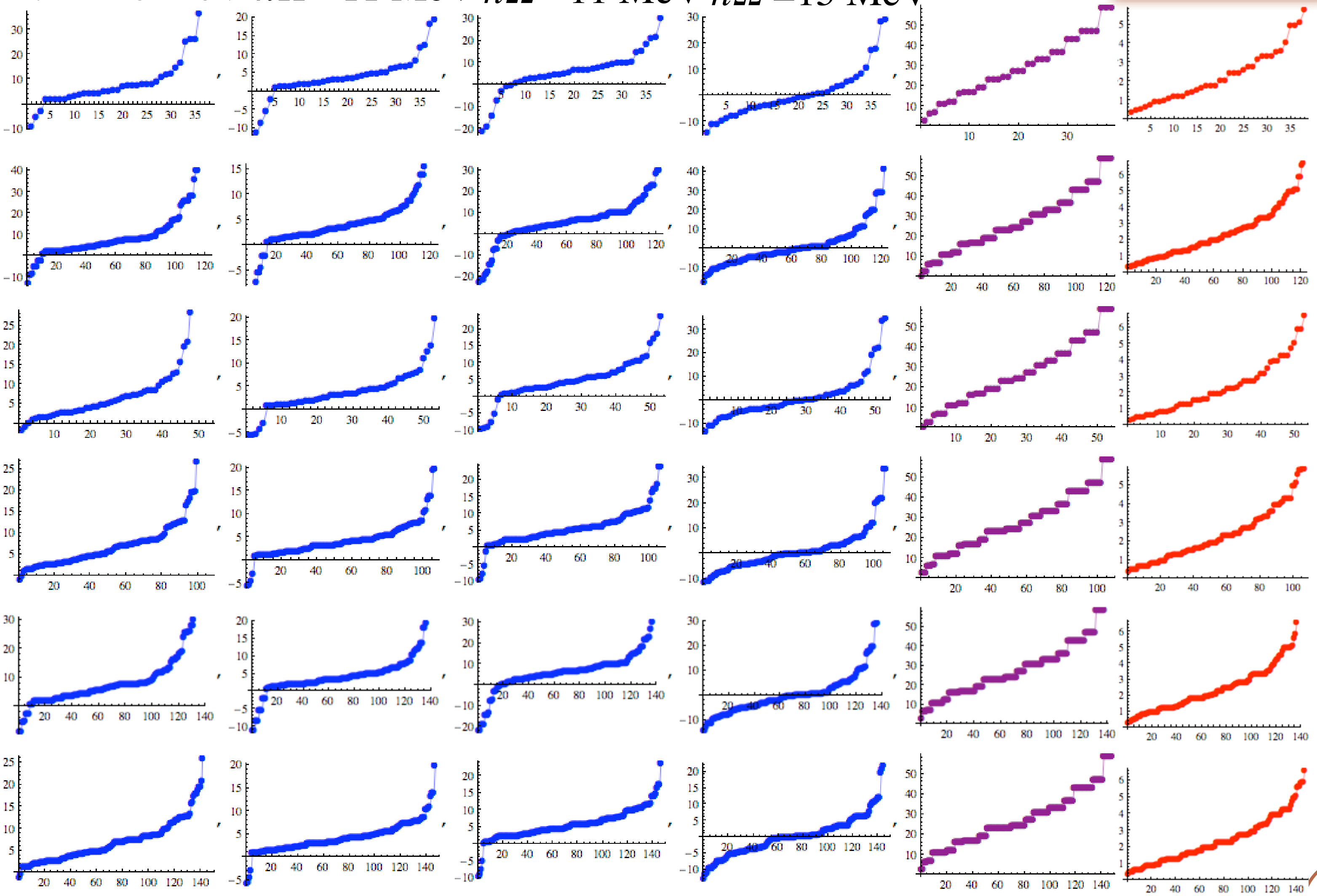
J=1, T=0

J=0, T=1

J=1, T=1

J=2, T=0

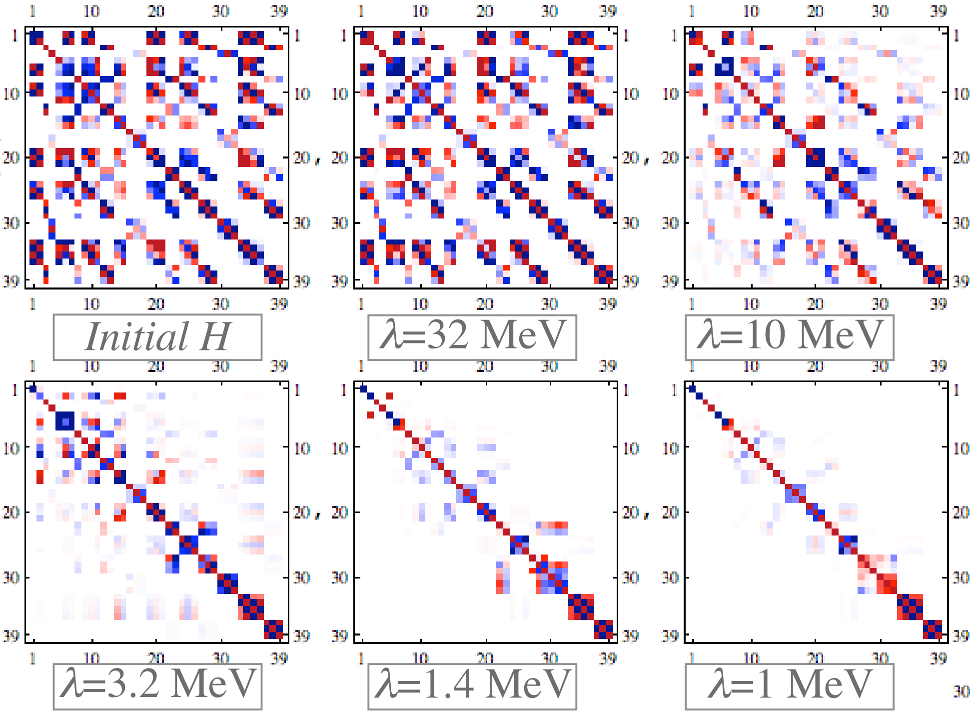
J=2, T=1



SRG with C_2 and HO

$$6C_2^{su(3)} = \underbrace{Q.Q + 3L^2}_{single\ shell}$$

HO Hamiltonian



Initial H

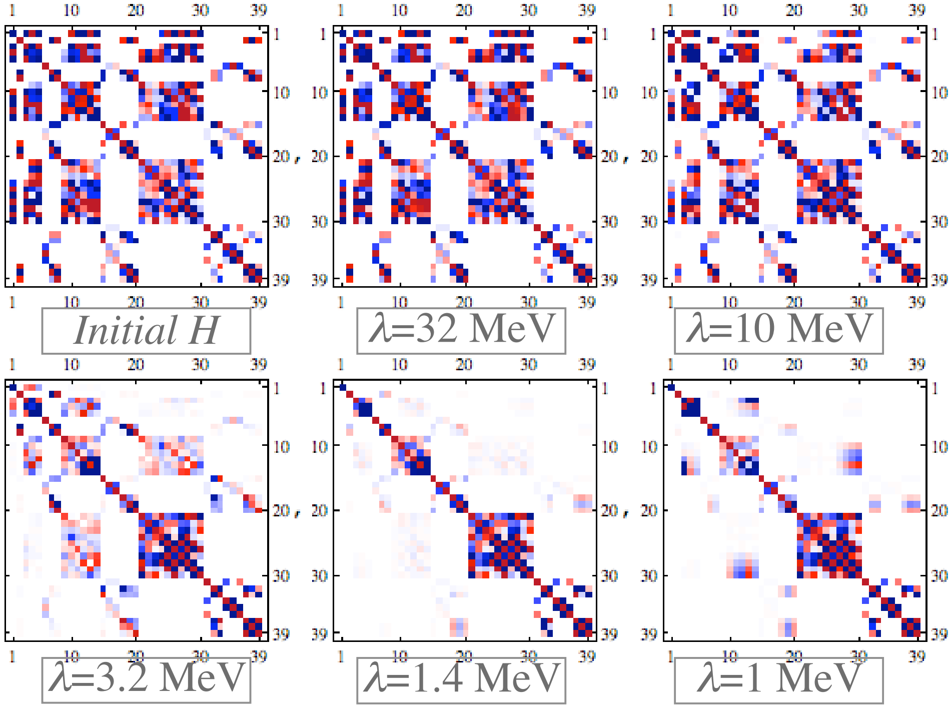
λ=32 MeV

λ=10 MeV

λ=3.2 MeV

λ=1.4 MeV

λ=1 MeV



Initial H

λ=32 MeV

λ=10 MeV

λ=3.2 MeV

λ=1.4 MeV

λ=1 MeV

N^3LO

$\hbar\Omega$ =11 MeV

J=0, T=0

Detecting the Many-body Forces: Integrity Basis

$$\left[X_i, C_2^{su(3)}\right] = 0$$

$$H_D(s) = \alpha(s) C_{su(3)}^{(2)} + \beta(s)\{X\}^n$$

$$\{X\}^n : 1, X_i, X_i^2, \ldots, X_i^n,$$

$$X_i X_j, \ldots, X_i^p X_j^{n-p}, \ldots, (X_i^p X_j^q \ldots X_k^r)$$

- Linearly dependent
- Expressed in terms of a set of SU(3) preserving operators

many-body interactions	integrity basis dimension (S=0)
0	1
1	1
2	3
3	5
4	9
5	13
6	22

Integrity Basis, S=0

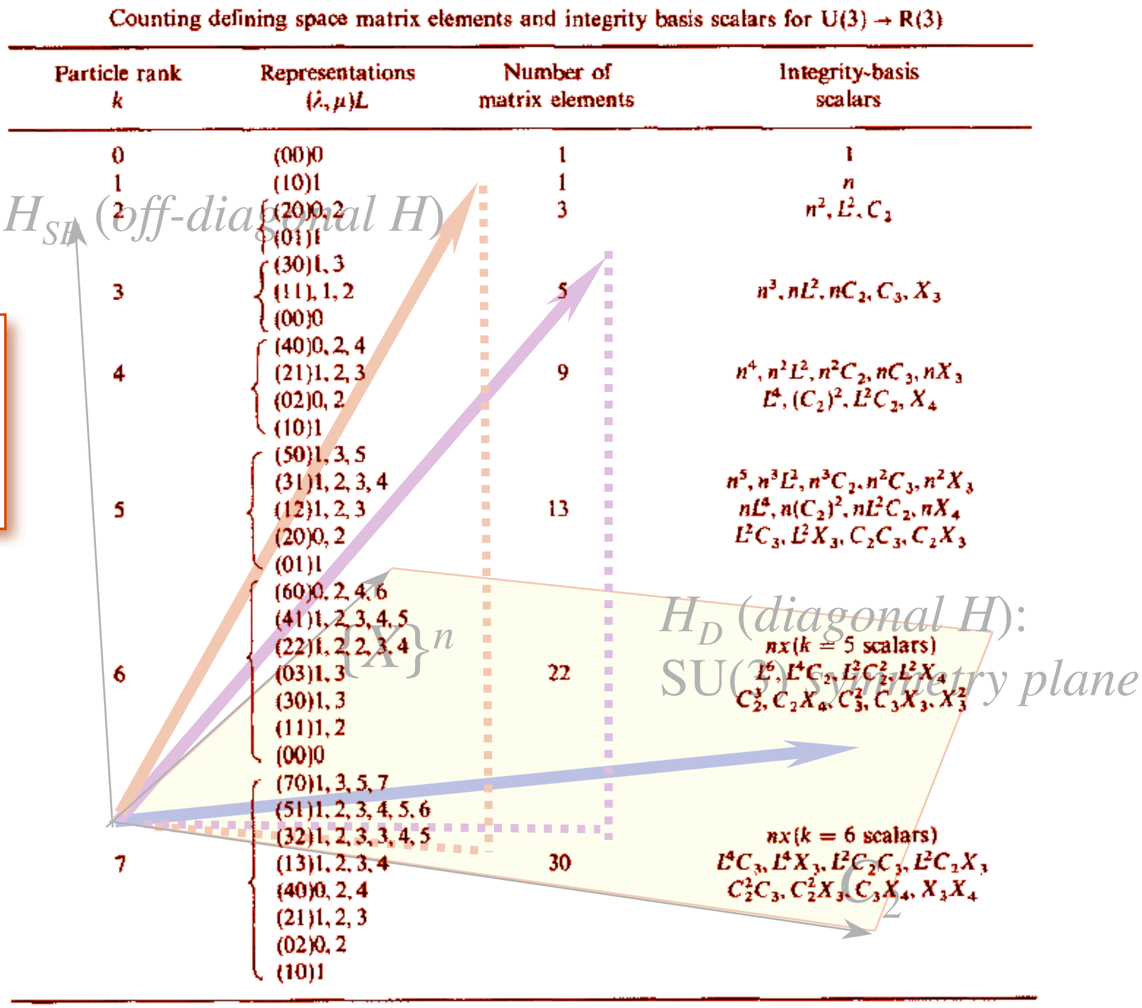


TABLE I

Counting defining space matrix elements and integrity basis scalars for U(3) → R(3)

Particle rank k	Representations $(\lambda, \mu)L$	Number of matrix elements	Integrity-basis scalars
0	(00)0	1	1
1	(10)1	1	n
2	(20)0, 2 (01)1	3	n^2, L^2, C_2
3	(30)1, 3 (11), 1, 2 (00)0	5	$n^3, nL^2, nC_2, C_3, X_3$
4	(40)0, 2, 4 (21)1, 2, 3 (02)0, 2 (10)1	9	$n^4, n^2L^2, n^2C_2, nC_3, nX_3$ $L^4, (C_2)^2, L^2C_2, X_4$
5	(50)1, 3, 5 (31)1, 2, 3, 4 (12)1, 2, 3 (20)0, 2 (01)1	13	$n^5, n^3L^2, n^3C_2, n^2C_3, n^2X_3$ $nL^4, n(C_2)^2, nL^2C_2, nX_4$ $L^2C_3, L^2X_3, C_2C_3, C_2X_3$
6	(60)0, 2, 4, 6 (41)1, 2, 3, 4, 5 (22)1, 2, 2, 3, 4 (03)1, 3 (30)1, 3 (11)1, 2 (00)0	22	$nx(k = 5$ scalars) $L^6, L^4C_2, L^2C_2^2, L^2X_4$ $C_2^3, C_2X_4, C_3^2, C_3X_3, X_3^2$
7	(70)1, 3, 5, 7 (51)1, 2, 3, 4, 5, 6 (32)1, 2, 3, 3, 4, 5 (13)1, 2, 3, 4 (40)0, 2, 4 (21)1, 2, 3 (02)0, 2 (10)1	30	$nx(k = 6$ scalars) $L^4C_3, L^4X_3, L^2C_2C_3, L^2C_2X_3$ $C_2^2C_3, C_2^2X_3, C_3X_4, X_3X_4$

J.P. Draayer and G. Rosensteel, Nucl. Phys. A 439 (1985) 61

Detecting the Many-body Forces: Integrity Basis

$$\left[X_i, C_2^{su(3)}\right] = 0$$

$$H_D(s) = \alpha(s) C^{(2)}_{su(3)} + \beta(s)\{X\}^n$$

$$\{X\}^n : 1, X_i, X_i^2, \ldots, X_i^n,$$

$$X_i X_j, \ldots, X_i^p X_j^{n-p}, \ldots, (X_i^p X_j^q \ldots X_k^r)$$

- Linearly dependent
- Expressed in terms of a set of SU(3) preserving operators

many-body interactions	integrity basis dimension (S=0)
0	1
1	1
2	3
3	5
4	9
5	13
6	22

SRG-induced 2*N*+3*N*+4*N* (*S*=0):

19 SU(3) preserving operators
[analytic MEs]

SRG-induced dominating many-body forces can be tracked during the evolution

Managing the SRG-Induced Many-body Forces: SU(3)-coupled SRG Evolution

$$\boxed{\frac{dH_s}{ds} = \left[\left[C_2^{su(3)}, H_s\right], H_s\right]}$$

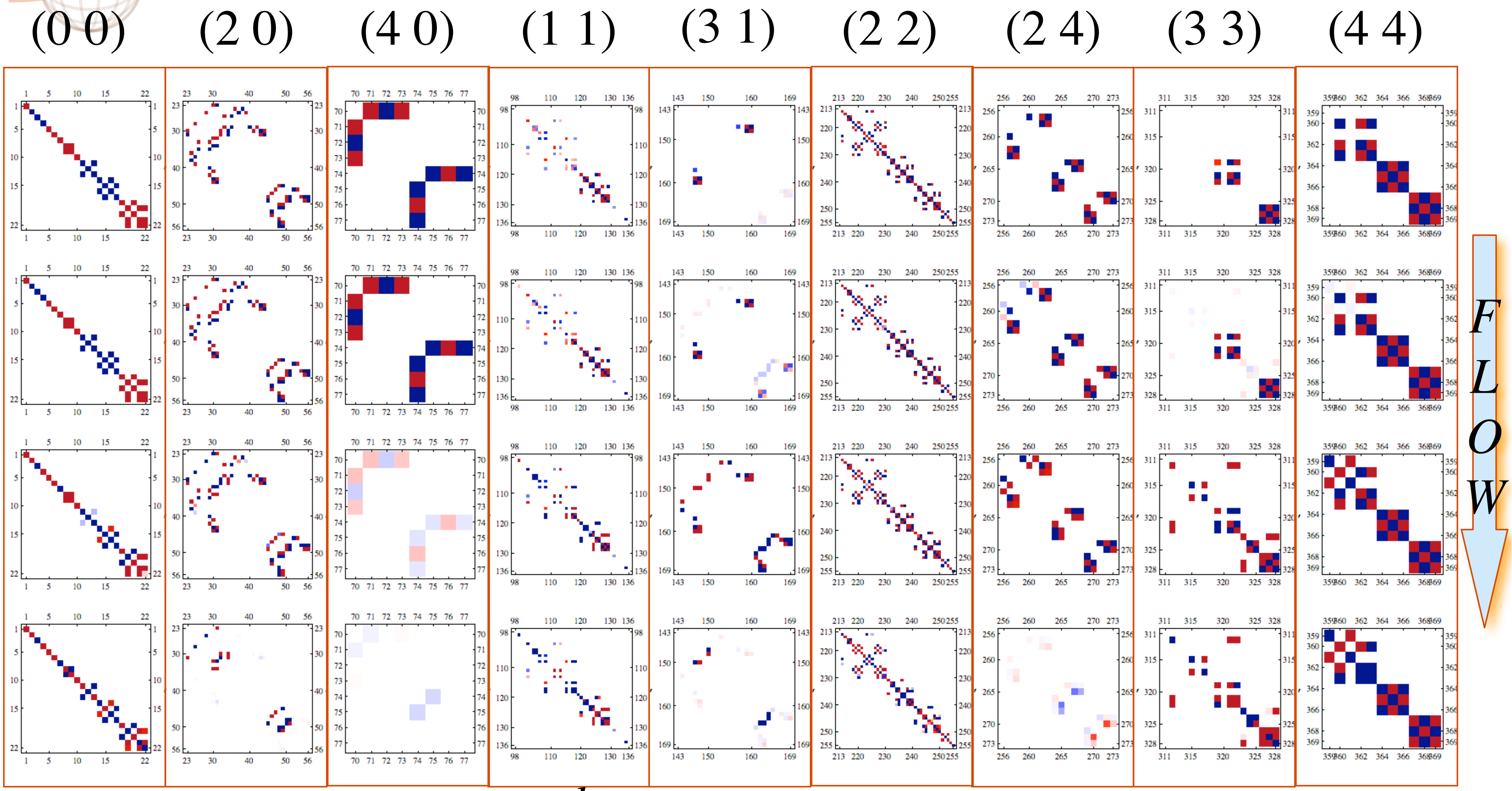
$$\frac{d}{ds} V^{\omega_0 S_0 T_0} = \left[[C_2^{(00)00}, V^{\omega_1 S_1 T_1}]^{\omega_1 S_1 T_1}, V^{\omega_2 S_2 T_2} \right]^{\omega_0 S_0 T_0} \qquad \boxed{\omega = (\lambda \mu)}$$

$$\left\langle n_1 n_2 \ldots n_A (\lambda \mu) S T \left\| V^{\omega_0 S_0 T_0} \right\| n_1' n_2' \ldots n_A' (\lambda' \mu') S' T' \right\rangle$$

$$\Downarrow$$

$$\left(V^{\omega_1 S_1 T_1} \right) \times \left(V^{\omega_2 S_2 T_2} \right)$$

SU(3)-coupled SRG



4 shells

CD-Bonn

$\hbar\Omega = 15$ MeV

$$\frac{d}{ds}V^{\omega_0 S_0 T_0} = [[C_2^{(00)00}, V^{\omega_1 S_1 T_1}]^{\omega_1 S_1 T_1}, V^{\omega_2 S_2 T_2}]^{\omega_0 S_0 T_0}$$

Managing the SRG-Induced Many-body Forces: SU(3)-coupled SRG Evolution

$$\boxed{\frac{dH_s}{ds} = \left[\left[C_2^{su(3)}, H_s\right], H_s\right]}$$

$$\frac{d}{ds} V^{\omega_0 S_0 T_0} = \left[[C_2^{(00)00}, V^{\omega_1 S_1 T_1}]^{\omega_1 S_1 T_1}, V^{\omega_2 S_2 T_2} \right]^{\omega_0 S_0 T_0} \qquad \boxed{\omega = (\lambda \mu)}$$

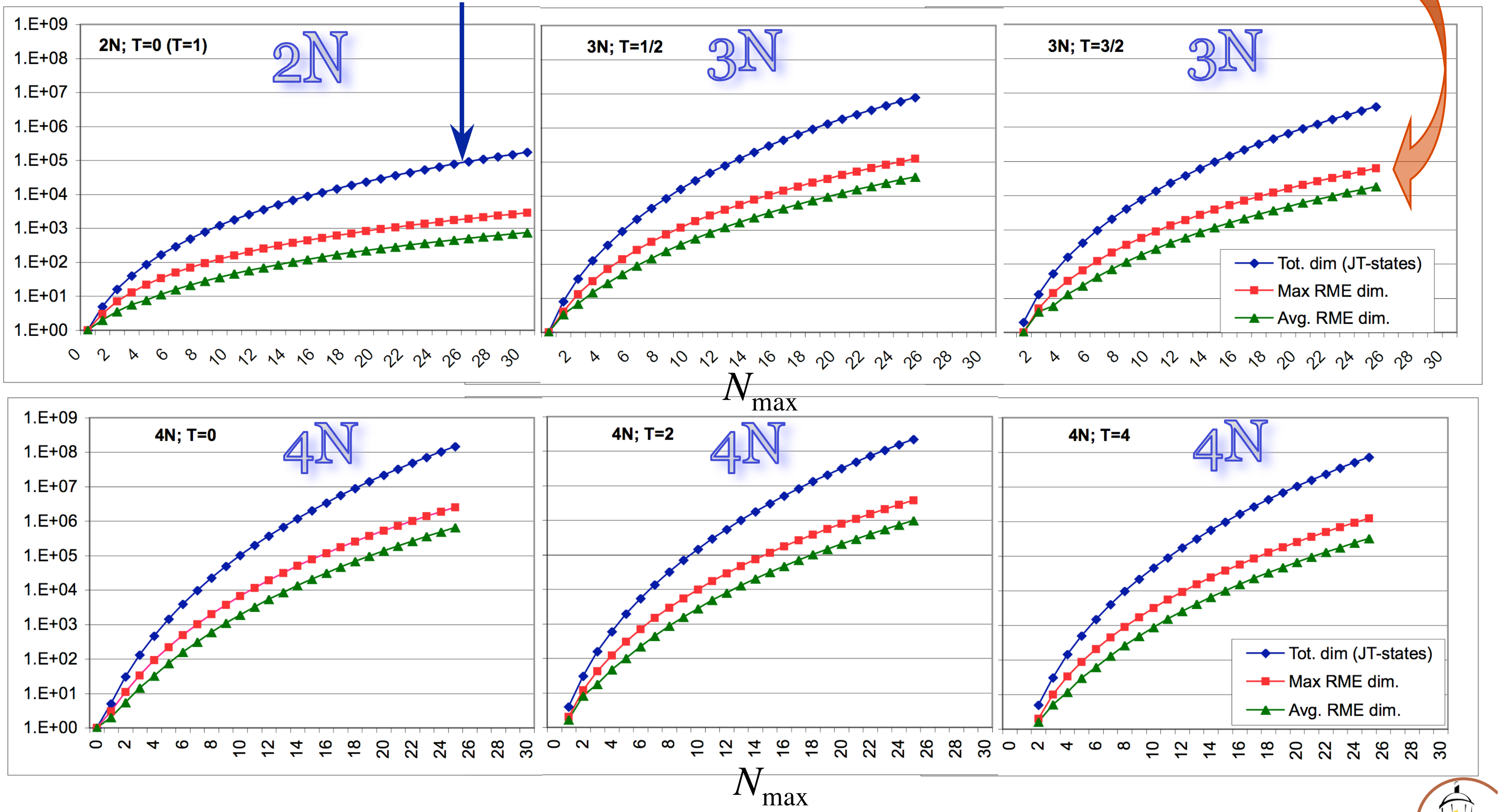
$$\left\langle n_1 n_2 \ldots n_A (\lambda \mu) ST \left\| V^{\omega_0 S_0 T_0} \right\| n_1' n_2' \ldots n_A' (\lambda' \mu') S'T' \right\rangle$$

↓

What are the dimensions of such matrices?

$$\begin{pmatrix} \\ V^{\omega_1 S_1 T_1} \\ \\ \end{pmatrix} \times \begin{pmatrix} \\ V^{\omega_2 S_2 T_2} \\ \\ \end{pmatrix}$$

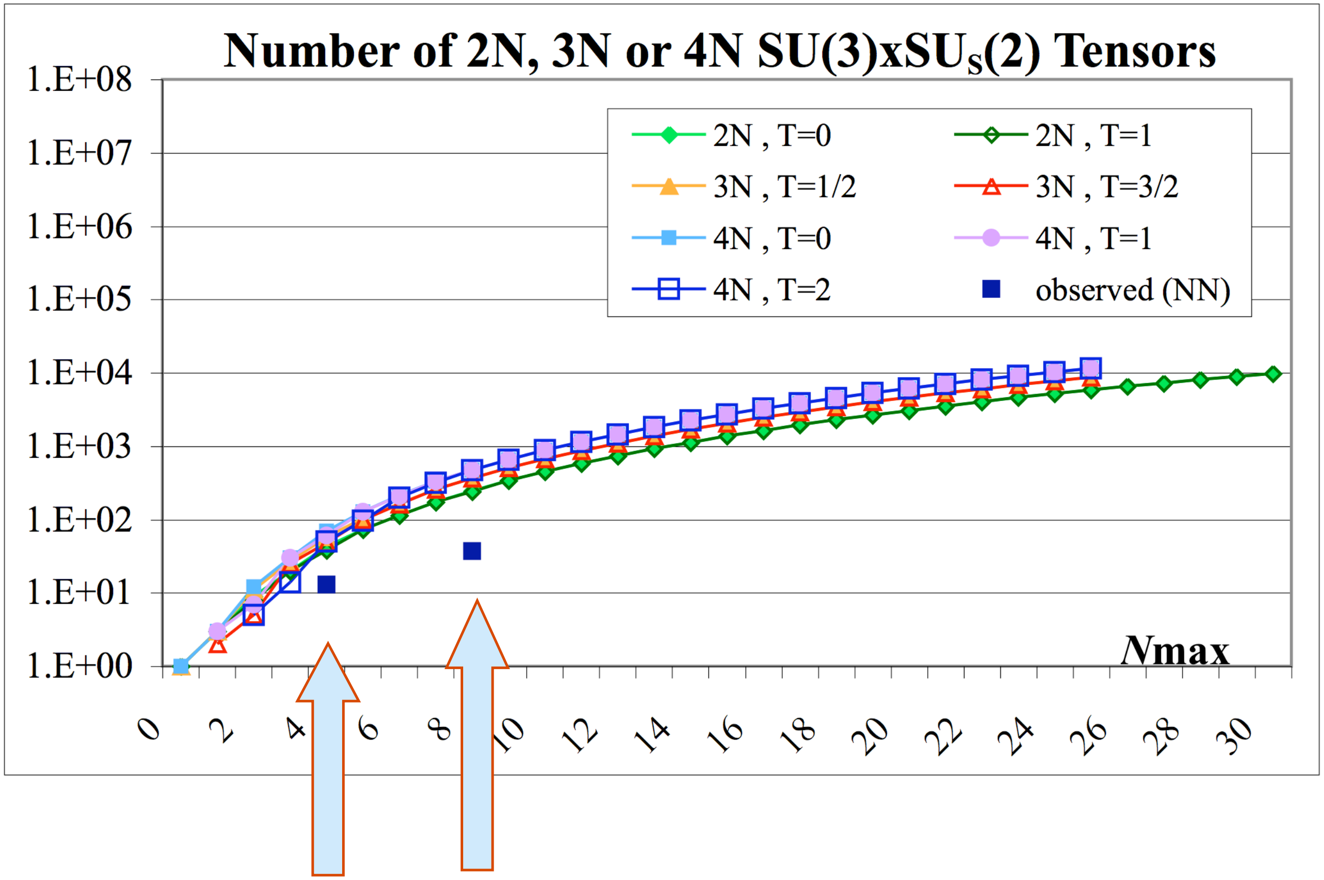
SU(3)-coupled SRG: Dimensions


Dimensions of matrices in JT-coupled basis

Dimensions of matrices in SU(3)x*S basis* (RME)

Number of $SU(3)xSU_S(2)$ 2N, 3N, 4N Tensors

$$V^{\omega_1 S_1 T_1}$$



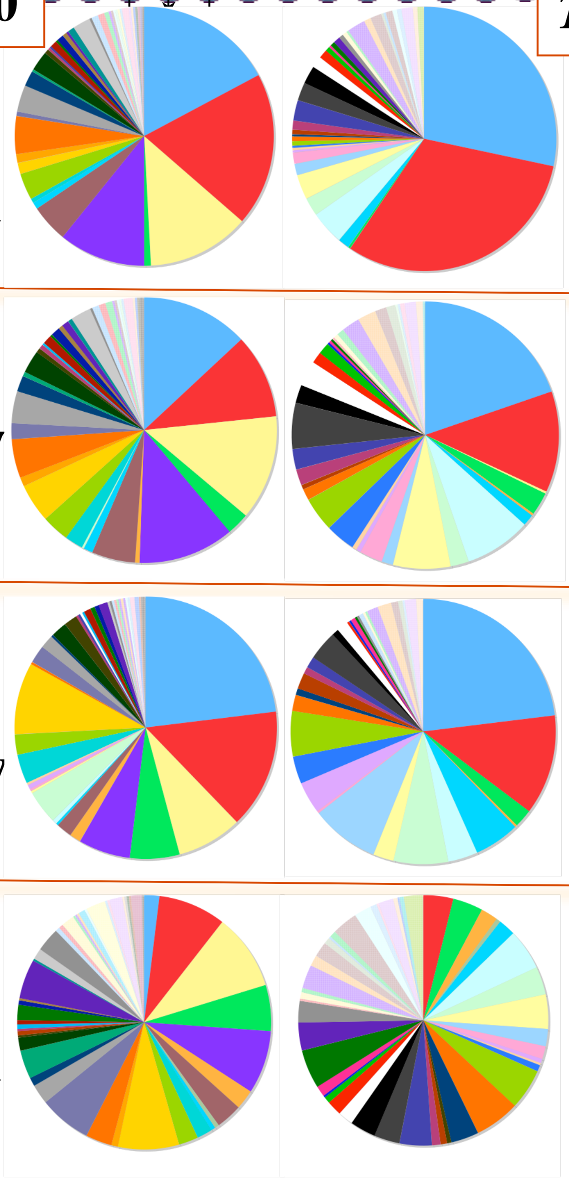
95% of *NN* interaction:

not all $SU(3)xSU_S(2)$ tensors appear to be significant in *NN*

Dominance of Some $SU(3)\times SU_S(2)$ Modes: SU(3) diagonal and Symplectic-like modes

8 shells

NN



***T*=0** ***T*=1**

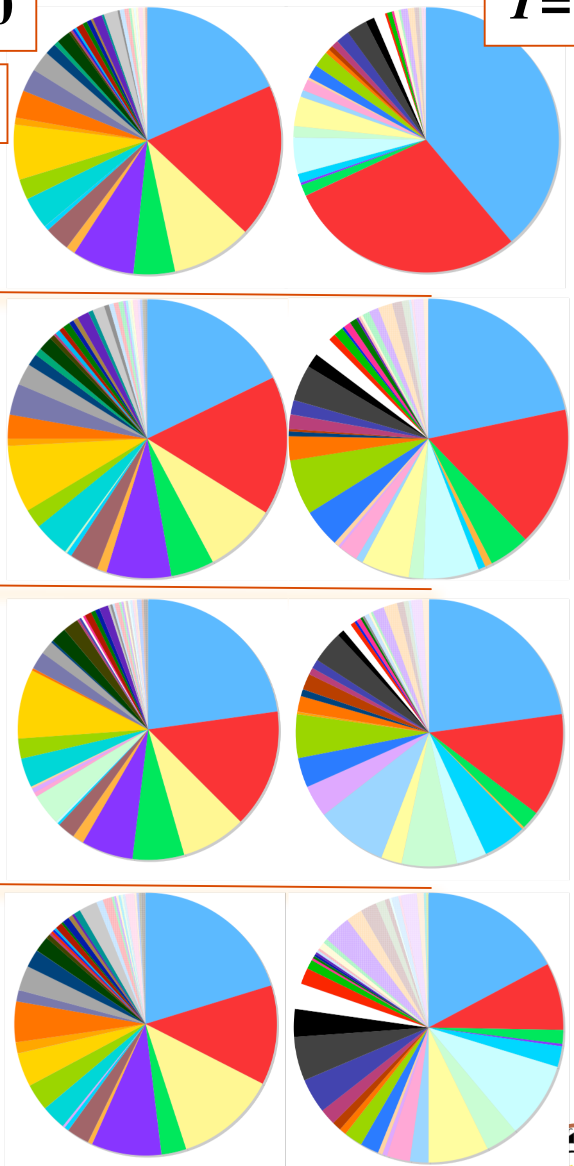
CD-Bonn
$\hbar\Omega$ =15MeV

N^3*LO*
$\hbar\Omega$ =11MeV

JISP16
$\hbar\Omega$ =15MeV

AV18
$\hbar\Omega$ =18MeV

Renormalized



***T*=0** ***T*=1**

Lee-Suzuki
$6\hbar\Omega$; C-12

$V_{\text{low-k}}$

Lee-Suzuki
$6\hbar\Omega$; O-16

UCOM

Summary

- Symmetries are found essential in interactions/many-body systems
- We suggest use of SU(3) basis together with the second-order SU(3) invariant as the evolution operator for the SRG approach
- SRG in SU(3) basis appears to be a very effective scheme for renormalization of the *NN* interaction: C_2 is a good choice-- physically relevant/ weak many-body forces expected
- SU(3)-coupled SRG: possible to be applied in 3-b and 4-b SU(3)-coupled basis and track many-body forces (integrity basis)
- Particularly suitable for the Sp-NCSM