EFT for the (NC)SM practitioner

applications to trapped atoms

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Effective Field Theories and the Many-Body Problem, INT-09-1 (2009)

Outline

- Motivation
- Few- and Many-Body Method: NCSM
- Formulation of EFT in a HO Basis:
  - Renormalization of the two-body interactions up to N^2LO
  - Comparison with other methods
  - Extension to include range
  - Three-body problem
- Summary and Outlook

Motivation

The nuclear physics problem:

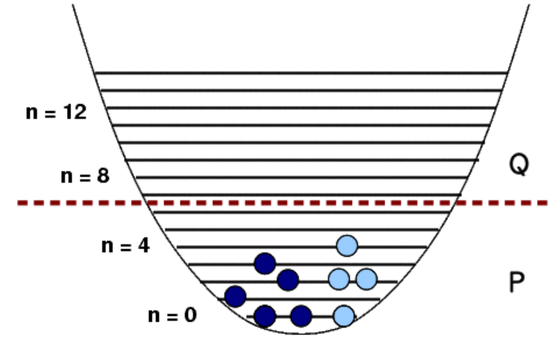
- connection to QCD
- all the current ab initio few- and many-body methods have limitations
- need for reliable methods to extrapolate outside the valley of stability

For the NCSM:

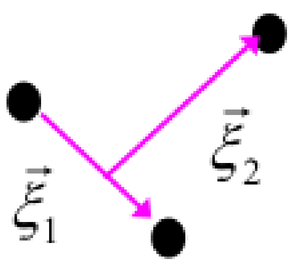
- *different types of interaction (motivates the cluster approximation)*
- *can mitigate the long- and short-range degrees of freedom (better description of long-range observables)*

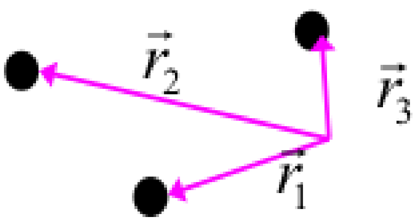
For The Greater Good

NCSM



- all particles are allowed to interact
- energy truncation in a HO basis (P&Q spaces)
- effective interaction constructed via a unitary transformation (energy independent, hermitian)
- "cluster approximation"
- short-range effects accounted by the effective interaction
- long-range and many-body effects accounted by increasing the model space
- *quite successful in describing low-energy properties of light nuclei*





$$\psi\left(\vec{\xi}_1,\vec{\xi}_2\right) = \mathrm{A}\left[\phi_{nlj}\left(\vec{\xi}_1\right)\phi_{n'l'j'}\left(\vec{\xi}_2\right)\right]_{JI}$$

$$\psi\left(\vec{r}_1,\vec{r}_2,\vec{r}_3\right) = \begin{vmatrix} \phi_{n_1l_1j_1}\left(\vec{r}_1\right) & \phi_{n_2l_2j_2}\left(\vec{r}_1\right) & \phi_{n_3l_3j_3}\left(\vec{r}_1\right) \\ \phi_{n_1l_1j_1}\left(\vec{r}_2\right) & \phi_{n_2l_2j_2}\left(\vec{r}_2\right) & \phi_{n_3l_3j_3}\left(\vec{r}_2\right) \\ \phi_{n_1l_1j_1}\left(\vec{r}_3\right) & \phi_{n_2l_2j_2}\left(\vec{r}_3\right) & \phi_{n_3l_3j_3}\left(\vec{r}_3\right) \end{vmatrix}$$

$$\boxed{2n + l + 2n' + l' \leq N_{max}}$$

$$\boxed{2n_1 + l_1 + 2n_2 + l_2 + 2n_3 + l_3 \leq N_{max}}$$

The nuclear many-body problem

$$H_{int} = \frac{1}{A}\sum_{i>j=1}^{A}\frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \ldots$$

$$H = H_{int} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2}mA\omega^2\vec{R}_{CM}^2$$

Lipkin 1958

$$= \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + \sum_{i<j=1}^{A}\left(V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right) + \sum_{i<j<k=1}^{A} V_{ijk} + \ldots$$

$$H_A = \sum_{i=1}^{A}\left(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 r_i^2\right) + C_0\sum_{i<j=1}^{A}\delta^{(3)}(\vec{r}_i - \vec{r}_j)$$

trapped fermions

$$a_0(^1S_0) \approx -20 \text{ fm}$$

$$r_0 \approx 2 \text{ fm}$$

$$a_0(^3S_1) \approx 5 \text{ fm}$$

EFT

Original motivation: to understand the gross features of nuclear systems from a QCD perspective.

- separation of scales: if $\rho = kr_0 << 1$ then expand all the observables in powers of ρ.
- captures relevant degrees of freedom (integrates out high momenta)
- long-distance physics included explicitly
- short-distance physics added as corrections in powers of relevant scales present in the problem (e.g., for NN interaction at low momentum, r_0/a_0)
- general application to other systems (nucleon-core interactions, clustering effects)

EFT approach:

- identify relevant degrees of freedom
- identify symmetries
- write the most general Lagrangian (infinite number of terms)
- organize the interaction (power counting)
- results are *improvable* order by order and *model independent*.

Interaction renormalization

from addition of the CM term

Unitary transformation (Lee-Suzuki)
[Navratil et. al.]

Effective Field Theory in HO basis
[Stetcu et. al.]

$$h = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + C_0 V_{12}$$

$$h^{eff} = U h U^{\dagger}$$

(preserves lowest *D* eigenvalues of *h*)

truncation introduces many-body forces

- form of interaction preserved
- the coupling constants adjusted in each model space to reproduce some observables

truncation introduces higher-order terms

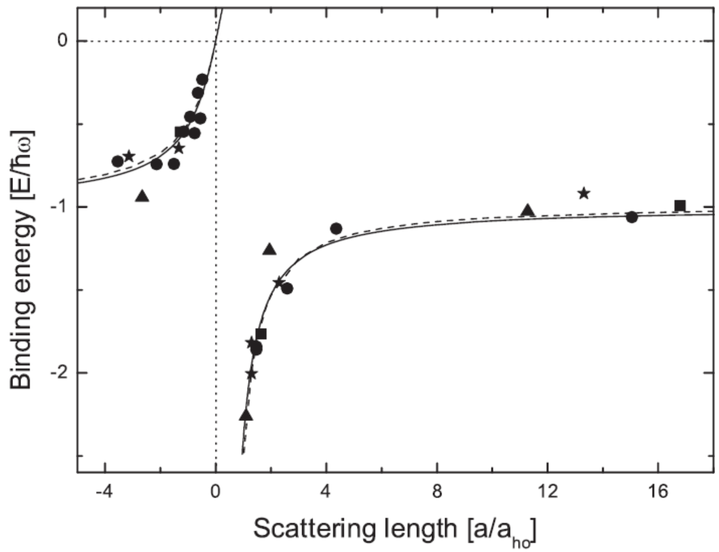
HERE WE COMBINE BOTH APPROACHES

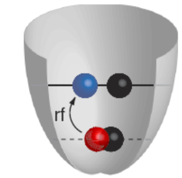
adjust C_0's to reproduce as many eigenvalues (but *no* unitary transformation)

preserve the form of the interaction (power counting)

Two-body problem in harmonic trap

T. Stöferle et. al., Phys. Rev. Lett. **96** (2006) 030401





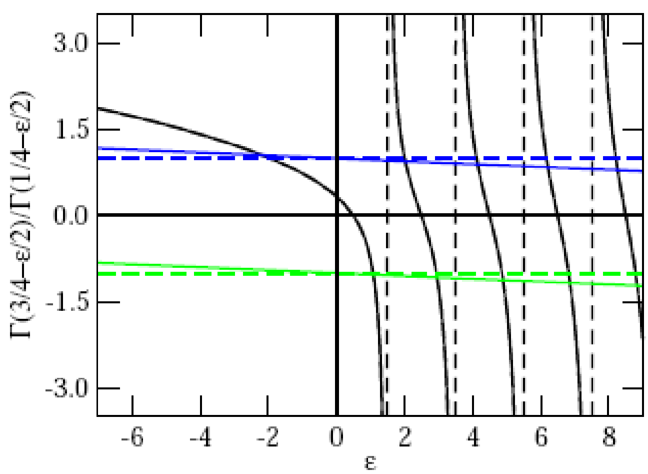
$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = \frac{b}{2a_2}$$

$$b = \frac{1}{\sqrt{\mu\omega}}$$

$$\varepsilon = E/\omega$$

T. Busch et. al., Found. Phys. **28** (1998) 549

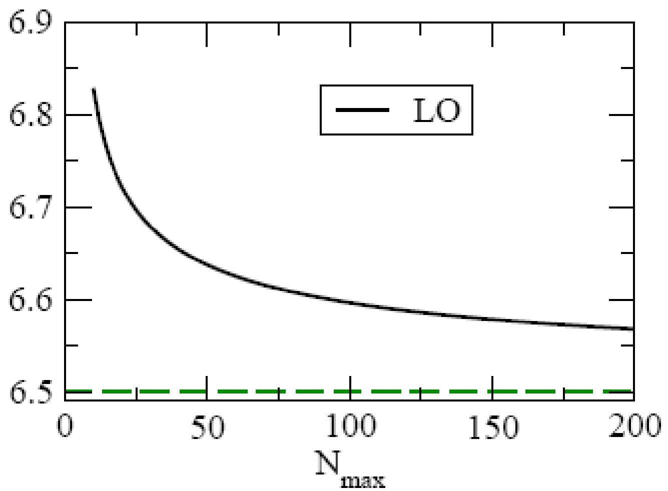
$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = -\frac{b}{2}\left(-\frac{1}{a_2} + r_0\varepsilon + \ldots\right)$$



In the following, our underlying theory is the pseudopotential

LO renormalization in a finite basis

$$V_{LO}(\vec{p},\vec{p}') = C_0$$



$$\psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \phi_n(\vec{r})$$

$$\left(\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + C_0(N_{max})\delta(\vec{r})\right)\psi(\vec{r}) = \varepsilon\omega\psi(\vec{r})$$

Eigenvalues:

$$\frac{1}{C_0(N_{max})} = -\sum_{n=0}^{N_{max}/2} \frac{|\phi_n(0)|^2}{2n+3/2-\varepsilon}$$

Fix C_0 to reproduce one observable, the others are predictions

$$\left(\frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2\right)\phi_n(\vec{r}) = (2n+3/2)\omega\phi_n(\vec{r})$$

Beyond LO

NLO:

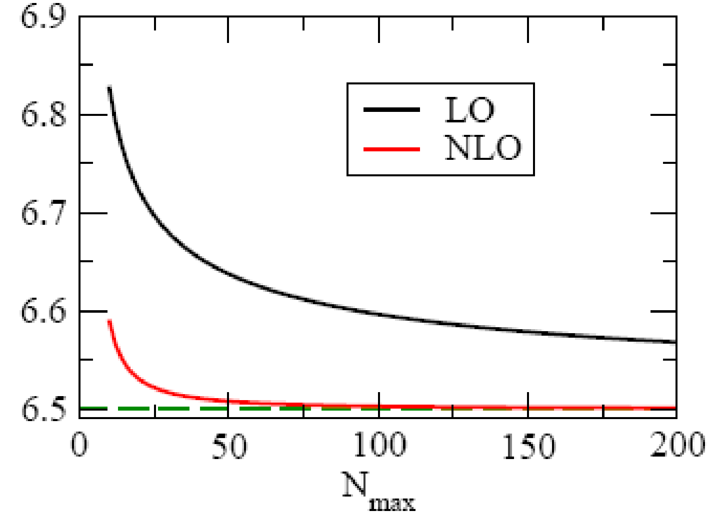
$$V_{NLO}(\vec{p},\vec{p}') = C_2(p^2 + p'^2)$$

$$\Delta\varepsilon_n = \langle\psi_n|V_{NLO}|\psi_n\rangle$$

$$\Delta\varepsilon_n = \langle\psi_n|C_2(p^2 + p'^2) + C_0^{(1)}|\psi_n\rangle$$

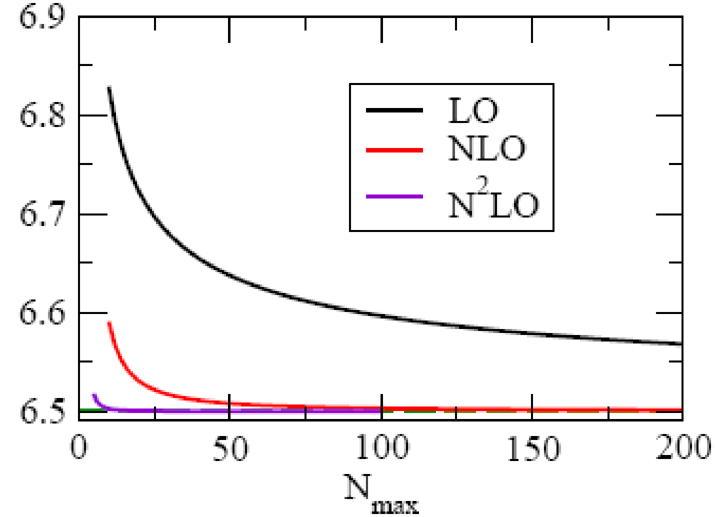
$$\Delta\varepsilon_0 = 0$$

$$\Delta\varepsilon_1 = \varepsilon_1^{exp} - \varepsilon_1^{LO}$$

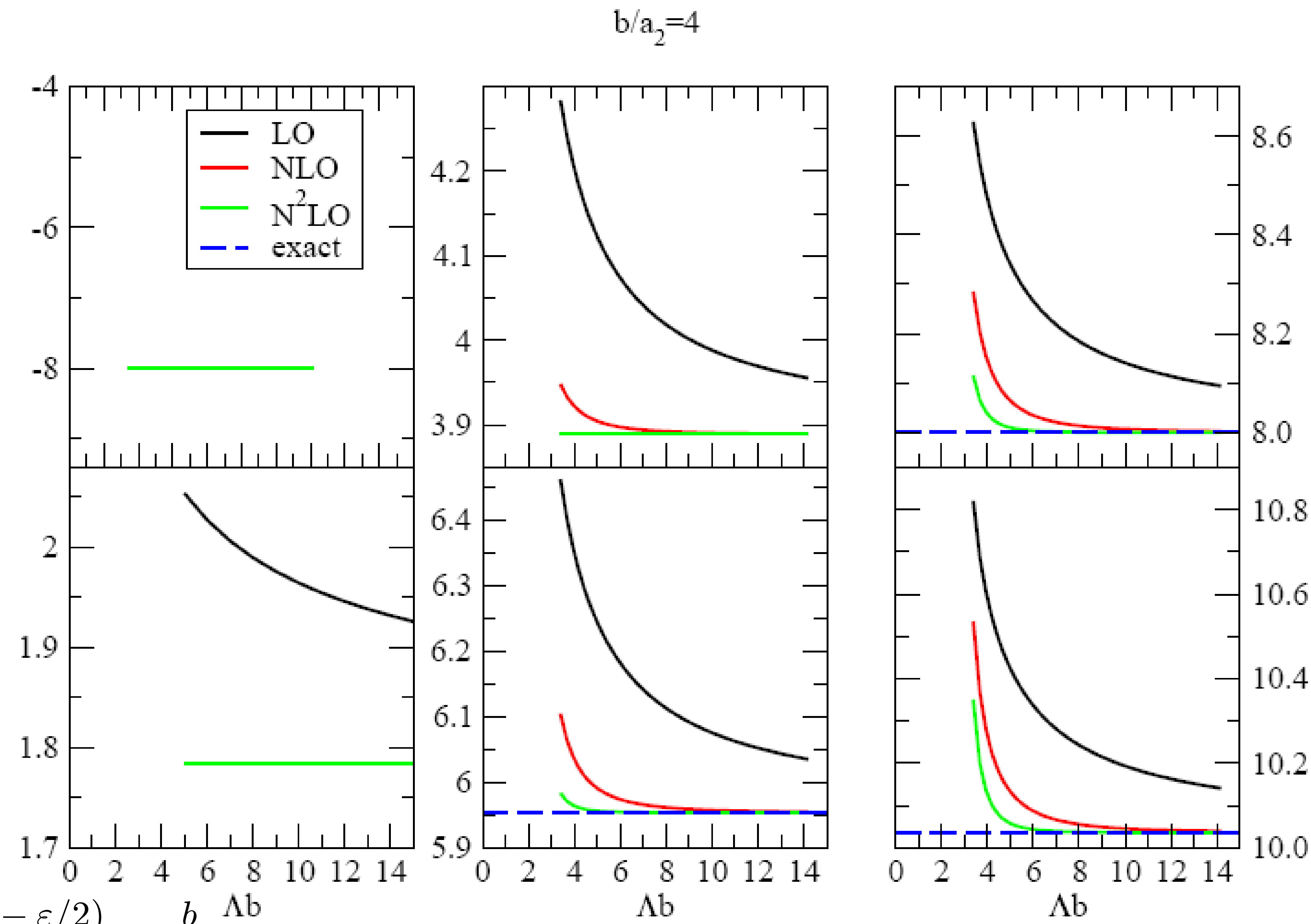


N2LO:

$$\Delta\varepsilon_n = \sum_{i\neq n}\frac{|\langle\psi_n|C_2(p^2 + p'^2) + C_0^{(1)}|\psi_i\rangle|^2}{\varepsilon_i - \varepsilon_n}$$
$$+\langle\psi_n|C_2^{(1)}(p^2 + p'^2) + C_0^{(2)}|\psi_n\rangle$$
$$+\langle\psi_n|C_4^{(1)}(p^2 + p'^2)^2|\psi_n\rangle$$

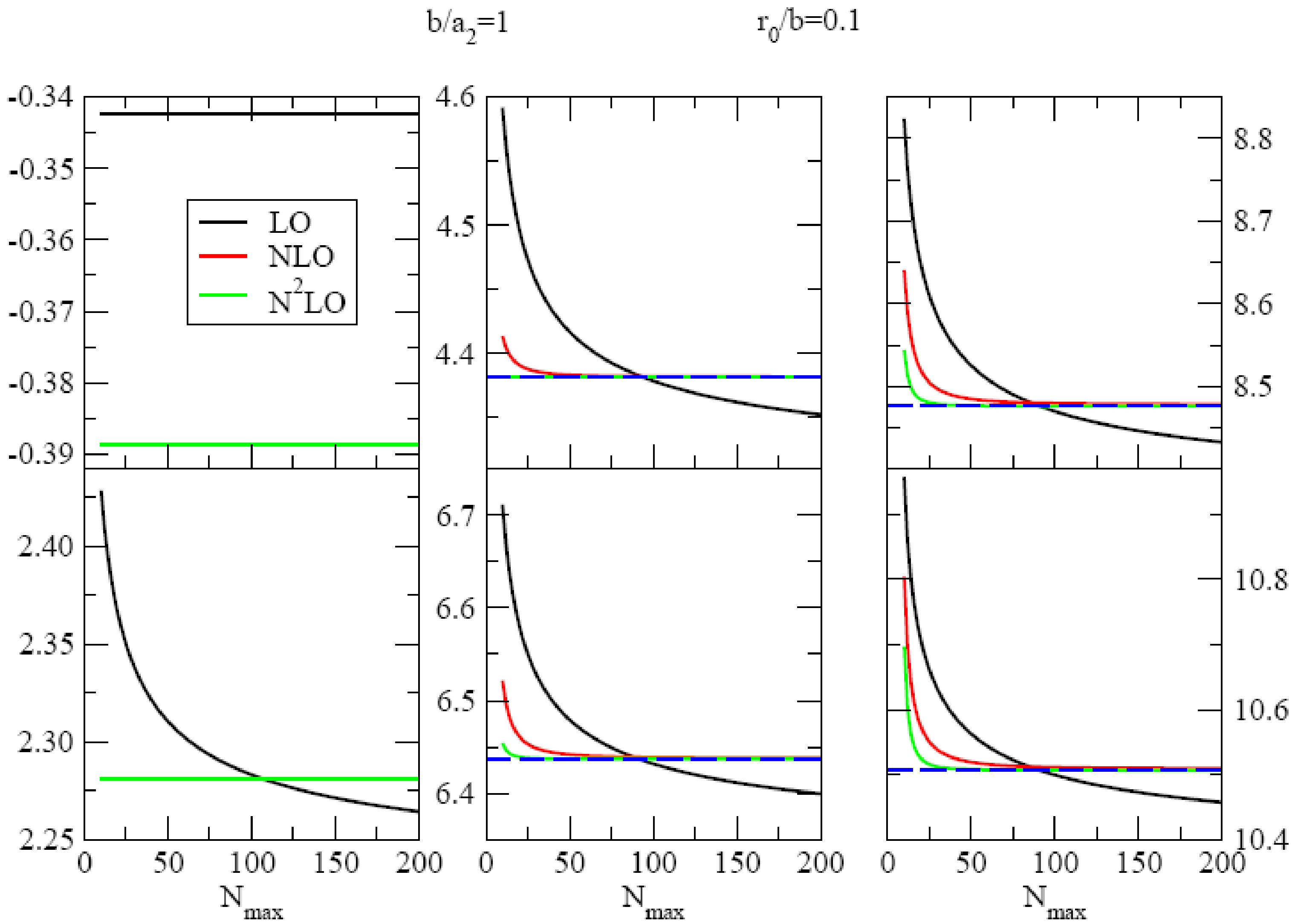


Running of two-body spectra: no range



$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = \frac{b}{2a_2}$$

Running of the two-body spectra w/ range



$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = \frac{b}{2a_2}$$

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)} = -\frac{b}{2}\left(-\frac{1}{a_2}+r_0\varepsilon\right)$$

Two-body problem: other approaches

Alhassid, Bertsch, Fang, PRL **100** (2008) 230401: separable interaction

$$V^q_{nn'} = -f^q_n \, f^q_{n'}$$

$$\sum_{n=0}^{q} \frac{f_n^2}{2n + 3/2 - \varepsilon_r} = 1, \quad r = 0, ..., q$$

M. Birse: fixed-point potential:

$$V(\varepsilon; \vec{r}) = C(\varepsilon, N_{max})\, \delta(\vec{r})$$

$$\frac{1}{C(\varepsilon, N_{max})} = \frac{1}{C_0(N_{max})} - \sum_{n=N_{max}/2+1}^{\infty} \left(\frac{1}{2n + 3/2 - \varepsilon_0} - \frac{1}{2n + 3/2 - \varepsilon} \right)$$

$$\psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \phi_n(\vec{r})$$

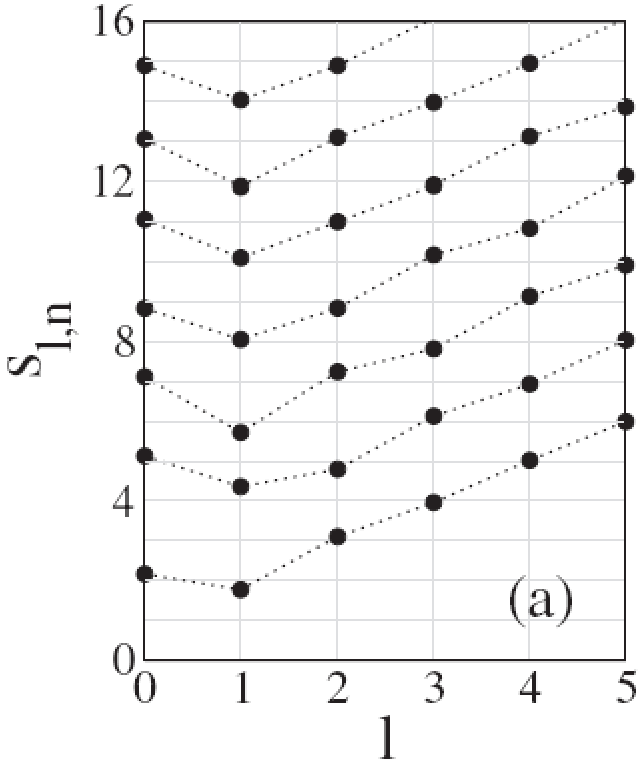
$$H\psi(\vec{r}) = \varepsilon \psi(\vec{r})$$

Three-body solution at unitarity

Solve the free Schrodinger Eq. w/ boundary condition:

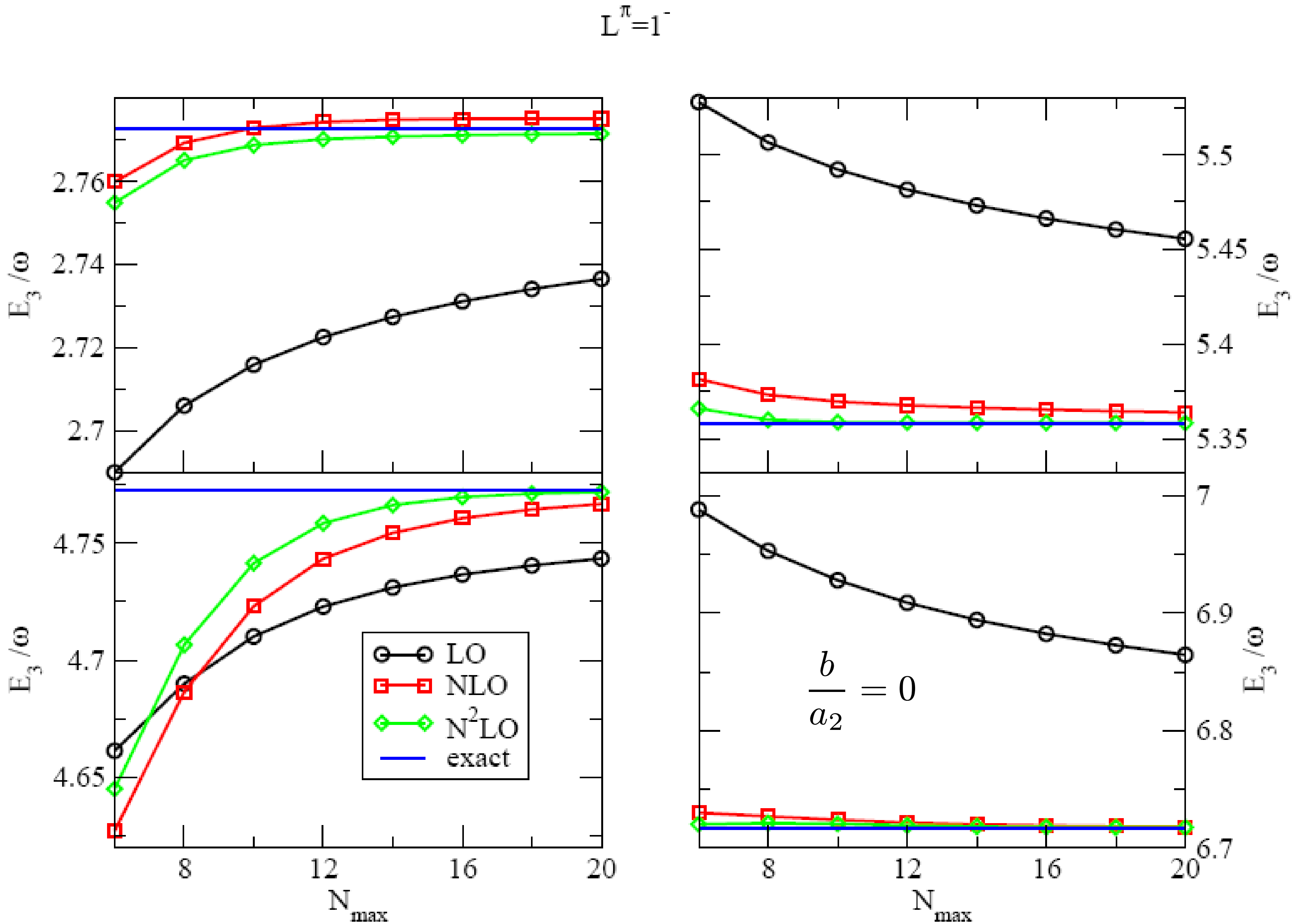
$$\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \left(\frac{1}{r_{ij}} - \frac{1}{a_2}\right) A(\vec{R}_{ij}, r_k) + \mathcal{O}(r_{ij})$$

$$E = E_{c.m.} + (s_{l,n} + 1 + 2q)\omega$$

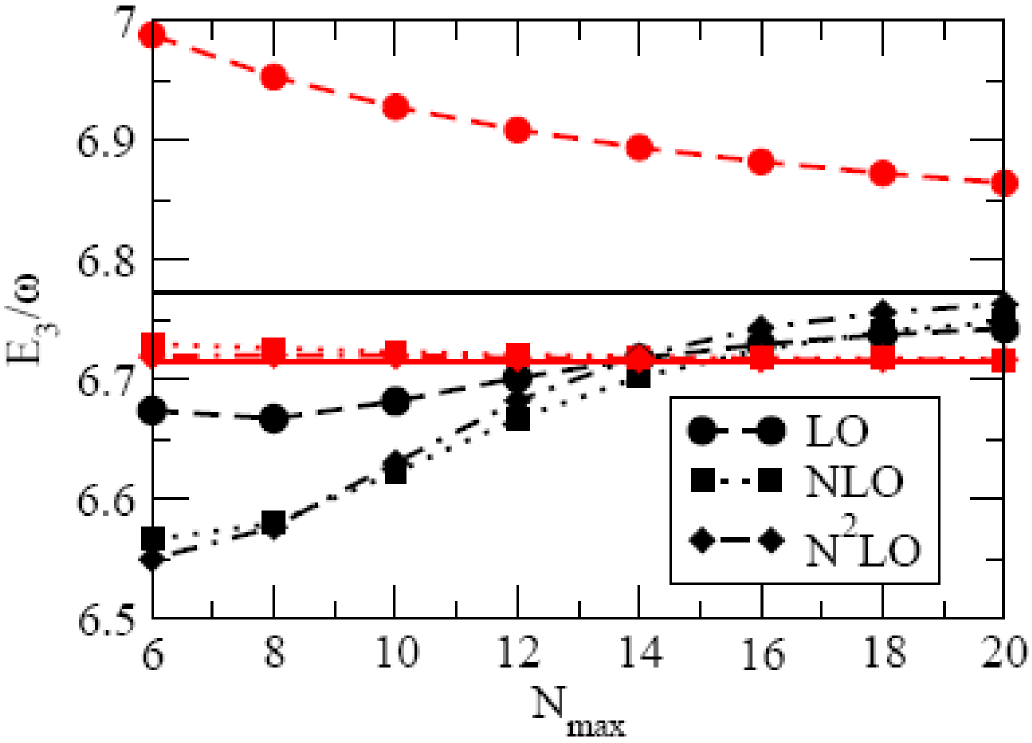


F. Werner and Y. Castin, Phys. Rev. Lett. **97**, 150401 (2006)

Three-body problem up to N^{2}LO

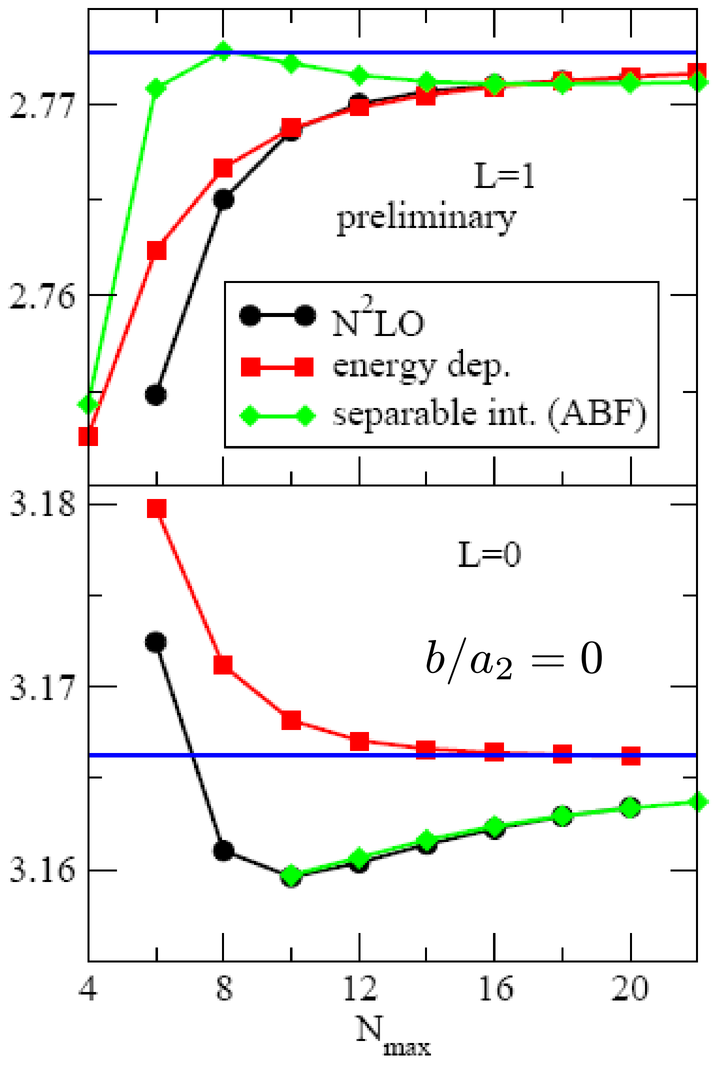


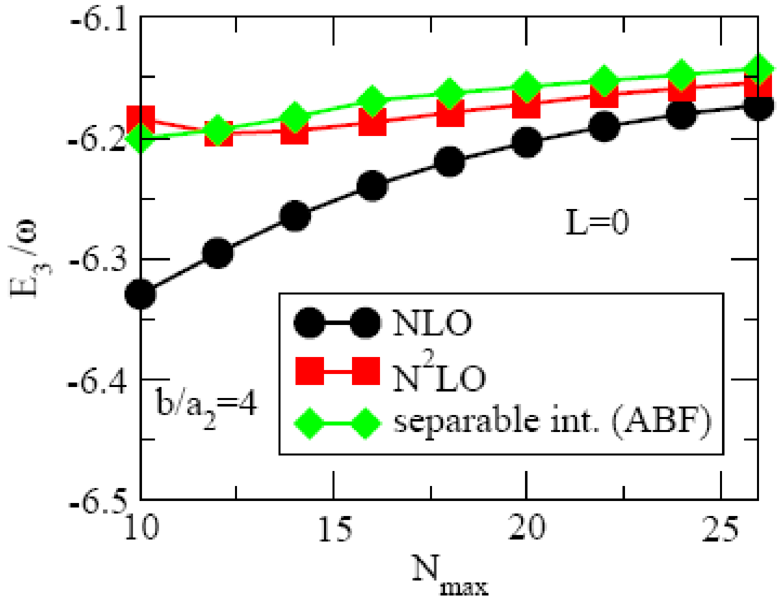
LO order inversion



LO: wrong ordering
NLO and N^2LO: restore the correct ordering

Comparison w/ other approaches

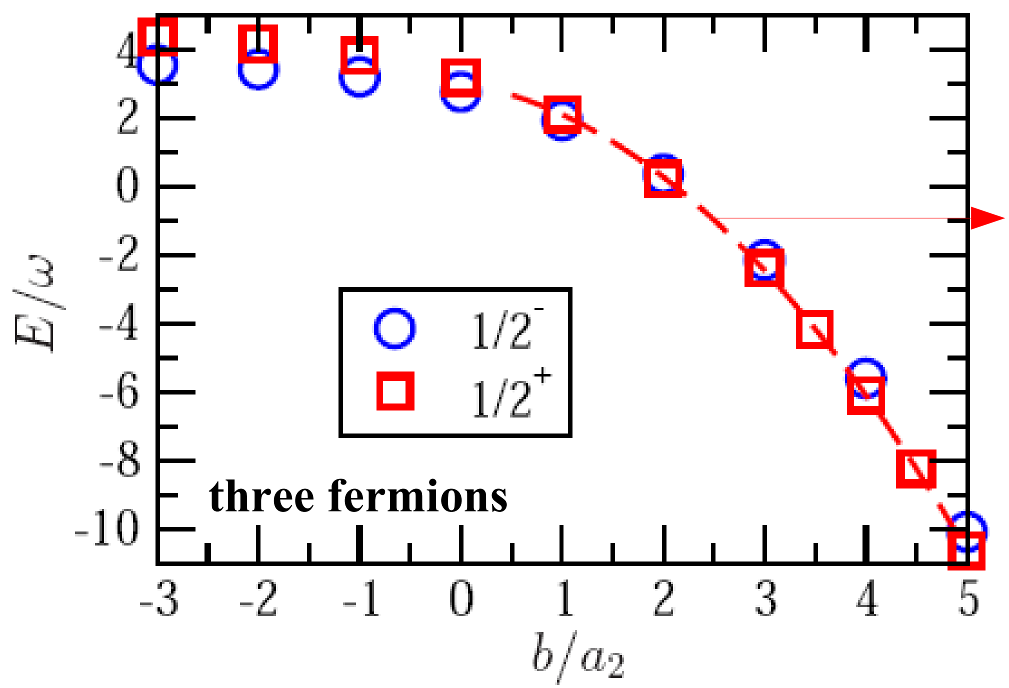




Three-body results away from unitarity

Stetcu et. al., Phys.Rev. A76 (2007) 063613

no interaction limit



fit to: $E/\omega = \alpha b^2 + \beta b + \gamma$

(no collapse)

untrapped limit

Untrapped particle limit: $b/a_2 \to \infty$

three-particle energy: $E \simeq -\dfrac{1}{2\mu a_2^2}$

Summary and outlook

- Applications of EFT principles directly into a many-body method
- Renormalization of the interaction intimately related with the model space used to solve the many-body problem
- Only LO iterated to all orders, beyond LO treated in PT
- Future applications to more particles (M-scheme) and to the nuclear problem