Three-nucleon forces and neutron-rich nuclei

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TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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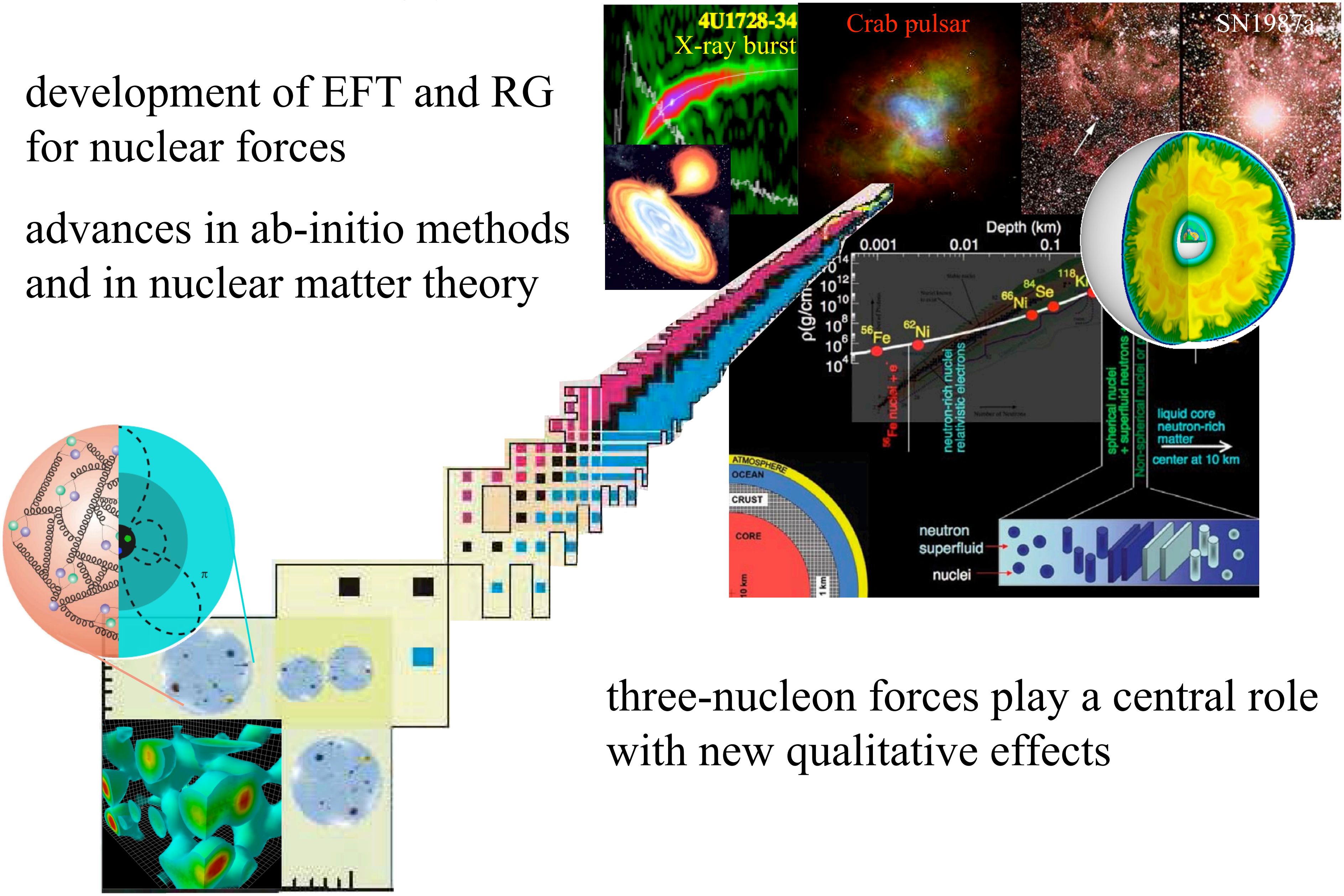
Outline

1. EFT and RG for nuclear forces
2. Three-nucleon forces in light nuclei
3. Three-nucleon forces towards heavier and neutron-rich nuclei
4. Three-nucleon forces in nuclear and neutron matter
5. Summary

Towards a unified description from light to heavy nuclei and strongly-interacting matter at the extremes

development of EFT and RG for nuclear forces

advances in ab-initio methods and in nuclear matter theory



three-nucleon forces play a central role with new qualitative effects

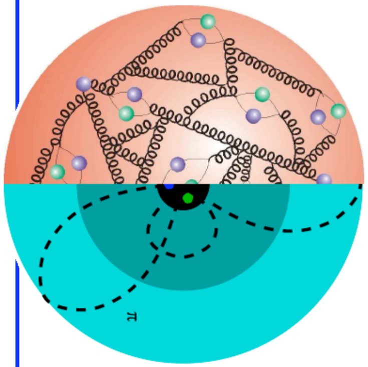
Outline

1. EFT and RG for nuclear forces

Λ / Resolution dependence of nuclear forces

with high-energy probes:
quarks+gluons

cf. scale/scheme dependence
of parton distribution functions

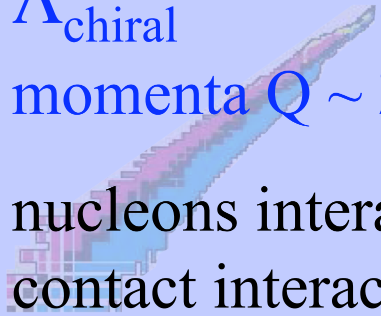


Lattice QCD

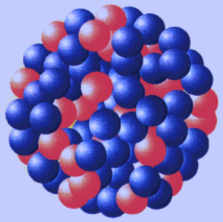
Effective theory for NN, 3N, many-N interactions and electroweak operators: resolution scale/Λ-dependent

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

$\Lambda_{\rm chiral}$



momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral EFT - typical momenta in nuclei

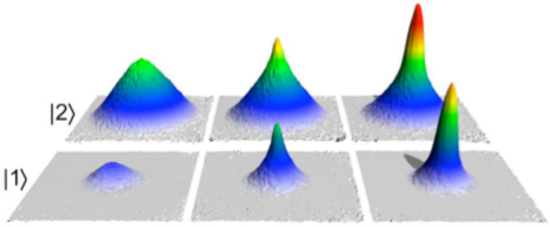


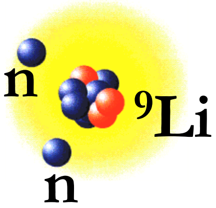
nucleons interacting via pion exchanges and shorter-range contact interactions

$\Lambda_{\rm pionless}$

$Q << m_\pi$=140 MeV: pionless EFT

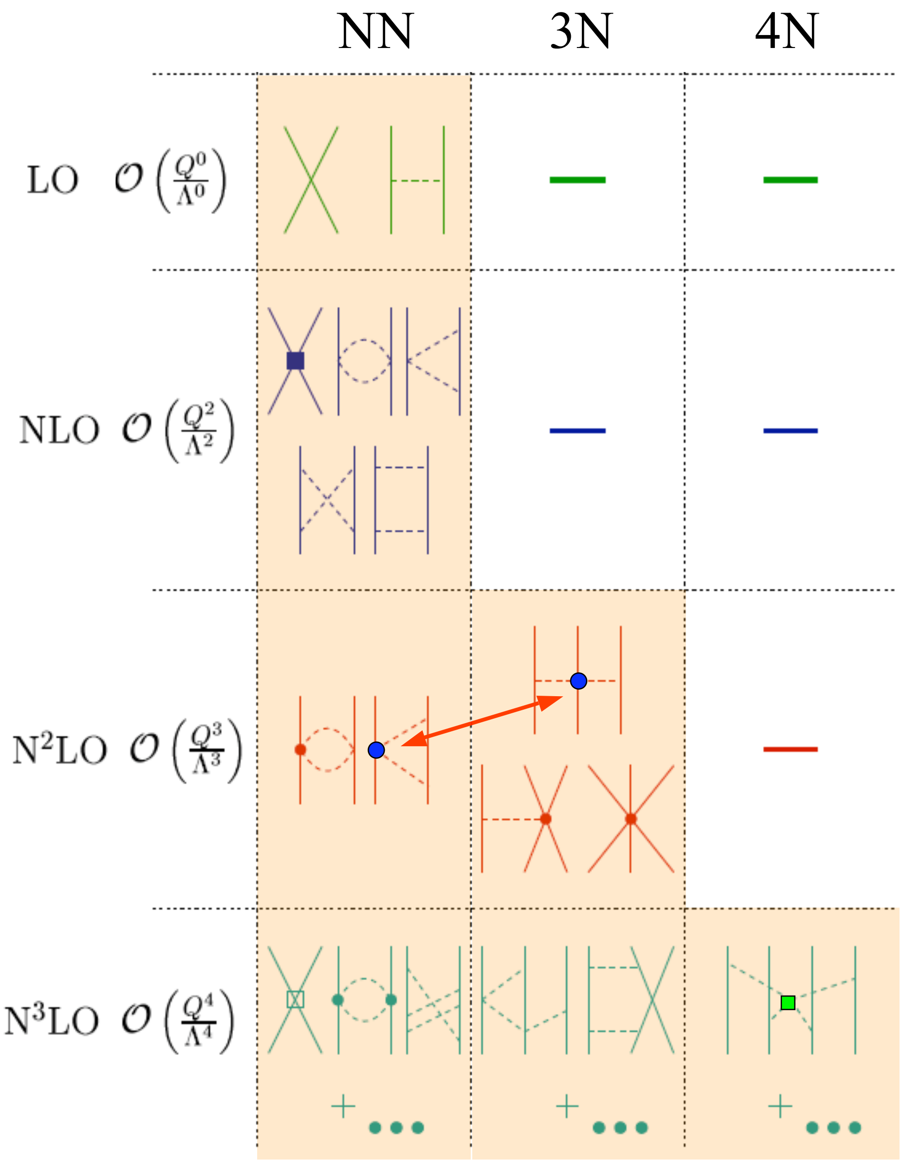
large scattering length physics and non-universal corrections





Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



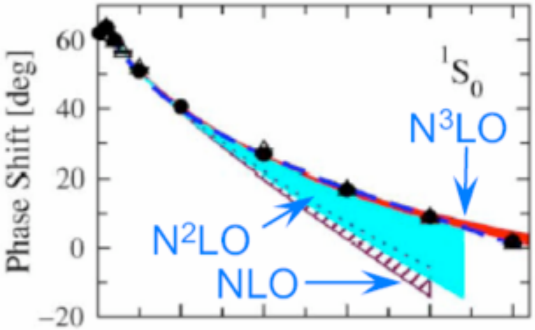
explains pheno hierarchy:
NN > 3N > 4N > …

consistent NN-3N, πN, ππ,
electroweak operators

3N,4N: 2 new couplings to N^{3}LO

resolution/Λ-dep. contact interactions

error estimates from truncation order
and resolution/Λ-variation



Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral EFT for nuclear forces

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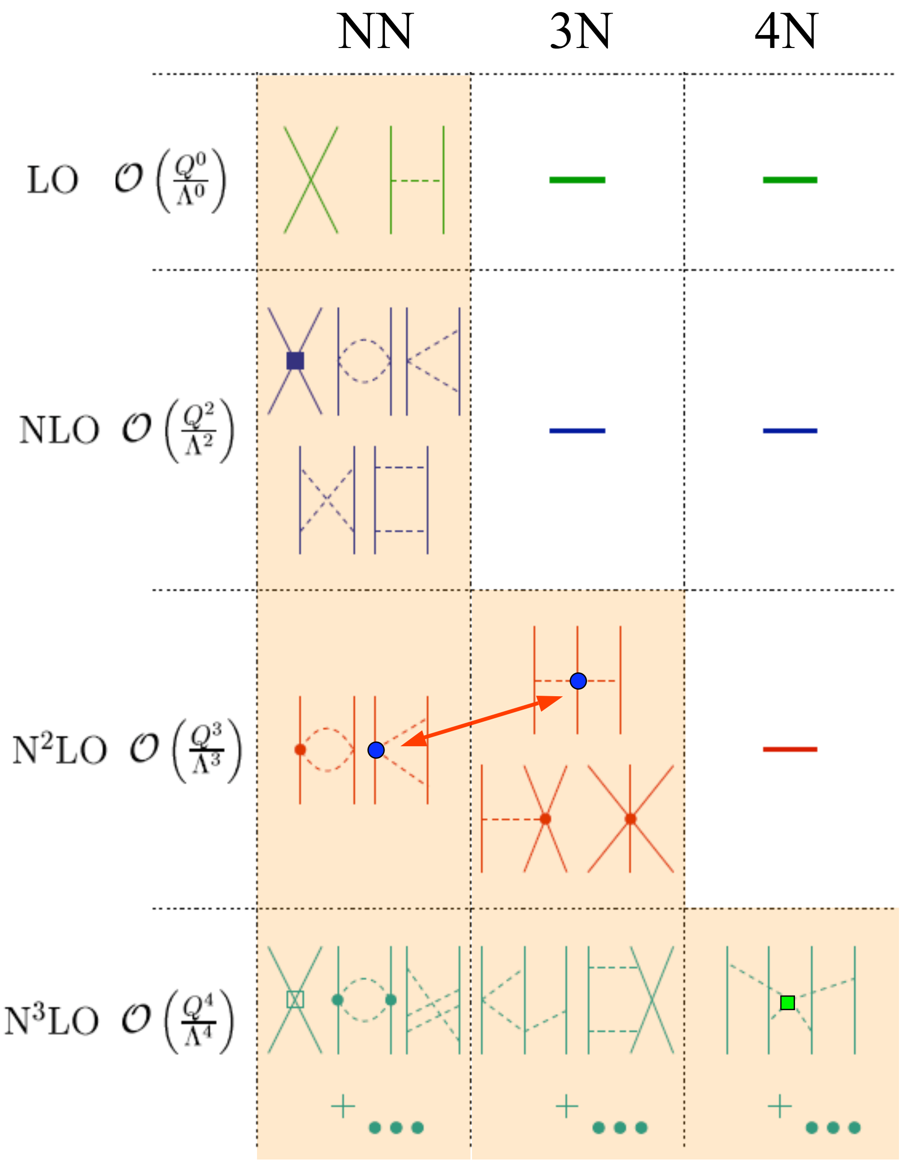


Open questions:

Power counting with singular pion exchanges, tensor parts ~1/r^n promotion of contact interactions

Nogga, Timmermans, van Kolck (2005)

Delta-full m_Δ-$m_N \sim m_\pi$ → 3N at NLO vs. Delta-less EFT

Counting of 1/m corrections, $m \sim \Lambda^2$ or Λ, could be important for A_y

This talk: 3N forces from N^{2}LO Delta-less chiral EFT

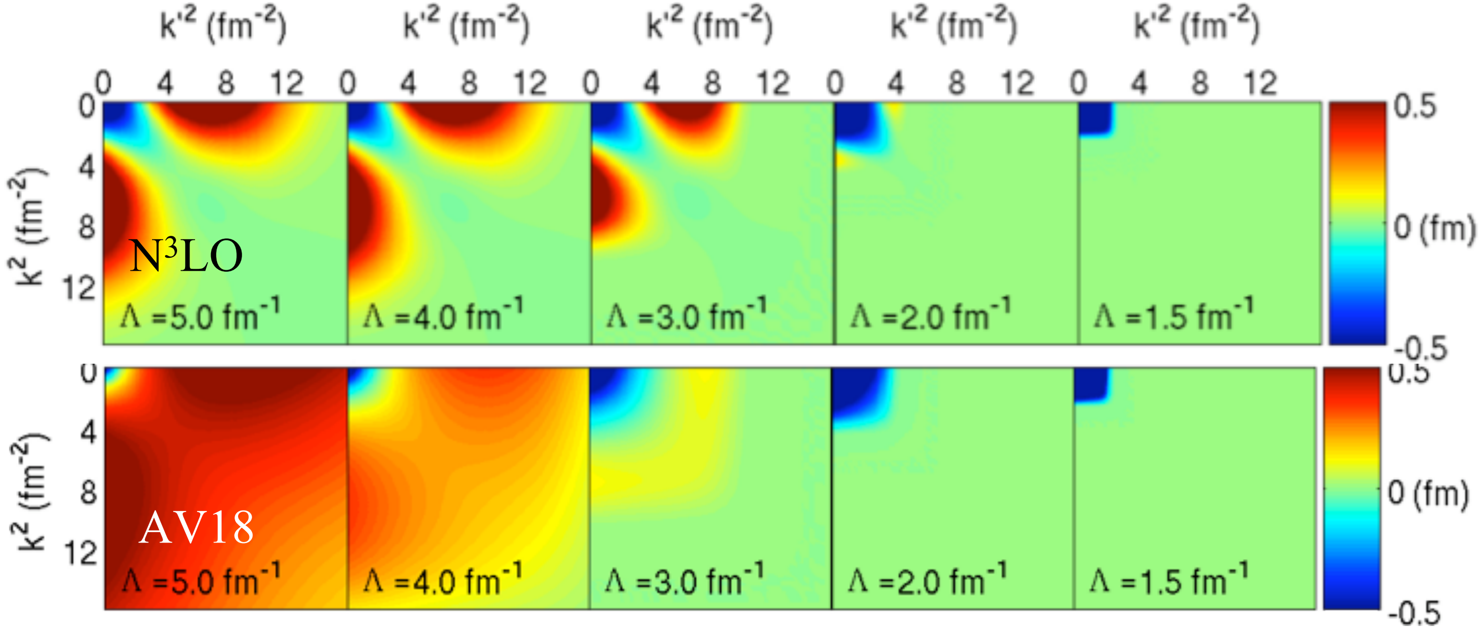
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Low-momentum interactions from the Renormalization Group

RG evolution to lower resolution/cutoffs

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{3\mathrm{N}}(\Lambda) + V_{4\mathrm{N}}(\Lambda) + \ldots$$

exact RG for NN interactions Bogner, Kuo, AS, Brown, Furnstahl,… (2003…)



≈ universal low-momentum interaction $V_{\mathrm{low}\,k}(\Lambda)$, sharp/smooth regulators

RG evolution decouples high momenta (short-range repulsion and tensor parts)

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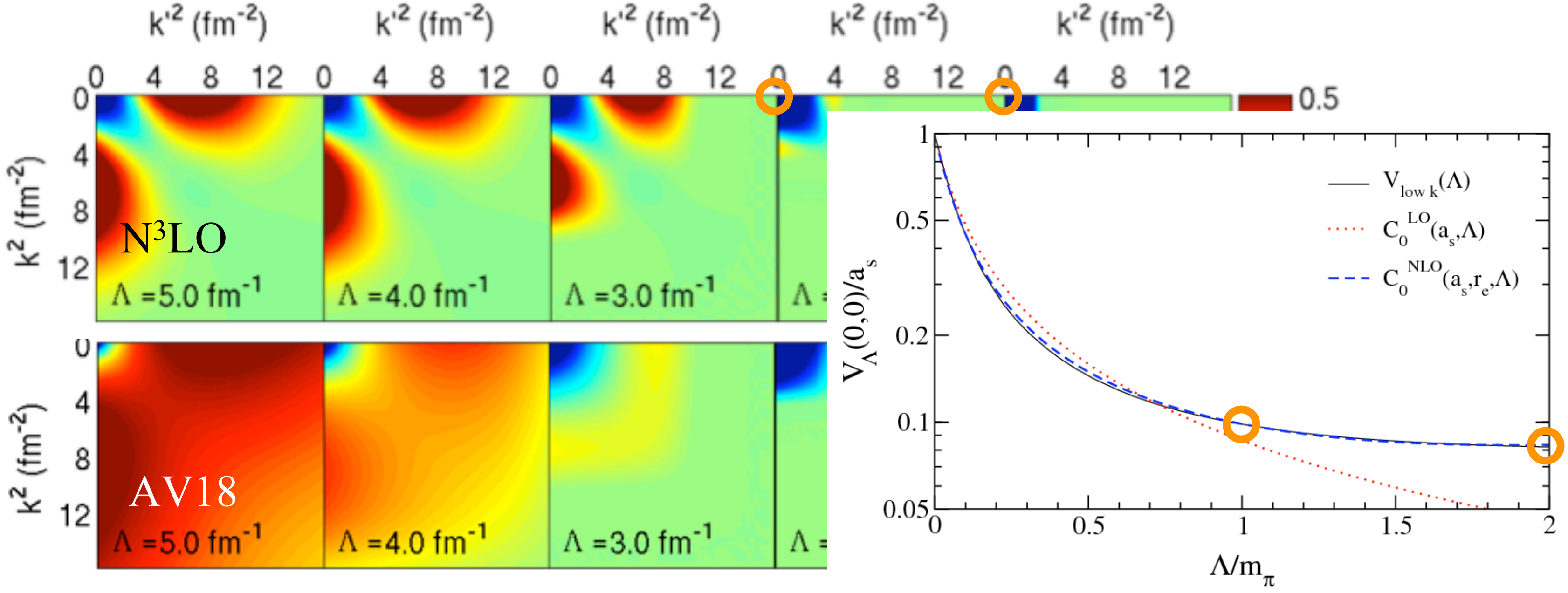


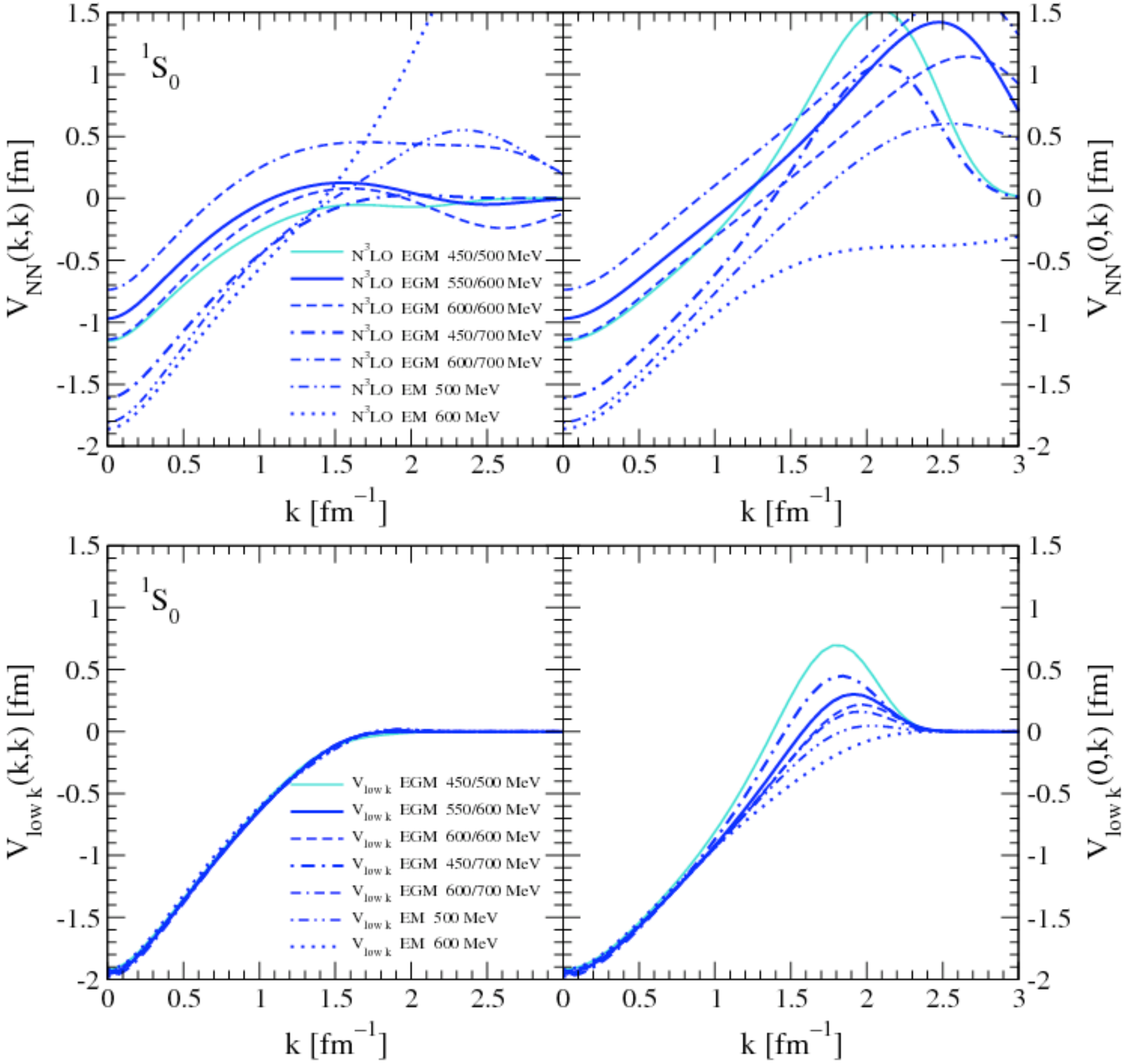
≈ universal low-momentum interaction $V_{\mathrm{low}\,k}(\Lambda)$, sharp/smooth regulators

RG evolution decouples high momenta (short-range repulsion and tensor parts)

same RG evolution as in EFT, but without expansion on operators

$V_{\mathrm{low}\,k}(0,0;\Lambda)$ follows contact interaction $c_0(\Lambda)$ at NLO in pionless EFT regime

Chiral EFT and RG



≈ universality from different N^{3}LO potentials

RG preserves long range and generates higher-order contact interactions

Weinberg eigenvalue diagnostic

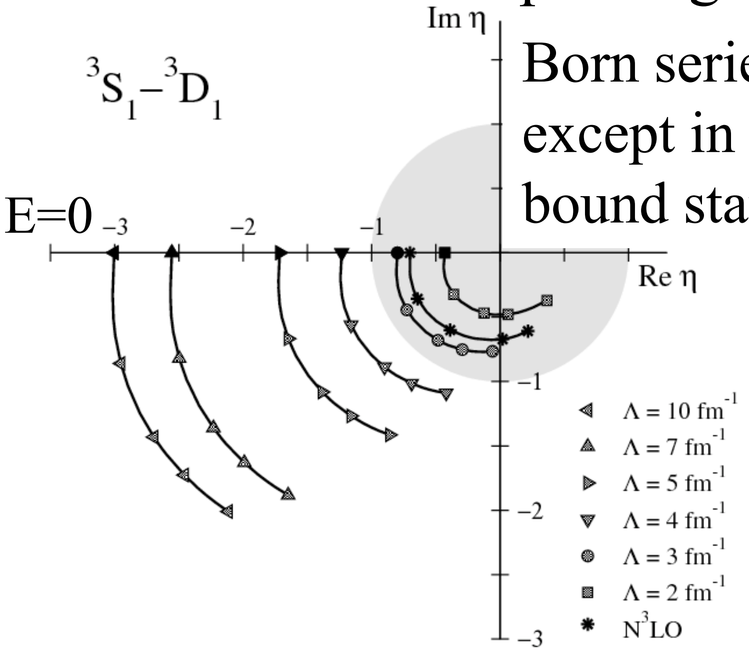
study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

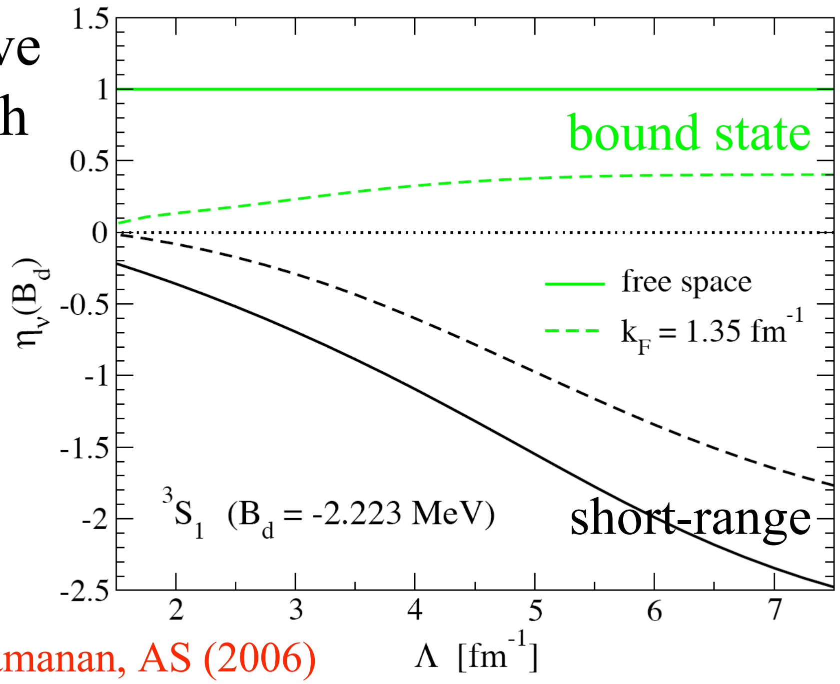
can write as Schrödinger equation $\Big(H_0 + \dfrac{1}{\eta_\nu(z)}V\Big)\,|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

high momenta/large cutoffs lead to flipped-potential bound states of -λV for small λ/large η → strong coupling to high momenta/short range and Born series always nonperturbative

RG evolution decouples high momenta (short-range repulsion and tensor parts)



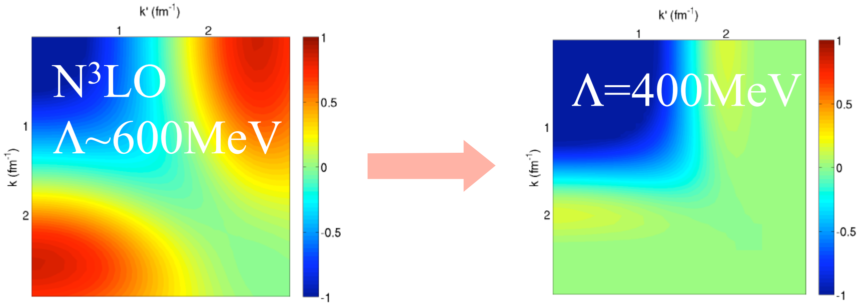
Born series perturbative except in channels with bound states



Bogner, AS, Furnstahl, Nogga (2005), Bogner, Furnstahl, Ramanan, AS (2006)

Advantages of low-momentum interactions for nuclei

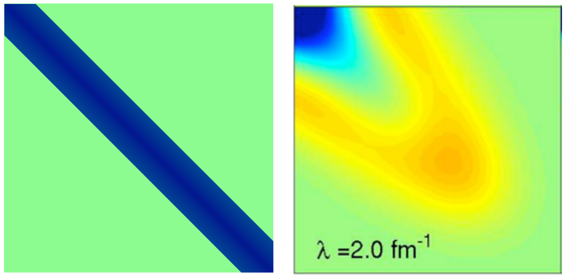
lower cutoffs need smaller basis



also via similarity RG (SRG)
evolution towards band diagonal

Bogner, Furnstahl, Perry, Roth, Hergert,… (2007…)

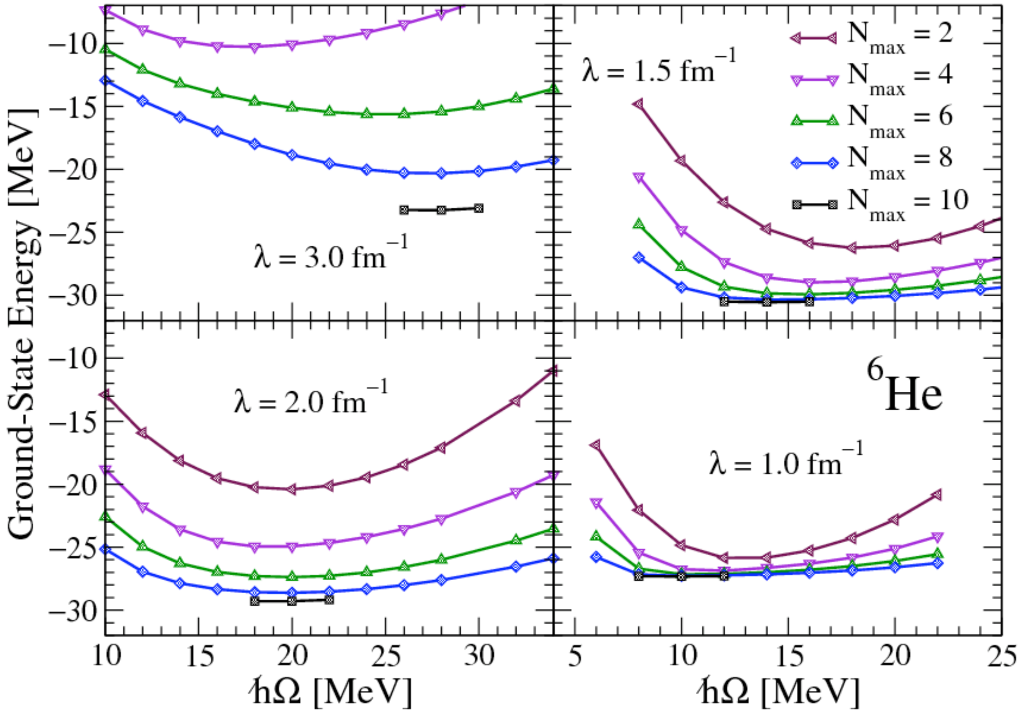
connection to EFT?



improved convergence for nuclei

Bogner Furnstahl, Maris, Perry, AS, Vary (2008)

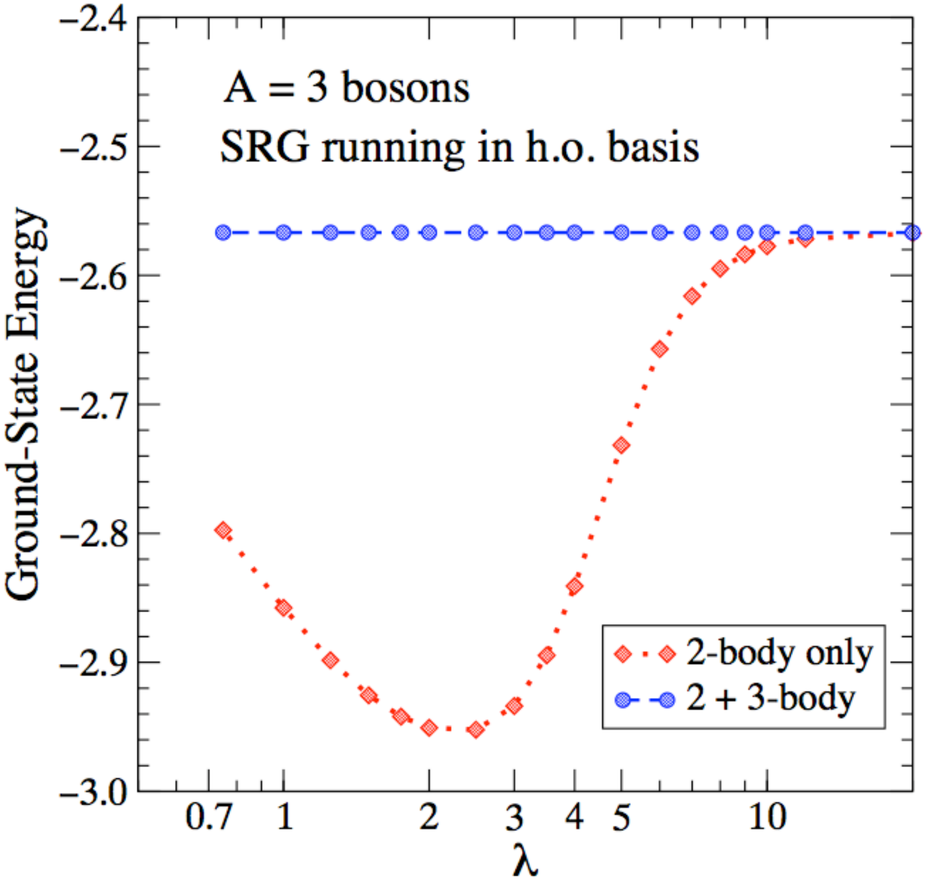
10^3 states for N_{max}=2 vs.
10^7 states for N_{max}=10

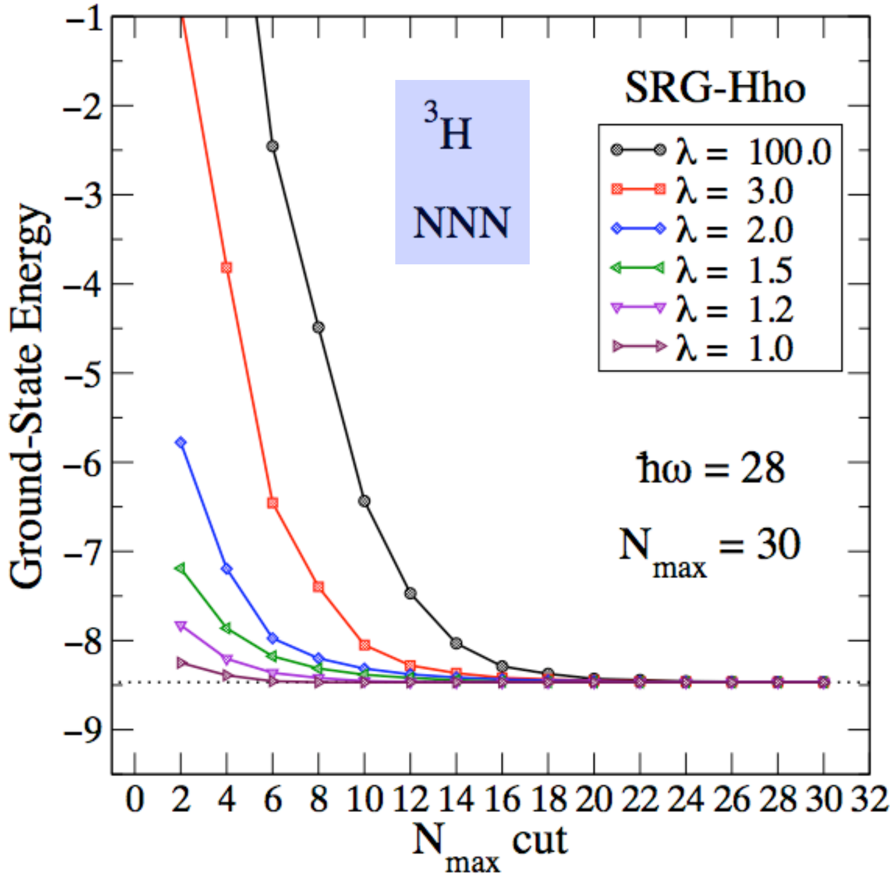


Towards evolving 3N interactions

SRG evolution with harmonic oscillator Hamiltonian as generator for bosons in 1d and for chiral 3N interactions

Jurgenson, Furnstahl (2008) and from Eric Jurgenson





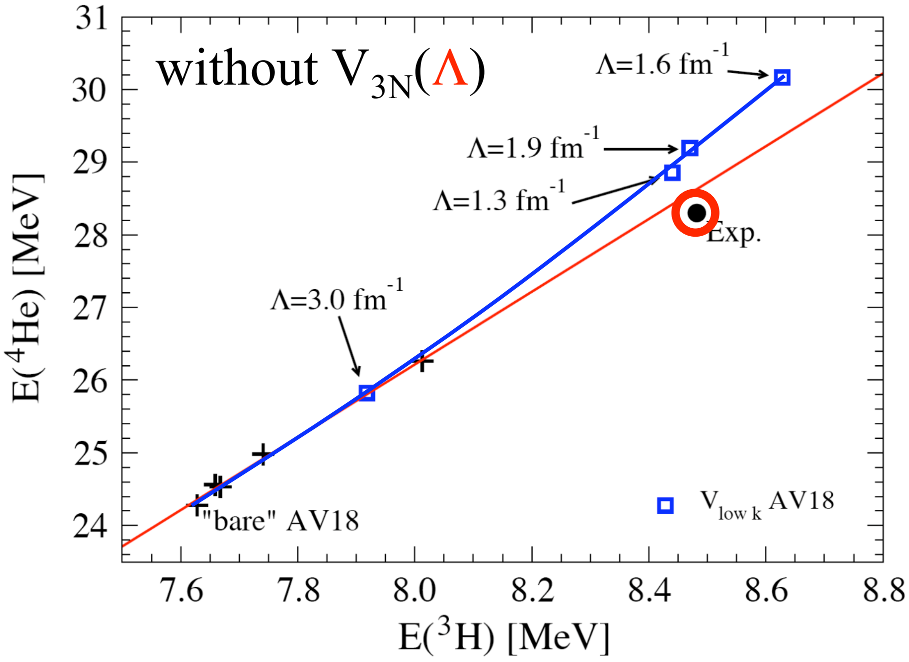
This talk: chiral EFT is complete basis, $V_{3N}(\Lambda)$ fits for lower cutoffs

3N interactions required for renormalization

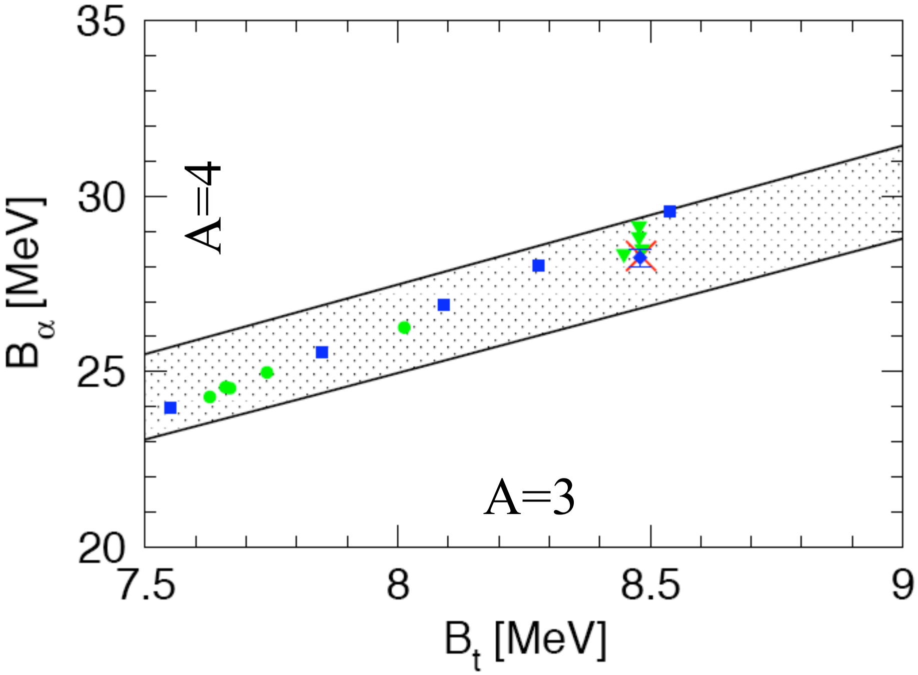
$V_{NN}(\Lambda)$ defines NN interactions with cutoff-independent NN observables

cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

cutoff dependence explains “Tjon line” → 3N for renormalization



Nogga, Bogner, AS (2004)



pionless EFT: Platter, Hammer, Meissner (2005)

large scattering lengths drive correlation

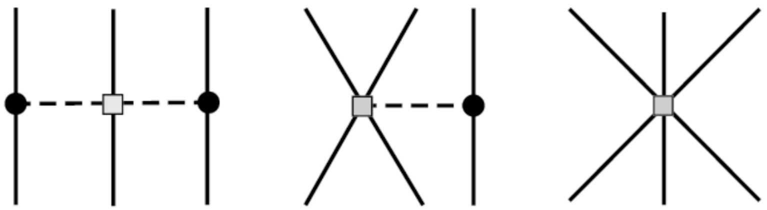
Tjon lines in ^{16}O±1 Hagen et al., in prep.

Outline

2. Three-nucleon forces in light nuclei

Chiral EFT 3N interactions

leading $N^2LO \sim (Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)



long (2π) intermed. (π) short-range

c_1,c_3,c_4 terms $D(\Lambda)$ term $E(\Lambda)$ term

c_i from πN, consistent with NN $\boxed{c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.2}_{-1.0}\,,\ c_4 = 3.5^{+0.5}_{-0.2}}$ Meissner et al.

c_3,c_4 important for structure, large uncertainties at present

$V_{3N}(\Lambda)$ based on fits of D,E couplings to A=3,4 for range of cutoffs

chiral EFT is a complete low-mom basis → 3N up to truncation errors

D term can be fixed by tritium beta decay Gardestig, Phillips (2006), Gazit et al. (2008)

generally improves 3N scattering Gloeckle, Kamada, Witala,…. (2002…)

Subleading chiral EFT 3N interactions

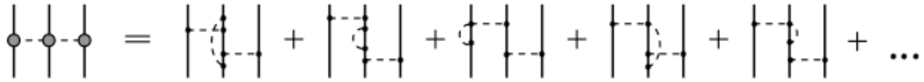
parameter-free N^{3}LO from Epelbaum; Bernard et al. (2007), Ishikawa, Robilotta (2007)

- 1/m-corrections to 1 insertion from $\mathcal{L}^{(2)}_{1/m} =$  $+\, \mathcal{O}(\pi^3)$

  — rich operator structure (includes spin-orbit interactions)

- 1-loop diagrams with all vertices from $\mathcal{L}^{(0)}_{\text{eff}}$

2π – exchange

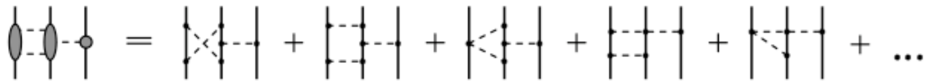


decrease c$_i$ constants → Δ

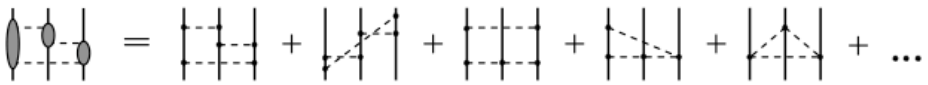
The calculated corrections simply shift the LECs c_i as follows:

$$\delta c_1 = \frac{g_A^2 M_\pi}{64\pi F_\pi^2} \sim 0.13\ \text{GeV}^{-1} \qquad \delta c_3 = \frac{3 g_A^4 M_\pi}{16\pi F_\pi^2} \sim 2.5\ \text{GeV}^{-1} \qquad \delta c_4 = -\frac{g_A^4 M_\pi}{16\pi F_\pi^2} \sim -0.85\ \text{GeV}^{-1}$$

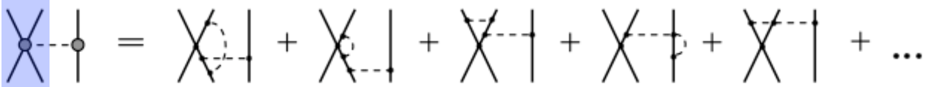
2π-1π – exchange



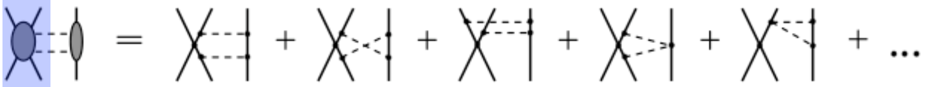
ring diagrams



contact-1π – exchange



contact-2π – exchange



3N interactions involving NN contacts in progress

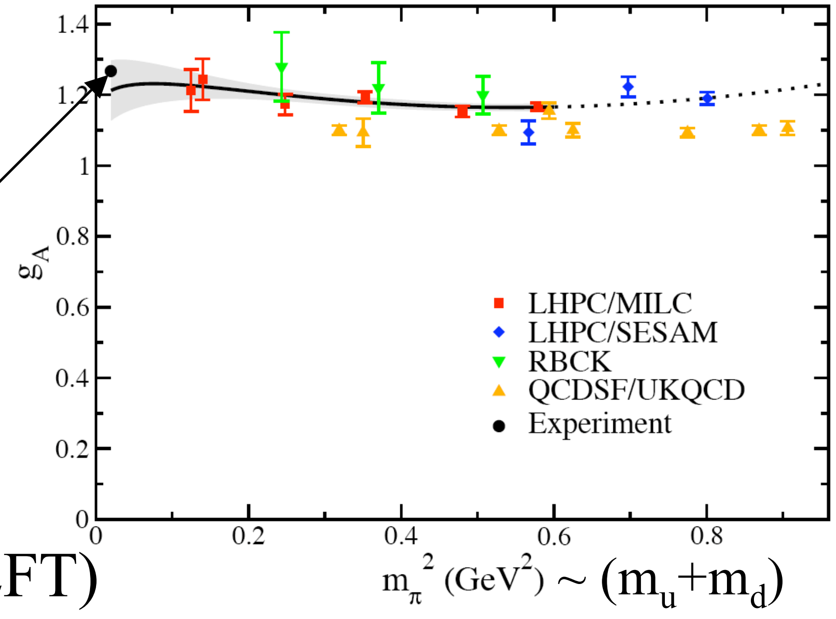
Epelbaum et al.

Lattice QCD and 3N forces

Long-range couplings:

pion-NN coupling g_a from full QCD

Edwards et al. (2006)



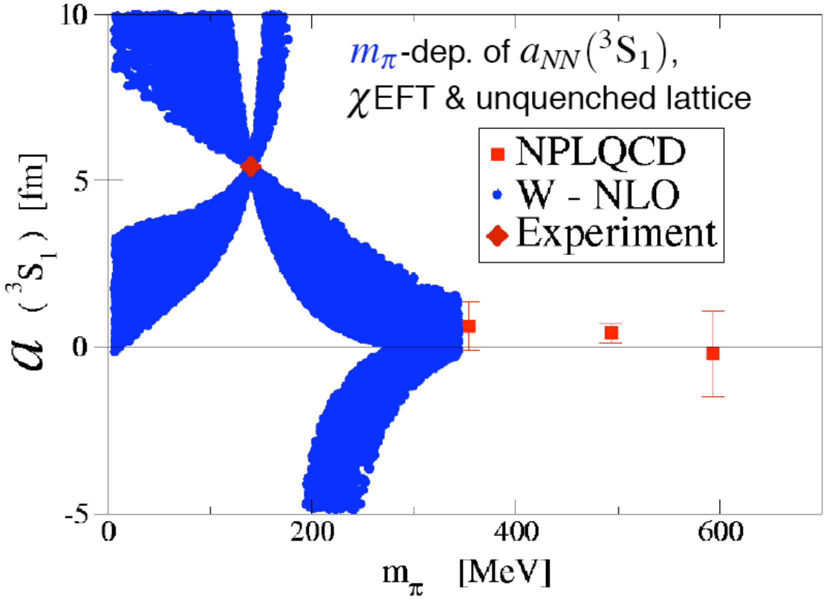
chiral EFT extrapolation to physical pion mass agrees with experiment

constrain higher-order long-range pion-, N-,… couplings (scheme: chiral EFT)

c_i from lattice QCD?

Few-nucleon observables:

NN scattering lengths from full QCD, dependence on quark masses Beane et al. (2006)

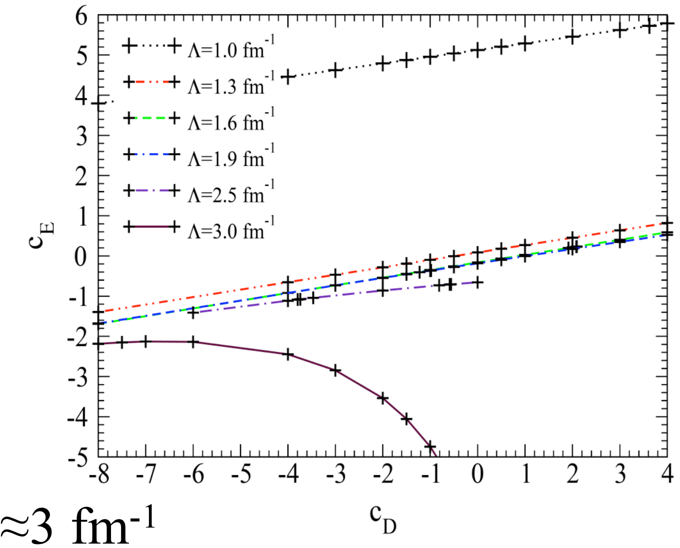


Constrain experimentally difficult observables: 3-neutron properties

first steps: 3 pions on a lattice Detmold et al. (2008)

Low-momentum 3N fits

fit D, E couplings to ^{3}H, ^{4}He binding energies for range of cutoffs



linear dependences in fits to triton binding

3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$

Nogga, Bogner, AS (2004)

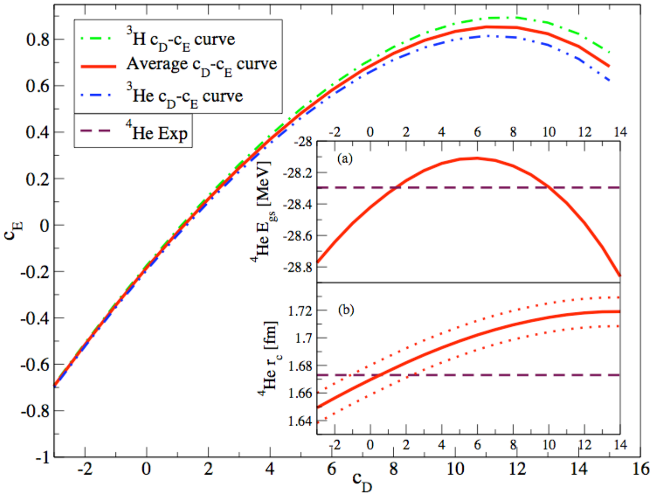
nonperturbative at larger cutoffs, cf. chiral EFT Λ≈3 fm^{-1}

new 3N fits to ^{3}H binding energy, ^{4}He radius

Bogner, Furnstahl, Nogga, AS, 0903.3366.

	$V_{\mathrm{low}\,k}$		SRG	
Λ or $\lambda/\Lambda_{\mathrm{3NF}}$ [fm^{-1}]	c_D	c_E	c_D	c_E
1.8/2.0	−0.0112	−0.2212		
2.0/2.0	−0.3000	−0.2761	−1.023	−0.3397
2.0/2.5	−2.000	−0.7564	−2.991	−0.8797
2.2/2.0	−0.9000	−0.3673		
2.8/2.0	−1.552	−0.4058		

TABLE I: Results for the c_D and c_E couplings fit to $E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV}$ and $r_{^4\mathrm{He}} = 1.46-1.47\,\mathrm{fm}$ for the NN/3N cutoffs used here. For $V_{\mathrm{low}\,k}$ (SRG) interactions, the 3NF fits lead to $E_{^4\mathrm{He}} = -28.22\ldots28.45\,\mathrm{MeV}$ ($-28.53\ldots28.71\,\mathrm{MeV}$).

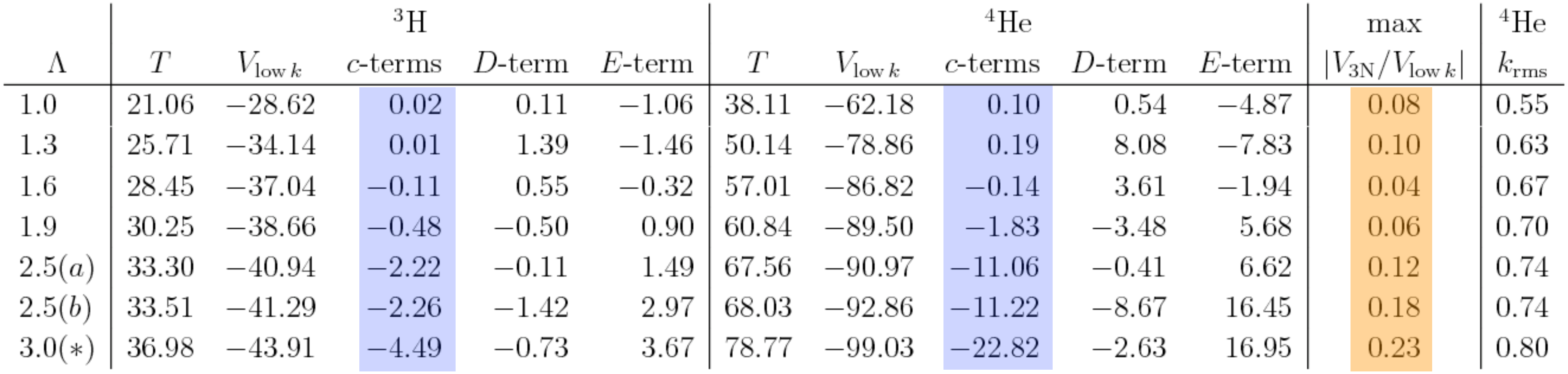


Navratil et al. (2007)

perturbative in A=3,4 for these cutoffs and generally $\Lambda_{\mathrm{3NF}} \lesssim \Lambda$

Low-momentum 3N interactions in light nuclei

Nogga, Bogner, AS (2004)

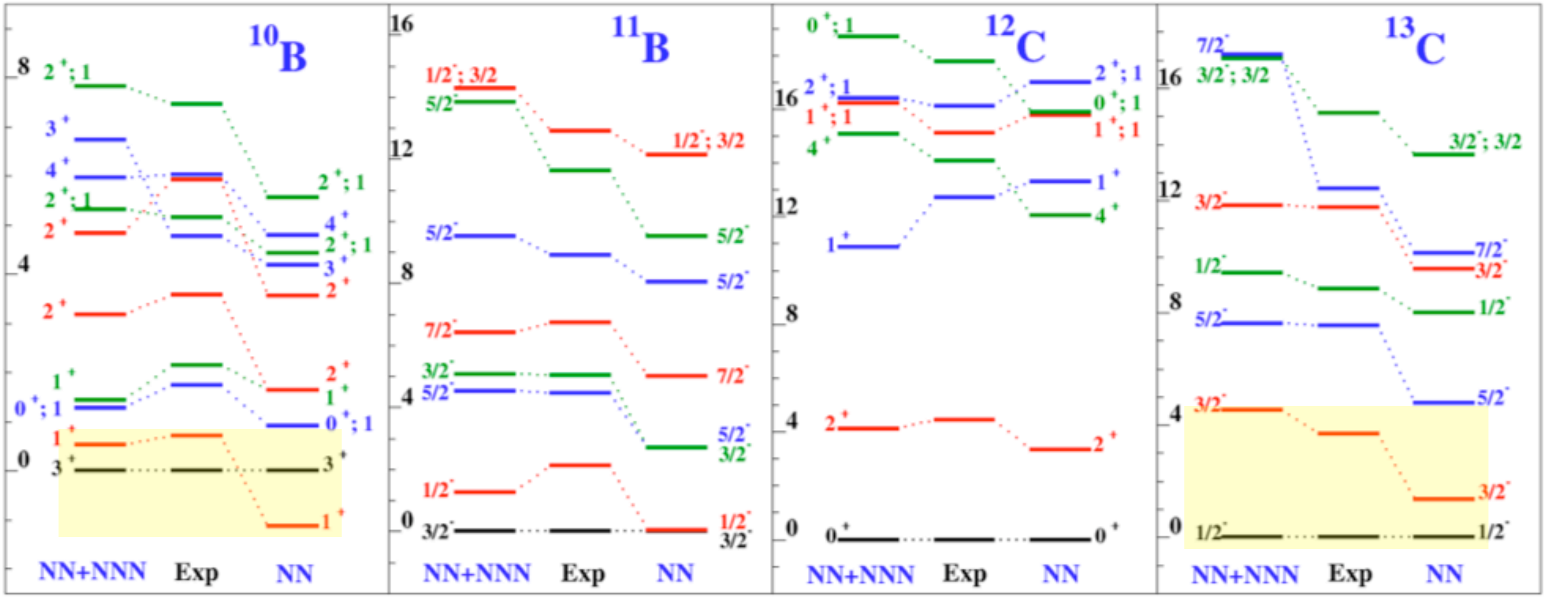


	^{3}H					^{4}He					max	^{4}He
Λ	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term	$\lvert V_{3\mathrm{N}}/V_{\mathrm{low}\,k}\rvert$	k_{rms}
1.0	21.06	−28.62	0.02	0.11	−1.06	38.11	−62.18	0.10	0.54	−4.87	0.08	0.55
1.3	25.71	−34.14	0.01	1.39	−1.46	50.14	−78.86	0.19	8.08	−7.83	0.10	0.63
1.6	28.45	−37.04	−0.11	0.55	−0.32	57.01	−86.82	−0.14	3.61	−1.94	0.04	0.67
1.9	30.25	−38.66	−0.48	−0.50	0.90	60.84	−89.50	−1.83	−3.48	5.68	0.06	0.70
2.5(a)	33.30	−40.94	−2.22	−0.11	1.49	67.56	−90.97	−11.06	−0.41	6.62	0.12	0.74
2.5(b)	33.51	−41.29	−2.26	−1.42	2.97	68.03	−92.86	−11.22	−8.67	16.45	0.18	0.74
3.0($*$)	36.98	−43.91	−4.49	−0.73	3.67	78.77	−99.03	−22.82	−2.63	16.95	0.23	0.80

natural size of individual 3N expectation values ~0.1 of NN, consistent with chiral power-counting estimates

long-range c_i-terms attractive in A=3,4 for most cutoffs, but repulsive in nuclear/neutron matter and drive saturation

3N interactions and nuclear structure



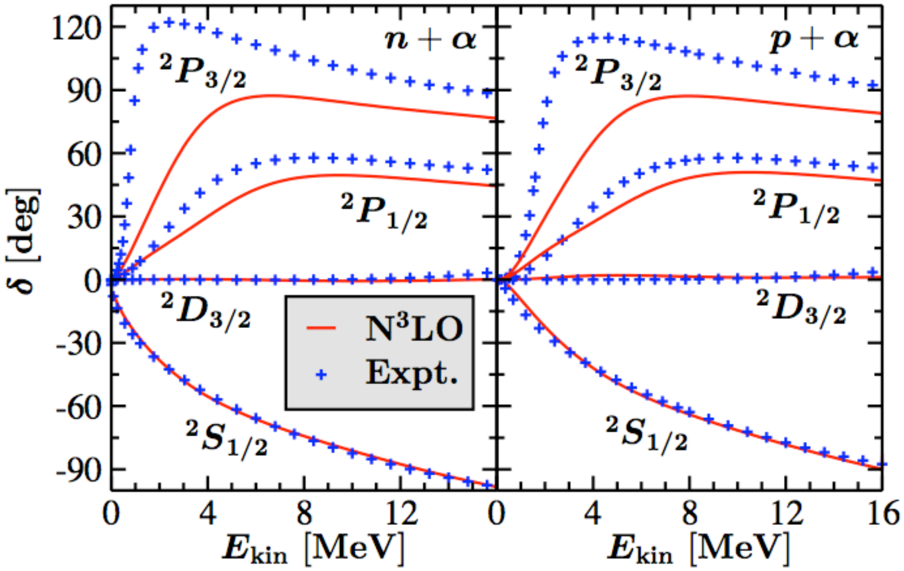
Navratil et al. (2007)

highlights importance of 3N forces

crucial for spin-orbit splittings

spin-orbit shell closures (^{48}Ca,…)

see, e.g., Zuker, AS (2006)



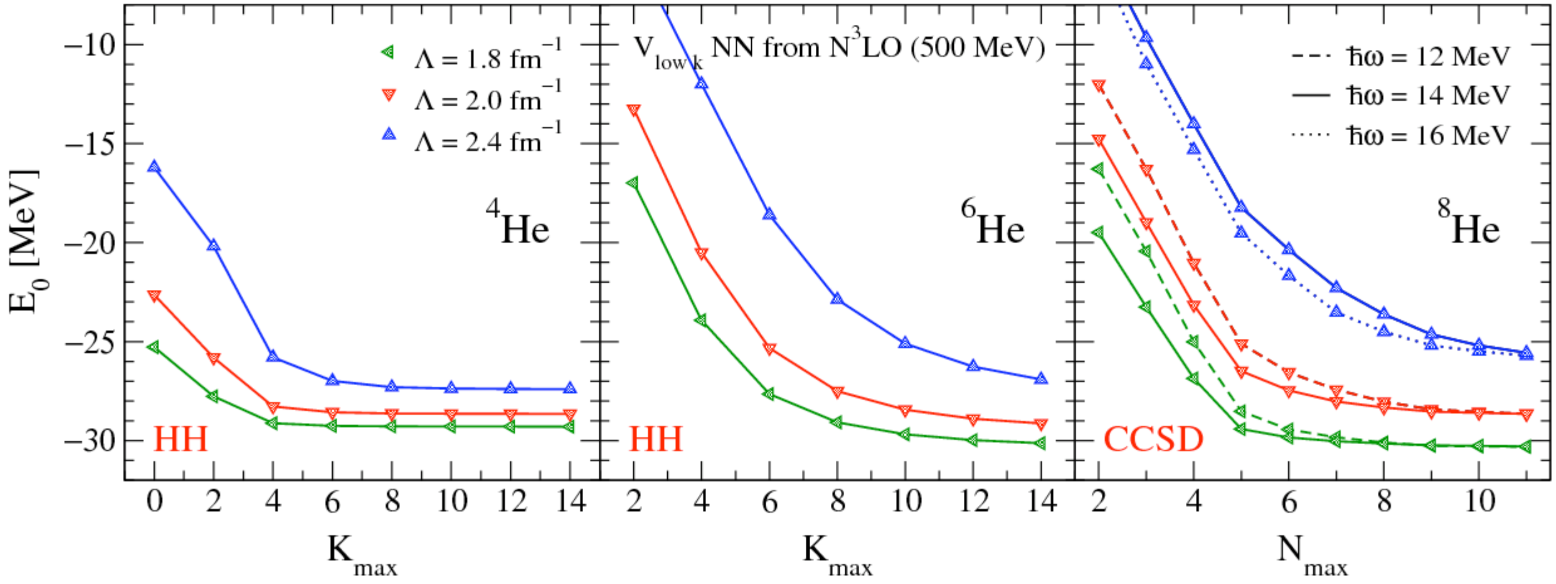
Quaglioni, Navratil (2008)

Helium halos

Hyperspherical Harmonics for ^{6}He and Coupled-Cluster theory for ^{8}He

Bacca et al., 0902.1696; see talk by Sonia Bacca.

based on N^{3}LO NN potential and RG,
describe weakly-bound nuclei with correct asymptotics



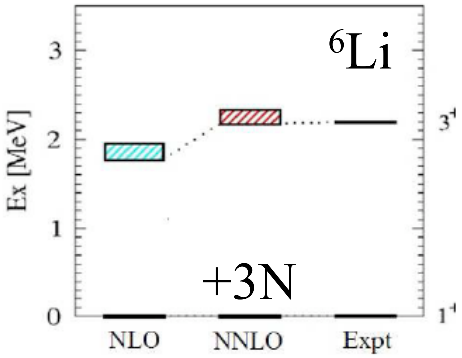
Λ [fm^{-1}]	$E_0(^4\mathrm{He})$ [MeV]	$E_0(^6\mathrm{He})$ extrapolated [MeV]	$E_0(^8\mathrm{He})$ Λ-CCSD(T) [MeV]
1.8	-29.30	-30.23(7)	-31.20
2.0	-28.65	-29.28(2)	-29.84
2.4	-27.40	-27.99(40)	-25.71 (CCSD)
experiment	-28.296	-29.268 [43]	-31.395 [4]

need attractive part of 3N interaction in neutron-rich Helium halos

Long-term goal

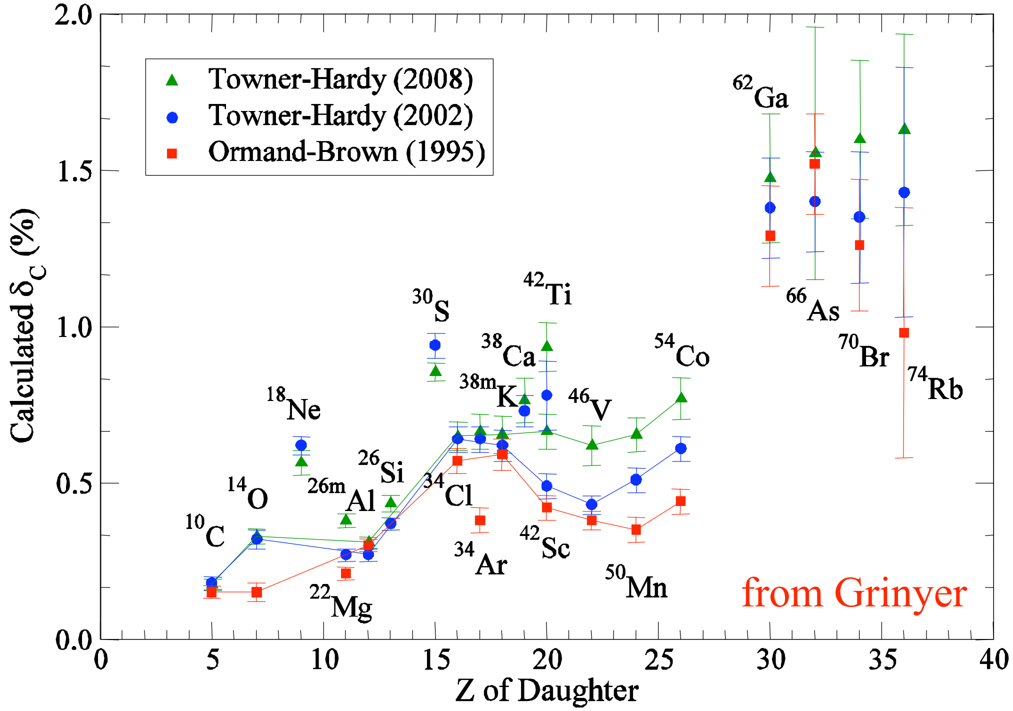
EFT/RG: more accurate with higher orders and
cutoff variation estimates theoretical uncertainties

apply to pivotal matrix elements needed
to constrain beyond Standard Model physics



from Nogga

isospin-symmetry breaking corrections for superallowed beta decay



from Grinyer

new 2008 Towner-Hardy calculation
of ISB corrections (~enlarged model space)
3.1σ shift in world-average Ft, 1.5σ in V_{ud}
largest changes in 20 years

^{62}Ga: 0.04% precise ft value ~lighter A

Miller, AS (2008)
Towner-Hardy formalism violates
isospin commutation relations,
cannot separate ISB corrections into
radial wf + SM space parts

nuclear matrix elements for $0\nu\beta\beta$ decay,
octupole EDM enhancement,… limited by nuclear theory uncertainties

Outline

3. Three-nucleon forces towards heavier and neutron-rich nuclei

Pushing the limits

First ab-initio calculations for heavier systems:

Coupled-cluster theory based on $V_{\text{low k}}(\Lambda)$

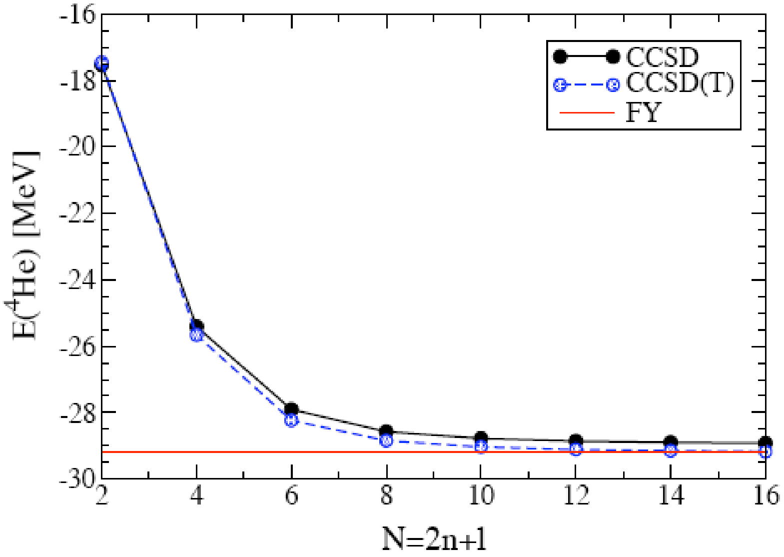
Hagen, Dean, Hjorth-Jensen, Papenbrock, AS (2007)

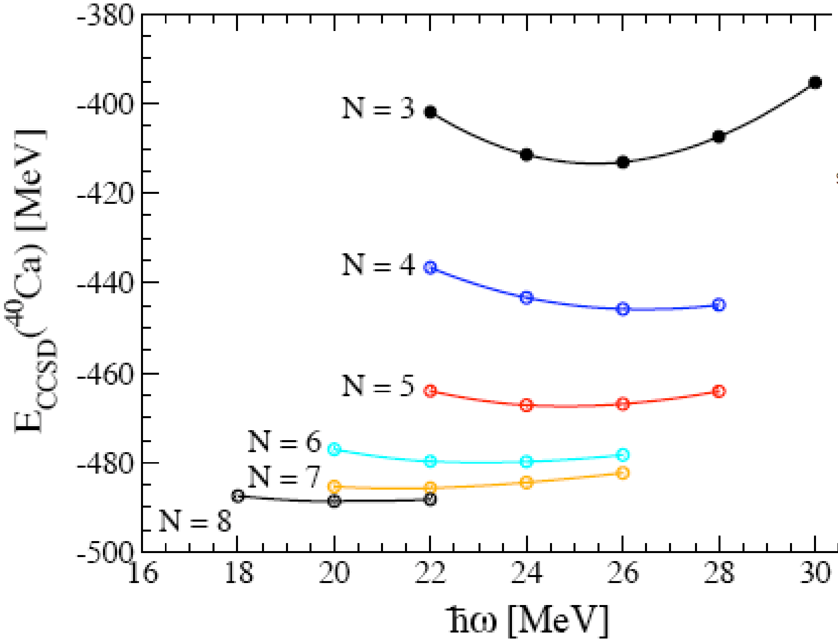
meets and sets benchmarks:

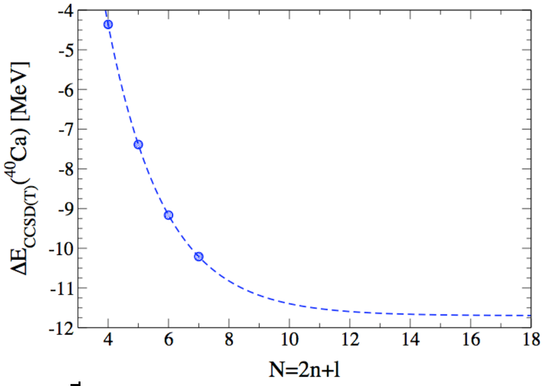
within 10 keV of exact FY for ^{4}He

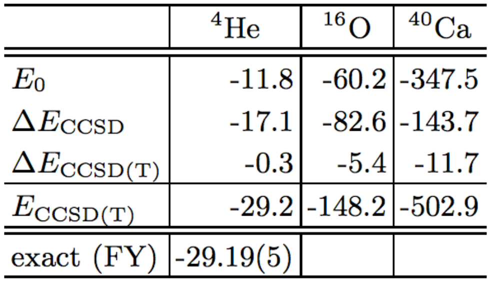
accurate for ^{16}O and ^{40}Ca

N=8: basis dimension ~10^{63}









	^{4}He	^{16}O	^{40}Ca
E_0	-11.8	-60.2	-347.5
ΔE_{CCSD}	-17.1	-82.6	-143.7
$\Delta E_{\text{CCSD(T)}}$	-0.3	-5.4	-11.7
$E_{\text{CCSD(T)}}$	-29.2	-148.2	-502.9
exact (FY)	-29.19(5)		

< 2 MeV triples corrections

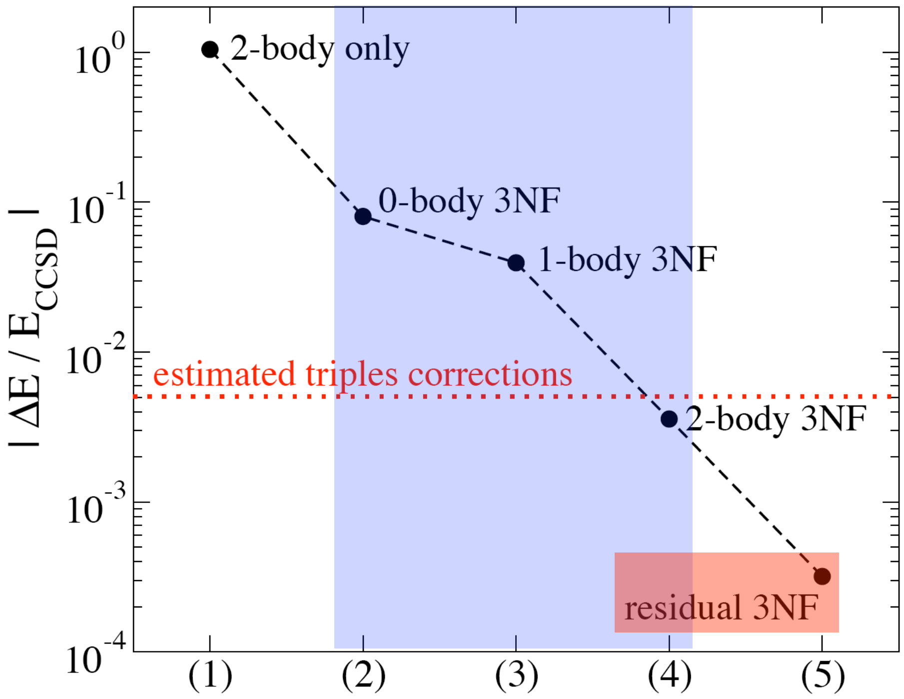
in ^{40}Ca extrapolated

for CC developments, see talk by Gaute Hagen

Towards 3N interactions in medium-mass nuclei

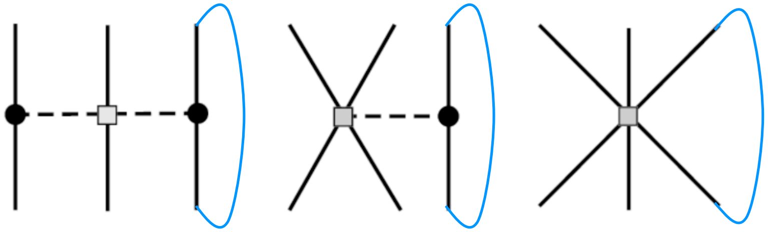
developed coupled-cluster theory with 3N interactions Hagen et al. (2007)

first benchmark for ^{4}He based on low-momentum interactions



show that 0-, 1- and 2-body parts of 3N interaction dominate

2-body part



occupied orbits

residual 3N interaction can be neglected: very promising

supports that monopole shifts are due to 3N interactions cf. Zuker (2003)

3N contribution to two-body monopole interaction

take into account 2-body part of 3N interactions to two-body monopoles

determines interaction of s with t orbit

$$\mathcal{V}^T_{st} = \frac{\sum_J \mathcal{V}^{JT}_{stst}(2J+1)[1-(-)^{J+T}\,\delta_{st}]}{\sum_J (2J+1)[1-(-)^{J+T}\,\delta_{st}]}$$

small changes in monopoles enhanced by number operators

monopoles from $V_{\text{low k}}(\Lambda)$ + 2nd order (6hw), cutoff independent in T=1

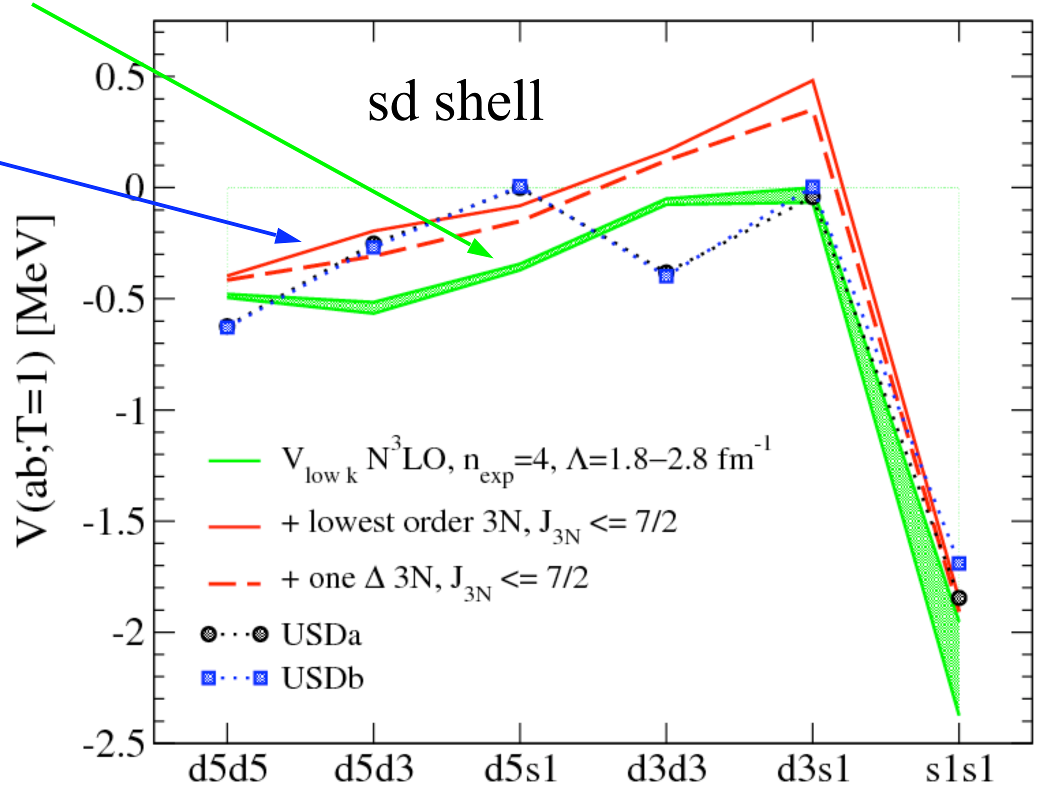
repulsive contributions from 3N interactions fit to A=3,4

reproduces hierarchy for low orbits

$(d_{3/2})^2$ with ^{40}Ca core?

dominated by c_i-terms

→ Δ-hole contributions



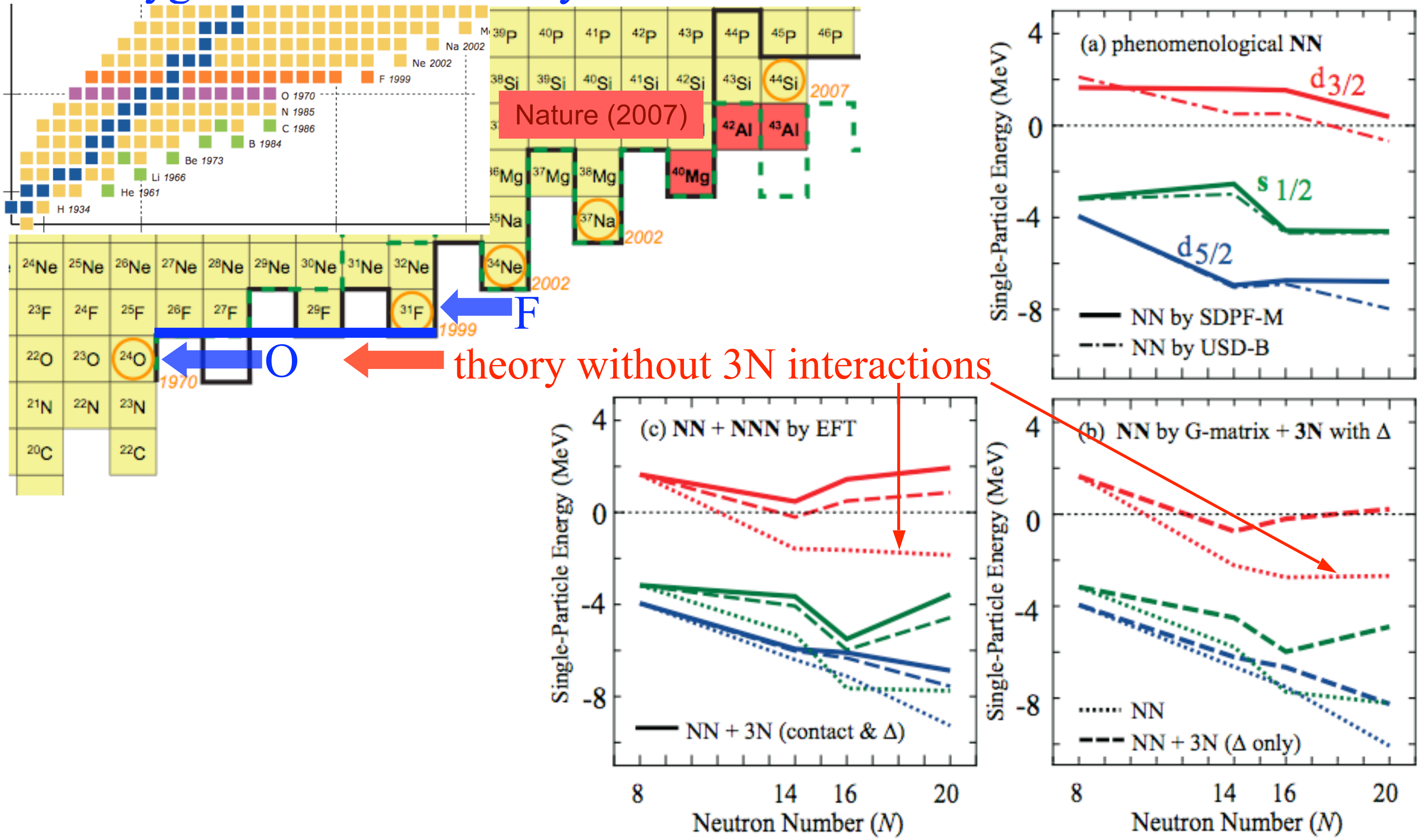
Otsuka, Suzuki, Holt, AS, Akaishi, in prep.

shell model matrix elements for different cores probe 3N interactions

3N forces and neutron-rich nuclei

evolution to the neutron drip-line: limits of existence and shell structure

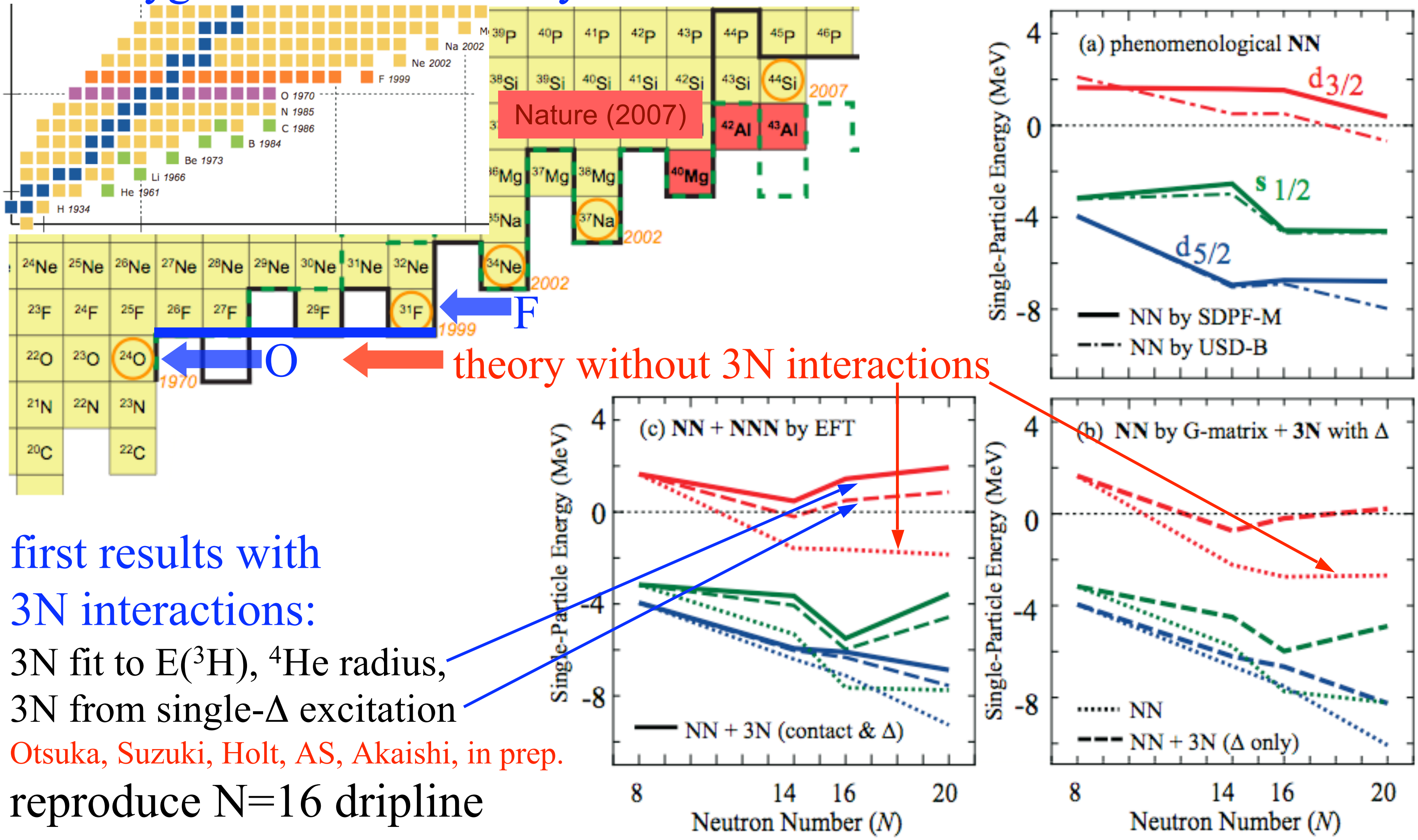
The oxygen-fluorine anomaly



3N forces and neutron-rich nuclei

evolution to the neutron drip-line: limits of existence and shell structure

The oxygen-fluorine anomaly



theory without 3N interactions

first results with
3N interactions:

3N fit to E(^{3}H), ^{4}He radius,
3N from single-Δ excitation

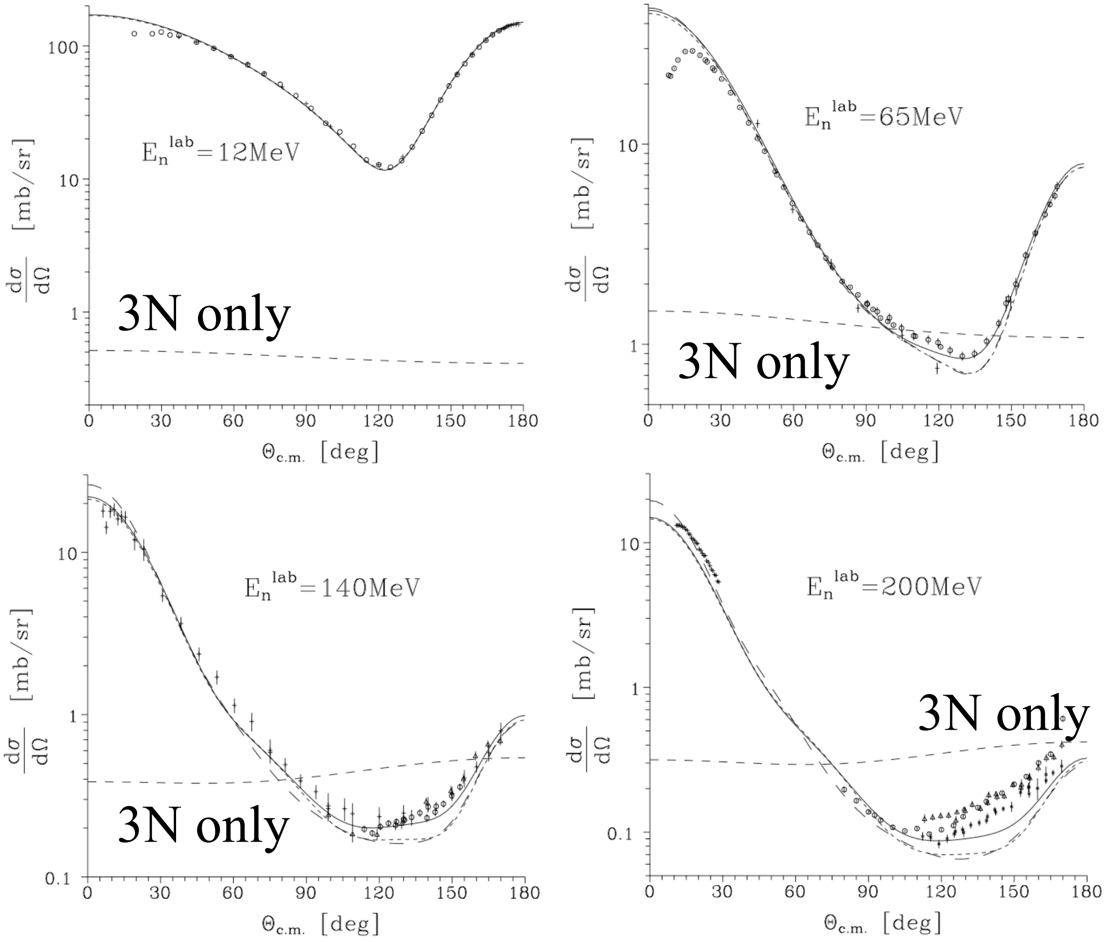
Otsuka, Suzuki, Holt, AS, Akaishi, in prep.

reproduce N=16 dripline

neutron-proton interaction stronger, binds $d_{3/2}$ neutrons in fluorine

Is there an EFT argument/counting for the hierarchy of n-body parts of many-body forces? for the SM multipole expansion?

maybe interesting to understand connection to
weak dependence on angle in N-d scattering



Witala et al. (1998)

Outline

4. Three-nucleon forces in nuclear and neutron matter

Weinberg eigenvalue diagnostic

study spectrum of $G_0(z)V\,|\Psi_\nu(z)\rangle = \eta_\nu(z)\,|\Psi_\nu(z)\rangle$ at fixed energy z

governs convergence $T(z)\,|\Psi_\nu(z)\rangle = (1+\eta_\nu(z)+\eta_\nu(z)^2+\ldots)\,V\,|\Psi_\nu(z)\rangle$

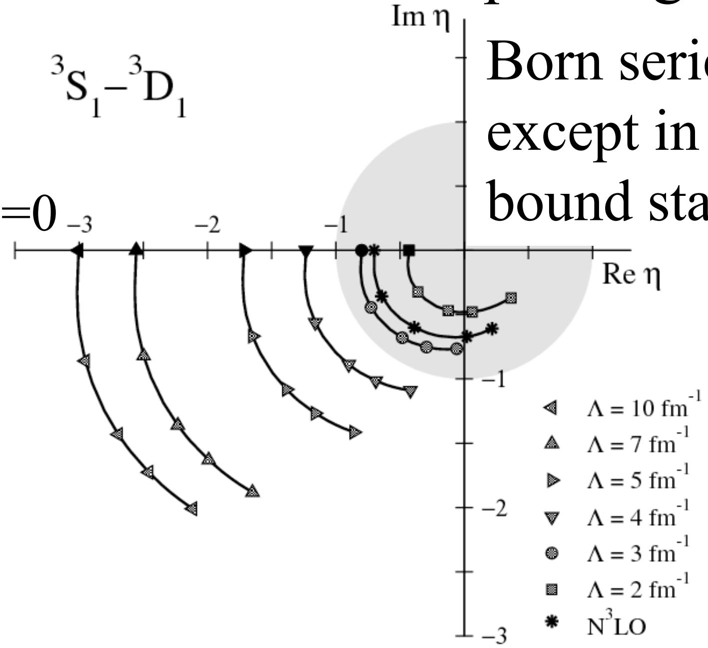
can write as Schrödinger equation $\Big(H_0+\dfrac{1}{\eta_\nu(z)}V\Big)\,|\Psi_\nu(z)\rangle = z\,|\Psi_\nu(z)\rangle$

high momenta/large cutoffs lead to flipped-potential bound states of -λV for small λ/large η → strong coupling to high momenta/short range and Born series always nonperturbative

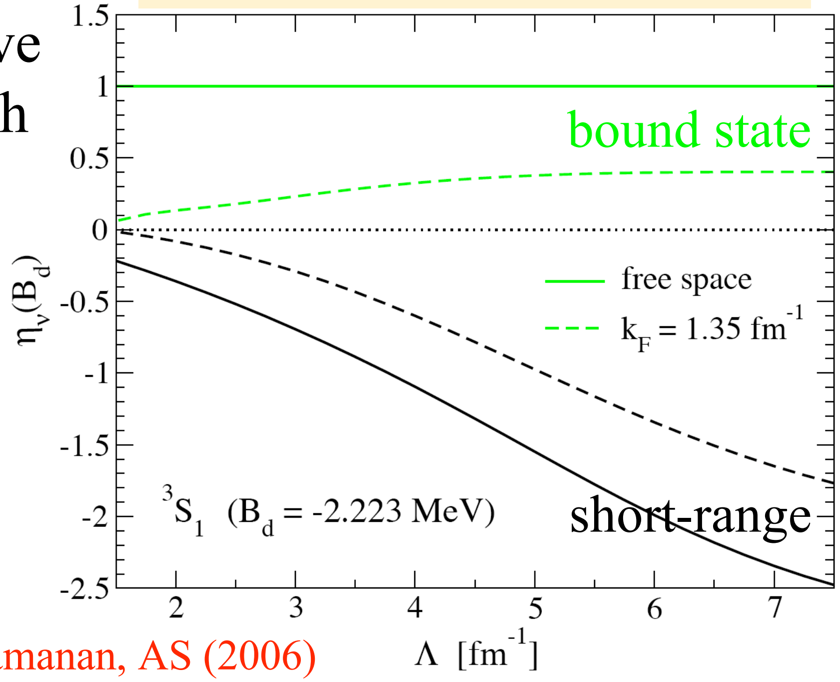
RG evolution decouples high momenta

loosely-bound state is dissolved in nuclear matter

E=0



Born series perturbative except in channels with bound states



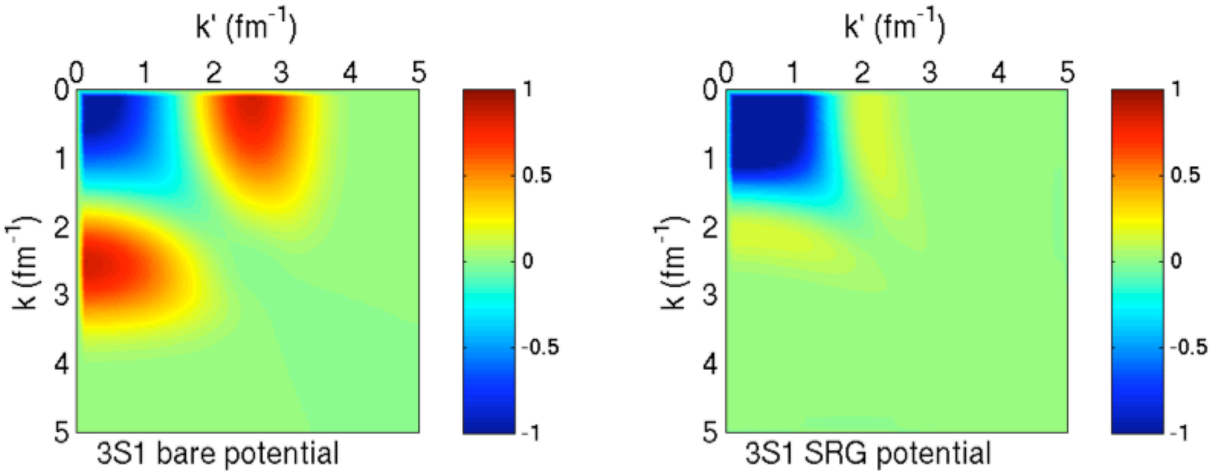
bound state

short-range

Bogner, AS, Furnstahl, Nogga (2005), Bogner, Furnstahl, Ramanan, AS (2006)

Is nuclear matter perturbative with chiral EFT and RG?

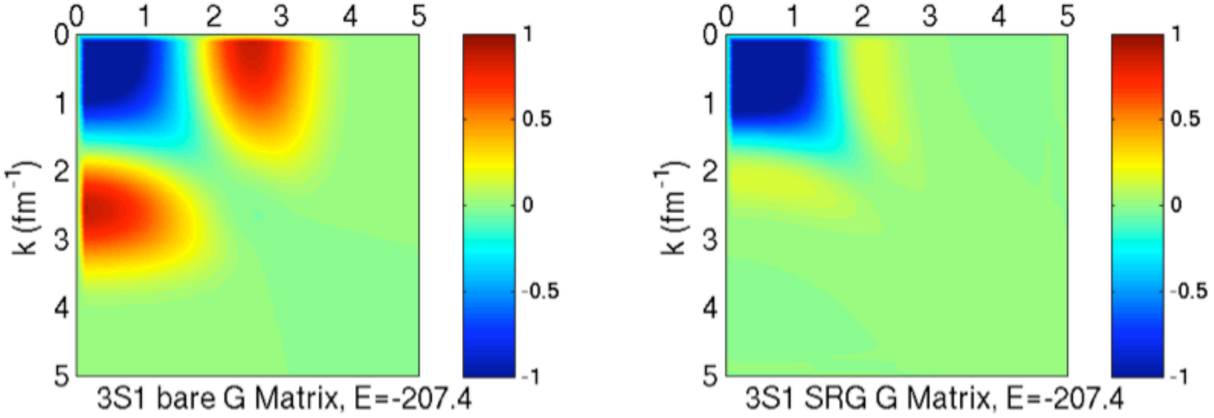
conventional Bethe-Brueckner-Goldstone expansion:
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling



chiral N^{3}LO

SRG/RG evolved

G matrix



conventional G-matrix approach does not solve off-diagonal coupling

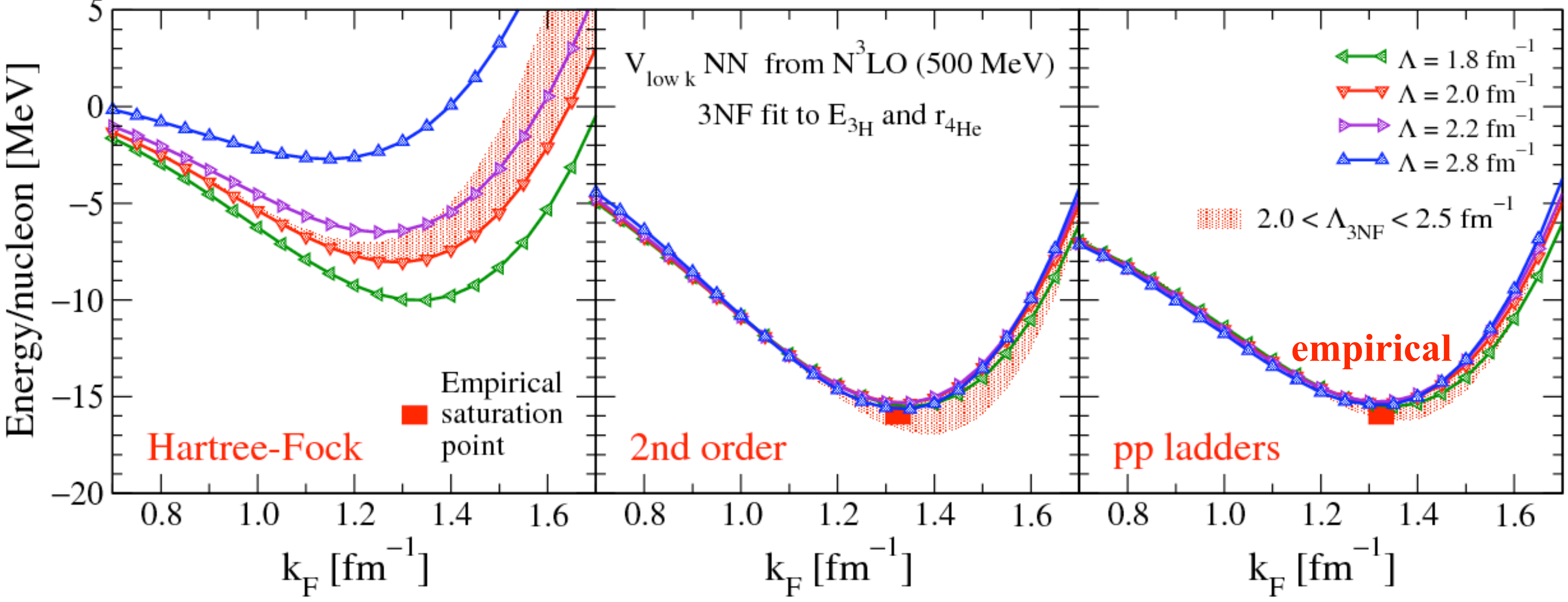
Is nuclear matter perturbative with chiral EFT and RG?

conventional Bethe-Brueckner-Goldstone expansion:
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling

start from chiral EFT and RG evolution:
nuclear matter converged at ≈ 2nd order, 3N drives saturation

weak cutoff dependence, improved by 3N fits to ^{4}He radius

exciting: empirical saturation within theoretical uncertainties



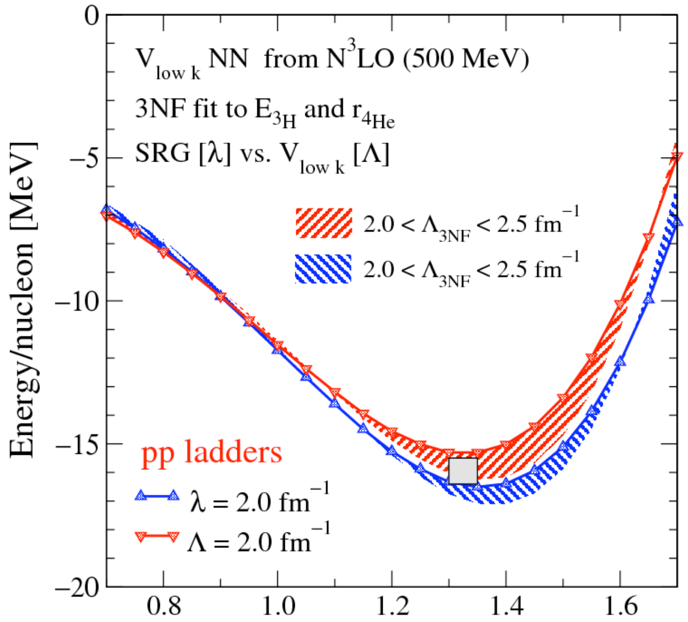
Bogner, AS, Furnstahl, Nogga (2005) and 0903.3366.

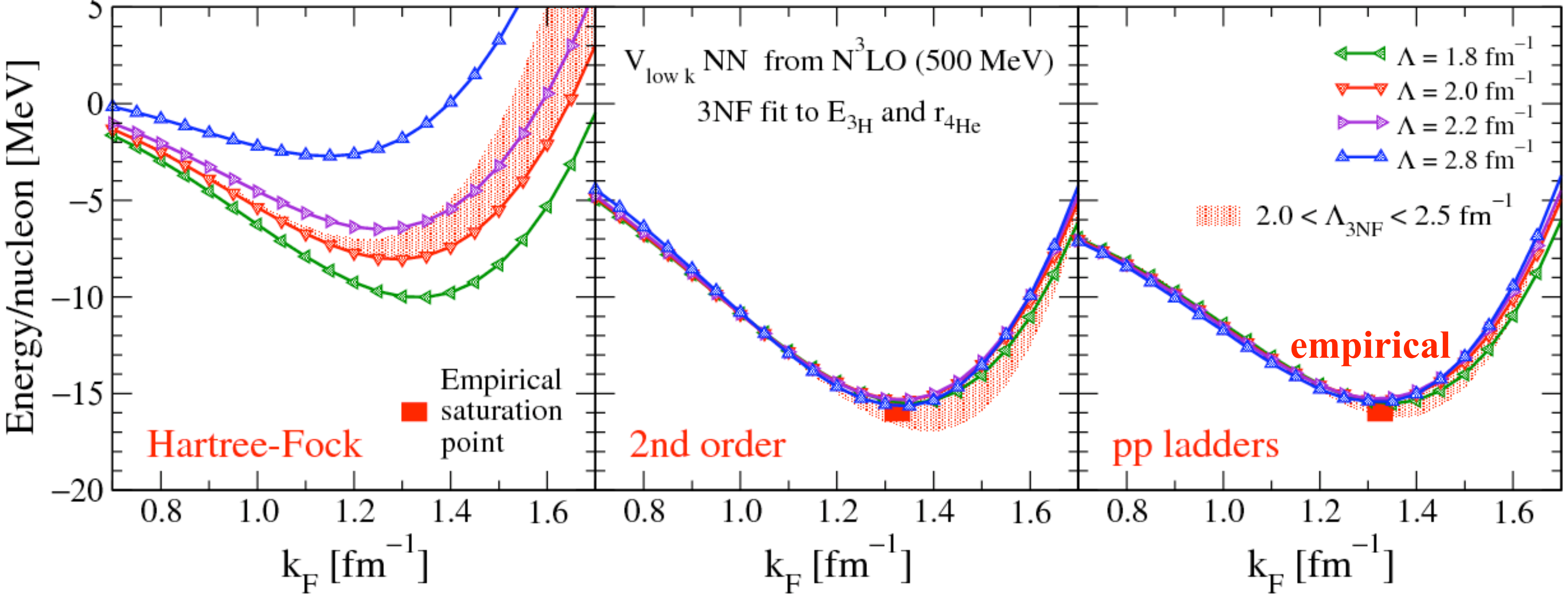
Is nuclear matter perturbative with chiral EFT and RG?

SRG nuclear matter results similar,
supports general nature of 3N fit and results

theoretical uncertainty ~1-2 MeV per particle

compressibility $K = 190 - 240\,\mathrm{MeV}$
mainly from $\Lambda_{3\mathrm{NF}} = 2.0 - 2.5\,\mathrm{fm}^{-1}$



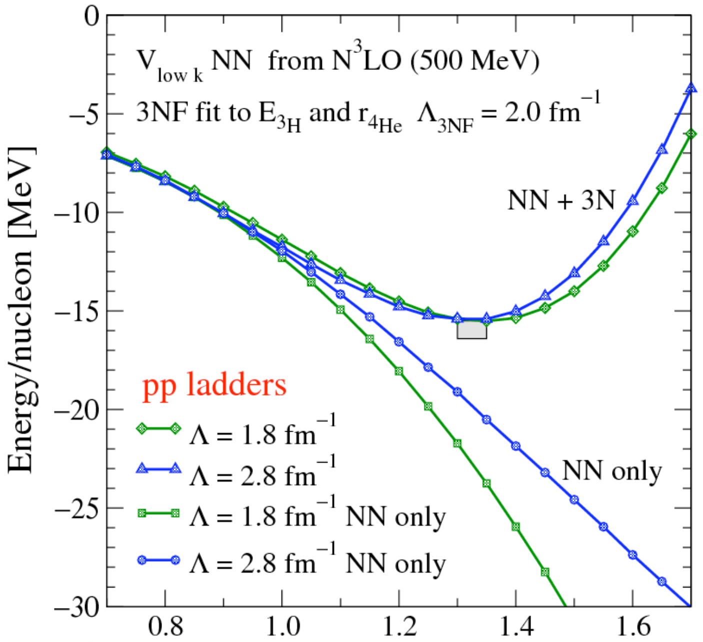


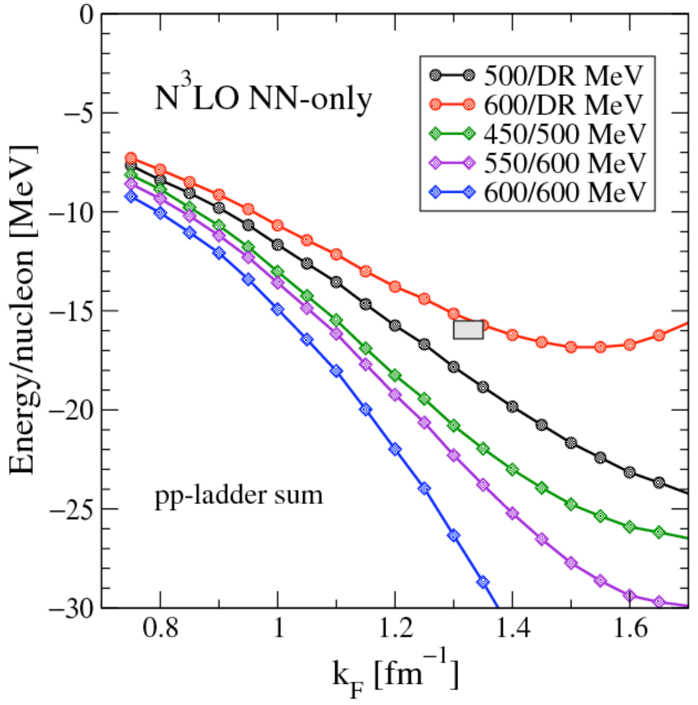
Impact of 3N interactions on nuclear matter

3N contributions natural but not small for RG-evolved and “bare” N^3LO interactions

long-range c_i-terms repulsive in nuclear matter and drive saturation

		Hartree-Fock					Hartree-Fock + dominant second order				
k_F	Λ	T	$V_{\text{low }k}$	V_c	V_D	V_E	T	$V_{\text{low }k}$	V_c	V_D	V_E
1.0	1.6	12.44	−19.62	1.65	0.42	−0.22	15.50	−26.58	1.49	0.34	−0.29
	1.9	12.44	−18.18	1.67	−0.25	0.40	16.29	−26.81	0.85	−0.09	0.55
	2.1	12.44	−17.35	1.67	−0.42	0.62	16.92	−27.04	0.11	0.05	0.79
	2.3	12.44	−16.56	1.67	−0.56	0.81	17.60	−27.27	−0.89	0.43	0.85
1.2	1.6	17.92	−31.47	5.37	1.31	−0.64	20.86	−37.66	4.59	1.03	−0.65
	1.9	17.92	−28.95	5.61	−0.81	1.18	21.80	−37.38	3.99	−0.50	1.28
	2.1	17.92	−27.51	5.67	−1.37	1.84	22.87	−37.53	2.27	−0.37	1.82
	2.3	17.92	−26.13	5.70	−1.86	2.42	24.32	−37.95	−0.38	0.51	1.78
1.35	1.6	22.67	−42.47	10.75	2.59	−1.21	26.09	−47.85	8.73	1.96	−1.12
	1.9	22.67	−38.82	11.95	−1.69	2.34	26.75	−46.72	9.14	−1.16	2.24
	2.1	22.67	−36.74	12.19	−2.91	3.68	28.05	−46.47	6.99	−1.33	3.22
	2.3	22.67	−34.77	12.30	−3.97	4.89	30.06	−46.45	3.10	−0.35	3.26





Impact of 3N interactions on neutron matter

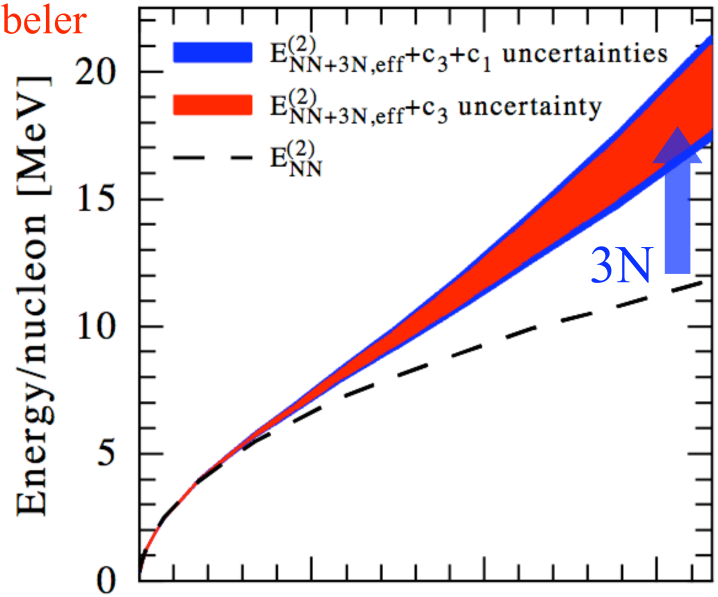
Tolos, Friman, AS (2007); Hebeler, AS, in prep., see talk by Kai Hebeler

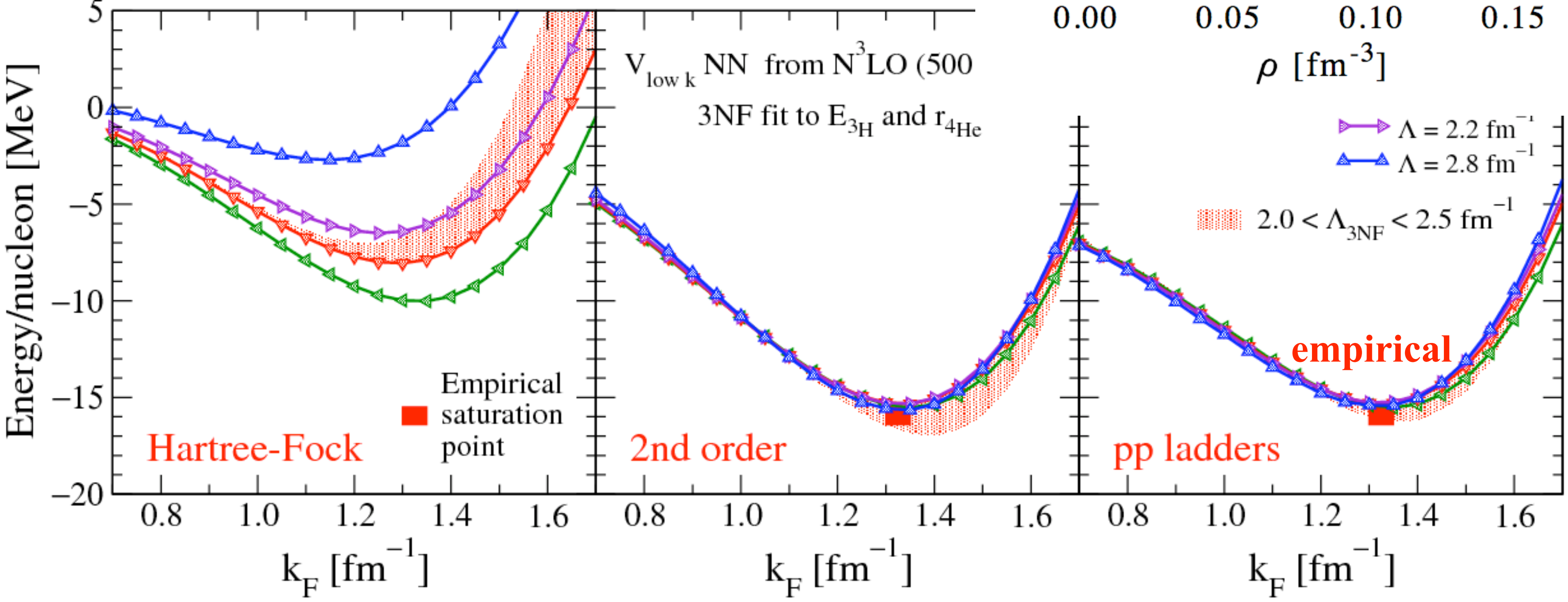
symmetry energy and density dependences

c_i uncertainties overwhelm cutoff variation

D, E terms vanish in spin-sat. neutron matter

developing density-dep. NN interactions
to include 3N effects



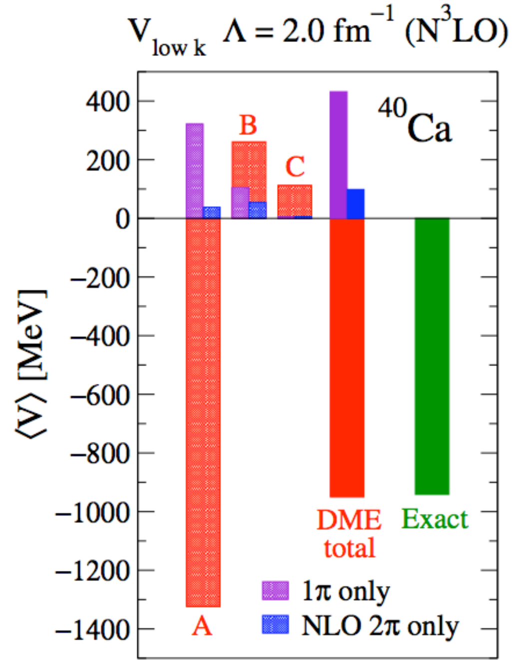


Towards ab-initio density functional theory

low-momentum interactions key to develop universal density functional

first DFT results with microscopic pairing functional

Lesinski, Duguet, Bennaceur, Meyer (2008), see talk by Thomas Lesinski



density matrix expansion

Bogner, Furnstahl, Platter (2008), see also Finelli, Kaiser, Weise (2003)

$$\mathcal{E} = \frac{\tau}{2M} + A[\rho] + B[\rho]\tau + C[\rho]|\nabla\rho|^2 + \cdots$$

improved DME: use phase space averaging for nuclei

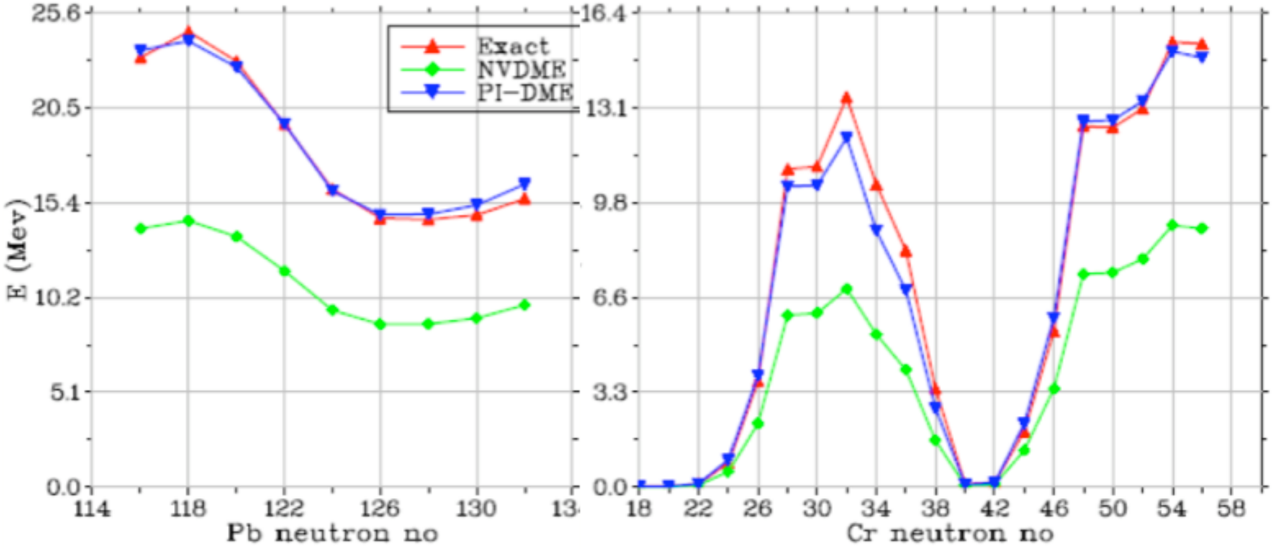
Gebremarian, Bogner, Duguet, in prep.

use EFT/RG interactions:

to identify new terms

to quantify errors

to benchmark with ab-initio methods



Outline

5. Summary: impact of three-nucleon forces

Thanks to collaborators

TRIUMF S. Bacca, S. Baroni, J. Braun, K. Hally, K. Hebeler, J. Holt, M. Ku

THE OHIO STATE UNIVERSITY R.J. Furnstahl, R.J. Perry, S. Ramanan, E. Anderson, E. Jurgenson

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東京大学 THE UNIVERSITY OF TOKYO T. Otsuka 日本大学 T. Suzuki

Summary

development of EFT and RG for nuclear forces

advances in ab-initio methods and in nuclear matter theory

EFT and RG interactions enable a unified description
from light to heavy nuclei and matter in astrophysics

three-nucleon forces play a central role
spin-orbit splittings and spin-orbit shell closures
helium halo nuclei, location of the neutron drip line
saturation, symmetry energy, density dependences

individual 3N parts can have interesting dependences
c_i-terms attractive in light nuclei, repulsive in bulk matter
(scheme dependent)