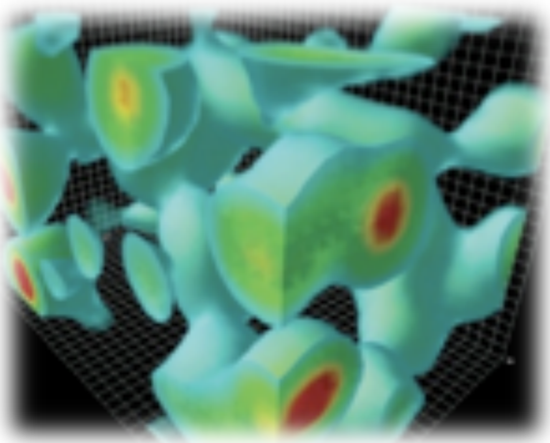
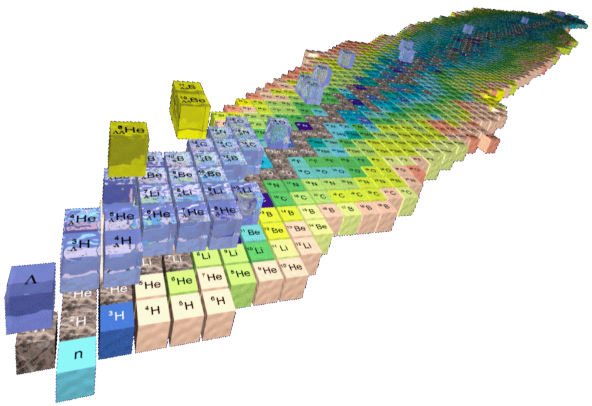




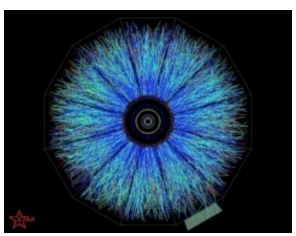
A First Look at Three Baryon Systems with Lattice QCD

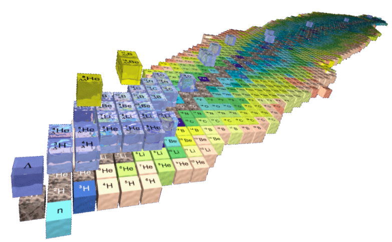
Martin J. Savage
University of Washington

(June 2009, INT)

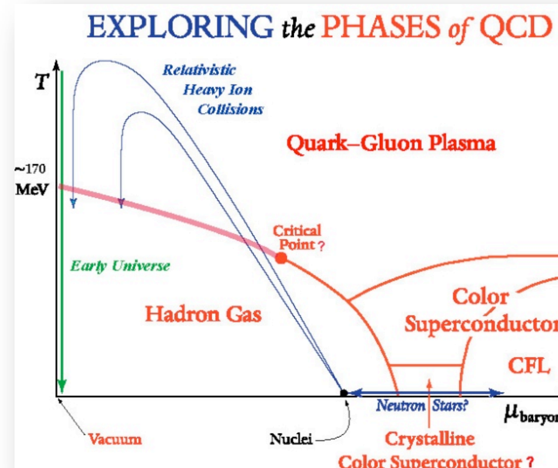
Solving QCD : What is to be gained ?
- Nuclear Physics IS QCD + electroweak

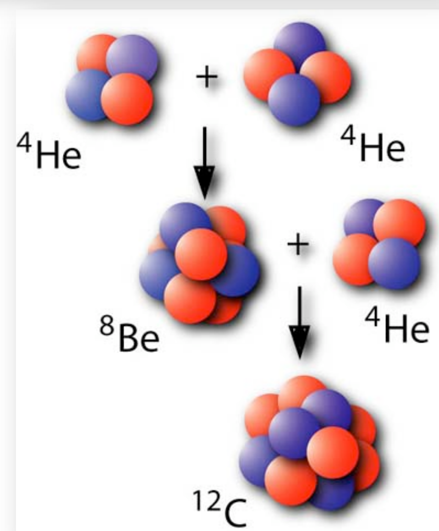
- Predictions without experiment :
  - interpolation is the tradition
    - *e.g. NN phase-shifts and NN potentials*



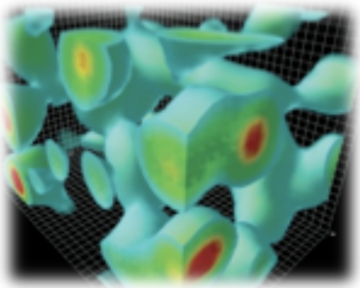


- Allow exploration of other ``*universes*”
  - *different quark masses, electroweak couplings,* θ

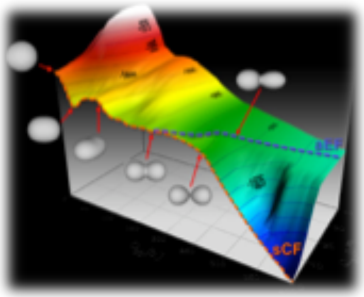




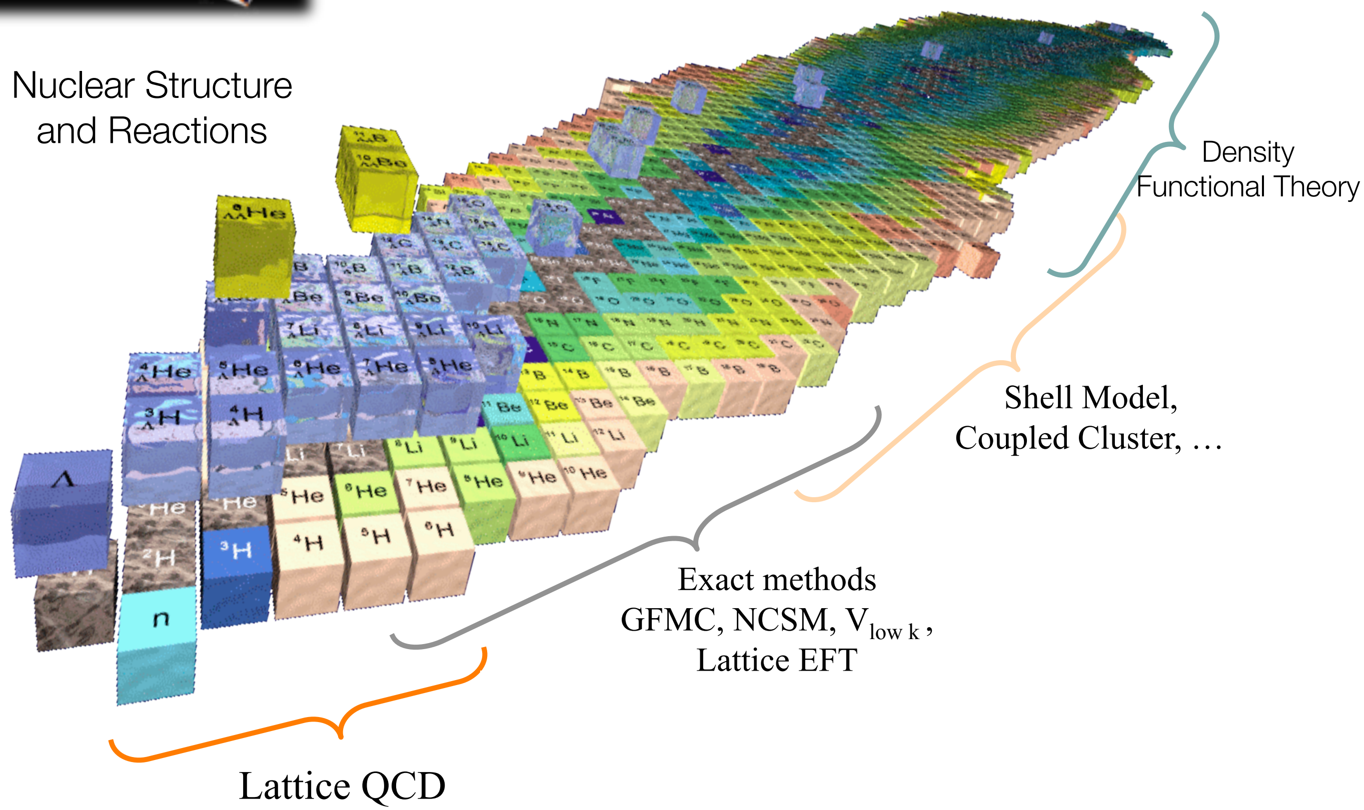
Exa-scale computing will unify Nuclear Physics



Cold QCD and
Nuclear Forces



Nuclear Structure
and Reactions



A Lattice QCD Calculation

Wilson

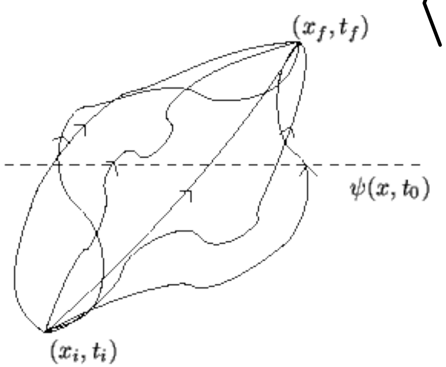
Feynman

Numerical evaluation of the Euclidean-space path integral

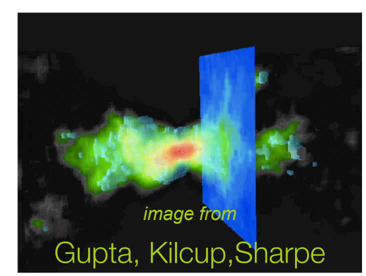
- quarks and gluons → First-principles QCD calculation

e.g. triton correlation function

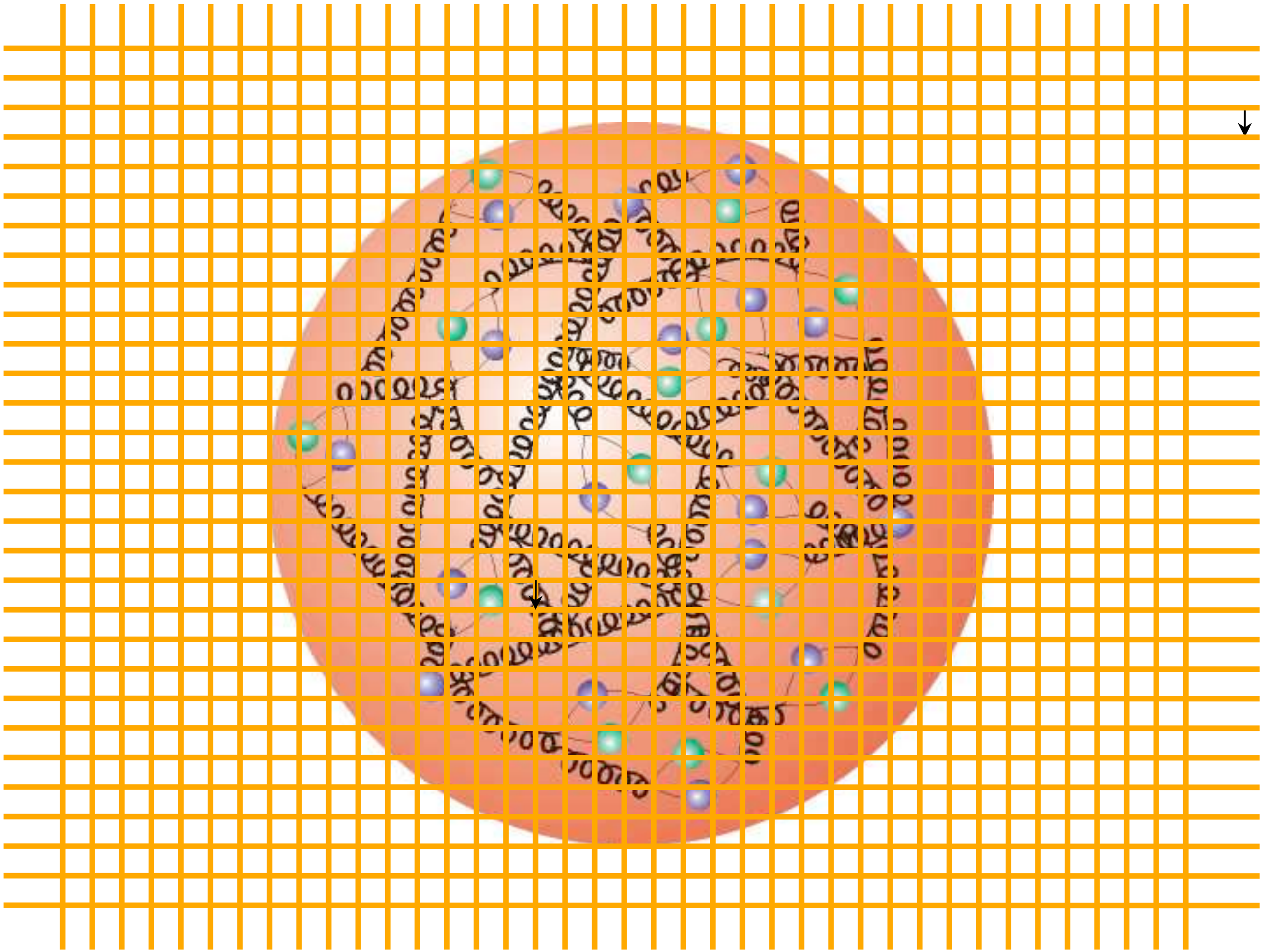
$$\langle \hat{\theta} \rangle \sim \int \mathcal{D}\mathcal{U}_\mu \; \hat{\theta}[\mathcal{U}_\mu] \; \det[\kappa[\mathcal{U}_\mu]] \; e^{-S_{YM}}$$



$$\rightarrow \frac{1}{N} \sum_{\text{gluonfields}}^{N} \hat{\theta}[\mathcal{U}_\mu]$$



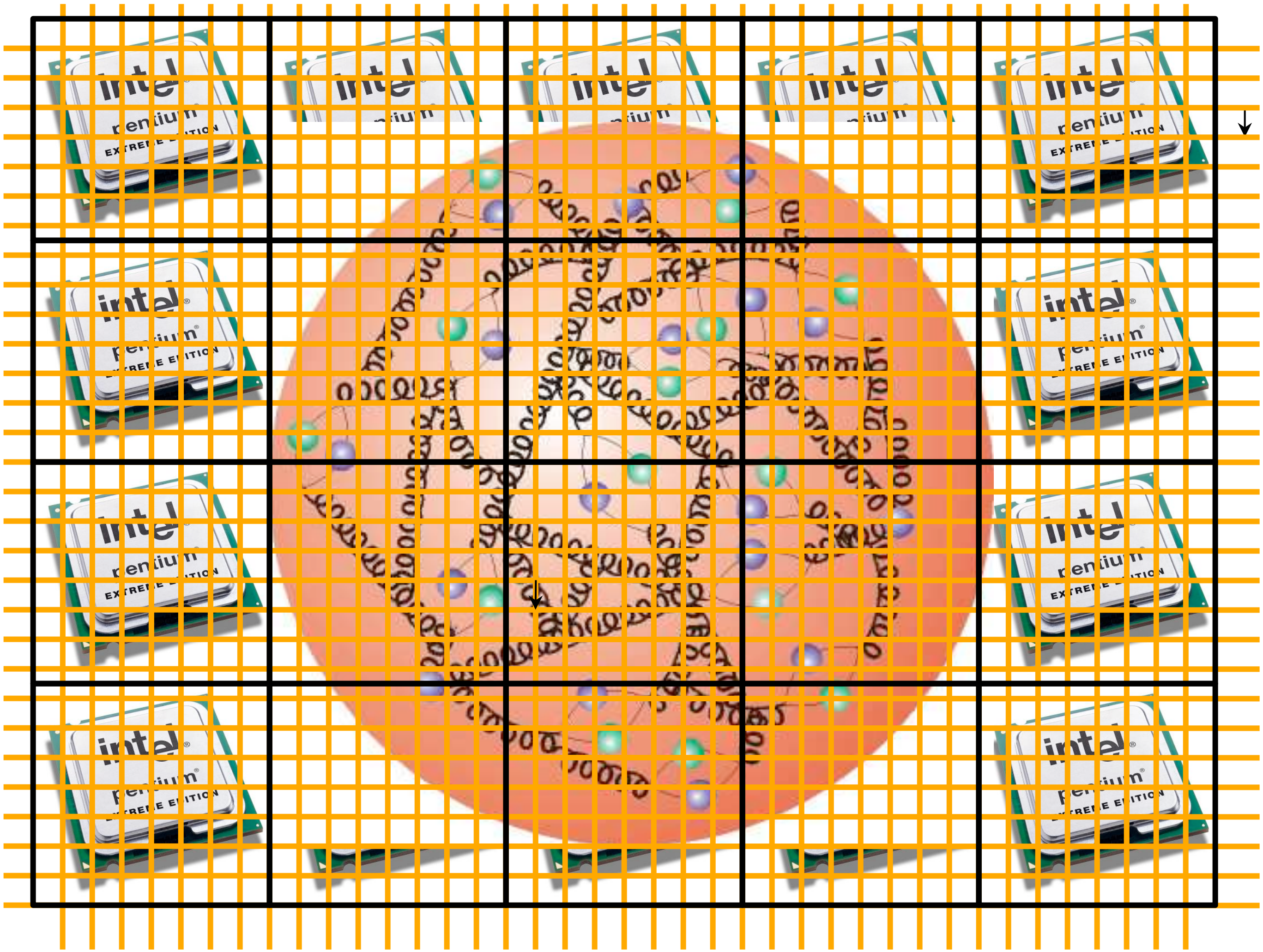
Lattice QCD



Lattice Spacing :

$a << 1/\Lambda\chi$

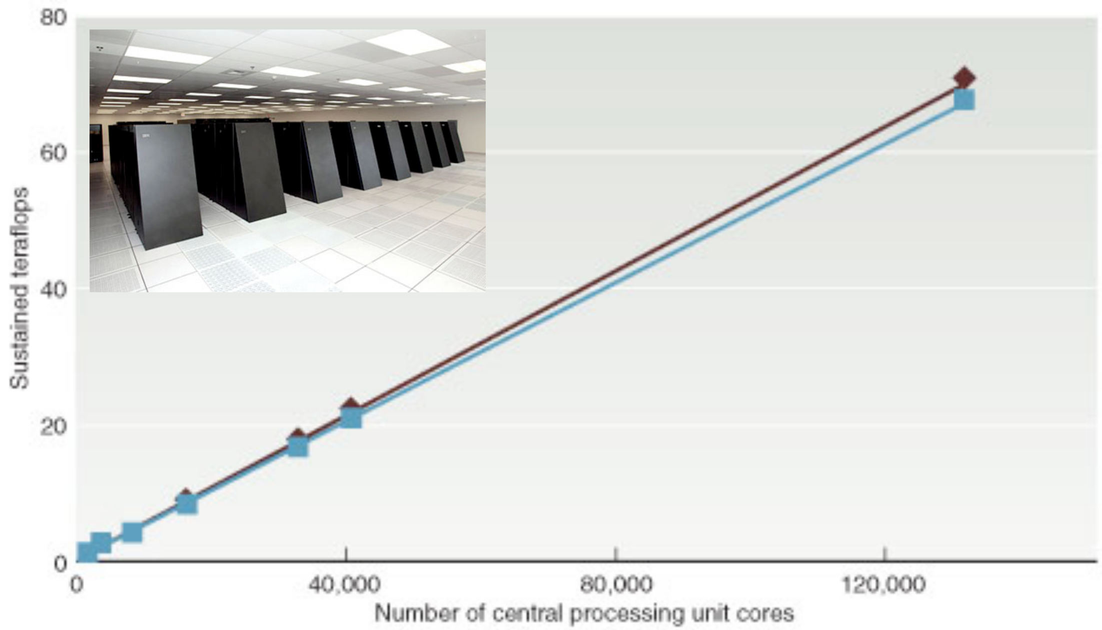
Lattice QCD



Lattice Spacing :

$$a << 1/\Lambda\chi$$

Significant Computational Needs

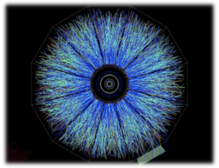


- e.g. – 1 gluon field cfg
  - m_π = 260 MeV
  - Lattice spacing = 0.10 fm
  - Lattice extent = 4.8 fm
  - Domain-wall sea quarks
  - $48^3 \times 64 \times 16$

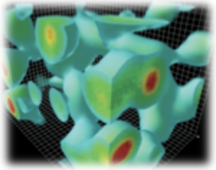
- 7 Tflop days

Need thousands for T=0 Nuclear Physics

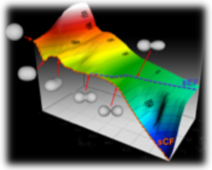
Near-Term LQCD Goals in Nuclear Physics



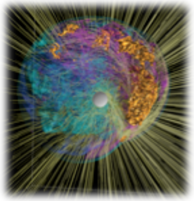
- Finite-Temperature QCD
  - Phase diagram : T and μ



- Spectrum and Structure of Matter
  - Hadronic structure
  - Baryon excitations
  - Exotics

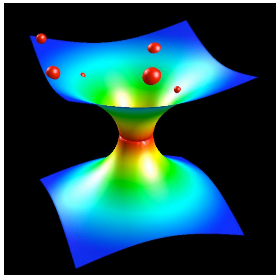


- Nuclear Reactions
  - NN, NNN, YN, MEC's



- Astrophysics
  - YN , Kaon condensation

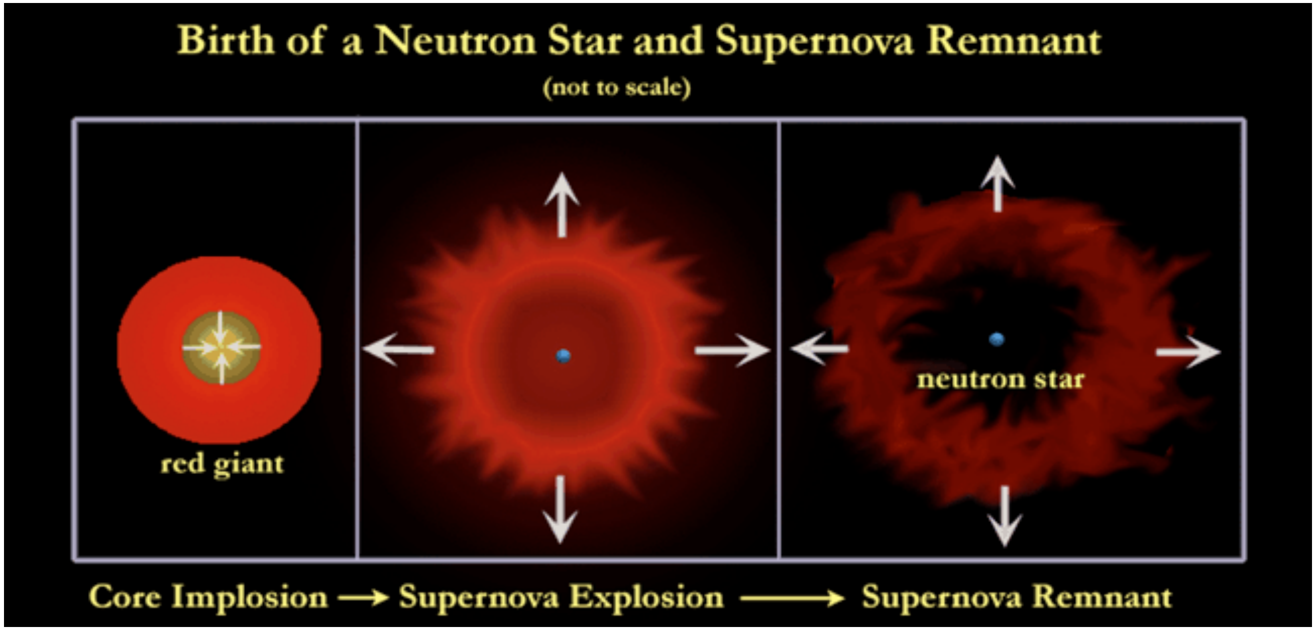
Dense Matter



Equation of State of Nuclear Material at High Densities ??

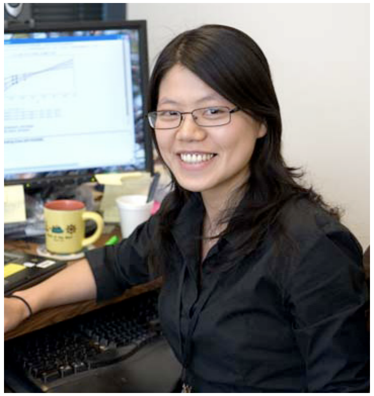
- Kaon (strange meson) Condensate ?
- Sigma (strange) Baryons ?

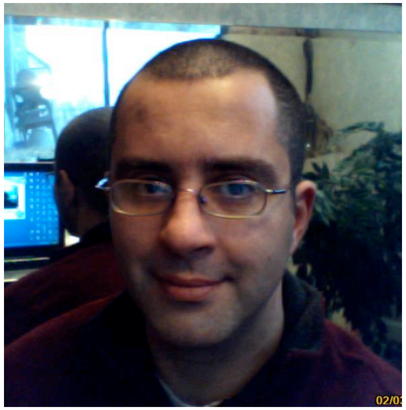
Is SN1987A a Black Hole or a Neutron Star ?

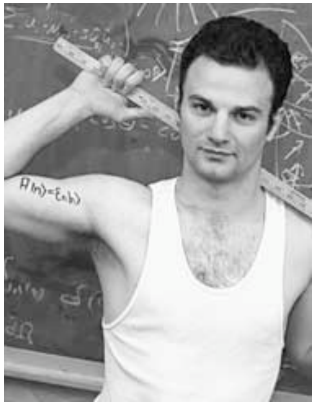


NPLQCD

+







Barcelona, LLNL, New Hampshire, Washington, William+Mary

... to make predictions for the structure and interactions of nuclei using lattice QCD.

USQCD

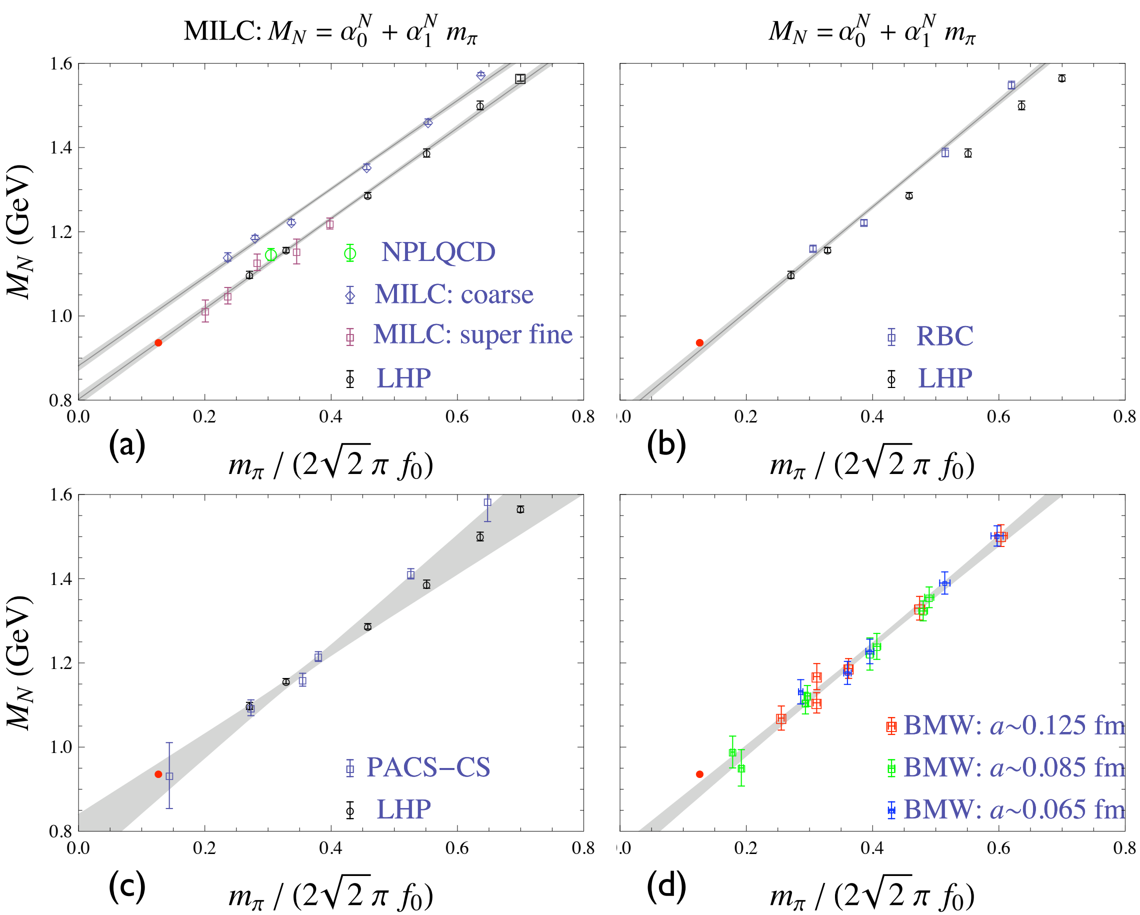
US Lattice Quantum Chromodynamics

SciDAC

Scientific Discovery through Advanced Computing

2008 (JLab)

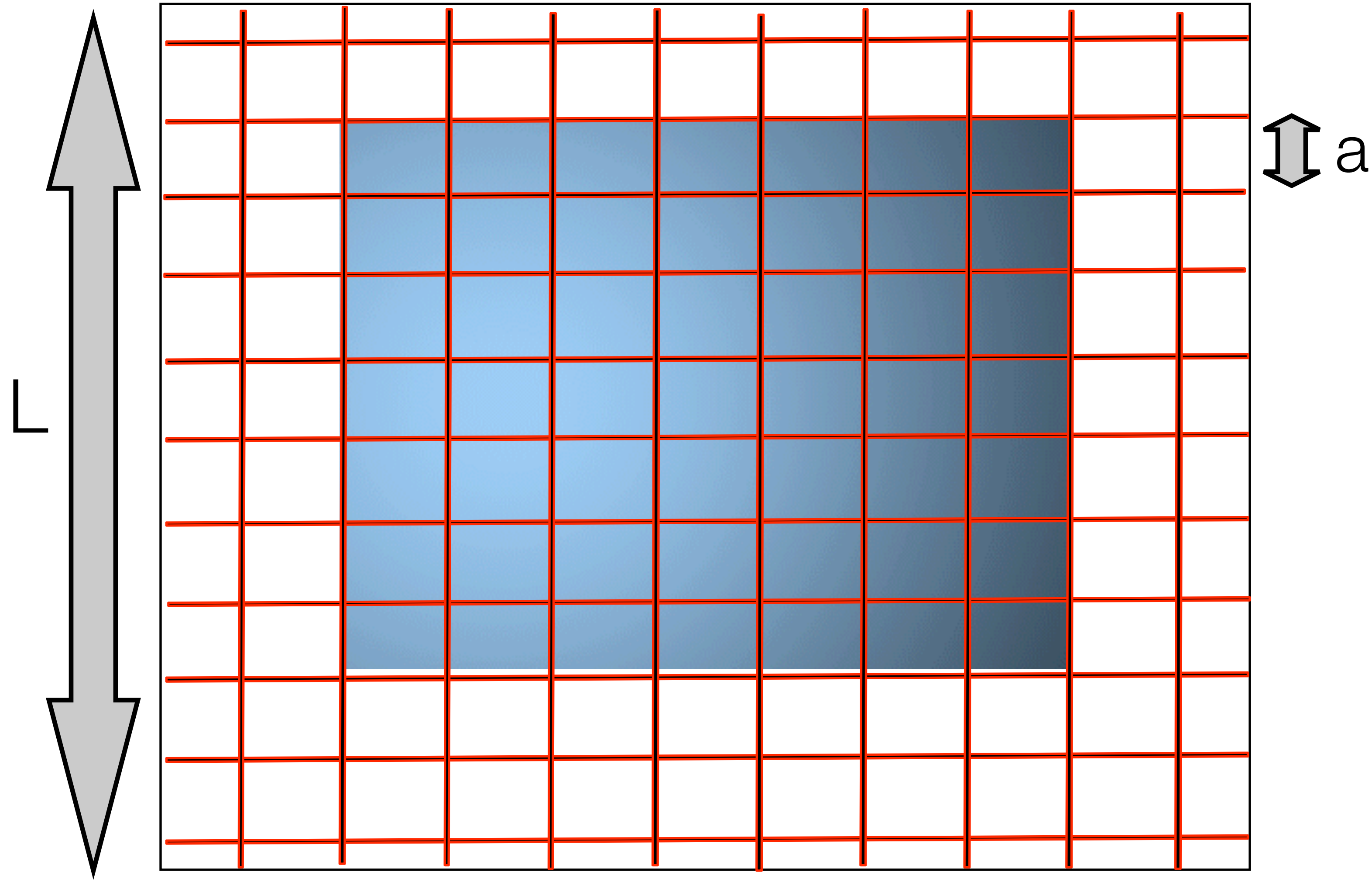
Extrapolations



Nucleon Mass -- Walker-Loud

- m_q ⟶ Interpolation
  - Rigorous for small m_q
  - physical quark masses
- lattice spacing
  - a~ 0.12, 0.09, 0.06 fm today
- lattice volume
  - L~ 2.5, 3.0, 3.5 fm today
  - expect L >> 5 fm very soon
  - Smaller volumes for scattering

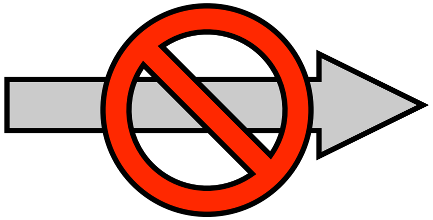
Hadronic Interactions from Lattice QCD



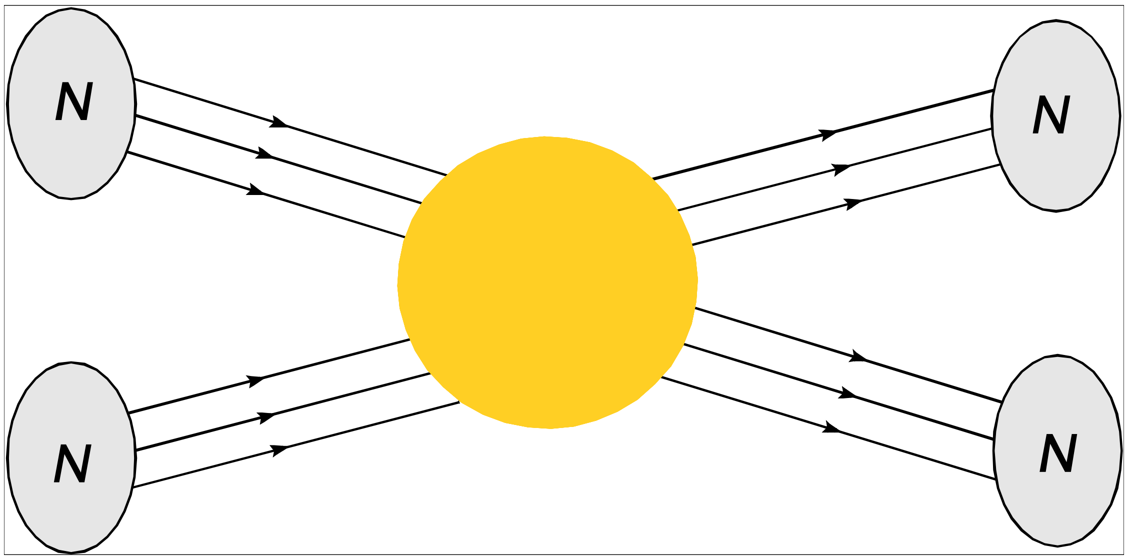
Maiani-Testa Theorem

Implications for Nuclear Physics !

Away from Kinematic Thresholds



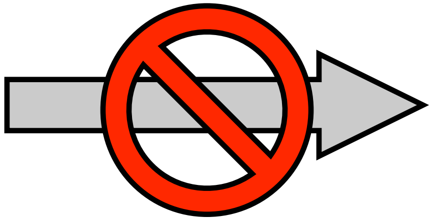
Maiani-Testa Theorem
Implications for Nuclear Physics !



$\delta(s)$?

Away from Kinematic Thresholds

$G_{NN}(s)^{\text{Euclidean}}$



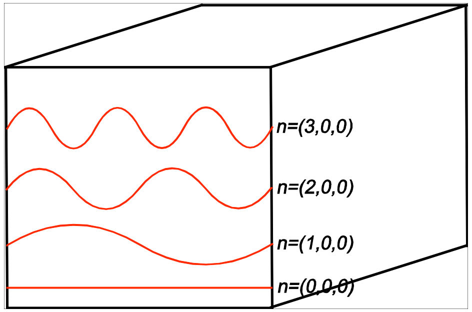
$G_{NN}(s)^{\text{Minkowski}}$

Two-Particle Energy Levels (Luscher)

Below Inelastic Thresholds :

Measure on lattice $\Rightarrow$ $$\delta E = 2\sqrt{p^2 + m^2} - 2m$$

UV regulator



Two-Particle Energy Levels (Luscher)

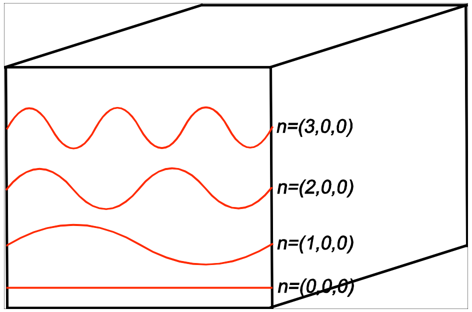
Below Inelastic Thresholds :

Measure on lattice ⟹ $$\delta E = 2\sqrt{p^2 + m^2} - 2m$$

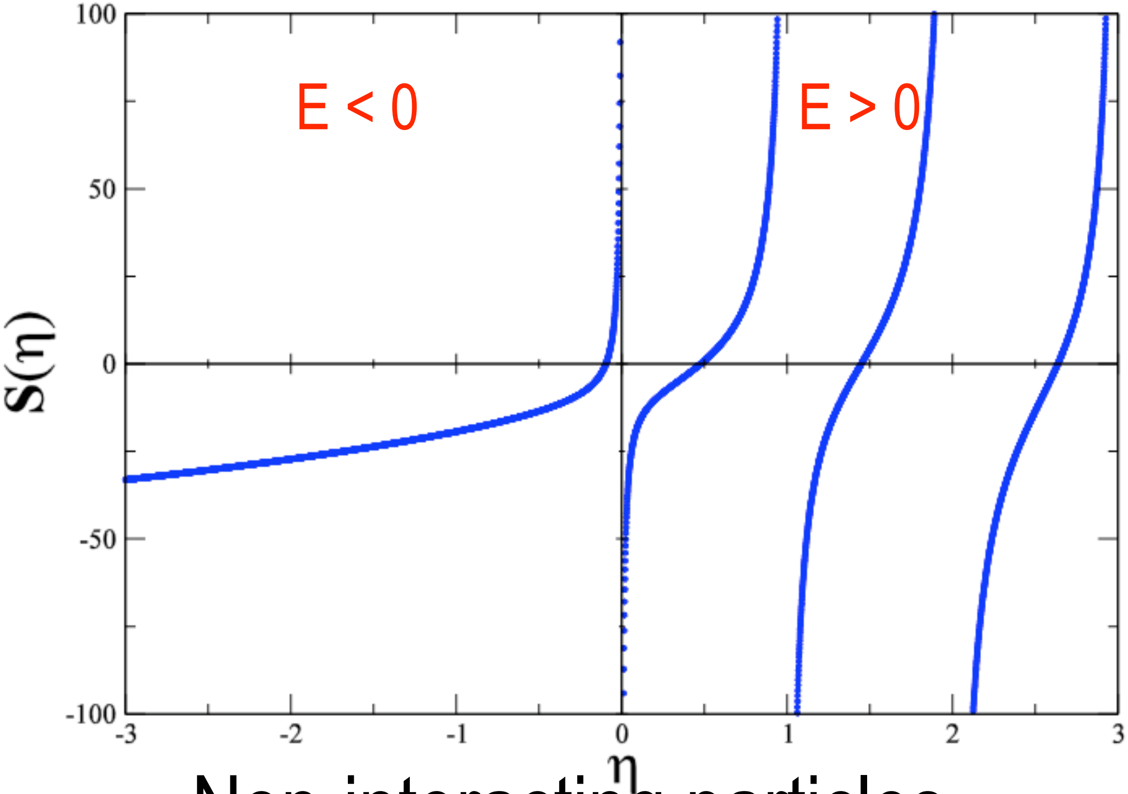
$$p \cot \delta(p) \;=\; \frac{1}{\pi L}\, \mathbf{S}\left(\left(\frac{Lp}{2\pi}\right)^2 \right)$$

UV regulator

$$\mathbf{S}(\eta) \;\equiv\; \sum_{\mathbf{j}}^{\Lambda_j} \frac{1}{|\mathbf{j}|^2 - \eta} \;-\; 4\pi\Lambda_j$$



Luscher Relation



Bound-state or
Scattering state ?

Non-interacting particles

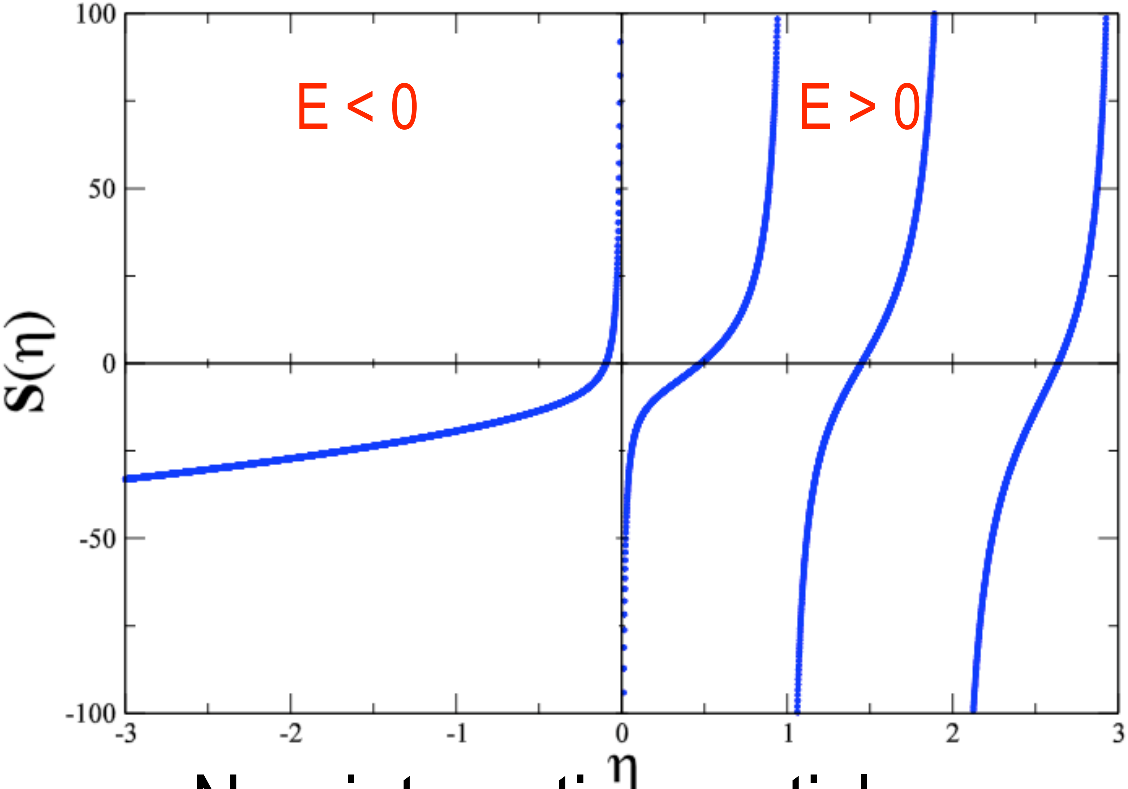
$$V = 0 \quad \rightarrow \quad a = r = 0$$

$$S = \infty$$

$$k = \frac{2\pi}{L} n$$

$$n = (nx, ny, nz)$$

Luscher Relation



Bound-state or
Scattering state ?

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S} \left(\left(\frac{Lp}{2\pi} \right)^2 \right)$$

Non-interacting particles

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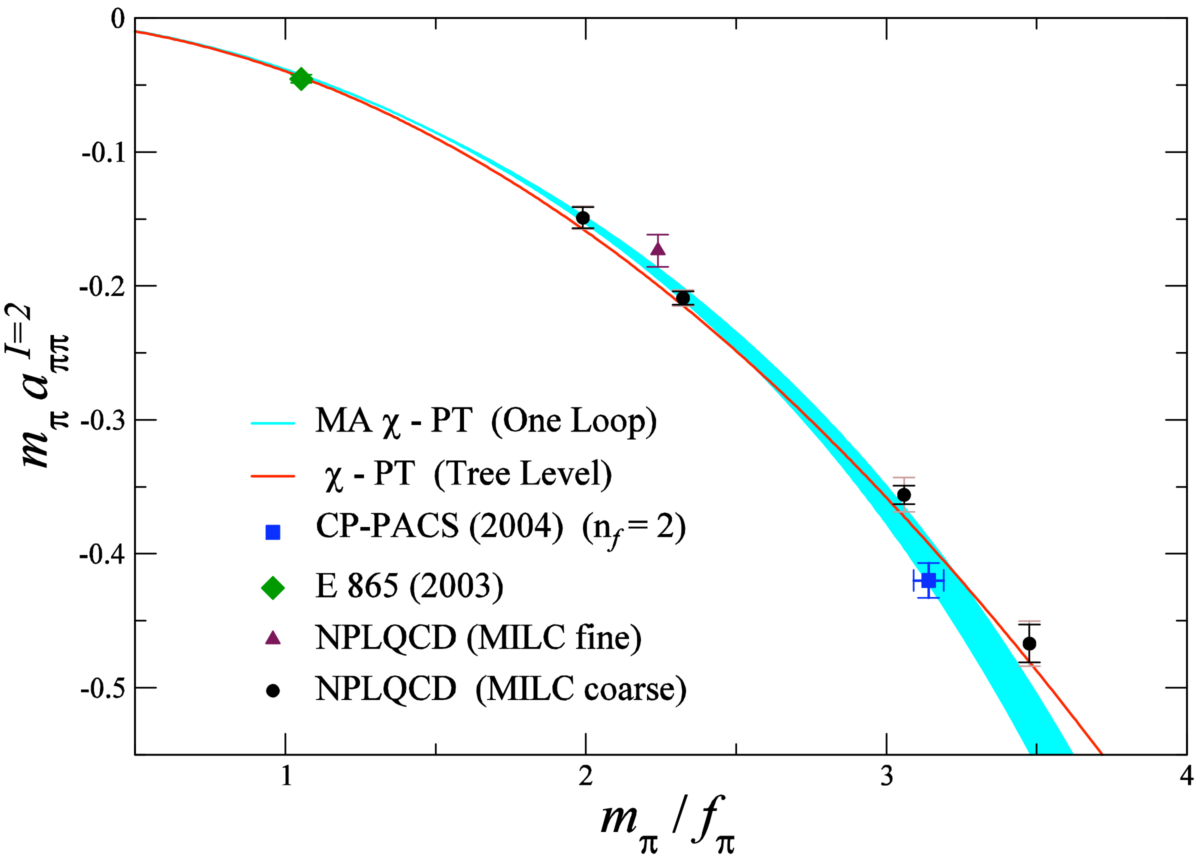
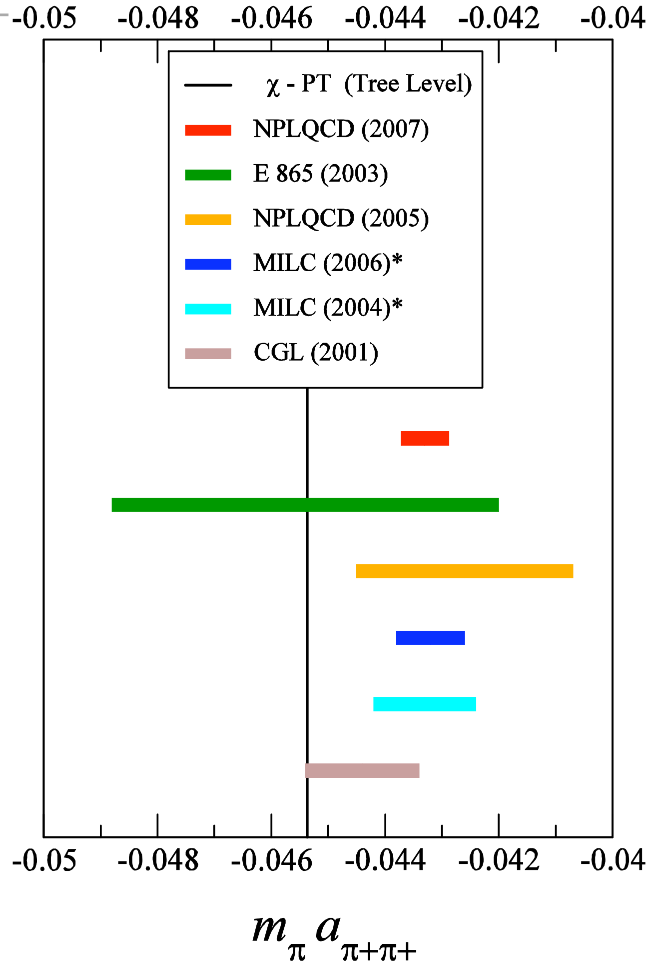
Meson-Meson Scattering

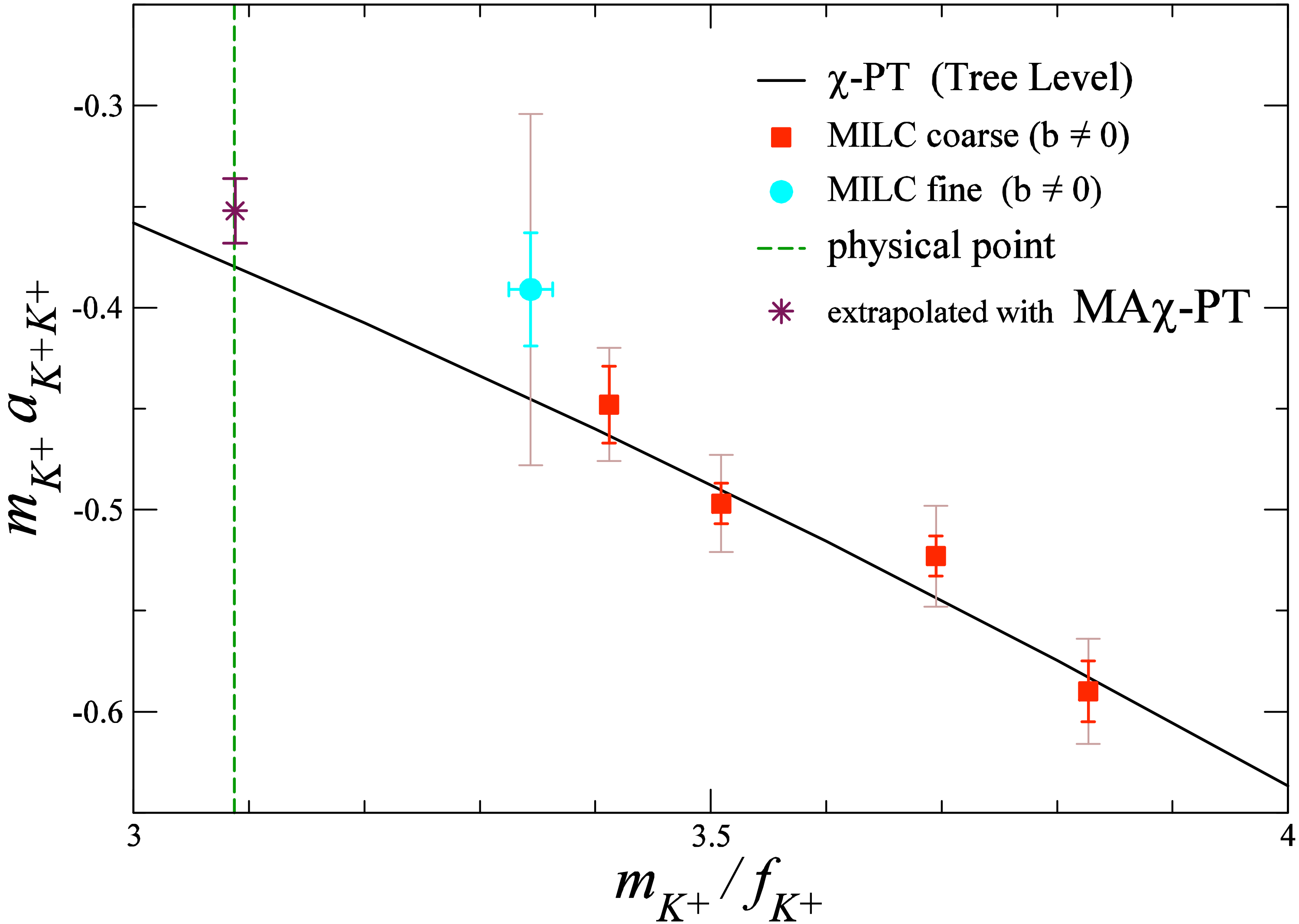




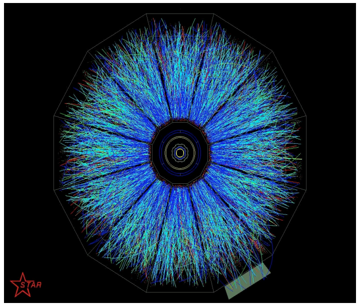
Weinberg's tree-level predictions agree with LQCD !!!

- Why only trees ???

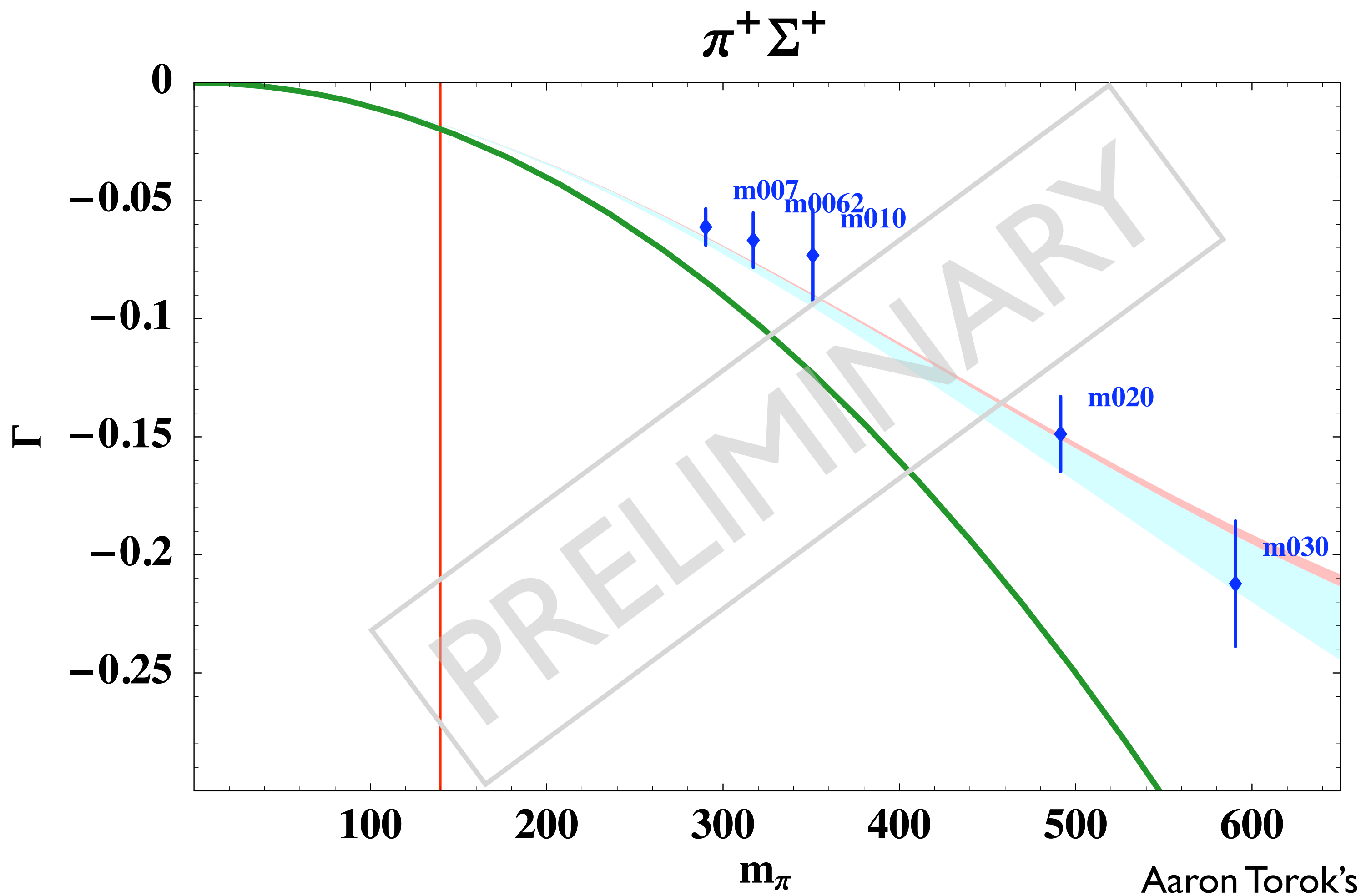
$K^+ K^+$



χ-PT (Tree Level)
MILC coarse (b ≠ 0)
MILC fine (b ≠ 0)
physical point
extrapolated with MAχ-PT
-0.3
-0.4
-0.5
-0.6
3
3.5
4
$m_{K^+} a_{K^+K^+}$
m_{K^+} / f_{K^+}

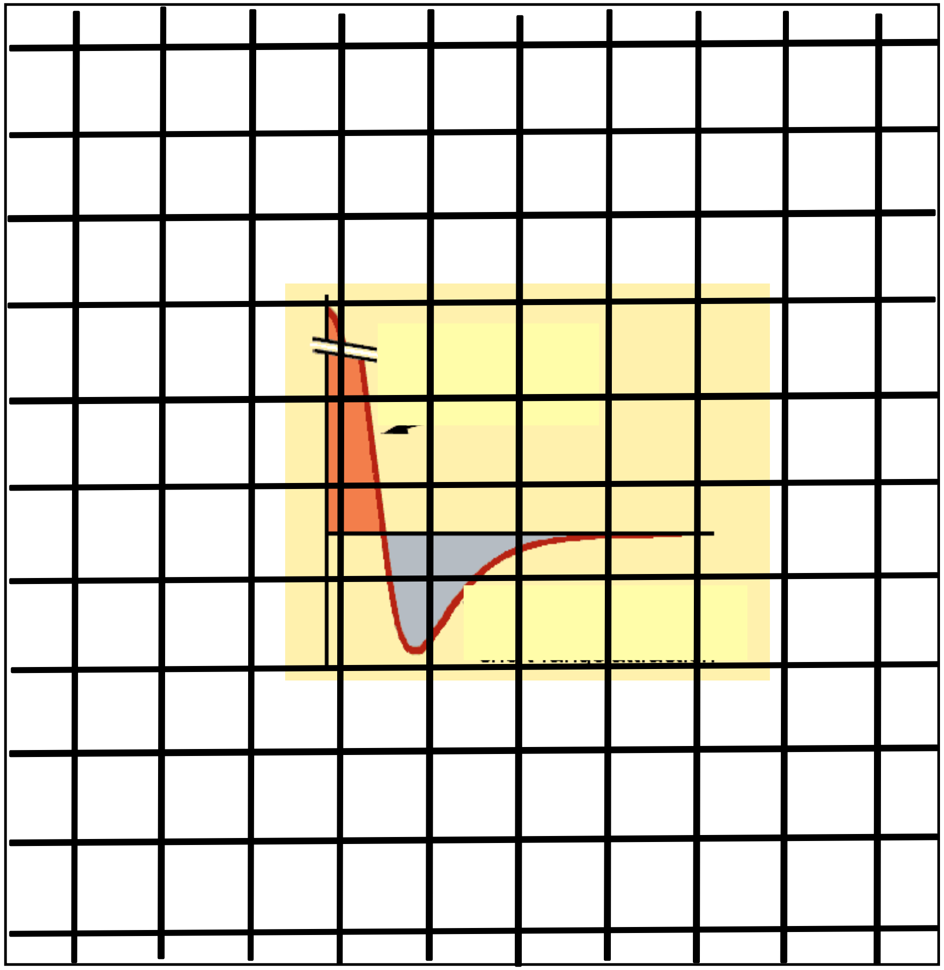


Meson-Baryon



Aaron Torok's PhD thesis
University of New Hampshire

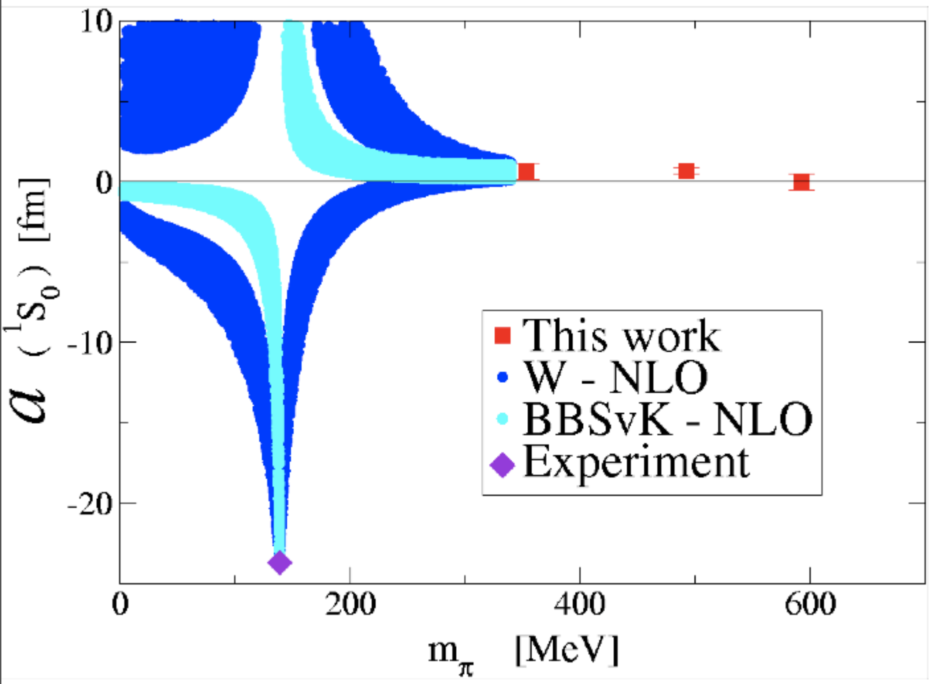
Large Scattering Lengths are OK !

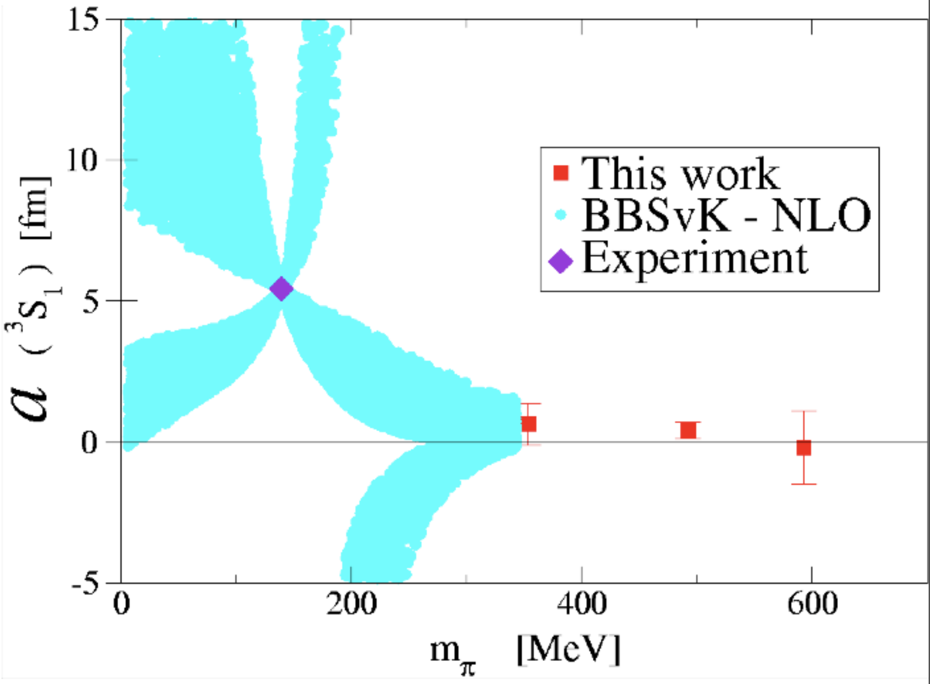


Require : L >> r_0

but ANY a

NN Scattering

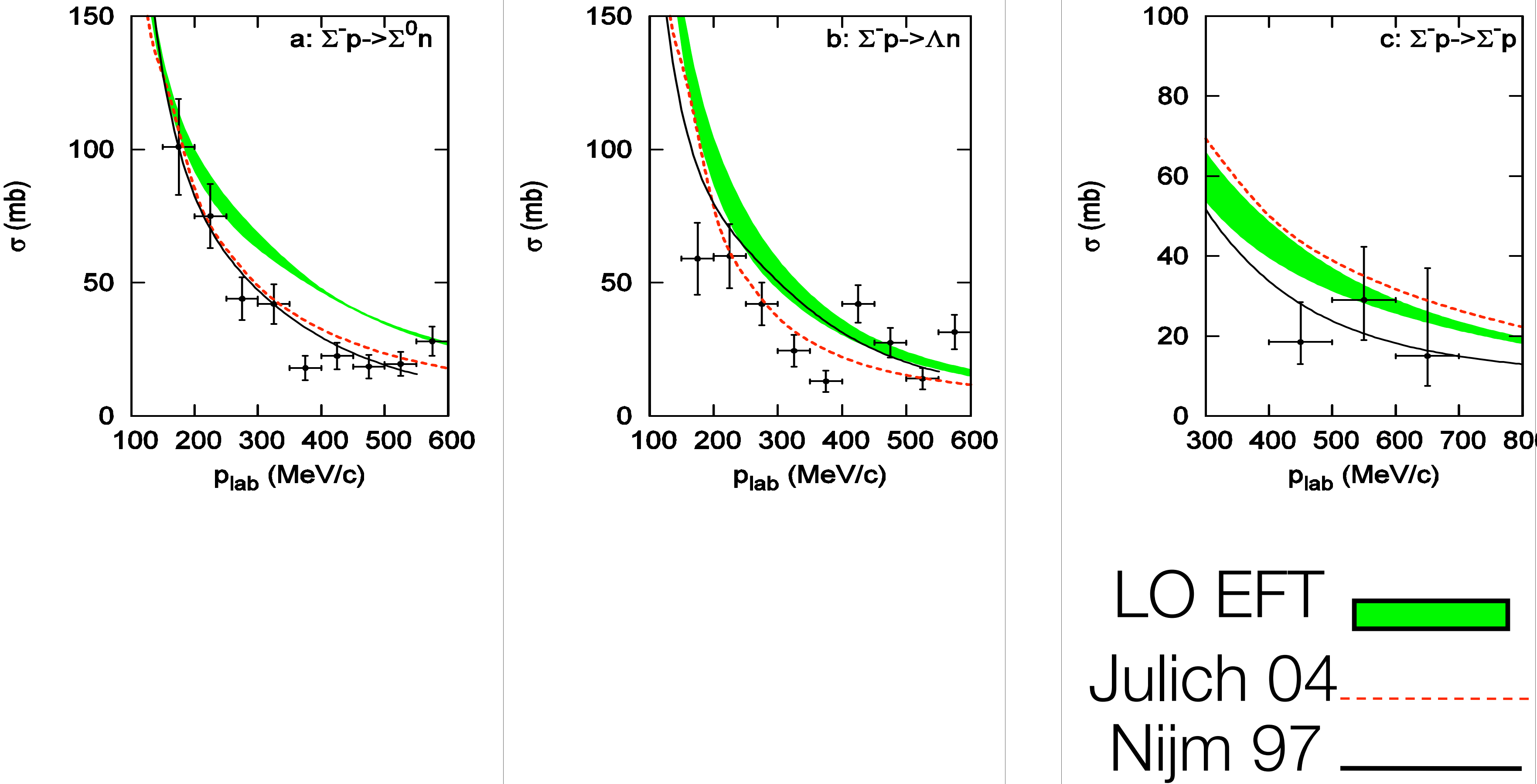




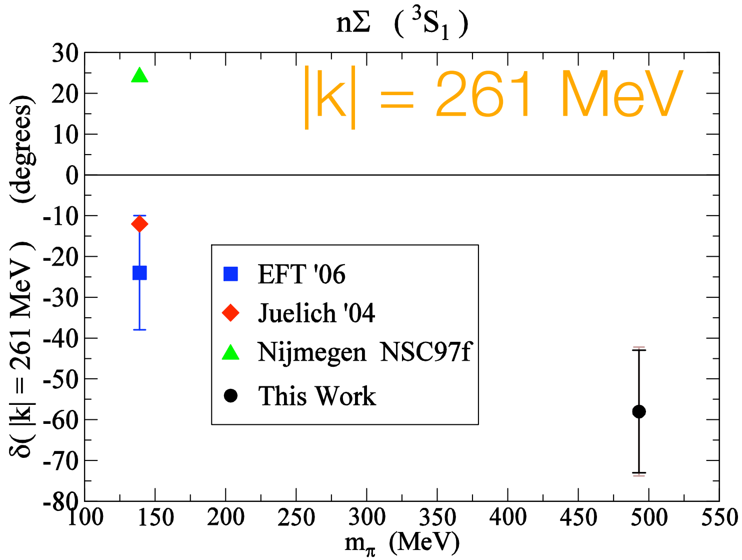
$^1S_0$: pp , pn , nn

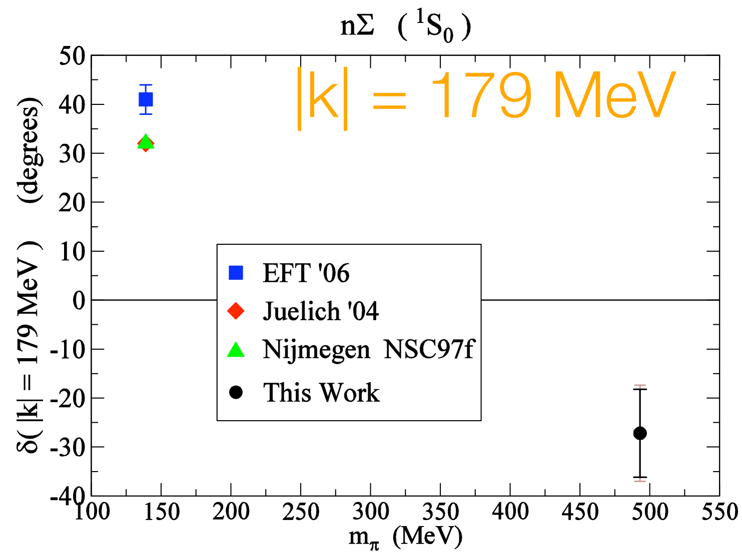
$^3S_1$-$^3D_1$: pn : deuteron

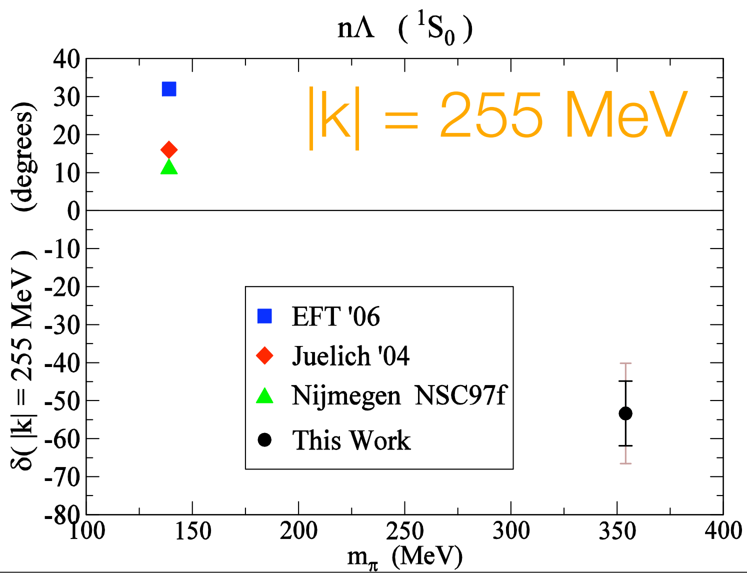
Some of the Experimental Hyperon-Nucleon Data

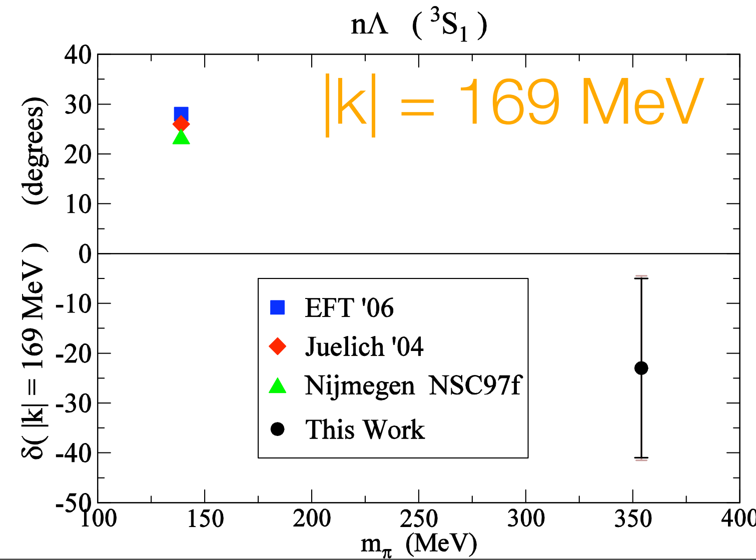


Hyperon-N Interactions

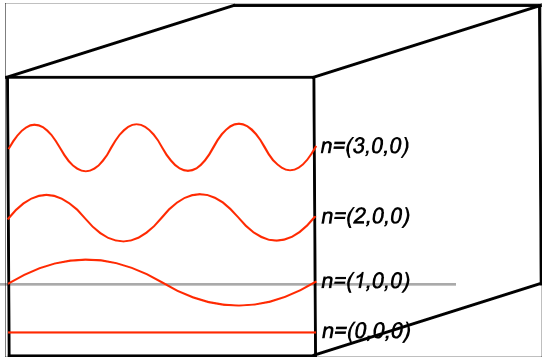


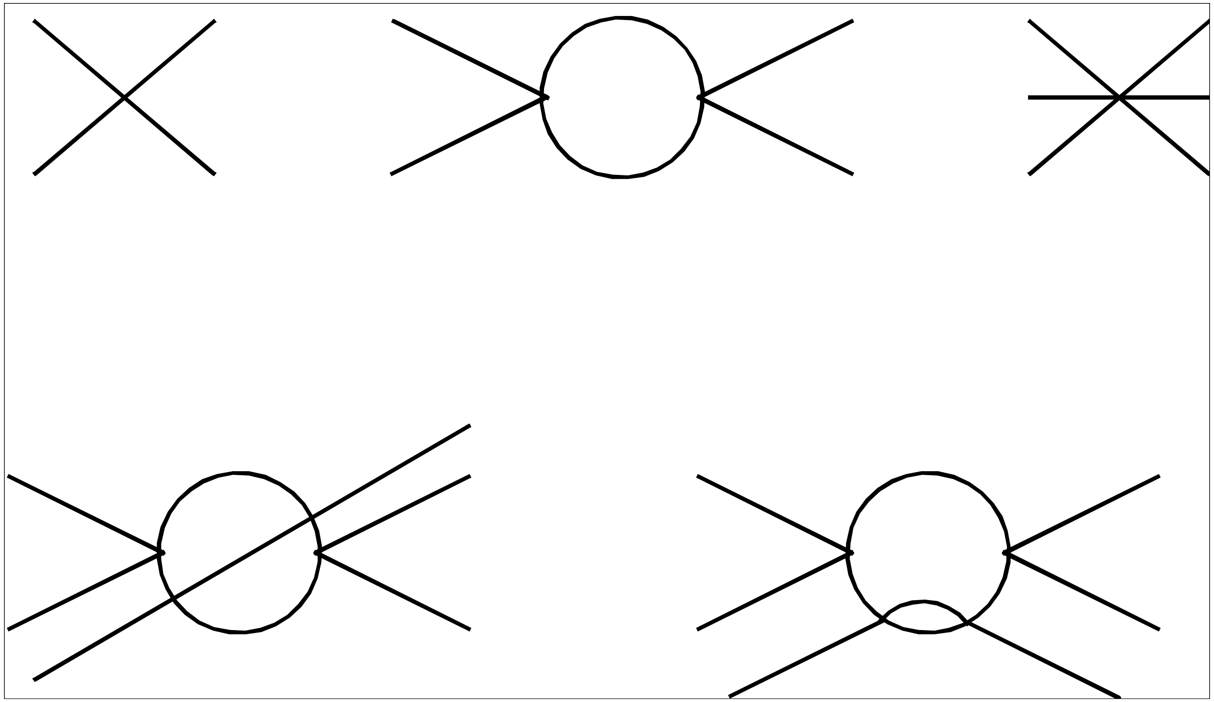






Multi-π's and Many-Body





3-π interaction

divergent

n-Bosons in a Finite Volume

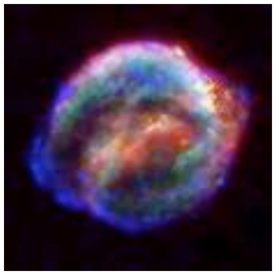
$$
\begin{aligned}
E_0(n,L) \ = \ & \frac{4\pi\, a}{M\, L^3}\binom{n}{2}\Bigg\{1-\left(\frac{a}{\pi\, L}\right)\mathcal{I}+\left(\frac{a}{\pi\, L}\right)^2\left[\mathcal{I}^2+(2n-5)\mathcal{J}\right] \\
& -\left(\frac{a}{\pi\, L}\right)^3\left[\mathcal{I}^3+(2n-7)\mathcal{I}\mathcal{J}+\left(5n^2-41n+63\right)\mathcal{K}\right] \\
& +\left(\frac{a}{\pi\, L}\right)^4\Big[\mathcal{I}^4-6\mathcal{I}^2\mathcal{J}+(4+n-n^2)\mathcal{J}^2+4(27-15n+n^2)\mathcal{I}\,\mathcal{K} \\
& +(14n^3-227n^2+919n-1043)\mathcal{L}\ \Big]\Bigg\} \\
& +\binom{n}{2}\frac{8\pi^2 a^3 r}{M\, L^6}\left[\ 1\ +\ \left(\frac{a}{\pi\, L}\right)3(n-3)\mathcal{I}\ \right] \\
& +\binom{n}{3}\frac{1}{L^6}\left[\ \eta_3(\mu)\ +\ \frac{64\pi a^4}{M}\left(3\sqrt{3}-4\pi\right)\ \log\left(\mu L\right)\ -\ \frac{96 a^4}{\pi^2 M}\ \mathcal{S}\ \right]\left[1\ -\ 6\ \left(\frac{a}{\pi L}\right)\ \mathcal{I}\ \right] \\
& +\binom{n}{3}\left[\ \frac{192\ a^5}{M\pi^3 L^7}\left(\mathcal{T}_0\ +\ \mathcal{T}_1\ n\right)\ +\ \frac{6\pi a^3}{M^3 L^7}\ (n+3)\ \mathcal{I}\ \right]\ +\ \mathcal{O}\left(L^{-8}\right) \quad .
\end{aligned}
$$

$$\mathcal{I} \ = \ -8.9136329 \qquad \mathcal{T}_0 \ = -4116.2338$$

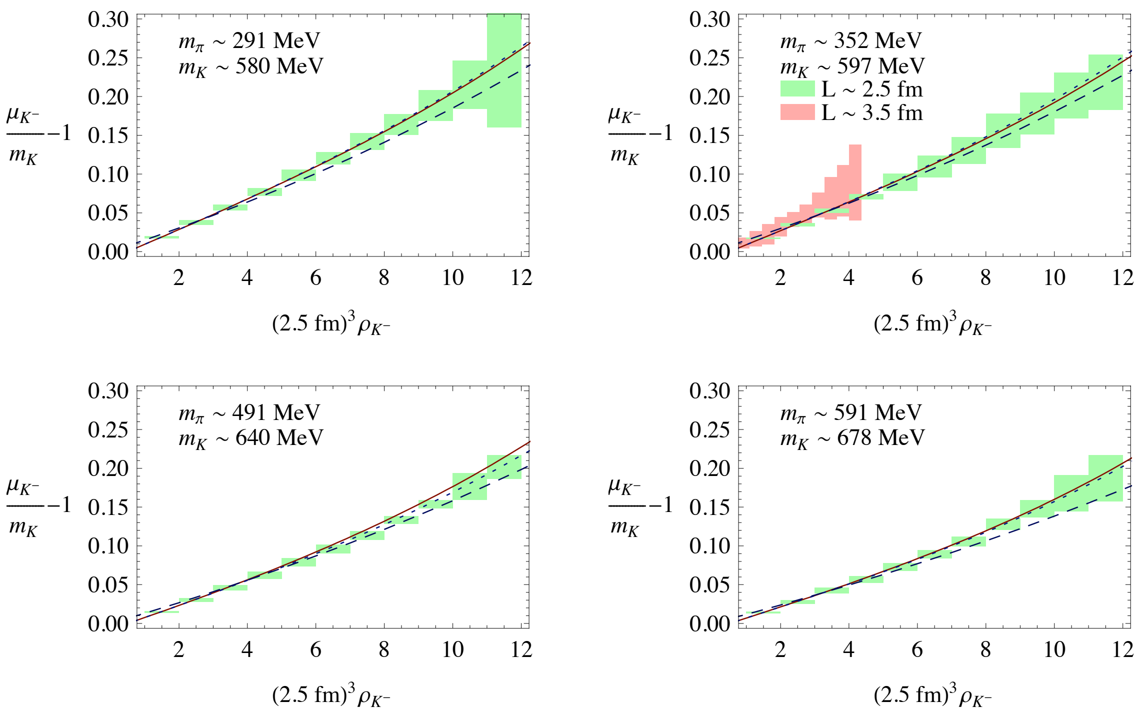
$$\mathcal{J} \ = \ 16.532316 \qquad \mathcal{T}_1 \ = \ 450.6392$$

$$\mathcal{K} \ = \ 8.4019240 \qquad \mathcal{S}_{\rm MS} \ = \ -185.12506$$

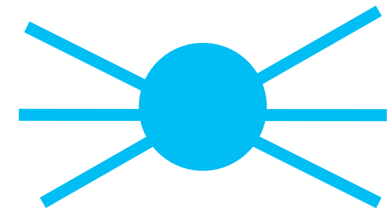
Bose-Einstein Condensates of Mesons in LQCD: Many-Body Systems

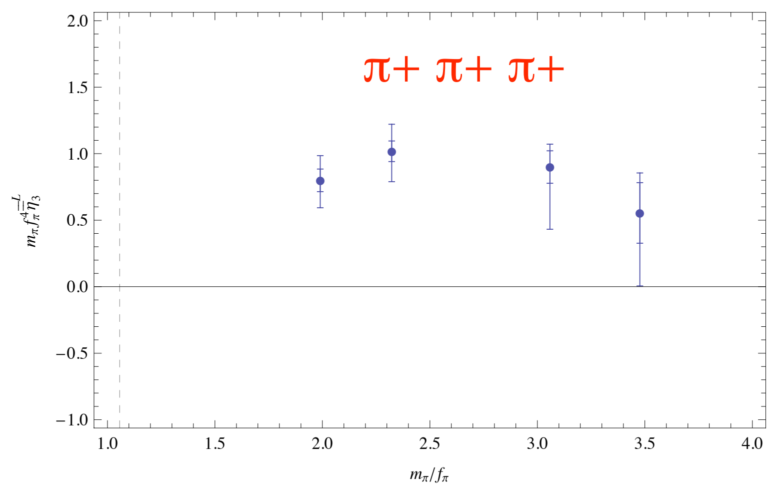


Kaon Condensates



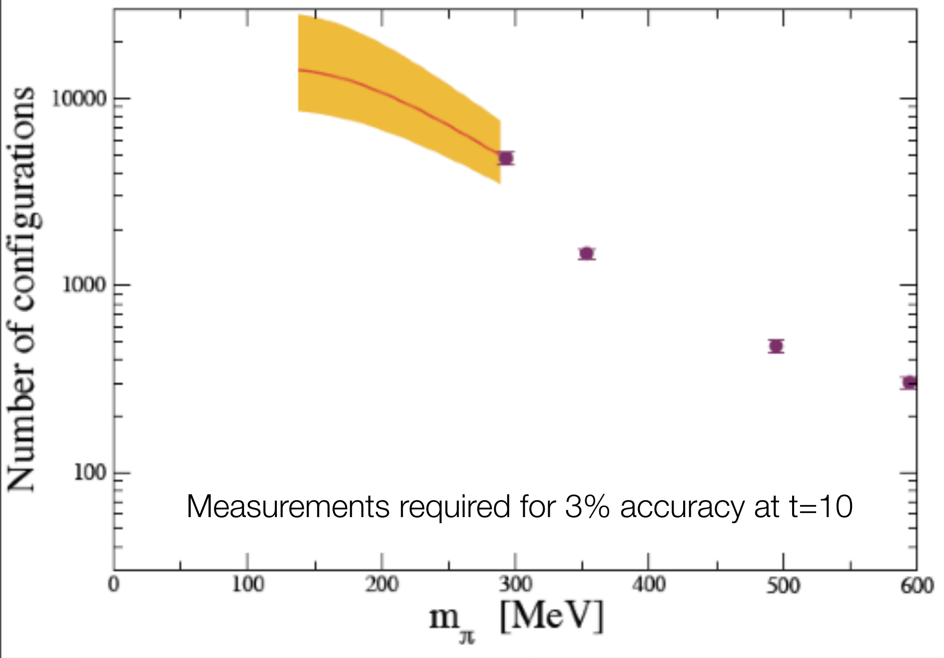
Pion 3-Body Interaction



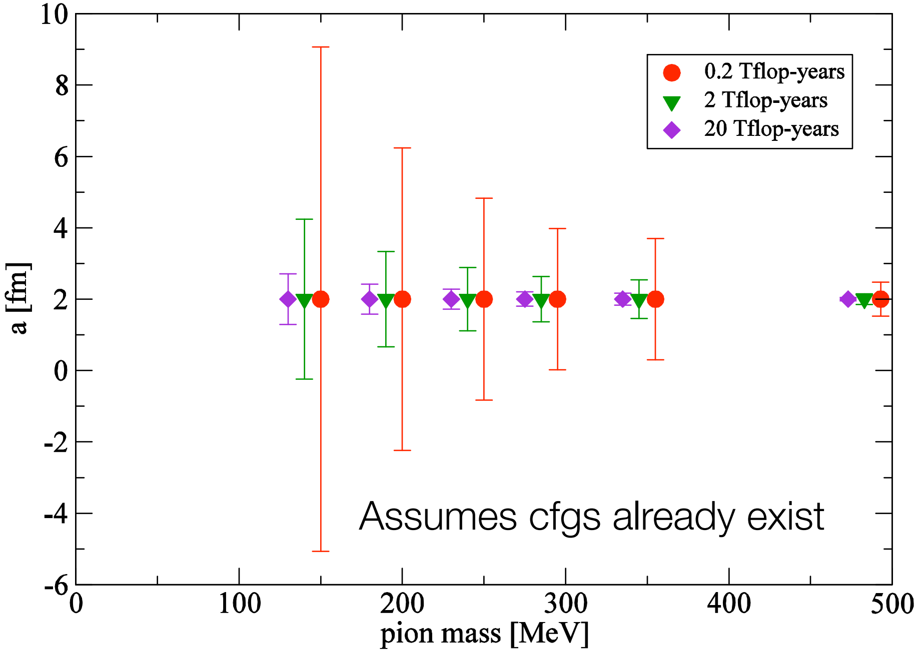


Resource Requirements with Current Technology

g_A

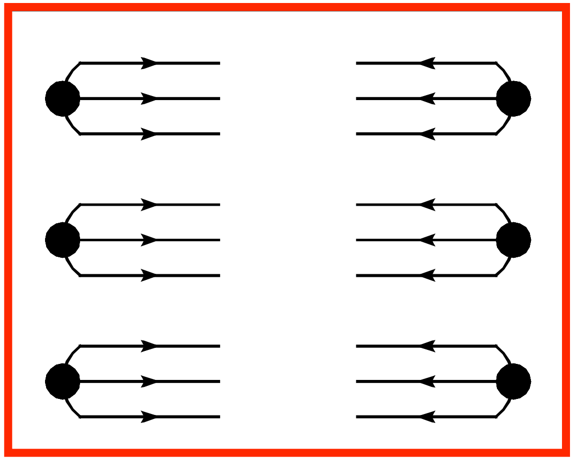


NN Scattering Length fixed at 2 fm for demonstrative purposes



Many Nucleons (Baryons)

Large number of quarks in initial and final states



Proton : N $^{\text{cont}}$ = 2

^{235}U : N $^{\text{cont}}$ = 10^{1494}

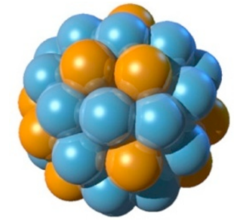
$$N_{\text{cont.}} = u!d!s! \quad (\text{Naive})$$
$$= (A+Z)!(2A-Z)!s!$$
$$\sim A^3 \ (\text{Kaplan})$$

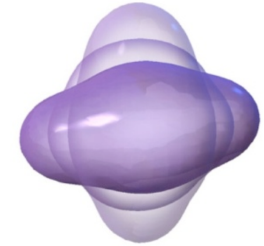
Need high statistics due to exponential growth of uncertainties

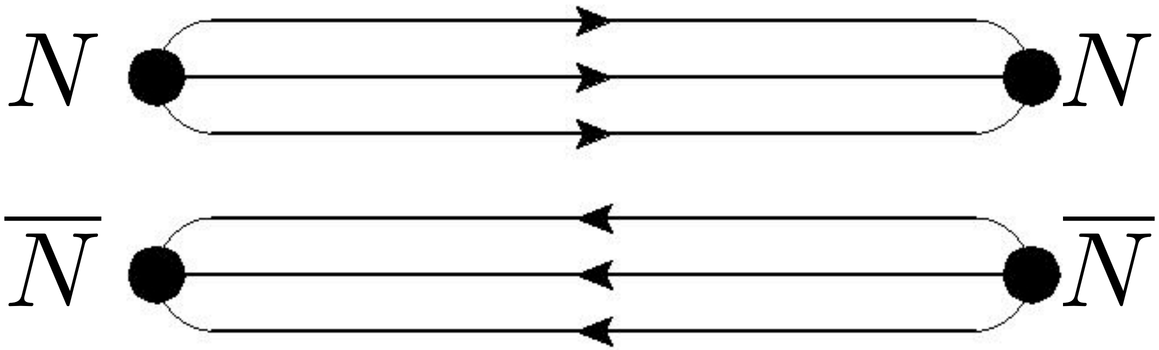
$$\frac{\sigma}{\overline{x}} = \frac{\sqrt{\langle G^2 \rangle - \langle G \rangle^2}}{\langle G \rangle} \sim e^{A(M_N - \frac{3}{2}m_\pi)t}$$

Nuclear Physics is exponentially more costly than particle physics !!

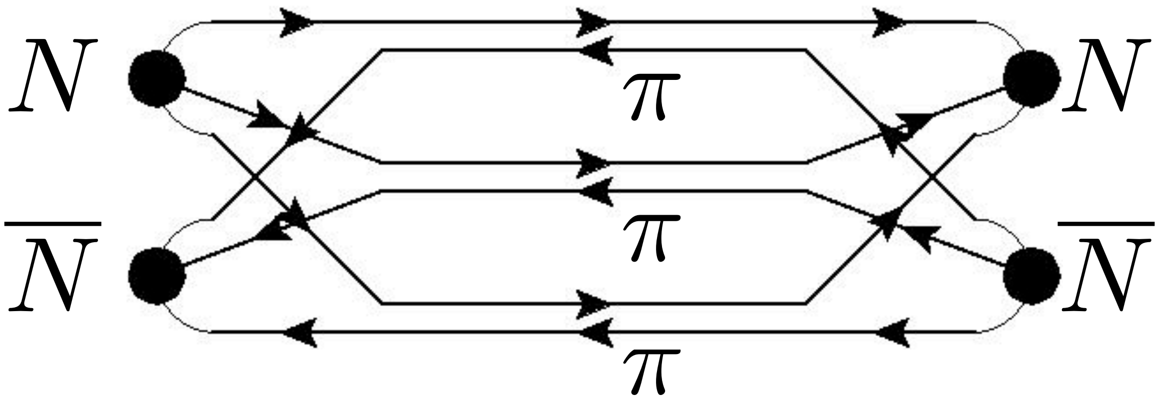
Baryon Noise Correlators





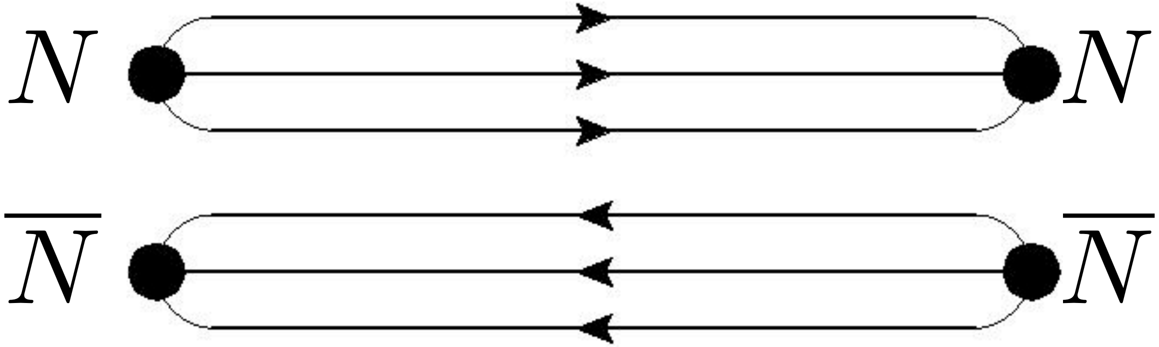


$$\sim \exp\left(-2M_N t\right)$$



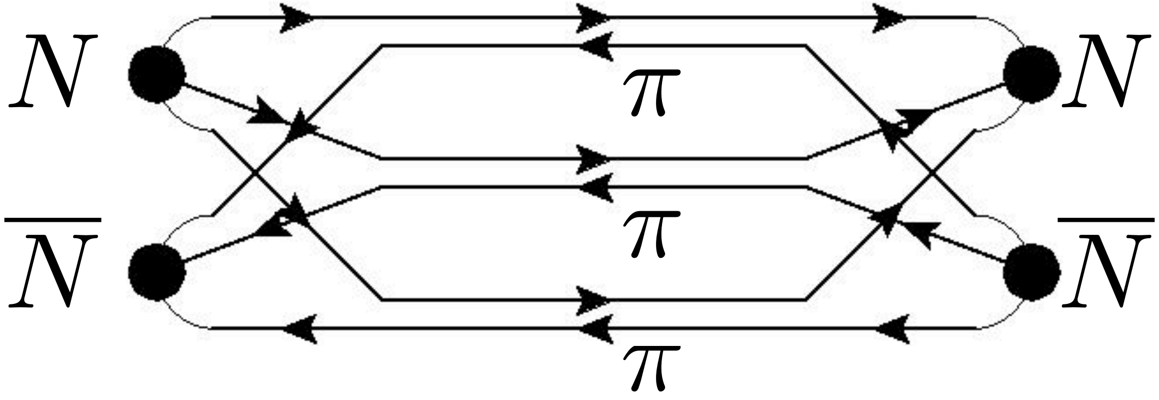
$$\sim \exp\left(-3m_\pi t\right)$$

Baryon Noise Correlators



$\sim \exp\left(-2M_N t\right)$

Maximize Amplitude

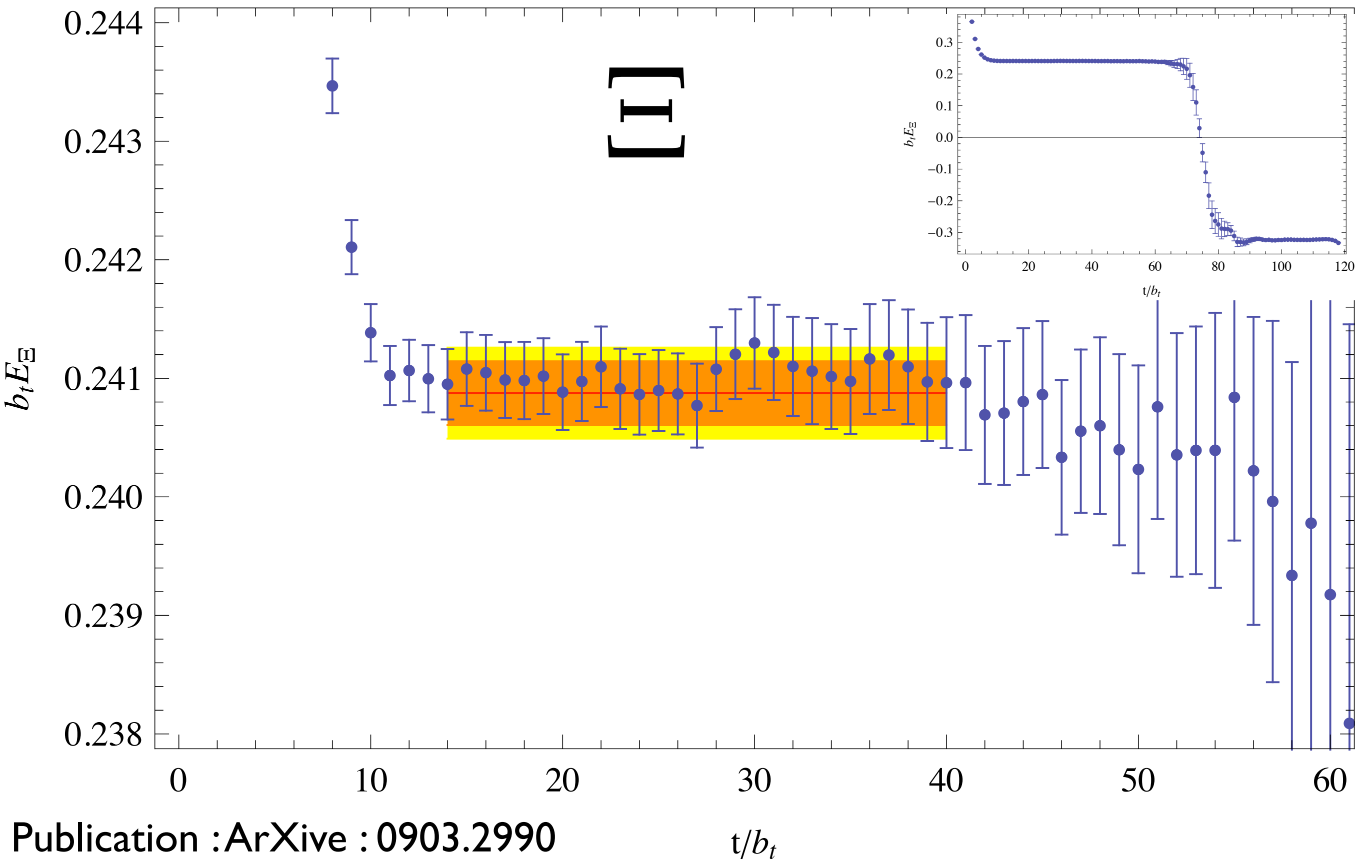


$\sim \exp\left(-3m_\pi t\right)$

Minimize Amplitude

High Statistics Calculations on the Jefferson Lab Anisotropic Clover Lattices : Single Baryon Systems

b~0.123 fm
20x20x20 x 128
pion ~ 390 MeV



~0.2% Precision !

> 350,000 measurements

Publication : ArXive : 0903.2990

High Statistics Calculations on the Jefferson Lab Anisotropic Clover Lattices : Single Baryon Systems

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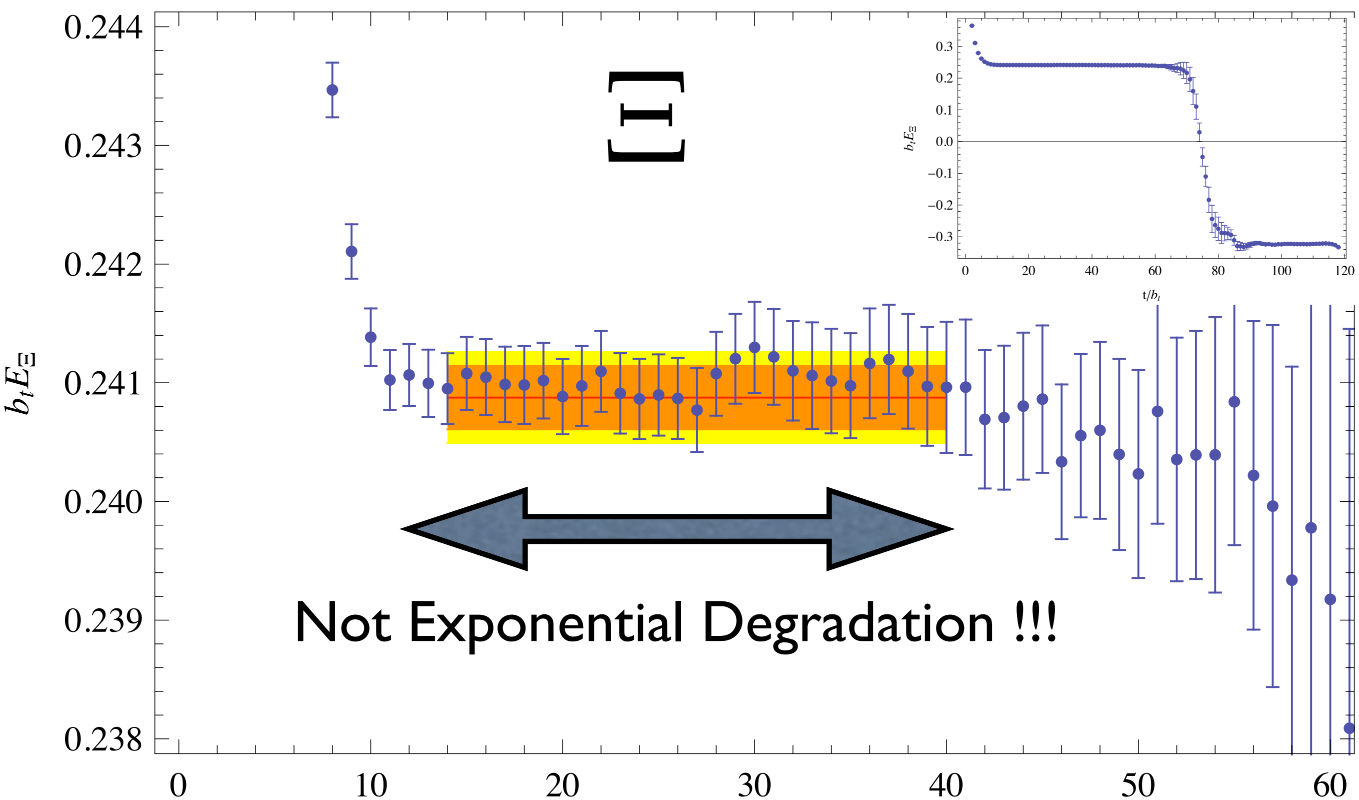

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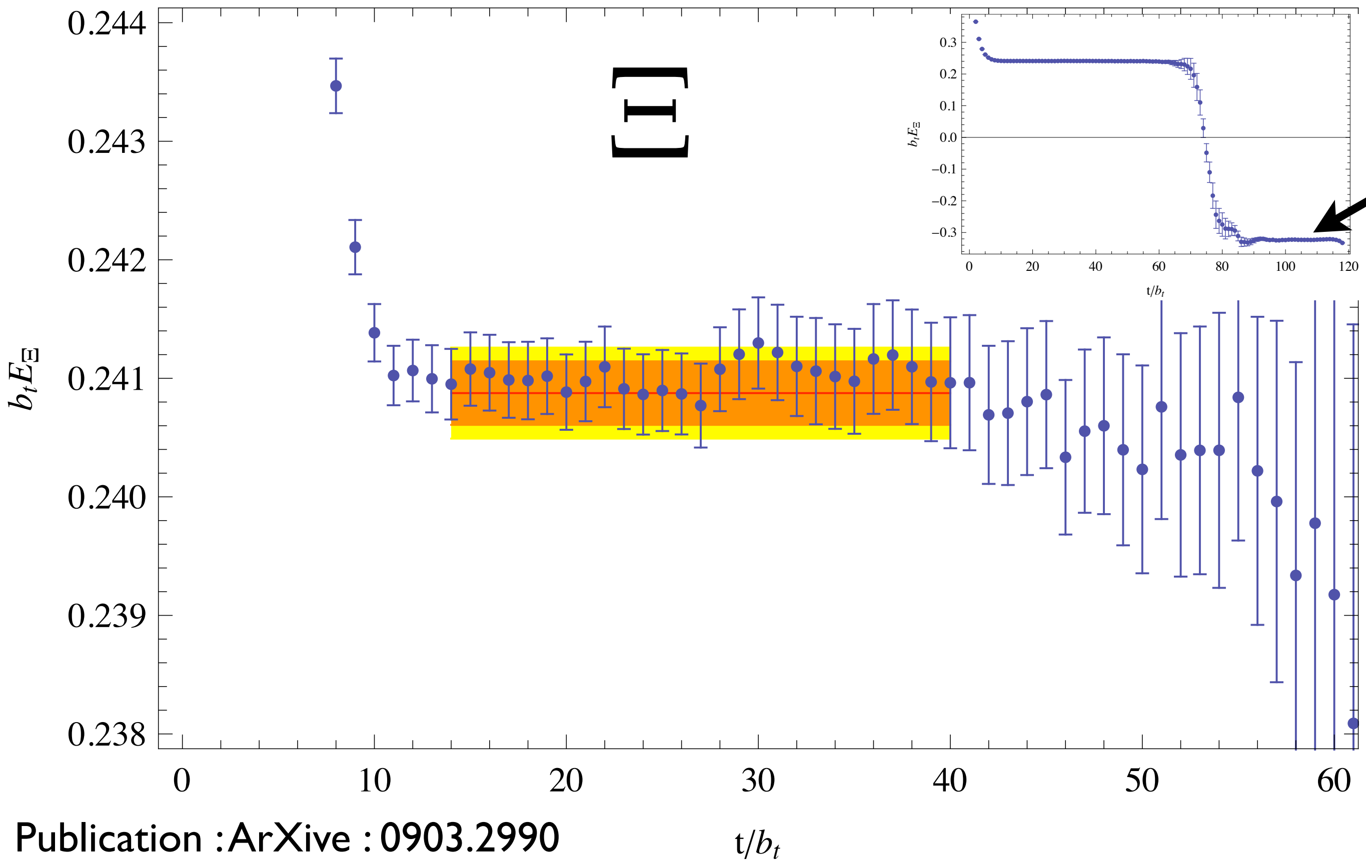
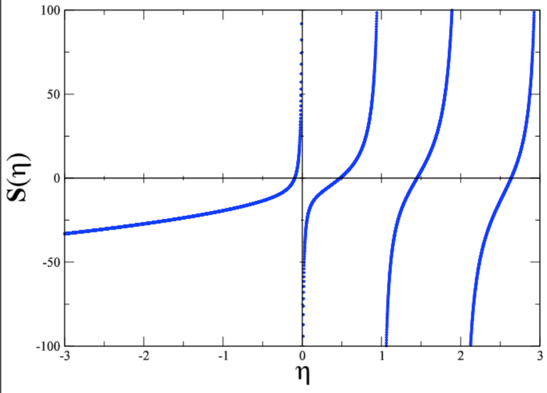


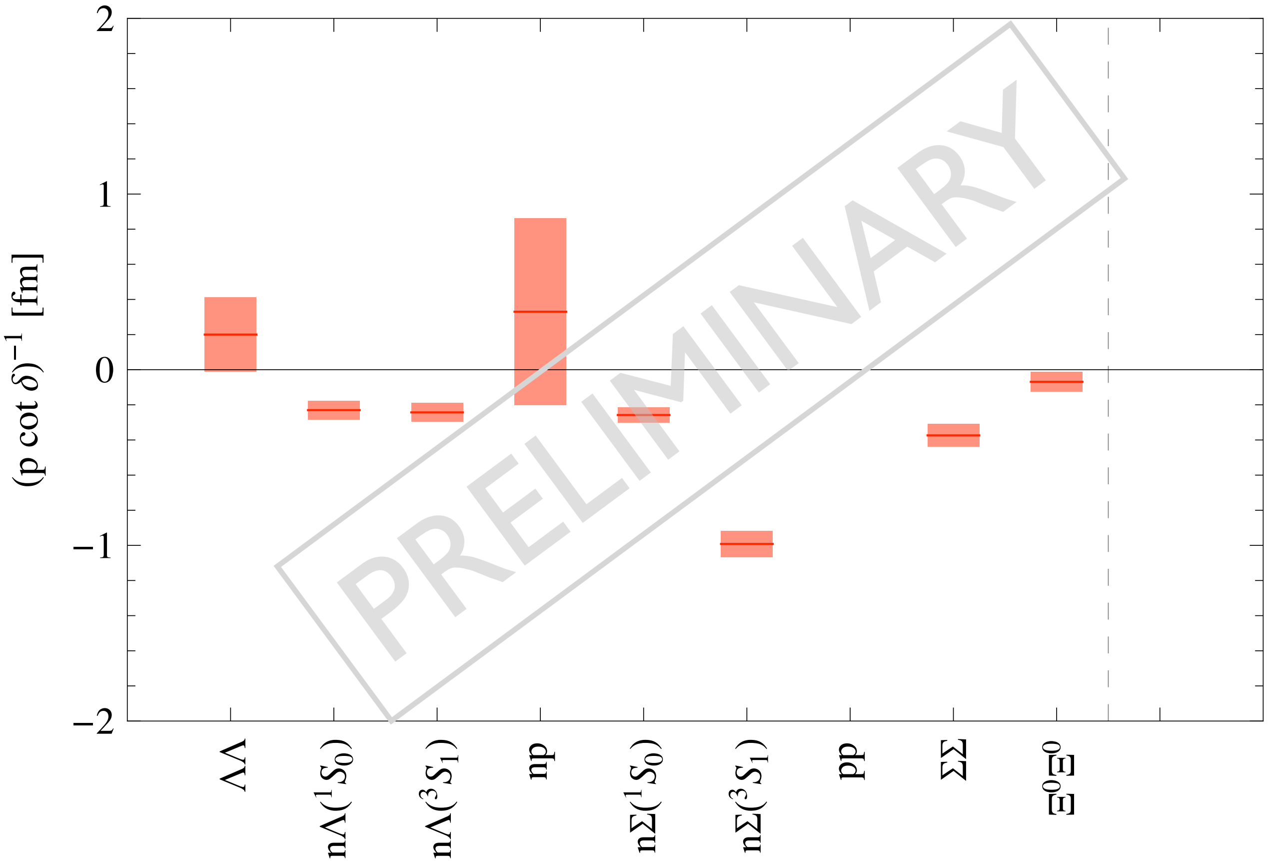
s-wave $\Xi\pi$

~0.2% Precision !

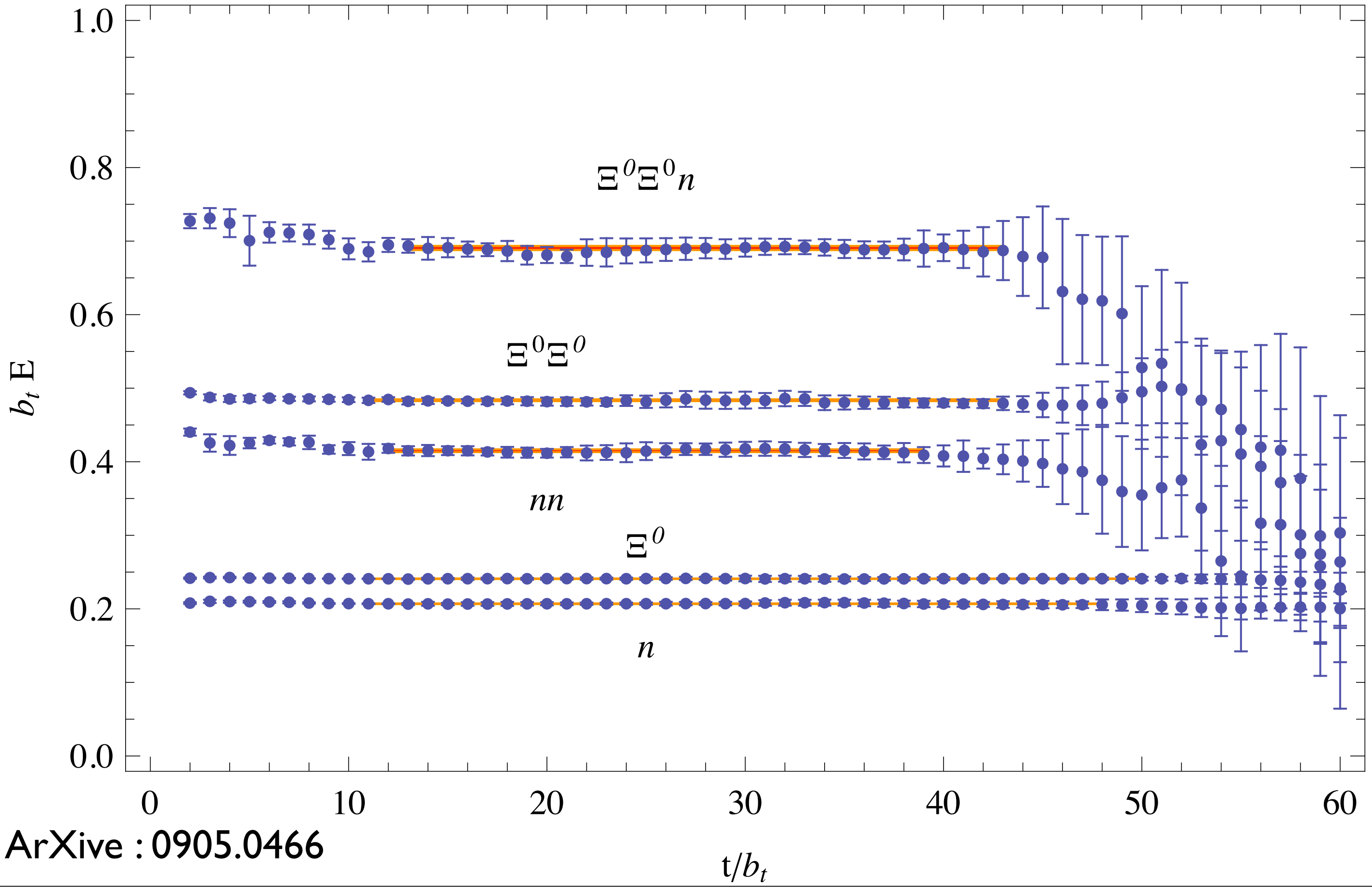
Publication : ArXive : 0903.2990

Two Baryon Systems





Three Baryon Systems

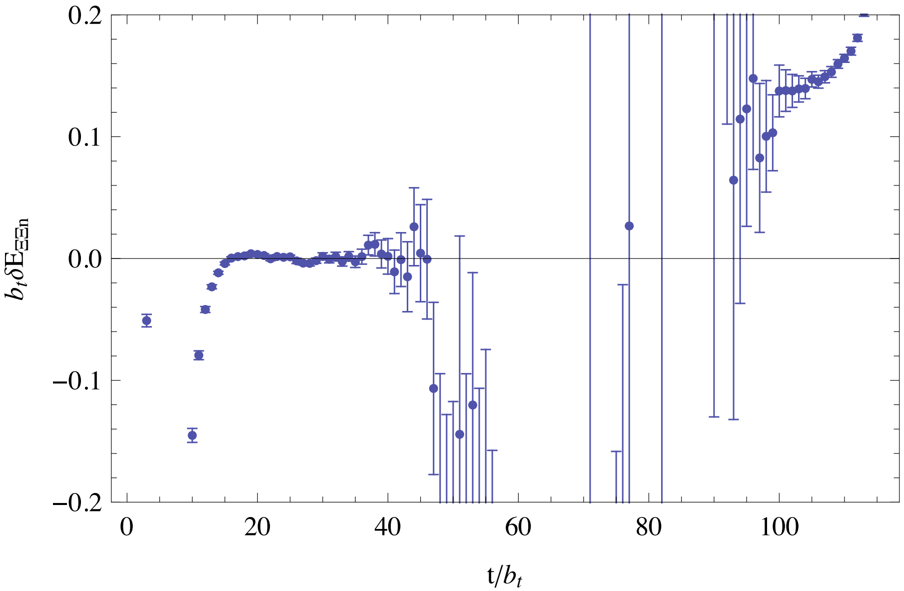


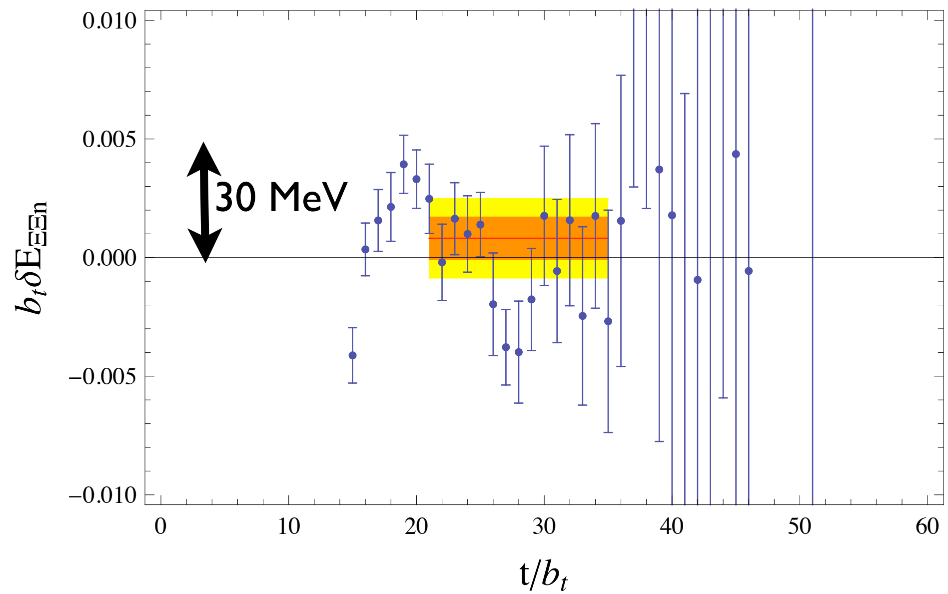
Publication : ArXive : 0905.0466

Three Baryon Systems

Energy-shift/Binding Energy

$$\Xi^0 \Xi^0 n$$



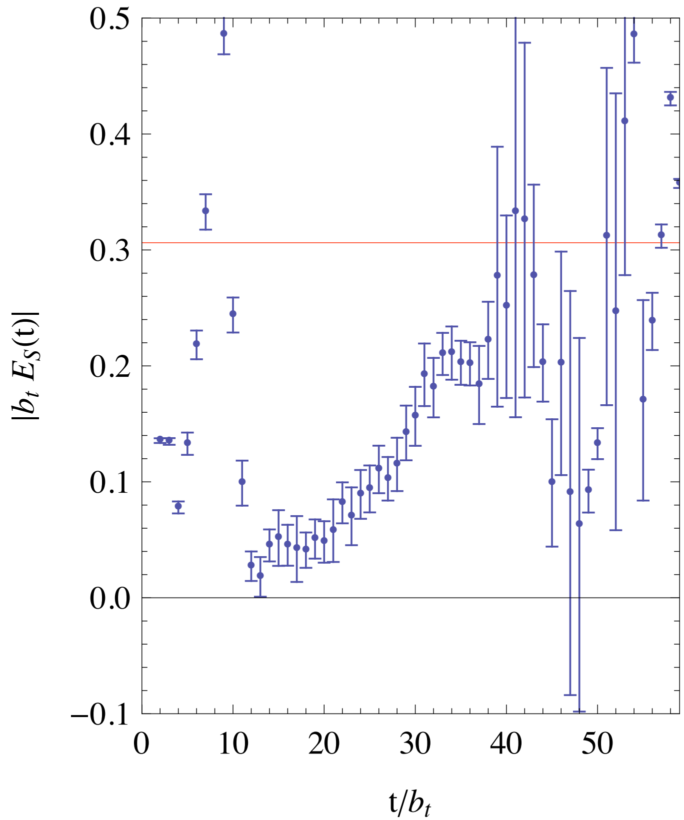


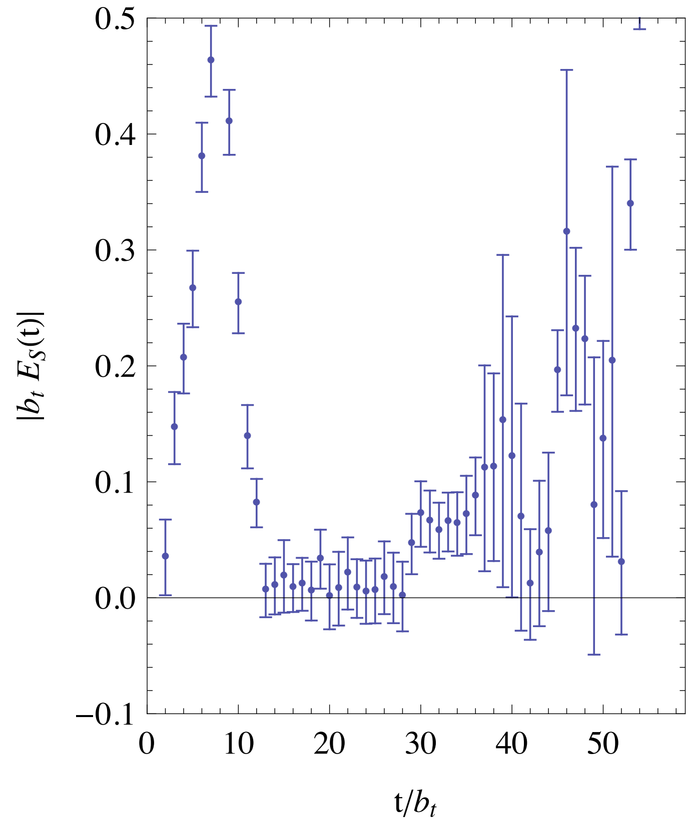
Publication : ArXive : 0905.0466

Three Baryon Systems

Signal-to-Noise Mass-Scale

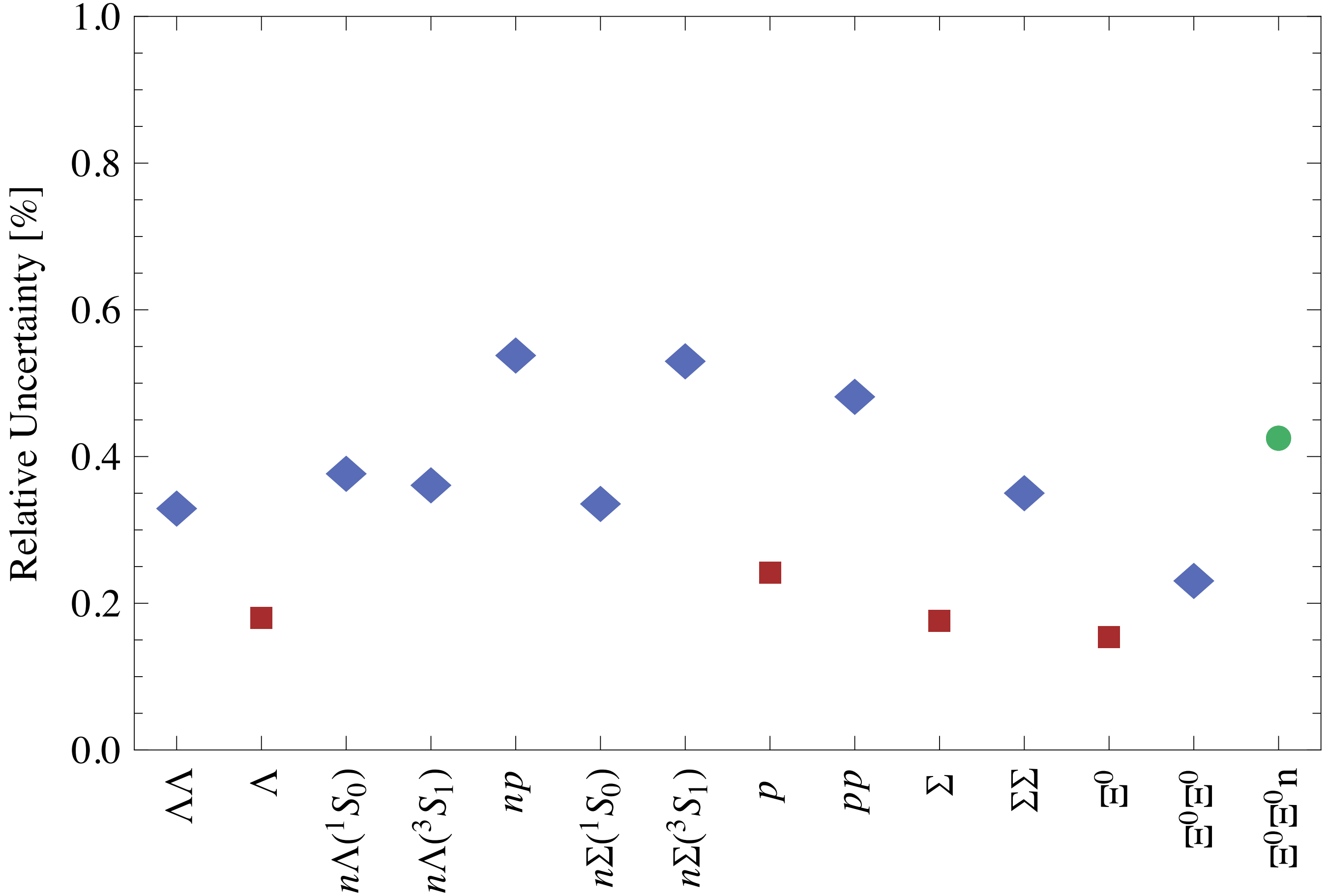
$\Xi^0\Xi^0 n$





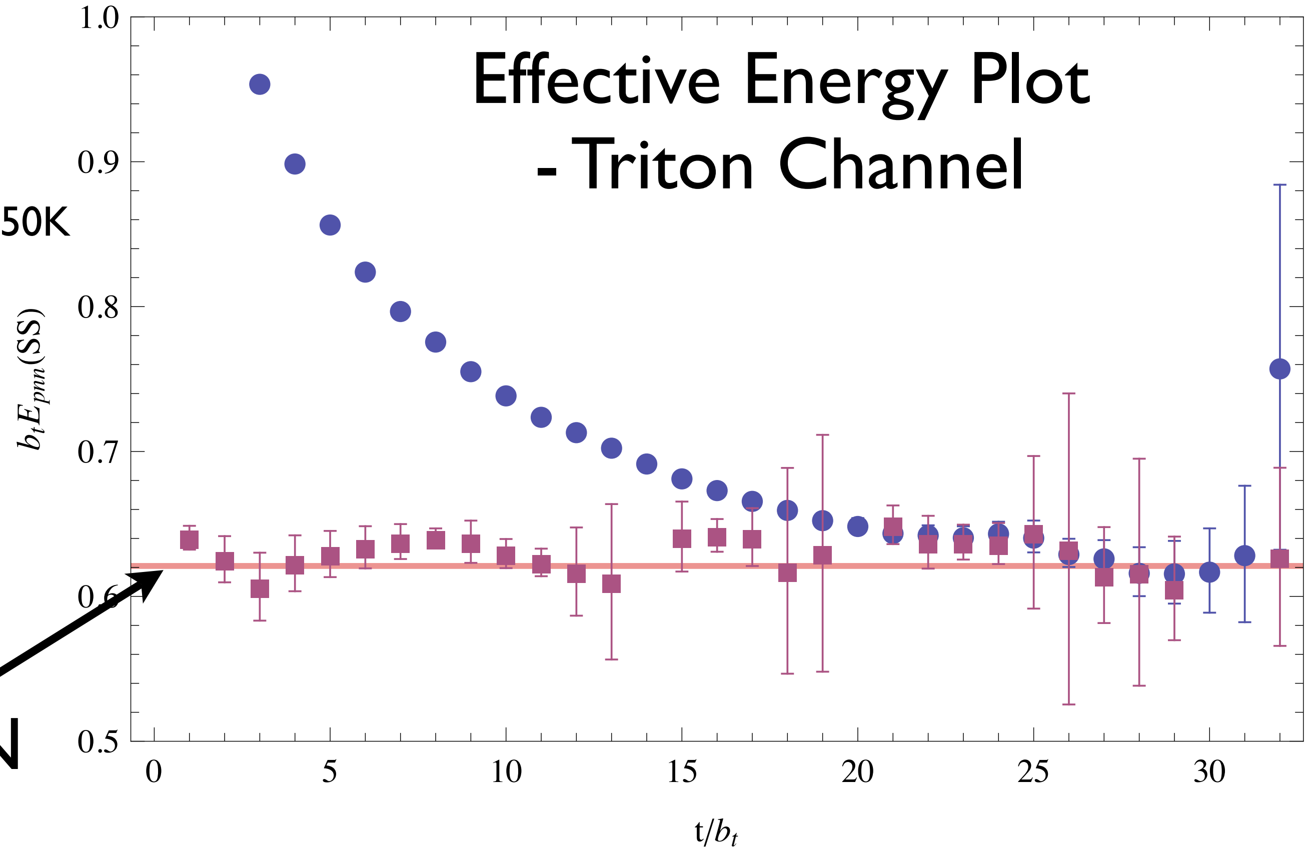
Optimize ``Golden Window''
for Nuclear Physics

Three Baryon Systems



Three Baryon Systems
nnp

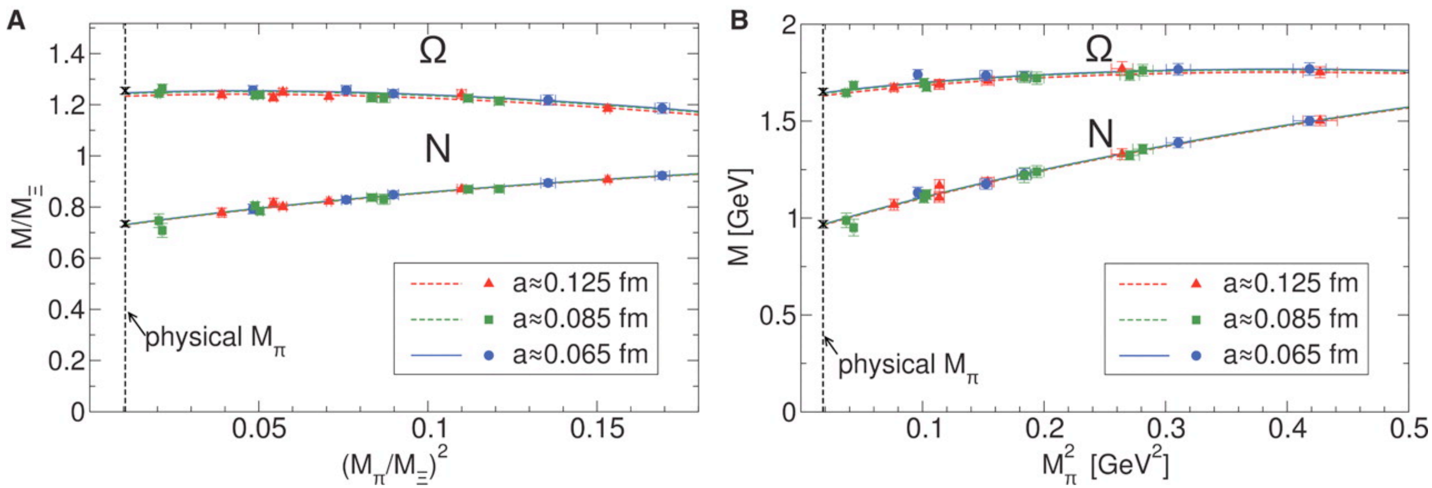
~10,000 measurements
2880 contractions
proton has 2
100M core-hrs required to do 350K

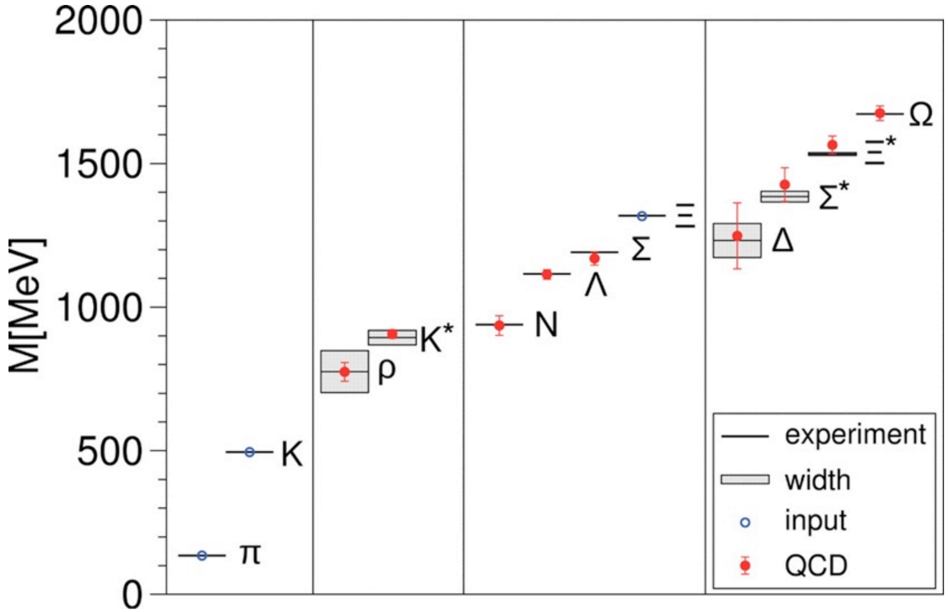


3 MN

Near Physical Quark Masses

(2008)

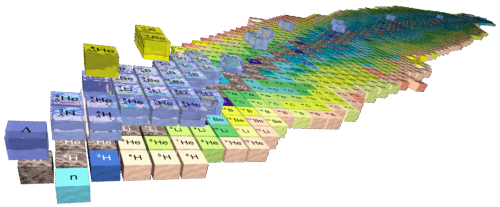




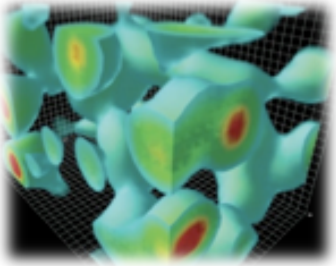
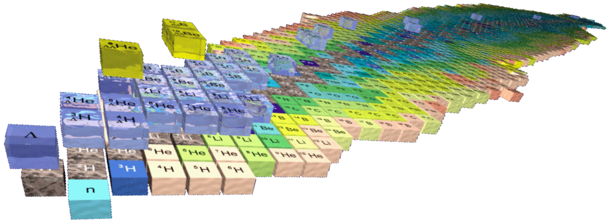
Volumes ?

Smearings ?

Nuclear Theory Wishes (physical m_π)



- What do we expect to observe in LQCD calculations in a finite volume
    - Need 3-body and 4-body spectrum (cubic symmetry)
        - With and without background electroweak fields
- How to optimally invert lattice data
    - what volumes should we calculate with ?
- Quark-mass dependence
    - Higher orders in EFT for few-body systems

It is clear ....

Lattice QCD is starting to make significant contributions to Nuclear Physics.

- The next 5 years will see remarkable things
- Lattice QCD calculations
  - at the physical pion mass,
  - in large volumes
  - at small lattice spacings

are close at hand

The END